代码库

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目录

1 数论

1.1 快速求逆元

返回结果：

$$x^{-1}(mod)$$

使用条件：$x \in [0, mod)$ 并且 x 与 mod 互质

```
LL inv(LL a,LL p){
	LL d,x,y;
	d=exgcd(a,p,x,y);
	return d==1?(x+p)%p:-1;
}
```

1.2 扩展欧几里德算法

返回结果：

$$ax + by = gcd(a, b)$$

时间复杂度：$\mathcal{O}(nlogn)$

```
LL exgcd(LL a,LL b,LL &x,LL &y){
	if(!b){
		x=1;y=0;return a;
	}else{
		LL d=exgcd(b,a%b,x,y);
		LL t=x;x=y;y=t-a/b*y;
		return d;
	}
}
```

1.3 中国剩余定理

返回结果：

$$x \equiv r_i(mod\ \ p_i)\ \ (0 \le i < n)$$

使用条件：p_i 需两两互质

```
LL china(int n,int *a,int *m){
	LL M=1,d,x=0,y;
	for(int i=0;i<n;i++)
		M*=m[i];
```

```
        for(int i=0;i<n;i++){
                LL w=M/m[i];
                d=exgcd(m[i],w,d,y);
                y=(y%M+M)%M;
                x=(x+y*w%M*a[i])%M;
        }
        while(x<0)x+=M;
        return x;
}
```

1.4 中国剩余定理 2

```
//merge Ax=B and ax=b to A'x=B'
void merge(LL &A,LL &B,LL a,LL b){
    LL x,y;
    sol(A,-a,b-B,x,y);
    A=lcm(A,a);
    B=(a*y+b)%A;
    B=(B+A)%A;
}
```

1.5 组合数取模

```
LL prod=1,P;
pair<LL,LL> comput(LL n,LL p,LL k){
    if(n<=1)return make_pair(0,1);
    LL ans=1,cnt=0;
    ans=pow(prod,n/P,P);
    cnt=n/p;
    pair<LL,LL>res=comput(n/p,p,k);
    cnt+=res.first;
    ans=ans*res.second%P;
    for(int i=n-n%P+1;i<=n;i++)if(i%p){

        ans=ans*i%P;
    }
    return make_pair(cnt,ans);
}
pair<LL,LL> calc(LL n,LL p,LL k){
    prod=1;P=pow(p,k,1e18);
    for(int i=1;i<P;i++)if(i%p)prod=prod*i%P;
```

```
    pair<LL,LL> res=comput(n,p,k);
//  res.second=res.second*pow(p,res.first%k,P)%P;
//  res.first-=res.first%k;
    return res;
}
LL calc(LL n,LL m,LL p,LL k){
    pair<LL,LL>A,B,C;
    LL P=pow(p,k,1e18);
    A=calc(n,p,k);
    B=calc(m,p,k);
    C=calc(n-m,p,k);
    LL ans=1;
    ans=pow(p,A.first-B.first-C.first,P);
    ans=ans*A.second%P*inv(B.second,P)%P*inv(C.second,P)%P;
    return ans;
}
```

1.6 扩展小步大步

```
LL solve2(LL a,LL b,LL p){
    //a^x=b (mod p)
    b%=p;
    LL e=1%p;
    for(int i=0;i<100;i++){
        if(e==b)return i;
        e=e*a%p;
    }
    int r=0;
    while(gcd(a,p)!=1){
        LL d=gcd(a,p);
        if(b%d)return -1;
        p/=d;b/=d;b=b*inv(a/d,p);
        r++;
    }LL res=BSGS(a,b,p);
    if(res==-1)return -1;
    return res+r;
}
```

1.7 卢卡斯定理

```cpp
LL Lucas(LL n,LL m,LL p){
	LL ans=1;
	while(n&&m){
		LL a=n%p,b=m%p;
		if(a<b)return 0;
		ans=(ans*C(a,b,p))%p;
		n/=p;m/=p;
	}return ans%p;
}
```

1.8 小步大步

返回结果:

$$a^x = b\ \ (mod\ \ p)$$

使用条件:p 为质数

时间复杂度:$\mathcal{O}(\sqrt{n})$

```cpp
LL BSGS(LL a,LL b,LL p){
	LL m=sqrt(p)+.5,v=inv(pw(a,m,p),p),e=1;
	map<LL,LL>hash;hash[1]=0;
	for(int i=1;i<m;i++)
		e=e*a%p,hash[e]=i;
	for(int i=0;i<=m;i++){
		if(hash.count(b))return i*m+hash[b];
		b=b*v%p;
	}return -1;
}
```

1.9 Miller Rabin 素数测试

```cpp
const int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool check(long long n,int base) {
    long long n2=n-1,res;
    int s=0;
    while(n2%2==0) n2>>=1,s++;
    res=pw(base,n2,n);
    if((res==1)||(res==n-1)) return 1;
    while(s--) {
        res=mul(res,res,n);
        if(res==n-1) return 1;
    }
```

```
    return 0; // n is not a strong pseudo prime
}
bool isprime(const long long &n) {
    if(n==2)
        return true;
    if(n<2 || n%2==0)
        return false;
    for(int i=0;i<12&&BASE[i]<n;i++){
        if(!check(n,BASE[i]))
            return false;
    }
    return true;
}
```

1.10 Pollard Rho 大数分解

时间复杂度：$\mathcal{O}(n^{1/4})$

```
LL prho(LL n,LL c){
        LL i=1,k=2,x=rand()%(n-1)+1,y=x;
        while(1){
                i++;x=(x*x%n+c)%n;
                LL d=__gcd((y-x+n)%n,n);
                if(d>1&&d<n)return d;
                if(y==x)return n;
                if(i==k)y=x,k<<=1;
        }
}
void factor(LL n,vector<LL>&fat){
        if(n==1)return;
        if(isprime(n)){
                fat.push_back(n);
                return;
        }LL p=n;
        while(p>=n)p=prho(p,rand()%(n-1)+1);
        factor(p,fat);
        factor(n/p,fat);
}
```

1.11 快速数论变换 (zky)

返回结果：

$$c_i = \sum_{0 \le j \le i} a_j \cdot b_{i-j}(mod) \ \ (0 \le i < n)$$

使用说明：$magic$ 是 mod 的原根

时间复杂度：$\mathcal{O}(nlogn)$

```
/*
{(mod,G)}={(81788929,7),(101711873,3),(167772161,3)
                        ,(377487361,7),(998244353,3),(1224736769,3)
                        ,(1300234241,3),(1484783617,5)}
*/
int mo=998244353,G=3;
void NTT(int a[],int n,int f){
        for(register int i=0;i<n;i++)
                if(i<rev[i])
                        swap(a[i],a[rev[i]]);
        for (register int i=2;i<=n;i<<=1){
                static int exp[maxn];
                exp[0]=1;exp[1]=pw(G,(mo-1)/i);
                if(f==-1)exp[1]=pw(exp[1],mo-2);
                for(register int k=2;k<(i>>1);k++)
                        exp[k]=1LL*exp[k-1]*exp[1]%mo;
                for(register int j=0;j<n;j+=i){
                        for(register int k=0;k<(i>>1);k++){
                                register int &pA=a[j+k],&pB=a[j+k+(i>>1)];
                                register int A=pA,B=1LL*pB*exp[k]%mo;
                                pA=(A+B)%mo;
                                pB=(A-B+mo)%mo;
                        }
                }
        }
        if(f==-1){
                int rv=pw(n,mo-2)%mo;
                for(int i=0;i<n;i++)
                        a[i]=1LL*a[i]*rv%mo;
        }
}
void mul(int m,int a[],int b[],int c[]){
        int n=1,len=0;
        while(n<m)n<<=1,len++;
        for (int i=1;i<n;i++)
```

```
                rev[i]=(rev[i>>1]>>1)|((i&1)<<(len-1));
        NTT(a,n,1);
        NTT(b,n,1);
        for(int i=0;i<n;i++)
                c[i]=1LL*a[i]*b[i]%mo;
        NTT(c,n,-1);
}
```

1.12 快速数论变换 (lyx)

```
int Pow(int x,int y,int z){
        if (y==0) return 1;
        LL ret=Pow(x,y>>1,z); (ret*=ret)%=z;
        if (y & 1) (ret*=x)%=z;
        return ret;
}


void Prep(){
        for (len=1, ci=0; len<=N+N; len<<=1, ci++);
        Wi[0]=1, Wi[1]=Pow(G,(Mo-1)/len,Mo);
        for (int i=2; i<=len; i++) Wi[i]=(Wi[i-1]*Wi[1])% Mo;
        for (int i=0; i<len; i++){
                int tmp=0;
                for (int j=i, c=0; c<ci; c++, j>>=1 ) tmp=(tmp <<= 1)|= (j & 1);
                Bel[i]=tmp;
        }
}

void Dft(Arr &a,int sig)
{
        for (int i=0; i<len; i++) tp[Bel[i]]=a[i];
        for (int m=1; m<=len; m<<=1){
                int half=m>>1, bei=len/m;
                for (int i=0; i<half; i++){
                        LL wi=(sig>0)?Wi[i*bei]:Wi[len-i*bei];
                        for (int j=i; j<len; j+=m){
                                int u=tp[j],v=wi*LL(tp[j+half]) % Mo;
                                tp[j]=(u+v) % Mo; tp[j+half]=(u-v+Mo)% Mo;
                        }
                }
        }
```

```
	for (int i=0; i<len; i++) a[i]=tp[i];
}

void Mul(Arr &x,Arr &y,Arr &c,bool same)
{
	if (!same){
		for(int i=0; i<len; i++) a[i]=x[i], b[i]=y[i];
		Dft(a,1),Dft(b,1);
		for(int i=0; i<len; i++) a[i]=a[i]*1ll*b[i] % Mo;
		Dft(a,-1);
		for(int i=0; i<=M; i++) c[i]=a[i]*1ll*Rev % Mo;
	} else
	{
		for(int i=0; i<len; i++) a[i]=x[i];
		Dft(a,1);
		for(int i=0; i<len; i++) a[i]=a[i]*1ll*a[i] % Mo;
		Dft(a,-1);
		for(int i=0; i<=M; i++) c[i]=a[i]*1ll*Rev % Mo;
	}
}

Prep();
Ans[0]=1; Rev=Pow(len,Mo-2,Mo);
for(; K; K>>=1){
	if (K & 1) Mul(Ans,F,Ans,0);
	if (K > 1) Mul(F,F,F,1);
}

printf("%d\n",Ans[M]);
```

1.13 原根

```
vector<LL>fct;
bool check(LL x,LL g){
	for(int i=0;i<fct.size();i++)
		if(pw(g,(x-1)/fct[i],x)==1)
			return 0;
	return 1;
}
LL findrt(LL x){
	LL tmp=x-1;
	for(int i=2;i*i<=tmp;i++){
```

```
                if(tmp%i==0){
                        fct.push_back(i);
                        while(tmp%i==0)tmp/=i;
                }
        }if(tmp>1)fct.push_back(tmp);
        // x is 1,2,4,p^n,2p^n
        // x has phi(phi(x)) primitive roots
        for(int i=2;i<int(1e9);i++)if(check(x,i))
                return i;
        return -1;
}
const int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool check(long long n,int base) {
    long long n2=n-1,res;
    int s=0;
    while(n2%2==0) n2>>=1,s++;
    res=pw(base,n2,n);
    if((res==1)||(res==n-1)) return 1;
    while(s--) {
        res=mul(res,res,n);
        if(res==n-1) return 1;
    }
    return 0; // n is not a strong pseudo prime
}
bool isprime(const long long &n) {
    if(n==2)
        return true;
    if(n<2 || n%2==0)
        return false;
    for(int i=0;i<12&&BASE[i]<n;i++){
        if(!check(n,BASE[i]))
            return false;
    }
    return true;
}
```

1.14 线性递推

```
//已知 $a_0,a_1,...,a_{m-1}$\\
        $a_n = c_0 * a_{n-m} + ... + c_{m-1} * a_{n-1}$\\
        求 $a_n = v_0 * a_0 + v_1 * a_1 + ... + v_{m-1} * a_{m-1}$\\

```

```cpp
void linear_recurrence(long long n, int m, int a[], int c[], int p) {
	long long v[M] = {1 % p}, u[M << 1], msk = !!n;
	for(long long i(n); i > 1; i >>= 1) {
		msk <<= 1;
	}
	for(long long x(0); msk; msk >>= 1, x <<= 1) {
		fill_n(u, m << 1, 0);
		int b(!!(n & msk));
		x |= b;
		if(x < m) {
			u[x] = 1 % p;
		}else {
			for(int i(0); i < m; i++) {
				for(int j(0), t(i + b); j < m; j++, t++) {
					u[t] = (u[t] + v[i] * v[j]) % p;
				}
			}
			for(int i((m << 1) - 1); i >= m; i--) {
				for(int j(0), t(i - m); j < m; j++, t++) {
					u[t] = (u[t] + c[j] * u[i]) % p;
				}
			}
		}
		copy(u, u + m, v);
	}
	//a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
	for(int i(m); i < 2 * m; i++) {
		a[i] = 0;
		for(int j(0); j < m; j++) {
			a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
		}
	}
	for(int j(0); j < m; j++) {
		b[j] = 0;
		for(int i(0); i < m; i++) {
			b[j] = (b[j] + v[i] * a[i + j]) % p;
		}
	}
	for(int j(0); j < m; j++) {
		a[j] = b[j];
	}
}
```

1.15 线性筛

```
void sieve(){
	f[1]=mu[1]=phi[1]=1;
	for(int i=2;i<maxn;i++){
		if(!minp[i]){
			minp[i]=i;
			minpw[i]=i;
			mu[i]=-1;
			phi[i]=i-1;
			f[i]=i-1;
			p[++p[0]]=i;//Case 1 prime
		}
		for(int j=1;j<=p[0]&&(LL)i*p[j]<maxn;j++){
			minp[i*p[j]]=p[j];
			if(i%p[j]==0){
				//Case 2 not coprime
				minpw[i*p[j]]=minpw[i]*p[j];
				phi[i*p[j]]=phi[i]*p[j];
				mu[i*p[j]]=0;
				if(i==minpw[i]){
					f[i*p[j]]=i*p[j]-i;//Special Case for f(p^k)
				}else{
					f[i*p[j]]=f[i/minpw[i]]*f[minpw[i]*p[j]];
				}
				break;
			}else{
				//Case 3 coprime
				minpw[i*p[j]]=p[j];
				f[i*p[j]]=f[i]*f[p[j]];
				phi[i*p[j]]=phi[i]*(p[j]-1);
				mu[i*p[j]]=-mu[i];
			}
		}
	}
}
```

1.16 直线下整点个数

返回结果：

$$\sum_{0\le i<n}\lfloor\frac{a+b\cdot i}{m}\rfloor$$

使用条件：$n,m>0$，$a,b\geq 0$

时间复杂度：$\mathcal{O}(nlogn)$

```
//calc \sum_{i=0}^{n-1} [(a+bi)/m]
// n,a,b,m >0
LL solve(LL n,LL a,LL b,LL m){
    if(b==0)
        return n*(a/m);
    if(a>=m || b>=m)
        return n*(a/m)+(n-1)*n/2*(b/m)+solve(n,a%m,b%m,m);
    return solve((a+b*n)/m,(a+b*n)%m,m,b);
}
```

2 数值

2.1 高斯消元

```
void Gauss(){
        int r,k;
        for(int i=0;i<n;i++){
                r=i;
                for(int j=i+1;j<n;j++)
                        if(fabs(A[j][i])>fabs(A[r][i]))r=j;
                if(r!=i)for(int j=0;j<=n;j++)swap(A[i][j],A[r][j]);
                for(int k=i+1;k<n;k++){
                        double f=A[k][i]/A[i][i];
                        for(int j=i;j<=n;j++)A[k][j]-=f*A[i][j];
                }
        }
        for(int i=n-1;i>=0;i--){
                for(int j=i+1;j<n;j++)
                        A[i][n]-=A[j][n]*A[i][j];
                A[i][n]/=A[i][i];
        }
        for(int i=0;i<n-1;i++)
                cout<<fixed<<setprecision(3)<<A[i][n]<<" ";
        cout<<fixed<<setprecision(3)<<A[n-1][n];
}
bool Gauss(){
        for(int i=1;i<=n;i++){
                int r=0;
                for(int j=i;j<=m;j++)
```

```
			if(a[j][i]){r=j;break;}
			if(!r)return 0;
			ans=max(ans,r);
			swap(a[i],a[r]);
			for(int j=i+1;j<=m;j++)
			if(a[j][i])a[j]^=a[i];
	}for(int i=n;i>=1;i--){
			for(int j=i+1;j<=n;j++)if(a[i][j])
			a[i][n+1]=a[i][n+1]^a[j][n+1];
	}return 1;
}
LL Gauss(){
	for(int i=0;i<n;i++)for(int j=0;j<n;j++)A[i][j]%=m;
	for(int i=0;i<n;i++)for(int j=0;j<n;j++)A[i][j]=(A[i][j]+m)%m;
	LL ans=n%2?-1:1;
	for(int i=0;i<n;i++){
		for(int j=i+1;j<n;j++){
			while(A[j][i]){
				LL t=A[i][i]/A[j][i];
				for(int k=0;k<n;k++)
				A[i][k]=(A[i][k]-A[j][k]*t%m+m)%m;
				swap(A[i],A[j]);
				ans=-ans;
			}
		}ans=ans*A[i][i]%m;
	}return (ans%m+m)%m;
}
int Gauss(){//求秩
	int r,now=-1;
	int ans=0;
	for(int i = 0; i <n; i++){
		r = now + 1;
		for(int j = now + 1; j < m; j++)
			if(fabs(A[j][i]) > fabs(A[r][i]))
				r = j;
		if (!sgn(A[r][i])) continue;
		ans++;
		now++;
		if(r != now)
			for(int j = 0; j < n; j++)
				swap(A[r][j], A[now][j]);

```

```
                for(int k = now + 1; k < m; k++){
                        double t = A[k][i] / A[now][i];
                        for(int j = 0; j < n; j++){
                                A[k][j] -= t * A[now][j];
                        }
                }
        }
        return ans;
}
```

