

DC Bike Sharing Analysis

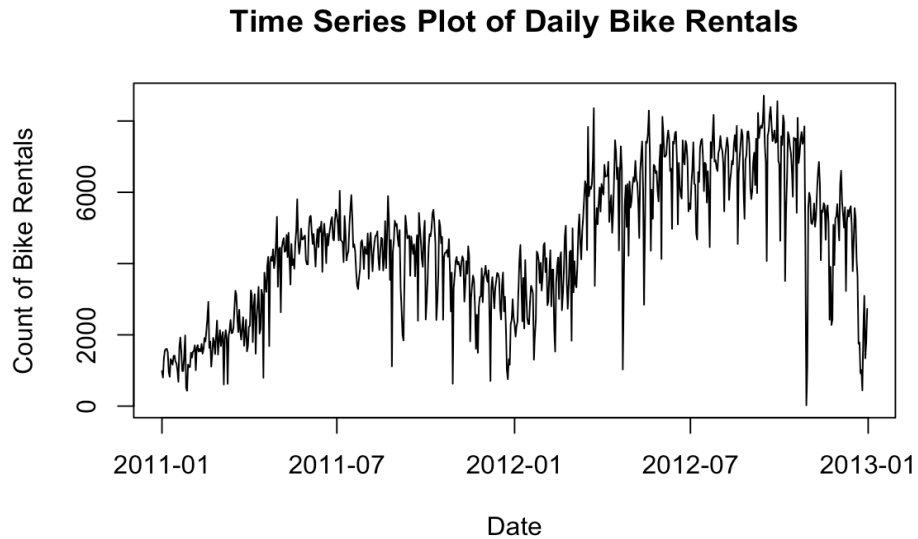
**Chia Yen Chen, Ibrahim Mohamed, Jiyoung Lee, Shalini Sah, Yu Jiyun Chang
(Group 1)**

1. Introduction and Overview

The DC bike-sharing dataset spans from January 1st, 2011 to December 31st, 2012, and involves 9 variables. These include counts of bike share rentals, date day, month, holiday, weekday, and weather features. Our project focuses on the counts of bike share rentals as the dependent variable. In Section 2 (univariate time series model), we will consider the month variable and sine and cosine pairs as the independent variables, while in Section 3 (time series regression model), we will use weather features as the independent variables. Additionally, our holdout sample will consist of 106 observations, which is 15% of the total dataset.

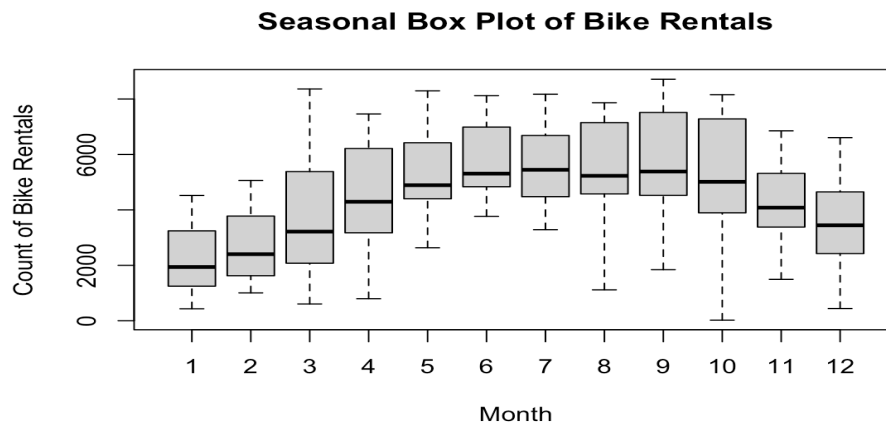
We started examining **Exhibit 1**, a Time Series Plot of Daily Bike Rentals, we observed some fluctuations in bike rental counts over time. These fluctuations suggest the presence of both trend and seasonality in the data. The plot does not follow a flat line, which implies the series is non-stationary. Furthermore, during certain periods, particularly around the middle of the year, there's a visible spike in rentals, likely an indication of seasonal trends such as better weather and increased outdoor activities.

Exhibit 1: *Time Series Plot of Daily Bike Rentals*



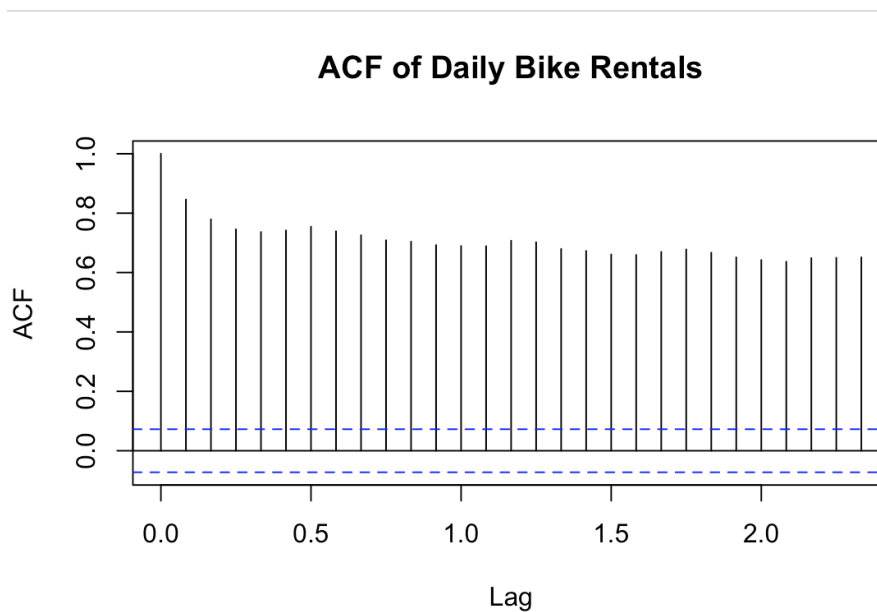
Moving to **Exhibit 2**, the Seasonal Box Plot of Bike Rentals, we can see the changeability of bike rentals throughout different months. The box plots show the typical values and range of rentals, with each box showing the middle 50% of the data and the line inside marking the middle value. From this visual, it's clear that rental patterns are not constant across the year. The higher median and larger range during warmer months suggest a surge in rental activity, possibly due to favorable weather conditions that encourage biking. Conversely, the lower median during the colder months indicates a decrease, likely due to the less conducive weather for outdoor activities.

Exhibit 2: *Seasonal Box Plot of Bike Rentals*



Lastly **Exhibit 3**, the Autocorrelation Function (ACF) of Daily Bike Rentals, reveals the correlation between the counts of bike rentals over a series of time lags. The plot shows a gradually declining pattern, indicating a strong positive correlation that decreases as the lags increase. This suggests that past values have an influence on future values in the series. The slow decay of the autocorrelation reconfirms that the series is nonstationary.

Exhibit 3: *Autocorrelation Function (ACF) of Daily Bike Rentals*



Now that we've looked at these patterns, we're ready to dive into Section 2 and 3 for more in depth analysis of the Daily Bike Rentals.

2. Univariate Time-series models.

2.1 Deterministic Time Series Models (Seasonal Dummies and Trend, Cyclical Trend)

Seasonal Dummies with Trends

```
Call:
lm(formula = n_CNT ~ time + as.factor(n_MONTH))

Residuals:
    Min       1Q   Median       3Q      Max
-6433.9  -441.9   135.6   609.2  3451.6

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    903.6547   130.7060   6.914 1.11e-11 ***
time             6.4115     0.2127  30.141 < 2e-16 ***
as.factor(n_MONTH)2    267.6610   178.8674   1.496  0.1350
as.factor(n_MONTH)3   1134.4347   175.3922   6.468 1.93e-10 ***
as.factor(n_MONTH)4   1731.5257   177.4226   9.759 < 2e-16 ***
as.factor(n_MONTH)5   2400.8490   176.8029  13.579 < 2e-16 ***
as.factor(n_MONTH)6   2627.8905   179.2875  14.657 < 2e-16 ***
as.factor(n_MONTH)7   2223.6503   179.1448  12.413 < 2e-16 ***
as.factor(n_MONTH)8   2125.6356   180.6807  11.765 < 2e-16 ***
as.factor(n_MONTH)9   2032.1820   183.8048  11.056 < 2e-16 ***
as.factor(n_MONTH)10  1269.3402   184.3557   6.885 1.34e-11 ***
as.factor(n_MONTH)11   424.2023   209.1335   2.028  0.0429 *
as.factor(n_MONTH)12  -330.8109   216.6608  -1.527  0.1273
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 974 on 662 degrees of freedom
Multiple R-squared:  0.7577,    Adjusted R-squared:  0.7533
F-statistic: 172.5 on 12 and 662 DF,  p-value: < 2.2e-16
```

Exhibit 4: Seasonal Dummies Model

As seen in **Exhibit 4**, the p-value associated with time is less than 0.05, meaning that there exists a trend term in this model, and the p-values associated with each month are also less than 0.05, indicating that there is a seasonality in the model. Also, the multiple R-squared

is 0.7577, which is satisfied. We will further investigate this in the next section by calculating its MAPE and correlatio

Cyclical Trend:

