EN2570 - Batch 17 - 2019 Nov
Quiz 1 - Answers

(Q1) A signal $x[n]$ is periodic if $x[n+N] = x[n]$ for all n, where $N \in \mathbb{Z}^+$. For a sinusoidal signal of the form $\cos[\omega_0 n]$,

$$N = \frac{2\pi k}{\omega_0}, \text{ where } k \in \mathbb{Z}.$$

(a) $\omega_0 = \frac{3\pi}{5}$. 1 So, $N = \frac{2\pi k}{3\pi/5} = \frac{10k}{3}$

for $k=3$, $N \in \mathbb{Z}^+$. So the signal is perodic.

The period is 10 samples. 1

(b) $x[n] = \underbrace{2\cos[\pi n/8]}_{x_1[n]} + \underbrace{\cos[3\pi n/4]}_{x_2[n]}$

for $x_1[n] \longrightarrow \omega_0 = \frac{\pi}{8}$. So, $N_1 = \frac{2\pi k}{\pi/8} = 16k$.

for $k=1$, $N_1 \in \mathbb{Z}^+$. So $x_1[n]$ is periodic with a period of 16 samples. 1

for $x_2[n] \longrightarrow \omega_0 = \frac{3\pi}{4}$. So, $N_2 = \frac{2\pi k}{3\pi/4} = \frac{8k}{3}$.

for $k=3$, $N_2 \in \mathbb{Z}^+$. So, $x_2[n]$ is periodic with a period of 8 samples. 1

Since $x_1[n]$ and $x_2[n]$ are periodic, $x[n]$ is also periodic. 2

The period is least common multiple (LCM) of N_1 and N_2.

$LCM(16, 8) = 16$. So, $N = 16$ samples. 2

Q2) $S_1\{x[n]\} = x[n] + n\,x[n-1]$

Let's consider output of the system for two signals $x_1[n]$ and $x_2[n]$ as $y_1[n]$ and $y_2[n]$. Then,

$$y_1[n] = S_1\{x_1[n]\} = x_1[n] + n\,x_1[n-1]$$
$$y_2[n] = S_1\{x_2[n]\} = x_2[n] + n\,x_2[n-1].$$

Now, let's consider the signal, $a x_1[n] + b x_2[n]$, where $a, b \in \mathbb{C}$.

$$y[n] = S_1\{a x_1[n] + b x_2[n]\} = a x_1[n] + b x_2[n] + n[a x_1[n-1] + b x_2[n-1]] \quad \text{— (A)}$$

1

$$a y_1[n] + b y_2[n] = a[x_1[n] + n x_1[n-1]] + b[x_2[n] + n x_2[n-1]]$$ 1
$$= a_1 x_1[n] + b x_2[n] + n[a x_1[n-1] + b x_2[n-1]] \quad \text{— (B)}$$

1

(A) = (B), So $a S_1\{x_1[n]\} + b S_1\{x_2[n]\} = S_1\{a x_1[n] + b x_2[n]\}$.

The system is <u>liner</u>. 1

Let the output ~~for~~ $x_1[n-k]$, where $k \in \mathbb{Z}$, be $y_k[n]$. Then,

$$y_k[n] = S_1\{x_1[n-k]\} = x_1[n-k] + n\,x[n-k-1] \quad \text{— (A)}$$ 1

Further, ~~Also~~

$$y_1[n-k] = x_1[n-k] + (n-k)\,x_1[n-k-1] \quad \text{— (B)}$$ 1

1 (A) $\neq$ (B). So, $y_1[n-k] \neq S_1\{x_1[n-k]\}$.

The system is not <u>time-invariant</u>.

1

$$S_2\{x[n]\} = \ln(x[n]) + x[n+1].$$

Let the output of the system for two signals $x_1[n]$ and $x_2[n]$ be $y_1[n]$ and $y_2[n]$. Then,

$$y_1[n] = S_2\{x_1[n]\} = \ln(x_1[n]) + x_1[n+1]$$
$$y_2[n] = S_2\{x_2[n]\} = \ln(x_2[n]) + x_2[n+1]$$

Now, let the output of the system for $a x_1[n] + b x_2[n]$, where $a, b \in \mathbb{C}$ be $y[n]$.

$$y[n] = S_2\{a x_1[n] + b x_2[n]\} = \ln(a x_1[n] + b x_2[n]) + (a x_1[n+1] + b x_2[n+1]) \quad \text{— (A)}$$

$$a y_1[n] + b y_2[n] = a[\ln(x_1[n]) + x_1[n+1]] + b[\ln(x_2[n]) + x_2[n+1]]$$
$$= a\ln(x_1[n]) + b\ln(x_2[n]) + a x_1[n+1] + b x_2[n+1] \quad \text{— (B)}$$

(A) $\neq$ (B), So, $a S_2\{x_1[n]\} + b S_2\{x_2[n]\} \neq S_2\{a x_1[n] + b x_2[n]\}$.