2.2 快速傅立叶变换

返回结果：

$$c_i = \sum_{0 \le j \le i} a_j \cdot b_{i-j} \quad (0 \le i < n)$$

时间复杂度：$\mathcal{O}(nlogn)$

```
typedef complex<double> cp;
const double pi = acos(-1);
void FFT(vector<cp>&num,int len,int ty){
    for(int i=1,j=0;i<len-1;i++){
        for(int k=len;j^=k>>=1,~j&k;);
        if(i<j)
            swap(num[i],num[j]);
    }
    for(int h=0;(1<<h)<len;h++){
        int step=1<<h,step2=step<<1;
        cp w0(cos(2.0*pi/step2),ty*sin(2.0*pi/step2));
        for(int i=0;i<len;i+=step2){
            cp w(1,0);
            for(int j=0;j<step;j++){
                cp &x=num[i+j+step];
                cp &y=num[i+j];
                cp d=w*x;
                x=y-d;
                y=y+d;
                w=w*w0;
            }
        }
    }
    if(ty==-1)
        for(int i=0;i<len;i++)
            num[i]=cp(num[i].real()/(double)len,num[i].imag());
```

```
}
vector<cp> mul(vector<cp>a,vector<cp>b){
    int len=a.size()+b.size();
    while((len&-len)!=len)len++;
    while(a.size()<len)a.push_back(cp(0,0));
    while(b.size()<len)b.push_back(cp(0,0));
    FFT(a,len,1);
    FFT(b,len,1);
    vector<cp>ans(len);
    for(int i=0;i<len;i++)
        ans[i]=a[i]*b[i];
    FFT(ans,len,-1);
    return ans;
}
```

2.3 单纯形法求解线性规划

返回结果:

$$max\{c_{1\times m}\cdot x_{m\times 1}\ |\ x_{m\times 1}\geq 0_{m\times 1},a_{n\times m}\cdot x_{m\times 1}\leq b_{n\times 1}\}$$

```
namespace LP{
        const int maxn=233;
        double a[maxn][maxn];
        int Ans[maxn],pt[maxn];
        int n,m;
        void pivot(int l,int i){
                double t;
                swap(Ans[l+n],Ans[i]);
                t=-a[l][i];
                a[l][i]=-1;
                for(int j=0;j<=n;j++)a[l][j]/=t;
                for(int j=0;j<=m;j++){
                        if(a[j][i]&&j!=l){
                                t=a[j][i];
                                a[j][i]=0;
                                for(int k=0;k<=n;k++)a[j][k]+=t*a[l][k];
                        }
                }
        }
        vector<double> solve(vector<vector<double> >A,vector<double>B,vector<double>C){
                n=C.size();
                m=B.size();
                for(int i=0;i<C.size();i++)
```

```
		a[0][i+1]=C[i];
	for(int i=0;i<B.size();i++)
		a[i+1][0]=B[i];

	for(int i=0;i<m;i++)
		for(int j=0;j<n;j++)
			a[i+1][j+1]=-A[i][j];

	for(int i=1;i<=n;i++)Ans[i]=i;

	double t;
	for(;;){
		int l=0;t=-eps;
		for(int j=1;j<=m;j++)if(a[j][0]<t)t=a[l=j][0];
		if(!l)break;
		int i=0;
		for(int j=1;j<=n;j++)if(a[l][j]>eps){i=j;break;}
		if(!i){
			puts("Infeasible");
			return vector<double>();
		}
		pivot(l,i);
	}
	for(;;){
		int i=0;t=eps;
		for(int j=1;j<=n;j++)if(a[0][j]>t)t=a[0][i=j];
		if(!i)break;
		int l=0;
		t=1e30;
		for(int j=1;j<=m;j++)if(a[j][i]<-eps){
			double tmp;
			tmp=-a[j][0]/a[j][i];
			if(t>tmp)t=tmp,l=j;
		}
		if(!l){
			puts("Unbounded");
			return vector<double>();
		}
		pivot(l,i);
	}
	vector<double>x;
	for(int i=n+1;i<=n+m;i++)pt[Ans[i]]=i-n;
```

```
                for(int i=1;i<=n;i++)x.push_back(pt[i]?a[pt[i]][0]:0);
                return x;
        }
}
```

2.4 自适应辛普森

```
double area(const double &left, const double &right) {
    double mid = (left + right) / 2;
    return (right - left) * (calc(left) + 4 * calc(mid) + calc(right)) / 6;
}

double simpson(const double &left, const double &right,
               const double &eps, const double &area_sum) {
    double mid = (left + right) / 2;
    double area_left = area(left, mid);
    double area_right = area(mid, right);
    double area_total = area_left + area_right;
    if (std::abs(area_total - area_sum) < 15 * eps) {
        return area_total + (area_total - area_sum) / 15;
    }
    return simpson(left, mid, eps / 2, area_left)
         + simpson(mid, right, eps / 2, area_right);
}

double simpson(const double &left, const double &right, const double &eps) {
    return simpson(left, right, eps, area(left, right));
}
```

2.5 多项式求根

```
const double eps=1e-12;
double a[10][10];
typedef vector<double> vd;
int sgn(double x) { return x < -eps ? -1 : x > eps; }
double mypow(double x,int num){
        double ans=1.0;
        for(int i=1;i<=num;++i)ans*=x;
        return ans;
}
double f(int n,double x){
```

```
	double ans=0;
	for(int i=n;i>=0;--i)ans+=a[n][i]*mypow(x,i);
	return ans;
}
double getRoot(int n,double l,double r){
	if(sgn(f(n,l))==0)return l;
	if(sgn(f(n,r))==0)return r;
	double temp;
	if(sgn(f(n,l))>0)temp=-1;else temp=1;
	double m;
	for(int i=1;i<=10000;++i){
		m=(l+r)/2;
		double mid=f(n,m);
		if(sgn(mid)==0){
			return m;
		}
		if(mid*temp<0)l=m;else r=m;
	}
	return (l+r)/2;
}
vd did(int n){
	vd ret;
	if(n==1){
		ret.push_back(-1e10);
		ret.push_back(-a[n][0]/a[n][1]);
		ret.push_back(1e10);
		return ret;
	}
	vd mid=did(n-1);
	ret.push_back(-1e10);
	for(int i=0;i+1<mid.size();++i){
		int t1=sgn(f(n,mid[i])),t2=sgn(f(n,mid[i+1]));
		if(t1*t2>0)continue;
		ret.push_back(getRoot(n,mid[i],mid[i+1]));
	}
	ret.push_back(1e10);
	return ret;
}
int main(){
	int n; scanf("%d",&n);
	for(int i=n;i>=0;--i){
		scanf("%lf",&a[n][i]);
```

```
	}
	for(int i=n-1;i>=0;--i)
		for(int j=0;j<=i;++j)a[i][j]=a[i+1][j+1]*(j+1);
	vd ans=did(n);
	sort(ans.begin(),ans.end());
	for(int i=1;i+1<ans.size();++i)printf("%.10f\n",ans[i]);
	return 0;
}
```

3 数据结构

3.1 平衡的二叉查找树

3.1.1 Treap

```
const int maxn=1e5+5;
#define sz(x) (x?x->siz:0)
struct Treap{
	struct node{
		int key,val;
		int siz,s;
		node *c[2];
		node(int v=0){
			val=v;
			key=rand();
			siz=1,s=1;
			c[0]=c[1]=0;
		}
		void rz(){siz=s;if(c[0])siz+=c[0]->siz;if(c[1])siz+=c[1]->siz;}
	}pool[maxn],*cur,*root;
	Treap(){cur=pool;}
	node* newnode(int val){return *cur=node(val),cur++;}
	void rot(node *&t,int d){
		if(!t->c[d])t=t->c[!d];
		else{
			node *p=t->c[d];t->c[d]=p->c[!d];
			p->c[!d]=t;t->rz();p->rz();t=p;
		}
	}
	void insert(node *&t,int x){
		if(!t){t=newnode(x);return;}
		if(t->val==x){t->s++;t->siz++;return;}
```

```
			insert(t->c[x>t->val],x);
			if(t->key<t->c[x>t->val]->key)
				rot(t,x>t->val);
			else t->rz();
		}
		int pre(node *t,int x){
			if(!t)return INT_MIN;
			int ans=pre(t->c[x>t->val],x);
			if(t->val<x)ans=max(ans,t->val);
			return ans;
		}
		int nxt(node *t,int x){
			if(!t)return INT_MAX;
			int ans=nxt(t->c[x>=t->val],x);
			if(t->val>x)ans=min(ans,t->val);
			return ans;
		}
		int rank(node *t,int x){
			if(!t)return 0;
			if(t->val==x)return sz(t->c[0]);
			if(t->val<x)return sz(t->c[0])+t->s+rank(t->c[1],x);
			if(t->val>x)return rank(t->c[0],x);
		}
		int kth(node *t,int x){
			if(sz(t->c[0])>=x)return kth(t->c[0],x);
			if(sz(t->c[0])+t->s>=x)return t->val;
			return kth(t->c[1],x-t->s-sz(t->c[0]));
		}
		void deb(node *t){
			if(!t)return;
			deb(t->c[0]);
			printf("%d ",t->val);
			deb(t->c[1]);
		}
}T;
```

3.1.2 Splay

```
void Rotate(int x, int c){
	int y = T[x].c[c];
	int z = T[y].c[1 - c];

```

```
        if (T[x].fa){
                if (T[T[x].fa].c[0] == x) T[T[x].fa].c[0] = y;
                else T[T[x].fa].c[1] = y;
        }

        T[z].fa = x; T[x].c[c] = z;
        T[y].fa = T[x].fa; T[x].fa = y; T[y].c[1 - c] = x;

        Update(x);
        Update(y);
}

int stack[M], fx[M];

void Splay(int x, int fa){
        int top = 0;
        for (int  u = x; u != fa; u = T[u].fa)
                stack[++top] = u;
        for (int i = 2; i <= top; i++)
                if (T[stack[i]].c[0] == stack[i - 1]) fx[i] = 0;
                else fx[i] = 1;

        for (int i = 2; i <= top; i += 2){
                if (i == top) Rotate(stack[i], fx[i]);
                else {
                        if (fx[i] == fx[i + 1]){
                                Rotate(stack[i + 1], fx[i + 1]);
                                Rotate(stack[i], fx[i]);
                        } else {
                                Rotate(stack[i], fx[i]);
                                Rotate(stack[i + 1], fx[i + 1]);
                        }
                }
        }

        if (fa == 0) Root = x;
}
```

3.2 坚固的数据结构

3.2.1 坚固的平衡树

```cpp
#define sz(x) (x?x->siz:0)
struct node{
    int siz,key;
    LL val,sum;
    LL mu,a,d;
    node *c[2],*f;
    void split(int ned,node *&p,node *&q);
    node* rz(){
        sum=val;siz=1;
        if(c[0])sum+=c[0]->sum,siz+=c[0]->siz;
        if(c[1])sum+=c[1]->sum,siz+=c[1]->siz;
        return this;
    }
    void make(LL _mu,LL _a,LL _d){
        sum=sum*_mu+_a*siz+_d*siz*(siz-1)/2;
        val=val*_mu+_a+_d*sz(c[0]);
        mu*=_mu;a=a*_mu+_a;d=d*_mu+_d;
    }
    void pd(){
        if(mu==1&&a==0&&d==0)return;
        if(c[0])c[0]->make(mu,a,d);
        if(c[1])c[1]->make(mu,a+d+d*sz(c[0]),d);
        mu=1;a=d=0;
    }
    node(){mu=1;}
}nd[maxn*2],*root;
node *merge(node *p,node *q){
    if(!p||!q)return p?p->rz():(q?q->rz():0);
    p->pd();q->pd();
    if(p->key<q->key){
        p->c[1]=merge(p->c[1],q);
        return p->rz();
    }else{
        q->c[0]=merge(p,q->c[0]);
        return q->rz();
    }
}
void node::split(int ned,node *&p,node *&q){
    if(!ned){p=0;q=this;return;}
    if(ned==siz){p=this;q=0;return;}
    pd();
    if(sz(c[0])>=ned){
```

```
            c[0]->split(ned,p,q);c[0]=0;rz();
            q=merge(q,this);
        }else{
            c[1]->split(ned-sz(c[0])-1,p,q);c[1]=0;rz();
            p=merge(this,p);
        }
}
int main(){
    for(int i=1;i<=n;i++){
        nd[i].val=in();
        nd[i].key=rand();
        nd[i].rz();
        root=merge(root,nd+i);
    }
}
```

3.2.2 坚固的字符串

1. ext 库中的 rope

```
#include <ext/rope>

using __gnu_cxx::crope;
using __gnu_cxx::rope;

crope a, b;

int main(void) {
    a = b.substr(pos, len);     // [pos, pos + len)
    a = b.substr(pos);          // [pos, pos]
    b.c_str();                  // might lead to memory leaks
    b.delete_c_str();           // delete the c_str that created before
    a.insert(pos, text);        // insert text before position pos
    a.erase(pos, len);          // erase [pos, pos + len)
}
```

2. 可持久化平衡树实现的 rope

```
class Rope {
private:
    class Node {
    public:
        Node *left, *right;
```

```
    int size;
    char key;

    Node(char key = 0, Node *left = NULL, Node *right = NULL)
            : key(key), left(left), right(right) {
        update();
    }

    void update() {
        size = (left ? left->size : 0) + 1 + (right ? right->size : 0);
    }

    std::string to_string() {
        return (left ? left->to_string() : "") + key
             + (right ? right->to_string() : "");
    }
};

bool random(int a, int b) {
    return rand() % (a + b) < a;
}

Node* merge(Node *x, Node *y) {
    if (!x) {
        return y;
    }
    if (!y) {
        return x;
    }
    if (random(x->size, y->size)) {
        return new Node(x->key, x->left, merge(x->right, y));
    } else {
        return new Node(y->key, merge(x, y->left), y->right);
    }
}

std::pair<Node*, Node*> split(Node *x, int size) {
    if (!x) {
        return std::make_pair<Node*, Node*>(NULL, NULL);
    }
    if (size == 0) {
        return std::make_pair<Node*, Node*>(NULL, x);
```

```
        }
        if (size > x->size) {
            return std::make_pair<Node*, Node*>(x, NULL);
        }
        if (x->left && size <= x->left->size) {
            std::pair<Node*, Node*> part =
                split(x->left, size);
            return std::make_pair(part.first, new Node(x->key, part.second, x->right))
        } else {
            std::pair<Node*, Node*> part =
                split(x->right, size - (x->left ? x->left->size : 0) - 1);
            return std::make_pair(new Node(x->key, x->left, part.first), part.second);
        }
    }

    Node* build(const std::string &text, int left, int right) {
        if (left > right) {
            return NULL;
        }
        int mid = left + right >> 1;
        return new Node(text[mid],
                        build(text, left, mid - 1),
                        build(text, mid + 1, right));
    }

public:
    Node *root;

    Rope() {
        root = NULL;
    }

    Rope(const std::string &text) {
        root = build(text, 0, (int)text.length() - 1);
    }

    Rope(const Rope &other) {
        root = other.root;
    }

    Rope& operator = (const Rope &other) {
        if (this == &other) {
```

```
            return *this;
        }
        root = other.root;
        return *this;
    }

    int size() {
        return root ? root->size : 0;
    }

    void insert(int pos, const std::string &text) {
        if (pos < 0 || pos > size()) {
            throw "Out of range";
        }
        std::pair<Node*, Node*> part = split(root, pos);
        root = merge(merge(part.first, build(text, 0, (int)text.length() - 1)),
                     part.second);
    }

    void erase(int left, int right) {
        if (left < 0 || left >= size() ||
            right < 1 || right > size()) {
            throw "Out of range";
        }
        if (left >= right) {
            return;
        }
        std::pair<Node*, Node*> part = split(root, left);
        root = merge(part.first, split(part.second, right - left).second);
    }

    std::string substr(int left, int right) {
        if (left < 0 || left >= size() ||
            right < 1 || right > size()) {
            throw "Out of range";
        }
        if (left >= right) {
            return "";
        }
        return split(split(root, left).second, right - left).first->to_string();
    }

```

```
    void copy(int left, int right, int pos) {
        if (left < 0 || left >= size() ||
            right < 1 || right > size() ||
            pos < 0 || pos > size()) {
            throw "Out of range";
        }
        if (left >= right) {
            return;
        }
        std::pair<Node*, Node*> part = split(root, pos);
        root = merge(merge(part.first,
                           split(split(root, left).second, right - left).first),
                     part.second);
    }
};
```

3.2.3 坚固的左偏树

```
int Merge(int x, int y){
  if (x == 0 || y == 0) return x + y;
  if (Heap[x].Key < Heap[y].Key) swap(x, y);
  Heap[x].Ri = Merge(Heap[x].Ri, y);
  if (Heap[Heap[x].Le].Dis < Heap[Heap[x].Ri].Dis) swap(Heap[x].Le, Heap[x].Ri);
  if (Heap[x].Ri == 0) Heap[x].Dis = 0;
  else Heap[x].Dis = Heap[Heap[x].Ri].Dis + 1;
  return x;
}

for (int i = 0; i <= n; i++){
        Heap[i].Le = Heap[i].Ri = 0;
        Heap[i].Dis = 0;
        Heap[i].Key = Cost[i];
}
Heap[0].Dis = -1;
```

3.2.4 不坚固的斜堆

```
struct node;
node *Null,*root[maxn];
struct node{
        node* c[2];
```

```
	int val,ind;
	node(int _val=0,int _ind=0){
		val=_val;c[0]=c[1]=Null;ind=_ind;
	}
};
node* merge(node *p,node *q){
	if(p==Null)return q;
	if(q==Null)return p;
	if(p->val>q->val)swap(p,q);
	p->c[1]=merge(p->c[1],q);
	swap(p->c[0],p->c[1]);
	return p;
}

Null=new node(0);
Null->c[0]=Null->c[1]=Null;
```