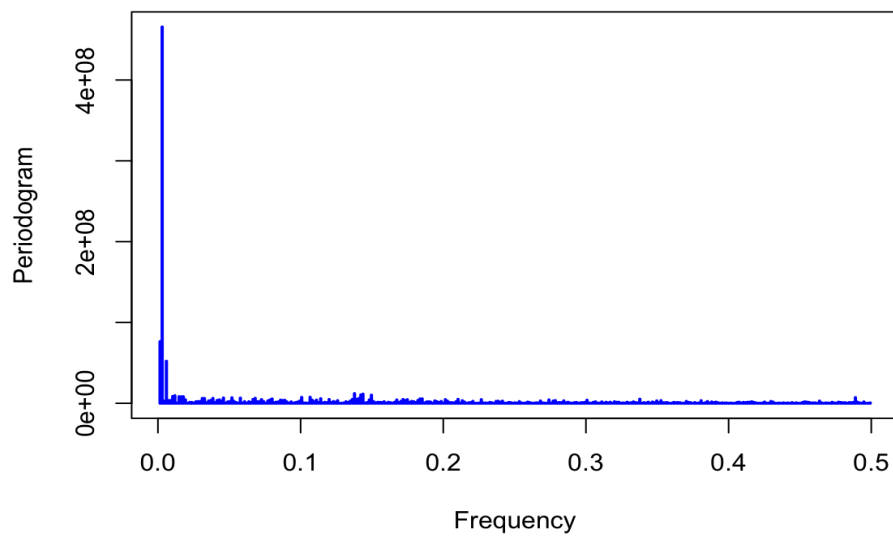


Exhibit 5: *Frequency of Cyclical Model*

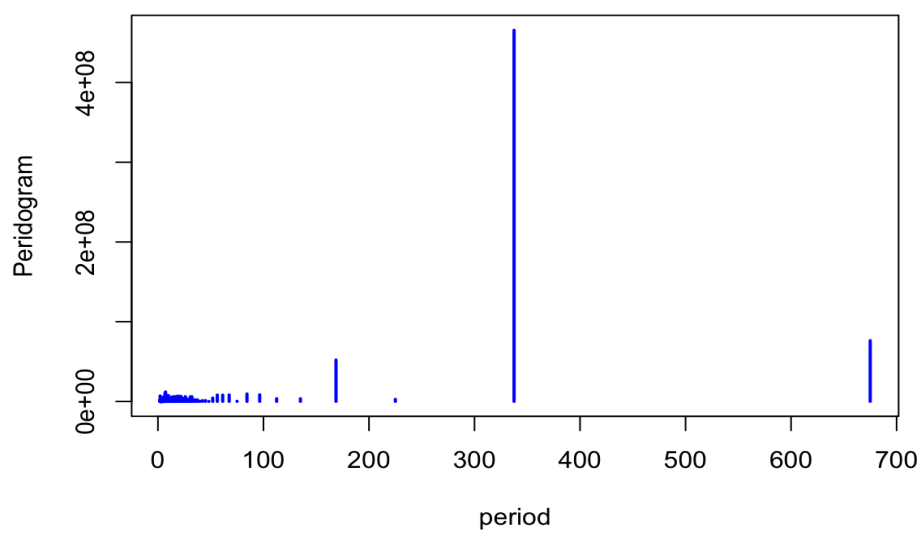


Exhibit 6: *Period of Cyclical Model*

	period	frequency	amplitude
[1,]	675.000000	0.001481481	7.634832e+07
[2,]	337.500000	0.002962963	4.656379e+08
[3,]	225.000000	0.004444444	2.751548e+06
[4,]	168.750000	0.005925926	5.207984e+07
[5,]	135.000000	0.007407407	3.502209e+06
[6,]	112.500000	0.008888889	3.565971e+06
[7,]	96.428571	0.010370370	8.474886e+06
[8,]	84.375000	0.011851852	9.505479e+06
[9,]	75.000000	0.013333333	3.525000e+05
[10,]	67.500000	0.014814815	8.228072e+06
[11,]	61.363636	0.016296296	8.073377e+06
[12,]	56.250000	0.017777778	8.226449e+06
[13,]	51.923077	0.019259259	4.368144e+06
[14,]	48.214286	0.020740741	1.922530e+05
[15,]	45.000000	0.022222222	1.242386e+06
[16,]	42.187500	0.023703704	1.190091e+06
[17,]	39.705882	0.025185185	4.141979e+05
[18,]	37.500000	0.026666667	2.011438e+06
[19,]	35.526316	0.028148148	2.166398e+06
[20,]	33.750000	0.029629630	2.328648e+06

Exhibit 7: *Periodogram of Cyclical Model*

As seen in **Exhibit 7**, we observe the highest amplitude, which is 465637855 at Period 337.5. To create sine and cosine pairs, we consider periods associated with the top 6 highest amplitude by sorting the amplitude in a descending order.

	period	frequency	amplitude
[1,]	337.500000	0.002962963	465637855
[2,]	675.000000	0.001481481	76348321
[3,]	168.750000	0.005925926	52079837
[4,]	7.258065	0.137777778	12021327
[5,]	6.958763	0.143703704	11349454
[6,]	7.031250	0.142222222	10442903

Exhibit 8: Top 6 Amplitudes

Call:

```
lm(formula = n_CNT ~ time + cos1 + sin1 + cos2 + sin2 + cos3 +
    sin3 + cos4 + sin4 + cos5 + sin5 + cos6 + sin6)
```

Residuals:

Min	1Q	Median	3Q	Max
-5799.3	-395.9	118.8	565.9	2799.4

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	2197.1743	140.8044	15.604	< 2e-16	***
time	6.8400	0.4033	16.961	< 2e-16	***
cos1	-1059.0836	49.9313	-21.211	< 2e-16	***
sin1	-567.6951	66.1043	-8.588	< 2e-16	***
cos2	364.2059	49.9313	7.294	8.63e-13	***
sin2	184.6996	100.0027	1.847	0.065202	.
cos3	6.1943	49.9313	0.124	0.901309	
sin3	-423.2475	54.4251	-7.777	2.87e-14	***
cos4	-99.9742	49.9313	-2.002	0.045667	*
sin4	-160.9466	49.9373	-3.223	0.001331	**
cos5	-122.7573	49.9313	-2.459	0.014206	*
sin5	134.5470	49.9366	2.694	0.007232	**
cos6	-173.6605	49.9313	-3.478	0.000538	***
sin6	-25.4048	49.9367	-0.509	0.611104	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 917.3 on 661 degrees of freedom

Multiple R-squared: 0.7854, Adjusted R-squared: 0.7812

F-statistic: 186.1 on 13 and 661 DF, p-value: < 2.2e-16

Exhibit 9: Cyclical Model with Sine and Cosine Pairs

According to the top 6 amplitudes, the periods are provided in **Exhibit 8**, associating with the Harmonic 2, 1, 4, 93, 97, 96 in the overall periodogram. We then create the sine and cosine pairs based on these values to run a cyclical model. As seen in **Exhibit 9**, most p-values associated with each cosine and sine pair are less than 0.05, meaning that the coefficients are different than 0, and that the periods associated with these pairs contribute to the model.

