

代码库

上海交通大学

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1 数论

1.1 快速求逆元

返回结果:

$$x^{-1}(\text{mod})$$

使用条件: $x \in [0, \text{mod})$ 并且 x 与 mod 互质

```
1 LL inv(LL a, LL p){
2     LL d, x, y;
3     d = exgcd(a, p, x, y);
4     return d == 1 ? (x + p) % p : -1;
5 }
```

1.2 扩展欧几里德算法

返回结果:

$$ax + by = \text{gcd}(a, b)$$

时间复杂度: $\mathcal{O}(n \log n)$

```
1 LL exgcd(LL a, LL b, LL &x, LL &y){
2     if(!b){
3         x = 1; y = 0; return a;
4     } else {
5         LL d = exgcd(b, a % b, x, y);
6         LL t = x; x = y; y = t - a / b * y;
7         return d;
8     }
9 }
```

1.3 中国剩余定理

返回结果:

$$x \equiv r_i(\text{mod } p_i) \quad (0 \leq i < n)$$

使用条件: p_i 需两两互质

```
1 LL china(int n, int *a, int *m){
2     LL M = 1, d, x = 0, y;
3     for(int i = 0; i < n; i++)
4         M *= m[i];
```

```

5         for(int i=0;i<n;i++){
6             LL w=M/m[i];
7             d=exgcd(m[i],w,d,y);
8             y=(y%M+M)%M;
9             x=(x+y*w%M*a[i])%M;
10        }
11        while(x<0)x+=M;
12        return x;
13    }

```

1.4 中国剩余定理 2

```

1  //merge Ax=B and ax=b to A'x=B'
2  void merge(LL &A,LL &B,LL a,LL b){
3      LL x,y;
4      sol(A,-a,b-B,x,y);
5      A=lcm(A,a);
6      B=(a*y+b)%A;
7      B=(B+A)%A;
8  }

```

1.5 组合数取模

```

1  LL prod=1,P;
2  pair<LL,LL> comput(LL n,LL p,LL k){
3      if(n<=1)return make_pair(0,1);
4      LL ans=1,cnt=0;
5      ans=pow(prod,n/P,P);
6      cnt=n/p;
7      pair<LL,LL>res=comput(n/p,p,k);
8      cnt+=res.first;
9      ans=ans*res.second%P;
10     for(int i=n-n%P+1;i<=n;i++)if(i%p){
11
12         ans=ans*i%P;
13     }
14     return make_pair(cnt,ans);
15 }
16 pair<LL,LL> calc(LL n,LL p,LL k){
17     prod=1;P=pow(p,k,1e18);
18     for(int i=1;i<P;i++)if(i%p)prod=prod*i%P;

```

```

19     pair<LL,LL> res=comput(n,p,k);
20     // res.second=res.second*pow(p,res.first%k,P)%P;
21     // res.first-=res.first%k;
22     return res;
23 }
24 LL calc(LL n,LL m,LL p,LL k){
25     pair<LL,LL>A,B,C;
26     LL P=pow(p,k,1e18);
27     A=calc(n,p,k);
28     B=calc(m,p,k);
29     C=calc(n-m,p,k);
30     LL ans=1;
31     ans=pow(p,A.first-B.first-C.first,P);
32     ans=ans*A.second%P*inv(B.second,P)%P*inv(C.second,P)%P;
33     return ans;
34 }

```

1.6 扩展小步大步

```

1 LL solve2(LL a,LL b,LL p){
2     //a^x=b (mod p)
3     b%=p;
4     LL e=1%p;
5     for(int i=0;i<100;i++){
6         if(e==b)return i;
7         e=e*a%p;
8     }
9     int r=0;
10    while(gcd(a,p)!=1){
11        LL d=gcd(a,p);
12        if(b%d)return -1;
13        p/=d;b/=d;b=b*inv(a/d,p);
14        r++;
15    }LL res=BSGS(a,b,p);
16    if(res==-1)return -1;
17    return res+r;
18 }

```

1.7 卢卡斯定理

```

1 LL Lucas(LL n,LL m,LL p){
2     LL ans=1;
3     while(n&& m){
4         LL a=n%p,b=m%p;
5         if(a<b) return 0;
6         ans=(ans*C(a,b,p))%p;
7         n/=p;m/=p;
8     }return ans%p;
9 }

```

1.8 小步大步

返回结果:

$$a^x = b \pmod{p}$$

使用条件: p 为质数

时间复杂度: $\mathcal{O}(\sqrt{n})$

```

1 LL BSGS(LL a,LL b,LL p){
2     LL m=sqrt(p)+.5,v=inv(pw(a,m,p),p),e=1;
3     map<LL,LL>hash;hash[1]=0;
4     for(int i=1;i<m;i++)
5         e=e*a%p,hash[e]=i;
6     for(int i=0;i<=m;i++){
7         if(hash.count(b)) return i*m+hash[b];
8         b=b*v%p;
9     }return -1;
10 }

```

1.9 Miller Rabin 素数测试

```

1 const int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
2 bool check(long long n,int base) {
3     long long n2=n-1,res;
4     int s=0;
5     while(n2%2==0) n2>>=1,s++;
6     res=pw(base,n2,n);
7     if((res==1)|| (res==n-1)) return 1;
8     while(s--){
9         res=mul(res,res,n);
10        if(res==n-1) return 1;
11    }

```



```

12     return 0; // n is not a strong pseudo prime
13 }
14 bool isprime(const long long &n) {
15     if(n==2)
16         return true;
17     if(n<2 || n%2==0)
18         return false;
19     for(int i=0;i<12&&BASE[i]<n;i++){
20         if(!check(n,BASE[i]))
21             return false;
22     }
23     return true;
24 }

```

1.10 Pollard Rho 大数分解

时间复杂度: $\mathcal{O}(n^{1/4})$

```

1 LL prho(LL n,LL c){
2     LL i=1,k=2,x=rand()%(n-1)+1,y=x;
3     while(1){
4         i++;x=(x*x%n+c)%n;
5         LL d=__gcd((y-x+n)%n,n);
6         if(d>1&&d<n)return d;
7         if(y==x)return n;
8         if(i==k)y=x,k<<=1;
9     }
10 }
11 void factor(LL n,vector<LL>&fat){
12     if(n==1)return;
13     if(isprime(n)){
14         fat.push_back(n);
15         return;
16     }LL p=n;
17     while(p>=n)p=prho(p,rand()%(n-1)+1);
18     factor(p,fat);
19     factor(n/p,fat);
20 }

```

1.11 快速数论变换 (zky)

返回结果:

$$c_i = \sum_{0 \leq j \leq i} a_j \cdot b_{i-j}(\text{mod}) \quad (0 \leq i < n)$$

使用说明: *magic* 是 *mod* 的原根

时间复杂度: $\mathcal{O}(n \log n)$

```
1  /*
2  {(mod,G)}={(81788929,7),(101711873,3),(167772161,3)
3              ,(377487361,7),(998244353,3),(1224736769,3)
4              ,(1300234241,3),(1484783617,5)}
5  */
6  int mo=998244353,G=3;
7  void NTT(int a[],int n,int f){
8      for(register int i=0;i<n;i++){
9          if(i<rev[i])
10             swap(a[i],a[rev[i]]);
11      for (register int i=2;i<=n;i<=1){
12          static int exp[maxn];
13          exp[0]=1;exp[1]=pw(G,(mo-1)/i);
14          if(f==-1)exp[1]=pw(exp[1],mo-2);
15          for(register int k=2;k<(i>>1);k++){
16              exp[k]=1LL*exp[k-1]*exp[1]%mo;
17          for(register int j=0;j<n;j+=i){
18              for(register int k=0;k<(i>>1);k++){
19                  register int &pA=a[j+k],&pB=a[j+k+(i>>1)];
20                  register int A=pA,B=1LL*pB*exp[k]%mo;
21                  pA=(A+B)%mo;
22                  pB=(A-B+mo)%mo;
23              }
24          }
25      }
26      if(f==-1){
27          int rv=pw(n,mo-2)%mo;
28          for(int i=0;i<n;i++){
29              a[i]=1LL*a[i]*rv%mo;
30          }
31      }
32  void mul(int m,int a[],int b[],int c[]){
33      int n=1,len=0;
34      while(n<m)n<=1,len++;
35      for (int i=1;i<n;i++){
```

```

36         rev[i]=(rev[i>>1]>>1)|((i&1)<<(len-1));
37     NTT(a,n,1);
38     NTT(b,n,1);
39     for(int i=0;i<n;i++)
40         c[i]=1LL*a[i]*b[i]%mo;
41     NTT(c,n,-1);
42 }

```

1.12 快速数论变换 (lyx)

```

1  int Pow(int x,int y,int z){
2      if (y==0) return 1;
3      LL ret=Pow(x,y>>1,z); (ret*=ret)%=z;
4      if (y & 1) (ret*=x)%=z;
5      return ret;
6  }
7
8
9  void Prep(){
10     for (len=1, ci=0; len<=N+N; len<=<1, ci++);
11     Wi[0]=1, Wi[1]=Pow(G,(Mo-1)/len,Mo);
12     for (int i=2; i<=len; i++) Wi[i]=(Wi[i-1]*Wi[1])% Mo;
13     for (int i=0; i<len; i++){
14         int tmp=0;
15         for (int j=i, c=0; c<ci; c++, j>>=1 ) tmp=(tmp <= 1)|= (j & 1);
16         Bel[i]=tmp;
17     }
18 }
19
20 void Dft(Arr &a,int sig)
21 {
22     for (int i=0; i<len; i++) tp[Bel[i]]=a[i];
23     for (int m=1; m<=len; m<=<1){
24         int half=m>>1, bei=len/m;
25         for (int i=0; i<half; i++){
26             LL wi=(sig>0)?Wi[i*bei]:Wi[len-i*bei];
27             for (int j=i; j<len; j+=m){
28                 int u=tp[j],v=wi*LL(tp[j+half]) % Mo;
29                 tp[j]=(u+v) % Mo; tp[j+half]=(u-v+Mo)% Mo;
30             }
31         }
32     }

```

```

33         for (int i=0; i<len; i++) a[i]=tp[i];
34     }
35
36     void Mul(Arr &x,Arr &y,Arr &c,bool same)
37     {
38         if (!same){
39             for(int i=0; i<len; i++) a[i]=x[i], b[i]=y[i];
40             Dft(a,1),Dft(b,1);
41             for(int i=0; i<len; i++) a[i]=a[i]*111*b[i] % Mo;
42             Dft(a,-1);
43             for(int i=0; i<=M; i++) c[i]=a[i]*111*Rev % Mo;
44         } else
45         {
46             for(int i=0; i<len; i++) a[i]=x[i];
47             Dft(a,1);
48             for(int i=0; i<len; i++) a[i]=a[i]*111*a[i] % Mo;
49             Dft(a,-1);
50             for(int i=0; i<=M; i++) c[i]=a[i]*111*Rev % Mo;
51         }
52     }
53
54     Prep();
55     Ans[0]=1; Rev=Pow(len,Mo-2,Mo);
56     for(; K; K>>=1){
57         if (K & 1) Mul(Ans,F,Ans,0);
58         if (K > 1) Mul(F,F,F,1);
59     }
60
61     printf("%d\n",Ans[M]);

```

1.13 原根

```

1     vector<LL>fct;
2     bool check(LL x,LL g){
3         for(int i=0;i<fct.size();i++)
4             if(pw(g,(x-1)/fct[i],x)==1)
5                 return 0;
6         return 1;
7     }
8     LL findrt(LL x){
9         LL tmp=x-1;
10        for(int i=2;i*i<=tmp;i++){

```

```

11         if(tmp%i==0){
12             fct.push_back(i);
13             while(tmp%i==0)tmp/=i;
14         }
15     }if(tmp>1)fct.push_back(tmp);
16     // x is 1,2,4,p^n,2p^n
17     // x has phi(phi(x)) primitive roots
18     for(int i=2;i<int(1e9);i++)if(check(x,i))
19         return i;
20     return -1;
21 }
22 const int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
23 bool check(long long n,int base) {
24     long long n2=n-1,res;
25     int s=0;
26     while(n2%2==0) n2>>=1,s++;
27     res=pw(base,n2,n);
28     if((res==1)|| (res==n-1)) return 1;
29     while(s--){
30         res=mul(res,res,n);
31         if(res==n-1) return 1;
32     }
33     return 0; // n is not a strong pseudo prime
34 }
35 bool isprime(const long long &n) {
36     if(n==2)
37         return true;
38     if(n<2 || n%2==0)
39         return false;
40     for(int i=0;i<12&&BASE[i]<n;i++){
41         if(!check(n,BASE[i]))
42             return false;
43     }
44     return true;
45 }

```

1.14 线性递推

```

1 //已知  $a_0, a_1, \dots, a_{m-1} \setminus \setminus$ 
2  $a_n = c_0 * a_{n-m} + \dots + c_{m-1} * a_{n-1} \setminus \setminus$ 
3 求  $a_n = v_0 * a_0 + v_1 * a_1 + \dots + v_{m-1} * a_{m-1} \setminus \setminus$ 
4

```

```

5 void linear_recurrence(long long n, int m, int a[], int c[], int p) {
6     long long v[M] = {1 % p}, u[M << 1], msk = !!n;
7     for(long long i(n); i > 1; i >>= 1) {
8         msk <<= 1;
9     }
10    for(long long x(0); msk; msk >>= 1, x <<= 1) {
11        fill_n(u, m << 1, 0);
12        int b(!!(n & msk));
13        x |= b;
14        if(x < m) {
15            u[x] = 1 % p;
16        }else {
17            for(int i(0); i < m; i++) {
18                for(int j(0), t(i + b); j < m; j++, t++) {
19                    u[t] = (u[t] + v[i] * v[j]) % p;
20                }
21            }
22            for(int i((m << 1) - 1); i >= m; i--) {
23                for(int j(0), t(i - m); j < m; j++, t++) {
24                    u[t] = (u[t] + c[j] * u[i]) % p;
25                }
26            }
27        }
28        copy(u, u + m, v);
29    }
30    //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
31    for(int i(m); i < 2 * m; i++) {
32        a[i] = 0;
33        for(int j(0); j < m; j++) {
34            a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
35        }
36    }
37    for(int j(0); j < m; j++) {
38        b[j] = 0;
39        for(int i(0); i < m; i++) {
40            b[j] = (b[j] + v[i] * a[i + j]) % p;
41        }
42    }
43    for(int j(0); j < m; j++) {
44        a[j] = b[j];
45    }
46 }

```

1.15 线性筛

```
1 void get_p(){
2     phi[1]=mu[1]=1;
3     for(int i=2;i<=n;i++){
4         if(!p[i]){
5             prime[++prime[0]]=i;
6             phi[i]=i-1;mu[i]=-1;
7             for(int j=1;j<=prime[0];j++){
8                 if(i*prime[j]>n)break;
9                 p[i*prime[j]]=1;
10                if(i%prime[j]==0){
11                    mu[i*prime[j]]=0;
12                    phi[i*prime[j]]=phi[i]*prime[j];
13                    break;
14                }else{
15                    phi[i*prime[j]]=phi[i]*(prime[j]-1);
16                    mu[i*prime[j]]=-mu[i];
17                }
18            }
19        }
20    }
```

1.16 直线下整点个数

返回结果:

$$\sum_{0 \leq i < n} \lfloor \frac{a+b \cdot i}{m} \rfloor$$

使用条件: $n, m > 0, a, b \geq 0$

时间复杂度: $\mathcal{O}(n \log n)$

```
1 //calc \sum_{i=0}^{n-1} [(a+bi)/m]
2 // n,a,b,m >0
3 LL solve(LL n,LL a,LL b,LL m){
4     if(b==0)
5         return n*(a/m);
6     if(a>=m || b>=m)
7         return n*(a/m)+(n-1)*n/2*(b/m)+solve(n,a%m,b%m,m);
8     return solve((a+b*n)/m,(a+b*n)%m,m,b);
9 }
```

2 数值

2.1 高斯消元

```
1 void Gauss(){
2     int r,k;
3     for(int i=0;i<n;i++){
4         r=i;
5         for(int j=i+1;j<n;j++)
6             if(fabs(A[j][i])>fabs(A[r][i]))r=j;
7         if(r!=i)for(int j=0;j<=n;j++)swap(A[i][j],A[r][j]);
8         for(int k=i+1;k<n;k++){
9             double f=A[k][i]/A[i][i];
10            for(int j=i;j<=n;j++)A[k][j]-=f*A[i][j];
11        }
12    }
13    for(int i=n-1;i>=0;i--){
14        for(int j=i+1;j<n;j++)
15            A[i][n]-=A[j][n]*A[i][j];
16        A[i][n]/=A[i][i];
17    }
18    for(int i=0;i<n-1;i++)
19        cout<<fixed<<setprecision(3)<<A[i][n]<<" ";
20    cout<<fixed<<setprecision(3)<<A[n-1][n];
21 }
22 bool Gauss(){
23     for(int i=1;i<=n;i++){
24         int r=0;
25         for(int j=i;j<=m;j++)
26             if(a[j][i]){r=j;break;}
27         if(!r)return 0;
28         ans=max(ans,r);
29         swap(a[i],a[r]);
30         for(int j=i+1;j<=m;j++)
31             if(a[j][i])a[j]^=a[i];
32     }for(int i=n;i>=1;i--){
33         for(int j=i+1;j<=n;j++)if(a[i][j])
34             a[i][n+1]=a[i][n+1]^a[j][n+1];
35     }return 1;
36 }
37 LL Gauss(){
38     for(int i=0;i<n;i++)for(int j=0;j<n;j++)A[i][j]%=m;
```



```

39     for(int i=0;i<n;i++)for(int j=0;j<n;j++)A[i][j]=(A[i][j]+m)%m;
40     LL ans=n%2?-1:1;
41     for(int i=0;i<n;i++){
42         for(int j=i+1;j<n;j++){
43             while(A[j][i]){
44                 LL t=A[i][i]/A[j][i];
45                 for(int k=0;k<n;k++)
46                     A[i][k]=(A[i][k]-A[j][k]*t%m+m)%m;
47                 swap(A[i],A[j]);
48                 ans=-ans;
49             }
50             ans=ans*A[i][i]%m;
51         }return (ans%m+m)%m;
52     }
53     int Gauss(){//求秩
54         int r,now=-1;
55         int ans=0;
56         for(int i = 0; i < n; i++){
57             r = now + 1;
58             for(int j = now + 1; j < m; j++)
59                 if(fabs(A[j][i]) > fabs(A[r][i]))
60                     r = j;
61             if (!sgn(A[r][i])) continue;
62             ans++;
63             now++;
64             if(r != now)
65                 for(int j = 0; j < n; j++)
66                     swap(A[r][j], A[now][j]);
67
68             for(int k = now + 1; k < m; k++){
69                 double t = A[k][i] / A[now][i];
70                 for(int j = 0; j < n; j++){
71                     A[k][j] -= t * A[now][j];
72                 }
73             }
74         }
75         return ans;
76     }

