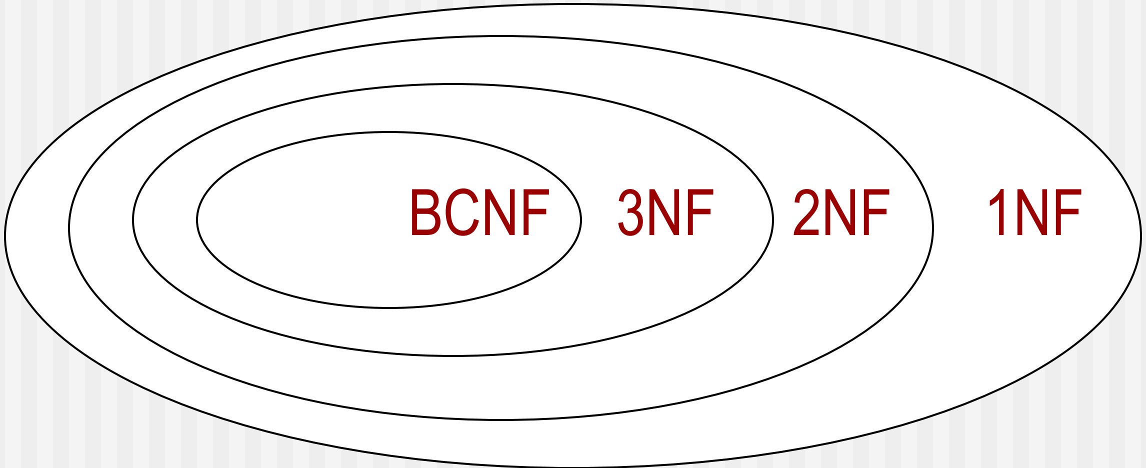
Normal Forms for Relational Data

- If a relation schema is in some normal form, it means it is in some level of quality in the sense that *certain kinds of problems and anomalies (related to redundancy) will not arise*

- Given a relation schema **R** with FDs **F**, how to determine in what normal form it is? And if it is not in a particular normal form, how to convert it into a few smaller relations all of which are in that normal form.
- To address these issues, we study definitions and algorithms for different **normal forms**.

Normal Forms

- The normal forms as defined and captured by FD's:
  - First normal form (1NF)
  - Second normal form (2NF)
  - √ Third normal form (3NF)
  - √ Boyce-Codd normal form (BCNF)
- The relationships among these normal forms:



Third Normal Form (3NF)

Let **R** be a relation schema with a set of FD's **F**.

- We say **R** w.r.t. **F** is in 3NF (**third normal form**), if for every FD **X** $\rightarrow$ **A** in **F**, at least one of the following conditions holds:
  - **X** $\rightarrow$ **A** is a trivial, i.e., **A** $\in$ **X,** or
  - **X** is a superkey, or
  - **X** is not a key but **A** is part of some key of **R**

➔ Therefore, to determine if **R** is in 3NF w.r.t. F, we need to:
  - Check if the LHS of each nontrivial FD in **F** is a superkey
  - If not, check if its RHS is part of any key of **R**

Boyce-Codd Normal Form

Given: A relation schema **R** with a set of FD's **F** on **R.**

- We say **R** w.r.t. **F** is in **Boyce-Codd normal form**, if for every FD $\mathbf{X} \rightarrow \mathbf{Y}$ in **F**, at least one of the following 2 conditions holds
  - $\mathbf{Y} \subseteq \mathbf{X}$, that is, $\mathbf{X} \rightarrow \mathbf{Y}$ is a trivial FD or
  - **X** is a superkey
- To determine if **R** is in BCNF w.r.t. **F**,

  For every FD $\mathbf{X} \rightarrow \mathbf{Y}$, check if its LHS **X** is a superkey.

  That is, compute $\mathbf{X}^+$.

  If $\mathbf{X}^+ = \mathbf{R}$, then no problem; the FD $\mathbf{X} \rightarrow \mathbf{Y}$ passes.

  If $\mathbf{X}^+ \neq \mathbf{R}$, then **R** is not in BCNF.