2.2 Comparison of "candidate" models in terms of fit and hold-out sample.

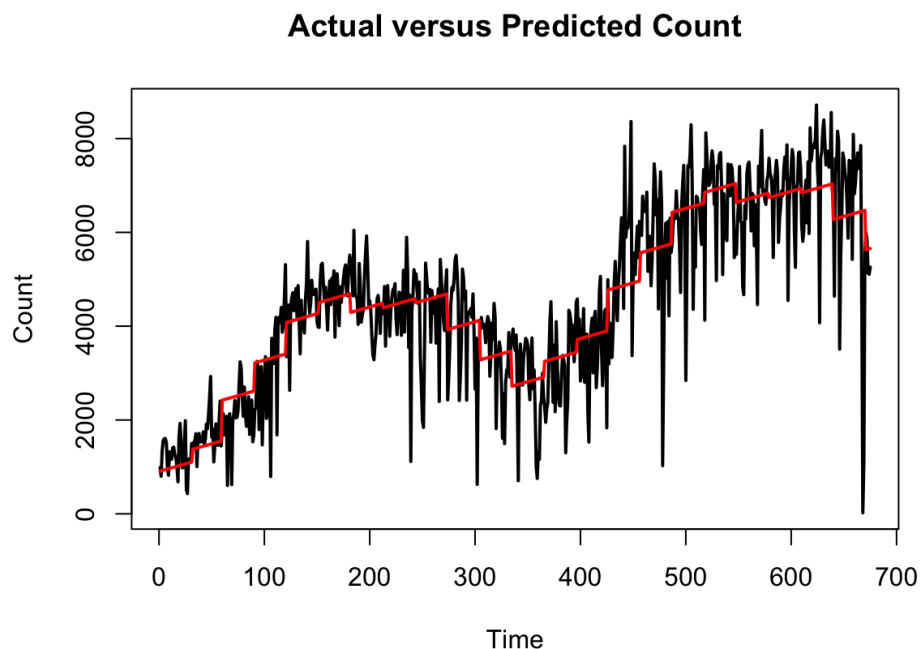


Exhibit 10: *Plot of Actual Versus Predicted of Seasonal Dummies Model*

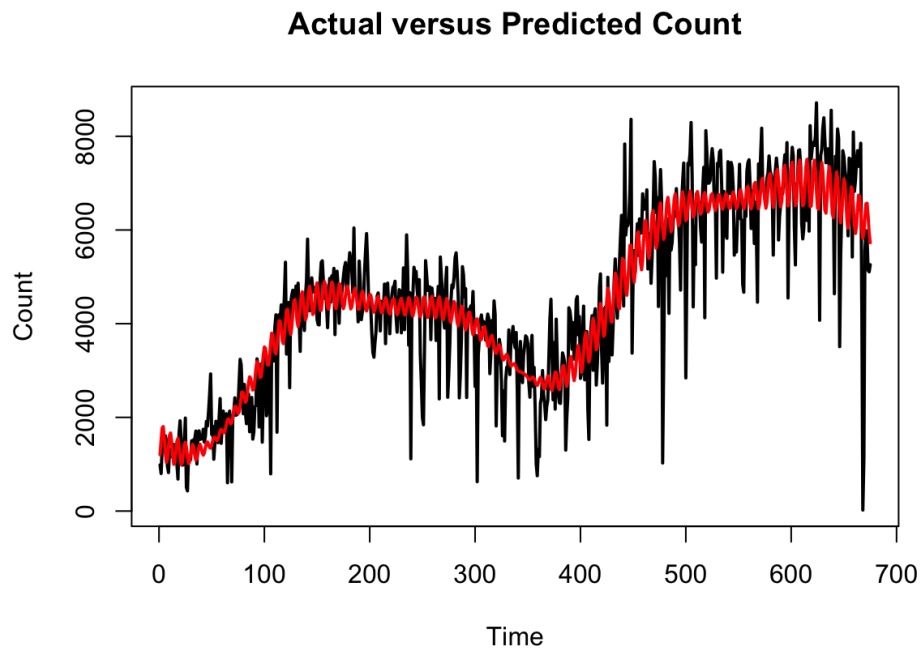


Exhibit 11: *Plot of Actual Versus Predicted of Cyclical Model*

Based on the plots, we can see that the plot of the actual and predicted counts of rental for the cyclical model fits relatively better than the seasonal dummies model. To consider statistically, you can refer to the following.

	MAPE (Training)	MAPE (Out-of-sample)	Correlation
Seasonal Dummies with Trends	66.37%	74.27%	0.8705
Cyclical Model	60.73%	79.28%	0.8862

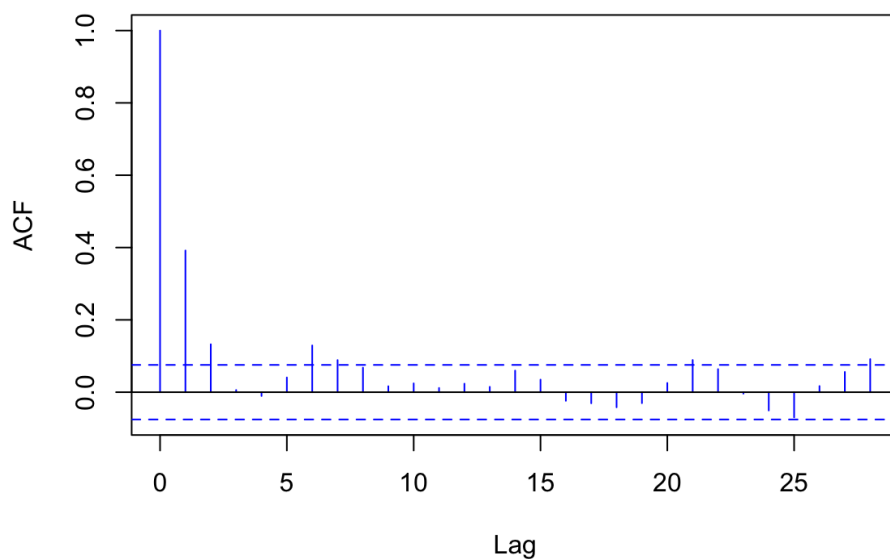
The lower MAPE for the training dataset of the cyclical model, which is 60.73%, suggests that this model fits better compared to the seasonal dummies model, whose MAPE is 66.37%.

However, the MAPE for the out-of-sample dataset of the cyclical model, which is 79.28%, is higher than the MAPE of the seasonal dummies model, meaning that the cyclical model performs worse when applied to the out-of-sample dataset. This suggests that while the model captures certain cyclic patterns well in the training data, it might be overfitting to the training data or may not be capturing all relevant factors influencing the data. However, the correlation of the cyclical model is slightly higher than that of the seasonal dummies model, indicating that the cyclical model demonstrates a stronger relationship between predictions and observations. Therefore, we then look at the residuals of the model for further analysis.

2.3 Looking at residuals of the model(s).

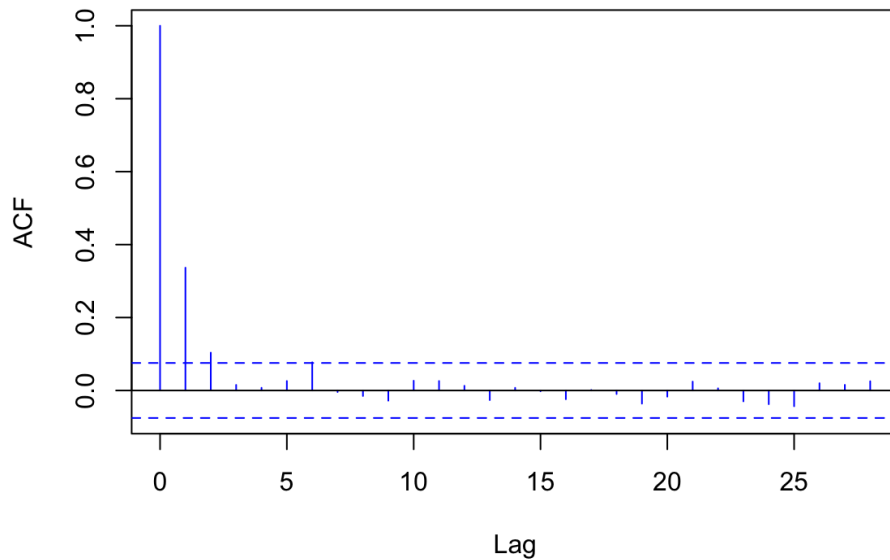
Seasonal Dummies with Trends

ACF of the residuals of the seasonal model



Cyclical Model

ACF of the residuals of the cyclical model



The residuals of both models are not white noise, as evidenced by the Box-Pierce test p-value being lower than 0.05, but stationary after Lag 1.

3. Time Series Regression Models

3.1 Discussion of independent variables. Correlation analysis and scatter plots

Correlation matrix:

	temp	hum	windspeed	cnt
temp	1.0000000	0.1269629	-0.1579441	0.6274940
hum	0.1269629	1.0000000	-0.2484891	-0.1006586
windspeed	-0.1579441	-0.2484891	1.0000000	-0.2345450
cnt	0.6274940	-0.1006586	-0.2345450	1.0000000

