

## 3203 Assignment 2

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1.

a)

$k = 4, n = 7$ . 3\*3 combos

a b c 4 e f g

$k = 4, n = 8$ . 3\*4 combos

a b c 4 e f g h

$(k-1)*(n-k)$  pairs.

b)

1.  $n$  is odd,  $k=(n+1)/2$ .

2.  $n$  is even,  $k=n/2$ , or  $n/2+1$

2.

a)

$n-d$  pairs with hop distance  $d$ .

b)

Average dist = Weight of all paths / # paths

$$\begin{aligned}\text{net weight} &= \sum_{i=1}^{n-1} (\text{path weight}) * (\# \text{paths of length } n-i) \\ &= \sum_{i=1}^{n-1} (n-i)i\end{aligned}$$

$$\begin{aligned}\# \text{ paths} &= \sum_{i=1}^{n-1} (\# \text{paths of length } n-i) \\ &= \sum_{i=1}^{n-1} i\end{aligned}$$

$$\text{Average Dist} = \frac{\sum_{i=1}^{n-1} (n-i)i}{\sum_{i=1}^{n-1} i}$$

c)

Average Dist = net weight / # paths

$$\begin{aligned}\text{net weight} &= \sum_{i=1}^{n-1} (n-i)i \\&= \sum_{i=1}^{n-1} ni + \sum_{i=1}^{n-1} i^2 \\&= n(n(n-1)/2) - n(n-1)(2n-1)/6 \\&= n(n^2-n)/2 - (n^2-n)(2n-1)/6 \\&= (n^3-n^2)/2 - (2n^3-2n^2-n^2+n)/6 \\&= (n^3-n^2)/2 - (2n^3-3n^2+n)/6 \\&= (3n^3-3n^2)/6 - (2n^3-3n^2+n)/6 \\&= (n^3-n)/6 \\&= n(n-1)(n+1)/6\end{aligned}$$

$$\begin{aligned}\# \text{ paths} &= \sum_{i=1}^{n-1} i \\&= 1 + 2 + 3 + \dots + n-3 + n-2 + n-1 \\&= (1+n-1) + (2+n-2) + (3+n-3) + \dots + ((n-1)/2 + n - (n-1)/2) \\&= n + n + n + \dots \\&= n(n-1)/2\end{aligned}$$

$$\begin{aligned}\text{Average Dist} &= (n(n-1)(n+1)/6) / (n(n-1)/2) \\&= 2n(n-1)(n+1)/(6n(n-1)) \\&= (n+1)/3\end{aligned}$$

Therefore,  $= (n+1)/3$

3.

Sending at most L bits in one packet, each packet lasts 2 minutes, sequence number is S bits long.

Maximum Sequence number =  $2^S$

Data Rate R =  $(L * (2^S) * 8) / 120s$

Max in 2 minutes =  $R * 120s = L * (2^S) * 8$

Max in 60 minutes =  $30 * \text{Max in 2 minutes} = 30 * L * (2^S) * 8$

4.

a)

$$\begin{aligned}
 \text{PR(Exactly One Lost)} &= (4 \text{ choose } 1) \text{PR(One Lost)} * \text{PR(4 Not Lost)} \\
 &= 4 * .1 * (1-.1) * (1-.1) * (1-.1) * (1-.1) \\
 &= 4 * .1 * .9 * .9 * .9 * .9 \\
 &= 4 * .06561 \\
 &= .26244
 \end{aligned}$$

b)

$$\begin{aligned}
 \text{PR(1 of 4 lost \&\& Last Arrived)} &= \text{PR}(1 < x \leq 4 \text{ fragments lost}) * (\text{PR}(\text{Last Arrived})) \\
 &= (1 - \text{PR}(4 \text{ Fragments Arrived})) * (1 - \text{PR}(\text{Last Lost})) \\
 &= (1 - .9 * .9 * .9 * .9) * (1 - .1) \\
 &= .3439 * .9 \\
 &= .30951
 \end{aligned}$$

5.