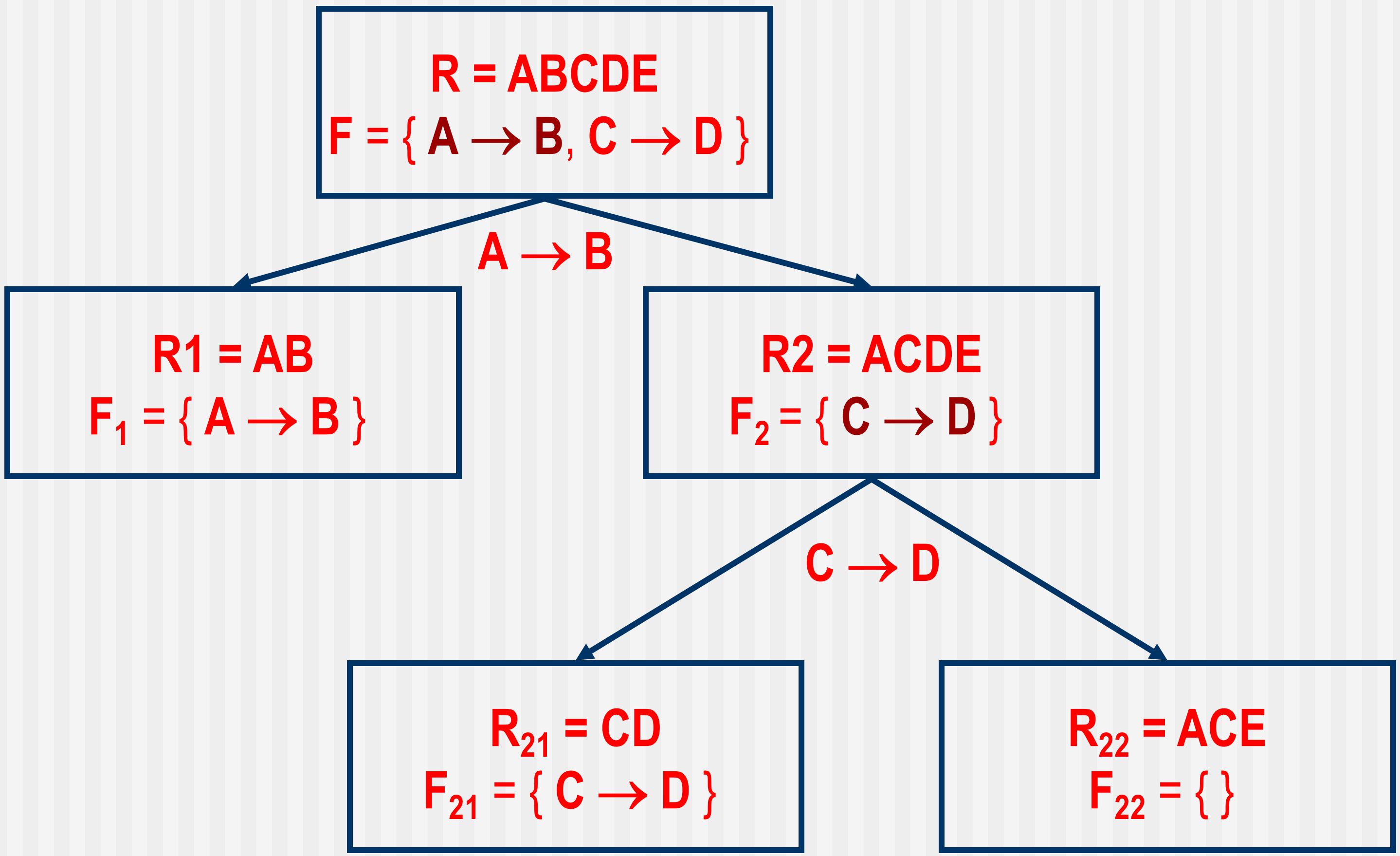
Decomposition into BCNF relations

- Suppose relation R is in 1NF. Consider <**R**, **F**>
- If **R** is not in BCNF w.r.t. **F**, we can always obtain a *lossless-join decomposition* of **R** into a collection of BCNF relations
- However, this decomposition may not always be dependency preserving.
- The basic step of a BCNF decomposition alg. (done **recursively**):

  Pick an FD $\mathbf{X} \rightarrow \mathbf{A} \in \mathbf{F}$ that violates the BCNF requirement:

  1. Decompose **R** into two relations: R1=$\mathbf{X}^+$ and R2=$(\mathbf{R} - \mathbf{X}^+) \cup \mathbf{X}$
  2. Project F onto R1 and R2. Let's call them as F1 and F2.
  3. If <R1,F1> or <R2,F2> is not in BCNF, decompose further.