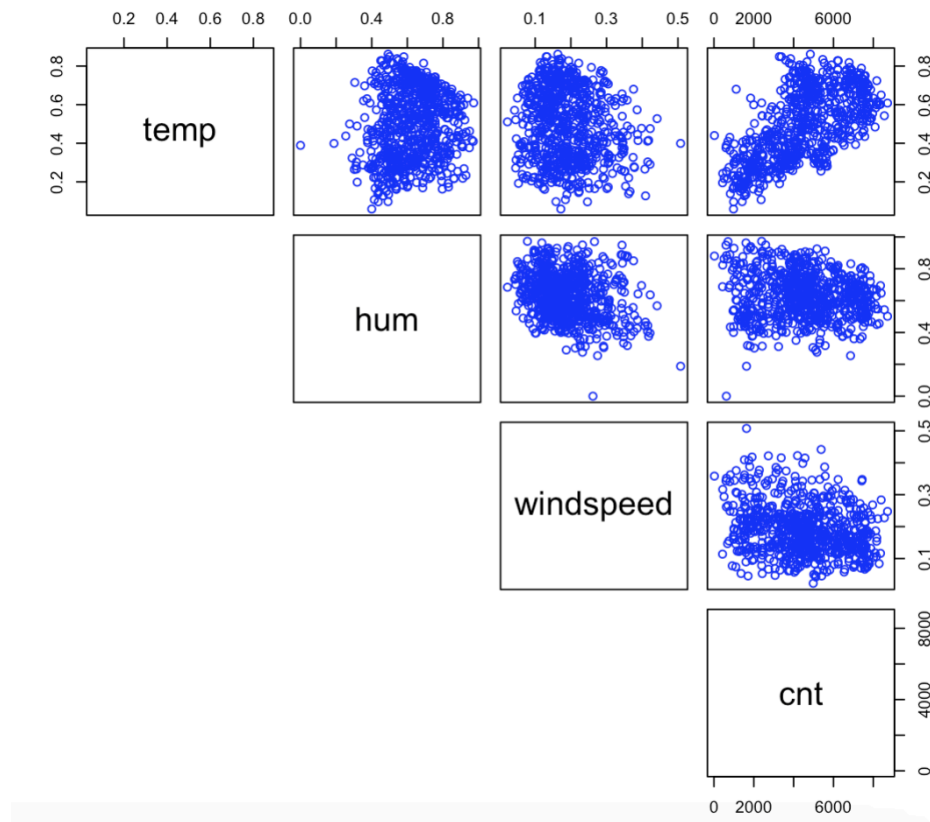
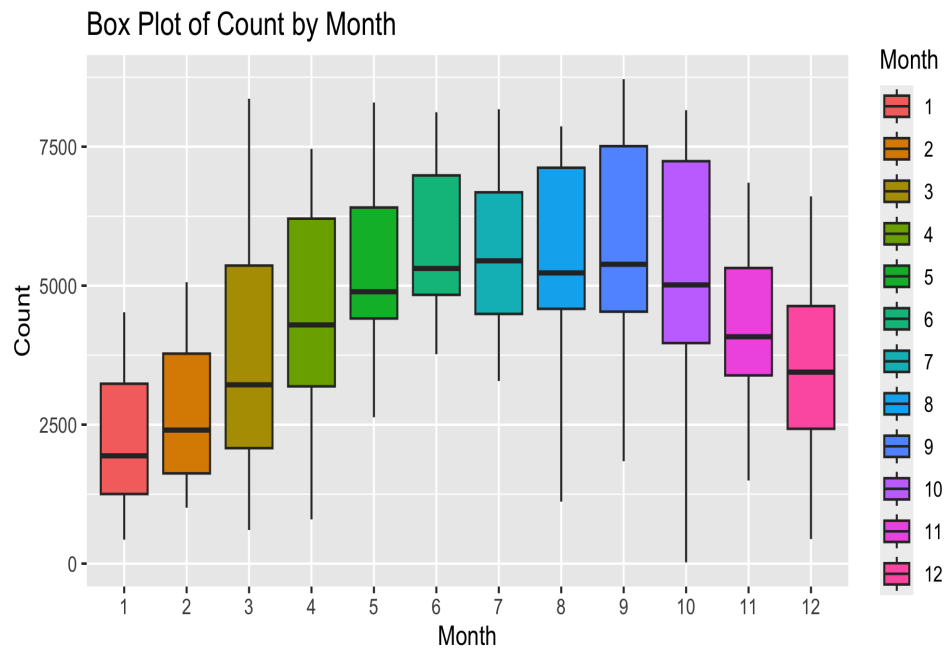
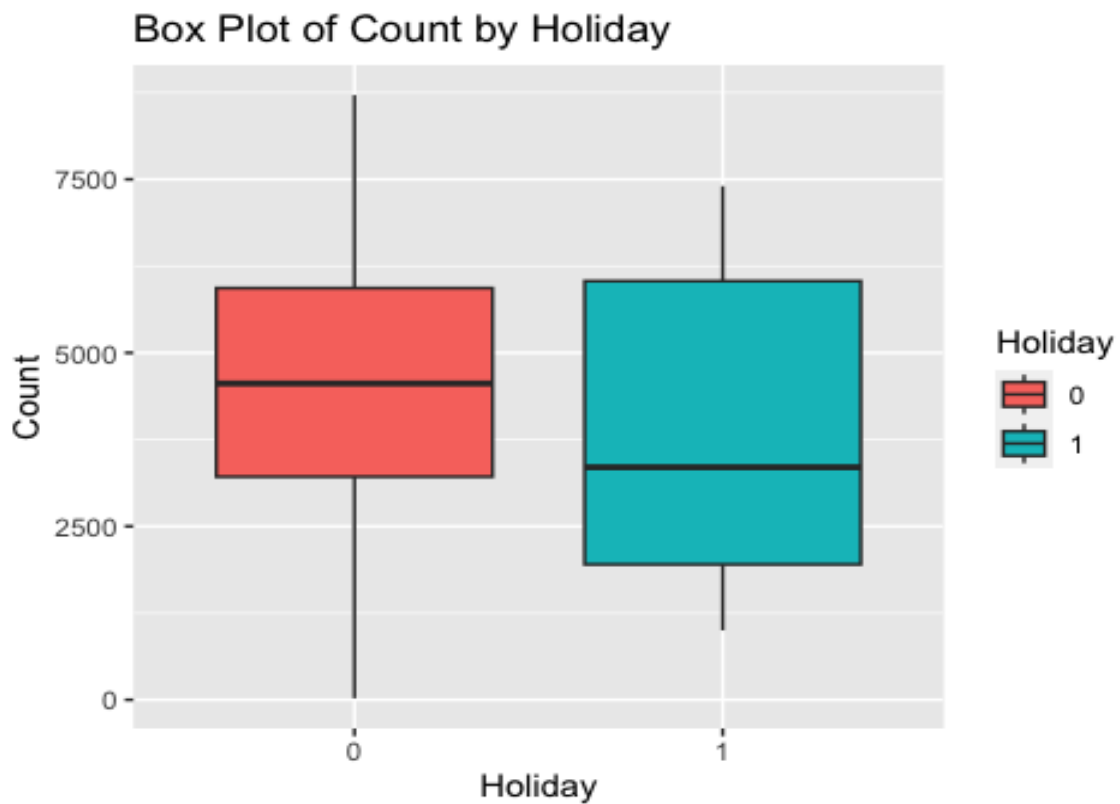


Exhibit 12: *Correlation Analysis and Scatter Plots*

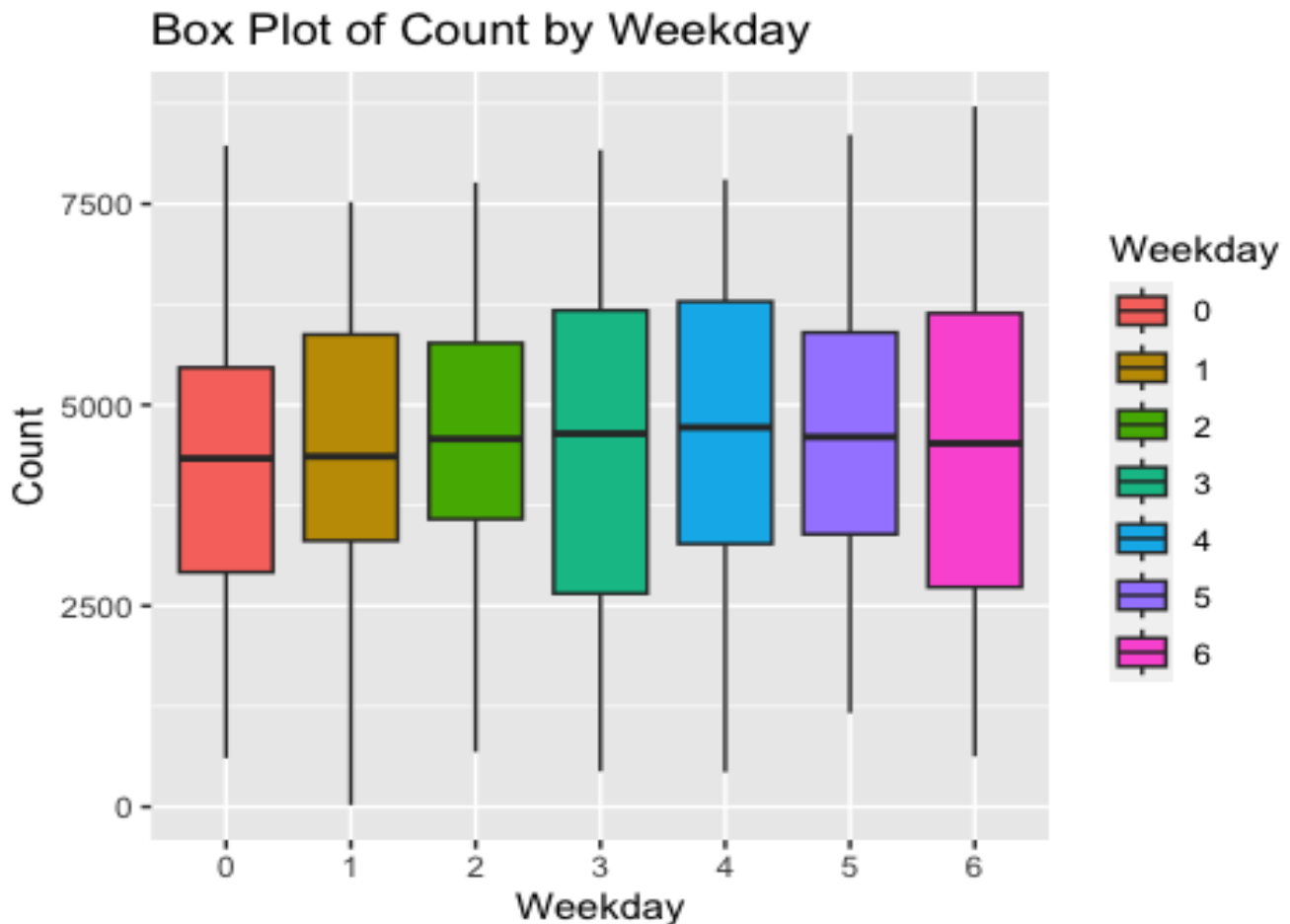
In the correlation matrix, we checked how each variable is positively and negatively correlated with each other, as seen in **Exhibit 12**. We have focused primarily on the relationship between count and other variables to determine how their unique trends contribute. Comparing month, weekday, and temperature showed a positive correlation. Since the month, holiday, and weather situation are based on the categorical data set, the following regression models will be implemented with the following box plots.