3.3 树上的魔术师

3.3.1 轻重树链剖分 (zky)

```
vector<int>G[maxn];
int fa[maxn],top[maxn],siz[maxn],son[maxn],mp[maxn],z,dep[maxn];
void dfs(int u){
	siz[u]=1;
	for(int i=0;i<G[u].size();i++){
		int v=G[u][i];
		if(v!=fa[u]){
			fa[v]=u;dep[v]=dep[u]+1;
			dfs(v);
			siz[u]+=siz[v];
			if(siz[son[u]]<siz[v])son[u]=v;
		}
	}
}
void build(int u,int tp){
	top[u]=tp;mp[u]=++z;
	if(son[u])build(son[u],tp);
	for(int v,i=0;i<G[u].size();i++)if((v=G[u][i])!=son[u]&&v!=fa[u])build(v,v);
}
```

3.3.2 轻重树链剖分 (lyx)

```
void Prep(int x){
        dep[x] = dep[fa[x]] + 1;
        size[x] = 1;
        son[x] = 0;
        for (int i = g[x]; i; i = nxt[i]){
                int y = adj[i];
                if (y == fa[x]) continue;
                fa[y] = x;
                Prep(y);
                size[x] += size[y];
                if (size[y] > size[son[x]]) son[x] = y;
        }
}
void Dfs(int x){
        dfn[x] = ++dfc;
        if (son[x] != 0){
                top[son[x]] = top[x];
                Dfs(son[x]);
        }
        for (int i = g[x]; i; i = nxt[i]){
                int y = adj[i];
                if (y != fa[x] && y != son[x]){
                        top[y] = y;
                        Dfs(y);
                }
                if (y != fa[x]){
                        Bel[(i + 1) >> 1] = dfn[y];
                        val[dfn[y]] = len[i];
                }
        }
}
int Ask(int x, int y){
        int Ret = -1000000001;
        while (top[x] != top[y]){
                if (dep[top[y]] > dep[top[x]]) swap(x, y);
                Ret = max(Ret, Query(1, 1, n, dfn[top[x]], dfn[x]));
                x = fa[top[x]];
        }
        if (dep[y] > dep[x]) swap(x, y);
        if (x != y)
```

```
                Ret = max(Ret, Query(1, 1, n, dfn[y] + 1, dfn[x]));
        return Ret;
}
//Hints : Ask 部分具体的求值或者修改要稍作变动
```

3.3.3 Link Cut Tree(zky)

```
struct LCT{
    struct node{
        bool rev;
        int mx,val;
        node *f,*c[2];
        bool d(){return this==f->c[1];}
        bool rt(){return !f||(f->c[0]!=this&&f->c[1]!=this);}
        void sets(node *x,int d){pd();if(x)x->f=this;c[d]=x;rz();}
        void makerv(){rev^=1;swap(c[0],c[1]);}
        void pd(){
            if(rev){
                if(c[0])c[0]->makerv();
                if(c[1])c[1]->makerv();
                rev=0;
            }
        }
        void rz(){
            mx=val;
            if(c[0])mx=max(mx,c[0]->mx);
            if(c[1])mx=max(mx,c[1]->mx);
        }
    }nd[int(1e4)+1];
    void rot(node *x){
        node *y=x->f;if(!y->rt())y->f->pd();
        y->pd();x->pd();bool d=x->d();
        y->sets(x->c[!d],d);
        if(y->rt())x->f=y->f;
        else y->f->sets(x,y->d());
        x->sets(y,!d);
    }
    void splay(node *x){
        while(!x->rt())
            if(x->f->rt())rot(x);
            else if(x->d()==x->f->d())rot(x->f),rot(x);
            else rot(x),rot(x);
```

```
    }
    node* access(node *x){
        node *y=0;
        for(;x;x=x->f){
            splay(x);
            x->sets(y,1);y=x;
        }return y;
    }
    void makert(node *x){
        access(x)->makerv();
        splay(x);
    }
    void link(node *x,node *y){
        makert(x);
        x->f=y;
        access(x);
    }
    void cut(node *x,node *y){
        makert(x);access(y);splay(y);
        y->c[0]=x->f=0;
        y->rz();
    }
    void link(int x,int y){link(nd+x,nd+y);}
    void cut(int x,int y){cut(nd+x,nd+y);}
}T;
```

3.3.4 Link Cut Tree(lyx)

```
struct node{
        bool Rev;
        int c[2], fa, Chain, Aux, Val;
}T[N];

inline int Sum(int x){
        return T[x].Chain ^ T[x].Aux;
}

inline void Rev(int x){
        if (!x) return;
        swap(T[x].c[0],T[x].c[1]);
        T[x].Rev ^= 1;
}
```

```
inline void Update(int x){
        T[x].Chain = Sum(T[x].c[0]) ^ Sum(T[x].c[1]) ^ T[x].Val;
}

inline void Lazy_Down(int x){
        if (!x) return;
        if (T[x].Rev) Rev(T[x].c[0]), Rev(T[x].c[1]), T[x].Rev = 0;
}

inline void Rotate(int x,int c){
        int fa = T[x].fa, ft = T[fa].fa;
        T[x].fa = ft, T[fa].fa = x;
        if (ft) T[ft].c[T[ft].c[1] == fa] = x;
        T[fa].c[c] = T[x].c[!c];
        if (T[x].c[!c]) T[T[x].c[!c]].fa = fa;
        T[x].c[!c] = fa;
        if (Par[fa]) Par[x] = Par[fa], Par[fa] = 0;
        Update(fa);
}

inline void Splay(int x){
        int top = 0;
        for (int u = x; u; u = T[u].fa) Stack[++top] = u;

        for( ; top; top--) Lazy_Down(Stack[top]);

        for( ; T[x].fa; ){
                int fa = T[x].fa, ft = T[fa].fa;
                if (!ft) Rotate(x, T[fa].c[1] == x); else
                {
                        if (T[fa].c[1] == x)
                        {
                                if (T[ft].c[1] == fa) Rotate(fa, 1),Rotate(x, 1);
                                else Rotate(x, 1),Rotate(x, 0);
                        } else
                                if (T[ft].c[0] == fa) Rotate(fa, 0),Rotate(x, 0);
                                else Rotate(x, 0),Rotate(x, 1);
                }
        }
        Update(x);
}
```

```
inline int Access(int u){

        int Nxt = 0;

        while (u){
                Splay(u);
                if (T[u].c[1]){
                        T[T[u].c[1]].fa = 0;
                        Par[T[u].c[1]] = u;
                        T[u].Aux ^= Sum(T[u].c[1]);
                }
                T[u].c[1] = Nxt;
                if (Nxt){
                        T[Nxt].fa = u;
                        Par[Nxt] = 0;
                        T[u].Aux ^= Sum(Nxt);
                }
                Update(u);
                Nxt = u;
                u = Par[u];
        }

        return Nxt;

}

inline void Root(int u){
        Rev(Access(u));
}

inline void Mark(int x, int col){
        Access(x);
        Splay(x);
        T[x].Val ^= col;
        Update(x);
}

inline void Link(int u, int v){
        Root(v);
        Access(v);
        Access(u);
```

```
        Splay(v);
        Splay(u);
        Par[v] = u;
        T[u].Aux ^= Sum(v);
        Access(v);
}

inline void Cut(int u, int v){
        Root(v);
        Access(u);
        Splay(u);
        T[T[u].c[0]].fa = 0;
        T[u].c[0] = 0;
        Update(u);
}
```

3.3.5 AAA Tree

```
#define rep(i,a,n) for(int i=a;i<n;i++)
int n,m;
struct info{
    int mx,mn,sum,sz;
    info(){}
    info(int mx,int mn,int sum,int sz):
        mx(mx),mn(mn),sum(sum),sz(sz){}
    void deb(){printf("sum:%d size:%d",(int)sum,sz);}
};
struct flag{
    int mul,add;
    flag(){mul=1;}
    flag(int mul,int add):
        mul(mul),add(add){}
    bool empty(){return mul==1&&add==0;}
};
info operator+(const info &a,const flag &b) {
    return a.sz?info(a.mx*b.mul+b.add,a.mn*b.mul+b.add,a.sum*b.mul+b.add*a.sz,a.sz):a;
}
info operator+(const info &a,const info &b) {
    return info(max(a.mx,b.mx),min(a.mn,b.mn),a.sum+b.sum,a.sz+b.sz);
}
flag operator+(const flag &a,const flag &b) {
    return flag(a.mul*b.mul,a.add*b.mul+b.add);
```

```
}
struct node{
    node *c[4],*f;
    flag Cha,All;
    info cha,sub,all;
    bool rev,inr;
    int val;
    void makerev(){rev^=1;swap(c[0],c[1]);}
    void makec(const flag &a){
        Cha=Cha+a;cha=cha+a;val=val*a.mul+a.add;
        all=cha+sub;
    }
    void makes(const flag &a,bool _=1){
        All=All+a;all=all+a;sub=sub+a;
        if(_)makec(a);
    }
    void rz(){
        cha=all=sub=info(-(1<<30),1<<30,0,0);
        if(!inr)all=cha=info(val,val,val,1);
        rep(i,0,2)if(c[i])cha=cha+c[i]->cha,sub=sub+c[i]->sub;
        rep(i,0,4)if(c[i])all=all+c[i]->all;
        rep(i,2,4)if(c[i])sub=sub+c[i]->all;
    }
    void pd(){
        if(rev){
            if(c[0])c[0]->makerev();
            if(c[1])c[1]->makerev();
            rev=0;
        }
        if(!All.empty()){
            rep(i,0,4)if(c[i])c[i]->makes(All,i>=2);
            All=flag(1,0);
        }
        if(!Cha.empty()){
            rep(i,0,2)if(c[i])c[i]->makec(Cha);
            Cha=flag(1,0);
        }

    }
    node *C(int i){if(c[i])c[i]->pd();return c[i];}
    bool d(int ty){return f->c[ty+1]==this;}
    int D(){rep(i,0,4)if(f->c[i]==this)return i;}
```

```
    void sets(node *x,int d){if(x)x->f=this;c[d]=x;}
    bool rt(int ty){
        if(ty==0)return !f||(f->c[0]!=this&&f->c[1]!=this);
        else return !f||!f->inr||!inr;
    }
}nd[maxn*2],*cur=nd+maxn,*pool[maxn],**Cur=pool;
int _cnt;
node *newnode(){
    _cnt++;
    node *x=(Cur==pool)?cur++:*(--Cur);
    rep(i,0,4)x->c[i]=0;x->f=0;
    x->All=x->Cha=flag(1,0);
    x->all=x->cha=info(-(1<<30),(1<<30),0,0);
    x->inr=1;x->rev=0;x->val=0;
    return x;
}
void dele(node *x){*(Cur++)=x;}
void rot(node *x,int ty){
    node *p=x->f;int d=x->d(ty);
    if(!p->f)x->f=0;else p->f->sets(x,p->D());
    p->sets(x->c[!d+ty],d+ty);x->sets(p,!d+ty);p->rz();
}
void splay(node *x,int ty=0){
    while(!x->rt(ty)){
        if(x->f->rt(ty))rot(x,ty);
        else if(x->d(ty)==x->f->d(ty))rot(x->f,ty),rot(x,ty);
        else rot(x,ty),rot(x,ty);
    }x->rz();
}
void add(node *u,node *w){
    w->pd();
    rep(i,2,4)if(!w->c[i]){w->sets(u,i);return;}
    node *x=newnode(),*v;
    for(v=w;v->c[2]->inr;v=v->C(2));
    x->sets(v->c[2],2);x->sets(u,3);
    v->sets(x,2);splay(x,2);
}
void del(node *w){
    if(w->f->inr){
        w->f->f->sets(w->f->c[5-w->D()],w->f->D());
        dele(w->f);splay(w->f->f,2);
    }else w->f->sets(0,w->D());
```

```
    w->f=0;
}
void access(node *w){
    static node *sta[maxn];
    static int top=0;
    node *v=w,*u;
    for(u=w;u;u=u->f)sta[top++]=u;
    while(top)sta[--top]->pd();
    splay(w);
    if(w->c[1])u=w->c[1],w->c[1]=0,add(u,w),w->rz();
    while(w->f){
        for(u=w->f;u->inr;u=u->f);
        splay(u);
        if(u->c[1])w->f->sets(u->c[1],w->D()),splay(w->f,2);
        else del(w);
        u->sets(w,1);
        (w=u)->rz();
    }splay(v);
}
void makert(node *x){
    access(x);x->makerev();
}
node *findp(node *u){
    access(u);u=u->C(0);
    while(u&&u->c[1])u=u->C(1);
    return u;
}
node *findr(node *u){for(;u->f;u=u->f);return u;}
node* cut(node *u){
    node *v=findp(u);
    if(v)access(v),del(u),v->rz();
    return v;
}
void link(node *u,node *v) {
    node* p=cut(u);
    if(findr(u)!=findr(v))p=v;
    if(p)access(p),add(u,p),p->rz();
}
int main(){
//  freopen("bzoj3153.in","r",stdin);
    n=getint();m=getint();
    static int _u[maxn],_v[maxn];
```

```
    rep(i,1,n)_u[i]=getint(),_v[i]=getint();
    rep(i,1,n+1){
        nd[i].val=getint();
        nd[i].rz();
    }
    rep(i,1,n)makert(nd+_u[i]),link(nd+_u[i],nd+_v[i]);
    int root=getint();
    makert(nd+root);
//  deb();
    int x,y,z;
    node *u,*v;
    while(m--){
        int k=getint();x=getint();
        u=nd+x;
        if(k==0||k==3||k==4||k==5||k==11){
            access(u);
            if(k==3||k==4||k==11){
                int ans=u->val;
                rep(i,2,4)if(u->c[i]){
                    info res=u->c[i]->all;
                    if(k==3) ans=min(ans,res.mn);
                    else if(k==4) ans=max(ans,res.mx);
                    else if(k==11) ans+=res.sum;
                }printf("%d\n",ans);
            }else{
                y=getint();
                flag fg(k==5,y);
                u->val=u->val*fg.mul+fg.add;
                rep(i,2,4)if(u->c[i])u->c[i]->makes(fg);
                u->rz();
            }
        }else if(k==2||k==6||k==7||k==8||k==10){
            y=getint();
            makert(u),access(nd+y),splay(u);
            if (k==7||k==8||k==10) {
                info ans=u->cha;
                if (k==7) printf("%d\n",ans.mn);
                else if (k==8) printf("%d\n",ans.mx);
                else printf("%d\n",ans.sum);
            }else u->makec(flag(k==6,getint()));
            makert(nd+root);
        }else if(k==9)link(u,nd+getint());
```

```
        else if(k==1)makert(u),root=x;
    }
    return 0;
}
```

3.4 ST

```
for (int i = 1; i <= n; i++)
        Log[i] = int(log2(i));

for (int i = 1; i <= n; i++)
        Rmq[i][0] = i;

for (int k = 1; (1 << k) <= n; k++)
        for (int i = 1; i + (1 << k) - 1 <= n; i++){
                int x = Rmq[i][k - 1], y = Rmq[i + (1 << (k - 1))][k - 1];
                if (a[x] < a[y])
                        Rmq[i][k] = x;
                else
                        Rmq[i][k] = y;
        }

int Smallest(int l, int r){
        int k = Log[r - l + 1];

        int x = Rmq[l][k];
        int y = Rmq[r - (1 << k) + 1][k];

        if (a[x] < a[y]) return x;
        else return y;
}
```

3.5 可持久化线段树

```
struct node1 {
        int L, R, Lson, Rson, Sum;
} tree[N * 40];
int root[N], a[N], b[N];
int tot, n, m;
int Real[N];
int Same(int x) {
```

```
        ++tot;
        tree[tot] = tree[x];
        return tot;
}
int build(int L, int R) {
        ++tot;
        tree[tot].L = L;
        tree[tot].R = R;
        tree[tot].Lson = tree[tot].Rson = tree[tot].Sum = 0;
        if (L == R) return tot;
        int s = tot;
        int mid = (L + R) >> 1;
        tree[s].Lson = build(L, mid);
        tree[s].Rson = build(mid + 1, R);
        return s;
}
int Ask(int Lst, int Cur, int L, int R, int k) {
        if (L == R) return L;
        int Mid = (L + R) >> 1;
        int Left = tree[tree[Cur].Lson].Sum - tree[tree[Lst].Lson].Sum;
        if (Left >= k) return Ask(tree[Lst].Lson, tree[Cur].Lson, L, Mid, k);
        k -= Left;
        return Ask(tree[Lst].Rson, tree[Cur].Rson, Mid + 1, R, k);
}
int Add(int Lst, int pos) {
        int root = Same(Lst);
        tree[root].Sum++;
        if (tree[root].L == tree[root].R) return root;
        int mid = (tree[root].L + tree[root].R) >> 1;
        if (pos <= mid) tree[root].Lson = Add(tree[root].Lson, pos);
        else tree[root].Rson = Add(tree[root].Rson, pos);
        return root;
}
int main() {
        scanf("%d%d", &n, &m);
        int up = 0;
        for (int i = 1; i <= n; i++){
                scanf("%d", &a[i]);
                b[i] = a[i];
        }
        sort(b + 1, b + n + 1);
        up = unique(b + 1, b + n + 1) - b - 1;
```

```
        for (int i = 1; i <= n; i++){
                int tmp = lower_bound(b + 1, b + up + 1, a[i]) - b;
                Real[tmp] = a[i];
                a[i] = tmp;
        }
        tot = 0;
        root[0] = build(1, up);
        for (int i = 1; i <= n; i++){
                root[i] = Add(root[i - 1], a[i]);
        }
        for (int i = 1; i <= m; i++){
                int u, v, w;
                scanf("%d%d%d", &u, &v, &w);
                printf("%d\n", Real[Ask(root[u - 1], root[v], 1, up, w)]);
        }
        return 0;
}
```