Example (Decomposition into BCNF relations)



Decomposition into 3NF

- We can always obtain a lossless-join, dependency-preserving decomposition of a relation into 3NF relations. How?
- We discuss 2 solution approaches for 3NF decomposition.
- **Approach 1:** using the *binary decomposition* method.

  Let **R** = { $\mathbf{R_1}$, $\mathbf{R_2}$, . . . $\mathbf{R_n}$} be the result. Recall that this is always lossless-join, but may not preserve all the FD's ➔ need to fix this!
  - Identify the set **N** of FD's in **F** which we lost in the decomposition proc.
  - For each FD $\mathbf{X} \rightarrow \mathbf{A}$ in **N**, create a relation schema **XA** and add it to **R**
  - A refinement step to avoid creating MANY relations: if there are several FD's with the same LHS, e.g., $\mathbf{X} \rightarrow \mathbf{A_1}$, $\mathbf{X} \rightarrow \mathbf{A_2}$, . . . , $\mathbf{X} \rightarrow \mathbf{A_k}$, create **just one relation** with schema $\mathbf{XA_1 \ldots A_k}$

Example (3NF Decomposition)

$R = ABCDE$

$F = \{ BD \rightarrow E, C \rightarrow B, CE \rightarrow A \}$

$BD \rightarrow E$

$R_1 = BDE$

$F_1 = \{ BD \rightarrow E \}$

$R_2 = ABCD$

$F_2 = \{ C \rightarrow B, CD \rightarrow A \}$

$C \rightarrow B$

$R_{21} = CB$

$F_{21} = \{ C \rightarrow B \}$

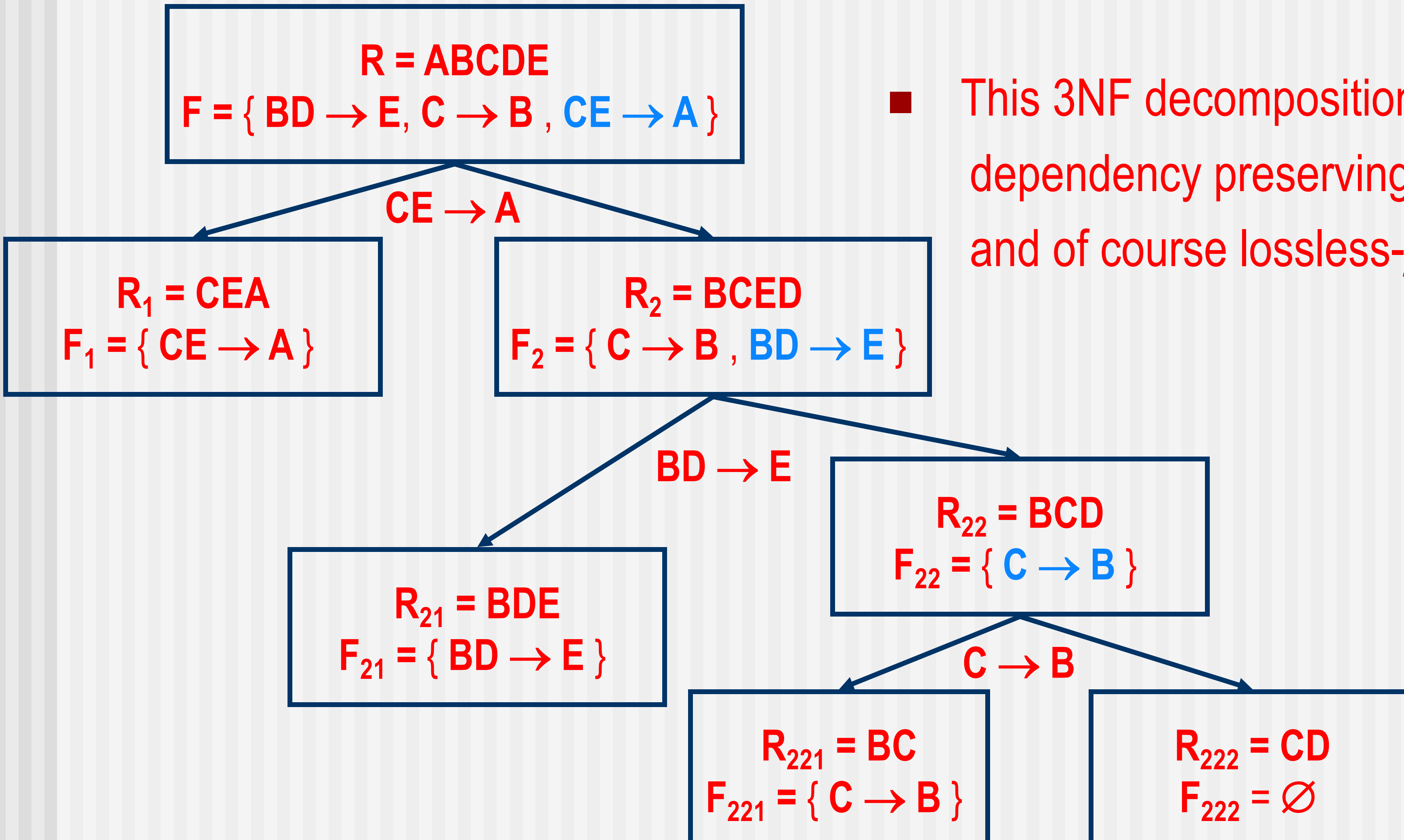
$R_{22} = ACD$

$F_{22} = \{ CD \rightarrow A \}$

- **CE → A** is not preserved, since **A** $\notin$ **{CE}**$^+$ w.r.t. $F_1 \cup F_{21} \cup F_{22}$

➔ To fix this, We add to **<u>R,</u>** a new relation R_3 = **CEA** with $F_3 = \{CE \rightarrow A\}$

Example (using a *different order*)



- This 3NF decomposition is dependency preserving, and of course lossless-join

