Q1.) What is time complexity of below code & how?

```
void fun (int n)
{ int j=1; int i=0;
  while (i<n) {
        i+=j;
        j++;
    }
}
```

$j=1 \quad i=1$

$j=2 \quad i=1+2$

$j=3 \quad i=1+2+3$

m - level

for (i)

$\because 1+2+3+ \ldots + m < n$

$1+2+3+ \ldots m < n$

$$\frac{m(m+1)}{2} < n$$

$$m \approx \sqrt{n}$$

By summation method

$$\sum_{i=1}^{m} 1 = 1+1+ \ldots \ldots + \sqrt{n} \text{ times}$$

$$\boxed{T(n) = \sqrt{n}}$$

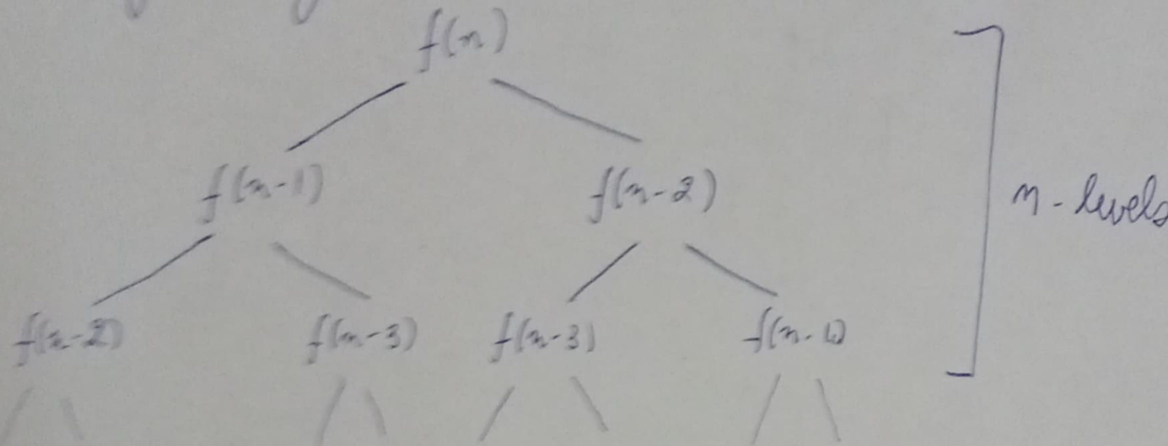
Q5) Write recurrence relation for function that prints Fibonacci series Solve it to get the time complexity what will be space complexity & why?

For Fibonacci Series -

$f(n) = f(n-1) + f(n-2)$ $\quad\quad$ $f(0) = 0$

$f(1) = 1$

By forming tree



$\because$ At every function call we get 2 function calls

$\therefore$ for n levels

We have $= 2 \times 2 \ldots$ n times

$T(n) = 2^n$

<u>Maximum Space</u> -

Considering Recursion

Stack:

no of calls maximum $= n$

For each call we have space complexity $O(1)$

$\therefore \boxed{T(n) = O(n)}$

Without considering Recursive stack:

each call we have time complexity $O(1)$

$\therefore \boxed{T(n) = O(1)}$

Q3) Write a program which have complexity $n(\log n)$, n^3, $\log(\log n)$ ③

1) $n \log n$

```
void ~~int~~ main(){
    int i, j; int n;
  for (i=0; i<n, i++)
    for(j=1; j<n; j*=2)
        printf("*");
  }
```

2) n^3

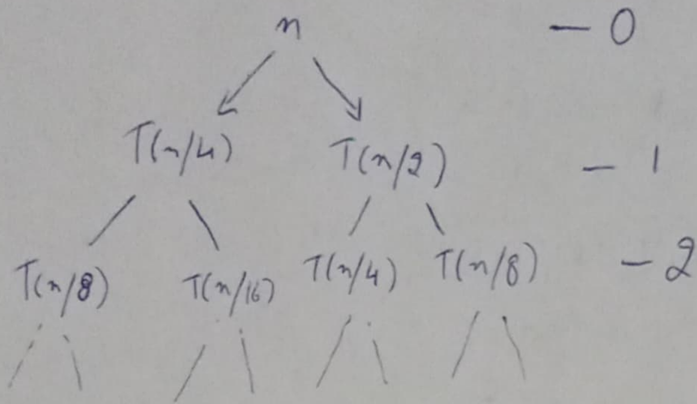
```
void main(){
    int i, j, K; int n;
    for (i=0, i<n, i++)
    for (j=0; j<n; j++)
    for (K=0, K<n, K++)
        printf("+");
    }
```

3) $\log(\log n)$

```
~~int main~~ void main()
    { int count=0; int i;
  for (i=2, i<n, i=i*i)
    { count++;
    }
  }
```