3.6 可持久化 Trie

```
int Pre[N];
int n, q, Len, cnt, Lstans;
char s[N];
int First[N], Last[N];
int Root[N];
int Trie_tot;
struct node{
    int To[30];
    int Lst;
}Trie[N];
int tot;
struct node1{
    int L, R, Lson, Rson, Sum;
}tree[N * 25];
int Build(int L, int R){
    ++tot;
    tree[tot].L = L;
    tree[tot].R = R;
    tree[tot].Lson = tree[tot].Rson = tree[tot].Sum = 0;
    if (L == R) return tot;
    int s = tot;
    int mid = (L + R) >> 1;
```

```
    tree[s].Lson = Build(L, mid);
    tree[s].Rson = Build(mid + 1, R);
    return s;
}
int Same(int x){
    ++tot;
    tree[tot] = tree[x];
    return tot;
}
int Add(int Lst, int pos){
    int s = Same(Lst);
    tree[s].Sum++;
    if (tree[s].L == tree[s].R) return s;
    int Mid = (tree[s].L + tree[s].R) >> 1;
    if (pos <= Mid) tree[s].Lson = Add(tree[Lst].Lson, pos);
    else tree[s].Rson = Add(tree[Lst].Rson, pos);
    return s;
}

int Ask(int Lst, int Cur, int L, int R, int pos){
    if (L >= pos) return 0;
    if (R < pos) return tree[Cur].Sum - tree[Lst].Sum;
    int Mid = (L + R) >> 1;
    int Ret = Ask(tree[Lst].Lson, tree[Cur].Lson, L, Mid, pos);
    Ret += Ask(tree[Lst].Rson, tree[Cur].Rson, Mid + 1, R, pos);
    return Ret;
}

int main(){
    while (scanf("%d", &n) == 1){
        for (int i = 1; i <= Trie_tot; i++){
            for (int j = 1; j <= 26; j++)
                Trie[i].To[j] = 0;
            Trie[i].Lst = 0;
        }
        Trie_tot = 1;
        cnt = 0;
        for (int ii = 1; ii <= n; ii++){
            scanf("%s", s + 1);
            Len = strlen(s + 1);
            int Cur = 1;
            First[ii] = cnt + 1;
```

```cpp
            for (int i = 1; i <= Len; i++){
                int ch = s[i] - 'a' + 1;
                if (Trie[Cur].To[ch] == 0){
                    ++Trie_tot;
                    Trie[Cur].To[ch] = Trie_tot;
                }
                Cur = Trie[Cur].To[ch];
                Pre[++cnt] = Trie[Cur].Lst;
                Trie[Cur].Lst = ii;
            }
            Last[ii] = cnt;
        }
        tot = 0;
        Root[0] = Build(0, n);
        for (int i = 1; i <= cnt; i++){
            Root[i] = Add(Root[i - 1], Pre[i]);
        }
        Lstans = 0;
        scanf("%d", &q);
        for (int ii = 1; ii <= q; ii++){
            int L, R;
            scanf("%d%d", &L, &R);
            L = (L + Lstans) % n + 1;
            R = (R + Lstans) % n + 1;
            if (L > R) swap(L, R);
            int Ret = Ask(Root[First[L] - 1], Root[Last[R]], 0, n, L);
            printf("%d\n", Ret);
            Lstans = Ret;
        }
    }
    return 0;
}
```

3.7 k-d 树

```cpp
long long norm(const long long &x) {
    //    For manhattan distance
    return std::abs(x);
    //    For euclid distance
    return x * x;
}

```

```cpp
struct Point {
    int x, y, id;

    const int& operator [] (int index) const {
        if (index == 0) {
            return x;
        } else {
            return y;
        }
    }

    friend long long dist(const Point &a, const Point &b) {
        long long result = 0;
        for (int i = 0; i < 2; ++i) {
            result += norm(a[i] - b[i]);
        }
        return result;
    }
} point[N];

struct Rectangle {
    int min[2], max[2];

    Rectangle() {
        min[0] = min[1] = INT_MAX;
        max[0] = max[1] = INT_MIN;
    }

    void add(const Point &p) {
        for (int i = 0; i < 2; ++i) {
            min[i] = std::min(min[i], p[i]);
            max[i] = std::max(max[i], p[i]);
        }
    }

    long long dist(const Point &p) {
        long long result = 0;
        for (int i = 0; i < 2; ++i) {
            //    For minimum distance
            result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
            //    For maximum distance
            result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
```

```
        }
        return result;
    }
};

struct Node {
    Point seperator;
    Rectangle rectangle;
    int child[2];

    void reset(const Point &p) {
        seperator = p;
        rectangle = Rectangle();
        rectangle.add(p);
        child[0] = child[1] = 0;
    }
} tree[N << 1];

int size, pivot;

bool compare(const Point &a, const Point &b) {
    if (a[pivot] != b[pivot]) {
        return a[pivot] < b[pivot];
    }
    return a.id < b.id;
}

int build(int l, int r, int type = 1) {
    pivot = type;
    if (l >= r) {
        return 0;
    }
    int x = ++size;
    int mid = l + r >> 1;
    std::nth_element(point + l, point + mid, point + r, compare);
    tree[x].reset(point[mid]);
    for (int i = l; i < r; ++i) {
        tree[x].rectangle.add(point[i]);
    }
    tree[x].child[0] = build(l, mid, type ^ 1);
    tree[x].child[1] = build(mid + 1, r, type ^ 1);
    return x;
```

```cpp
}

int insert(int x, const Point &p, int type = 1) {
    pivot = type;
    if (x == 0) {
        tree[++size].reset(p);
        return size;
    }
    tree[x].rectangle.add(p);
    if (compare(p, tree[x].seperator)) {
        tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
    } else {
        tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
    }
    return x;
}

//    For minimum distance
void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
    pivot = type;
    if (x == 0 || tree[x].rectangle.dist(p) > answer.first) {
        return;
    }
    answer = std::min(answer,
             std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
    if (compare(p, tree[x].seperator)) {
        query(tree[x].child[0], p, answer, type ^ 1);
        query(tree[x].child[1], p, answer, type ^ 1);
    } else {
        query(tree[x].child[1], p, answer, type ^ 1);
        query(tree[x].child[0], p, answer, type ^ 1);
    }
}

std::priority_queue<std::pair<long long, int> > answer;

void query(int x, const Point &p, int k, int type = 1) {
    pivot = type;
    if (x == 0 ||
        (int)answer.size() == k && tree[x].rectangle.dist(p) > answer.top().first) {
        return;
    }
```

```
    answer.push(std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
    if ((int)answer.size() > k) {
        answer.pop();
    }
    if (compare(p, tree[x].seperator)) {
        query(tree[x].child[0], p, k, type ^ 1);
        query(tree[x].child[1], p, k, type ^ 1);
    } else {
        query(tree[x].child[1], p, k, type ^ 1);
        query(tree[x].child[0], p, k, type ^ 1);
    }
}
```

3.8 莫队算法

```
struct node{
        int l, r, id;
        friend bool operator < (const node &a, const node &b){
                if (a.l / Block == b.l / Block) return a.r / Block < b.r / Block;
                return a.l / Block < b.l / Block;
        }
}q[N];
Block = int(sqrt(n));
for (int i = 1; i <= m; i++){
        scanf("%d%d", &q[i].l, &q[i].r);
        q[i].id = i;
}
sort(q + 1, q + 1 + m);
Cur = a[1]; /// Hints: adjust by yourself
Le = Ri = 1;
for (int i = 1; i <= m; i++){
        while (q[i].r > Ri) Ri++, ChangeRi(1, Le, Ri);
        while (q[i].l > Le) ChangeLe(-1, Le, Ri), Le++;
        while (q[i].l < Le) Le--, ChangeLe(1, Le, Ri);
        while (q[i].r < Ri) ChangeRi(-1, Le, Ri), Ri--;
        Ans[q[i].id] = Cur;
}
```

3.9 整体二分

```
struct BIT{
	LL d[maxn];
	inline int lowbit(int x){return x&-x;}
	LL get(int x){
		LL ans=0;
		while(x)ans+=d[x],x-=lowbit(x);
		return ans;
	}
	void updata(int x,LL f){
		while(x<=m)d[x]+=f,x+=lowbit(x);
	}
	void add(int l,int r,LL f){
		updata(l,f);
		updata(r+1,-f);
	}
}T,T2;
int anss[maxn],wana[maxn];
struct qes{
	LL x,y,z;
	qes(LL _x=0,LL _y=0,LL _z=0):
		x(_x),y(_y),z(_z){}
}q[maxn],p[maxn];
bool part(qes &q){
	if(q.y+q.z>=wana[q.x])return 1;
	q.z+=q.y;q.y=0;return 0;
}
void solve(int lef,int rig,int l,int r){
	if(l==r){
		for(int i=lef;i<=rig;i++)if(anss[p[i].x]!=-1)
		anss[p[i].x]=l;return;
	}int mid=(l+r)>>1;
	for(int i=l;i<=mid;i++){
		if(q[i].x<=q[i].y)T.add(q[i].x,q[i].y,q[i].z);
		else T.add(1,q[i].y,q[i].z),T.add(q[i].x,m,q[i].z);
	}for(int i=lef;i<=rig;i++){
		p[i].y=0;
		for(int j=0;j<O[p[i].x].size()&&p[i].y<=int(1e9)+1;j++)
		p[i].y+=T.get(O[p[i].x][j]);
	}for(int i=l;i<=mid;i++){
		if(q[i].x<=q[i].y)T.add(q[i].x,q[i].y,-q[i].z);
		else T.add(1,q[i].y,-q[i].z),T.add(q[i].x,m,-q[i].z);
	}int dv=stable_partition(p+lef,p+rig+1,part)-p-1;
```

```
        if(lef<=dv)
        solve(lef,dv,l,mid);
        if(dv+1<=rig)
        solve(dv+1,rig,mid+1,r);
}
```

3.10 树状数组 kth

```
int find(int k){
    int cnt=0,ans=0;
    for(int i=22;i>=0;i--){
        ans+=(1<<i);
        if(ans>n || cnt+d[ans]>=k)ans-=(1<<i);
        else cnt+=d[ans];
    }
    return ans+1;
}
```

4 图论

4.1 强连通分量

```
int stamp, comps, top;
int dfn[N], low[N], comp[N], stack[N];

void tarjan(int x) {
    dfn[x] = low[x] = ++stamp;
    stack[top++] = x;
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (!dfn[y]) {
            tarjan(y);
            low[x] = std::min(low[x], low[y]);
        } else if (!comp[y]) {
            low[x] = std::min(low[x], dfn[y]);
        }
    }
    if (low[x] == dfn[x]) {
        comps++;
        do {
            int y = stack[--top];
```

```
                comp[y] = comps;
            } while (stack[top] != x);
        }
}

void solve() {
    stamp = comps = top = 0;
    std::fill(dfn, dfn + n, 0);
    std::fill(comp, comp + n, 0);
    for (int i = 0; i < n; ++i) {
        if (!dfn[i]) {
            tarjan(i);
        }
    }
}
```

4.1.1 点双连通分量

```
struct Edge{
        int To, id;
        Edge(){}
        Edge(int _To, int _id){
                To = _To;
                id = _id;
        }
};

int n, m, dfc, block, top;
vector<Edge> G[N];
vector<int> H[N];

int dfn[N], low[N], stack[N], belong[N];

void Tarjan(int x, int lst){
        dfn[x] = low[x] = ++dfc;
        stack[top++] = x;
        for (int i = 0; i < (int)G[x].size(); i++){
                int y = G[x][i].To;
                if (!dfn[y]){
                        Tarjan(y, G[x][i].id);
                        low[x] = min(low[x], low[y]);
                } else if (!belong[y] && G[x][i].id != lst){
```

```
                        low[x] = min(low[x], dfn[y]);
                }
        }
        if (low[x] == dfn[x]){
                block++;
                do{
                        int y = stack[--top];
                        belong[y] = block;
                } while (stack[top] != x);
        }
}

//bridge
                for (int i = 1; i <= n; i++)
                        for (int j = 0; j < G[i].size(); j++){
                                int y = G[i][j].To;
                                if (belong[i] == belong[y]) continue;
                                H[belong[i]].push_back(belong[y]);
                        }
```

4.2 2-SAT 问题

```
int stamp, comps, top;
int dfn[N], low[N], comp[N], stack[N];

void add(int x, int a, int y, int b) {
    edge[x << 1 | a].push_back(y << 1 | b);
}

void tarjan(int x) {
    dfn[x] = low[x] = ++stamp;
    stack[top++] = x;
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        if (!dfn[y]) {
            tarjan(y);
            low[x] = std::min(low[x], low[y]);
        } else if (!comp[y]) {
            low[x] = std::min(low[x], dfn[y]);
        }
    }
    if (low[x] == dfn[x]) {
```

```
            comps++;
            do {
                int y = stack[--top];
                comp[y] = comps;
            } while (stack[top] != x);
        }
}

bool solve() {
    int counter = n + n + 1;
    stamp = top = comps = 0;
    std::fill(dfn, dfn + counter, 0);
    std::fill(comp, comp + counter, 0);
    for (int i = 0; i < counter; ++i) {
        if (!dfn[i]) {
            tarjan(i);
        }
    }
    for (int i = 0; i < n; ++i) {
        if (comp[i << 1] == comp[i << 1 | 1]) {
            return false;
        }
        answer[i] = (comp[i << 1 | 1] < comp[i << 1]);
    }
    return true;
}
```

4.3 二分图最大匹配

4.3.1 Hungary 算法

时间复杂度：$\mathcal{O}(V \cdot E)$

```
vector<int>G[maxn];
int Link[maxn],vis[maxn],T;
bool find(int x){
        for(int i=0;i<G[x].size();i++){
                int v=G[x][i];
                if(vis[v]==T)continue;
                vis[v]=T;
                if(!Link[v]||find(Link[v])){
                        Link[v]=x;
                        return 1;
```

```
                }
        }return 0;
}
int Hungarian(int n){
        int ans=0;
        memset(Link,0,sizeof Link);
        for(int i=1;i<=n;i++){
                T++;
                ans+=find(i);
        }return ans;
}
```

4.3.2 Hopcroft Karp 算法

时间复杂度：$\mathcal{O}(\sqrt{V} \cdot E)$

```
int matchx[N], matchy[N], level[N];

bool dfs(int x) {
    for (int i = 0; i < (int)edge[x].size(); ++i) {
        int y = edge[x][i];
        int w = matchy[y];
        if (w == -1 || level[x] + 1 == level[w] && dfs(w)) {
            matchx[x] = y;
            matchy[y] = x;
            return true;
        }
    }
    level[x] = -1;
    return false;
}

int solve() {
    std::fill(matchx, matchx + n, -1);
    std::fill(matchy, matchy + m, -1);
    for (int answer = 0; ; ) {
        std::vector<int> queue;
        for (int i = 0; i < n; ++i) {
            if (matchx[i] == -1) {
                level[i] = 0;
                queue.push_back(i);
            } else {
                level[i] = -1;
```

```
            }
        }
        for (int head = 0; head < (int)queue.size(); ++head) {
            int x = queue[head];
            for (int i = 0; i < (int)edge[x].size(); ++i) {
                int y = edge[x][i];
                int w = matchy[y];
                if (w != -1 && level[w] < 0) {
                    level[w] = level[x] + 1;
                    queue.push_back(w);
                }
            }
        }
        int delta = 0;
        for (int i = 0; i < n; ++i) {
            if (matchx[i] == -1 && dfs(i)) {
                delta++;
            }
        }
        if (delta == 0) {
            return answer;
        } else {
            answer += delta;
        }
    }
}
```

4.4 二分图最大权匹配

时间复杂度：$\mathcal{O}(V^4)$

```
int labelx[N], labely[N], match[N], slack[N];
bool visitx[N], visity[N];

bool dfs(int x) {
    visitx[x] = true;
    for (int y = 0; y < n; ++y) {
        if (visity[y]) {
            continue;
        }
        int delta = labelx[x] + labely[y] - graph[x][y];
        if (delta == 0) {
            visity[y] = true;
```

```cpp
            if (match[y] == -1 || dfs(match[y])) {
                match[y] = x;
                return true;
            }
        } else {
            slack[y] = std::min(slack[y], delta);
        }
    }
    return false;
}

int solve() {
    for (int i = 0; i < n; ++i) {
        match[i] = -1;
        labelx[i] = INT_MIN;
        labely[i] = 0;
        for (int j = 0; j < n; ++j) {
            labelx[i] = std::max(labelx[i], graph[i][j]);
        }
    }
    for (int i = 0; i < n; ++i) {
        while (true) {
            std::fill(visitx, visitx + n, 0);
            std::fill(visity, visity + n, 0);
            for (int j = 0; j < n; ++j) {
                slack[j] = INT_MAX;
            }
            if (dfs(i)) {
                break;
            }
            int delta = INT_MAX;
            for (int j = 0; j < n; ++j) {
                if (!visity[j]) {
                    delta = std::min(delta, slack[j]);
                }
            }
            for (int j = 0; j < n; ++j) {
                if (visitx[j]) {
                    labelx[j] -= delta;
                }
                if (visity[j]) {
                    labely[j] += delta;
```

```
                } else {
                    slack[j] -= delta;
                }
            }
        }
    }
    int answer = 0;
    for (int i = 0; i < n; ++i) {
        answer += graph[match[i]][i];
    }
    return answer;
}
```

4.5 最大流 (dinic)

时间复杂度：$\mathcal{O}(V^2 \cdot E)$

```
struct edge{int u,v,cap,flow;};
vector<edge>edges;
vector<int>G[maxn];
int s,t;
int cur[maxn],d[maxn];
void add(int u,int v,int cap){
        edges.push_back((edge){u,v,cap,0});
        G[u].push_back(edges.size()-1);
        edges.push_back((edge){v,u,0,0});
        G[v].push_back(edges.size()-1);
}
bool bfs(){
        static int vis[maxn];
        memset(vis,0,sizeof vis);vis[s]=1;
        queue<int>q;q.push(s);d[s]=0;
        while(!q.empty()){
                int u=q.front();q.pop();
                for(int i=0;i<G[u].size();i++){
                        edge e=edges[G[u][i]];if(vis[e.v]||e.cap==e.flow)continue;
                        d[e.v]=d[u]+1;vis[e.v]=1;q.push(e.v);
                }
        }return vis[t];
}
int dfs(int u,int a){
        if(u==t||!a)return a;
        int flow=0,f;
```

```
	for(int &i=cur[u];i<G[u].size();i++){
		edge e=edges[G[u][i]];
		if(d[e.v]==d[u]+1&&(f=dfs(e.v,min(a,e.cap-e.flow)))>0){
			edges[G[u][i]].flow+=f;
			edges[G[u][i]^1].flow-=f;
			flow+=f;a-=f;if(!a)break;
		}
	}return flow;
}
int dinic(){
	int flow=0,x;
	while(bfs()){
		memset(cur,0,sizeof cur);
		while(x=dfs(s,INT_MAX)){
			flow+=x;
			memset(cur,0,sizeof cur);
		}
	}return flow;
}
```