The system is <u>not linear.</u>

Let the output for $x_1[n-k]$, where $k \in \mathbb{Z}$, be $y_k[n]$. then,

$$y_k[n] = S_2\{x_1[n-k]\} = \ln(x_1[n-k]) + x_1[n-k+1] \quad \text{— (A)}$$

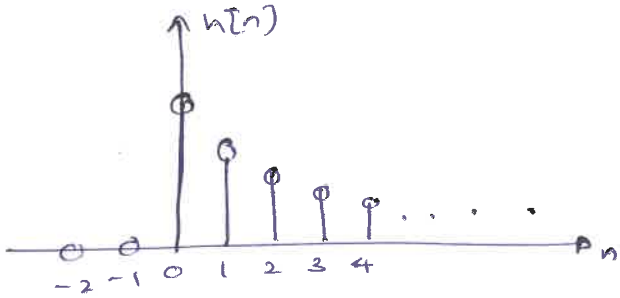
Further, $y_1[n-k] = \ln(x_1[n-k]) + x_1[n-k+1]$ — (B)

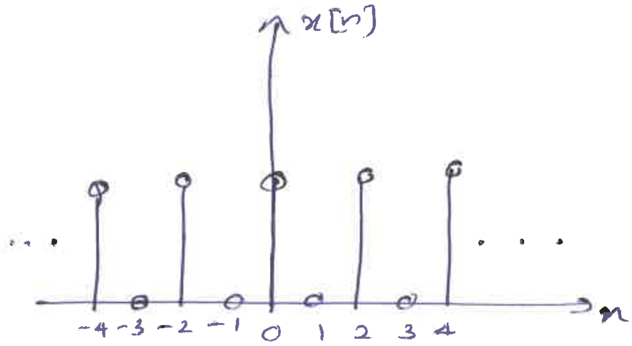
(A) $=$ (B). So, $y_1[n-k] = S_2\{x_1[n-k]\}$.

The system is <u>time-invariant.</u>

(Q3) $h[n] = a^n u[n]$, $a \in \mathbb{R}$ and $|a| < 1$.

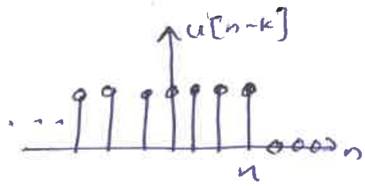
$$x[n] = \begin{cases} 1, & \text{for } n = 2m, \; m \in \mathbb{Z} \\ 0, & \text{otherwise.} \end{cases}$$





$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$ (2)

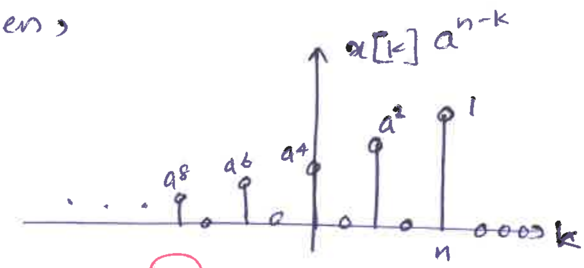
$$= \sum_{k=-\infty}^{\infty} x[k]\, a^{n-k}\, u[n-k]$$



$$= \sum_{k=-\infty}^{\infty} x[k]\, a^{n-k}$$ (2)

identification of two cases
n = even and n = odd (4)

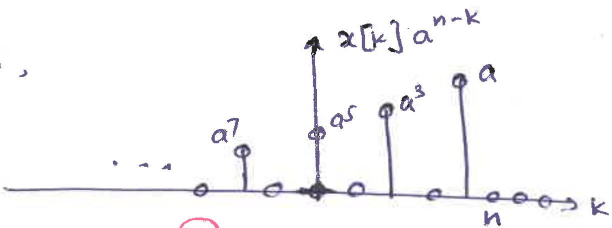
for $n = $ even,



(2)

$$y[n] = \sum_{k=0}^{\infty} a^{2k} = \frac{1}{1-a^2}$$ (2)

for $n = $ odd,



(2)

$$y[n] = \sum_{k=0}^{\infty} a^{(2k+1)} = \frac{a}{1-a^2}$$ (2)

So,

$$y[n] = \begin{cases} \dfrac{1}{1-a^2}, & n = \text{even} \\ \dfrac{a}{1-a^2}, & n = \text{odd} \end{cases}$$