INST0072 Logic and Knowledge Representation: Exercise Sheet 3

INST0072 Exercise Sheet 3 - Question 1

Let

$$KB = \{(\neg red \vee \neg big \vee soft \vee new), (\neg red \vee \neg wet \vee \neg new), (\neg wet \vee big), (\neg wet \vee red), (wet)\}$$

Use the Resolution Soundness and Completeness theorem (Lecture 3, Slide 12) to prove that $KB \models soft$.

[Example Answer]

INST0072 Exercise Sheet 3 - Question 2

The following two inference rules are from a calculus called *natural deduction*:

$$\frac{A,\ B}{A \wedge B}\ (\wedge\text{-introduction)} \qquad \frac{\neg\neg A}{A}\ (\neg\neg\text{-elimination)}$$

Let

$$KB = \{(red \wedge big \rightarrow (\neg soft \rightarrow new)), (red \wedge wet \rightarrow \neg new), (wet \rightarrow big), (wet \rightarrow red), (wet)\}$$

Use $\wedge$-introduction, $\neg\neg$-elimination, Modus Ponens and Modus Tollens (see Lecture 1, Slide 16) to demonstrate that $KB \vdash soft$.

[Example Answer]

INST0072 Exercise Sheet 3 - Question 3

(This question is similar to Exercise 3.1 on page 63 of Ertel.)

Using the predicates *child*/3, *female*/1, *male*/1, *father*/2, *mother*/2, *siblings*/2, *parents*/3, and reading "$child(A, B, C)$" as "B is the mother and C is the father of A", express the following statements as predicate logic sentences:

(a) A person is male if and only if they are not female.

(b) A person is the father of another person if and only if they are male and the other person is their child.

(c) A person is the mother of another person if and only if they are female and the other person is their child.

(d) Siblings have either the same father or the same mother.

(e) The parents of a child are their mother and father.

Now complete the following sentences to define the predicates *uncle*/2 and *ancestor*/2 according to their intended meanings, using the predicates from parts (a)–(e).

(f) $\forall x \forall y.[uncle(x, y) \leftrightarrow (\ldots)]$

(g) $\forall x \forall y.[ancestor(x, y) \leftrightarrow (\ldots)]$

[Example Answer]

INST0072 Exercise Sheet 3 - Question 4

(This question is similar to Exercise 3.2 on page 64 of Ertel.)

Using the predicates *child*/3, *father*/2, *mother*/2, *bird*/1, *flies*/1, *animal*/1, *eats*/2, *grain*/1, *plant*/1 and *much_smaller_than*/2 express the following statements as predicate logic sentences:

(a) Every person has a father and a mother.

(b) Some people have children.

(c) All birds fly.

(d) There is an animal that eats (some) grain-eating animals.

(e) Every animal eats either plants, or eats much smaller plant-eating animals.

[Example Answer]

INST0072 Exercise Sheet 3 - Question 5

(This question is similar to Exercise 3.3 on page 64 of Ertel.)

Express the statement of Question 1(b) – "a person is the father of another person if and only if they are male and the other person is their child" as a predicate logic sentence, but using the function *father_of*/1 instead of the predicate *father*/2.

[Example Answer]

INST0072 Exercise Sheet 3 - Question 6

Let the domain of discourse $\mathbb{D} = \{\heartsuit, \diamondsuit\}$. Write out:

(a) The set $\mathbb{D}^2$.

(b) The set $\mathbb{D}^3$.

(c) An example of a non-empty relation of arity 3 over $\mathbb{D}$.

(d) An example of a function of arity 1 over $\mathbb{D}$.

(e) An example of a function of arity 2 over $\mathbb{D}$.

[Example Answer]