Decomposition into 3NF

- **Method 1:** (binary decomposition):
  - Lossless-join √
  - May not be dependency preserving. If so, then add extra relations **XA**, for every FD **X** $\longrightarrow$ **A** we lost
- **Method 2:** the *synthesis* approcah
  - Dependency preservation √
  - However, may not be lossless-join. If so, we must add to **<u>R</u>**, one extra relation that includes whose attributes form a key of **R**

  What would be the FDs on this newly added relation?

Decomposition into 3NF (Using the synthesis approach)

Consider <**R, F**>

- The synthesis approach:
  - Get a minimal cover $\mathbf{F}^c$ of **F**
  - For each FD $\mathbf{X} \rightarrow \mathbf{A}$ in $\mathbf{F}^c$, add schema **XA** to $\underline{\mathbf{R}}$
  - If none of the relation schemas in $\underline{\mathbf{R}}$ from previous step is a superkey for **R**, add a relation whose schema is a key for **R**

Example

- **R** = (**A**, **B**, **C**) with **F** = { **A** $\rightarrow$ **B**, **C** $\rightarrow$ **B** } is not in 3NF, so we decompose **R**. This yields:
- **R** = {R_1, R_2}, where R_1 = (**A**, **B**) with FDs F1={A $\rightarrow$ B} and R_2 = (**B**, **C**) with F2 = { C$\rightarrow$B}
- AC is the key for **R**, and since neither AB nor BC is a *superky* for R, we add R_3 = (**A**,**C**) to **R**.
- The decomposition **R** = {R_1, R_2, R_3} is both lossless and dependency-preserving. (The FD on AC is Ø).

The Chase test to check lossless join

Suppose relation $\mathbf{R}\{A_1, \ldots, A_k\}$ is decomposed into $\mathbf{R}_1, \ldots, \mathbf{R}_n$
To determine if this decomposition is lossless, we use table $L[1 \ldots n][1 \ldots k]$, and initialize it as follows.