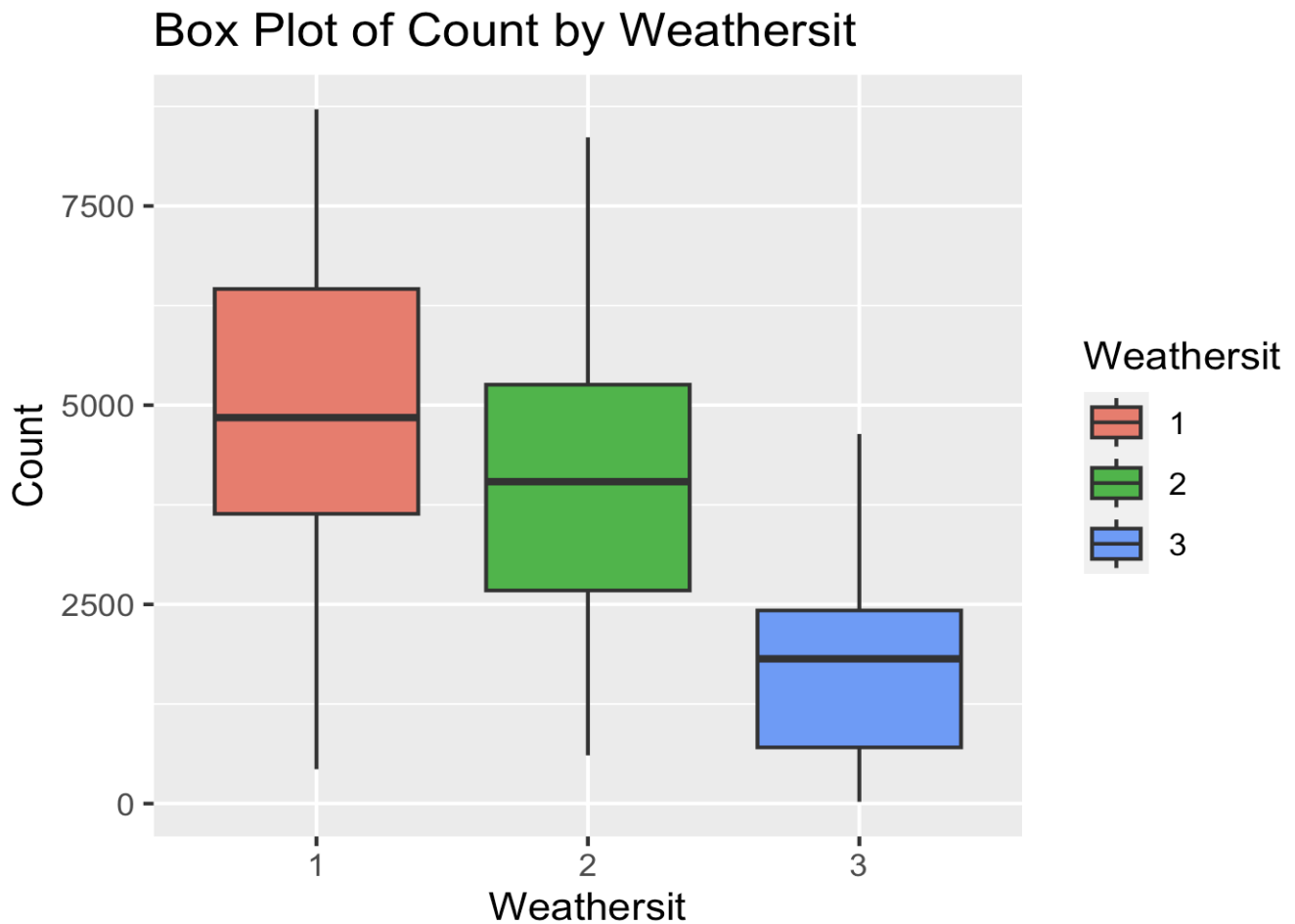
Our data analysis has revealed a distinct relationship between Bike rentals and month. Specifically, we've found that March, April, September, and October exhibit a significant degree of variability, indicating potential seasonal patterns in bike rentals.



The Plot shows the relationship between variable bike counts and Holidays. By checking the Box plot results we could check that customers are more willing to pick up and drop them not during the holiday seasons.



The plot shows the relationship between the weekdays and bike rentals. From the box plots and data set from 1/1/2011, we see that there is not much difference between the weekdays for the bike rental by checking their counts which all show a range between 2,500 to 5,000.



The box plot shows the relationship between bike rentals and weather situations. We could check from the graph that when the weather improves (1: excellent, 2: worse, 3: worst), customers are willing to rent more bicycles; however, when the weather gets worse, the situation goes vice versa.

3.2 Comparison of "candidate" models in terms of fit and hold-out sample

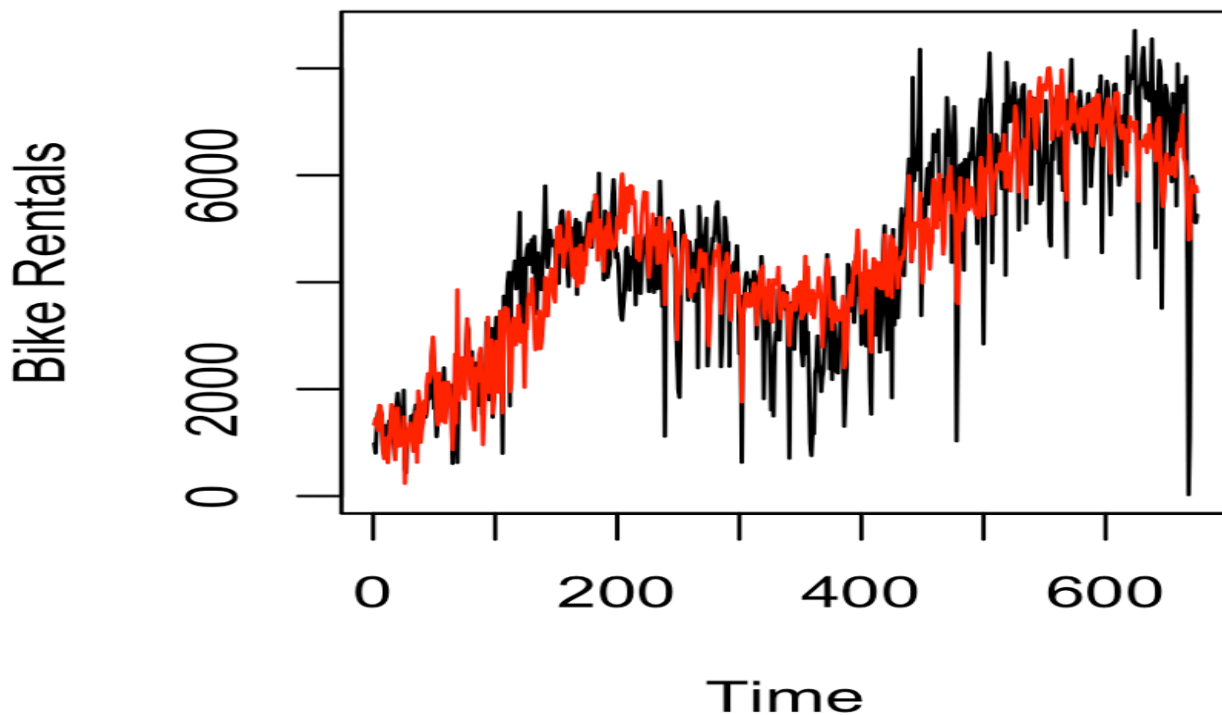


Exhibit 13: *Plot of Actual Versus Predicted of Regression Model*

Exhibit 13 depicts the actual and the predicted value of the regression model.

```
lm(formula = n_cnt ~ time + n_temp + n_hum + n_wind, data = pro)
```

Residuals:

Min	1Q	Median	3Q	Max
-4764.8	-505.6	69.3	563.4	2796.8

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2349.8833	231.6468	10.144	< 2e-16 ***
time	5.7306	0.1916	29.916	< 2e-16 ***
n_temp	5109.9788	207.0025	24.686	< 2e-16 ***
n_hum	-2782.1197	255.0215	-10.909	< 2e-16 ***
n_wind	-3325.7920	484.1633	-6.869	1.48e-11 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 915.3 on 670 degrees of freedom

Multiple R-squared: 0.7834, Adjusted R-squared: 0.7821

F-statistic: 605.9 on 4 and 670 DF, p-value: < 2.2e-16

Exhibit 14: Regression Model

As seen in **Exhibit 14**, all the coefficients of the regression model are significant, since p value for temperature, humidity and wind speed is less than .05.

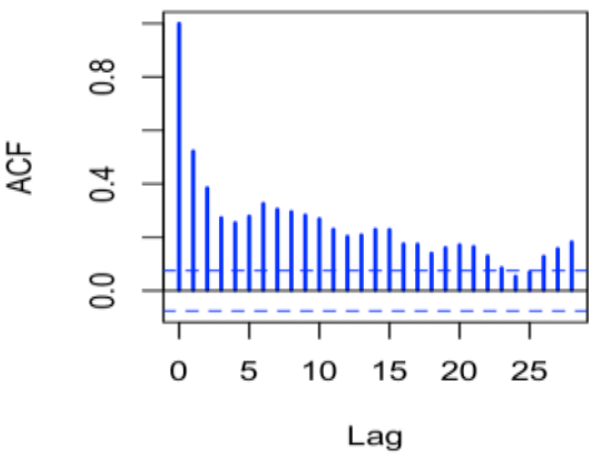
	MAPE (Train)	MAPE (Out-of-sample)	Correlation
Time Series Regression model	52.83569%	72.48018%	0.8843642

From the regression model we found that the MAPE for in sample data is 53% which is less than the holdout sample of 72%.The correlation coefficient in Actual and Predicted value is 88% which suggests a good correlation.