```

2.2 快速傅立叶变换

返回结果:

$$c_i = \sum_{0 \leq j \leq i} a_j \cdot b_{i-j} \quad (0 \leq i < n)$$

时间复杂度: $\mathcal{O}(n \log n)$

```
1  typedef complex<double> cp;
2  const double pi = acos(-1);
3  void FFT(vector<cp>&num, int len, int ty){
4      for(int i=1, j=0; i<len-1; i++){
5          for(int k=len; j^=k>>=1, ~j&k);
6          if(i<j)
7              swap(num[i], num[j]);
8      }
9      for(int h=0; (1<<h)<len; h++){
10         int step=1<<h, step2=step<<1;
11         cp w0(cos(2.0*pi/step2), ty*sin(2.0*pi/step2));
12         for(int i=0; i<len; i+=step2){
13             cp w(1, 0);
14             for(int j=0; j<step; j++){
15                 cp &x=num[i+j+step];
16                 cp &y=num[i+j];
17                 cp d=w*x;
18                 x=y-d;
19                 y=y+d;
20                 w=w*w0;
21             }
22         }
23     }
24     if(ty== -1)
25         for(int i=0; i<len; i++)
26             num[i]=cp(num[i].real()/(double)len, num[i].imag());
27 }
28 vector<cp> mul(vector<cp>a, vector<cp>b){
29     int len=a.size()+b.size();
30     while((len&-len)!=len)len++;
31     while(a.size()<len)a.push_back(cp(0, 0));
32     while(b.size()<len)b.push_back(cp(0, 0));
33     FFT(a, len, 1);
34     FFT(b, len, 1);
35     vector<cp>ans(len);
36     for(int i=0; i<len; i++)
```

```

37     ans[i]=a[i]*b[i];
38     FFT(ans,len,-1);
39     return ans;
40 }

```

2.3 单纯形法求解线性规划

返回结果:

$$\max\{c_{1 \times m} \cdot x_{m \times 1} \mid x_{m \times 1} \geq 0_{m \times 1}, a_{n \times m} \cdot x_{m \times 1} \leq b_{n \times 1}\}$$

```

1  namespace LP{
2      const int maxn=233;
3      double a[maxn][maxn];
4      int Ans[maxn],pt[maxn];
5      int n,m;
6      void pivot(int l,int i){
7          double t;
8          swap(Ans[l+n],Ans[i]);
9          t=-a[l][i];
10         a[l][i]=-1;
11         for(int j=0;j<=n;j++)a[l][j]/=t;
12         for(int j=0;j<=m;j++){
13             if(a[j][i]&&j!=l){
14                 t=a[j][i];
15                 a[j][i]=0;
16                 for(int k=0;k<=n;k++)a[j][k]+=t*a[l][k];
17             }
18         }
19     }
20     vector<double> solve(vector<vector<double> >A,vector<double>B,vector<double>C){
21         n=C.size();
22         m=B.size();
23         for(int i=0;i<C.size();i++)
24             a[0][i+1]=C[i];
25         for(int i=0;i<B.size();i++)
26             a[i+1][0]=B[i];
27
28         for(int i=0;i<m;i++)
29             for(int j=0;j<n;j++)
30                 a[i+1][j+1]=-A[i][j];
31
32         for(int i=1;i<=n;i++)Ans[i]=i;
33     }

```

```

34     double t;
35     for(;;){
36         int l=0;t=-eps;
37         for(int j=1;j<=m;j++)if(a[j][0]<t)t=a[l=j][0];
38         if(!l)break;
39         int i=0;
40         for(int j=1;j<=n;j++)if(a[l][j]>eps){i=j;break;}
41         if(!i){
42             puts("Infeasible");
43             return vector<double>();
44         }
45         pivot(l,i);
46     }
47     for(;;){
48         int i=0;t=eps;
49         for(int j=1;j<=n;j++)if(a[0][j]>t)t=a[0][i=j];
50         if(!i)break;
51         int l=0;
52         t=1e30;
53         for(int j=1;j<=m;j++)if(a[j][i]<-eps){
54             double tmp;
55             tmp=-a[j][0]/a[j][i];
56             if(t>tmp)t=tmp,l=j;
57         }
58         if(!l){
59             puts("Unbounded");
60             return vector<double>();
61         }
62         pivot(l,i);
63     }
64     vector<double>x;
65     for(int i=n+1;i<=n+m;i++)pt[Ans[i]]=i-n;
66     for(int i=1;i<=n;i++)x.push_back(pt[i]?a[pt[i]][0]:0);
67     return x;
68 }
69 }

```

2.4 自适应辛普森

```

1 double area(const double &left, const double &right) {
2     double mid = (left + right) / 2;
3     return (right - left) * (calc(left) + 4 * calc(mid) + calc(right)) / 6;

```

```

4  }
5
6  double simpson(const double &left, const double &right,
7               const double &eps, const double &area_sum) {
8      double mid = (left + right) / 2;
9      double area_left = area(left, mid);
10     double area_right = area(mid, right);
11     double area_total = area_left + area_right;
12     if (std::abs(area_total - area_sum) < 15 * eps) {
13         return area_total + (area_total - area_sum) / 15;
14     }
15     return simpson(left, mid, eps / 2, area_left)
16         + simpson(mid, right, eps / 2, area_right);
17 }
18
19 double simpson(const double &left, const double &right, const double &eps) {
20     return simpson(left, right, eps, area(left, right));
21 }

```

2.5 多项式求根

```

1  const double eps=1e-12;
2  double a[10][10];
3  typedef vector<double> vd;
4  int sgn(double x) { return x < -eps ? -1 : x > eps; }
5  double mypow(double x,int num){
6      double ans=1.0;
7      for(int i=1;i<=num;++i)ans*=x;
8      return ans;
9  }
10 double f(int n,double x){
11     double ans=0;
12     for(int i=n;i>=0;--i)ans+=a[n][i]*mypow(x,i);
13     return ans;
14 }
15 double getRoot(int n,double l,double r){
16     if(sgn(f(n,l))==0)return l;
17     if(sgn(f(n,r))==0)return r;
18     double temp;
19     if(sgn(f(n,l))>0)temp=-1;else temp=1;
20     double m;
21     for(int i=1;i<=10000;++i){

```

```

22         m=(l+r)/2;
23         double mid=f(n,m);
24         if(sgn(mid)==0){
25             return m;
26         }
27         if(mid*temp<0)l=m;else r=m;
28     }
29     return (l+r)/2;
30 }
31 vd did(int n){
32     vd ret;
33     if(n==1){
34         ret.push_back(-1e10);
35         ret.push_back(-a[n][0]/a[n][1]);
36         ret.push_back(1e10);
37         return ret;
38     }
39     vd mid=did(n-1);
40     ret.push_back(-1e10);
41     for(int i=0;i+1<mid.size();++i){
42         int t1=sgn(f(n,mid[i])),t2=sgn(f(n,mid[i+1]));
43         if(t1*t2>0)continue;
44         ret.push_back(getRoot(n,mid[i],mid[i+1]));
45     }
46     ret.push_back(1e10);
47     return ret;
48 }
49 int main(){
50     int n; scanf("%d",&n);
51     for(int i=n;i>=0;--i){
52         scanf("%lf",&a[n][i]);
53     }
54     for(int i=n-1;i>=0;--i)
55         for(int j=0;j<=i;++j)a[i][j]=a[i+1][j+1]*(j+1);
56     vd ans=did(n);
57     sort(ans.begin(),ans.end());
58     for(int i=1;i+1<ans.size();++i)printf("%.10f\n",ans[i]);
59     return 0;
60 }

```

3 数据结构

3.1 平衡的二叉查找树

3.1.1 Treap

```
1  const int maxn=1e5+5;
2  #define sz(x) (x?x->siz:0)
3  struct Treap{
4      struct node{
5          int key,val;
6          int siz,s;
7          node *c[2];
8          node(int v=0){
9              val=v;
10             key=rand();
11             siz=1,s=1;
12             c[0]=c[1]=0;
13         }
14         void rz(){siz=s;if(c[0])siz+=c[0]->siz;if(c[1])siz+=c[1]->siz;}
15     }pool[maxn],*cur,*root;
16     Treap(){cur=pool;}
17     node* newnode(int val){return *cur=node(val),cur++;}
18     void rot(node *&t,int d){
19         if(!t->c[d])t=t->c[!d];
20         else{
21             node *p=t->c[d];t->c[d]=p->c[!d];
22             p->c[!d]=t;t->rz();p->rz();t=p;
23         }
24     }
25     void insert(node *&t,int x){
26         if(!t){t=newnode(x);return;}
27         if(t->val==x){t->s++;t->siz++;return;}
28         insert(t->c[x>t->val],x);
29         if(t->key<t->c[x>t->val]->key)
30             rot(t,x>t->val);
31         else t->rz();
32     }
33     int pre(node *t,int x){
34         if(!t)return INT_MIN;
35         int ans=pre(t->c[x>t->val],x);
36         if(t->val<x)ans=max(ans,t->val);
37         return ans;
```

```

38     }
39     int nxt(node *t, int x){
40         if(!t) return INT_MAX;
41         int ans=nxt(t->c[x>=t->val],x);
42         if(t->val>x) ans=min(ans,t->val);
43         return ans;
44     }
45     int rank(node *t, int x){
46         if(!t) return 0;
47         if(t->val==x) return sz(t->c[0]);
48         if(t->val<x) return sz(t->c[0])+t->s+rank(t->c[1],x);
49         if(t->val>x) return rank(t->c[0],x);
50     }
51     int kth(node *t, int x){
52         if(sz(t->c[0])>=x) return kth(t->c[0],x);
53         if(sz(t->c[0])+t->s>=x) return t->val;
54         return kth(t->c[1],x-t->s-sz(t->c[0]));
55     }
56     void deb(node *t){
57         if(!t) return;
58         deb(t->c[0]);
59         printf("%d ", t->val);
60         deb(t->c[1]);
61     }
62 }T;

```

3.1.2 Splay

```

1 void Rotate(int x, int c){
2     int y = T[x].c[c];
3     int z = T[y].c[1 - c];
4
5     if (T[x].fa){
6         if (T[T[x].fa].c[0] == x) T[T[x].fa].c[0] = y;
7         else T[T[x].fa].c[1] = y;
8     }
9
10    T[z].fa = x; T[x].c[c] = z;
11    T[y].fa = T[x].fa; T[x].fa = y; T[y].c[1 - c] = x;
12
13    Update(x);
14    Update(y);

```



```

15 }
16
17 int stack[M], fx[M];
18
19 void Splay(int x, int fa){
20     int top = 0;
21     for (int u = x; u != fa; u = T[u].fa)
22         stack[++top] = u;
23     for (int i = 2; i <= top; i++)
24         if (T[stack[i]].c[0] == stack[i - 1]) fx[i] = 0;
25         else fx[i] = 1;
26
27     for (int i = 2; i <= top; i += 2){
28         if (i == top) Rotate(stack[i], fx[i]);
29         else {
30             if (fx[i] == fx[i + 1]){
31                 Rotate(stack[i + 1], fx[i + 1]);
32                 Rotate(stack[i], fx[i]);
33             } else {
34                 Rotate(stack[i], fx[i]);
35                 Rotate(stack[i + 1], fx[i + 1]);
36             }
37         }
38     }
39
40     if (fa == 0) Root = x;
41 }

```

3.2 坚固的数据结构

3.2.1 坚固的平衡树

```

1 #define sz(x) (x?x->siz:0)
2 struct node{
3     int siz,key;
4     LL val,sum;
5     LL mu,a,d;
6     node *c[2],*f;
7     void split(int ned,node *&p,node *&q);
8     node* rz(){
9         sum=val;siz=1;
10        if(c[0])sum+=c[0]->sum,siz+=c[0]->siz;

```

```

11         if(c[1])sum+=c[1]->sum,siz+=c[1]->siz;
12         return this;
13     }
14     void make(LL _mu,LL _a,LL _d){
15         sum=sum*_mu+_a*siz+_d*siz*(siz-1)/2;
16         val=val*_mu+_a+_d*sz(c[0]);
17         mu=_mu;a=_a;d=_d;
18     }
19     void pd(){
20         if(mu==1&&a==0&&d==0)return;
21         if(c[0])c[0]->make(mu,a,d);
22         if(c[1])c[1]->make(mu,a+d+d*sz(c[0]),d);
23         mu=1;a=d=0;
24     }
25     node(){mu=1;}
26 }nd[maxn*2],*root;
27 node *merge(node *p,node *q){
28     if(!p||!q)return p?p->rz():(q?q->rz():0);
29     p->pd();q->pd();
30     if(p->key<q->key){
31         p->c[1]=merge(p->c[1],q);
32         return p->rz();
33     }else{
34         q->c[0]=merge(p,q->c[0]);
35         return q->rz();
36     }
37 }
38 void node::split(int ned,node *&p,node *&q){
39     if(!ned){p=0;q=this;return;}
40     if(ned==siz){p=this;q=0;return;}
41     pd();
42     if(sz(c[0])>=ned){
43         c[0]->split(ned,p,q);c[0]=0;rz();
44         q=merge(q,this);
45     }else{
46         c[1]->split(ned-sz(c[0])-1,p,q);c[1]=0;rz();
47         p=merge(this,p);
48     }
49 }
50 int main(){
51     for(int i=1;i<=n;i++){
52         nd[i].val=in();

```

```

53         nd[i].key=rand();
54         nd[i].rz();
55         root=merge(root,nd+i);
56     }
57 }

```

3.2.2 坚固的字符串

1. ext 库中的 rope

```

1  #include <ext/rope>
2
3  using __gnu_cxx::crope;
4  using __gnu_cxx::rope;
5
6  crope a, b;
7
8  int main(void) {
9      a = b.substr(pos, len);    // [pos, pos + len)
10     a = b.substr(pos);         // [pos, pos]
11     b.c_str();                 // might lead to memory leaks
12     b.delete_c_str();          // delete the c_str that created before
13     a.insert(pos, text);        // insert text before position pos
14     a.erase(pos, len);         // erase [pos, pos + len)
15 }

```

2. 可持久化平衡树实现的 rope

```

1  class Rope {
2  private:
3      class Node {
4      public:
5          Node *left, *right;
6          int size;
7          char key;
8
9          Node(char key = 0, Node *left = NULL, Node *right = NULL)
10             : key(key), left(left), right(right) {
11              update();
12          }
13
14          void update() {
15              size = (left ? left->size : 0) + 1 + (right ? right->size : 0);

```

```

16         }
17
18     std::string to_string() {
19         return (left ? left->to_string() : "") + key
20             + (right ? right->to_string() : "");
21     }
22 };
23
24 bool random(int a, int b) {
25     return rand() % (a + b) < a;
26 }
27
28 Node* merge(Node *x, Node *y) {
29     if (!x) {
30         return y;
31     }
32     if (!y) {
33         return x;
34     }
35     if (random(x->size, y->size)) {
36         return new Node(x->key, x->left, merge(x->right, y));
37     } else {
38         return new Node(y->key, merge(x, y->left), y->right);
39     }
40 }
41
42 std::pair<Node*, Node*> split(Node *x, int size) {
43     if (!x) {
44         return std::make_pair<Node*, Node*>(NULL, NULL);
45     }
46     if (size == 0) {
47         return std::make_pair<Node*, Node*>(NULL, x);
48     }
49     if (size > x->size) {
50         return std::make_pair<Node*, Node*>(x, NULL);
51     }
52     if (x->left && size <= x->left->size) {
53         std::pair<Node*, Node*> part =
54             split(x->left, size);
55         return std::make_pair(part.first, new Node(x->key, part.second, x->right));
56     } else {
57         std::pair<Node*, Node*> part =

```

```

58         split(x->right, size - (x->left ? x->left->size : 0) - 1);
59         return std::make_pair(new Node(x->key, x->left, part.first), part.second);
60     }
61 }
62
63 Node* build(const std::string &text, int left, int right) {
64     if (left > right) {
65         return NULL;
66     }
67     int mid = left + right >> 1;
68     return new Node(text[mid],
69                     build(text, left, mid - 1),
70                     build(text, mid + 1, right));
71 }
72
73 public:
74     Node *root;
75
76     Rope() {
77         root = NULL;
78     }
79
80     Rope(const std::string &text) {
81         root = build(text, 0, (int)text.length() - 1);
82     }
83
84     Rope(const Rope &other) {
85         root = other.root;
86     }
87
88     Rope& operator = (const Rope &other) {
89         if (this == &other) {
90             return *this;
91         }
92         root = other.root;
93         return *this;
94     }
95
96     int size() {
97         return root ? root->size : 0;
98     }
99

```

```

100 void insert(int pos, const std::string &text) {
101     if (pos < 0 || pos > size()) {
102         throw "Out of range";
103     }
104     std::pair<Node*, Node*> part = split(root, pos);
105     root = merge(merge(part.first, build(text, 0, (int)text.length() - 1)),
106                 part.second);
107 }
108
109 void erase(int left, int right) {
110     if (left < 0 || left >= size() ||
111         right < 1 || right > size()) {
112         throw "Out of range";
113     }
114     if (left >= right) {
115         return;
116     }
117     std::pair<Node*, Node*> part = split(root, left);
118     root = merge(part.first, split(part.second, right - left).second);
119 }
120
121 std::string substr(int left, int right) {
122     if (left < 0 || left >= size() ||
123         right < 1 || right > size()) {
124         throw "Out of range";
125     }
126     if (left >= right) {
127         return "";
128     }
129     return split(split(root, left).second, right - left).first->to_string();
130 }
131
132 void copy(int left, int right, int pos) {
133     if (left < 0 || left >= size() ||
134         right < 1 || right > size() ||
135         pos < 0 || pos > size()) {
136         throw "Out of range";
137     }
138     if (left >= right) {
139         return;
140     }
141     std::pair<Node*, Node*> part = split(root, pos);

```

```

142         root = merge(merge(part.first,
143                           split(split(root, left).second, right - left).first),
144                           part.second);
145     }
146 };

```

3.2.3 坚固的左偏树

```

1  int Merge(int x, int y){
2      if (x == 0 || y == 0) return x + y;
3      if (Heap[x].Key < Heap[y].Key) swap(x, y);
4      Heap[x].Ri = Merge(Heap[x].Ri, y);
5      if (Heap[Heap[x].Le].Dis < Heap[Heap[x].Ri].Dis) swap(Heap[x].Le, Heap[x].Ri);
6      if (Heap[x].Ri == 0) Heap[x].Dis = 0;
7      else Heap[x].Dis = Heap[Heap[x].Ri].Dis + 1;
8      return x;
9  }
10
11 for (int i = 0; i <= n; i++){
12     Heap[i].Le = Heap[i].Ri = 0;
13     Heap[i].Dis = 0;
14     Heap[i].Key = Cost[i];
15 }
16 Heap[0].Dis = -1;

```

3.2.4 不坚固的斜堆

```

1  struct node;
2  node *Null,*root[maxn];
3  struct node{
4      node* c[2];
5      int val,ind;
6      node(int _val=0,int _ind=0){
7          val=_val;c[0]=c[1]=Null;ind=_ind;
8      }
9  };
10 node* merge(node *p,node *q){
11     if(p==Null)return q;
12     if(q==Null)return p;
13     if(p->val>q->val)swap(p,q);
14     p->c[1]=merge(p->c[1],q);

```

```

15         swap(p->c[0],p->c[1]);
16         return p;
17     }
18
19     Null=new node(0);
20     Null->c[0]=Null->c[1]=Null;

```

3.3 树上的魔术师

3.3.1 轻重树链剖分 (zky)

```

1  vector<int>G[maxn];
2  int fa[maxn],top[maxn],siz[maxn],son[maxn],mp[maxn],z,dep[maxn];
3  void dfs(int u){
4      siz[u]=1;
5      for(int i=0;i<G[u].size();i++){
6          int v=G[u][i];
7          if(v!=fa[u]){
8              fa[v]=u;dep[v]=dep[u]+1;
9              dfs(v);
10             siz[u]+=siz[v];
11             if(siz[son[u]]<siz[v])son[u]=v;
12         }
13     }
14 }
15 void build(int u,int tp){
16     top[u]=tp;mp[u]=++z;
17     if(son[u])build(son[u],tp);
18     for(int v,i=0;i<G[u].size();i++)if((v=G[u][i])!=son[u]&&v!=fa[u])build(v,v);
19 }