4.6 最大流 (sap)

时间复杂度：$\mathcal{O}(V^2 \cdot E)$

```
int g[T], adj[M], nxt[M], f[M];
int cnt[T], dist[T], cur[T], fa[T], dat[T];
void Ins(int x, int y, int ff, int rf){
	adj[++tot] = y; nxt[tot] = g[x]; g[x] = tot; f[tot] = ff;
	adj[++tot] = x; nxt[tot] = g[y]; g[y] = tot; f[tot] = rf;
}
int sap(int s, int t){
	int x, sum;
	for (int i = 1; i <= t; i++){
		dist[i] = 1;
		cur[i] = g[i];
		fa[i] = 0;
		dat[i] = 0;
		cnt[i] = 0;
	}
	cnt[0] = 1; cnt[1] = t - 1;
	dist[t] = 0;
	dat[s] = INF;
	x = s;
```

```
    sum = 0;
    while (1){
        int p;
        for (p = cur[x]; p; p = nxt[p]){
            if (f[p] > 0 && dist[adj[p]] == dist[x] - 1) break;
        }
        if (p > 0){
            cur[x] = p;
            fa[adj[p]] = p;
            dat[adj[p]] = min(dat[x], f[p]);
            x = adj[p];
            if (x == t){
                sum += dat[x];
                while (x != s){
                    f[fa[x]] -= dat[t];
                    f[fa[x] ^ 1] += dat[t];
                    x = adj[fa[x] ^ 1];
                }
            }
        } else {
            cnt[dist[x]] --;
            if (cnt[dist[x]] == 0) return sum;
            dist[x] = t + 1;
            for (int p = g[x]; p; p = nxt[p]){
                if (f[p] > 0 && dist[adj[p]] + 1 < dist[x]){
                    dist[x] = dist[adj[p]] + 1;
                    cur[x] = p;
                }
            }
            cnt[dist[x]]++;
            if (dist[s] > t) return sum;
            if (x != s) x = adj[fa[x] ^ 1];
        }
    }
}
/*
tot = 1
edges' id start from 2
remember to clean g
t is the number of points
*/
```

4.7 上下界网络流

$B(u,v)$ 表示边 (u,v) 流量的下界，$C(u,v)$ 表示边 (u,v) 流量的上界，$F(u,v)$ 表示边 (u,v) 的流量。设 $G(u,v)=F(u,v)-B(u,v)$，显然有

$$0 \le G(u,v) \le C(u,v)-B(u,v)$$

4.7.1 无源汇的上下界可行流

建立超级源点 S^* 和超级汇点 T^*，对于原图每条边 (u,v) 在新网络中连如下三条边：$S^* \to v$，容量为 $B(u,v)$；$u \to T^*$，容量为 $B(u,v)$；$u \to v$，容量为 $C(u,v)-B(u,v)$。最后求新网络的最大流，判断从超级源点 S^* 出发的边是否都满流即可，边 (u,v) 的最终解中的实际流量为 $G(u,v)+B(u,v)$。

4.7.2 有源汇的上下界可行流

从汇点 T 到源点 S 连一条上界为 ∞，下界为 0 的边。按照无源汇的上下界可行流一样做即可，流量即为 $T \to S$ 边上的流量。

4.7.3 有源汇的上下界最大流

1. 在有源汇的上下界可行流中，从汇点 T 到源点 S 的边改为连一条上界为 ∞，下届为 x 的边。x 满足二分性质，找到最大的 x 使得新网络存在无源汇的上下界可行流即为原图的最大流。

2. 从汇点 T 到源点 S 连一条上界为 ∞，下界为 0 的边，变成无源汇的网络。按照无源汇的上下界可行流的方法，建立超级源点 S^* 和超级汇点 T^*，求一遍 $S^* \to T^*$ 的最大流，再将从汇点 T 到源点 S 的这条边拆掉，求一次 $S \to T$ 的最大流即可。

4.7.4 有源汇的上下界最小流

1. 在有源汇的上下界可行流中，从汇点 T 到源点 S 的边改为连一条上界为 x，下界为 0 的边。x 满足二分性质，找到最小的 x 使得新网络存在无源汇的上下界可行流即为原图的最小流。

2. 按照无源汇的上下界可行流的方法，建立超级源点 S^* 与超级汇点 T^*，求一遍 $S^* \to T^*$ 的最大流，但是注意这一次不加上汇点 T 到源点 S 的这条边，即不使之改为无源汇的网络去求解。求完后，再加上那条汇点 T 到源点 S 上界 ∞ 的边。因为这条边下界为 0，所以 S^*，T^* 无影响，再直接求一次 $S^* \to T^*$ 的最大流。若超级源点 S^* 出发的边全部满流，则 $T \to S$ 边上的流量即为原图的最小流，否则无解。

4.8 最小费用最大流

4.8.1 稀疏图

时间复杂度：$\mathcal{O}(V \cdot E^2)$

```cpp
struct EdgeList {
    int size;
    int last[N];
    int succ[M], other[M], flow[M], cost[M];
    void clear(int n) {
        size = 0;
        std::fill(last, last + n, -1);
    }
    void add(int x, int y, int c, int w) {
        succ[size] = last[x];
        last[x] = size;
        other[size] = y;
        flow[size] = c;
        cost[size++] = w;
    }
} e;

int n, source, target;
int prev[N];

void add(int x, int y, int c, int w) {
    e.add(x, y, c, w);
    e.add(y, x, 0, -w);
}

bool augment() {
    static int dist[N], occur[N];
    std::vector<int> queue;
    std::fill(dist, dist + n, INT_MAX);
    std::fill(occur, occur + n, 0);
    dist[source] = 0;
    occur[source] = true;
    queue.push_back(source);
    for (int head = 0; head < (int)queue.size(); ++head) {
        int x = queue[head];
        for (int i = e.last[x]; ~i; i = e.succ[i]) {
            int y = e.other[i];
            if (e.flow[i] && dist[y] > dist[x] + e.cost[i]) {
                dist[y] = dist[x] + e.cost[i];
                prev[y] = i;
                if (!occur[y]) {
                    occur[y] = true;
```

```
                        queue.push_back(y);
                    }
                }
            }
            occur[x] = false;
        }
        return dist[target] < INT_MAX;
    }

    std::pair<int, int> solve() {
        std::pair<int, int> answer = std::make_pair(0, 0);
        while (augment()) {
            int number = INT_MAX;
            for (int i = target; i != source; i = e.other[prev[i] ^ 1]) {
                number = std::min(number, e.flow[prev[i]]);
            }
            answer.first += number;
            for (int i = target; i != source; i = e.other[prev[i] ^ 1]) {
                e.flow[prev[i]] -= number;
                e.flow[prev[i] ^ 1] += number;
                answer.second += number * e.cost[prev[i]];
            }
        }
        return answer;
    }
```

4.8.2 稠密图

使用条件：费用非负

时间复杂度：$\mathcal{O}(V \cdot E^2)$

```
struct EdgeList {
    int size;
    int last[N];
    int succ[M], other[M], flow[M], cost[M];
    void clear(int n) {
        size = 0;
        std::fill(last, last + n, -1);
    }
    void add(int x, int y, int c, int w) {
        succ[size] = last[x];
        last[x] = size;
        other[size] = y;
```

```
        flow[size] = c;
        cost[size++] = w;
    }
} e;

int n, source, target, flow, cost;
int slack[N], dist[N];
bool visit[N];

void add(int x, int y, int c, int w) {
    e.add(x, y, c, w);
    e.add(y, x, 0, -w);
}

bool relabel() {
    int delta = INT_MAX;
    for (int i = 0; i < n; ++i) {
        if (!visit[i]) {
            delta = std::min(delta, slack[i]);
        }
        slack[i] = INT_MAX;
    }
    if (delta == INT_MAX) {
        return true;
    }
    for (int i = 0; i < n; ++i) {
        if (visit[i]) {
            dist[i] += delta;
        }
    }
    return false;
}

int dfs(int x, int answer) {
    if (x == target) {
        flow += answer;
        cost += answer * (dist[source] - dist[target]);
        return answer;
    }
    visit[x] = true;
    int delta = answer;
    for (int i = e.last[x]; ~i; i = e.succ[i]) {
```

```cpp
            int y = e.other[i];
            if (e.flow[i] > 0 && !visit[y]) {
                if (dist[y] + e.cost[i] == dist[x]) {
                    int number = dfs(y, std::min(e.flow[i], delta));
                    e.flow[i] -= number;
                    e.flow[i ^ 1] += number;
                    delta -= number;
                    if (delta == 0) {
                        dist[x] = INT_MIN;
                        return answer;
                    }
                } else {
                    slack[y] = std::min(slack[y], dist[y] + e.cost[i] - dist[x]);
                }
            }
        }
        return answer - delta;
}

std::pair<int, int> solve() {
    flow = cost = 0;
    std::fill(dist, dist + n, 0);
    do {
        do {
            fill(visit, visit + n, 0);
        } while (dfs(source, INT_MAX));
    } while (!relabel());
    return std::make_pair(flow, cost);
}
```

4.9 一般图最大匹配

时间复杂度：$\mathcal{O}(V^3)$

```cpp
int match[N], belong[N], next[N], mark[N], visit[N];
std::vector<int> queue;

int find(int x) {
    if (belong[x] != x) {
        belong[x] = find(belong[x]);
    }
    return belong[x];
}
```

```cpp

void merge(int x, int y) {
    x = find(x);
    y = find(y);
    if (x != y) {
        belong[x] = y;
    }
}

int lca(int x, int y) {
    static int stamp = 0;
    stamp++;
    while (true) {
        if (x != -1) {
            x = find(x);
            if (visit[x] == stamp) {
                return x;
            }
            visit[x] = stamp;
            if (match[x] != -1) {
                x = next[match[x]];
            } else {
                x = -1;
            }
        }
        std::swap(x, y);
    }
}

void group(int a, int p) {
    while (a != p) {
        int b = match[a], c = next[b];
        if (find(c) != p) {
            next[c] = b;
        }
        if (mark[b] == 2) {
            mark[b] = 1;
            queue.push_back(b);
        }
        if (mark[c] == 2) {
            mark[c] = 1;
            queue.push_back(c);
```

```
        }
        merge(a, b);
        merge(b, c);
        a = c;
    }
}

void augment(int source) {
    queue.clear();
    for (int i = 0; i < n; ++i) {
        next[i] = visit[i] = -1;
        belong[i] = i;
        mark[i] = 0;
    }
    mark[source] = 1;
    queue.push_back(source);
    for (int head = 0; head < (int)queue.size() && match[source] == -1; ++head) {
        int x = queue[head];
        for (int i = 0; i < (int)edge[x].size(); ++i) {
            int y = edge[x][i];
            if (match[x] == y || find(x) == find(y) || mark[y] == 2) {
                continue;
            }
            if (mark[y] == 1) {
                int r = lca(x, y);
                if (find(x) != r) {
                    next[x] = y;
                }
                if (find(y) != r) {
                    next[y] = x;
                }
                group(x, r);
                group(y, r);
            } else if (match[y] == -1) {
                next[y] = x;
                for (int u = y; u != -1; ) {
                    int v = next[u];
                    int mv = match[v];
                    match[v] = u;
                    match[u] = v;
                    u = mv;
                }
```

```
                break;
            } else {
                next[y] = x;
                mark[y] = 2;
                mark[match[y]] = 1;
                queue.push_back(match[y]);
            }
        }
    }
}

int solve() {
    std::fill(match, match + n, -1);
    for (int i = 0; i < n; ++i) {
        if (match[i] == -1) {
            augment(i);
        }
    }
    int answer = 0;
    for (int i = 0; i < n; ++i) {
        answer += (match[i] != -1);
    }
    return answer;
}
```

4.10 无向图全局最小割

时间复杂度：$\mathcal{O}(V^3)$
注意事项：处理重边时，应该对边权累加

```
int node[N], dist[N];
bool visit[N];

int solve(int n) {
    int answer = INT_MAX;
    for (int i = 0; i < n; ++i) {
        node[i] = i;
    }
    while (n > 1) {
        int max = 1;
        for (int i = 0; i < n; ++i) {
            dist[node[i]] = graph[node[0]][node[i]];
            if (dist[node[i]] > dist[node[max]]) {
```

```
                max = i;
            }
        }
        int prev = 0;
        memset(visit, 0, sizeof(visit));
        visit[node[0]] = true;
        for (int i = 1; i < n; ++i) {
            if (i == n - 1) {
                answer = std::min(answer, dist[node[max]]);
                for (int k = 0; k < n; ++k) {
                    graph[node[k]][node[prev]] =
                        (graph[node[prev]][node[k]] += graph[node[k]][node[max]]);
                }
                node[max] = node[--n];
            }
            visit[node[max]] = true;
            prev = max;
            max = -1;
            for (int j = 1; j < n; ++j) {
                if (!visit[node[j]]) {
                    dist[node[j]] += graph[node[prev]][node[j]];
                    if (max == -1 || dist[node[max]] < dist[node[j]]) {
                        max = j;
                    }
                }
            }
        }
    }
    return answer;
}
```

4.11 有根树的同构

时间复杂度：$\mathcal{O}(VlogV)$

```
const unsigned long long MAGIC = 4423;

unsigned long long magic[N];
std::pair<unsigned long long, int> hash[N];

void solve(int root) {
    magic[0] = 1;
    for (int i = 1; i <= n; ++i) {
```

```
            magic[i] = magic[i - 1] * MAGIC;
        }
        std::vector<int> queue;
        queue.push_back(root);
        for (int head = 0; head < (int)queue.size(); ++head) {
            int x = queue[head];
            for (int i = 0; i < (int)son[x].size(); ++i) {
                int y = son[x][i];
                queue.push_back(y);
            }
        }
        for (int index = n - 1; index >= 0; --index) {
            int x = queue[index];
            hash[x] = std::make_pair(0, 0);

            std::vector<std::pair<unsigned long long, int> > value;
            for (int i = 0; i < (int)son[x].size(); ++i) {
                int y = son[x][i];
                value.push_back(hash[y]);
            }
            std::sort(value.begin(), value.end());

            hash[x].first = hash[x].first * magic[1] + 37;
            hash[x].second++;
            for (int i = 0; i < (int)value.size(); ++i) {
                hash[x].first = hash[x].first * magic[value[i].second] + value[i].first;
                hash[x].second += value[i].second;
            }
            hash[x].first = hash[x].first * magic[1] + 41;
            hash[x].second++;
        }
}
```

4.12 哈密尔顿回路（ORE 性质的图）

ORE 性质：

$$\forall x, y \in V \wedge (x, y) \notin E \quad s.t. \quad deg_x + deg_y \geq n$$

返回结果：从顶点 1 出发的一个哈密尔顿回路

使用条件：$n \geq 3$

```
int left[N], right[N], next[N], last[N];

```

```
void cover(int x) {
    left[right[x]] = left[x];
    right[left[x]] = right[x];
}

int adjacent(int x) {
    for (int i = right[0]; i <= n; i = right[i]) {
        if (graph[x][i]) {
            return i;
        }
    }
    return 0;
}

std::vector<int> solve() {
    for (int i = 1; i <= n; ++i) {
        left[i] = i - 1;
        right[i] = i + 1;
    }
    int head, tail;
    for (int i = 2; i <= n; ++i) {
        if (graph[1][i]) {
            head = 1;
            tail = i;
            cover(head);
            cover(tail);
            next[head] = tail;
            break;
        }
    }
    while (true) {
        int x;
        while (x = adjacent(head)) {
            next[x] = head;
            head = x;
            cover(head);
        }
        while (x = adjacent(tail)) {
            next[tail] = x;
            tail = x;
            cover(tail);
        }
```

```
        if (!graph[head][tail]) {
            for (int i = head, j; i != tail; i = next[i]) {
                if (graph[head][next[i]] && graph[tail][i]) {
                    for (j = head; j != i; j = next[j]) {
                        last[next[j]] = j;
                    }
                    j = next[head];
                    next[head] = next[i];
                    next[tail] = i;
                    tail = j;
                    for (j = i; j != head; j = last[j]) {
                        next[j] = last[j];
                    }
                    break;
                }
            }
        }
        next[tail] = head;
        if (right[0] > n) {
            break;
        }
        for (int i = head; i != tail; i = next[i]) {
            if (adjacent(i)) {
                head = next[i];
                tail = i;
                next[tail] = 0;
                break;
            }
        }
    }
    std::vector<int> answer;
    for (int i = head; ; i = next[i]) {
        if (i == 1) {
            answer.push_back(i);
            for (int j = next[i]; j != i; j = next[j]) {
                answer.push_back(j);
            }
            answer.push_back(i);
            break;
        }
        if (i == tail) {
            break;
```

```
        }
    }
    return answer;
}
```

4.13 必经点树

```
vector<int>G[maxn],rG[maxn],dom[maxn];
int n,m;
int dfn[maxn],rdfn[maxn],dfs_c,semi[maxn],idom[maxn],fa[maxn];
struct ufsets{
    int fa[maxn],best[maxn];
    int find(int x){
        if(fa[x]==x)
            return x;
        int f=find(fa[x]);
        if(dfn[semi[best[x]]]>dfn[semi[best[fa[x]]]])
            best[x]=best[fa[x]];
        fa[x]=f;
        return f;
    }
    int getbest(int x){
        find(x);
        return best[x];
    }
    void init(){
        for(int i=1;i<=n;i++)
            fa[i]=best[i]=i;
    }
}uf;
void init(){
    uf.init();
    for(int i=1;i<=n;i++){
        semi[i]=i;
        idom[i]=0;
        fa[i]=0;
        dfn[i]=rdfn[i]=0;
    }
    dfs_c=0;
}
void dfs(int u){
    dfn[u]=++dfs_c;
```

```
    rdfn[dfn[u]]=u;
    for(int i=0;i<G[u].size();i++){
        int v=G[u][i];
        if(!dfn[v]){
            fa[v]=u;
            dfs(v);
        }
    }
}

void tarjan(){
    for(int i=n;i>1;i--){
        int tmp=1e9;
        int y=rdfn[i];
        for(int i=0;i<rG[y].size();i++){
            int x=rG[y][i];
            tmp=min(tmp,dfn[semi[uf.getbest(x)]]);
        }
        semi[y]=rdfn[tmp];
        int x=fa[y];
        dom[semi[y]].push_back(y);
        uf.fa[y]=x;
        for(int i=0;i<dom[x].size();i++){
            int z=dom[x][i];
            if(dfn[semi[uf.getbest(z)]]<dfn[x])
                idom[z]=uf.getbest(z);
            else
                idom[z]=semi[z];
        }
        dom[x].clear();
    }
    semi[rdfn[1]]=1;
    for(int i=2;i<=n;i++){
        int x=rdfn[i];
        if(idom[x]!=semi[x])
            idom[x]=idom[idom[x]];

    }
    idom[rdfn[1]]=0;
}
init();
dfs(1);
```

```
tarjan();
```