RMSE of the Time Series Regression Model :911.9317

3.3 Looking at residuals of the model(s).

Residuals of the Regression model	Box-Pierce p-value	White Noise or not
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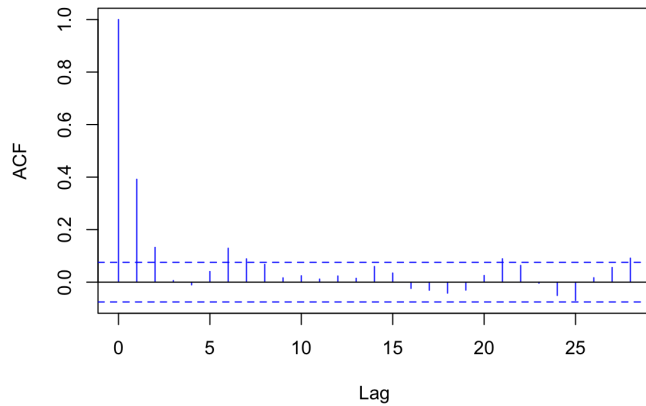
	<p><2.2e-16</p>	<p>Reject the null hypothesis → Not WN</p>
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4. Stochastic Time Series Models

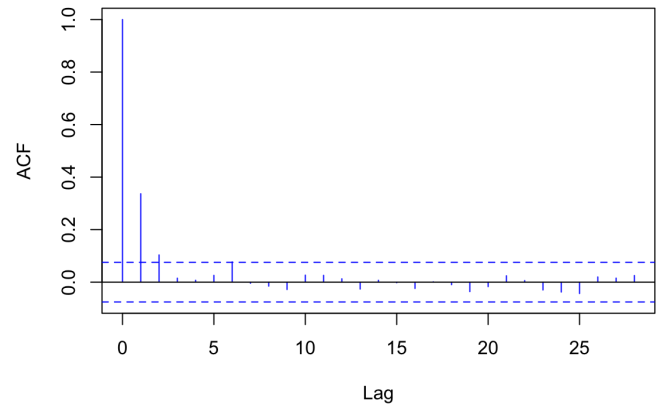
4.1 Analysis and modeling of deterministic time series model residuals

Seasonal Dummies Model	Cyclical Model
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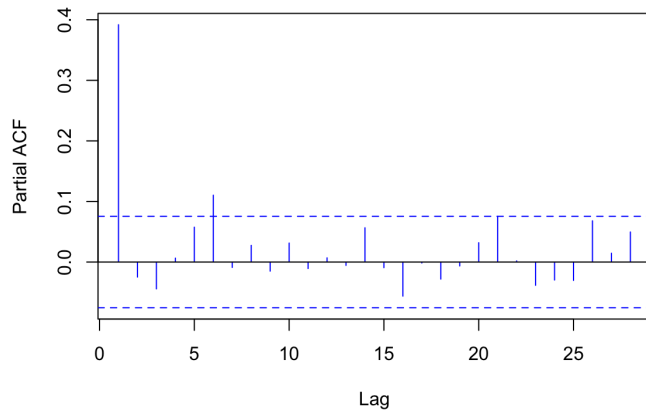
ACF of Seasonal Residuals



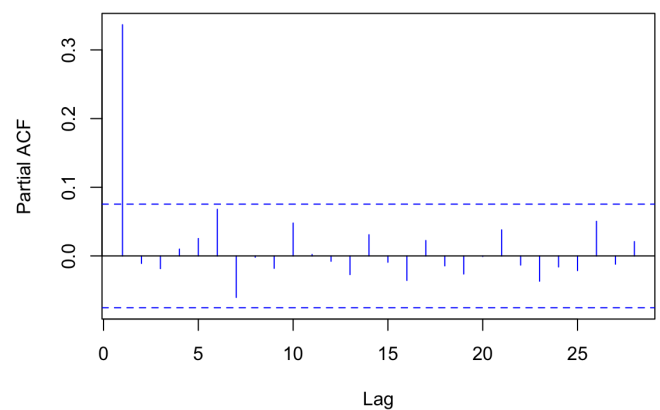
ACF of Cyclical Residuals



PACF of Seasonal Residuals



PACF of Cyclical Residuals



As seen in the above graphs, the ACF decays quickly, and PACF chops off to 0 after lag 1.

Therefore, we will consider the **AR(1)**

AR(1)

Series: cr
ARIMA(1,0,0) with non-zero mean

Coefficients:

	ar1	mean
	0.3363	-0.0753
s.e.	0.0362	49.5270

sigma^2 = 732653: log likelihood = -5514.59
AIC=11035.17 AICc=11035.21 BIC=11048.72

process.

```
Series: resforfinal1  
ARIMA(1,0,0) with non-zero mean
```

```
Coefficients:  
      ar1      mean  
    0.3913  -0.0958  
s.e.  0.0354  56.0629
```

```
sigma^2 = 789872: log likelihood = -5539.99  
AIC=11085.98  AICc=11086.01  BIC=11099.52
```

Box-Pierce test

```
data: fit_arar$resid  
X-squared = 19.809, df = 20, p-value = 0.47
```

Since the ratio of ϕ_1 and s.e., which is $0.3913/0.0354$, is greater than 2, we can conclude that the coefficient at lag 1 is statistically different from 0.

Moreover, according to the Box-Pierce Test, the p-value, 0.47, is greater than 0.05, meaning that the residuals from this AR model are white noise after we model the series as AR(1).

Box-Pierce test

```
data: cc$resid  
X-squared = 11.568, df = 20, p-value = 0.9301
```

Since the ratio of ϕ_1 and s.e., which is $0.3363/0.0362$, is greater than 2, we can conclude that the coefficient at lag 1 is statistically different from 0.

Moreover, according to the Box-Pierce Test, the p-value, 0.9301, is greater than 0.05, meaning that the residuals from this AR model are white noise after we model the series as AR(1).

MA(2)

```
Series: res3  
ARIMA(0,0,2) with non-zero mean
```

```
Coefficients:  
      ma1      ma2      mean  
    0.3376  0.1040  -0.0973  
s.e.  0.0380  0.0376  47.3936
```

```
sigma^2 = 733652: log likelihood = -5514.54  
AIC=11037.09  AICc=11037.15  BIC=11055.15
```

Box-Pierce test

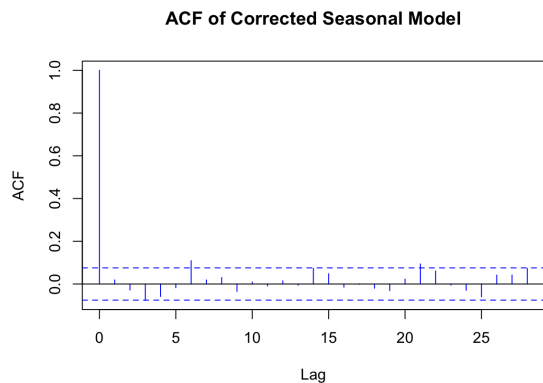
```
data: res_armi  
X-squared = 0.0046962, df = 1, p-value = 0.9454
```

Also, we can use the MA(2) model, since the t-stats are $0.3376/0.0380$ and $0.1040/0.0376$, which are greater than 2, we can conclude that the t-stat is significant.

Additionally, according to the Box-Pierce Test, the p-value, 0.9454, is greater than 0.05, meaning that the residuals from this MA model are white noise after we model the series as MA(2).

Corrected Model:

ARIMA(2,0,1)



The ACF falls within 2 standard error bounds, meaning that the series is white noise.

Series: n_CNT
Regression with ARIMA(2,0,1) errors

Coefficients:

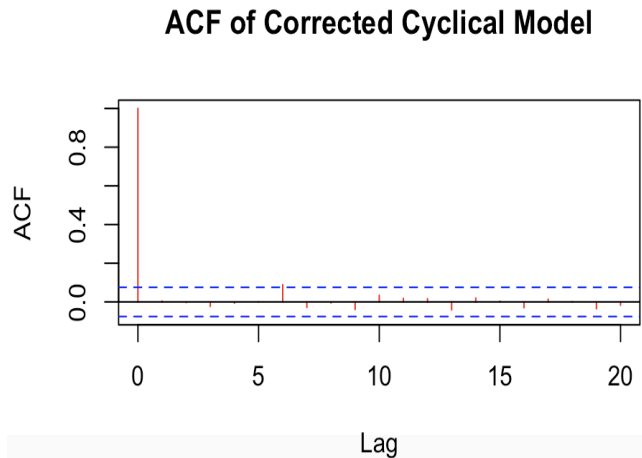
	ar1	ar2	ma1	intercept	time	
	1.3176	-0.3238	-0.9000	2026.4730	7.0990	-18.0626
s.e.	0.0430	0.0421	0.0188	833.9941	2.0245	45.2946

sigma^2 = 826387: log likelihood = -5553.85
AIC=11121.7 AICc=11121.87 BIC=11153.3

After trying the ARIMA process, we find that ARIMA(2,0,1) is an appropriate model. The t-stats are all greater than 2, indicating the

Corrected Model:

ARIMA(1,0,0)



The ACF falls within 2 standard error bounds, meaning that the series is white noise.

