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Part 1

Could not find sufficient information in slides or online to make progress on Question 3.

Ran out of time to complete Question 8 due to constraints on other assignments.

3009 A1 - Shalin Lathigra

① 1) Translate object $O(O_x, O_y, O_z)$ about point $P(P_x, P_y, P_z)$ by angle α

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{\text{point to origin}} = \begin{bmatrix} -P_x & 0 & 0 \\ 0 & -P_y & 0 \\ 0 & 0 & -P_z \end{bmatrix}$$

$$R_{\text{point origin } \alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$T_{\text{origin to point}} = \begin{bmatrix} P_x & 0 & 0 \\ 0 & P_y & 0 \\ 0 & 0 & P_z \end{bmatrix}$$

$$T_{\text{object}} = \begin{bmatrix} O_x & 0 & 0 \\ 0 & O_y & 0 \\ 0 & 0 & O_z \end{bmatrix}$$

composed in OpenGL order ⇒

$$T_{\text{object}} \cdot T_{\text{origin to point}} \cdot R_{\text{point origin } \alpha} \cdot T_{\text{point to origin}}$$

2)
$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

② $v = (4, 3, 2)$ $u = (5, 1, 7)$

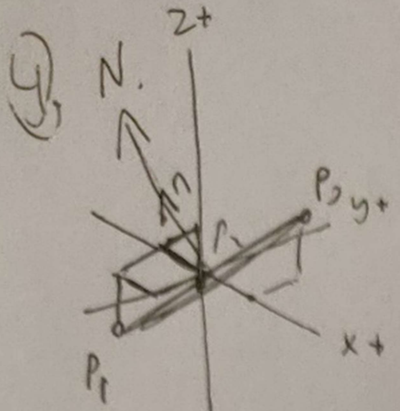
$u \cdot v = |u||v| \cos\theta$

$$\theta = \arccos\left(\frac{u \cdot v}{|v||u|}\right) = \arccos\left(\frac{20+3+14}{|v||u|}\right)$$

$$= 37.499 \text{ deg}$$

Figure 1: Questions 1 and 2

4)

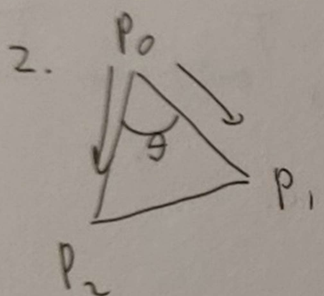


$$N = (P_1 - P_0) \times (P_1 - P_2)$$
$$= (-2,-2,-2) \times (0,-2,0)$$
$$= (-4,0,4)$$

1.
$$n = \frac{N}{|N|} = \frac{(-4,0,4)}{5.6568}$$
$$= (-0.7, 0, 0.7)$$

!

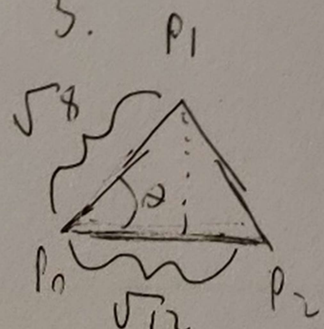
2.



$$\theta = \arccos\left(\frac{(P_1 - P_0) \text{ dot } (P_2 - P_0)}{|P_1 - P_0| \times |P_2 - P_0|}\right)$$
$$= 0.615$$
$$= 61.5°$$

2.

3.

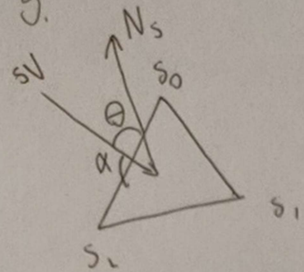


$$A = \frac{1}{2}\left(\sqrt{12} \cdot \sqrt{8} \sin(61.5)\right)$$
$$= 4.305$$

3.

Figure 2: Question 4

5.



$N_s = (s_1 - s_0) \times (s_2 - s_0) = (0,1,5) \times (5,2,10) = (0,-25,3)$

$n_s = \frac{N_s}{|N_s|} = \frac{(0,-25,3)}{|N_s|} = (0,-0.99,0.11)$

$S_V = 0,-12,11$

$\theta = \arccos\left(\frac{n_s \cdot S_V}{|n_s||S_V|}\right) = \arccos\left(\frac{0+11.88+1.21}{1 \cdot 16.278}\right)$

$= 63.60^\circ$

$\alpha = 90^\circ - \theta = 26.40 < 60^\circ$

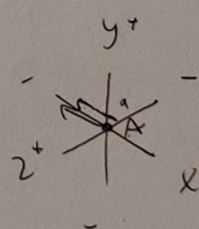
$\therefore$ as $\alpha < 60^\circ$, heroine is safe

Rotate some deg about $(P_1 - P_0)$, where $P_0 = (1,1,1)$, & $P_1 = (2,3,4)$

General Form:

$$T^{-1}(-P_0)\, R_x^{-1}(\alpha)\, R_y^{-1}(\beta)\, R_z(\theta)\, R_y(\beta)\, R_x(\alpha)\, T(-P_0)$$

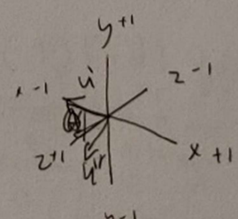
① Translate point $_0$ to origin / $T(-P_0)$



From u to u'.

$$T(-P_0) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② Rotate axis to xz Plane.