5 字符串

5.1 模式匹配

5.1.1 KMP 算法

```
void build(char *pattern) {
    int length = (int)strlen(pattern + 1);
    fail[0] = -1;
    for (int i = 1, j; i <= length; ++i) {
        for (j = fail[i - 1]; j != -1 && pattern[i] != pattern[j + 1]; j = fail[j]);
        fail[i] = j + 1;
    }
}

void solve(char *text, char *pattern) {
    int length = (int)strlen(text + 1);
    for (int i = 1, j; i <= length; ++i) {
        for (j = match[i - 1]; j != -1 && text[i] != pattern[j + 1]; j = fail[j]);
        match[i] = j + 1;
    }
}
///Hint: 1 - Base
```

5.1.2 扩展 KMP 算法

返回结果：

$$next_i = lcp(text, text_{i...n-1})$$

```
void solve(char *text, int length, int *next) {
    int j = 0, k = 1;
    for (; j + 1 < length && text[j] == text[j + 1]; j++);
    next[0] = length - 1;
    next[1] = j;
    for (int i = 2; i < length; ++i) {
        int far = k + next[k] - 1;
        if (next[i - k] < far - i + 1) {
            next[i] = next[i - k];
        } else {
            j = std::max(far - i + 1, 0);
```

```
            for (; i + j < length && text[j] == text[i + j]; j++);
            next[i] = j;
            k = i;
        }
    }
}
/// 0 - Base
```

5.1.3 AC 自动机

```
struct Node{
        int Next[30], fail, mark;
}Tree[N];

void Init(){
        memset(Tree, 0, sizeof Tree);
        cnt = 1;

        for (int i = 1; i <= n; i++){
                char c;
                int now = 1;
                scanf("%s", s + 1);
                int Length = strlen(s + 1);
                for (int j = 1; j <= Length; j++){
                        c = s[j];
                        if (Tree[now].Next[c - 'a']) now = Tree[now].Next[c - 'a']; else
                                Tree[now].Next[c - 'a'] = ++ cnt, now = cnt;
                }
        }
}

void Build_Ac(){
        int en = 0;
        Q[0] = 1;
        for (int fi = 0; fi <= en; fi++){
                int now = Q[fi];
                for (int next = 0; next < 26; next++)
                        if (Tree[now].Next[next])
                        {
                                int k = Tree[now].Next[next];
                                if (now == 1) Tree[k].fail = 1; else
                                {
```

```
                                        int h = Tree[now].fail;
                                        while (h && !Tree[h].Next[next]) h = Tree[h].fail;
                                        if (!h) Tree[k].fail = 1;
                                        else Tree[k].fail = Tree[h].Next[next];
                                }
                                Q[++ en] = k;
                        }
        }
}

/// Hints : when not match , fail = 1
```

5.2 后缀三姐妹

5.2.1 后缀数组

```
struct Sa{
        int heap[N],s[N],sa[N],r[N],tr[N],sec[N],m,cnt;
        int h[19][N];

        void Prep(){
                for (int i=1; i<=m; i++) heap[i]=0;
                for (int i=1; i<=n; i++) heap[s[i]]++;
                for (int i=2; i<=m; i++) heap[i]+=heap[i-1];
                for (int i=n; i>=1; i--) sa[heap[s[i]]--]=i;
                r[sa[1]]=1; cnt=1;
                for (int i=2; i<=n; i++){
                        if (s[sa[i]]!=s[sa[i-1]]) cnt++;
                        r[sa[i]]=cnt;
                }
                m=cnt;
        }

        void Suffix(){
                int j=1;
                while (cnt<n){
                        cnt=0;
                        for (int i=n-j+1; i<=n; i++) sec[++cnt]=i;
                        for (int i=1; i<=n; i++) if (sa[i]>j)
                                sec[++cnt]=sa[i]-j;
                        for (int i=1; i<=n; i++) tr[i]=r[sec[i]];
                        for (int i=1; i<=m; i++) heap[i]=0;
```

```
                for (int i=1; i<=n; i++) heap[tr[i]]++;
                for (int i=2; i<=m; i++) heap[i]+=heap[i-1];
                for (int i=n; i>=1; i--)
                        sa[heap[tr[i]]--]=sec[i];
                tr[sa[1]]=1; cnt=1;
                for (int i=2; i<=n; i++){
                    if ((r[sa[i]]!=r[sa[i-1]]) || (r[sa[i]+j]!=r[sa[i-1]+j]))
                        cnt++;
                    tr[sa[i]]=cnt;
                }
                for (int i=1; i<=n; i++) r[i]=tr[i];
                m=cnt; j=j+j;
            }
    }

    void Calc(){
        int k=0;
        for (int i=1; i<=n; i++){
            if (r[i]==1) continue;
            int j=sa[r[i]-1];
            while ((i+k<=n) && (j+k<=n) && (s[i+k]==s[j+k])) k++;
            h[0][r[i]]=k;
            if (k) k--;
        }
        for (int i=1; i<19; i++)
            for (int j=1; j+(1 << i)-1<=n; j++)
                h[i][j]=min(h[i-1][j],h[i-1][j + (1 << (i - 1)) + 1]);
    }

    int Query(int L,int R){
        L=r[L], R=r[R];
        if (L>R) swap(L,R);
        L++;
        int l0 = Lg[R-L+1];
        return min(h[l0][L],h[l0][R-(1 << l0)+1]);
    }

    void Work(){
        Prep(); Suffix(); Calc();
    }
}P,S;
```

```
/// Hints : 1 - Base
```

5.2.2 后缀数组 (dc3)

```
//`DC3 待排序的字符串放在 r 数组中，从 r[0] 到 r[n-1]，长度为 n，且最大值小于 m.`
//`约定除 r[n-1] 外所有的 r[i] 都大于 0, r[n-1]=0。`
//`函数结束后，结果放在 sa 数组中，从 sa[0] 到 sa[n-1]。`
//`r 必须开长度乘 3`
#define maxn 10000
#define F(x) ((x)/3+((x)%3==1?0:tb))
#define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)

int wa[maxn],wb[maxn],wv[maxn],wss[maxn];
int s[maxn*3],sa[maxn*3];
int c0(int *r,int a,int b)
{
        return r[a]==r[b]&&r[a+1]==r[b+1]&&r[a+2]==r[b+2];
}
int c12(int k,int *r,int a,int b)
{
        if(k==2) return r[a]<r[b]||r[a]==r[b]&&c12(1,r,a+1,b+1);
        else return r[a]<r[b]||r[a]==r[b]&&wv[a+1]<wv[b+1];
}
void sort(int *r,int *a,int *b,int n,int m)
{
        int i;
        for(i=0;i<n;i++) wv[i]=r[a[i]];
        for(i=0;i<m;i++) wss[i]=0;
        for(i=0;i<n;i++) wss[wv[i]]++;
        for(i=1;i<m;i++) wss[i]+=wss[i-1];
        for(i=n-1;i>=0;i--) b[--wss[wv[i]]]=a[i];
}
void dc3(int *r,int *sa,int n,int m)
{
        int i,j,*rn=r+n,*san=sa+n,ta=0,tb=(n+1)/3,tbc=0,p;
        r[n]=r[n+1]=0;
        for(i=0;i<n;i++)
                if(i%3!=0) wa[tbc++]=i;
        sort(r+2,wa,wb,tbc,m);
        sort(r+1,wb,wa,tbc,m);
        sort(r,wa,wb,tbc,m);
        for(p=1,rn[F(wb[0])]=0,i=1;i<tbc;i++)
```

```
			rn[F(wb[i])]=c0(r,wb[i-1],wb[i])?p-1:p++;
		if (p<tbc) dc3(rn,san,tbc,p);
		else for (i=0;i<tbc;i++) san[rn[i]]=i;
		for (i=0;i<tbc;i++)
			if(san[i]<tb) wb[ta++]=san[i]*3;
		if(n%3==1) wb[ta++]=n-1;
		sort(r,wb,wa,ta,m);
		for(i=0;i<tbc;i++)
			wv[wb[i]=G(san[i])]=i;
		for(i=0,j=0,p=0;i<ta && j<tbc;p++)
			sa[p]=c12(wb[j]%3,r,wa[i],wb[j])?wa[i++]:wb[j++];
		for(;i<ta;p++) sa[p]=wa[i++];
		for(;j<tbc;p++) sa[p]=wb[j++];
}

int main(){
		int n,m=0;
		scanf("%d",&n);
		for (int i=0;i<n;i++) scanf("%d",&s[i]),s[i]++,m=max(s[i]+1,m);
		printf("%d\n",m);
		s[n++]=0;
		dc3(s,sa,n,m);
		for (int i=0;i<n;i++) printf("%d ",sa[i]);printf("\n");
}
```

5.2.3 后缀自动机-多串 LCS

对一个串建后缀自动机，其他串在上面匹配，因为是求所有串的公共子串，所以每个点记录每个串最长匹配长度的最小值，最后找到所有点中最长的一个即可。一个注意事项就是，当走到一个点时，还要更新它的 parent 树上的祖先的匹配长度，数组开两倍啦啦啦!

```
struct Node{
		int len, fail;
		int To[30];
}T[N];
int Lst, Root, tot, ans;
char s[N];
int Len[N], Ans[N], Ord[N];
void Add(int x, int l){
		int Nt = ++tot, p = Lst;
		T[Nt].len = l;
		for (;p && !T[p].To[x]; p = T[p].fail) T[p].To[x] = Nt;
		if (!p) T[Nt].fail = Root; else
```

```
        if (T[T[p].To[x]].len == T[p].len + 1) T[Nt].fail = T[p].To[x];
        else{
                int q = ++tot, qt = T[p].To[x];
                T[q] = T[qt];
                T[q].len = T[p].len + 1;
                T[qt].fail = T[Nt].fail = q;
                for (;p && T[p].To[x] == qt; p = T[p].fail) T[p].To[x] = q;
        }
        Lst = Nt;
}
bool cmp(int a, int b){
        return T[a].len < T[b].len;
}
int main(){
        scanf("%s", s + 1);
        int n = strlen(s + 1);
        ans = n;
        Root = tot = Lst = 1;
        for (int i = 1; i <= n; i++)
                Add(s[i] - 'a' + 1, i);
        for (int i = 1; i <= tot; i++)
                Ord[i] = i;
        sort(Ord + 1, Ord + tot + 1, cmp);
        for (int i = 1; i <= tot; i++)
                Ans[i] = T[i].len;
        bool flag = 0;
        while (scanf("%s", s + 1) != EOF){
                flag = 1;
                int n = strlen(s + 1);
                int p = Root, len = 0;
                for (int i = 1; i <= tot; i++) Len[i] = 0;
                for (int i = 1; i <= n; i++){
                        int x = s[i] - 'a' + 1;
                        if (T[p].To[x]) len++, p = T[p].To[x];
                        else {
                                while (p && !T[p].To[x]) p = T[p].fail;
                                if (!p) p = Root, len = 0;
                                else len = T[p].len + 1, p = T[p].To[x];
                        }
                        Len[p] = max(Len[p], len);
                }
                for (int i = tot; i >= 1; i--){
```

```
                int Cur = Ord[i];
                Ans[Cur] = min(Ans[Cur], Len[Cur]);
                if (Len[Cur] && T[Cur].fail)
                    Len[T[Cur].fail] = T[T[Cur].fail].len;
            }
        }
        if (flag){
            ans = 0;
            for (int i = 1; i <= tot; i++){
                ans = max(ans, Ans[i]);
            }
        }
        printf("%d\n", ans);
        return 0;
}
```

5.2.4 后缀自动机-各长度字串出现次数最大值

给一个字符串 S，令 F(x) 表示 S 的所有长度为 x 的子串中，出现次数的最大值。 构建字符串的自动机，对于每个节点，right 集合大小就是出现次数，maxs 就是它代表的最长长度，那么我们用 |right(x)| 去更新 f[maxs[x]] 的值，最后从大到小用 f[i] 去更新 f[i-1] 的值即可

```
struct Node{
        int len, fail;
        int To[30];
}T[N];
int Lst, Root, tot, n;
char s[N];
int Ord[N], Ans[N], Ways[N], heap[N];
void Add(int x, int l){
        int Nt = ++tot, p = Lst;
        T[Nt].len = l;
        for (;p && !T[p].To[x]; p = T[p].fail) T[p].To[x] = Nt;
        if (!p) T[Nt].fail = Root; else
        if (T[T[p].To[x]].len == T[p].len + 1) T[Nt].fail = T[p].To[x];
        else{
            int q = ++tot, qt = T[p].To[x];
            T[q] = T[qt];
            T[q].len = T[p].len + 1;
            T[qt].fail = T[Nt].fail = q;
            for (;p && T[p].To[x] == qt; p = T[p].fail) T[p].To[x] = q;
        }
```

```
        Lst = Nt;
}
bool cmp(int a, int b){
        return T[a].len < T[b].len;
}
void sort(){
        for (int i = 1; i <= tot; i++) heap[T[i].len]++;
        for (int i = 1; i <= n; i++) heap[i] += heap[i-1];
        for (int i = 1; i <= tot; i++) Ord[heap[T[i].len]--]=i;
}
int main(){
        scanf("%s", s + 1);
        n = strlen(s + 1);
        Root = tot = Lst = 1;
        for (int i = 1; i <= n; i++)
                Add(s[i] - 'a' + 1, i);
        sort();
        memset(Ways , 0, sizeof(Ways));
        for (int i = 1, p = Root; i <= n; i++)
                p = T[p].To[s[i] - 'a' + 1], Ways[p] = 1;
        for (int i = tot; i >= 1; i--){
                int Cur = Ord[i];
                if (T[Cur].fail == 0) continue;
                Ways[T[Cur].fail] += Ways[Cur];
        }
        for (int i = 1; i <= tot; i++)
                Ans[T[i].len] = max(Ans[T[i].len], Ways[i]);
        for (int i = n; i >= 1; i--)
                Ans[i] = max(Ans[i + 1], Ans[i]);
        for (int i = 1; i <= n; i++)
                printf("%d\n", Ans[i]);
        return 0;
}
```

5.2.5 后缀自动机-两串 LCS

```
struct node{
        int len, fail;
        int To[27];
}T[N];
char a[N], b[N];
int Lst, Root, tot;
```

```
void add(int x, int l){
	int Nt = ++tot, p = Lst;
	T[Nt].len = l;
	for (;p && !T[p].To[x]; p = T[p].fail) T[p].To[x] = Nt;
	if (!p) T[Nt].fail = Root;
	else
	if (T[T[p].To[x]].len == T[p].len + 1) T[Nt].fail = T[p].To[x];
	else{
		int q = ++tot, qt = T[p].To[x];
		T[q] = T[qt];
		T[q].len  = T[p].len + 1;
		T[qt].fail = T[Nt].fail = q;
		for (;p && T[p].To[x] == qt; p = T[p].fail) T[p].To[x] = q;
	}
	Lst = Nt;
}
int main(){
	while (scanf("%s%s", a + 1, b + 1) == 2){
		int n = strlen(a + 1);
		Lst = Root = tot = 1;
		for (int i = 1; i <= n; i++)
			add(a[i] - 'a' + 1, i);
		int m = strlen(b + 1);
		int p = Root, len = 0;
		int Ans = 0;
		for (int i = 1; i <= m; i++){
			int x = b[i] - 'a' + 1;
			if (T[p].To[x]) len++, p = T[p].To[x];
			else {
				while (p && !T[p].To[x]) p = T[p].fail;
				if (!p) p = Root, len = 0;
				else len = T[p].len + 1, p = T[p].To[x];
			}
			if (len > Ans) Ans = len;
		}
		printf("%d\n", Ans);
		for (int i = 1; i <= tot; i++){
			T[i].len = T[i].fail = 0;
			for (int j = 1; j <= 26; j++)
				T[i].To[j] = 0;
		}
	}
```

```
        return 0;
}
//Hints£ºSAM + Longest common subsequence
```

5.3 回文三兄弟

5.3.1 马拉车

```
void Manacher(){
        R[1] = 1;
        for (int i = 2, j = 1; i <= length; i++){
                if (j + R[j] <= i){
                        R[i] = 0;
                } else {
                        R[i] = min(R[j * 2 - i], j + R[j] - i);
                }
                while (i - R[i] >= 1 && i + R[i] <= length
                        && text[i - R[i]] == text[i + R[i]]){
                        R[i]++;
                }
                if (i + R[i] > j + R[j]){
                        j = i;
                }
        }
}
        length = 0;
        int n = strlen(s + 1);
        for (int i = 1; i <= n; i++){
                text[++length] = '*';
                text[++length] = s[i];
        }
        text[++length] = '*';
/// Hints: 1 - Base
```

5.3.2 回文树 (lyx)

```
const int N = 400005;

char s[N];
int Len;

struct Palindromic_Tree {
```

```
int next[N][27];
int fail[N];
int cnt[N];
int num[N];
int len[N];
char S[N];
int last;
int n;
int p;

int newnode(int l)
{
    for(int i = 1; i <= 26; i++) next[p][i] = 0;
    cnt[p] = 0;
    num[p] = 0;
    len[p] = l;
            fail[p] = 0;
    return p++;
}
void init()
{
    p = 0;
    newnode(0);
    newnode(-1);
    last = 0;
    n = 0;
    S[n] = -1;
    fail[0] = 1;
}
int get_fail(int x)
{
    while (S[n - len[x] - 1] != S[n]) x = fail[x];
    return x;
}
void add(char c, int pos)
{
    c = c - 'a' + 1;
    S[++ n] = c ;
    int cur = get_fail(last);
    if (!next[cur][c])
    {
        int now = newnode(len[cur] + 2);
```

```
            fail[now] = next[get_fail(fail[cur])][c];
            next[cur][c] = now;
            num[now] = num[fail[now]] + 1;
        }
                last = next[cur][c] ;
        cnt[last]++;
    }
    void count()
    {
        for (int i = p - 1 ; i >= 0 ; -- i) cnt[fail[i]] += cnt[i] ;
    }
}T;

Len = strlen(s + 1);

T.init();
for (int i = 1; i <= Len; i++)
        T.add(s[i], i);
T.count();
```