Slot 1	2	4	10	12	13	16	COLLISION			
Slot 2	2	4	COLLISION							
Slot 3	2	4	COLLISION							
Slot 4	2	SUCCESS								
Slot 5	4	SUCCESS								
Slot 6	10	12						13	16	COLLISION
Slot 7	10	12						COLLISION		
Slot 8	10	SUCCESS								
Slot 9	12	SUCCESS								
Slot 10	13	16						COLLISION		
Slot 11	13	SUCCESS								
Slot 12	16	SUCCESS								

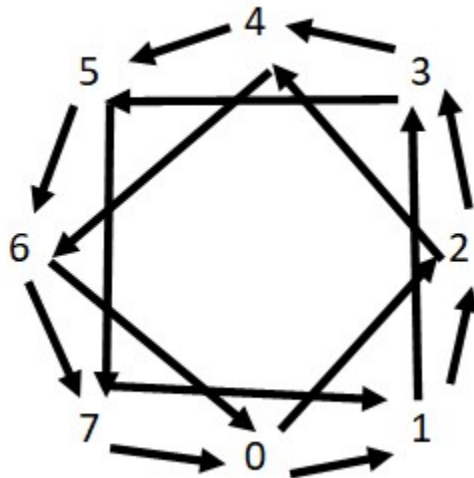
6.

a)

case 1:  $i < j$

# hops =  $j - i$ . Will be clamped between 1 and 6 hops

b)



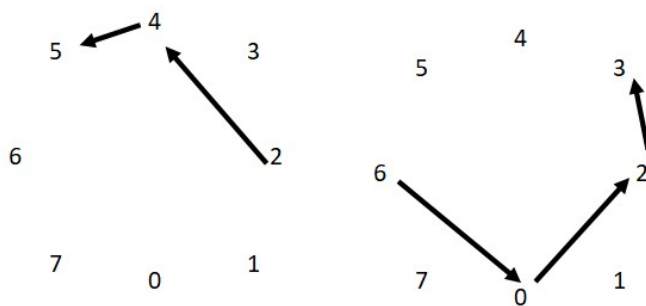
c)

2 to 5 takes 2 hops. One from 2 to 4, then 4 to 5.

6 to 0 takes 3. 6 to 0, 0 to 2, 2 to 3.

2 to 5

6 to 3



7.

$$a = 01110101 = -1+1+1+1-1+1-1+1$$

$$b = 10101001 = +1-1+1-1+1-1-1+1$$

$$c = 11100101 = +1+1+1-1-1+1-1+1$$

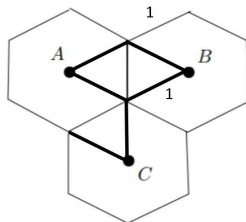
$$\begin{aligned}\{a,b\} &= (-1*1)+(1*-1)+(1*1)+(1*-1)+(-1*1)+(1*-1)+(-1-1)+(1*1) \\ &= -1-1+1-1-1-1+1+1=-5+2=-2\end{aligned}$$

$$\begin{aligned}\{a,c\} &= (-1*1)+(1*1)+(1*1)+(1*-1)+(-1*-1)+(1*1)+(-1-1)+(1*1) \\ &= -1+1+1-1+1+1+1+1=-2+6=4\end{aligned}$$

$$\begin{aligned}\{b,c\} &= (1*1)+(-1*1)+(1*1)+(-1*-1)+(1*-1)+(-1*1)+(-1-1)+(1*1) \\ &= 1-1+1+1-1-1+1+1=-3+5=2\end{aligned}$$

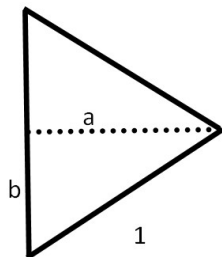
None are orthogonal because their dot products are not zero.

