

## Useful Stuff

### Trig Functions

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$
$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

### Area & Volume

**Volume by slicing**  $A(x)$  = area of cross section at  $x$ ,  $V(S) = \int_a^b A(x) dx$

**Volume by disk** If  $f(x)$  rotated around  $x$ -axis,  $V(S) = \int_a^b \pi [f(x)]^2 dx$

**Volume by cylindrical shells** If  $f(x)$  rotated around  $y$ -axis,  $V(S) = \int_a^b (2\pi x f(x)) dx$

### Applications

**Arc Length**  $\int_a^b \sqrt{1 + [f'(x)]^2} dx$  \*Note use of derivative!

### Surface Area of Revolution

Revolve  $f(x)$  around  $x$  axis,  
 $SA(x) = \int_a^b (2\pi f(x) \sqrt{1 + [f'(x)]^2}) dx$

**Mass-density for 1-d object** If  $p(x)$  linear density for given  $x$ ,  $m = \int_a^b p(x) dx$

### Mass-density for circular object

If  $p(x)$  radial density for given  $x$ , and radius =  $r$ ,  $m = \int_0^r 2\pi x p(x) dx$

**Work done** If  $F(X)$  = force at point  $x$ ,  
 $W = \int_a^b F(x) dx$  \*Recall constant force yields  $F * d$

### Hyperbolic Functions

$f(x)$	$\frac{d}{dx} f(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\sec^2(x)$
$\coth(x)$	$-\operatorname{csch}^2(x)$
$\operatorname{sech}(x)$	$-\operatorname{sech}(x)\tanh(x)$
$\operatorname{csch}(x)$	$-\operatorname{csch}(x)\coth(x)$

## Integration Techniques

**Int by parts**  $\int u dv = uv - \int v du$

Pick  $u$  using LIATE (log, inv trig, alg, trig, exp)

$$\int \cos^j(x) \sin^k(x) dx$$

**If  $k$  odd** keep 1  $\sin(x)$ , convert rest using  $\sin^2 x = 1 - \cos^2 x$ . u-sub with  $u = \cos(x)$ .

**If  $j$  odd** keep 1  $\cos(x)$ , convert rest using  $\cos^2 x = 1 - \sin^2 x$ . u-sub with  $u = \sin(x)$ .

**If both even** use  $\sin^2 x = \frac{1 - \cos(2x)}{2}$ .

$$\int \tan^k(x) \sec^j(x) dx$$

**If  $j$  even and  $\geq 2$**  keep  $\sec^2(x)$ , convert rest using  $\sec^2 x = \tan^2 x + 1$ . u-sub with  $u = \tan x$ .

**If  $k$  odd,  $j \geq 1$**  keep  $\sec(x)\tan(x)$ , convert rest using  $\tan^2 x = \sec^2 x$ . u-sub with  $u = \sec x$ .

**If  $k$  odd,  $k \geq 3$  and  $j = 0$**  turn one  $\tan^2 x$  into  $\sec^2 x - 1$ . Repeat process. If  $k$  even,  $j$  odd, use  $\tan^2 x = \sec^2 x - 1$  to turn  $\tan^k x$  to  $\sec x$ .

### Reductions

$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx$$

**Sub  $a^2 - x^2$**  Use  $x = a \sin \theta$

**Sub  $a^2 + x^2$**  Use  $x = a \tan \theta$

**Sub  $x^2 - a^2$**  Use  $x = a \sec \theta$