

Useful Stuff

Completing the square

$$ax^2 + bx + c = 0$$

$$a(x + d)^2 + e = 0$$

$$d = \frac{b}{2a}$$

$$e = c - \frac{b^2}{4a}$$

Trig Functions

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\int \sec(x) = \ln|\sec(x) + \tan(x)|$$

Area & Volume

Volume by slicing A(x) = area of cross section at x,

$$V(S) = \int_a^b A(x) dx$$

Volume by disk If f(x) rotated around x-axis,

$$V(S) = \int_a^b \pi[f(x)]^2 dx$$

Volume by cylindrical shells If f(x) rotated around y-axis,

$$V(S) = \int_a^b (2\pi x f(x)) dx$$

Applications

$$\text{Arc Length } \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Note use of derivative!

Surface Area of Revolution

Revolve f(x) around x axis,

$$SA(x) = \int_a^b (2\pi f(x) \sqrt{1 + [f'(x)]^2}) dx$$

Mass-density for 1-d object

If p(x) linear density for given x,

$$m = \int_a^b p(x) dx$$

Mass-density for circular object

If p(x) radial density for given x, and radius = r,

$$m = \int_0^r 2\pi x p(x) dx$$

Work done

If F(X) = force at point x,

$$W = \int_a^b F(x) dx$$

*Recall constant force yields $F * d$

Hyperbolic Functions

$f(x)$	$\frac{d}{dx} f(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\sec^2(x)$
$\coth(x)$	$-\operatorname{csch}^2(x)$
$\operatorname{sech}(x)$	$-\operatorname{sech}(x)\tanh(x)$
$\operatorname{csch}(x)$	$-\operatorname{csch}(x)\coth(x)$

Integration Techniques

Int by parts $\int u dv = uv - \int v du$

Pick u using LIATE (log, inv trig, alg, trig, exp)

$$\int \cos^j(x) \sin^k(x) dx$$

If k odd keep 1 $\sin(x)$, convert rest using $\sin^2 x = 1 - \cos^2 x$. u-sub with $u = \cos(x)$.

If j odd keep 1 $\cos(x)$, convert rest using $\cos^2 x = 1 - \sin^2 x$. u-sub with $u = \sin(x)$.

If both even use $\sin^2 x = \frac{1 - \cos(2x)}{2}$.

$$\int \tan^k(x) \sec^j(x) dx$$

If j even and ≥ 2 keep $\sec^2(x)$, convert rest using $\sec^2 x = \tan^2 x + 1$. u-sub with $u = \tan x$.

If k odd, $j \geq 1$ keep $\sec(x)\tan(x)$, convert rest using $\tan^2 x = \sec^2 x$. u-sub with $u = \sec x$.

If k odd, $k \geq 3$ and $j = 0$ turn one $\tan^2 x$ into $\sec^2 x - 1$. Repeat process. If k even, j odd, use $\tan^2 x = \sec^2 x - 1$ to turn $\tan^k x$ to $\sec x$.

Reductions

$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx$$

Trig subs Don't forget to change dx to dθ

$a^2 - x^2$	Use $x = a \sin \theta$
$a^2 + x^2$	Use $x = a \tan \theta$
$x^2 - a^2$	Use $x = a \sec \theta$

Arclength ax^2

$$\frac{x\sqrt{1+4a^2x^2}}{2} + \frac{\ln(|\sqrt{1+4a^2x^2}+2ax|)}{4a}$$

Note: if evaluating over interval [a,b], if a=0, just plug in b

$$\text{Reduction } \int \frac{1}{(ax^2+b)^n} = \frac{2n-3}{2b(n-1)} \int \frac{1}{(ax^2+b)^{n-1}} dx + \frac{x}{2b(n-1)(ax^2+b)^{n-1}}$$