Carleton University School of Computer Science

COMP 3804 /MATH 3804

Design and Analysis of Algorithms I

Assignment 3

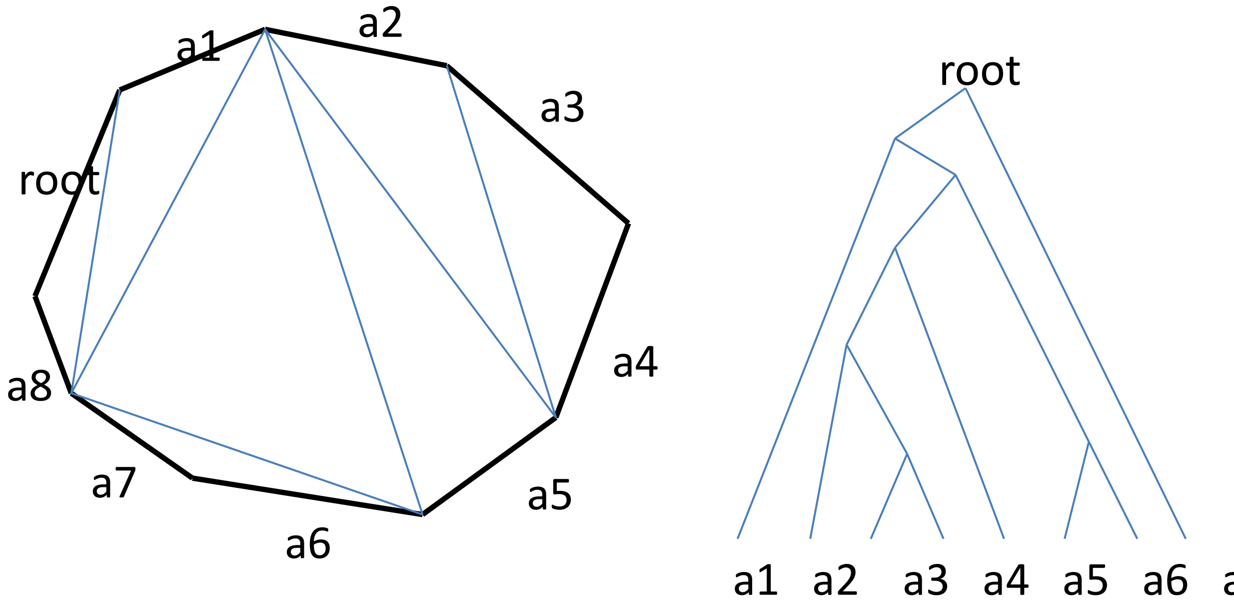
Date Due: Sunday, Dec 1st, 23:59

Your assignment should be submitted online on cuLearn as a single .pdf file. No late assignments will be accepted. You can type your assignment or you can upload a scanned copy of it. Please, use a good image capturing device. Make sure that your upload is clearly readable. If it is difficult to read, it will not be graded!

Question 1: [20] Consider the triangulation problem for convex polygons. We claim that there is a 1-1 correspondence between different triangulations of convex polygons and binary trees. Below is an illustration of such a correspondence.

a. formalize that relationship i.e., give the **precise** construction of a binary tree from a given triangulation. Illustrate your general construction on an example.
b. then give the precise construction how to construct a triangulation from a given binary tree. Illustrate your general construction on an example.
c. Now, state precisely the relationship between the triangulations and binary trees.

d. In the Minimum Weight Triangulation Algorithm, a loop index is called ``gap''. What is the exact meaning of that loop index?



Question 2: [15] Consider cutting a strip of metal of length *n*. You are paid as follows: for a piece of length **i** you obtain a price of *pi* dollars. You wish to maximize your total earnings (see class notes).

a) Example:

length *i*	1	2	3	4	5	6	7	8	9
price *pi*	1	5	8	9	10	17	17	20	24

Describe all ways of cutting a strip of length 6 into pieces and compute the respective earnings.

b) Describe a dynamic programming algorithm for the problem

c) Show how your algorithm works on a strip of length 15 for the above example. Illustrate the execution by filling in a table step by step.

Question 3: [20] A *contiguous subsequence* of a list S is a subsequence made up of consecutive elements of S. For instance, if S is 5, 15, −30, 10, −5, 40, 10, then 15, −30, 10 is a contiguous subsequence but 5, 15, 40 is not. Give a linear-time algorithm for the following task:

Input: A list of numbers, $a_1, a_2, \ldots, a_n$.

Output: The contiguous subsequence of maximum sum (a subsequence of length zero has sum zero).

For the preceding example, the answer would be 10, −5, 40, 10, with a sum of 55.
(*Hint:* For each $j \in \{1, 2, \ldots, n\}$, consider contiguous subsequences ending exactly at position j.)

Question 4: [20] You are going on a long trip. You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1 < a_2 < \cdots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance a_n), which is your destination.

You would ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the *penalty* for that day is $(200 - x)^2$. You want to plan your trip so as to minimize the total penalty—that is, the sum, over all travel days, of the daily penalties.

Give an efficient algorithm that determines the optimal sequence of hotels at which to stop.

Question 5: [15] Consider the following linear program:

- Maximize $5x+3y$
- With $5x - 2y \geq 0$ $\quad$ $x+y \leq 7$ $\quad$ $x \leq 5$ $\quad$ $x \geq 0$ $\quad$ $y \geq 0$

Plot the feasible region and identify the optimal solution.

Question 6: [15] A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, up to the maximum available limits given below.

Material 1 has density 2 tons/cubic meter, maximum available amount 40 cubic meters, and revenue $2,000 per cubic meter.

Material 2 has density 1 ton/cubic meter, maximum available amount 30 cubic meters, and revenue $3,400 per cubic meter.

Material 3 has density 3 tons/cubic meter, maximum available amount 20 cubic meters, and revenue $24,000 per cubic meter.

Set up the LP so that you can enter these data into a linear programming solver. Print the results and include them with your solution (LP set-up).

Question 7: [5] Give an example of a linear program in two variables whose feasible region is infinite, but such that there is an optimum solution of bounded cost.