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UNIVERSITY OF MORATUWA

Faculty of Engineering

Department of Electronic and Telecommunication Engineering

B.Sc.Engineering

Semester 5 (11 Batch) Examination

EN3322 DIGITAL SIGNAL PROCESSING

October 2014

Time allowed: *Two* hours

ADDITIONAL MATERIAL:

- No additional material is provided.

INSTRUCTIONS TO CANDIDATES:

- This paper contains 4 questions on 6 pages.
- Answer ALL questions.
- This examination accounts for 50% of the module assessment. The total maximum mark attainable is 100. The marks assigned for each question & sections there of are indicated in square brackets.
- Relevant formulas are given in the last page.
- Pay attention to the marks indicated for each part of the question.
- This is a closed book examination.
- Derivations are not required if they are not explicitly requested for in the question.
- Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions.
- If you have any doubt as to the interpretation of the wording of a question, make your own decision, and clearly state it.
- Answers must be given in point form. Marks will be deducted for unnecessarily descriptive answers.
- All questions carry equal marks.

Continued ...

Question 1.

(a) Describe the significance of Finite Precision Arithmetic in Digital Signal Processing. Illustrate your answer giving appropriate examples. [5]

(b) Derive an equation showing the relationship between number of bits required to represent a decimal fraction and explain its importance in practical implementation of filters. [4]

(c) Consider the second order transfer function given below: [6]

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

You have been requested to investigate the effects of coefficient quantization (Finite Word Length Effects) when implementing the above transfer function. Outline the steps of your investigation that shows the effect of quantized value on poles and zeroes. Explain each step.

(d) In addition to the coefficient quantization referred to above in Q1(c), briefly describe the other sources of errors that affect accuracy of digital filter implementation. [4]

(e) You have been asked to implement a digital filter in hardware. Briefly describe the factors to consider when choosing a suitable hardware platform for this purpose. [6]

Question 2.

(a) Critically analyze the need for different filter realization techniques. [4]

(b) Consider the following partial fraction representation of a 2nd order transfer function.

$$H(z) = -1.39 + \frac{2.1}{1 + 0.4z^{-1}} + \frac{-0.21}{1 + 0.9z^{-1}}$$

i. Draw the parallel realization of this transfer function. [3]

ii. Draw the cascade realization of the transfer function. [5]

iii. Comment on the Finite Word Length effects of the above two realizations. If you are to choose between the two, which realization wold you choose if the requirement is to minimize the effects of finite precision. [4]

(c) i. Obtain the state space representation of the transfer function in Q2(b). [5]

ii. Comment on the use of state space representation. [4]

Question 3.

(a) Consider the following transfer function of an analog normalized low pass prototype Filter. (1)

$$H(s) = \frac{1}{1+s}$$

You are required to design a digital Bandpass filter using the above prototype filter considering the following factors. The design technique should

- ensure stable analog filter always results in a stable digital filter.
- use the minimum allowed sampling frequency.

i. What do you understand by the term "Normalized Low Pass Prototype Filter" [3]

ii. Describe the basis of your design technique specifically highlighting how it satisfies the factors given above. [8]

iii. Graphically illustrate the basis for converting the low pass filter to a Bandpass Filter. The relevant mapping is s is replaced with $\frac{s^2+w_0{}^2}{W_s}$ where $w_0{}^2 = w_{p1}w_{p2}$ and $W_s = w_{p2} - w_{p1}$. Symbols have their usual meaning. [4]

(b) When a continuous time system needs to be mapped to discrete time system, one needs to find an appropriate relationship between s and z domain. Use of First forward difference leads to the following relationship.

$$s = \frac{z-1}{T}$$

T is the sampling interval.

i. Show how the above relationship is derived from the first forward difference. [4]

ii. Analyze the suitability of this mapping for IIR filter design. Illustrate by showing the mapping from s plane to z plane. [6]

Question 4.

Consider an LTI system with the impulse response

$$h[n] = \delta[n] - \frac{1}{4}\delta[n-2].$$

Assume $N = 4$. Notations carry their usual meaning (unless otherwise specified).

(a) Suppose that the following input sequence is applied to the system with certain modifications.

$$x[n] = \sin\left(\frac{\pi n}{2}\right), \quad n = 0, 1, 2, 3.$$

The system response is given by $y[n] = h[n] \otimes x[n]$, where $\otimes$ denotes the N-point circular convolution.

i. Find the N-point DFT of $x[n]$. [3]

ii. Consider the periodic sequence given by [3]

$$\tilde{x}[n] = x[(n \text{ modulo } N)].$$

Determine the DFS coefficients $\tilde{X}[k]$ of $\tilde{x}[n]$ and sketch the magnitude of $\tilde{X}[k]$ for $-8 \leq k \leq 8$.

iii. Calculate $y[n]$ by performing the circular convolution directly. [3]

iv. Determine the z-transform $H(z)$ and hence, the N-point DFT $H[k]$ of the impulse response $h[n]$. [3]

v. Calculate $y[n]$ by performing the inverse DFT of $Y[k]$. [4]

(b) It is required to remove the distortion introduced by the system in order to successfully recover $x[n]$ from $y[n]$. Suppose for this purpose, $y[n]$ is sent through an inverse filter having a transfer function $H_i(z) = 1/H(z)$. It can be shown that the impulse response of the inverse filter is given by $h_i[n] = \sum_{m=0}^{\infty}(1/2)^n\delta[n-2m]$.

i. In an attempt to implement the inverse filter $h_i[n]$, an FIR filter of length N, whose N-point DFT is given by [3]

$$G[k] = \frac{1}{H[k]} \quad k = 0, 1, 2, 3,$$

is used. Find the impulse response $g[n]$ of the FIR filter.

ii. Is this FIR filter $G[k]$ capable of removing distortion at all frequencies $0 \leq \omega \leq 2\pi$? Justify your answer. [2]

iii. Find the (resultant) total impulse response $h_T[n] = h[n] * g[n]$ and thus, comment on how well the FIR filter $G[k]$ implements the inverse filter $H_i(z)$. [4]

Formulas

- Discrete-Time Fourier Transform (DTFT) of $x[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

- z-Transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- N-point DFT of $x[n]$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{N}},\ 0 \leq k \leq N-1$$

- N-point Inverse DFT (IDFT) of $X[k]$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi kn}{N}},\ 0 \leq n \leq N-1.$$

End of Paper.