```

3.3.2 轻重树链剖分 (lyx)

```

1  void Prep(int x){
2      dep[x] = dep[fa[x]] + 1;
3      size[x] = 1;
4      son[x] = 0;
5      for (int i = g[x]; i; i = nxt[i]){
6          int y = adj[i];
7          if (y == fa[x]) continue;
8          fa[y] = x;
9          Prep(y);

```



```

10         size[x] += size[y];
11         if (size[y] > size[son[x]]) son[x] = y;
12     }
13 }
14 void Dfs(int x){
15     dfn[x] = ++dfc;
16     if (son[x] != 0){
17         top[son[x]] = top[x];
18         Dfs(son[x]);
19     }
20     for (int i = g[x]; i; i = nxt[i]){
21         int y = adj[i];
22         if (y != fa[x] && y != son[x]){
23             top[y] = y;
24             Dfs(y);
25         }
26         if (y != fa[x]){
27             Bel[(i + 1) >> 1] = dfn[y];
28             val[dfn[y]] = len[i];
29         }
30     }
31 }
32 int Ask(int x, int y){
33     int Ret = -1000000001;
34     while (top[x] != top[y]){
35         if (dep[top[y]] > dep[top[x]]) swap(x, y);
36         Ret = max(Ret, Query(1, 1, n, dfn[top[x]], dfn[x]));
37         x = fa[top[x]];
38     }
39     if (dep[y] > dep[x]) swap(x, y);
40     if (x != y)
41         Ret = max(Ret, Query(1, 1, n, dfn[y] + 1, dfn[x]));
42     return Ret;
43 }
44 //Hints : Ask 部分具体的求值或者修改要稍作变动

```

3.3.3 Link Cut Tree(zky)

```

1 struct LCT{
2     struct node{
3         bool rev;
4         int mx,val;

```

```

5     node *f,*c[2];
6     bool d(){return this==f->c[1];}
7     bool rt(){return !f||(f->c[0]!=this&&f->c[1]!=this);}
8     void sets(node *x,int d){pd();if(x)x->f=this;c[d]=x;rz();}
9     void makerv(){rev^=1;swap(c[0],c[1]);}
10    void pd(){
11        if(rev){
12            if(c[0])c[0]->makerv();
13            if(c[1])c[1]->makerv();
14            rev=0;
15        }
16    }
17    void rz(){
18        mx=val;
19        if(c[0])mx=max(mx,c[0]->mx);
20        if(c[1])mx=max(mx,c[1]->mx);
21    }
22 }nd[int(1e4)+1];
23 void rot(node *x){
24     node *y=x->f;if(!y->rt())y->f->pd();
25     y->pd();x->pd();bool d=x->d();
26     y->sets(x->c[!d],d);
27     if(y->rt())x->f=y->f;
28     else y->f->sets(x,y->d());
29     x->sets(y,!d);
30 }
31 void splay(node *x){
32     while(!x->rt())
33         if(x->f->rt())rot(x);
34         else if(x->d()==x->f->d())rot(x->f),rot(x);
35         else rot(x),rot(x);
36 }
37 node* access(node *x){
38     node *y=0;
39     for(;x;x=x->f){
40         splay(x);
41         x->sets(y,1);y=x;
42     }return y;
43 }
44 void makert(node *x){
45     access(x)->makerv();
46     splay(x);

```

```

47     }
48     void link(node *x,node *y){
49         makert(x);
50         x->f=y;
51         access(x);
52     }
53     void cut(node *x,node *y){
54         makert(x);access(y);splay(y);
55         y->c[0]=x->f=0;
56         y->rz();
57     }
58     void link(int x,int y){link(nd+x,nd+y);}
59     void cut(int x,int y){cut(nd+x,nd+y);}
60 }T;

```

3.3.4 Link Cut Tree(lyx)

```

1  struct node{
2      bool Rev;
3      int c[2], fa, Chain, Aux, Val;
4  }T[N];
5
6  inline int Sum(int x){
7      return T[x].Chain ^ T[x].Aux;
8  }
9
10 inline void Rev(int x){
11     if (!x) return;
12     swap(T[x].c[0],T[x].c[1]);
13     T[x].Rev ^= 1;
14 }
15
16 inline void Update(int x){
17     T[x].Chain = Sum(T[x].c[0]) ^ Sum(T[x].c[1]) ^ T[x].Val;
18 }
19
20 inline void Lazy_Down(int x){
21     if (!x) return;
22     if (T[x].Rev) Rev(T[x].c[0]), Rev(T[x].c[1]), T[x].Rev = 0;
23 }
24
25 inline void Rotate(int x,int c){

```

```

26     int fa = T[x].fa, ft = T[fa].fa;
27     T[x].fa = ft, T[fa].fa = x;
28     if (ft) T[ft].c[T[ft].c[1] == fa] = x;
29     T[fa].c[c] = T[x].c[!c];
30     if (T[x].c[!c]) T[T[x].c[!c]].fa = fa;
31     T[x].c[!c] = fa;
32     if (Par[fa]) Par[x] = Par[fa], Par[fa] = 0;
33     Update(fa);
34 }
35
36 inline void Splay(int x){
37     int top = 0;
38     for (int u = x; u; u = T[u].fa) Stack[++top] = u;
39
40     for( ; top; top--) Lazy_Down(Stack[top]);
41
42     for( ; T[x].fa; ){
43         int fa = T[x].fa, ft = T[fa].fa;
44         if (!ft) Rotate(x, T[fa].c[1] == x); else
45         {
46             if (T[fa].c[1] == x)
47             {
48                 if (T[ft].c[1] == fa) Rotate(fa, 1), Rotate(x, 1);
49                 else Rotate(x, 1), Rotate(x, 0);
50             } else
51                 if (T[ft].c[0] == fa) Rotate(fa, 0), Rotate(x, 0);
52                 else Rotate(x, 0), Rotate(x, 1);
53         }
54     }
55     Update(x);
56 }
57
58 inline int Access(int u){
59
60     int Nxt = 0;
61
62     while (u){
63         Splay(u);
64         if (T[u].c[1]){
65             T[T[u].c[1]].fa = 0;
66             Par[T[u].c[1]] = u;
67             T[u].Aux ^= Sum(T[u].c[1]);

```

```

68         }
69         T[u].c[1] = Nxt;
70         if (Nxt){
71             T[Nxt].fa = u;
72             Par[Nxt] = 0;
73             T[u].Aux ^= Sum(Nxt);
74         }
75         Update(u);
76         Nxt = u;
77         u = Par[u];
78     }
79
80     return Nxt;
81
82 }
83
84 inline void Root(int u){
85     Rev(Access(u));
86 }
87
88 inline void Mark(int x, int col){
89     Access(x);
90     Splay(x);
91     T[x].Val ^= col;
92     Update(x);
93 }
94
95 inline void Link(int u, int v){
96     Root(v);
97     Access(v);
98     Access(u);
99     Splay(v);
100    Splay(u);
101    Par[v] = u;
102    T[u].Aux ^= Sum(v);
103    Access(v);
104 }
105
106 inline void Cut(int u, int v){
107     Root(v);
108     Access(u);
109     Splay(u);

```

```

110         T[T[u].c[0]].fa = 0;
111         T[u].c[0] = 0;
112         Update(u);
113     }

```

3.3.5 AAA Tree

```

1  #define rep(i,a,n) for(int i=a;i<n;i++)
2  int n,m;
3  struct info{
4      int mx,mn,sum,sz;
5      info(){}
6      info(int mx,int mn,int sum,int sz):
7          mx(mx),mn(mn),sum(sum),sz(sz){}
8      void deb(){printf("sum:%d size:%d",(int)sum,sz);}
9  };
10 struct flag{
11     int mul,add;
12     flag(){mul=1;}
13     flag(int mul,int add):
14         mul(mul),add(add){}
15     bool empty(){return mul==1&&add==0;}
16 };
17 info operator+(const info &a,const flag &b) {
18     return a.sz?info(a.mx*b.mul+b.add,a.mn*b.mul+b.add,a.sum*b.mul+b.add*a.sz,a.sz):a;
19 }
20 info operator+(const info &a,const info &b) {
21     return info(max(a.mx,b.mx),min(a.mn,b.mn),a.sum+b.sum,a.sz+b.sz);
22 }
23 flag operator+(const flag &a,const flag &b) {
24     return flag(a.mul*b.mul,a.add*b.mul+b.add);
25 }
26 struct node{
27     node *c[4],*f;
28     flag Cha,All;
29     info cha,sub,all;
30     bool rev,inr;
31     int val;
32     void makerev(){rev^=1;swap(c[0],c[1]);}
33     void makec(const flag &a){
34         Cha=Cha+a;cha=cha+a;val=val*a.mul+a.add;
35         all=cha+sub;

```

```

36     }
37     void makes(const flag &a, bool _=1){
38         All=All+a; all=all+a; sub=sub+a;
39         if(_)makec(a);
40     }
41     void rz(){
42         cha=all=sub=info(-(1<<30),1<<30,0,0);
43         if(!inr)all=cha=info(val,val,val,1);
44         rep(i,0,2)if(c[i])cha=cha+c[i]->cha, sub=sub+c[i]->sub;
45         rep(i,0,4)if(c[i])all=all+c[i]->all;
46         rep(i,2,4)if(c[i])sub=sub+c[i]->all;
47     }
48     void pd(){
49         if(rev){
50             if(c[0])c[0]->makerev();
51             if(c[1])c[1]->makerev();
52             rev=0;
53         }
54         if(!All.empty()){
55             rep(i,0,4)if(c[i])c[i]->makes(All,i>=2);
56             All=flag(1,0);
57         }
58         if(!Cha.empty()){
59             rep(i,0,2)if(c[i])c[i]->makec(Cha);
60             Cha=flag(1,0);
61         }
62     }
63 }
64 node *C(int i){if(c[i])c[i]->pd();return c[i];}
65 bool d(int ty){return f->c[ty+1]==this;}
66 int D(){rep(i,0,4)if(f->c[i]==this)return i;}
67 void sets(node *x, int d){if(x)x->f=this;c[d]=x;}
68 bool rt(int ty){
69     if(ty==0)return !f|| (f->c[0]!=this&&f->c[1]!=this);
70     else return !f|| !f->inr|| !inr;
71 }
72 }nd[maxn*2], *cur=nd+maxn, *pool[maxn], **Cur=pool;
73 int _cnt;
74 node *newnode(){
75     _cnt++;
76     node *x=(Cur==pool)?cur++:*(--Cur);
77     rep(i,0,4)x->c[i]=0;x->f=0;

```

```

78     x->All=x->Cha=flag(1,0);
79     x->all=x->cha=info(-(1<<30),(1<<30),0,0);
80     x->inr=1;x->rev=0;x->val=0;
81     return x;
82 }
83 void dele(node *x){*(Cur++)=x;}
84 void rot(node *x,int ty){
85     node *p=x->f;int d=x->d(ty);
86     if(!p->f)x->f=0;else p->f->sets(x,p->D());
87     p->sets(x->c[!d+ty],d+ty);x->sets(p,!d+ty);p->rz();
88 }
89 void splay(node *x,int ty=0){
90     while(!x->rt(ty)){
91         if(x->f->rt(ty))rot(x,ty);
92         else if(x->d(ty)==x->f->d(ty))rot(x->f,ty),rot(x,ty);
93         else rot(x,ty),rot(x,ty);
94     }x->rz();
95 }
96 void add(node *u,node *w){
97     w->pd();
98     rep(i,2,4)if(!w->c[i]){w->sets(u,i);return;}
99     node *x=newnode(),*v;
100     for(v=w;v->c[2]->inr;v=v->C(2));
101     x->sets(v->c[2],2);x->sets(u,3);
102     v->sets(x,2);splay(x,2);
103 }
104 void del(node *w){
105     if(w->f->inr){
106         w->f->f->sets(w->f->c[5-w->D()],w->f->D());
107         dele(w->f);splay(w->f->f,2);
108     }else w->f->sets(0,w->D());
109     w->f=0;
110 }
111 void access(node *w){
112     static node *sta[maxn];
113     static int top=0;
114     node *v=w,*u;
115     for(u=w;u;u=u->f)sta[top++]=u;
116     while(top)sta[--top]->pd();
117     splay(w);
118     if(w->c[1])u=w->c[1],w->c[1]=0,add(u,w),w->rz();
119     while(w->f){

```



```

120         for(u=w->f;u->innr;u=u->f);
121         splay(u);
122         if(u->c[1])w->f->sets(u->c[1],w->D()),splay(w->f,2);
123         else del(w);
124         u->sets(w,1);
125         (w=u)->rz();
126     }splay(v);
127 }
128 void makert(node *x){
129     access(x);x->makerev();
130 }
131 node *findp(node *u){
132     access(u);u=u->C(0);
133     while(u&&u->c[1])u=u->C(1);
134     return u;
135 }
136 node *findr(node *u){for(;u->f;u=u->f);return u;}
137 node* cut(node *u){
138     node *v=findp(u);
139     if(v)access(v),del(u),v->rz();
140     return v;
141 }
142 void link(node *u,node *v) {
143     node* p=cut(u);
144     if(findr(u)!=findr(v))p=v;
145     if(p)access(p),add(u,p),p->rz();
146 }
147 int main(){
148     // freopen("bzoj3153.in","r",stdin);
149     n=getint();m=getint();
150     static int _u[maxn],_v[maxn];
151     rep(i,1,n)_u[i]=getint(),_v[i]=getint();
152     rep(i,1,n+1){
153         nd[i].val=getint();
154         nd[i].rz();
155     }
156     rep(i,1,n)makert(nd+_u[i]),link(nd+_u[i],nd+_v[i]);
157     int root=getint();
158     makert(nd+root);
159     // deb();
160     int x,y,z;
161     node *u,*v;

```

```

162 while(m--){
163     int k=getint();x=getint();
164     u=nd+x;
165     if(k==0 || k==3 || k==4 || k==5 || k==11){
166         access(u);
167         if(k==3 || k==4 || k==11){
168             int ans=u->val;
169             rep(i,2,4)if(u->c[i]){
170                 info res=u->c[i]->all;
171                 if(k==3) ans=min(ans,res.mn);
172                 else if(k==4) ans=max(ans,res.mx);
173                 else if(k==11) ans+=res.sum;
174             }printf("%d\n",ans);
175         }else{
176             y=getint();
177             flag fg(k==5,y);
178             u->val=u->val*fg.mul+fg.add;
179             rep(i,2,4)if(u->c[i])u->c[i]->makes(fg);
180             u->rz();
181         }
182     }else if(k==2 || k==6 || k==7 || k==8 || k==10){
183         y=getint();
184         makert(u),access(nd+y),splay(u);
185         if (k==7 || k==8 || k==10) {
186             info ans=u->cha;
187             if (k==7) printf("%d\n",ans.mn);
188             else if (k==8) printf("%d\n",ans.mx);
189             else printf("%d\n",ans.sum);
190         }else u->makec(flag(k==6,getint()));
191         makert(nd+root);
192     }else if(k==9)link(u,nd+getint());
193     else if(k==1)makert(u),root=x;
194 }
195 return 0;
196 }

```

3.4 ST

```

1 for (int i = 1; i <= n; i++)
2     Log[i] = int(log2(i));
3
4 for (int i = 1; i <= n; i++)

```

```

5         Rmq[i][0] = i;
6
7     for (int k = 1; (1 << k) <= n; k++)
8         for (int i = 1; i + (1 << k) - 1 <= n; i++){
9             int x = Rmq[i][k - 1], y = Rmq[i + (1 << (k - 1))][k - 1];
10            if (a[x] < a[y])
11                Rmq[i][k] = x;
12            else
13                Rmq[i][k] = y;
14        }
15
16    int Smallest(int l, int r){
17        int k = Log[r - l + 1];
18
19        int x = Rmq[l][k];
20        int y = Rmq[r - (1 << k) + 1][k];
21
22        if (a[x] < a[y]) return x;
23        else return y;
24    }

```

3.5 可持久化线段树

```

1    struct node1 {
2        int L, R, Lson, Rson, Sum;
3    } tree[N * 40];
4    int root[N], a[N], b[N];
5    int tot, n, m;
6    int Real[N];
7    int Same(int x) {
8        ++tot;
9        tree[tot] = tree[x];
10       return tot;
11    }
12    int build(int L, int R) {
13        ++tot;
14        tree[tot].L = L;
15        tree[tot].R = R;
16        tree[tot].Lson = tree[tot].Rson = tree[tot].Sum = 0;
17        if (L == R) return tot;
18        int s = tot;
19        int mid = (L + R) >> 1;

```

```

20     tree[s].Lson = build(L, mid);
21     tree[s].Rson = build(mid + 1, R);
22     return s;
23 }
24 int Ask(int Lst, int Cur, int L, int R, int k) {
25     if (L == R) return L;
26     int Mid = (L + R) >> 1;
27     int Left = tree[tree[Cur].Lson].Sum - tree[tree[Lst].Lson].Sum;
28     if (Left >= k) return Ask(tree[Lst].Lson, tree[Cur].Lson, L, Mid, k);
29     k -= Left;
30     return Ask(tree[Lst].Rson, tree[Cur].Rson, Mid + 1, R, k);
31 }
32 int Add(int Lst, int pos) {
33     int root = Same(Lst);
34     tree[root].Sum++;
35     if (tree[root].L == tree[root].R) return root;
36     int mid = (tree[root].L + tree[root].R) >> 1;
37     if (pos <= mid) tree[root].Lson = Add(tree[root].Lson, pos);
38     else tree[root].Rson = Add(tree[root].Rson, pos);
39     return root;
40 }
41 int main() {
42     scanf("%d%d", &n, &m);
43     int up = 0;
44     for (int i = 1; i <= n; i++){
45         scanf("%d", &a[i]);
46         b[i] = a[i];
47     }
48     sort(b + 1, b + n + 1);
49     up = unique(b + 1, b + n + 1) - b - 1;
50     for (int i = 1; i <= n; i++){
51         int tmp = lower_bound(b + 1, b + up + 1, a[i]) - b;
52         Real[tmp] = a[i];
53         a[i] = tmp;
54     }
55     tot = 0;
56     root[0] = build(1, up);
57     for (int i = 1; i <= n; i++){
58         root[i] = Add(root[i - 1], a[i]);
59     }
60     for (int i = 1; i <= m; i++){
61         int u, v, w;

```

```

62         scanf("%d%d%d", &u, &v, &w);
63         printf("%d\n", Real[Ask(root[u - 1], root[v], 1, up, w)]);
64     }
65     return 0;
66 }

```

3.6 可持久化 Trie

```

1  int Pre[N];
2  int n, q, Len, cnt, Lstans;
3  char s[N];
4  int First[N], Last[N];
5  int Root[N];
6  int Trie_tot;
7  struct node{
8      int To[30];
9      int Lst;
10 }Trie[N];
11 int tot;
12 struct node1{
13     int L, R, Lson, Rson, Sum;
14 }tree[N * 25];
15 int Build(int L, int R){
16     ++tot;
17     tree[tot].L = L;
18     tree[tot].R = R;
19     tree[tot].Lson = tree[tot].Rson = tree[tot].Sum = 0;
20     if (L == R) return tot;
21     int s = tot;
22     int mid = (L + R) >> 1;
23     tree[s].Lson = Build(L, mid);
24     tree[s].Rson = Build(mid + 1, R);
25     return s;
26 }
27 int Same(int x){
28     ++tot;
29     tree[tot] = tree[x];
30     return tot;
31 }
32 int Add(int Lst, int pos){
33     int s = Same(Lst);
34     tree[s].Sum++;