Series: n_usage
Regression with ARIMA(1,0,0) errors

Coefficients:

	ar1	intercept	time	cos1	sin1	cos2	sin2	cos3
	0.3364	2200.0345	6.8301	363.2670	182.5726	-1060.0224	-568.7695	5.2559
s.e.	0.0362	195.6294	0.5599	70.0012	139.2343	69.9943	92.3593	69.9667

	sin3	cos4	sin4	cos5	sin5	cos6	sin6
	-423.8064	-100.5792	-161.5714	-174.2441	-26.0417	-123.3338	133.9063
s.e.	76.2405	56.5054	56.5532	55.9129	55.9578	55.7166	55.7606

sigma^2 = 747083: log likelihood = -5514.59
AIC=11061.17 AICc=11062 BIC=11133.41

After trying the ARIMA process, we find that ARIMA(1,0,0) is an appropriate model. The t-stats are all greater than 2, indicating the

significance, thus validating the use of this model.

Box-Pierce test

```
data:  sfit_corrected$resid
X-squared = 23.899, df = 20, p-value = 0.2468
```

Additionally, we assess the residuals using the Box-Pierce test. The p-value is greater than 0.05, leading to the conclusion that they present white noise.

significance, thus validating the use of this model.

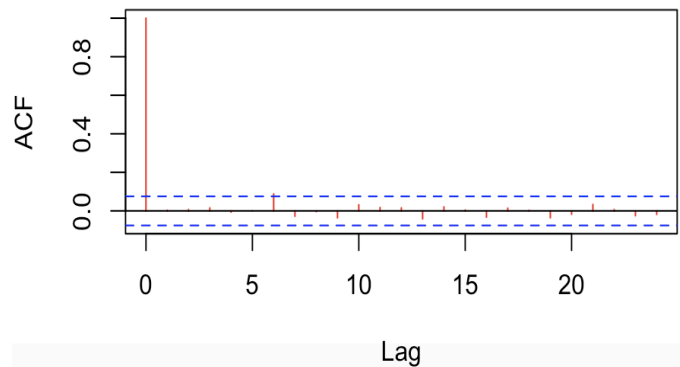
Box-Pierce test

```
data:  cfit_corrected$resid
X-squared = 11.56, df = 20, p-value = 0.9304
```

Additionally, we assess the residuals using the Box-Pierce test. The p-value is greater than 0.05, leading to the conclusion that they present white noise.

ARIMA(0,0,2)

ACF of Corrected Cyclical Model



The ACF falls within 2 standard error bounds, meaning that the series is white noise.

```
Series: n_usage
Regression with ARIMA(0,0,2) errors

Coefficients:
      ma1      ma2  intercept      time      cos1      sin1      cos2      sin2
0.3376  0.1040  2198.1207    6.8363  363.5864  183.8950 -1059.7032 -568.1047
s.e.    0.0380  0.0376   187.6344   0.5371   66.9885   133.4738   66.9852   88.4625
      cos3      sin3      cos4      sin4      cos5      sin5      cos6      sin6
5.5750 -423.4670 -100.4378 -161.3747 -174.1135 -25.8431 -123.2067 134.1055
s.e.    66.9721   72.9832   58.3358   58.3823   57.8268   57.8706   57.6551   57.6982

sigma^2 = 748124: log likelihood = -5514.54
AIC=11063.09  AICc=11064.02  BIC=11139.84
```


After trying the ARIMA process, we find that ARIMA(0,0,2) is an appropriate model. The t-stats are all greater than 2, indicating the significance, thus validating the use of this model.

Box-Pierce test

```
data: cfit_corrected$resid
X-squared = 11.14, df = 20, p-value = 0.9425
```

Additionally, we assess the residuals using the Box-Pierce test. The p-value is greater than 0.05, leading to the conclusion that they present white noise.

4.2 Analysis and modeling of regression model residuals

Model of Regression Residual

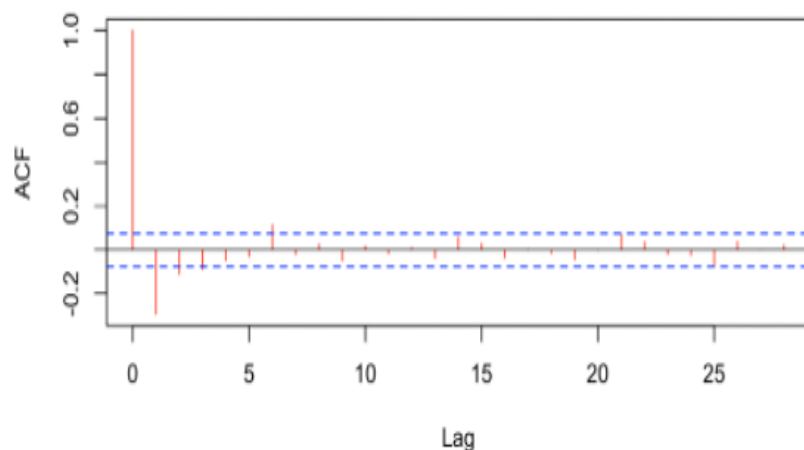
Since the series obtained from the Regression model is nonstationary, we differentiate the series.

The difference series ACF is chopped after lag 2.

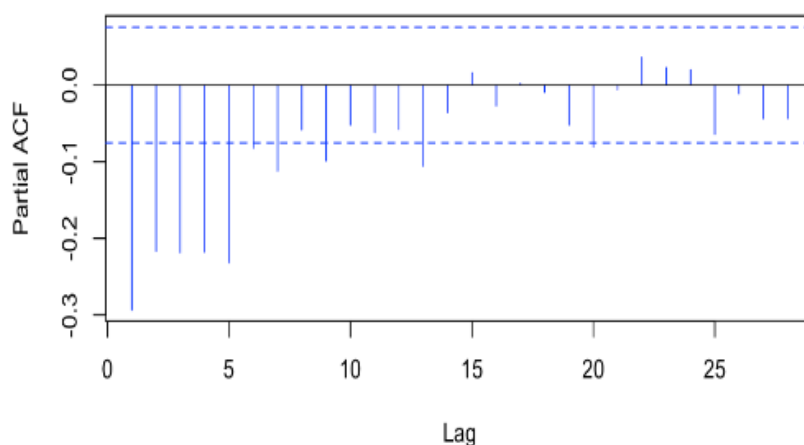
PACF is decaying quickly.

We fit an MA(2) with corrected residuals.

First difference of series



First difference of series



Regression coefficients of the corrected Model

Fitting the model with ARIMA(0,1,2) we found that the absolute value of coefficients of the model are greater than 1.96.

Regression Coefficients with Residuals

Regression with ARIMA(0,1,2) errors

Coefficients:

	ma1	ma2	n_temp	n_hum	n_wind
	-0.6154	-0.2044	5.2233	4850.1841	-3306.1891
s.e.	0.0358	0.0350	5.3138	480.7439	239.9195

sigma^2 = 580735: log likelihood = -5426.53

AIC=10867.06 AICc=10867.23 BIC=10898.65

Hence, all the coefficients in the model are significant.

We further found the accuracy of the model for In sample and out of Sample.

For holdout Sample

Correlation Coefficient = 0.78642

RMSE: 1002.372

MAPE :

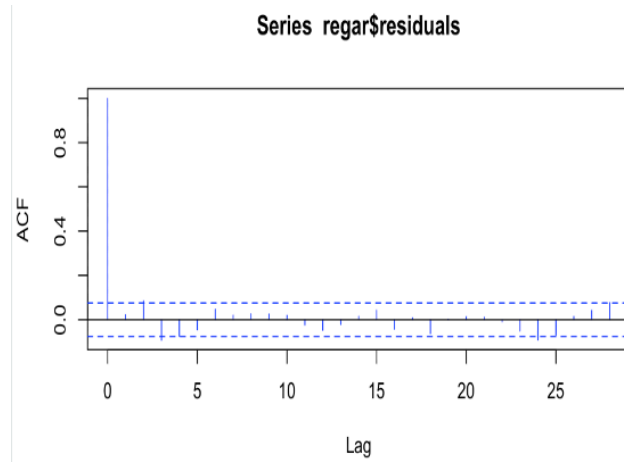
29.55824%

For In Sample

Correlation Coefficient
=0.7864226

RMSE: 758.0977

MAPE : 49.49%



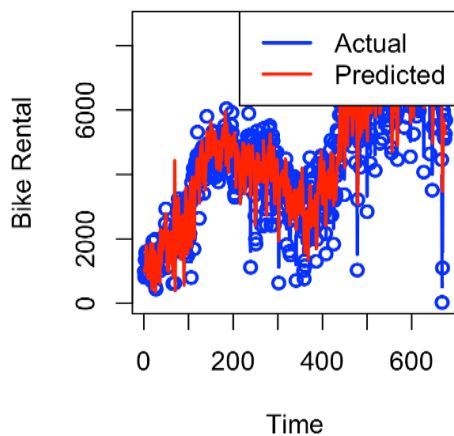
From Box Pierce test,
 $p = .08408$ and ACF is also
between $2se$ bounds hence it's
a White Noise Series

Box-Pierce test

```
data: regar$residuals  
X-squared = 33.857, df = 24, p-value = 0.08724
```

