UNIVERSITY OF MORATUWA
Faculty of Engineering
Department of Electronic and Telecommunication Engineering
B.Sc.Engineering
Semester 4 (2016 Batch) Examination

EN2570 DIGITAL SIGNAL PROCESSING

Time allowed: *Two (2)* hours December 2018

INSTRUCTIONS TO CANDIDATES:

- This paper contains 4 questions on 6 pages.
- Answer **all** questions.
- All questions carry **equal** marks.
- This examination accounts for **60%** of the module assessment. The total maximum mark attainable is **100**. The marks assigned for each question & sections thereof are indicated in square brackets.
- The symbols and abbreviations used in this paper have their usual meanings.
- This is a **closed-book** examination.
- Electronic/communication devices are not permitted. Only equipment allowed is a calculator approved and labeled by the Faculty of Engineering.
- Derivations are not required if they are not explicitly requested in the question.
- Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions.
- If you have any doubt as to the interpretation of the wording of a question, make your own decision, and clearly state it.

ADDITIONAL MATERIAL:

- Useful formulae and discrete-time Fourier transform theorems are given in the last page (page 6) of the paper.

Question 1.

Consider the processing of a discrete-time signal $x[n]$ with a discrete-time LTI system having the impulse response

$$h[n] = a^n u[n],$$

where $a \in \mathbb{R}$ and $u[n]$ is the discrete-time unit-step function.

(a) Determine the condition that should be satisfied by a in order to have a BIBO stable system. [4 marks]

(b) For the following parts, assume that the system is BIBO stable. The system processes an input signal $x_M[n]$ of the form

$$x_M[n] = \begin{cases} 1, & \text{for } n = rM, \text{ where } r, M \in \mathbb{Z} \\ 0, & \text{otherwise,} \end{cases}$$

and generates an output $y_M[n]$. For the case $M = 3$, compute the output $y_3[n]$ of the system using the convolution summation. [10 marks]

(c) Determine the frequency response $H(e^{j\omega})$ of the system. [4 marks]

(d) Assume that a new signal $w[n]$, defined as

$$w[n] = h[n]x_M[n],$$

is processed by the system resulting an output $y_w[n]$. Let the DTFT of $y_w[n]$ be denoted by $Y_w(e^{j\omega})$. Furthermore, the DTFT $X_M(e^{j\omega})$ of $x_M[n]$ is given by

$$X_M(e^{j\omega}) = \frac{2\pi}{M} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{M}\right),$$

where $\delta(\cdot)$ is the Dirac delta function. Using relevant theorems of the DTFT (refer to page 6), show that

$$Y_w(e^{j\omega}) = \frac{H(e^{j\omega})}{M} \sum_{k=0}^{M-1} H\left(e^{j\left[\omega - \frac{2\pi k}{M}\right]}\right),$$

if $0 \leq \omega < 2\pi$. [7 marks]

Question 2.

(a) Two continuous-time signals $x_1^c(t) = 2\cos(30\pi t)$ and $x_2^c(t) = 2\cos(70\pi t)$ are sampled with 100π rad/s in order to generate the discrete-time signals $x_1[n]$ and $x_2[n]$, respectively.

i. Find the digital angular frequencies of the discrete-time signals $x_1[n]$ and $x_2[n]$. [4 marks]

ii. Assume that the discrete-time signals $x_1[n]$ and $x_2[n]$ are separately applied to an ideal reconstruction filter of which the outputs are $\hat{x}_1(t)$ and $\hat{x}_2(t)$, respectively. Briefly explain whether $\hat{x}_1(t)$ and $\hat{x}_2(t)$ are distinguishable. [5 marks]

(b) Compute the z-transforms (with the regions of convergence) of the following signals:

i. $x[n] = a^n u[n]$, [3 marks]

ii. $x[n] = a^{|n|}$, [4 marks]

where $a \in \mathbb{R}$ and $u[n]$ is the discrete-time unit-step function.

(c) Consider a discrete-time LTI system having the transfer function

$$H(z) = \frac{1}{\left(1 + \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{5}z^{-1}\right)}, \qquad |z| > \frac{1}{3}.$$

Using the relevant properties of the z-transform and the answer obtained for **Q2(b)**, determine the output signal $y[n]$ of the system if the input signal is $x[n] = \left(\frac{1}{2}\right)^n u[n]$. [9 marks]

Question 3.