8.



a)

Equilateral Triangles. Height of one triangle =  $\sqrt{3} / 2$ .



$$a = 1 * \sin(60) = \sqrt{3}/2$$

$$b = 1 * \cos(60) = 1/2$$

Therefore dist from A to B =  $2*a = \sqrt{3} = 1.73$

b)

As per diagram of hexes above, dist from C = side length +  $\frac{1}{2}$  side length =  $1 + .5 = 1.5$

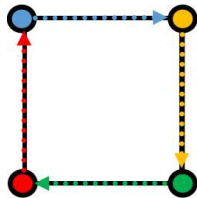
9.

a)

min sensor range for every pair to communicate:  $r = \sqrt{2}$

b)

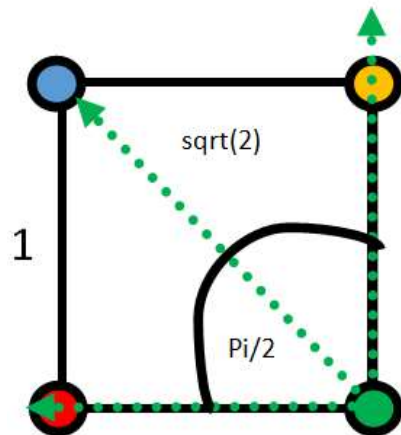
Orient each such that it is looking at it's left neighbor. Networks can communicate cyclically



c)

Angle such that each sensor is pointing directly at the opposite corner, with  $\pi/4$  range to the left and right.

This requires  $r$  to be  $\sqrt{2}$  as in part 1 of this question



d)

No orientation with angle less than  $\pi/2$  will allow a complete network.

Each node must be able to communicate with ALL OTHER nodes, as in the previous section.

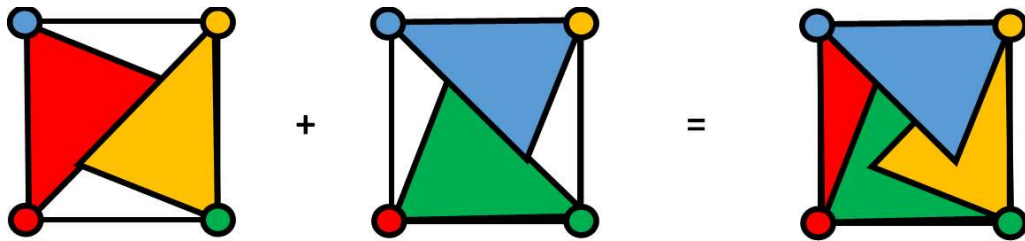
With angle less than  $\pi/2$ , this cannot be attained.

10.

a)

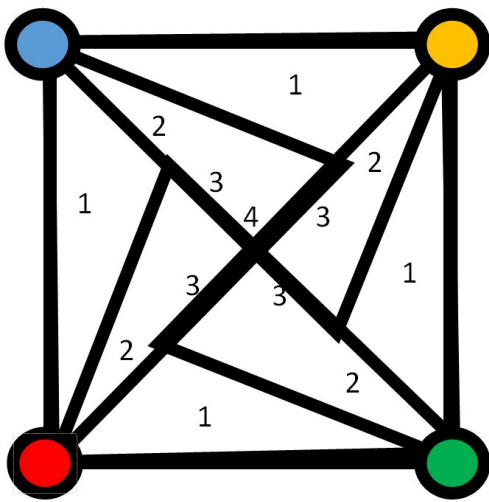
The edge of each antenna's range must be perfectly along exactly one edge of the square and be facing towards the antenna's left neighbor. This results in a bit of overlap between

every antenna and its left and right neighbors, but will ensure complete coverage.



b)

See image. Assuming that even the slightest overlap counts as multiple antennae reaching a location



11.

a)

$$E = c \cdot \pi \cdot r^2$$

Solve for  $r$ .

b)

$$E = c \cdot \pi \cdot r^2 (\phi/360)$$

Solve for  $r$