From u' to u''

$\alpha = ?$

$u' = P_1 - P_0 = (1,2,3)$

$u'_0 = (0,2,3)$

$\alpha = \arccos\left(\frac{u' \cdot u'_0}{|u'||u'_0|}\right) = 15.5^\circ$

$$R_x\alpha = R_x 15.5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(15.5) & -\sin(15.5) & 0 \\ 0 & \sin(15.5) & \cos(15.5) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

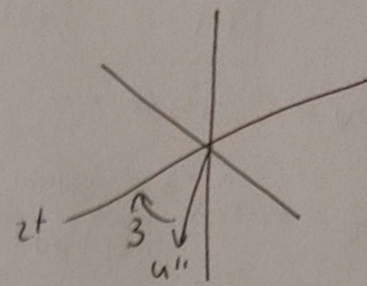
Figure 3: Question 5, Beginning of Question 6.

③ Rotate u'' about y axis onto z axis.

$R_y \beta = ?$

$z = (0,0,1)$

$u''_z = (1,0,3)$



$$\beta = \arccos\left(\frac{z \cdot u''_z}{|z|\,|u''_z|}\right)$$

$$= 43.88^\circ$$

$$R_y\beta = R_y(43.88^\circ) = \begin{bmatrix} \cos(43.88) & 0 & \sin(43.88) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(43.88) & 0 & \cos(43.88) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

④ Rotate about z axis by θ

$$R_z\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Formally:

$$T^{-1}(-P_1)\, R_x^{-1}(\alpha)\, R_y^{-1}(\beta)\, R_z(\theta)\, R_y(\beta)\, R_x(\alpha)\, T(-P_1)$$

$$= T^{-1}(-P_1)\, R_x^{-1}(\alpha)\, R_y^{-1}(\beta) \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(43.88) & 0 & \sin(43.88) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(43.88) & 0 & \cos(43.88) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(15.5) & -\sin(15.5) & 0 \\ 0 & \sin(15.5) & \cos(15.5) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 4: Question 6 conclusion

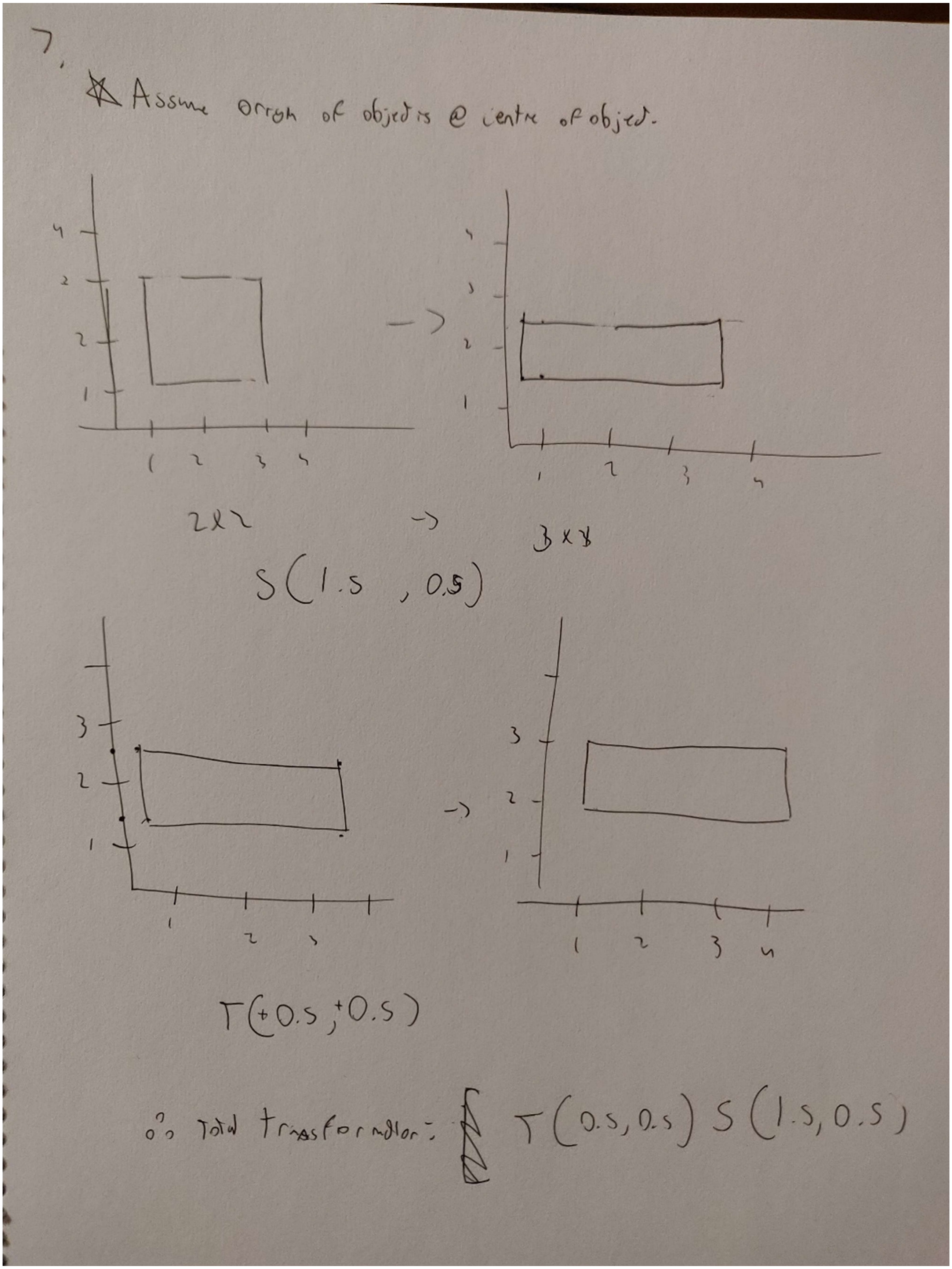


Figure 5: Question 7