```

```

35     if (tree[s].L == tree[s].R) return s;
36     int Mid = (tree[s].L + tree[s].R) >> 1;
37     if (pos <= Mid) tree[s].Lson = Add(tree[Lst].Lson, pos);
38     else tree[s].Rson = Add(tree[Lst].Rson, pos);
39     return s;
40 }
41
42 int Ask(int Lst, int Cur, int L, int R, int pos){
43     if (L >= pos) return 0;
44     if (R < pos) return tree[Cur].Sum - tree[Lst].Sum;
45     int Mid = (L + R) >> 1;
46     int Ret = Ask(tree[Lst].Lson, tree[Cur].Lson, L, Mid, pos);
47     Ret += Ask(tree[Lst].Rson, tree[Cur].Rson, Mid + 1, R, pos);
48     return Ret;
49 }
50
51 int main(){
52     while (scanf("%d", &n) == 1){
53         for (int i = 1; i <= Trie_tot; i++){
54             for (int j = 1; j <= 26; j++){
55                 Trie[i].To[j] = 0;
56                 Trie[i].Lst = 0;
57             }
58             Trie_tot = 1;
59             cnt = 0;
60             for (int ii = 1; ii <= n; ii++){
61                 scanf("%s", s + 1);
62                 Len = strlen(s + 1);
63                 int Cur = 1;
64                 First[ii] = cnt + 1;
65                 for (int i = 1; i <= Len; i++){
66                     int ch = s[i] - 'a' + 1;
67                     if (Trie[Cur].To[ch] == 0){
68                         ++Trie_tot;
69                         Trie[Cur].To[ch] = Trie_tot;
70                     }
71                     Cur = Trie[Cur].To[ch];
72                     Pre[++cnt] = Trie[Cur].Lst;
73                     Trie[Cur].Lst = ii;
74                 }
75                 Last[ii] = cnt;
76             }

```

```

77     tot = 0;
78     Root[0] = Build(0, n);
79     for (int i = 1; i <= cnt; i++){
80         Root[i] = Add(Root[i - 1], Pre[i]);
81     }
82     Lstans = 0;
83     scanf("%d", &q);
84     for (int ii = 1; ii <= q; ii++){
85         int L, R;
86         scanf("%d%d", &L, &R);
87         L = (L + Lstans) % n + 1;
88         R = (R + Lstans) % n + 1;
89         if (L > R) swap(L, R);
90         int Ret = Ask(Root[First[L] - 1], Root[Last[R]], 0, n, L);
91         printf("%d\n", Ret);
92         Lstans = Ret;
93     }
94 }
95 return 0;
96 }

```

3.7 k-d 树

```

1  long long norm(const long long &x) {
2      // For manhattan distance
3      return std::abs(x);
4      // For euclid distance
5      return x * x;
6  }
7
8  struct Point {
9      int x, y, id;
10
11     const int& operator [] (int index) const {
12         if (index == 0) {
13             return x;
14         } else {
15             return y;
16         }
17     }
18
19     friend long long dist(const Point &a, const Point &b) {

```

```

20         long long result = 0;
21         for (int i = 0; i < 2; ++i) {
22             result += norm(a[i] - b[i]);
23         }
24         return result;
25     }
26 } point[N];
27
28 struct Rectangle {
29     int min[2], max[2];
30
31     Rectangle() {
32         min[0] = min[1] = INT_MAX;
33         max[0] = max[1] = INT_MIN;
34     }
35
36     void add(const Point &p) {
37         for (int i = 0; i < 2; ++i) {
38             min[i] = std::min(min[i], p[i]);
39             max[i] = std::max(max[i], p[i]);
40         }
41     }
42
43     long long dist(const Point &p) {
44         long long result = 0;
45         for (int i = 0; i < 2; ++i) {
46             // For minimum distance
47             result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
48             // For maximum distance
49             result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
50         }
51         return result;
52     }
53 };
54
55 struct Node {
56     Point separator;
57     Rectangle rectangle;
58     int child[2];
59
60     void reset(const Point &p) {
61         separator = p;

```



```

62         rectangle = Rectangle();
63         rectangle.add(p);
64         child[0] = child[1] = 0;
65     }
66 } tree[N << 1];
67
68 int size, pivot;
69
70 bool compare(const Point &a, const Point &b) {
71     if (a[pivot] != b[pivot]) {
72         return a[pivot] < b[pivot];
73     }
74     return a.id < b.id;
75 }
76
77 int build(int l, int r, int type = 1) {
78     pivot = type;
79     if (l >= r) {
80         return 0;
81     }
82     int x = ++size;
83     int mid = l + r >> 1;
84     std::nth_element(point + l, point + mid, point + r, compare);
85     tree[x].reset(point[mid]);
86     for (int i = l; i < r; ++i) {
87         tree[x].rectangle.add(point[i]);
88     }
89     tree[x].child[0] = build(l, mid, type ^ 1);
90     tree[x].child[1] = build(mid + 1, r, type ^ 1);
91     return x;
92 }
93
94 int insert(int x, const Point &p, int type = 1) {
95     pivot = type;
96     if (x == 0) {
97         tree[++size].reset(p);
98         return size;
99     }
100     tree[x].rectangle.add(p);
101     if (compare(p, tree[x].separator)) {
102         tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
103     } else {

```

```

104         tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
105     }
106     return x;
107 }
108
109 // For minimum distance
110 void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
111     pivot = type;
112     if (x == 0 || tree[x].rectangle.dist(p) > answer.first) {
113         return;
114     }
115     answer = std::min(answer,
116         std::make_pair(dist(tree[x].separator, p), tree[x].separator.id));
117     if (compare(p, tree[x].separator)) {
118         query(tree[x].child[0], p, answer, type ^ 1);
119         query(tree[x].child[1], p, answer, type ^ 1);
120     } else {
121         query(tree[x].child[1], p, answer, type ^ 1);
122         query(tree[x].child[0], p, answer, type ^ 1);
123     }
124 }
125
126 std::priority_queue<std::pair<long long, int> > answer;
127
128 void query(int x, const Point &p, int k, int type = 1) {
129     pivot = type;
130     if (x == 0 ||
131         ((int)answer.size() == k && tree[x].rectangle.dist(p) > answer.top().first) {
132         return;
133     }
134     answer.push(std::make_pair(dist(tree[x].separator, p), tree[x].separator.id));
135     if ((int)answer.size() > k) {
136         answer.pop();
137     }
138     if (compare(p, tree[x].separator)) {
139         query(tree[x].child[0], p, k, type ^ 1);
140         query(tree[x].child[1], p, k, type ^ 1);
141     } else {
142         query(tree[x].child[1], p, k, type ^ 1);
143         query(tree[x].child[0], p, k, type ^ 1);
144     }
145 }

```

3.8 必经点树

```
1 vector<int>G[maxn],rG[maxn],dom[maxn];
2 int n,m;
3 int dfn[maxn],rdfs[maxn],dfs_c,semi[maxn],idom[maxn],fa[maxn];
4 struct ufsets{
5     int fa[maxn],best[maxn];
6     int find(int x){
7         if(fa[x]==x)
8             return x;
9         int f=find(fa[x]);
10        if(dfn[semi[best[x]]]>dfn[semi[best[fa[x]]]])
11            best[x]=best[fa[x]];
12        fa[x]=f;
13        return f;
14    }
15    int getbest(int x){
16        find(x);
17        return best[x];
18    }
19    void init(){
20        for(int i=1;i<=n;i++)
21            fa[i]=best[i]=i;
22    }
23 }uf;
24 void init(){
25     uf.init();
26     for(int i=1;i<=n;i++){
27         semi[i]=i;
28         idom[i]=0;
29         fa[i]=0;
30         dfn[i]=rdfs[i]=0;
31     }
32     dfs_c=0;
33 }
34 void dfs(int u){
35     dfn[u]=++dfs_c;
36     rdfs[dfn[u]]=u;
37     for(int i=0;i<G[u].size();i++){
38         int v=G[u][i];
39         if(!dfn[v]){
40             fa[v]=u;
```

```

41         dfs(v);
42     }
43 }
44 }
45
46 void tarjan(){
47     for(int i=n;i>1;i--){
48         int tmp=1e9;
49         int y=rdfn[i];
50         for(int i=0;i<rG[y].size();i++){
51             int x=rG[y][i];
52             tmp=min(tmp,dfn[semi[uf.getbest(x)]]);
53         }
54         semi[y]=rdfn[tmp];
55         int x=fa[y];
56         dom[semi[y]].push_back(y);
57         uf.fa[y]=x;
58         for(int i=0;i<dom[x].size();i++){
59             int z=dom[x][i];
60             if(dfn[semi[uf.getbest(z)]]<dfn[x])
61                 idom[z]=uf.getbest(z);
62             else
63                 idom[z]=semi[z];
64         }
65         dom[x].clear();
66     }
67     semi[rdfn[1]]=1;
68     for(int i=2;i<=n;i++){
69         int x=rdfn[i];
70         if(idom[x]!=semi[x])
71             idom[x]=idom[idom[x]];
72     }
73     idom[rdfn[1]]=0;
74 }
75 }
76
77 int main(){
78     while(scanf("%d%d",&n,&m)==2){
79         for(int i=1;i<=n;i++){
80             G[i].clear();
81             rG[i].clear();
82             dom[i].clear();

```

```

83     }
84
85     for(int i=1;i<=m;i++){
86         int u,v;
87         scanf("%d%d",&u,&v);
88         G[u].push_back(v);
89         rG[v].push_back(u);
90     }
91     init();
92     dfs(1);
93
94     tarjan();
95
96     static int vis[maxn];
97     memset(vis,0,sizeof vis);
98     int ans=0;
99     for(int i=2;i<=n;i++){
100         if(!vis[idom[i]]){
101             vis[idom[i]]=1;
102             ans++;
103         }
104     }
105     cout<<ans<<endl;
106     for(int i=1;i<=n;i++)
107         if(vis[i]){
108             ans--;
109             printf("%d%c",i," \n"[ans==0]);
110         }
111     }
112     return 0;
113 }

```

3.9 莫队算法

```

1 struct node{
2     int l, r, id;
3     friend bool operator < (const node &a, const node &b){
4         if (a.l / Block == b.l / Block) return a.r / Block < b.r / Block;
5         return a.l / Block < b.l / Block;
6     }
7 }q[N];
8 Block = int(sqrt(n));

```

```

9  for (int i = 1; i <= m; i++){
10      scanf("%d%d", &q[i].l, &q[i].r);
11      q[i].id = i;
12  }
13  sort(q + 1, q + 1 + m);
14  Cur = a[1]; /// Hints: adjust by yourself
15  Le = Ri = 1;
16  for (int i = 1; i <= m; i++){
17      while (q[i].r > Ri) Ri++, ChangeRi(1, Le, Ri);
18      while (q[i].l > Le) ChangeLe(-1, Le, Ri), Le++;
19      while (q[i].l < Le) Le--, ChangeLe(1, Le, Ri);
20      while (q[i].r < Ri) ChangeRi(-1, Le, Ri), Ri--;
21      Ans[q[i].id] = Cur;
22  }

```

3.10 整体二分

```

1  struct BIT{
2      LL d[maxn];
3      inline int lowbit(int x){return x&-x;}
4      LL get(int x){
5          LL ans=0;
6          while(x)ans+=d[x],x-=lowbit(x);
7          return ans;
8      }
9      void updata(int x,LL f){
10         while(x<=m)d[x]+=f,x+=lowbit(x);
11     }
12     void add(int l,int r,LL f){
13         updata(l,f);
14         updata(r+1,-f);
15     }
16 }T,T2;
17 int anss[maxn],wana[maxn];
18 struct qes{
19     LL x,y,z;
20     qes(LL _x=0,LL _y=0,LL _z=0):
21         x(_x),y(_y),z(_z){}
22 }q[maxn],p[maxn];
23 bool part(qes &q){
24     if(q.y+q.z>=wana[q.x])return 1;
25     q.z+=q.y;q.y=0;return 0;

```

```

26 }
27 void solve(int lef,int rig,int l,int r){
28     if(l==r){
29         for(int i=lef;i<=rig;i++)if(anss[p[i].x]!=-1)
30             anss[p[i].x]=1;return;
31     }int mid=(l+r)>>1;
32     for(int i=l;i<=mid;i++){
33         if(q[i].x<=q[i].y)T.add(q[i].x,q[i].y,q[i].z);
34         else T.add(1,q[i].y,q[i].z),T.add(q[i].x,m,q[i].z);
35     }for(int i=lef;i<=rig;i++){
36         p[i].y=0;
37         for(int j=0;j<O[p[i].x].size()&&p[i].y<=int(1e9)+1;j++)
38             p[i].y+=T.get(O[p[i].x][j]);
39     }for(int i=l;i<=mid;i++){
40         if(q[i].x<=q[i].y)T.add(q[i].x,q[i].y,-q[i].z);
41         else T.add(1,q[i].y,-q[i].z),T.add(q[i].x,m,-q[i].z);
42     }int dv=stable_partition(p+lef,p+rig+1,part)-p-1;
43     if(lef<=dv)
44         solve(lef,dv,l,mid);
45     if(dv+1<=rig)
46         solve(dv+1,rig,mid+1,r);
47 }

```

4 图论

4.1 强连通分量

```

1 int stamp, comps, top;
2 int dfn[N], low[N], comp[N], stack[N];
3
4 void tarjan(int x) {
5     dfn[x] = low[x] = ++stamp;
6     stack[top++] = x;
7     for (int i = 0; i < (int)edge[x].size(); ++i) {
8         int y = edge[x][i];
9         if (!dfn[y]) {
10             tarjan(y);
11             low[x] = std::min(low[x], low[y]);
12         } else if (!comp[y]) {
13             low[x] = std::min(low[x], dfn[y]);
14         }
15     }
16 }

```

```

16     if (low[x] == dfn[x]) {
17         comps++;
18         do {
19             int y = stack[--top];
20             comp[y] = comps;
21         } while (stack[top] != x);
22     }
23 }
24
25 void solve() {
26     stamp = comps = top = 0;
27     std::fill(dfn, dfn + n, 0);
28     std::fill(comp, comp + n, 0);
29     for (int i = 0; i < n; ++i) {
30         if (!dfn[i]) {
31             tarjan(i);
32         }
33     }
34 }

```

4.1.1 点双连通分量

```

1  struct Edge{
2      int To, id;
3      Edge(){}
4      Edge(int _To, int _id){
5          To = _To;
6          id = _id;
7      }
8  };
9
10 int n, m, dfc, block, top;
11 vector<Edge> G[N];
12 vector<int> H[N];
13
14 int dfn[N], low[N], stack[N], belong[N];
15
16 void Tarjan(int x, int lst){
17     dfn[x] = low[x] = ++dfc;
18     stack[top++] = x;
19     for (int i = 0; i < (int)G[x].size(); i++){
20         int y = G[x][i].To;

```



```

21         if (!dfn[y]){
22             Tarjan(y, G[x][i].id);
23             low[x] = min(low[x], low[y]);
24         } else if (!belong[y] && G[x][i].id != 1st){
25             low[x] = min(low[x], dfn[y]);
26         }
27     }
28     if (low[x] == dfn[x]){
29         block++;
30         do{
31             int y = stack[--top];
32             belong[y] = block;
33         } while (stack[top] != x);
34     }
35 }
36
37 //bridge
38 for (int i = 1; i <= n; i++)
39     for (int j = 0; j < G[i].size(); j++){
40         int y = G[i][j].To;
41         if (belong[i] == belong[y]) continue;
42         H[belong[i]].push_back(belong[y]);
43     }

```

4.2 2-SAT 问题

```

1  int stamp, comps, top;
2  int dfn[N], low[N], comp[N], stack[N];
3
4  void add(int x, int a, int y, int b) {
5      edge[x << 1 | a].push_back(y << 1 | b);
6  }
7
8  void tarjan(int x) {
9      dfn[x] = low[x] = ++stamp;
10     stack[top++] = x;
11     for (int i = 0; i < (int)edge[x].size(); ++i) {
12         int y = edge[x][i];
13         if (!dfn[y]) {
14             tarjan(y);
15             low[x] = std::min(low[x], low[y]);
16         } else if (!comp[y]) {

```

```

17         low[x] = std::min(low[x], dfn[y]);
18     }
19 }
20 if (low[x] == dfn[x]) {
21     comps++;
22     do {
23         int y = stack[--top];
24         comp[y] = comps;
25     } while (stack[top] != x);
26 }
27 }
28
29 bool solve() {
30     int counter = n + n + 1;
31     stamp = top = comps = 0;
32     std::fill(dfn, dfn + counter, 0);
33     std::fill(comp, comp + counter, 0);
34     for (int i = 0; i < counter; ++i) {
35         if (!dfn[i]) {
36             tarjan(i);
37         }
38     }
39     for (int i = 0; i < n; ++i) {
40         if (comp[i << 1] == comp[i << 1 | 1]) {
41             return false;
42         }
43         answer[i] = (comp[i << 1 | 1] < comp[i << 1]);
44     }
45     return true;
46 }

```

4.3 二分图最大匹配

4.3.1 Hungary 算法

时间复杂度: $\mathcal{O}(V \cdot E)$

```

1 vector<int>G[maxn];
2 int Link[maxn],vis[maxn],T;
3 bool find(int x){
4     for(int i=0;i<G[x].size();i++){
5         int v=G[x][i];
6         if(vis[v]==T)continue;

```

```

7         vis[v]=T;
8         if(!Link[v]||find(Link[v])){
9             Link[v]=x;
10            return 1;
11        }
12    }return 0;
13 }
14 int Hungarian(int n){
15     int ans=0;
16     memset(Link,0,sizeof Link);
17     for(int i=1;i<=n;i++){
18         T++;
19         ans+=find(i);
20     }return ans;
21 }

```

4.3.2 Hopcroft Karp 算法

时间复杂度: $\mathcal{O}(\sqrt{V} \cdot E)$

```

1  int matchx[N], matchy[N], level[N];
2
3  bool dfs(int x) {
4      for (int i = 0; i < (int)edge[x].size(); ++i) {
5          int y = edge[x][i];
6          int w = matchy[y];
7          if (w == -1 || level[x] + 1 == level[w] && dfs(w)) {
8              matchx[x] = y;
9              matchy[y] = x;
10             return true;
11         }
12     }
13     level[x] = -1;
14     return false;
15 }
16
17 int solve() {
18     std::fill(matchx, matchx + n, -1);
19     std::fill(matchy, matchy + m, -1);
20     for (int answer = 0; ; ) {
21         std::vector<int> queue;
22         for (int i = 0; i < n; ++i) {
23             if (matchx[i] == -1) {

```

```

24         level[i] = 0;
25         queue.push_back(i);
26     } else {
27         level[i] = -1;
28     }
29 }
30 for (int head = 0; head < (int)queue.size(); ++head) {
31     int x = queue[head];
32     for (int i = 0; i < (int)edge[x].size(); ++i) {
33         int y = edge[x][i];
34         int w = matchy[y];
35         if (w != -1 && level[w] < 0) {
36             level[w] = level[x] + 1;
37             queue.push_back(w);
38         }
39     }
40 }
41 int delta = 0;
42 for (int i = 0; i < n; ++i) {
43     if (matchx[i] == -1 && dfs(i)) {
44         delta++;
45     }
46 }
47 if (delta == 0) {
48     return answer;
49 } else {
50     answer += delta;
51 }
52 }
53 }