(a) A second-order discrete-time filter $H(z)$ with real-valued coefficients can be expressed as

$$H(z) = \frac{a_2 z^2 + a_1 z + a_0}{z^2 + b_1 z + b_0}.$$

Derive the conditions that should be satisfied by the coefficients of $H(z)$ using the Jury-Marden stability criterion. Assume that no common zeros and poles exist.

[5 marks]

(b) Consider the design of an FIR filter using the windowing method. The ideal frequency response $H_I(e^{j\omega})$ of a *zero-phase bandstop* filter $H(z)$ is defined as

$$H_I(e^{j\omega}) = \begin{cases} 1, & \text{for } 0 \leq |\omega| \leq \omega_c^l \text{ and } \omega_c^u \leq |\omega| \leq \pi \\ 0, & \text{for } \omega_c^l < |\omega| < \omega_c^u, \end{cases}$$

where ω_c^l and ω_c^u are the lower and upper cutoff frequencies, respectively.

i. Derive a closed-form expression for the infinite-extent ideal impulse response $h_I[n]$.

[8 marks]

ii. The cutoff frequencies of the zero-phase bandstop filter are selected as $\omega_c^l = 0.25\pi$ rad/sample and $\omega_c^u = 0.6\pi$ rad/sample. Furthermore, the order N of the filter is 6 and the Hamming window is selected as the window function $w[n]$. Compute the finite-extent impulse response $h[n]$ (or the coefficients) of the corresponding zero-phase bandstop filter. The Hamming window of length $(N+1)$ is defined as

$$w[n] = \begin{cases} 0.54 + 0.46\cos\left(\frac{2\pi n}{N}\right), & \text{for } |n| \leq \frac{N}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Provide your answer in a table having columns for $h_I[n]$, $w[n]$ and $h[n]$ for required n.

[6 marks]

iii. The impulse response $h[n]$ computed in **Q3(b)ii** is noncausal. How can a causal impulse response $h_c[n]$ be obtained from $h[n]$.

[2 marks]

iv. Show that the operation of obtaining a causal impulse response from a noncausal impulse response does not change the magnitude response of the filter.

[4 marks]

Question 4.

(a) A discrete-time IIR filter $H(z)$ can be designed using a continuous-time filter $H_c(s)$ with the bilinear transform, which is defined as

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right),$$

where T is the sampling interval. Assuming that both $H_c(s)$ and $H(z)$ are causal, show that $H(z)$ is stable if $H_c(s)$ is stable. [8 marks]

(b) A continuous-time elliptic lowpass filter $H_c(s)$ having the transfer function

$$H_c(s) = \frac{0.07(s^2 + 2.58)}{(s + 0.38)(s^2 + 0.31s + 0.51)}$$

is employed to design a discrete-time IIR lowpass filter $H(z)$ using the bilinear transform method. The passband edge Ω_p of $H_c(s)$ is 0.71 rad/s, and the sampling frequency is 10π rad/s. Furthermore, $H(z)$ is realized as a *cascade structure* of a first-order section and a second-order section.

i. Derive the transfer function $H(z)$. [8 marks]

ii. Determine the passband edge ω_p of $H(z)$. [3 marks]

iii. Draw the realization of $H(z)$ as a cascade structure. Note that the first-order and the second-order sections should be realized using the *direct form II* realizations. [6 marks]

Useful Formulae

- The sum of the first $N+1$ terms of a geometric series

$$a + ar + ar^2 + \ldots + ar^N = \frac{a\left(1 - r^{N+1}\right)}{1 - r},$$

where a and $r(\neq 1)$ are constants.

- Discrete-Time Fourier Transform (DTFT) of $x[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

- Inverse Discrete-Time Fourier Transform (IDTFT) of $X(e^{j\omega})$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

- z-Transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Discrete-Time Fourier Transform Theorems

Index	Sequence	Discrete-time Fourier transform
	$x[n]$	$X(e^{j\omega})$
	$y[n]$	$Y(e^{j\omega})$
1	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2	$x[n - n_0]$, $n_0 \in \mathbb{Z}$	$e^{-j\omega n_0}X(e^{j\omega})$
3	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
4	$x[-n]$	$X(e^{-j\omega})$
5	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
6	$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})\,d\theta$

- End of Question Paper -