5.3.3 回文自动机 (zky)

```
struct PAM{
        int tot,last,str[maxn],nxt[maxn][26],n;
        int len[maxn],suf[maxn],cnt[maxn];
        int newnode(int l){
                len[tot]=l;
                return tot++;
        }
        void init(){
                tot=0;
                newnode(0);// tree0 is node 0
                newnode(-1);// tree-1 is node 1
                str[0]=-1;
                suf[0]=1;
        }
        int find(int x){
                while(str[n-len[x]-1]!=str[n])x=suf[x];
                return x;
        }
        void add(int c){
                str[++n]=c;
```

```
            int u=find(last);
            if(!nxt[u][c]){
                int v=newnode(len[u]+2);
                suf[v]=nxt[find(suf[u])][c];
                nxt[u][c]=v;
            }last=nxt[u][c];
            cnt[last]++;
        }
        void count(){
            for(int i=tot-1;i>=0;i--)cnt[suf[i]]+=cnt[i];
        }
}P;
int main(){
        P.init();
        for(int i=0;i<n;i++)
            P.add(s[i]-'a');
        P.count();
```

5.4 循环串最小表示

```
string sol(char *s){
    int n=strlen(s);
    int i=0,j=1,k=0,p;
    while(i<n&&j<n&&k<n){
        int t=s[(i+k)%n]-s[(j+k)%n];
        if(t==0)k++;
        else if(t<0)j+=k+1,k=0;
        else i+=k+1,k=0;
        if(i==j)j++;
    }p=min(i,j);
    string S;
    for(int i=p;i<p+n;i++)S.push_back(s[i%n]);
    return S;
}
```

6 计算几何

6.1 二维基础

6.1.1 点类

```cpp
int sgn(double x){return (x>eps)-(x<-eps);}
int sgn(double a,double b){return sgn(a-b);}
double sqr(double x){return x*x;}
struct P{
	double x,y;
	P(){}
	P(double x,double y):x(x),y(y){}
	double len2(){
		return sqr(x)+sqr(y);
	}
	double len(){
		return sqrt(len2());
	}
	void print(){
		printf("(%.3f,%.3f)\n",x,y);
	}
	P turn90(){return P(-y,x);}
	P norm(){return P(x/len(),y/len());}
};
bool operator==(P a,P b){
	return !sgn(a.x-b.x) and !sgn(a.y-b.y);
}
P operator+(P a,P b){
	return P(a.x+b.x,a.y+b.y);
}
P operator-(P a,P b){
	return P(a.x-b.x,a.y-b.y);
}
P operator*(P a,double b){
	return P(a.x*b,a.y*b);
}
P operator/(P a,double b){
	return P(a.x/b,a.y/b);
}
double operator^(P a,P b){
	return a.x*b.x + a.y*b.y;
}
double operator*(P a,P b){
	return a.x*b.y - a.y*b.x;
}
double det(P a,P b,P c){
	return (b-a)*(c-a);
```

```
}
double dis(P a,P b){
        return (b-a).len();
}
double Area(vector<P>poly){
        double ans=0;
        for(int i=1;i<poly.size();i++)
                ans+=(poly[i]-poly[0])*(poly[(i+1)%poly.size()]-poly[0]);
        return fabs(ans)/2;
}
struct L{
        P a,b;
        L(){}
        L(P a,P b):a(a),b(b){}
        P v(){return b-a;}
};
bool onLine(P p,L l){
        return sgn((l.a-p)*(l.b-p))==0;
}
bool onSeg(P p,L s){
        return onLine(p,s) and sgn((s.b-s.a)^(p-s.a))>=0 and sgn((s.a-s.b)^(p-s.b))>=0;
}
bool parallel(L l1,L l2){
        return sgn(l1.v()*l2.v())==0;
}
P intersect(L l1,L l2){
        double s1=det(l1.a,l1.b,l2.a);
        double s2=det(l1.a,l1.b,l2.b);
        return (l2.a*s2-l2.b*s1)/(s2-s1);
}
P project(P p,L l){
        return l.a+l.v()*((p-l.a)^l.v())/l.v().len2();
}
double dis(P p,L l){
        return fabs((p-l.a)*l.v())/l.v().len();
}
```

6.1.2 凸包

```
vector<P> convex(vector<P>p){
        sort(p.begin(),p.end());
        vector<P>ans,S;
```

```
    for(int i=0;i<p.size();i++){
            while(S.size()>=2
                            && sgn(det(S[S.size()-2],S.back(),p[i]))<=0)
                                    S.pop_back();
            S.push_back(p[i]);
    }//dw
    ans=S;
    S.clear();
        for(int i=(int)p.size()-1;i>=0;i--){
            while(S.size()>=2
                            && sgn(det(S[S.size()-2],S.back(),p[i]))<=0)
                                    S.pop_back();
            S.push_back(p[i]);
        }//up
        for(int i=1;i+1<S.size();i++)
                ans.push_back(S[i]);
        return ans;
}
```

6.1.3 半平面交

```
struct P{
        int quad() const { return sgn(y) == 1 || (sgn(y) == 0 && sgn(x) >= 0);}
};
struct L{
        bool onLeft(const P &p) const { return sgn((b - a)*( p - a)) > 0; }
        L push() const{ // push out eps
                const double eps = 1e-10;
                P delta = (b - a).turn90().norm() * eps;
                return L(a - delta, b - delta);
        }
};
bool sameDir(const L &l0, const L &l1) {
        return parallel(l0, l1) && sgn((l0.b - l0.a)^(l1.b - l1.a)) == 1;
}
bool operator < (const P &a, const P &b) {
        if (a.quad() != b.quad())
                return a.quad() < b.quad();
        else
                return sgn((a*b)) > 0;
}
bool operator < (const L &l0, const L &l1) {
```

```
	if (sameDir(l0, l1))
		return l1.onLeft(l0.a);
	else
		return (l0.b - l0.a) < (l1.b - l1.a);
}
bool check(const L &u, const L &v, const L &w) {
	return w.onLeft(intersect(u, v));
}
vector<P> intersection(vector<L> &l) {
	sort(l.begin(), l.end());
	deque<L> q;
	for (int i = 0; i < (int)l.size(); ++i) {
		if (i && sameDir(l[i], l[i - 1])) {
			continue;
		}
		while (q.size() > 1
			&& !check(q[q.size() - 2], q[q.size() - 1], l[i]))
				q.pop_back();
		while (q.size() > 1
			&& !check(q[1], q[0], l[i]))
				q.pop_front();
		q.push_back(l[i]);
	}
	while (q.size() > 2
		&& !check(q[q.size() - 2], q[q.size() - 1], q[0]))
			q.pop_back();
	while (q.size() > 2
		&& !check(q[1], q[0], q[q.size() - 1]))
			q.pop_front();
	vector<P> ret;
	for (int i = 0; i < (int)q.size(); ++i)
	ret.push_back(intersect(q[i], q[(i + 1) % q.size()]));
	return ret;
}
```

6.1.4 最近点对

```
bool byY(P a,P b){return a.y<b.y;}
LL solve(P *p,int l,int r){
	LL d=1LL<<62;
	if(l==r)
		return d;
```

```
	if(l+1==r)
		return dis2(p[l],p[r]);
	int mid=(l+r)>>1;
	d=min(solve(l,mid),d);
	d=min(solve(mid+1,r),d);
	vector<P>tmp;
	for(int i=l;i<=r;i++)
		if(sqr(p[mid].x-p[i].x)<=d)
			tmp.push_back(p[i]);
	sort(tmp.begin(),tmp.end(),byY);
	for(int i=0;i<tmp.size();i++)
		for(int j=i+1;j<tmp.size()&&j-i<10;j++)
			d=min(d,dis2(tmp[i],tmp[j]));
	return d;
}
```

6.1.5 最小圆覆盖

```
struct line{
	point p,v;
};
point Rev(point v){return point(-v.y,v.x);}
point operator*(line A,line B){
	point u=B.p-A.p;
	double t=(B.v*u)/(B.v*A.v);
	return A.p+A.v*t;
}
point get(point a,point b){
	return (a+b)/2;
}
point get(point a,point b,point c){
	if(a==b)return get(a,c);
	if(a==c)return get(a,b);
	if(b==c)return get(a,b);
	line ABO=(line){(a+b)/2,Rev(a-b)};
	line BCO=(line){(c+b)/2,Rev(b-c)};
	return ABO*BCO;
}
int main(){
	scanf("%d",&n);
	for(int i=1;i<=n;i++)scanf("%lf%lf",&p[i].x,&p[i].y);
	random_shuffle(p+1,p+1+n);
```

```
        O=p[1];r=0;
        for(int i=2;i<=n;i++){
                if(dis(p[i],O)<r+1e-6)continue;
                O=get(p[1],p[i]);r=dis(O,p[i]);
                for(int j=1;j<i;j++){
                        if(dis(p[j],O)<r+1e-6)continue;
                        O=get(p[i],p[j]);r=dis(O,p[i]);
                        for(int k=1;k<j;k++){
                                if(dis(p[k],O)<r+1e-6)continue;
                                O=get(p[i],p[j],p[k]);r=dis(O,p[i]);
                        }
                }
        }printf("%.2lf %.2lf %.2lf\n",O.x,O.y,r);
        return 0;
}s
```

6.2 多边形

6.2.1 判断点在多边形内部

```
bool InPoly(P p,vector<P>poly){
        int cnt=0;
        for(int i=0;i<poly.size();i++){
                P a=poly[i],b=poly[(i+1)%poly.size()];
                if(OnLine(p,L(a,b)))
                        return false;
                int x=sgn(det(a,p,b));
                int y=sgn(a.y-p.y);
                int z=sgn(b.y-p.y);
                cnt+=(x>0&&y<=0&&z>0);
                cnt-=(x<0&&z<=0&&y>0);
        }
        return cnt;
}
```

7 其他

7.1 斯坦那树

```
priority_queue<pair<int, int> > Q;

```

```
// m is key point
// n is all point

for (int s = 0; s < (1 << m); s++){
        for (int i = 1; i <= n; i++){
                if (id[i]) continue;
                for (int s0 = 0; s0 < s; s0++)
                        if ( (s0 & s) == s0 ){
                                f[s][i] = min(f[s][i], f[s0][i] + f[s - s0][i]);
                        }
        }
        for (int i = 1; i <= n; i++) vis[i] = 0;
    while (!Q.empty()) Q.pop();
        for (int i = 1; i <= n; i++){
                if (id[i]) continue;
                Q.push(mp(-f[s][i], i));
        }
        while (!Q.empty()){
                while (!Q.empty() && Q.top().first != -f[s][Q.top().second]) Q.pop();
                        if (Q.empty()) break;
                        int Cur = Q.top().second; Q.pop();
                        for (int p = g[Cur]; p; p = nxt[p]){
                                int y = adj[p];
                                if ( f[s][y] > f[s][Cur] + 1){
                                        f[s][y] = f[s][Cur] + 1;
                                        Q.push(mp(-f[s][y], y));
                                }
                        }
        }
}
```

7.2 最小树形图

```
const int maxn=1100;

int n,m , g[maxn][maxn] , used[maxn] , pass[maxn] , eg[maxn] , more , queue[maxn];

void combine (int id , int &sum ) {
        int tot = 0 , from , i , j , k ;
        for ( ; id!=0 && !pass[ id ] ; id=eg[id] ) {
                queue[tot++]=id ; pass[id]=1;
        }
```

```
        for ( from=0; from<tot && queue[from]!=id ; from++);
        if  ( from==tot ) return ;
        more = 1 ;
        for ( i=from ; i<tot ; i++) {
                sum+=g[eg[queue[i]]][queue[i]] ;
                if ( i!=from ) {
                        used[queue[i]]=1;
                        for ( j = 1 ; j <= n ; j++) if ( !used[j] )
                                if ( g[queue[i]][j]<g[id][j] ) g[id][j]=g[queue[i]][j] ;
                }
        }
        for ( i=1; i<=n ; i++) if ( !used[i] && i!=id ) {
                for ( j=from ; j<tot ; j++){
                        k=queue[j];
                        if ( g[i][id]>g[i][k]-g[eg[k]][k] ) g[i][id]=g[i][k]-g[eg[k]][k];
                }
        }
}

int mdst( int root ) { // return the total length of MDST
        int i , j , k , sum = 0 ;
        memset ( used , 0 , sizeof ( used ) ) ;
        for ( more =1; more ; ) {
                more = 0 ;
                memset (eg,0,sizeof(eg)) ;
                for ( i=1 ; i <= n ; i ++) if ( !used[i] && i!=root ) {
                        for ( j=1 , k=0 ; j <= n ; j ++) if ( !used[j] && i!=j )
                                if ( k==0 || g[j][i] < g[k][i] ) k=j ;
                        eg[i] = k ;
                }
                memset(pass,0,sizeof(pass));
                for ( i=1; i<=n ; i++) if ( !used[i] && !pass[i] && i!= root ) combine ( i
        }
        for ( i =1; i<=n ; i ++) if ( !used[i] && i!= root ) sum+=g[eg[i]][i];
        return sum ;
}
```

7.3 DLX

```
int n,m,K;
struct DLX{
        int L[maxn],R[maxn],U[maxn],D[maxn];
```

```
int sz,col[maxn],row[maxn],s[maxn],H[maxn];
bool vis[233];
int ans[maxn],cnt;
void init(int m){
        for(int i=0;i<=m;i++){
                L[i]=i-1;R[i]=i+1;
                U[i]=D[i]=i;s[i]=0;
        }
        memset(H,-1,sizeof H);
        L[0]=m;R[m]=0;sz=m+1;
}
void Link(int r,int c){
        U[sz]=c;D[sz]=D[c];U[D[c]]=sz;D[c]=sz;
        if(H[r]<0)H[r]=L[sz]=R[sz]=sz;
        else{
                L[sz]=H[r];R[sz]=R[H[r]];
                L[R[H[r]]]=sz;R[H[r]]=sz;
        }
        s[c]++;col[sz]=c;row[sz]=r;sz++;
}
void remove(int c){
        for(int i=D[c];i!=c;i=D[i])
                L[R[i]]=L[i],R[L[i]]=R[i];
}
void resume(int c){
        for(int i=U[c];i!=c;i=U[i])
                L[R[i]]=R[L[i]]=i;
}
int A(){
        int res=0;
        memset(vis,0,sizeof vis);
        for(int i=R[0];i;i=R[i])if(!vis[i]){
                vis[i]=1;res++;
                for(int j=D[i];j!=i;j=D[j])
                        for(int k=R[j];k!=j;k=R[k])
                                vis[col[k]]=1;
        }
        return res;
}
void dfs(int d,int &ans){
        if(R[0]==0){ans=min(ans,d);return;}
        if(d+A()>=ans)return;
```

```
                int tmp=23333,c;
                for(int i=R[0];i;i=R[i])
                        if(tmp>s[i])tmp=s[i],c=i;
                for(int i=D[c];i!=c;i=D[i]){
                        remove(i);
                        for(int j=R[i];j!=i;j=R[j])remove(j);
                        dfs(d+1,ans);
                        for(int j=L[i];j!=i;j=L[j])resume(j);
                        resume(i);
                }
        }
        void del(int c){//exactly cover
        L[R[c]]=L[c];R[L[c]]=R[c];
                for(int i=D[c];i!=c;i=D[i])
                        for(int j=R[i];j!=i;j=R[j])
                                U[D[j]]=U[j],D[U[j]]=D[j],--s[col[j]];
    }
    void add(int c){  //exactly cover
        R[L[c]]=L[R[c]]=c;
                for(int i=U[c];i!=c;i=U[i])
                        for(int j=L[i];j!=i;j=L[j])
                                ++s[col[U[D[j]]=D[U[j]]=j]];
    }
        bool dfs2(int k){//exactly cover
        if(!R[0]){
            cnt=k;return 1;
        }
        int c=R[0];
                for(int i=R[0];i;i=R[i])
                        if(s[c]>s[i])c=i;
        del(c);
                for(int i=D[c];i!=c;i=D[i]){
                        for(int j=R[i];j!=i;j=R[j])
                                del(col[j]);
            ans[k]=row[i];if(dfs2(k+1))return true;
                        for(int j=L[i];j!=i;j=L[j])
                                add(col[j]);
        }
        add(c);
                return 0;
        }
}dlx;
```

```
int main(){
        dlx.init(n);
        for(int i=1;i<=m;i++)
                for(int j=1;j<=n;j++)
                        if(dis(station[i],city[j])<mid-eps)
                                dlx.Link(i,j);
                        dlx.dfs(0,ans);
}
```