```

4.4 二分图最大权匹配

时间复杂度: $\mathcal{O}(V^4)$

```

1  int labelx[N], labely[N], match[N], slack[N];
2  bool visitx[N], visity[N];
3
4  bool dfs(int x) {
5      visitx[x] = true;
6      for (int y = 0; y < n; ++y) {
7          if (visity[y]) {
8              continue;

```

```

9         }
10        int delta = labelx[x] + labely[y] - graph[x][y];
11        if (delta == 0) {
12            visity[y] = true;
13            if (match[y] == -1 || dfs(match[y])) {
14                match[y] = x;
15                return true;
16            }
17        } else {
18            slack[y] = std::min(slack[y], delta);
19        }
20    }
21    return false;
22 }
23
24 int solve() {
25     for (int i = 0; i < n; ++i) {
26         match[i] = -1;
27         labelx[i] = INT_MIN;
28         labely[i] = 0;
29         for (int j = 0; j < n; ++j) {
30             labelx[i] = std::max(labelx[i], graph[i][j]);
31         }
32     }
33     for (int i = 0; i < n; ++i) {
34         while (true) {
35             std::fill(visitx, visitx + n, 0);
36             std::fill(visity, visity + n, 0);
37             for (int j = 0; j < n; ++j) {
38                 slack[j] = INT_MAX;
39             }
40             if (dfs(i)) {
41                 break;
42             }
43             int delta = INT_MAX;
44             for (int j = 0; j < n; ++j) {
45                 if (!visity[j]) {
46                     delta = std::min(delta, slack[j]);
47                 }
48             }
49             for (int j = 0; j < n; ++j) {
50                 if (visitx[j]) {

```

```

51         labelx[j] -= delta;
52     }
53     if (visity[j]) {
54         labely[j] += delta;
55     } else {
56         slack[j] -= delta;
57     }
58 }
59 }
60 }
61 int answer = 0;
62 for (int i = 0; i < n; ++i) {
63     answer += graph[match[i]][i];
64 }
65 return answer;
66 }

```

4.5 最大流 (dinic)

时间复杂度: $\mathcal{O}(V^2 \cdot E)$

```

1  struct edge{int u,v,cap,flow;};
2  vector<edge>edges;
3  vector<int>G[maxn];
4  int s,t;
5  int cur[maxn],d[maxn];
6  void add(int u,int v,int cap){
7      edges.push_back((edge){u,v,cap,0});
8      G[u].push_back(edges.size()-1);
9      edges.push_back((edge){v,u,0,0});
10     G[v].push_back(edges.size()-1);
11 }
12 bool bfs(){
13     static int vis[maxn];
14     memset(vis,0,sizeof vis);vis[s]=1;
15     queue<int>q;q.push(s);d[s]=0;
16     while(!q.empty()){
17         int u=q.front();q.pop();
18         for(int i=0;i<G[u].size();i++){
19             edge e=edges[G[u][i]];if(vis[e.v]||e.cap==e.flow)continue;
20             d[e.v]=d[u]+1;vis[e.v]=1;q.push(e.v);
21         }
22     }return vis[t];

```

```

23 }
24 int dfs(int u,int a){
25     if(u==t||!a)return a;
26     int flow=0,f;
27     for(int &i=cur[u];i<G[u].size();i++){
28         edge e=edges[G[u][i]];
29         if(d[e.v]==d[u]+1&&(f=dfs(e.v,min(a,e.cap-e.flow)))>0){
30             edges[G[u][i]].flow+=f;
31             edges[G[u][i]^1].flow-=f;
32             flow+=f;a-=f;if(!a)break;
33         }
34     }return flow;
35 }
36 int dinic(){
37     int flow=0,x;
38     while(bfs()){
39         memset(cur,0,sizeof cur);
40         while(x=dfs(s,INT_MAX)){
41             flow+=x;
42             memset(cur,0,sizeof cur);
43         }
44     }return flow;
45 }

```

4.6 最大流 (sap)

时间复杂度: $\mathcal{O}(V^2 \cdot E)$

```

1 int g[T], adj[M], nxt[M], f[M];
2 int cnt[T], dist[T], cur[T], fa[T], dat[T];
3 void Ins(int x, int y, int ff, int rf){
4     adj[++tot] = y; nxt[tot] = g[x]; g[x] = tot; f[tot] = ff;
5     adj[++tot] = x; nxt[tot] = g[y]; g[y] = tot; f[tot] = rf;
6 }
7 int sap(int s, int t){
8     int x, sum;
9     for (int i = 1; i <= t; i++){
10         dist[i] = 1;
11         cur[i] = g[i];
12         fa[i] = 0;
13         dat[i] = 0;
14         cnt[i] = 0;
15     }

```

```

16     cnt[0] = 1; cnt[1] = t - 1;
17     dist[t] = 0;
18     dat[s] = INF;
19     x = s;
20     sum = 0;
21     while (1){
22         int p;
23         for (p = cur[x]; p; p = nxt[p]){
24             if (f[p] > 0 && dist[adj[p]] == dist[x] - 1) break;
25         }
26         if (p > 0){
27             cur[x] = p;
28             fa[adj[p]] = p;
29             dat[adj[p]] = min(dat[x], f[p]);
30             x = adj[p];
31             if (x == t){
32                 sum += dat[x];
33                 while (x != s){
34                     f[fa[x]] -= dat[t];
35                     f[fa[x] ^ 1] += dat[t];
36                     x = adj[fa[x] ^ 1];
37                 }
38             }
39         } else {
40             cnt[dist[x]]--;
41             if (cnt[dist[x]] == 0) return sum;
42             dist[x] = t + 1;
43             for (int p = g[x]; p; p = nxt[p]){
44                 if (f[p] > 0 && dist[adj[p]] + 1 < dist[x]){
45                     dist[x] = dist[adj[p]] + 1;
46                     cur[x] = p;
47                 }
48             }
49             cnt[dist[x]]++;
50             if (dist[s] > t) return sum;
51             if (x != s) x = adj[fa[x] ^ 1];
52         }
53     }
54 }
55 /*
56 tot = 1
57 edges' id start from 2

```


58 *remember to clean g*
59 *t is the number of points*
60 **/*

4.7 上下界网络流

$B(u, v)$ 表示边 (u, v) 流量的下界, $C(u, v)$ 表示边 (u, v) 流量的上界, $F(u, v)$ 表示边 (u, v) 的流量。设 $G(u, v) = F(u, v) - B(u, v)$, 显然有

$$0 \leq G(u, v) \leq C(u, v) - B(u, v)$$

4.7.1 无源汇的上下界可行流

建立超级源点 S^* 和超级汇点 T^* , 对于原图每条边 (u, v) 在新网络中连如下三条边: $S^* \rightarrow v$, 容量为 $B(u, v)$; $u \rightarrow T^*$, 容量为 $B(u, v)$; $u \rightarrow v$, 容量为 $C(u, v) - B(u, v)$ 。最后求新网络的最大流, 判断从超级源点 S^* 出发的边是否都满流即可, 边 (u, v) 的最终解中的实际流量为 $G(u, v) + B(u, v)$ 。

4.7.2 有源汇的上下界可行流

从汇点 T 到源点 S 连一条上界为 ∞ , 下界为 0 的边。按照无源汇的上下界可行流一样做即可, 流量即为 $T \rightarrow S$ 边上的流量。

4.7.3 有源汇的上下界最大流

1. 在有源汇的上下界可行流中, 从汇点 T 到源点 S 的边改为连一条上界为 ∞ , 下届为 x 的边。 x 满足二分性质, 找到最大的 x 使得新网络存在无源汇的上下界可行流即为原图的最大流。
2. 从汇点 T 到源点 S 连一条上界为 ∞ , 下界为 0 的边, 变成无源汇的网络。按照无源汇的上下界可行流的方法, 建立超级源点 S^* 和超级汇点 T^* , 求一遍 $S^* \rightarrow T^*$ 的最大流, 再将汇点 T 到源点 S 的这条边拆掉, 求一次 $S \rightarrow T$ 的最大流即可。

4.7.4 有源汇的上下界最小流

1. 在有源汇的上下界可行流中, 从汇点 T 到源点 S 的边改为连一条上界为 x , 下界为 0 的边。 x 满足二分性质, 找到最小的 x 使得新网络存在无源汇的上下界可行流即为原图的最小流。
2. 按照无源汇的上下界可行流的方法, 建立超级源点 S^* 与超级汇点 T^* , 求一遍 $S^* \rightarrow T^*$ 的最大流, 但是注意这一次不加上汇点 T 到源点 S 的这条边, 即不使之改为无源汇的网络去求解。求完后, 再加上那条汇点 T 到源点 S 上界 ∞ 的边。因为这条边下界为 0 , 所以 S^*, T^* 无影响, 再直接求一次 $S^* \rightarrow T^*$ 的最大流。若超级源点 S^* 出发的边全部满流, 则 $T \rightarrow S$ 边上的流量即为原图的最小流, 否则无解。

4.8 最小费用最大流

4.8.1 稀疏图

时间复杂度: $\mathcal{O}(V \cdot E^2)$

```
1 struct EdgeList {
2     int size;
3     int last[N];
4     int succ[M], other[M], flow[M], cost[M];
5     void clear(int n) {
6         size = 0;
7         std::fill(last, last + n, -1);
8     }
9     void add(int x, int y, int c, int w) {
10         succ[size] = last[x];
11         last[x] = size;
12         other[size] = y;
13         flow[size] = c;
14         cost[size++] = w;
15     }
16 } e;
17
18 int n, source, target;
19 int prev[N];
20
21 void add(int x, int y, int c, int w) {
22     e.add(x, y, c, w);
23     e.add(y, x, 0, -w);
24 }
25
26 bool augment() {
27     static int dist[N], occur[N];
28     std::vector<int> queue;
29     std::fill(dist, dist + n, INT_MAX);
30     std::fill(occur, occur + n, 0);
31     dist[source] = 0;
32     occur[source] = true;
33     queue.push_back(source);
34     for (int head = 0; head < (int)queue.size(); ++head) {
35         int x = queue[head];
36         for (int i = e.last[x]; ~i; i = e.succ[i]) {
37             int y = e.other[i];
38             if (e.flow[i] && dist[y] > dist[x] + e.cost[i]) {
```

```

39         dist[y] = dist[x] + e.cost[i];
40         prev[y] = i;
41         if (!occur[y]) {
42             occur[y] = true;
43             queue.push_back(y);
44         }
45     }
46 }
47 occur[x] = false;
48 }
49 return dist[target] < INT_MAX;
50 }
51
52 std::pair<int, int> solve() {
53     std::pair<int, int> answer = std::make_pair(0, 0);
54     while (augment()) {
55         int number = INT_MAX;
56         for (int i = target; i != source; i = e.other[prev[i] ^ 1]) {
57             number = std::min(number, e.flow[prev[i]]);
58         }
59         answer.first += number;
60         for (int i = target; i != source; i = e.other[prev[i] ^ 1]) {
61             e.flow[prev[i]] -= number;
62             e.flow[prev[i] ^ 1] += number;
63             answer.second += number * e.cost[prev[i]];
64         }
65     }
66     return answer;
67 }

```

4.8.2 稠密图

使用条件：费用非负

时间复杂度： $\mathcal{O}(V \cdot E^2)$

```

1 struct EdgeList {
2     int size;
3     int last[N];
4     int succ[M], other[M], flow[M], cost[M];
5     void clear(int n) {
6         size = 0;
7         std::fill(last, last + n, -1);
8     }

```

```

9     void add(int x, int y, int c, int w) {
10         succ[size] = last[x];
11         last[x] = size;
12         other[size] = y;
13         flow[size] = c;
14         cost[size++] = w;
15     }
16 } e;
17
18 int n, source, target, flow, cost;
19 int slack[N], dist[N];
20 bool visit[N];
21
22 void add(int x, int y, int c, int w) {
23     e.add(x, y, c, w);
24     e.add(y, x, 0, -w);
25 }
26
27 bool relabel() {
28     int delta = INT_MAX;
29     for (int i = 0; i < n; ++i) {
30         if (!visit[i]) {
31             delta = std::min(delta, slack[i]);
32         }
33         slack[i] = INT_MAX;
34     }
35     if (delta == INT_MAX) {
36         return true;
37     }
38     for (int i = 0; i < n; ++i) {
39         if (visit[i]) {
40             dist[i] += delta;
41         }
42     }
43     return false;
44 }
45
46 int dfs(int x, int answer) {
47     if (x == target) {
48         flow += answer;
49         cost += answer * (dist[source] - dist[target]);
50         return answer;

```

```

51     }
52     visit[x] = true;
53     int delta = answer;
54     for (int i = e.last[x]; ~i; i = e.succ[i]) {
55         int y = e.other[i];
56         if (e.flow[i] > 0 && !visit[y]) {
57             if (dist[y] + e.cost[i] == dist[x]) {
58                 int number = dfs(y, std::min(e.flow[i], delta));
59                 e.flow[i] -= number;
60                 e.flow[i ^ 1] += number;
61                 delta -= number;
62                 if (delta == 0) {
63                     dist[x] = INT_MIN;
64                     return answer;
65                 }
66             } else {
67                 slack[y] = std::min(slack[y], dist[y] + e.cost[i] - dist[x]);
68             }
69         }
70     }
71     return answer - delta;
72 }
73
74 std::pair<int, int> solve() {
75     flow = cost = 0;
76     std::fill(dist, dist + n, 0);
77     do {
78         do {
79             fill(visit, visit + n, 0);
80         } while (dfs(source, INT_MAX));
81     } while (!relabel());
82     return std::make_pair(flow, cost);
83 }

```

4.9 一般图最大匹配

时间复杂度: $\mathcal{O}(V^3)$

```

1  int match[N], belong[N], next[N], mark[N], visit[N];
2  std::vector<int> queue;
3
4  int find(int x) {
5      if (belong[x] != x) {

```

```

6         belong[x] = find(belong[x]);
7     }
8     return belong[x];
9 }
10
11 void merge(int x, int y) {
12     x = find(x);
13     y = find(y);
14     if (x != y) {
15         belong[x] = y;
16     }
17 }
18
19 int lca(int x, int y) {
20     static int stamp = 0;
21     stamp++;
22     while (true) {
23         if (x != -1) {
24             x = find(x);
25             if (visit[x] == stamp) {
26                 return x;
27             }
28             visit[x] = stamp;
29             if (match[x] != -1) {
30                 x = next[match[x]];
31             } else {
32                 x = -1;
33             }
34         }
35         std::swap(x, y);
36     }
37 }
38
39 void group(int a, int p) {
40     while (a != p) {
41         int b = match[a], c = next[b];
42         if (find(c) != p) {
43             next[c] = b;
44         }
45         if (mark[b] == 2) {
46             mark[b] = 1;
47             queue.push_back(b);

```

```

48     }
49     if (mark[c] == 2) {
50         mark[c] = 1;
51         queue.push_back(c);
52     }
53     merge(a, b);
54     merge(b, c);
55     a = c;
56 }
57 }
58
59 void augment(int source) {
60     queue.clear();
61     for (int i = 0; i < n; ++i) {
62         next[i] = visit[i] = -1;
63         belong[i] = i;
64         mark[i] = 0;
65     }
66     mark[source] = 1;
67     queue.push_back(source);
68     for (int head = 0; head < (int)queue.size() && match[source] == -1; ++head) {
69         int x = queue[head];
70         for (int i = 0; i < (int)edge[x].size(); ++i) {
71             int y = edge[x][i];
72             if (match[x] == y || find(x) == find(y) || mark[y] == 2) {
73                 continue;
74             }
75             if (mark[y] == 1) {
76                 int r = lca(x, y);
77                 if (find(x) != r) {
78                     next[x] = y;
79                 }
80                 if (find(y) != r) {
81                     next[y] = x;
82                 }
83                 group(x, r);
84                 group(y, r);
85             } else if (match[y] == -1) {
86                 next[y] = x;
87                 for (int u = y; u != -1; ) {
88                     int v = next[u];
89                     int mv = match[v];

```

```

90         match[v] = u;
91         match[u] = v;
92         u = mv;
93     }
94     break;
95 } else {
96     next[y] = x;
97     mark[y] = 2;
98     mark[match[y]] = 1;
99     queue.push_back(match[y]);
100 }
101 }
102 }
103 }
104
105 int solve() {
106     std::fill(match, match + n, -1);
107     for (int i = 0; i < n; ++i) {
108         if (match[i] == -1) {
109             augment(i);
110         }
111     }
112     int answer = 0;
113     for (int i = 0; i < n; ++i) {
114         answer += (match[i] != -1);
115     }
116     return answer;
117 }

```

4.10 无向图全局最小割

时间复杂度: $\mathcal{O}(V^3)$

注意事项: 处理重边时, 应该对边权累加

```

1  int node[N], dist[N];
2  bool visit[N];
3
4  int solve(int n) {
5      int answer = INT_MAX;
6      for (int i = 0; i < n; ++i) {
7          node[i] = i;
8      }
9      while (n > 1) {

```



```

10     int max = 1;
11     for (int i = 0; i < n; ++i) {
12         dist[node[i]] = graph[node[0]][node[i]];
13         if (dist[node[i]] > dist[node[max]]) {
14             max = i;
15         }
16     }
17     int prev = 0;
18     memset(visit, 0, sizeof(visit));
19     visit[node[0]] = true;
20     for (int i = 1; i < n; ++i) {
21         if (i == n - 1) {
22             answer = std::min(answer, dist[node[max]]);
23             for (int k = 0; k < n; ++k) {
24                 graph[node[k]][node[prev]] =
25                     (graph[node[prev]][node[k]] += graph[node[k]][node[max]]);
26             }
27             node[max] = node[--n];
28         }
29         visit[node[max]] = true;
30         prev = max;
31         max = -1;
32         for (int j = 1; j < n; ++j) {
33             if (!visit[node[j]]) {
34                 dist[node[j]] += graph[node[prev]][node[j]];
35                 if (max == -1 || dist[node[max]] < dist[node[j]]) {
36                     max = j;
37                 }
38             }
39         }
40     }
41 }
42 return answer;
43 }

```

4.11 有根树的同构

时间复杂度: $\mathcal{O}(V \log V)$

```

1  const unsigned long long MAGIC = 4423;
2
3  unsigned long long magic[N];
4  std::pair<unsigned long long, int> hash[N];

```

```

5
6 void solve(int root) {
7     magic[0] = 1;
8     for (int i = 1; i <= n; ++i) {
9         magic[i] = magic[i - 1] * MAGIC;
10    }
11    std::vector<int> queue;
12    queue.push_back(root);
13    for (int head = 0; head < (int)queue.size(); ++head) {
14        int x = queue[head];
15        for (int i = 0; i < (int)son[x].size(); ++i) {
16            int y = son[x][i];
17            queue.push_back(y);
18        }
19    }
20    for (int index = n - 1; index >= 0; --index) {
21        int x = queue[index];
22        hash[x] = std::make_pair(0, 0);
23
24        std::vector<std::pair<unsigned long long, int> > value;
25        for (int i = 0; i < (int)son[x].size(); ++i) {
26            int y = son[x][i];
27            value.push_back(hash[y]);
28        }
29        std::sort(value.begin(), value.end());
30
31        hash[x].first = hash[x].first * magic[1] + 37;
32        hash[x].second++;
33        for (int i = 0; i < (int)value.size(); ++i) {
34            hash[x].first = hash[x].first * magic[value[i].second] + value[i].first;
35            hash[x].second += value[i].second;
36        }
37        hash[x].first = hash[x].first * magic[1] + 41;
38        hash[x].second++;
39    }
40 }

