1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3, \quad b = 2, \quad f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$T(n) = \Theta(n^2)$

2) $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$T(n) = \Theta(n^2 \log n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2, \quad f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

4) $T(n) = 2^n T(n/2) + n^n$

a is not constant

5) $T(n) = 16\, T\left(\frac{n}{4}\right) + n$

$a = 16, \quad b = 4$

$c = \log_4 16 = 2$

$n^c = n^2$

$f(n) < n^c$

$T(n) = \Theta(n^2)$

6) $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2, \quad b = 2$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

7) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Does not apply

non-polynomial difference b/w $f(n)$ & $n^{\log_b a}$

8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a=2,\ b=4,\ f(n)=0.51$

$c = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$T(n) = \Theta(n^{0.51})$

9) $T(n) = 0.5\,T\left(\frac{n}{2}\right) + \frac{1}{n}$

Does not apply $(a<1)$

10) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a=16,\ b=4,\ f(n)=3!$

$c = \log_4 16 = 2$, $n^c = n^2$

$n! > n^2$

$T(n) = \Theta(n!)$

11) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a=4,\ b=2,\ f(n) = \log n$

$c = \log_2 a = 2$, $n^c = n^2$

$\log n < n^2$

$T(n) = \Theta(n^2)$

12) $T(n) = \text{sqrt}(2)\,T\left(\frac{n}{2}\right) + \log n$

$a = \sqrt{2},\ b=2$

$c = \log_2 \sqrt{2} = \frac{1}{2}$, $n^c = n^{\frac{1}{2}}$

$n^{\frac{1}{2}} > \log n$

$T(n) = \Theta(\sqrt{n})$

13) $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a=3,\ b=2,\ f(n)=n$

$c = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$T(n) = \Theta(n^{1.5849})$

14) $T(n) = 3T\left(\frac{n}{3}\right) + \text{sqrt}(n)$

$a=3,\ b=3$

$c = \log_3 3 = 1$, $n^c = n^1 = n$

$\text{sqrt}(n) < n$

$\Theta(n)$

15) $T(n) = 4T\left(\frac{n}{2}\right) + cn$

$a=4,\ b=2,\ f(n)=cn$

$c = \log_2 4 = 2,\ n^c = n^2$

$cn < n^2$

$T(n) = O(n^2)$

16) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$ (3)

$a=3,\ b=4,\ f(n)=n\log n$

$c = \log_4 3 = 0.792$

$n^{0.792} < n\log n$

$T(n) = O(n\log n)$

17) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a=3,\ b=3,\ f(n)=\frac{n}{2}$

$c = \log_3 3 = 1$

$n^c = n$

$T(n) = n\log n$

18) $T(n) = 6T(n/3) + n^2\log n$

$a=6,\ b=3,\ f(n)=n^2\log n$

$c = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$T(n) = O(n^2\log n)$

19) $T(n) = 4T\left(\frac{n}{2}\right) + n\log n$

$a=4,\ b=2,\ f(n)=\frac{n}{\log n}$

$c = 2$

$\frac{n}{\log n} < n^2$

$T(n) = O(n^2)$

20) $T(n) = 64\,T(n/8) - n^2\log n$

Does not apply

(f(n) is not positive)

21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a=7,\ b=3,\ f(n)=n^2$

$c = \log_3 7 = 1.7712,\ n^c = 1.7712$

$n^{1.7712} < n^2$

$T(n) = O(n^2)$

22) $T(n) = T\left(\frac{n}{2}\right) + n\,(2 - \cos n)$

Does not apply.