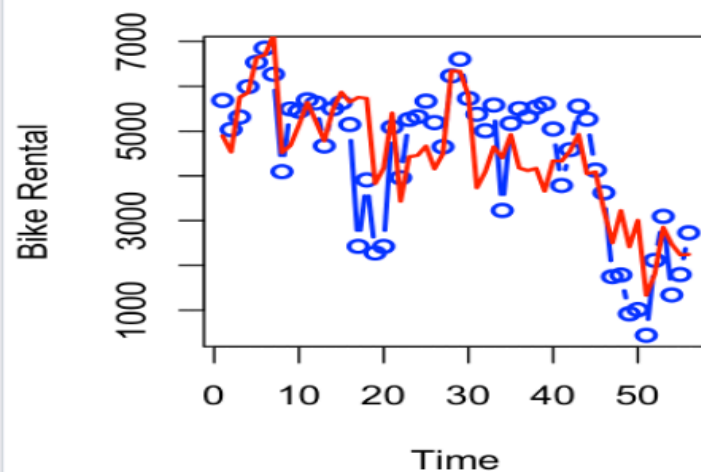
**Actual Vs predicted for In
sample**

Actual versus Predicted



Actual Vs Predicted for Holdout Sample

Actual versus Predicted

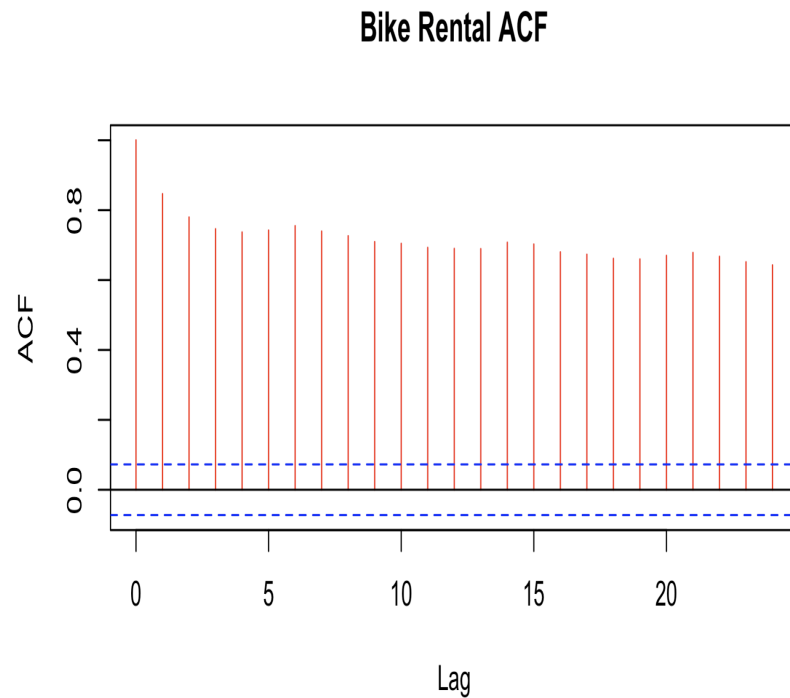


4.3 ARIMA models (for the variable of interest)

Model ARIMA(1,1,1)

Bike Rental residuals as shown in the plot are decaying slowly so we differenced the series and further modeled it with ARIMA(1,1,1).

Since in the section 4.2 value for Box plot p value is near to .05 so we tried fitting ARIMA(1,1,1) to the Bike rental series.



The Model Box plot is now
.3557 and ACF is between
2 standard error bounds.

In sample:

As the p value is greater
than the .05 so we can say
that we cannot reject the
null hypothesis and the
series is a white Noise
series.

MAPE: 48.91067%

RMSE : 753.9765

Correlation Coefficient:

0.9233345

Regression with ARIMA(1,1,1) errors

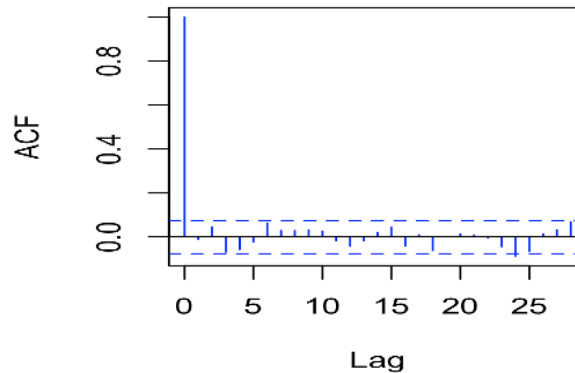
Coefficients:

	ar1	ma1	n_temp	n_hum	n_wind
	0.3116	-0.8912	5.3554	4961.0642	-3330.6928
s.e.	0.0477	0.0235	4.6501	485.6595	241.1232

sigma^2 = 574438: log likelihood = -5422.87

AIC=10859.75 AICc=10859.92 BIC=10891.34

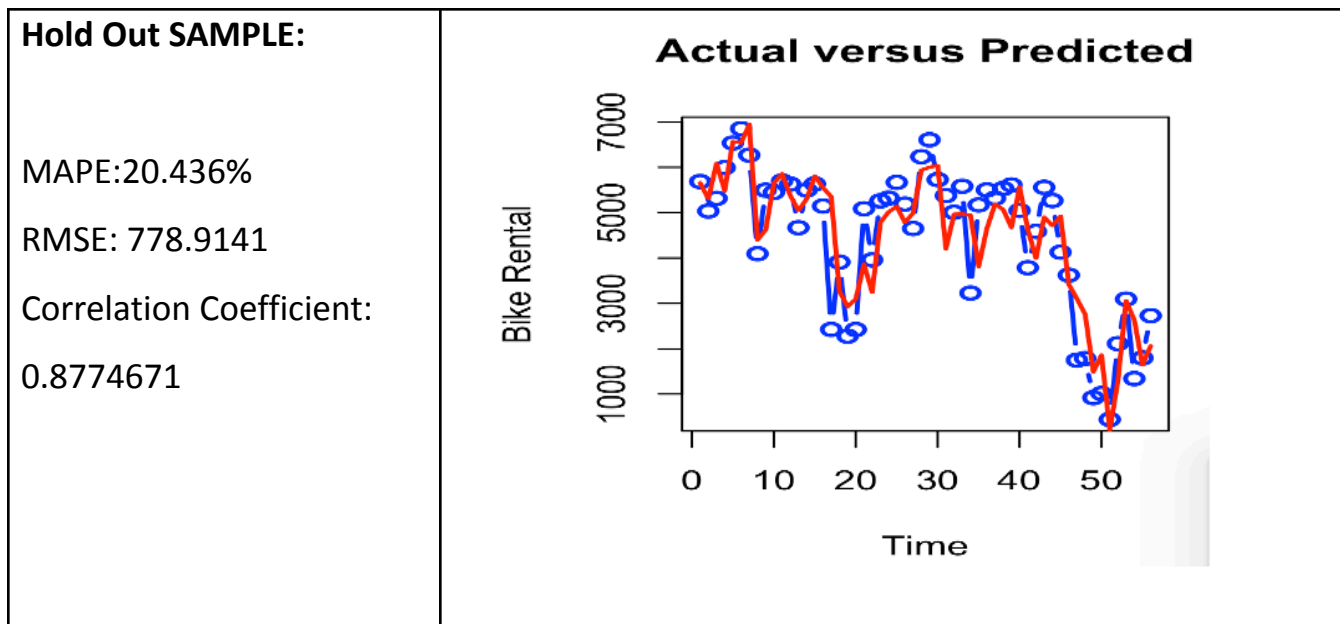
Series regar\$residuals



Box-Pierce test

data: regar\$residuals

X-squared = 25.911, df = 24, p-value = 0.3577



5. Conclusion summary of your findings and comparison of deterministic and stochastic model performance based on the hold-out sample.

In conclusion, our study provides insights into the deterministic and regression model that explain the forecast of rental counts.

In terms of the deterministic model, we conclude that the seasonal dummies model demonstrates a significant trend term and seasonality, as analyzed by low p-values associated with time and each month, and that the cyclical model captures cyclical patterns well, as evidenced by low p-values associated with time and sine and cosine pairs. However, the higher out-of-sample MAPE for the cyclical model suggests potential overfitting or inadequacy in capturing all relevant factors. Therefore, analyzing the residuals, we model the seasonal dummies model using ARIMA(2,0,1) process, and the cyclical model using AR(1) or MA(2) process, as the t-stats imply the significance of the coefficient associated with lags, and both models' residuals are considered as white noise, as analyzed by Box-Pierce test p-values greater than 0.05. This means that the models adequately capture the underlying structure of the data, providing a foundation for further refinement and application in rental count forecasting.

As for the regression model, we difference the series and conclude that the model with 3 independent variables, temperature, humidity, and wind speed, From the Regression models we found the MAPE to be 52% for in sample and 72.48018% for out sample but we fitted regression model with residuals with ARIMA(0,1,2) we found that the MAPE for Insample is 49.49% and for hold out sample is 29.5% .Thus we can say that by fitting the regression model with residuals the MAPE is reduced which means the model is improved by fitting the regression model with residuals. Further we see the RMSE of the regression model is 911.91 while the RMSE for corrected regression model is 753.97 for in sample and 1005 for hold out may be due to overfitting. Due to this overfitting we also have implemented ARIMA(1,1,1) which shows Insample RMSE is 753.9765 and MAPE of 48.91067% and also the holdout sample Accuracy is also improved, RMSE is 778.9141 and MAPE of 20.49%. This suggests that after taking the difference of the residuals and also fitting the model with residuals the performance of the model is increased. ARIMA(1,1,1) error is appropriate, as evidenced by the significant t-stats and residuals falling within the standard error bounds and MAPE and RMSE values.

Overall, our findings suggest that refining regression models through residual fitting and incorporating ARIMA processes can enhance forecasting accuracy, mitigating potential overfitting issues and improving model performance for rental count predictions.