```

4.12 哈密尔顿回路 (ORE 性质的图)

ORE 性质:

$$\forall x, y \in V \wedge (x, y) \notin E \quad s.t. \quad deg_x + deg_y \geq n$$

返回结果：从顶点 1 出发的一个哈密顿回路
使用条件： $n \geq 3$

```
1  int left[N], right[N], next[N], last[N];
2
3  void cover(int x) {
4      left[right[x]] = left[x];
5      right[left[x]] = right[x];
6  }
7
8  int adjacent(int x) {
9      for (int i = right[0]; i <= n; i = right[i]) {
10         if (graph[x][i]) {
11             return i;
12         }
13     }
14     return 0;
15 }
16
17 std::vector<int> solve() {
18     for (int i = 1; i <= n; ++i) {
19         left[i] = i - 1;
20         right[i] = i + 1;
21     }
22     int head, tail;
23     for (int i = 2; i <= n; ++i) {
24         if (graph[1][i]) {
25             head = 1;
26             tail = i;
27             cover(head);
28             cover(tail);
29             next[head] = tail;
30             break;
31         }
32     }
33     while (true) {
34         int x;
35         while (x = adjacent(head)) {
36             next[x] = head;
37             head = x;
38             cover(head);
39         }
40         while (x = adjacent(tail)) {
```

```

41         next[tail] = x;
42         tail = x;
43         cover(tail);
44     }
45     if (!graph[head][tail]) {
46         for (int i = head, j; i != tail; i = next[i]) {
47             if (graph[head][next[i]] && graph[tail][i]) {
48                 for (j = head; j != i; j = next[j]) {
49                     last[next[j]] = j;
50                 }
51                 j = next[head];
52                 next[head] = next[i];
53                 next[tail] = i;
54                 tail = j;
55                 for (j = i; j != head; j = last[j]) {
56                     next[j] = last[j];
57                 }
58                 break;
59             }
60         }
61     }
62     next[tail] = head;
63     if (right[0] > n) {
64         break;
65     }
66     for (int i = head; i != tail; i = next[i]) {
67         if (adjacent(i)) {
68             head = next[i];
69             tail = i;
70             next[tail] = 0;
71             break;
72         }
73     }
74 }
75 std::vector<int> answer;
76 for (int i = head; ; i = next[i]) {
77     if (i == 1) {
78         answer.push_back(i);
79         for (int j = next[i]; j != i; j = next[j]) {
80             answer.push_back(j);
81         }
82         answer.push_back(i);

```

```

83         break;
84     }
85     if (i == tail) {
86         break;
87     }
88 }
89 return answer;
90 }

```

5 字符串

5.1 模式匹配

5.1.1 KMP 算法

```

1 void build(char *pattern) {
2     int length = (int)strlen(pattern + 1);
3     fail[0] = -1;
4     for (int i = 1, j; i <= length; ++i) {
5         for (j = fail[i - 1]; j != -1 && pattern[i] != pattern[j + 1]; j = fail[j]);
6         fail[i] = j + 1;
7     }
8 }
9
10 void solve(char *text, char *pattern) {
11     int length = (int)strlen(text + 1);
12     for (int i = 1, j; i <= length; ++i) {
13         for (j = match[i - 1]; j != -1 && text[i] != pattern[j + 1]; j = fail[j]);
14         match[i] = j + 1;
15     }
16 }
17 ///Hint: 1 - Base

```

5.1.2 扩展 KMP 算法

返回结果:

$$next_i = lcp(text, text_{i...n-1})$$

```

1 void solve(char *text, int length, int *next) {
2     int j = 0, k = 1;
3     for (; j + 1 < length && text[j] == text[j + 1]; j++);
4     next[0] = length - 1;

```

```

5     next[1] = j;
6     for (int i = 2; i < length; ++i) {
7         int far = k + next[k] - 1;
8         if (next[i - k] < far - i + 1) {
9             next[i] = next[i - k];
10        } else {
11            j = std::max(far - i + 1, 0);
12            for (; i + j < length && text[j] == text[i + j]; j++);
13            next[i] = j;
14            k = i;
15        }
16    }
17 }
18 /// 0 - Base

```

5.1.3 AC 自动机

```

1 struct Node{
2     int Next[30], fail, mark;
3 }Tree[N];
4
5 void Init(){
6     memset(Tree, 0, sizeof Tree);
7     cnt = 1;
8
9     for (int i = 1; i <= n; i++){
10        char c;
11        int now = 1;
12        scanf("%s", s + 1);
13        int Length = strlen(s + 1);
14        for (int j = 1; j <= Length; j++){
15            c = s[j];
16            if (Tree[now].Next[c - 'a']) now = Tree[now].Next[c - 'a']; else
17                Tree[now].Next[c - 'a'] = ++ cnt, now = cnt;
18        }
19    }
20 }
21
22 void Build_Ac(){
23     int en = 0;
24     Q[0] = 1;
25     for (int fi = 0; fi <= en; fi++){

```

```

26     int now = Q[fi];
27     for (int next = 0; next < 26; next++)
28         if (Tree[now].Next[next])
29             {
30                 int k = Tree[now].Next[next];
31                 if (now == 1) Tree[k].fail = 1; else
32                 {
33                     int h = Tree[now].fail;
34                     while (h && !Tree[h].Next[next]) h = Tree[h].fail;
35                     if (!h) Tree[k].fail = 1;
36                     else Tree[k].fail = Tree[h].Next[next];
37                 }
38                 Q[++ en] = k;
39             }
40     }
41 }
42
43 /// Hints : when not match , fail = 1

```

5.2 后缀三姐妹

5.2.1 后缀数组

```

1 struct Sa{
2     int heap[N],s[N],sa[N],r[N],tr[N],sec[N],m,cnt;
3     int h[19][N];
4
5     void Prep(){
6         for (int i=1; i<=m; i++) heap[i]=0;
7         for (int i=1; i<=n; i++) heap[s[i]]++;
8         for (int i=2; i<=m; i++) heap[i]+=heap[i-1];
9         for (int i=n; i>=1; i--) sa[heap[s[i]]--]=i;
10        r[sa[1]]=1; cnt=1;
11        for (int i=2; i<=n; i++){
12            if (s[sa[i]]!=s[sa[i-1]]) cnt++;
13            r[sa[i]]=cnt;
14        }
15        m=cnt;
16    }
17
18    void Suffix(){
19        int j=1;

```

```

20     while (cnt<n){
21         cnt=0;
22         for (int i=n-j+1; i<=n; i++) sec[++cnt]=i;
23         for (int i=1; i<=n; i++) if (sa[i]>j)
24             sec[++cnt]=sa[i]-j;
25         for (int i=1; i<=n; i++) tr[i]=r[sec[i]];
26         for (int i=1; i<=m; i++) heap[i]=0;
27         for (int i=1; i<=n; i++) heap[tr[i]]++;
28         for (int i=2; i<=m; i++) heap[i]+=heap[i-1];
29         for (int i=n; i>=1; i--)
30             sa[heap[tr[i]]--]=sec[i];
31         tr[sa[1]]=1; cnt=1;
32         for (int i=2; i<=n; i++){
33             if ((r[sa[i]]!=r[sa[i-1]]) || (r[sa[i]+j]!=r[sa[i-1]+j]))
34                 cnt++;
35             tr[sa[i]]=cnt;
36         }
37         for (int i=1; i<=n; i++) r[i]=tr[i];
38         m=cnt; j=j+j;
39     }
40 }
41
42 void Calc(){
43     int k=0;
44     for (int i=1; i<=n; i++){
45         if (r[i]==1) continue;
46         int j=sa[r[i]-1];
47         while ((i+k<=n) && (j+k<=n) && (s[i+k]==s[j+k])) k++;
48         h[0][r[i]]=k;
49         if (k) k--;
50     }
51     for (int i=1; i<19; i++)
52         for (int j=1; j+(1<<i)-1<=n; j++)
53             h[i][j]=min(h[i-1][j],h[i-1][j + (1<<(i - 1)) + 1]);
54 }
55
56 int Query(int L,int R){
57     L=r[L], R=r[R];
58     if (L>R) swap(L,R);
59     L++;
60     int l0 = Lg[R-L+1];
61     return min(h[l0][L],h[l0][R-(1<<l0)+1]);

```



```

62     }
63
64     void Work(){
65         Prep(); Suffix(); Calc();
66     }
67 }P,S;
68
69 /// Hints : 1 - Base

```

5.2.2 后缀数组 (dc3)

```

1  ///`DC3` 待排序的字符串放在  $r$  数组中, 从  $r[0]$  到  $r[n-1]$ , 长度为  $n$ , 且最大值小于  $m$ .`
2  ///`约定除  $r[n-1]$  外所有的  $r[i]$  都大于  $0$ ,  $r[n-1]=0$ `.`
3  ///`函数结束后, 结果放在  $sa$  数组中, 从  $sa[0]$  到  $sa[n-1]$ `.`
4  ///` $r$  必须开长度乘 3`
5  #define maxn 10000
6  #define F(x) ((x)/3+((x)%3==1?0:tb))
7  #define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)
8
9  int wa[maxn],wb[maxn],wv[maxn],wss[maxn];
10 int s[maxn*3],sa[maxn*3];
11 int c0(int *r,int a,int b)
12 {
13     return r[a]==r[b]&&r[a+1]==r[b+1]&&r[a+2]==r[b+2];
14 }
15 int c12(int k,int *r,int a,int b)
16 {
17     if(k==2) return r[a]<r[b]||r[a]==r[b]&&c12(1,r,a+1,b+1);
18     else return r[a]<r[b]||r[a]==r[b]&&wv[a+1]<wv[b+1];
19 }
20 void sort(int *r,int *a,int *b,int n,int m)
21 {
22     int i;
23     for(i=0;i<n;i++) wv[i]=r[a[i]];
24     for(i=0;i<m;i++) wss[i]=0;
25     for(i=0;i<n;i++) wss[wv[i]]++;
26     for(i=1;i<m;i++) wss[i]+=wss[i-1];
27     for(i=n-1;i>=0;i--) b[--wss[wv[i]]]=a[i];
28 }
29 void dc3(int *r,int *sa,int n,int m)
30 {
31     int i,j,*rn=r+n,*san=sa+n,ta=0,tb=(n+1)/3,tbc=0,p;

```

```

32     r[n]=r[n+1]=0;
33     for(i=0;i<n;i++)
34         if(i%3!=0) wa[tbc++]=i;
35     sort(r+2,wa,wb,tbc,m);
36     sort(r+1,wb,wa,tbc,m);
37     sort(r,wa,wb,tbc,m);
38     for(p=1,rn[F(wb[0])]=0,i=1;i<tbc;i++)
39         rn[F(wb[i])]=c0(r,wb[i-1],wb[i])?p-1:p++;
40     if (p<tbc) dc3(rn,san,tbc,p);
41     else for (i=0;i<tbc;i++) san[rn[i]]=i;
42     for (i=0;i<tbc;i++)
43         if(san[i]<tb) wb[ta++]=san[i]*3;
44     if(n%3==1) wb[ta++]=n-1;
45     sort(r,wb,wa,ta,m);
46     for(i=0;i<tbc;i++)
47         wv[wb[i]=G(san[i])]=i;
48     for(i=0,j=0,p=0;i<ta && j<tbc;p++)
49         sa[p]=c12(wb[j]%3,r,wa[i],wb[j])?wa[i++]:wb[j++];
50     for(;i<ta;p++) sa[p]=wa[i++];
51     for(;j<tbc;p++) sa[p]=wb[j++];
52 }
53
54 int main(){
55     int n,m=0;
56     scanf("%d",&n);
57     for (int i=0;i<n;i++) scanf("%d",&s[i]),s[i]++,m=max(s[i]+1,m);
58     printf("%d\n",m);
59     s[n++]=0;
60     dc3(s,sa,n,m);
61     for (int i=0;i<n;i++) printf("%d ",sa[i]);printf("\n");
62 }

```

5.2.3 后缀自动机-多串 LCS

对一个串建后缀自动机，其他串在上面匹配，因为是求所有串的公共子串，所以每个点记录每个串最长匹配长度的最小值，最后找到所有点中最长的一个即可。一个注意事项就是，当走到一个点时，还要更新它的 `parent` 树上的祖先的匹配长度，数组开两倍啦啦啦！

```

1 struct Node{
2     int len, fail;
3     int To[30];
4 }T[N];
5 int Lst, Root, tot, ans;

```

```

6  char s[N];
7  int Len[N], Ans[N], Ord[N];
8  void Add(int x, int l){
9      int Nt = ++tot, p = Lst;
10     T[Nt].len = 1;
11     for (;p && !T[p].To[x]; p = T[p].fail) T[p].To[x] = Nt;
12     if (!p) T[Nt].fail = Root; else
13     if (T[T[p].To[x]].len == T[p].len + 1) T[Nt].fail = T[p].To[x];
14     else{
15         int q = ++tot, qt = T[p].To[x];
16         T[q] = T[qt];
17         T[q].len = T[p].len + 1;
18         T[qt].fail = T[Nt].fail = q;
19         for (;p && T[p].To[x] == qt; p = T[p].fail) T[p].To[x] = q;
20     }
21     Lst = Nt;
22 }
23 bool cmp(int a, int b){
24     return T[a].len < T[b].len;
25 }
26 int main(){
27     scanf("%s", s + 1);
28     int n = strlen(s + 1);
29     ans = n;
30     Root = tot = Lst = 1;
31     for (int i = 1; i <= n; i++)
32         Add(s[i] - 'a' + 1, i);
33     for (int i = 1; i <= tot; i++)
34         Ord[i] = i;
35     sort(Ord + 1, Ord + tot + 1, cmp);
36     for (int i = 1; i <= tot; i++)
37         Ans[i] = T[i].len;
38     bool flag = 0;
39     while (scanf("%s", s + 1) != EOF){
40         flag = 1;
41         int n = strlen(s + 1);
42         int p = Root, len = 0;
43         for (int i = 1; i <= tot; i++) Len[i] = 0;
44         for (int i = 1; i <= n; i++){
45             int x = s[i] - 'a' + 1;
46             if (T[p].To[x]) len++, p = T[p].To[x];
47             else {

```

```

48         while (p && !T[p].To[x]) p = T[p].fail;
49         if (!p) p = Root, len = 0;
50         else len = T[p].len + 1, p = T[p].To[x];
51     }
52     Len[p] = max(Len[p], len);
53 }
54 for (int i = tot; i >= 1; i--){
55     int Cur = Ord[i];
56     Ans[Cur] = min(Ans[Cur], Len[Cur]);
57     if (Len[Cur] && T[Cur].fail)
58         Len[T[Cur].fail] = T[Cur].fail.len;
59 }
60 }
61 if (flag){
62     ans = 0;
63     for (int i = 1; i <= tot; i++){
64         ans = max(ans, Ans[i]);
65     }
66 }
67 printf("%d\n", ans);
68 return 0;
69 }

```

5.2.4 后缀自动机-各长度字串出现次数最大值

给一个字符串 S ，令 $F(x)$ 表示 S 的所有长度为 x 的子串中，出现次数的最大值。构建字符串的自动机，对于每个节点， right 集合大小就是出现次数， maxs 就是它代表的最长长度，那么我们用 $|\text{right}(x)|$ 去更新 $f[\text{maxs}[x]]$ 的值，最后从大到小用 $f[i]$ 去更新 $f[i-1]$ 的值即可

```

1 struct Node{
2     int len, fail;
3     int To[30];
4 }T[N];
5 int Lst, Root, tot, n;
6 char s[N];
7 int Ord[N], Ans[N], Ways[N], heap[N];
8 void Add(int x, int l){
9     int Nt = ++tot, p = Lst;
10    T[Nt].len = l;
11    for (;p && !T[p].To[x]; p = T[p].fail) T[p].To[x] = Nt;
12    if (!p) T[Nt].fail = Root; else
13    if (T[T[p].To[x]].len == T[p].len + 1) T[Nt].fail = T[p].To[x];

```

```

14     else{
15         int q = ++tot, qt = T[p].To[x];
16         T[q] = T[qt];
17         T[q].len = T[p].len + 1;
18         T[qt].fail = T[Nt].fail = q;
19         for (;p && T[p].To[x] == qt; p = T[p].fail) T[p].To[x] = q;
20     }
21     Lst = Nt;
22 }
23 bool cmp(int a, int b){
24     return T[a].len < T[b].len;
25 }
26 void sort(){
27     for (int i = 1; i <= tot; i++) heap[T[i].len]++;
28     for (int i = 1; i <= n; i++) heap[i] += heap[i-1];
29     for (int i = 1; i <= tot; i++) Ord[heap[T[i].len]--]=i;
30 }
31 int main(){
32     scanf("%s", s + 1);
33     n = strlen(s + 1);
34     Root = tot = Lst = 1;
35     for (int i = 1; i <= n; i++)
36         Add(s[i] - 'a' + 1, i);
37     sort();
38     memset(Ways , 0, sizeof(Ways));
39     for (int i = 1, p = Root; i <= n; i++)
40         p = T[p].To[s[i] - 'a' + 1], Ways[p] = 1;
41     for (int i = tot; i >= 1; i--){
42         int Cur = Ord[i];
43         if (T[Cur].fail == 0) continue;
44         Ways[T[Cur].fail] += Ways[Cur];
45     }
46     for (int i = 1; i <= tot; i++)
47         Ans[T[i].len] = max(Ans[T[i].len], Ways[i]);
48     for (int i = n; i >= 1; i--)
49         Ans[i] = max(Ans[i + 1], Ans[i]);
50     for (int i = 1; i <= n; i++)
51         printf("%d\n", Ans[i]);
52     return 0;
53 }

```

5.2.5 后缀自动机-两串 LCS

```
1 struct node{
2     int len, fail;
3     int To[27];
4 }T[N];
5 char a[N], b[N];
6 int Lst, Root, tot;
7 void add(int x, int l){
8     int Nt = ++tot, p = Lst;
9     T[Nt].len = l;
10    for (;p && !T[p].To[x]; p = T[p].fail) T[p].To[x] = Nt;
11    if (!p) T[Nt].fail = Root;
12    else
13        if (T[T[p].To[x]].len == T[p].len + 1) T[Nt].fail = T[p].To[x];
14        else{
15            int q = ++tot, qt = T[p].To[x];
16            T[q] = T[qt];
17            T[q].len = T[p].len + 1;
18            T[qt].fail = T[Nt].fail = q;
19            for (;p && T[p].To[x] == qt; p = T[p].fail) T[p].To[x] = q;
20        }
21    Lst = Nt;
22 }
23 int main(){
24     while (scanf("%s%s", a + 1, b + 1) == 2){
25         int n = strlen(a + 1);
26         Lst = Root = tot = 1;
27         for (int i = 1; i <= n; i++)
28             add(a[i] - 'a' + 1, i);
29         int m = strlen(b + 1);
30         int p = Root, len = 0;
31         int Ans = 0;
32         for (int i = 1; i <= m; i++){
33             int x = b[i] - 'a' + 1;
34             if (T[p].To[x]) len++, p = T[p].To[x];
35             else {
36                 while (p && !T[p].To[x]) p = T[p].fail;
37                 if (!p) p = Root, len = 0;
38                 else len = T[p].len + 1, p = T[p].To[x];
39             }
40             if (len > Ans) Ans = len;
41         }
42     }
43 }
```

```

41     }
42     printf("%d\n", Ans);
43     for (int i = 1; i <= tot; i++){
44         T[i].len = T[i].fail = 0;
45         for (int j = 1; j <= 26; j++)
46             T[i].To[j] = 0;
47     }
48 }
49 return 0;
50 }
51 //Hints£°SAM + Longest common subsequence