7.4 插头 DP

```
int n,m,l;
struct L{
    int d[11];
    int& operator[](int x){return d[x];}
    int mc(int x){
        int an=1;
        if(d[x]==1){
            for(x++;x<l;x++)if(d[x]){
                an=an+(d[x]==1?1:-1);
                if(!an)return x;
            }
        }else{
            for(x--;x>=0;x--)if(d[x]){
                an=an+(d[x]==2?1:-1);
                if(!an)return x;
            }
        }
    }
    int h(){int an=0;for(int i=l-1;i>=0;i--)an=an*3+d[i];return an;}
    L s(int x,int y){
        L S=*this;
        S[x]=y;return S;
    }
    L operator>>(int _){
        L S=*this;
        for(int i=l-1;i>=1;i--)S[i]=S[i-1];
        S[0]=0;return S;
    }
};
struct Int{
    int len;
```

```
    int a[40];
    Int(){len=1;memset(a,0,sizeof a);}
    Int operator+=(const Int &o){
        int l=max(len,o.len);
        for(int i=0;i<l;i++)
            a[i]=a[i]+o.a[i];
        for(int i=0;i<l;i++)
            a[i+1]+=a[i]/10,a[i]%=10;
        if(a[l])l++;len=l;
        return *this;
    }
    void print(){
        for(int i=len-1;i>=0;i--)
            printf("%d",a[i]);
        puts("");
    }
};
struct hashtab{
    int sz;
    int tab[177147];
    Int w[177147];
    L s[177147];
    hashtab(){memset(tab,-1,sizeof tab);}
    void cl(){
        for(int i=0;i<sz;i++)tab[s[i].h()]=-1;
        sz=0;
    }
    Int& operator[](L S){
        int h=S.h();
        if(tab[h]==-1)tab[h]=sz,s[sz]=S,w[sz]=Int(),sz++;
        return w[tab[h]];
    }
}f[2];
bool check(L S){
    int cn1=0,cn2=0;
    for(int i=0;i<l;i++){
        cn1+=S[i]==1;
        cn2+=S[i]==2;
    }return cn1==1&&cn2==1;
}
int main(){
    Int One;One.a[0]=1;
```

```
    scanf("%d%d",&n,&m);if(n<m)swap(n,m);l=m+1;
    if(n==1||m==1){puts("1");return 0;}
    int cur=0;f[cur].cl();
    for(int i=1;i<=n;i++){
        for(int j=1;j<=m;j++){
            if(i==1&&j==1){
                f[cur][L().s(0,1).s(1,2)]+=One;
                continue;
            }
            cur^=1;f[cur].cl();
            for(int k=0;k<f[!cur].sz;k++){
                L S=f[!cur].s[k];Int w=f[!cur][S];
                int d1=S[j-1],d2=S[j];
                if(d1==0&&d2==0){
                    if(i!=n&&j!=m)f[cur][S.s(j-1,1).s(j,2)]+=w;
                }else
                if(d1==0||d2==0){
                    if(i!=n)f[cur][S.s(j-1,d1|d2).s(j,0)]+=w;
                    if(j!=m)f[cur][S.s(j-1,0).s(j,d1|d2)]+=w;
                }else
                if(d1==1&&d2==2){
                    if(i==n&&j==m&&check(S))
                        (w+=w).print();
                }else
                if(d1==2&&d2==1){
                    f[cur][S.s(j-1,0).s(j,0)]+=w;
                }else
                if((d1==1&&d2==1)||(d1==2&&d2==2)){
                    int m1=S.mc(j),m2=S.mc(j-1);
                    f[cur][S.s(j-1,0).s(j,0).s(m1,1).s(m2,2)]+=w;
                }
            }
        }
        cur^=1;f[cur].cl();
        for(int k=0;k<f[!cur].sz;k++){
            L S=f[!cur].s[k];Int w=f[!cur][S];
            f[cur][S>>1]=w;
        }
    }
    return 0;
}
```

7.5 某年某月某日是星期几

```
int solve(int year, int month, int day) {
    int answer;
    if (month == 1 || month == 2) {
        month += 12;
        year--;
    }
    if ((year < 1752) || (year == 1752 && month < 9) ||
        (year == 1752 && month == 9 && day < 3)) {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4 + 5) % 7;
    } else {
        answer = (day + 2 * month + 3 * (month + 1) / 5 + year + year / 4
               - year / 100 + year / 400) % 7;
    }
    return answer;
}
```

7.6 枚举大小为 k 的子集

使用条件：$k > 0$

```
void solve(int n, int k) {
    for (int comb = (1 << k) - 1; comb < (1 << n); ) {
        // ...
        int x = comb & -comb, y = comb + x;
        comb = (((comb & ~y) / x) >> 1) | y;
    }
}
```

7.7 环状最长公共子串

```
int n, a[N << 1], b[N << 1];

bool has(int i, int j) {
    return a[(i - 1) % n] == b[(j - 1) % n];
}

const int DELTA[3][2] = {{0, -1}, {-1, -1}, {-1, 0}};

int from[N][N];

```

```
int solve() {
    memset(from, 0, sizeof(from));
    int ret = 0;
    for (int i = 1; i <= 2 * n; ++i) {
        from[i][0] = 2;
        int left = 0, up = 0;
        for (int j = 1; j <= n; ++j) {
            int upleft = up + 1 + !!from[i - 1][j];
            if (!has(i, j)) {
                upleft = INT_MIN;
            }
            int max = std::max(left, std::max(upleft, up));
            if (left == max) {
                from[i][j] = 0;
            } else if (upleft == max) {
                from[i][j] = 1;
            } else {
                from[i][j] = 2;
            }
            left = max;
        }
        if (i >= n) {
            int count = 0;
            for (int x = i, y = n; y; ) {
                int t = from[x][y];
                count += t == 1;
                x += DELTA[t][0];
                y += DELTA[t][1];
            }
            ret = std::max(ret, count);
            int x = i - n + 1;
            from[x][0] = 0;
            int y = 0;
            while (y <= n && from[x][y] == 0) {
                y++;
            }
            for (; x <= i; ++x) {
                from[x][y] = 0;
                if (x == i) {
                    break;
                }
                for (; y <= n; ++y) {
```

```
                if (from[x + 1][y] == 2) {
                    break;
                }
                if (y + 1 <= n && from[x + 1][y + 1] == 1) {
                    y++;
                    break;
                }
            }
        }
    }
}
return ret;
}
```

7.8 LLMOD

```
LL multiplyMod(LL a, LL b, LL P) { // `需要保证 a 和 b 非负`
        LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
        return t < 0 : t + P : t;
}
```

8 Java

8.1 基础模板

```
import java.io.*;
import java.util.*;
import java.math.*;

public class Main {
    public static void main(String[] args) {
        InputStream inputStream = System.in;
        OutputStream outputStream = System.out;
        InputReader in = new InputReader(inputStream);
        PrintWriter out = new PrintWriter(outputStream);
        Task solver = new Task();
        solver.solve(0, in, out);
        out.close();
    }
}

```

```java
class Task {
    public void solve(int testNumber, InputReader in, PrintWriter out) {

    }
}

class InputReader {
    public BufferedReader reader;
    public StringTokenizer tokenizer;

    public InputReader(InputStream stream) {
        reader = new BufferedReader(new InputStreamReader(stream), 32768);
        tokenizer = null;
    }

    public String next() {
        while (tokenizer == null || !tokenizer.hasMoreTokens()) {
            try {
                tokenizer = new StringTokenizer(reader.readLine());
            } catch (IOException e) {
                throw new RuntimeException(e);
            }
        }
        return tokenizer.nextToken();
    }

    public int nextInt() {
        return Integer.parseInt(next());
    }

    public long nextLong() {
        return Long.parseLong(next());
    }
}
```

9 gedit

```
Compile:
#!/bin/sh
full=$GEDIT_CURRENT_DOCUMENT_NAME
name=`echo $full | cut -d. -f1`
```

```
g++ $full -o $name -g -Wall

Debug:
#!/bin/bash
name=`echo $GEDIT_CURRENT_DOCUMENT_NAME | cut -d. -f1`
gnome-terminal -x bash -c "gdb ./$name"

Run:
#!/bin/bash
name=`echo $GEDIT_CURRENT_DOCUMENT_NAME | cut -d. -f1`
gnome-terminal -x bash -c "time ./$name;echo 'Press any key to continue'; read"
```

10 数学

10.1 常用数学公式

10.1.1 求和公式

1. $\sum_{k=1}^{n}(2k-1)^2 = \frac{n(4n^2-1)}{3}$
2. $\sum_{k=1}^{n} k^3 = [\frac{n(n+1)}{2}]^2$
3. $\sum_{k=1}^{n}(2k-1)^3 = n^2(2n^2-1)$
4. $\sum_{k=1}^{n} k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
5. $\sum_{k=1}^{n} k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
6. $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$
7. $\sum_{k=1}^{n} k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
8. $\sum_{k=1}^{n} k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$

10.1.2 斐波那契数列

1. $fib_0 = 0, fib_1 = 1, fib_n = fib_{n-1} + fib_{n-2}$
2. $fib_{n+2} \cdot fib_n - fib_{n+1}^2 = (-1)^{n+1}$
3. $fib_{-n} = (-1)^{n-1} fib_n$
4. $fib_{n+k} = fib_k \cdot fib_{n+1} + fib_{k-1} \cdot fib_n$
5. $gcd(fib_m, fib_n) = fib_{gcd(m,n)}$
6. $fib_m | fib_n^2 \Leftrightarrow nfib_n | m$

10.1.3 错排公式

1. $D_n = (n-1)(D_{n-2} - D_{n-1})$
2. $D_n = n! \cdot (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + \frac{(-1)^n}{n!})$

10.1.4 莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & \text{若}n = 1 \\ (-1)^k & \text{若}n\text{无平方数因子，且}n = p_1 p_2 \ldots p_k \\ 0 & \text{若}n\text{有大于1的平方数因数} \end{cases}$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{若}n = 1 \\ 0 & \text{其他情况} \end{cases}$$

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(\frac{n}{d})$$

$$g(x) = \sum_{n=1}^{[x]} f(\frac{x}{n}) \Leftrightarrow f(x) = \sum_{n=1}^{[x]} \mu(n) g(\frac{x}{n})$$

10.1.5 伯恩赛德引理

设 G 是一个有限群，作用在集合 X 上。对每个 g 属于 G，令 X^g 表示 X 中在 g 作用下的不动元素，轨道数（记作 $|X/G|$）由如下公式给出：

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

10.1.6 五边形数定理

设 $p(n)$ 是 n 的拆分数，有

$$p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k-1} p\left(n - \frac{k(3k-1)}{2}\right)$$

10.1.7 树的计数

1. 有根树计数：$n+1$ 个结点的有根树的个数为

$$a_{n+1} = \frac{\sum_{j=1}^{n} j \cdot a_j \cdot S_{n,j}}{n}$$

其中，

$$S_{n,j} = \sum_{i=1}^{n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$$

2. 无根树计数：当 n 为奇数时，n 个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$$

当 n 为偶数时，n 个结点的无根树的个数为

$$a_n - \sum_{i=1}^{n/2} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$$

3. n 个结点的完全图的生成树个数为

$$n^{n-2}$$

4. 矩阵一树定理：图 G 由 n 个结点构成，设 $\boldsymbol{A}[G]$ 为图 G 的邻接矩阵、$\boldsymbol{D}[G]$ 为图 G 的度数矩阵，则图 G 的不同生成树的个数为 $\boldsymbol{C}[G] = \boldsymbol{D}[G] - \boldsymbol{A}[G]$ 的任意一个 $n-1$ 阶主子式的行列式值。

10.1.8 欧拉公式

平面图的顶点个数、边数和面的个数有如下关系：

$$V - E + F = C + 1$$

其中，V 是顶点的数目，E 是边的数目，F 是面的数目，C 是组成图形的连通部分的数目。当图是单连通图的时候，公式简化为：

$$V - E + F = 2$$

10.1.9 皮克定理

给定顶点坐标均是整点（或正方形格点）的简单多边形，其面积 A 和内部格点数目 i、边上格点数目 b 的关系：

$$A = i + \frac{b}{2} - 1$$

10.1.10 牛顿恒等式

设

$$\prod_{i=1}^{n} (x - x_i) = a_n + a_{n-1}x + \cdots + a_1 x^{n-1} + a_0 x^n$$

$$p_k = \sum_{i=1}^{n} x_i^k$$

则

$$a_0 p_k + a_1 p_{k-1} + \cdots + a_{k-1} p_1 + k a_k = 0$$

特别地，对于

$$|\boldsymbol{A} - \lambda \boldsymbol{E}| = (-1)^n (a_n + a_{n-1}\lambda + \cdots + a_1 \lambda^{n-1} + a_0 \lambda^n)$$

有

$$p_k = Tr(\boldsymbol{A}^k)$$

10.2 平面几何公式

10.2.1 三角形

1. 半周长

$$p = \frac{a+b+c}{2}$$

2. 面积

$$S = \frac{a \cdot H_a}{2} = \frac{ab \cdot sinC}{2} = \sqrt{p(p-a)(p-b)(p-c)}$$

3. 中线

$$M_a = \frac{\sqrt{2(b^2+c^2)-a^2}}{2} = \frac{\sqrt{b^2+c^2+2bc \cdot cosA}}{2}$$

4. 角平分线

$$T_a = \frac{\sqrt{bc \cdot [(b+c)^2-a^2]}}{b+c} = \frac{2bc}{b+c} cos\frac{A}{2}$$

5. 高线

$$H_a = bsinC = csinB = \sqrt{b^2 - (\frac{a^2+b^2-c^2}{2a})^2}$$

6. 内切圆半径

$$\begin{aligned} r = \frac{S}{p} = \frac{arcsin\frac{B}{2} \cdot sin\frac{C}{2}}{sin\frac{B+C}{2}} = 4R \cdot sin\frac{A}{2} sin\frac{B}{2} sin\frac{C}{2} \\ = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} = p \cdot tan\frac{A}{2} tan\frac{B}{2} tan\frac{C}{2} \end{aligned}$$

7. 外接圆半径

$$R = \frac{abc}{4S} = \frac{a}{2sinA} = \frac{b}{2sinB} = \frac{c}{2sinC}$$

10.2.2 四边形

D_1, D_2 为对角线，M 对角线中点连线，A 为对角线夹角，p 为半周长

1. $a^2+b^2+c^2+d^2 = D_1^2+D_2^2+4M^2$

2. $S = \frac{1}{2} D_1 D_2 sinA$

3. 对于圆内接四边形

$$ac + bd = D_1 D_2$$

4. 对于圆内接四边形

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$$

10.2.3 正 n 边形

R 为外接圆半径，r 为内切圆半径

1. 中心角

$$A = \frac{2\pi}{n}$$

2. 内角

$$C = \frac{n-2}{n}\pi$$

3. 边长

$$a = 2\sqrt{R^2 - r^2} = 2R \cdot sin\frac{A}{2} = 2r \cdot tan\frac{A}{2}$$

4. 面积

$$S = \frac{nar}{2} = nr^2 \cdot tan\frac{A}{2} = \frac{nR^2}{2} \cdot sinA = \frac{na^2}{4 \cdot tan\frac{A}{2}}$$

10.2.4 圆

1. 弧长

$$l = rA$$

2. 弦长

$$a = 2\sqrt{2hr - h^2} = 2r \cdot sin\frac{A}{2}$$

3. 弓形高

$$h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1 - cos\frac{A}{2}) = \frac{1}{2} \cdot arctan\frac{A}{4}$$

4. 扇形面积

$$S_1 = \frac{rl}{2} = \frac{r^2A}{2}$$

5. 弓形面积

$$S_2 = \frac{rl - a(r-h)}{2} = \frac{r^2}{2}(A - sinA)$$

10.2.5 棱柱

1. 体积

$$V = Ah$$

A 为底面积，h 为高

2. 侧面积

$$S = lp$$

l 为棱长，p 为直截面周长

3. 全面积

$$T = S + 2A$$

10.2.6 棱锥

1. 体积

$$V = Ah$$

   A 为底面积，h 为高

2. 正棱锥侧面积

$$S = lp$$

   l 为棱长，p 为直截面周长

3. 正棱锥全面积

$$T = S + 2A$$

10.2.7 棱台

1. 体积

$$V = (A_1 + A_2 + \sqrt{A_1 A_2}) \cdot \frac{h}{3}$$

   A_1, A_2 为上下底面积，h 为高

2. 正棱台侧面积

$$S = \frac{p_1 + p_2}{2} l$$

   p_1, p_2 为上下底面周长，l 为斜高

3. 正棱台全面积

$$T = S + A_1 + A_2$$

10.2.8 圆柱

1. 侧面积

$$S = 2\pi rh$$

2. 全面积

$$T = 2\pi r(h + r)$$

3. 体积

$$V = \pi r^2 h$$

10.2.9 圆锥

1. 母线

$$l = \sqrt{h^2 + r^2}$$

2. 侧面积

$$S = \pi rl$$

3. 全面积

$$T = \pi r(l + r)$$

4. 体积

$$V = \frac{\pi}{3} r^2 h$$

10.2.10 圆台

1. 母线

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

2. 侧面积

$$S = \pi(r_1 + r_2)l$$

3. 全面积

$$T = \pi r_1(l + r_1) + \pi r_2(l + r_2)$$

4. 体积

$$V = \frac{\pi}{3}(r_1^2 + r_2^2 + r_1 r_2)h$$

10.2.11 球

1. 全面积

$$T = 4\pi r^2$$

2. 体积

$$V = \frac{4}{3}\pi r^3$$

10.2.12 球台

1. 侧面积

$$S = 2\pi r h$$

2. 全面积

$$T = \pi(2rh + r_1^2 + r_2^2)$$

3. 体积

$$V = \frac{\pi h[3(r_1^2 + r_2^2) + h^2]}{6}$$

10.2.13 球扇形

1. 全面积

$$T = \pi r(2h + r_0)$$

h 为球冠高，r_0 为球冠底面半径

2. 体积

$$V = \frac{2}{3}\pi r^2 h$$

10.3 立体几何公式

10.3.1 球面三角公式

设 a,b,c 是边长，A,B,C 是所对的二面角，有余弦定理

$$cosa = cosb \cdot cosc + sinb \cdot sinc \cdot cosA$$

正弦定理

$$\frac{sinA}{sina} = \frac{sinB}{sinb} = \frac{sinC}{sinc}$$

三角形面积是 $A+B+C-\pi$

10.3.2 四面体体积公式

U,V,W,u,v,w 是四面体的 6 条棱，U,V,W 构成三角形，$(U,u),(V,v),(W,w)$ 互为对棱，则

$$V = \frac{\sqrt{(s-2a)(s-2b)(s-2c)(s-2d)}}{192uvw}$$

其中

$$\begin{cases} a &= \sqrt{xYZ}, \\ b &= \sqrt{yZX}, \\ c &= \sqrt{zXY}, \\ d &= \sqrt{xyz}, \\ s &= a+b+c+d, \\ X &= (w-U+v)(U+v+w), \\ x &= (U-v+w)(v-w+U), \\ Y &= (u-V+w)(V+w+u), \\ y &= (V-w+u)(w-u+V), \\ Z &= (v-W+u)(W+u+v), \\ z &= (W-u+v)(u-v+W) \end{cases}$$