Q4.) Solve the following recurrence relation

$$T(n) = T(n/4) + T(n/2) + cn^2$$



At level →

$$0 \to cn^2$$

$$1 \to \frac{n^2}{4^2} + \frac{n^2}{2^2} = \frac{c\,5n^2}{16}$$

$$2 \to \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$$

$$\text{max level} = \frac{n}{2^k} = 1$$

$$K = \log_2 n$$

$$T(n) = c\left(n^2 + \left(\frac{5}{16}\right)n^2 + \left(\frac{5}{16}\right)^2 n^2 + \cdots + \left(\frac{5}{16}\right)^{\log n} n^2\right)$$

$$T(n) = cn^2\left[1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \cdots + \left(\frac{5}{16}\right)^{\log n}\right)$$

$$= cn^2 \times 1 \times \left(\frac{1 - (5/16)^{\log n}}{1 - 5/16}\right)$$

$$= cn^2 \times \frac{11}{5} \times \left(1 - \left(\frac{5}{16}\right)^{\log n}\right)$$

$$T(n) = O(n^2 c)$$

$$O(n^2 c)$$

Q5) What is time complexity of following func()?

```
int fun (int n) {
    for (int i=1; i<=n; i++) {
    for (int j=1; j<n; j+=i) {
    // Some O(1) task
    } }
}
```

for	i	j
	1	1
	2	1+3+5
	3	1+4+7
	⋮	⋮
	n	1+5+9

$j = (n-1)/i$ times

$$\sum_{i=1}^{n} \frac{(n-1)}{i}$$

$$\therefore T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-1)}{3} + \dots \frac{(n-1)}{n}$$

$$T(n) = n\left[1 + \frac{1}{2} + \frac{1}{3} + \text{---} + \frac{1}{n}\right] - 1\left[1 + \frac{1}{2} + \frac{1}{3} + \text{---} + \frac{1}{n}\right]$$

$$= n\log n - \log n$$

$$T(n) = O(n\log n)$$

Q6) What should be time complexity of

```
for (int i=2; i<=n; i = pow(i,K))
{ // Some O(1)
}
```

where K is constant.

→ for i

2^1

2^k

2^{k^2}

2^{k^3}

$\vdots$

2^{k^m}

where

$2^{k^m} <= n$

$k^m = \log_2 n$

$m = \log k \log_2 n$

$\therefore \sum_{i=1}^{m} 1$

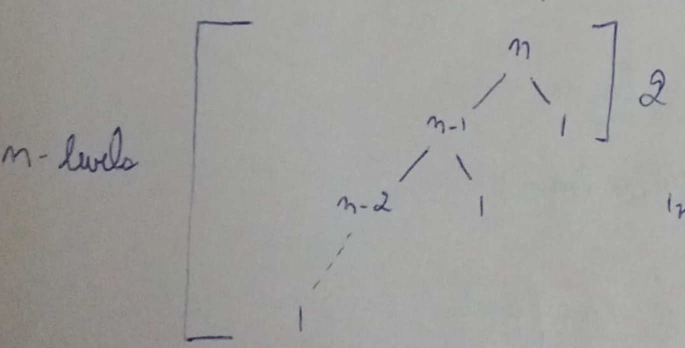
$1+1+1+ \dots + m$ times

$T(n) = O(\log_k \log n)$

Q7) Write a recurrence relation when quick sort repeatedly divides array into 2 parts of 99% & 1%. Derive time complexity in this case. Show the recurrence tree while deriving time complexity & find difference in heights of both extreme parts. What do you understand by this analysis?

Given algorithm divide array in 99% & 1% part.

$\therefore T(n) = T(n-1) + O(1)$



'n' work is done at each level

$T(n) = (T(n-1) + T(n-2) + \dots + T(1) + O(1)) \times n$

$= n \times n$

$\boxed{T(n) = O(n^2)}$

Lowest height = 2

highest height = n

$\therefore$ $\boxed{\text{difference} = n-2}$ $\quad n>1$

The given algorithm produces linear result

Q8.) Arrange following in increasing order of rate of growth:

a) $n, n!, \log n, \log\log n, \sqrt{n}, \log n!, n\log n, \log^{2n}, 2^n, 2^{2^n}, 4^n, n^2, 100$

$100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log n! < n^2$
$< 2^n < 4^n < 2^{2^n}$

b) $2(2^n), 4n, 2n, 1, \log n, \log(\log n), \sqrt{\log n}, \log 2n, 2\log n, n, \log(n!), n!, n2, n\log n$

$1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n <$
$4n < \log(n!) < n^2 < n! < 2^{2^n}$

c) $8^{2n}, \log_2 n, n\log_6 n, n\log_2 n, \log n!, n!, \log_8 n, 96, 8n^2, 7n^3, 5n$

$96 < \log_8 n < \log 2n < 5n < n\log_6(n) < n\log_2 n < \log n! < 8n^2$
$< 7n^3 < n! < 8^{2n}$