

Useful Stuff

Trig Functions

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$
$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Area & Volume

Volume by slicing $A(x)$ = area of cross section at x , $V(S) = \int_a^b A(x)dx$

Volume by disk If $f(x)$ rotated around x -axis, $V(S) = \int_a^b \pi[f(x)]^2 dx$

Volume by cylindrical shells If $f(x)$ rotated around y -axis, $V(S) = \int_a^b (2\pi x f(x))dx$

Applications

Arc Length $\int_a^b \sqrt{1 + [f'(x)]^2} dx$
Note use of derivative!

Surface Area of Revolution

Revolve $f(x)$ around x axis,
 $SA(x) = \int_a^b (2\pi f(x) \sqrt{1 + [f'(x)]^2}) dx$

Mass-density for 1-d object If $p(x)$ linear density for given x , $m = \int_a^b p(x)dx$

Mass-density for circular object

If $p(x)$ radial density for given x , and radius = r , $m = \int_0^r 2\pi x p(x)dx$

Work done If $F(X)$ = force at point x ,

$W = \int_a^b F(x)dx$ *Recall constant force yields $F * d$

Hyperbolic Functions

| $f(x)$ | $\frac{d}{dx} f(x)$ |
|--------------------------|-----------------------------------|
| $\sinh(x)$ | $\cosh(x)$ |
| $\cosh(x)$ | $\sinh(x)$ |
| $\tanh(x)$ | $\sec^2(x)$ |
| $\coth(x)$ | $-\operatorname{csch}^2(x)$ |
| $\operatorname{sech}(x)$ | $-\operatorname{sech}(x)\tanh(x)$ |
| $\operatorname{csch}(x)$ | $-\operatorname{csch}(x)\coth(x)$ |

Integration Techniques

Int by parts $\int u dv = uv - \int v du$

Pick u using LIATE (log, inv trig, alg, trig, exp)

$$\int \cos^j(x) \sin^k(x) dx$$

If k odd keep 1 $\sin(x)$, convert rest using $\sin^2 x = 1 - \cos^2 x$. u-sub with $u = \cos(x)$.

If j odd keep 1 $\cos(x)$, convert rest using $\cos^2 x = 1 - \sin^2 x$. u-sub with $u = \sin(x)$.

If both even use $\sin^2 x = \frac{1 - \cos(2x)}{2}$.

$$\int \tan^k(x) \sec^j(x) dx$$

If j even and ≥ 2 keep $\sec^2(x)$, convert rest using $\sec^2 x = \tan^2 x + 1$. u-sub with $u = \tan x$.

If k odd, $j \geq 1$ keep $\sec(x)\tan(x)$, convert rest using $\tan^2 x = \sec^2 x$. u-sub with $u = \sec x$.

If k odd, $k \geq 3$ and $j = 0$ turn one $\tan^2 x$ into $\sec^2 x - 1$. Repeat process. If k even, j odd, use $\tan^2 x = \sec^2 x - 1$ to turn $\tan^k x$ to $\sec x$.

Reductions

$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx$$

Trig subs Don't forget to change dx to $d\theta$

$$\begin{array}{ll} a^2 - x^2 & \text{Use } x = a \sin \theta \\ a^2 + x^2 & \text{Use } x = a \tan \theta \\ x^2 - a^2 & \text{Use } x = a \sec \theta \end{array}$$