```

5.3 回文三兄弟

5.3.1 马拉车

```

1 void Manacher(){
2     R[1] = 1;
3     for (int i = 2, j = 1; i <= length; i++){
4         if (j + R[j] <= i){
5             R[i] = 0;
6         } else {
7             R[i] = min(R[j * 2 - i], j + R[j] - i);
8         }
9         while (i - R[i] >= 1 && i + R[i] <= length
10             && text[i - R[i]] == text[i + R[i]]){
11             R[i]++;
12         }
13         if (i + R[i] > j + R[j]){
14             j = i;
15         }
16     }
17 }
18
19 length = 0;
20 int n = strlen(s + 1);
21 for (int i = 1; i <= n; i++){
22     text[++length] = '*';
23     text[++length] = s[i];
24 }
25 text[++length] = '*';
26
27 /// Hints: 1 - Base

```

5.3.2 回文树 (lyx)

```
1  const int N = 400005;
2
3  char s[N];
4  int Len;
5
6  struct Palindromic_Tree {
7      int next[N][27];
8      int fail[N];
9      int cnt[N];
10     int num[N];
11     int len[N];
12     char S[N];
13     int last;
14     int n;
15     int p;
16
17     int newnode(int l)
18     {
19         for(int i = 1; i <= 26; i++) next[p][i] = 0;
20         cnt[p] = 0;
21         num[p] = 0;
22         len[p] = l;
23         fail[p] = 0;
24         return p++;
25     }
26     void init()
27     {
28         p = 0;
29         newnode(0);
30         newnode(-1);
31         last = 0;
32         n = 0;
33         S[n] = -1;
34         fail[0] = 1;
35     }
36     int get_fail(int x)
37     {
38         while (S[n - len[x] - 1] != S[n]) x = fail[x];
39         return x;
40     }
```



```

41     void add(char c, int pos)
42     {
43         c = c - 'a' + 1;
44         S[++ n] = c ;
45         int cur = get_fail(last);
46         if (!next[cur][c])
47         {
48             int now = newnode(len[cur] + 2);
49             fail[now] = next[get_fail(fail[cur])][c];
50             next[cur][c] = now;
51             num[now] = num[fail[now]] + 1;
52         }
53         last = next[cur][c] ;
54         cnt[last]++;
55     }
56     void count()
57     {
58         for (int i = p - 1 ; i >= 0 ; -- i) cnt[fail[i]] += cnt[i] ;
59     }
60 }T;
61
62 Len = strlen(s + 1);
63
64 T.init();
65 for (int i = 1; i <= Len; i++)
66     T.add(s[i], i);
67 T.count();

```

5.3.3 回文自动机 (zky)

```

1  struct PAM{
2      int tot,last,str[maxn],nxt[maxn][26],n;
3      int len[maxn],suf[maxn],cnt[maxn];
4      int newnode(int l){
5          len[tot]=l;
6          return tot++;
7      }
8      void init(){
9          tot=0;
10         newnode(0);// tree0 is node 0
11         newnode(-1);// tree-1 is node 1
12         str[0]=-1;

```

```

13         suf[0]=1;
14     }
15     int find(int x){
16         while(str[n-len[x]-1]!=str[n])x=suf[x];
17         return x;
18     }
19     void add(int c){
20         str[++n]=c;
21         int u=find(last);
22         if(!nxt[u][c]){
23             int v=newnode(len[u]+2);
24             suf[v]=nxt[find(suf[u])][c];
25             nxt[u][c]=v;
26         }last=nxt[u][c];
27         cnt[last]++;
28     }
29     void count(){
30         for(int i=tot-1;i>=0;i--)cnt[suf[i]]+=cnt[i];
31     }
32 }P;
33 int main(){
34     P.init();
35     for(int i=0;i<n;i++)
36         P.add(s[i]-'a');
37     P.count();

```

5.4 循环串最小表示

```

1 string sol(char *s){
2     int n=strlen(s);
3     int i=0,j=1,k=0,p;
4     while(i<n&&j<n&&k<n){
5         int t=s[(i+k)%n]-s[(j+k)%n];
6         if(t==0)k++;
7         else if(t<0)j+=k+1,k=0;
8         else i+=k+1,k=0;
9         if(i==j)j++;
10    }p=min(i,j);
11    string S;
12    for(int i=p;i<p+n;i++)S.push_back(s[i%n]);
13    return S;
14 }

```

6 计算几何

6.1 二维基础

6.1.1 点类

```
1  int sgn(double x){return (x>eps)-(x<-eps);}
2  int sgn(double a,double b){return sgn(a-b);}
3  double sqr(double x){return x*x;}
4  struct P{
5      double x,y;
6      P(){}
7      P(double x,double y):x(x),y(y){}
8      double len2(){
9          return sqr(x)+sqr(y);
10     }
11     double len(){
12         return sqrt(len2());
13     }
14     void print(){
15         printf("%.3f,%.3f\n",x,y);
16     }
17     P turn90(){return P(-y,x);}
18     P norm(){return P(x/len(),y/len());}
19 };
20 bool operator==(P a,P b){
21     return !sgn(a.x-b.x) and !sgn(a.y-b.y);
22 }
23 P operator+(P a,P b){
24     return P(a.x+b.x,a.y+b.y);
25 }
26 P operator-(P a,P b){
27     return P(a.x-b.x,a.y-b.y);
28 }
29 P operator*(P a,double b){
30     return P(a.x*b,a.y*b);
31 }
32 P operator/(P a,double b){
33     return P(a.x/b,a.y/b);
34 }
35 double operator^(P a,P b){
```

```

36         return a.x*b.x + a.y*b.y;
37     }
38     double operator*(P a,P b){
39         return a.x*b.y - a.y*b.x;
40     }
41     double det(P a,P b,P c){
42         return (b-a)*(c-a);
43     }
44     double dis(P a,P b){
45         return (b-a).len();
46     }
47     double Area(vector<P>poly){
48         double ans=0;
49         for(int i=1;i<poly.size();i++)
50             ans+=(poly[i]-poly[0])*(poly[(i+1)%poly.size()]-poly[0]);
51         return fabs(ans)/2;
52     }
53     struct L{
54         P a,b;
55         L(){ }
56         L(P a,P b):a(a),b(b){ }
57         P v(){return b-a;}
58     };
59     bool onLine(P p,L l){
60         return sgn((l.a-p)*(l.b-p))==0;
61     }
62     bool onSeg(P p,L s){
63         return onLine(p,s) and sgn((s.b-s.a)^(p-s.a))>=0 and sgn((s.a-s.b)^(p-s.b))>=0;
64     }
65     bool parallel(L l1,L l2){
66         return sgn(l1.v()*l2.v())==0;
67     }
68     P intersect(L l1,L l2){
69         double s1=det(l1.a,l1.b,l2.a);
70         double s2=det(l1.a,l1.b,l2.b);
71         return (l2.a*s2-l2.b*s1)/(s2-s1);
72     }
73     P project(P p,L l){
74         return l.a+l.v()*((p-l.a)^l.v())/l.v().len2();
75     }
76     double dis(P p,L l){
77         return fabs((p-l.a)*l.v())/l.v().len();

```

78 }

6.1.2 凸包

```
1 vector<P> convex(vector<P>p){
2     sort(p.begin(),p.end());
3     vector<P>ans,S;
4     for(int i=0;i<p.size();i++){
5         while(S.size()>=2
6             && sgn(det(S[S.size()-2],S.back(),p[i]))<=0)
7             S.pop_back();
8         S.push_back(p[i]);
9     }//dw
10    ans=S;
11    S.clear();
12    for(int i=(int)p.size()-1;i>=0;i--){
13        while(S.size()>=2
14            && sgn(det(S[S.size()-2],S.back(),p[i]))<=0)
15            S.pop_back();
16        S.push_back(p[i]);
17    }//up
18    for(int i=1;i+1<S.size();i++)
19        ans.push_back(S[i]);
20    return ans;
21 }
```

6.1.3 半平面交

```
1 struct P{
2     int quad() const { return sgn(y) == 1 || (sgn(y) == 0 && sgn(x) >= 0); }
3 };
4 struct L{
5     bool onLeft(const P &p) const { return sgn((b - a)*(p - a)) > 0; }
6     L push() const { // push out eps
7         const double eps = 1e-10;
8         P delta = (b - a).turn90().norm() * eps;
9         return L(a - delta, b - delta);
10    }
11 };
12 bool sameDir(const L &l0, const L &l1) {
13     return parallel(l0, l1) && sgn((l0.b - l0.a)^(l1.b - l1.a)) == 1;
```

```

14 }
15 bool operator < (const P &a, const P &b) {
16     if (a.quad() != b.quad())
17         return a.quad() < b.quad();
18     else
19         return sgn((a*b)) > 0;
20 }
21 bool operator < (const L &l0, const L &l1) {
22     if (sameDir(l0, l1))
23         return l1.onLeft(l0.a);
24     else
25         return (l0.b - l0.a) < (l1.b - l1.a);
26 }
27 bool check(const L &u, const L &v, const L &w) {
28     return w.onLeft(intersect(u, v));
29 }
30 vector<P> intersection(vector<L> &l) {
31     sort(l.begin(), l.end());
32     deque<L> q;
33     for (int i = 0; i < (int)l.size(); ++i) {
34         if (i && sameDir(l[i], l[i - 1])) {
35             continue;
36         }
37         while (q.size() > 1
38             && !check(q[q.size() - 2], q[q.size() - 1], l[i]))
39             q.pop_back();
40         while (q.size() > 1
41             && !check(q[1], q[0], l[i]))
42             q.pop_front();
43         q.push_back(l[i]);
44     }
45     while (q.size() > 2
46         && !check(q[q.size() - 2], q[q.size() - 1], q[0]))
47         q.pop_back();
48     while (q.size() > 2
49         && !check(q[1], q[0], q[q.size() - 1]))
50         q.pop_front();
51     vector<P> ret;
52     for (int i = 0; i < (int)q.size(); ++i)
53         ret.push_back(intersect(q[i], q[(i + 1) % q.size()]));
54     return ret;
55 }

```

6.1.4 最近点对

```
1 bool byY(P a,P b){return a.y<b.y;}
2 LL solve(P *p,int l,int r){
3     LL d=1LL<<62;
4     if(l==r)
5         return d;
6     if(l+1==r)
7         return dis2(p[l],p[r]);
8     int mid=(l+r)>>1;
9     d=min(solve(l,mid),d);
10    d=min(solve(mid+1,r),d);
11    vector<P>tmp;
12    for(int i=l;i<=r;i++)
13        if(sqr(p[mid].x-p[i].x)<=d)
14            tmp.push_back(p[i]);
15    sort(tmp.begin(),tmp.end(),byY);
16    for(int i=0;i<tmp.size();i++)
17        for(int j=i+1;j<tmp.size()&&j-i<10;j++)
18            d=min(d,dis2(tmp[i],tmp[j]));
19    return d;
20 }
```

6.1.5 最小圆覆盖

```
1 struct line{
2     point p,v;
3 };
4 point Rev(point v){return point(-v.y,v.x);}
5 point operator*(line A,line B){
6     point u=B.p-A.p;
7     double t=(B.v*u)/(B.v*A.v);
8     return A.p+A.v*t;
9 }
10 point get(point a,point b){
11     return (a+b)/2;
12 }
13 point get(point a,point b,point c){
14     if(a==b)return get(a,c);
15     if(a==c)return get(a,b);
16     if(b==c)return get(a,b);
17     line ABO=(line){(a+b)/2,Rev(a-b)};
```

```

18     line BCO=(line){(c+b)/2,Rev(b-c)};
19     return ABO*BCO;
20 }
21 int main(){
22     scanf("%d",&n);
23     for(int i=1;i<=n;i++)scanf("%lf%lf",&p[i].x,&p[i].y);
24     random_shuffle(p+1,p+1+n);
25     O=p[1];r=0;
26     for(int i=2;i<=n;i++){
27         if(dis(p[i],O)<r+1e-6)continue;
28         O=get(p[1],p[i]);r=dis(O,p[i]);
29         for(int j=1;j<i;j++){
30             if(dis(p[j],O)<r+1e-6)continue;
31             O=get(p[i],p[j]);r=dis(O,p[i]);
32             for(int k=1;k<j;k++){
33                 if(dis(p[k],O)<r+1e-6)continue;
34                 O=get(p[i],p[j],p[k]);r=dis(O,p[i]);
35             }
36         }
37     }printf("%.21f %.21f %.21f\n",O.x,O.y,r);
38     return 0;
39 }s

```

6.2 多边形

6.2.1 判断点在多边形内部

```

1 bool InPoly(P p,vector<P>poly){
2     int cnt=0;
3     for(int i=0;i<poly.size();i++){
4         P a=poly[i],b=poly[(i+1)%poly.size()];
5         if(OnLine(p,L(a,b)))
6             return false;
7         int x=sgn(det(a,p,b));
8         int y=sgn(a.y-p.y);
9         int z=sgn(b.y-p.y);
10        cnt+=(x>0&&y<=0&&z>0);
11        cnt-=(x<0&&z<=0&&y>0);
12    }
13    return cnt;
14 }

```

7 其他

7.1 斯坦那树

```
1 priority_queue<pair<int, int> > Q;
2
3 // m is key point
4 // n is all point
5
6 for (int s = 0; s < (1 << m); s++){
7     for (int i = 1; i <= n; i++){
8         if (id[i]) continue;
9         for (int s0 = 0; s0 < s; s0++){
10             if ( (s0 & s) == s0 ){
11                 f[s][i] = min(f[s][i], f[s0][i] + f[s - s0][i]);
12             }
13         }
14         for (int i = 1; i <= n; i++) vis[i] = 0;
15         while (!Q.empty()) Q.pop();
16         for (int i = 1; i <= n; i++){
17             if (id[i]) continue;
18             Q.push(mp(-f[s][i], i));
19         }
20         while (!Q.empty()){
21             while (!Q.empty() && Q.top().first != -f[s][Q.top().second]) Q.pop();
22             if (Q.empty()) break;
23             int Cur = Q.top().second; Q.pop();
24             for (int p = g[Cur]; p; p = nxt[p]){
25                 int y = adj[p];
26                 if ( f[s][y] > f[s][Cur] + 1){
27                     f[s][y] = f[s][Cur] + 1;
28                     Q.push(mp(-f[s][y], y));
29                 }
30             }
31         }
32     }
```

7.2 最小树形图

```
1 const int maxn=1100;
2
3 int n,m , g[maxn][maxn] , used[maxn] , pass[maxn] , eg[maxn] , more , queue[maxn];
```

```

4
5 void combine (int id , int &sum ) {
6     int tot = 0 , from , i , j , k ;
7     for ( ; id!=0 && !pass[ id ] ; id=eg[id] ) {
8         queue[tot++]=id ; pass[id]=1;
9     }
10    for ( from=0; from<tot && queue[from]!=id ; from++);
11    if ( from==tot ) return ;
12    more = 1 ;
13    for ( i=from ; i<tot ; i++) {
14        sum+=g[eg[queue[i]]][queue[i]] ;
15        if ( i!=from ) {
16            used[queue[i]]=1;
17            for ( j = 1 ; j <= n ; j++) if ( !used[j] )
18                if ( g[queue[i]][j]<g[id][j] ) g[id][j]=g[queue[i]][j] ;
19        }
20    }
21    for ( i=1; i<=n ; i++) if ( !used[i] && i!=id ) {
22        for ( j=from ; j<tot ; j++){
23            k=queue[j];
24            if ( g[i][id]>g[i][k]-g[eg[k]][k] ) g[i][id]=g[i][k]-g[eg[k]][k];
25        }
26    }
27 }
28
29 int mdst( int root ) { // return the total length of MDST
30     int i , j , k , sum = 0 ;
31     memset ( used , 0 , sizeof ( used ) ) ;
32     for ( more =1; more ; ) {
33         more = 0 ;
34         memset (eg,0,sizeof(eg)) ;
35         for ( i=1 ; i <= n ; i ++ ) if ( !used[i] && i!=root ) {
36             for ( j=1 , k=0 ; j <= n ; j ++ ) if ( !used[j] && i!=j )
37                 if ( k==0 || g[j][i] < g[k][i] ) k=j ;
38             eg[i] = k ;
39         }
40         memset(pass,0,sizeof(pass));
41         for ( i=1; i<=n ; i++) if ( !used[i] && !pass[i] && i!= root ) combine ( i
42     }
43     for ( i =1; i<=n ; i ++ ) if ( !used[i] && i!= root ) sum+=g[eg[i]][i];
44     return sum ;
45 }

```

7.3 DLX

```
1  int n,m,K;
2  struct DLX{
3      int L[maxn],R[maxn],U[maxn],D[maxn];
4      int sz,col[maxn],row[maxn],s[maxn],H[maxn];
5      bool vis[233];
6      int ans[maxn],cnt;
7      void init(int m){
8          for(int i=0;i<=m;i++){
9              L[i]=i-1;R[i]=i+1;
10             U[i]=D[i]=i;s[i]=0;
11         }
12         memset(H,-1,sizeof H);
13         L[0]=m;R[m]=0;sz=m+1;
14     }
15     void Link(int r,int c){
16         U[sz]=c;D[sz]=D[c];U[D[c]]=sz;D[c]=sz;
17         if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
18         else{
19             L[sz]=H[r];R[sz]=R[H[r]];
20             L[R[H[r]]]=sz;R[H[r]]=sz;
21         }
22         s[c]++;col[sz]=c;row[sz]=r;sz++;
23     }
24     void remove(int c){
25         for(int i=D[c];i!=c;i=D[i])
26             L[R[i]]=L[i],R[L[i]]=R[i];
27     }
28     void resume(int c){
29         for(int i=U[c];i!=c;i=U[i])
30             L[R[i]]=R[L[i]]=i;
31     }
32     int A(){
33         int res=0;
34         memset(vis,0,sizeof vis);
35         for(int i=R[0];i;i=R[i])if(!vis[i]){
36             vis[i]=1;res++;
37             for(int j=D[i];j!=i;j=D[j])
38                 for(int k=R[j];k!=j;k=R[k])
39                     vis[col[k]]=1;
40         }
```

```

41         return res;
42     }
43     void dfs(int d,int &ans){
44         if(R[0]==0){ans=min(ans,d);return;}
45         if(d+A()>=ans)return;
46         int tmp=23333,c;
47         for(int i=R[0];i;i=R[i])
48             if(tmp>s[i])tmp=s[i],c=i;
49         for(int i=D[c];i!=c;i=D[i]){
50             remove(i);
51             for(int j=R[i];j!=i;j=R[j])remove(j);
52             dfs(d+1,ans);
53             for(int j=L[i];j!=i;j=L[j])resume(j);
54             resume(i);
55         }
56     }
57     void del(int c){//exactly cover
58         L[R[c]]=L[c];R[L[c]]=R[c];
59         for(int i=D[c];i!=c;i=D[i])
60             for(int j=R[i];j!=i;j=R[j])
61                 U[D[j]]=U[j],D[U[j]]=D[j],--s[col[j]]];
62     }
63     void add(int c){ //exactly cover
64         R[L[c]]=L[R[c]]=c;
65         for(int i=U[c];i!=c;i=U[i])
66             for(int j=L[i];j!=i;j=L[j])
67                 ++s[col[U[D[j]]=D[U[j]]=j]];
68     }
69     bool dfs2(int k){//exactly cover
70         if(!R[0]){
71             cnt=k;return 1;
72         }
73         int c=R[0];
74         for(int i=R[0];i;i=R[i])
75             if(s[c]>s[i])c=i;
76         del(c);
77         for(int i=D[c];i!=c;i=D[i]){
78             for(int j=R[i];j!=i;j=R[j])
79                 del(col[j]);
80             ans[k]=row[i];if(dfs2(k+1))return true;
81             for(int j=L[i];j!=i;j=L[j])
82                 add(col[j]);

