<u>Assignment – 01</u>
<u>SCS1110 – Discrete Mathematics</u>

1. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
    a. No one is perfect.
    b. Not everyone is perfect.
    c. All your friends are perfect.
    d. At least one of your friends is perfect.
    e. Everyone is your friend and is perfect.
    f. Not everybody is your friend or someone is not perfect.

2. Prove the following statements:
    a. For all integers n, $2n^2 - 1$ is odd (Hint – use the proof by cases method).
    b. For any real number x, $20(x-1) \leq 4x^2 + 5$.
    c. Between every two rational numbers there is an irrational number.
    d. There exists a pair of consecutive integers such that one of these integers is a perfect square and the other is a perfect cube.
    e. There is a positive integer that equals the sum of the positive integers not exceeding it.

3. Let A, B, and C be three sets. Prove the following identities:
    a. $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$
    b. $(B - A) \cup (C - A) = (B \cup C) - A$
    c. $A \cup (B \cup C) = (A \cup B) \cup C$
    d. $(A - B) - C = (A - C) - (B - C)$
    e. $(B - A) \cup (C - A) = (B \cup C) - A$.

4. Let f be a function from A to B. Let S be a subset of B. The inverse image of S is denoted by $f^{-1}(S)$ and is defined as $f^{-1}(S) = \{a \in A \mid f(a) \in S\}$.
   Let S and T be two subsets of B. Show that
    a. $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
    b. $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.