

## Statistics Exploration & Reasoning – Assignment 2

Name – Shalini Mishra

Section -2

### Question 1

Let,  $\mu$  be the mean amount of time taken to pay bills

$H_0: \mu \geq 22$

$H_a: \mu < 22$

		Ho True	Ho False
Decision	Reject Ho System is effective	Tell people: system is effective when it is not in reality. CFO is assumed to be right where in reality he's not (Type I Error)	Tell people: CFO is assumed to be right when the system is actually effective (Correct Decision)
	Failed to reject Ho System is not effective	Tell people: System is not effective; CFO's assumption is wrong and in reality also that's the case (Correct Decision)	Tell people: System is not effective when in reality it is effective (Type II Error)

Type 1 error would be a more dangerous in this scenario than type 2 since if CFO is proved right despite the system being unprofitable would cause the firm to incur extra expenses due to SSA envelopes. There will be added cost of these arrangements further hampering their margin. Whereas in case of Type 2, they will lose the opportunity for increasing their margin. Type 1 error is more hurting to the company. Hence, we go for the significance level of 5%.

At 5% significance level, there is not sufficient evidence to support the claim that the proposal of CFO to use SSA (self addressed stamped) envelopes is profitable for the company.

### Question 2

Let,  $p_r$  be the proportion of voters voting for Republicans

$H_0: p_r \leq 0.5$

$H_a: p_r > 0.5$

		Ho True	Ho False
Decision	Reject Ho Republican will win the election	Tell people: Announce that Republicans will win when in reality they will lose (Type I Error)	Tell people: Announce that Republicans will win and they actually win (Correct Decision)
	Failed to reject Ho Republican will lose the election	Tell people: Announce that Republicans will lose when in reality they will (Correct Decision)	Tell people: Announce that Republicans will lose when they will win (Type II Error)

Type 1 error would be a more dangerous in this scenario than type 2 because Type 1 error will hurt their reputation drastically. Viewers will lose their confidence on the television network. Hence, we go for the significance level of 5%.

At 5% significance level, there is sufficient evidence to support the claim that Republicans will win the elections. Based on the estimation, the network should announce at 8:01 p.m. that the Republican candidate will win.

### **Question 3**

**a.**

Let  $\mu$  be the weekly spent average for the town.

$H_0: \mu = 150$

$H_1: \mu \neq 150$

At 5% significance level, p value (say p) = 0.000176

$p < 0.10$  (10% significance)  $\Rightarrow$  Reject the null Hypothesis,  $H_0$

$p < 0.05$  (5% significance)  $\Rightarrow$  Reject the null Hypothesis,  $H_0$

$p < 0.01$  (1% significance)  $\Rightarrow$  Reject the null Hypothesis,  $H_0$

Yes, the sample evidence is statistically significant. We can reject the null hypothesis

From 1% to 10% significance level, there is sufficient evidence to support the claim that the weekly spent average for the town is different than the national average.

**b.**

As it's a two-sided test,

Any value of sample means below 143.7134 and any value above 156.2866 will reject the null hypothesis at **1%** significance level.

Any value of sample means below 146.0257 and any value above 153.9743 will reject the null hypothesis at **10%** significance level

### **Question 4**

Let  $\sigma^2$  be the variance of the daily demand of the product.

$H_0: \sigma^2 = 250$

$H_1: \sigma^2 \neq 250$

p-value = 0.708 > 0.05 (5% significance)  $\Rightarrow$  Failed to reject  $H_0$

The data doesn't provide enough at the 5% significance level to infer that the operations research analyst's assumption about the variance is wrong.

At 5% significance level, there is not sufficient evidence to support the claim that variance is different than 250.

### **Question 5**

**a.**

Let  $p_w$  be the proportion of PBS viewers watching the show.

$H_0: p_w \geq 0.14$

$H_1: p_w < 0.14$

From the alternate hypothesis, we can see it's a left tailed test.

To determine the sample proportion values that will lead to the show's cancellation, assuming a 5% significance level,

We will have to find the critical value

the sample proportion values that will lead to the show's cancellation, assuming a 5% significance level are:

**any sample proportion below 12.5% (Left tailed test)**

**b.**

Our goal is to find the significance level which will give me the same rejection region as above

Using the same p-critical value ie. 12.5%, we will try to obtain the corresponding z-score.

A sample will reject the null hypothesis if the proportion of people watching it will be below 12.5%

With a new population proportion of 13.4%, Substitute in the following equation

$$z = (\hat{p} - p) / \sqrt{p(1-p)/n}$$

(Ans solved in R )

The probability that a sample will lead to rejection of the null hypothesis when 13.4% of all viewing households are watching it is 15.3%

### Question 6

$H_0$ : Declare emergency, Not enough fuel as fuel < 15 mins flight time

$H_a$ : Enough fuel for 15 minutes, fuel >= 15 mins flight time

		Ho True	Ho False
Decision	Reject $H_0$ Enough Fuel to stay afloat	Tell people: Do not declare emergency when in reality there is no fuel (Type I Error)	Tell people: Do not declare emergency and there is fuel available in reality to stay afloat for 15 mins (Correct Decision)
	Failed to reject $H_0$ No Fuel, declare Emergency	Tell people: Declare emergency and in reality, there is no fuel (Correct Decision)	Tell people: Declare Emergency when there is enough fuel to stay aloft for 15 minutes (Type II Error)

Consequences of each of these error types:

**Type 1:** Do not declare an emergency when there is no fuel

Loss of lives

Human lives and property at danger

Drastic result is crashing of the aircraft which has irreversible consequences

**Type 2:** Declare an emergency and land immediately when there is enough fuel

FAA investigation will be launched

Other flights might be endangered

Will create commotion

**Type 1** error is more dangerous in this scenario than **Type 2**.

### Question 7

a.

$H_0$ : the accused is innocent | Acquitted by the jury

$H_a$ : the accused is guilty | Convicted by the jury

According to the question,

Evidence in the trial of an innocent suspect is enough to convince 42% of all jurors in the population that the suspect is guilty.

Each of these 12 jurors are picked from the population.

Probability that a juror is voting for an innocent suspect as guilty =  $p = 0.42$

As they are voting unanimously, I am considering each activity of voting as an event

Probability that a jury convicts an innocent suspect

$$= p \times p \times p \times \dots \times p \text{ (12 times)} = p^{12} = (0.42)^{12} \sim 0.00003$$

**Also solved in R, since it's a binomial distribution**

b. Type 1 error was committed by the Jury in part (a)

		Ho True	Ho False
--	--	---------	----------

Decision	Reject $H_0$ Accused is convicted	Tell people: Accused is convicted when in reality he's innocent (Type I Error)	Tell people: Accused is convicted and he's guilty (Correct Decision)
	Failed to reject $H_0$ Accused is acquitted	Tell people: Accused is acquitted and he's innocent (Correct Decision)	Tell people: Accused is acquitted when he's guilty (Type II Error)

c.

Evidence in the trial of a guilty suspect is enough to convince 99% of all jurors in the population that the suspect is guilty.

Probability of a juror voting that the suspect is guilty = 0.99

Probability of the jury voting that the suspect is guilty (conviction) =  $0.99 \times 0.99 \times \dots 12 \text{ times}$   
 $= 0.99^{12} = 0.88638487 \sim 0.886$

Probability of the jury voting that the suspect is not guilty (acquitted) =  $1 - \text{Prob}(\text{conviction by jury})$   
 $= 1 - 0.886 = 0.114$

d. From the error analysis table drawn in part(b)

We can conclude that the case in part (c) is a type 2 error.