```

```

83     }
84     add(c);
85     return 0;
86 }
87 }dlx;
88 int main(){
89     dlx.init(n);
90     for(int i=1;i<=m;i++)
91         for(int j=1;j<=n;j++)
92             if(dis(station[i],city[j])<mid-eps)
93                 dlx.Link(i,j);
94     dlx.dfs(0,ans);
95 }

```

7.4 插头 DP

```

1  int n,m,l;
2  struct L{
3      int d[11];
4      int& operator[](int x){return d[x];}
5      int mc(int x){
6          int an=1;
7          if(d[x]==1){
8              for(x++;x<1;x++)if(d[x]){
9                  an=an+(d[x]==1?1:-1);
10                 if(!an)return x;
11             }
12         }else{
13             for(x--;x>=0;x--)if(d[x]){
14                 an=an+(d[x]==2?1:-1);
15                 if(!an)return x;
16             }
17         }
18     }
19     int h(){int an=0;for(int i=l-1;i>=0;i--)an=an*3+d[i];return an;}
20     L s(int x,int y){
21         L S=*this;
22         S[x]=y;return S;
23     }
24     L operator>>(int _){
25         L S=*this;
26         for(int i=l-1;i>=1;i--)S[i]=S[i-1];

```

```

27         S[0]=0;return S;
28     }
29 };
30 struct Int{
31     int len;
32     int a[40];
33     Int(){len=1;memset(a,0,sizeof a);}
34     Int operator+=(const Int &o){
35         int l=max(len,o.len);
36         for(int i=0;i<l;i++)
37             a[i]=a[i]+o.a[i];
38         for(int i=0;i<l;i++)
39             a[i+1]+=a[i]/10,a[i]%=10;
40         if(a[1])l++;len=l;
41         return *this;
42     }
43     void print(){
44         for(int i=len-1;i>=0;i--)
45             printf("%d",a[i]);
46         puts("");
47     }
48 };
49 struct hashtab{
50     int sz;
51     int tab[177147];
52     Int w[177147];
53     L s[177147];
54     hashtab(){memset(tab,-1,sizeof tab);}
55     void cl(){
56         for(int i=0;i<sz;i++)tab[s[i].h()]=-1;
57         sz=0;
58     }
59     Int& operator[](L S){
60         int h=S.h();
61         if(tab[h]==-1)tab[h]=sz,s[sz]=S,w[sz]=Int(),sz++;
62         return w[tab[h]];
63     }
64 }f[2];
65 bool check(L S){
66     int cn1=0,cn2=0;
67     for(int i=0;i<l;i++){
68         cn1+=S[i]==1;

```

```

69         cn2+=S[i]==2;
70     }return cn1==1&&cn2==1;
71 }
72 int main(){
73     Int One;One.a[0]=1;
74     scanf("%d%d",&n,&m);if(n<m)swap(n,m);l=m+1;
75     if(n==1||m==1){puts("1");return 0;}
76     int cur=0;f[cur].cl();
77     for(int i=1;i<=n;i++){
78         for(int j=1;j<=m;j++){
79             if(i==1&&j==1){
80                 f[cur][L().s(0,1).s(1,2)]+=One;
81                 continue;
82             }
83             cur^=1;f[cur].cl();
84             for(int k=0;k<f[!cur].sz;k++){
85                 L S=f[!cur].s[k];Int w=f[!cur][S];
86                 int d1=S[j-1],d2=S[j];
87                 if(d1==0&&d2==0){
88                     if(i!=n&&j!=m)f[cur][S.s(j-1,1).s(j,2)]+=w;
89                 }else
90                 if(d1==0||d2==0){
91                     if(i!=n)f[cur][S.s(j-1,d1|d2).s(j,0)]+=w;
92                     if(j!=m)f[cur][S.s(j-1,0).s(j,d1|d2)]+=w;
93                 }else
94                 if(d1==1&&d2==2){
95                     if(i==n&&j==m&&check(S))
96                         (w+=w).print();
97                 }else
98                 if(d1==2&&d2==1){
99                     f[cur][S.s(j-1,0).s(j,0)]+=w;
100                 }else
101                 if((d1==1&&d2==1)|| (d1==2&&d2==2)){
102                     int m1=S.mc(j),m2=S.mc(j-1);
103                     f[cur][S.s(j-1,0).s(j,0).s(m1,1).s(m2,2)]+=w;
104                 }
105             }
106         }
107         cur^=1;f[cur].cl();
108         for(int k=0;k<f[!cur].sz;k++){
109             L S=f[!cur].s[k];Int w=f[!cur][S];
110             f[cur][S>>1]=w;

```

```

111     }
112 }
113 return 0;
114 }

```

7.5 某年某月某日是星期几

```

1 int solve(int year, int month, int day) {
2     int answer;
3     if (month == 1 || month == 2) {
4         month += 12;
5         year--;
6     }
7     if ((year < 1752) || (year == 1752 && month < 9) ||
8         (year == 1752 && month == 9 && day < 3)) {
9         answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4 + 5) % 7;
10    } else {
11        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4
12                - year / 100 + year / 400) % 7;
13    }
14    return answer;
15 }

```

7.6 枚举大小为 k 的子集

使用条件: $k > 0$

```

1 void solve(int n, int k) {
2     for (int comb = (1 << k) - 1; comb < (1 << n); ) {
3         // ...
4         int x = comb & -comb, y = comb + x;
5         comb = (((comb & ~y) / x) >> 1) | y;
6     }
7 }

```

7.7 环状最长公共子串

```

1 int n, a[N << 1], b[N << 1];
2
3 bool has(int i, int j) {
4     return a[(i - 1) % n] == b[(j - 1) % n];

```



```

5  }
6
7  const int DELTA[3][2] = {{0, -1}, {-1, -1}, {-1, 0}};
8
9  int from[N][N];
10
11 int solve() {
12     memset(from, 0, sizeof(from));
13     int ret = 0;
14     for (int i = 1; i <= 2 * n; ++i) {
15         from[i][0] = 2;
16         int left = 0, up = 0;
17         for (int j = 1; j <= n; ++j) {
18             int upleft = up + 1 + !!from[i - 1][j];
19             if (!has(i, j)) {
20                 upleft = INT_MIN;
21             }
22             int max = std::max(left, std::max(upleft, up));
23             if (left == max) {
24                 from[i][j] = 0;
25             } else if (upleft == max) {
26                 from[i][j] = 1;
27             } else {
28                 from[i][j] = 2;
29             }
30             left = max;
31         }
32         if (i >= n) {
33             int count = 0;
34             for (int x = i, y = n; y; ) {
35                 int t = from[x][y];
36                 count += t == 1;
37                 x += DELTA[t][0];
38                 y += DELTA[t][1];
39             }
40             ret = std::max(ret, count);
41             int x = i - n + 1;
42             from[x][0] = 0;
43             int y = 0;
44             while (y <= n && from[x][y] == 0) {
45                 y++;
46             }

```

```

47         for (; x <= i; ++x) {
48             from[x][y] = 0;
49             if (x == i) {
50                 break;
51             }
52             for (; y <= n; ++y) {
53                 if (from[x + 1][y] == 2) {
54                     break;
55                 }
56                 if (y + 1 <= n && from[x + 1][y + 1] == 1) {
57                     y++;
58                     break;
59                 }
60             }
61         }
62     }
63 }
64 return ret;
65 }

```

7.8 LLMOD

```

1 LL multiplyMod(LL a, LL b, LL P) { // `需要保证 a 和 b 非负`
2     LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
3     return t < 0 : t + P : t;
4 }

```

8 Java

8.1 基础模板

```

1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4
5 public class Main {
6     public static void main(String[] args) {
7         InputStream inputStream = System.in;
8         OutputStream outputStream = System.out;
9         InputReader in = new InputReader(inputStream);
10        PrintWriter out = new PrintWriter(outputStream);

```

```

11         Task solver = new Task();
12         solver.solve(0, in, out);
13         out.close();
14     }
15 }
16
17 class Task {
18     public void solve(int testNumber, InputReader in, PrintWriter out) {
19
20     }
21 }
22
23 class InputReader {
24     public BufferedReader reader;
25     public StringTokenizer tokenizer;
26
27     public InputReader(InputStream stream) {
28         reader = new BufferedReader(new InputStreamReader(stream), 32768);
29         tokenizer = null;
30     }
31
32     public String next() {
33         while (tokenizer == null || !tokenizer.hasMoreTokens()) {
34             try {
35                 tokenizer = new StringTokenizer(reader.readLine());
36             } catch (IOException e) {
37                 throw new RuntimeException(e);
38             }
39         }
40         return tokenizer.nextToken();
41     }
42
43     public int nextInt() {
44         return Integer.parseInt(next());
45     }
46
47     public long nextLong() {
48         return Long.parseLong(next());
49     }
50 }

```

9 数学

9.1 常用数学公式

9.1.1 求和公式

1. $\sum_{k=1}^n (2k-1)^2 = \frac{n(4n^2-1)}{3}$
2. $\sum_{k=1}^n k^3 = [\frac{n(n+1)}{2}]^2$
3. $\sum_{k=1}^n (2k-1)^3 = n^2(2n^2-1)$
4. $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
5. $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
6. $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
7. $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
8. $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$

9.1.2 斐波那契数列

1. $fib_0 = 0, fib_1 = 1, fib_n = fib_{n-1} + fib_{n-2}$
2. $fib_{n+2} \cdot fib_n - fib_{n+1}^2 = (-1)^{n+1}$
3. $fib_{-n} = (-1)^{n-1} fib_n$
4. $fib_{n+k} = fib_k \cdot fib_{n+1} + fib_{k-1} \cdot fib_n$
5. $gcd(fib_m, fib_n) = fib_{gcd(m,n)}$
6. $fib_m | fib_n^2 \Leftrightarrow n fib_n | m$

9.1.3 错排公式

1. $D_n = (n-1)(D_{n-2} - D_{n-1})$
2. $D_n = n! \cdot (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!})$

9.1.4 莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & \text{若 } n = 1 \\ (-1)^k & \text{若 } n \text{ 无平方数因子, 且 } n = p_1 p_2 \dots p_k \\ 0 & \text{若 } n \text{ 有大于1的平方数因数} \end{cases}$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{若 } n = 1 \\ 0 & \text{其他情况} \end{cases}$$

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g\left(\frac{n}{d}\right)$$

$$g(x) = \sum_{n=1}^{[x]} f\left(\frac{x}{n}\right) \Leftrightarrow f(x) = \sum_{n=1}^{[x]} \mu(n)g\left(\frac{x}{n}\right)$$

9.1.5 伯恩赛德引理

设 G 是一个有限群, 作用在集合 X 上. 对每个 g 属于 G , 令 X^g 表示 X 中在 g 作用下的不动元素, 轨道数 (记作 $|X/G|$) 由如下公式给出:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

9.1.6 五边形数定理

设 $p(n)$ 是 n 的拆分数, 有

$$p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k-1} p\left(n - \frac{k(3k-1)}{2}\right)$$

9.1.7 树的计数

1. 有根树计数: $n+1$ 个结点的有根树的个数为

$$a_{n+1} = \frac{\sum_{j=1}^n j \cdot a_j \cdot S_{n,j}}{n}$$

其中,

$$S_{n,j} = \sum_{i=1}^{n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$$

2. 无根树计数: 当 n 为奇数时, n 个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$$

当 n 为偶数时, n 个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$$

3. n 个结点的完全图的生成树个数为

$$n^{n-2}$$

4. 矩阵—树定理: 图 G 由 n 个结点构成, 设 $A[G]$ 为图 G 的邻接矩阵、 $D[G]$ 为图 G 的度数矩阵, 则图 G 的不同生成树的个数为 $C[G] = D[G] - A[G]$ 的任意一个 $n-1$ 阶主子式的行列式值。

9.1.8 欧拉公式

平面图的顶点个数、边数和面的个数有如下关系：

$$V - E + F = C + 1$$

其中， V 是顶点的数目， E 是边的数目， F 是面的数目， C 是组成图形的连通部分的数目。当图是单连通图的时候，公式简化为：

$$V - E + F = 2$$

9.1.9 皮克定理

给定顶点坐标均是整点（或正方形格点）的简单多边形，其面积 A 和内部格点数目 i 、边上格点数目 b 的关系：

$$A = i + \frac{b}{2} - 1$$

9.1.10 牛顿恒等式

设

$$\prod_{i=1}^n (x - x_i) = a_n + a_{n-1}x + \cdots + a_1x^{n-1} + a_0x^n$$

$$p_k = \sum_{i=1}^n x_i^k$$

则

$$a_0p_k + a_1p_{k-1} + \cdots + a_{k-1}p_1 + ka_k = 0$$

特别地，对于

$$|\mathbf{A} - \lambda \mathbf{E}| = (-1)^n (a_n + a_{n-1}\lambda + \cdots + a_1\lambda^{n-1} + a_0\lambda^n)$$

有

$$p_k = \text{Tr}(\mathbf{A}^k)$$

9.2 平面几何公式

9.2.1 三角形

1. 半周长

$$p = \frac{a + b + c}{2}$$

2. 面积

$$S = \frac{a \cdot H_a}{2} = \frac{ab \cdot \sin C}{2} = \sqrt{p(p-a)(p-b)(p-c)}$$

3. 中线

$$M_a = \frac{\sqrt{2(b^2 + c^2) - a^2}}{2} = \frac{\sqrt{b^2 + c^2 + 2bc \cdot \cos A}}{2}$$

4. 角平分线

$$T_a = \frac{\sqrt{bc \cdot [(b+c)^2 - a^2]}}{b+c} = \frac{2bc}{b+c} \cos \frac{A}{2}$$

5. 高线

$$H_a = b \sin C = c \sin B = \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}$$

6. 内切圆半径

$$\begin{aligned} r = \frac{S}{p} &= \frac{\arcsin \frac{B}{2} \cdot \sin \frac{C}{2}}{\sin \frac{B+C}{2}} = 4R \cdot \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \\ &= \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} = p \cdot \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \end{aligned}$$

7. 外接圆半径

$$R = \frac{abc}{4S} = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$$

9.2.2 四边形

D_1, D_2 为对角线, M 为对角线中点连线, A 为对角线夹角, p 为半周长

$$1. a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$$

$$2. S = \frac{1}{2} D_1 D_2 \sin A$$

3. 对于圆内接四边形

$$ac + bd = D_1 D_2$$

4. 对于圆内接四边形

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$$

9.2.3 正 n 边形

R 为外接圆半径, r 为内切圆半径

1. 中心角

$$A = \frac{2\pi}{n}$$

2. 内角

$$C = \frac{n-2}{n} \pi$$

3. 边长

$$a = 2\sqrt{R^2 - r^2} = 2R \cdot \sin \frac{A}{2} = 2r \cdot \tan \frac{A}{2}$$

4. 面积

$$S = \frac{nar}{2} = nr^2 \cdot \tan \frac{A}{2} = \frac{nR^2}{2} \cdot \sin A = \frac{na^2}{4 \cdot \tan \frac{A}{2}}$$

9.2.4 圆

1. 弧长

$$l = rA$$

2. 弦长

$$a = 2\sqrt{2hr - h^2} = 2r \cdot \sin \frac{A}{2}$$

3. 弓形高

$$h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1 - \cos \frac{A}{2}) = \frac{1}{2} \cdot \arctan \frac{A}{4}$$

4. 扇形面积

$$S_1 = \frac{rl}{2} = \frac{r^2 A}{2}$$

5. 弓形面积

$$S_2 = \frac{rl - a(r - h)}{2} = \frac{r^2}{2}(A - \sin A)$$

9.2.5 棱柱

1. 体积

$$V = Ah$$

A 为底面积, h 为高

2. 侧面积

$$S = lp$$

l 为棱长, p 为直截面周长

3. 全面积

$$T = S + 2A$$

9.2.6 棱锥

1. 体积

$$V = Ah$$

A 为底面积, h 为高

2. 正棱锥侧面积

$$S = lp$$

l 为棱长, p 为直截面周长

3. 正棱锥全面积

$$T = S + 2A$$

9.2.7 棱台

1. 体积

$$V = (A_1 + A_2 + \sqrt{A_1 A_2}) \cdot \frac{h}{3}$$

A_1, A_2 为上下底面积, h 为高

2. 正棱台侧面积

$$S = \frac{p_1 + p_2}{2} l$$

p_1, p_2 为上下底面周长, l 为斜高

3. 正棱台全面积

$$T = S + A_1 + A_2$$

9.2.8 圆柱

1. 侧面积

$$S = 2\pi r h$$

2. 全面积

$$T = 2\pi r(h + r)$$

3. 体积

$$V = \pi r^2 h$$

9.2.9 圆锥

1. 母线

$$l = \sqrt{h^2 + r^2}$$

2. 侧面积

$$S = \pi r l$$

3. 全面积

$$T = \pi r(l + r)$$

4. 体积

$$V = \frac{\pi}{3} r^2 h$$

9.2.10 圆台

1. 母线

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

2. 侧面积

$$S = \pi(r_1 + r_2)l$$

3. 全面积

$$T = \pi r_1(l + r_1) + \pi r_2(l + r_2)$$

4. 体积

$$V = \frac{\pi}{3}(r_1^2 + r_2^2 + r_1 r_2)h$$

9.2.11 球

1. 全面积

$$T = 4\pi r^2$$

2. 体积

$$V = \frac{4}{3}\pi r^3$$

9.2.12 球台

1. 侧面积

$$S = 2\pi r h$$

2. 全面积

$$T = \pi(2rh + r_1^2 + r_2^2)$$

3. 体积

$$V = \frac{\pi h[3(r_1^2 + r_2^2) + h^2]}{6}$$

9.2.13 球扇形

1. 全面积

$$T = \pi r(2h + r_0)$$

h 为球冠高, r_0 为球冠底面半径

2. 体积

$$V = \frac{2}{3}\pi r^2 h$$

9.3 立体几何公式

9.3.1 球面三角公式

设 a, b, c 是边长, A, B, C 是所对的二面角, 有余弦定理

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

正弦定理

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

三角形面积是 $A + B + C - \pi$

9.3.2 四面体体积公式

U, V, W, u, v, w 是四面体的 6 条棱, U, V, W 构成三角形, $(U, u), (V, v), (W, w)$ 互为对棱, 则

$$V = \frac{\sqrt{(s-2a)(s-2b)(s-2c)(s-2d)}}{192uvw}$$

其中

$$\left\{ \begin{array}{lcl} a & = & \sqrt{xYZ}, \\ b & = & \sqrt{yZX}, \\ c & = & \sqrt{zXY}, \\ d & = & \sqrt{xyz}, \\ s & = & a + b + c + d, \\ X & = & (w - U + v)(U + v + w), \\ x & = & (U - v + w)(v - w + U), \\ Y & = & (u - V + w)(V + w + u), \\ y & = & (V - w + u)(w - u + V), \\ Z & = & (v - W + u)(W + u + v), \\ z & = & (W - u + v)(u - v + W) \end{array} \right.$$