Initializing the table:

for each relation $\mathbf{R}_i$ **do**
  for each attribute A_j **do**
    if A_j is an attribute in $\mathbf{R}_i$
      then $L[i][j] \leftarrow a_j$
      else $L[i][j] \leftarrow b_{ij}$

Chase test (to check lossless join)

repeat

for each FD $\mathbf{X} \rightarrow \mathbf{Y}$ in **F do:**

for each pair of rows i and j in L that agree on X (that is, L [*i*] = L [*j*] for every attribute in X), modify every column t in L corresponding to attribute A_t in **Y** as follows:

if L [*i*][*t*] = a_t

then L [*j*][*t*] $\leftarrow$ a_t

else if L [*j*][*t*] = a_t

then L [*i*][*t*] $\leftarrow$ a_t

else L [*j*][*t*] $\leftarrow$ L [*i*][*t*]

until *no change*

The decomposition is lossless if, after this algorithm terminates, table L contains a row of all a's, that is, if there is a row *i* such that

L [*i*][*j*] = a_j for every column *j* corresponding to each attribute A_j in **R**

Examples

- Given $\prec$**R**,**F**$\succ$, where **R** = (**A**, **B**, **C**, **D**), and
  F = { **A** $\rightarrow$ **B**, **A** $\rightarrow$ **C**, **C** $\rightarrow$ **D** } is a set of FD's on **R**
- Is the decomposition **<u>R</u>** = {R_1, R_2} lossless, where
  R_1 = (**A**, **B**, **C**) and R_2 = (**C** , **D**)?
  - To be discussed in class
- Now consider **S** = (**A**, **B**, **C**, **D**, **E**) with the FD's:
  G = { **AB** $\rightarrow$ **CD**, **A** $\rightarrow$ **E**, **C** $\rightarrow$ **D** }
- Is decomposition of **<u>S</u>** = {S_1, S_2, S_3} lossless, where
  S_1 = (**A**, **B**, **C**), S_2 = (**B**, **C**, **D**), and S_3 = (**C**, **D**, **E**)?
  - To be discussed in class

Checking if a decomposition is Dependency-Preserving?

Inputs: Let $\prec$**R,F**$\succ$, where **F** = {$\mathbf{X_1 \rightarrow Y_1, \ldots, X_n \rightarrow Y_n}$}.

Suppose $\underline{\mathbf{R}}$ = { $\mathbf{R_1, \ldots, R_k}$ } is a decomposition of **R**

and $\mathbf{F_i}$ is the projection of **F** on schema $\mathbf{R_i}$

Method:

preserved $\leftarrow$ *TRUE*

for each FD $\mathbf{X \rightarrow Y}$ in **F** and while preserved = *TRUE*

do compute $\mathbf{X^+}$ under $\mathbf{F_1 \cup \ldots \cup F_k}$;

if $\mathbf{Y \not\subseteq X^+}$ then {*preserved* $\leftarrow$ *FALSE; exit* };

end

Example

- Consider **R** = (**A**, **B**, **C**, **D**), **F** = { **A** $\rightarrow$ **B**, **B** $\rightarrow$ **C**, **C** $\rightarrow$ **D** }
- Is the decomposition $\underline{\mathbf{R}}$ = {R_1, R_2} dependency-preserving, where

  $\mathbf{R_1}$ = (**A**, **B**), $\mathbf{F_1}$ = { **A** $\rightarrow$ **B** }, $\mathbf{R_2}$ = (**A**, **C** , **D**), AND $\mathbf{F_2}$ = { **C** $\rightarrow$ **D**, **A** $\rightarrow$ **D, A** $\rightarrow$ **C** }?

  - Check if **A** $\rightarrow$ **B** is preserved
    - Compute $\mathbf{A^+}$ under { **A** $\rightarrow$ **B** } $\cup$ { **C** $\rightarrow$ **D**, **A** $\rightarrow$ **D, A** $\rightarrow$ **C** }
      - $\mathbf{A^+}$ = { **A**, **B**, **C**, **D**}
      - Check if **B** $\in$ $\mathbf{A^+}$
      - **Yes**
    - **A** $\rightarrow$**B** is preserved
  - Check if **B** $\rightarrow$ **C** is preserved
    - Compute $\mathbf{B^+}$ under { **A** $\rightarrow$ **B** } $\cup$ { **C** $\rightarrow$ **D**, **A** $\rightarrow$ **D, A** $\rightarrow$ **C** }
      - $\mathbf{B^+}$ = { **B** }
    - Check if **C** $\in$ $\mathbf{B^+}$
      - **No**
    - **B** $\rightarrow$ **C** is not preserved

➔The decomposition is not dependency-preserving