

## Useful Stuff

### Completing the square

$$ax^2 + bx + c = 0$$

$$a(x + d)^2 + e = 0$$

$$d = \frac{b}{2a}$$

$$e = c - \frac{b^2}{4a}$$

## Trig Functions

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\int \sec(x) = \ln|\sec(x) + \tan(x)|$$

## Area & Volume

**Volume by slicing** A(x) = area of cross section at x,

$$V(S) = \int_a^b A(x) dx$$

**Volume by disk** If f(x) rotated around x-axis,

$$V(S) = \int_a^b \pi[f(x)]^2 dx$$

**Volume by cylindrical shells** If f(x) rotated around y-axis,

$$V(S) = \int_a^b (2\pi x f(x)) dx$$

## Applications

**Arc Length**  $\int_a^b \sqrt{1 + [f'(x)]^2} dx$

Note use of derivative!

### Surface Area of Revolution

Revolve f(x) around x axis,

$$SA(x) = \int_a^b (2\pi f(x) \sqrt{1 + [f'(x)]^2}) dx$$

**Mass-density for 1-d object** If p(x)

linear density for given x,

$$m = \int_a^b p(x) dx$$

**Mass-density for circular object** If

p(x) radial density for given x, and radius = r,

$$m = \int_0^r 2\pi x p(x) dx$$

**Work done** If F(X) = force at point x,

$$W = \int_a^b F(x) dx$$

\*Recall constant force yields  $F * d$

## Hyperbolic Functions

$f(x)$	$\frac{d}{dx} f(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\sec^2(x)$
$\coth(x)$	$-\operatorname{csch}^2(x)$
$\operatorname{sech}(x)$	$-\operatorname{sech}(x)\tanh(x)$
$\operatorname{csch}(x)$	$-\operatorname{csch}(x)\coth(x)$

## Integration Techniques

**Int by parts**  $\int u dv = uv - \int v du$

Pick u using LIATE (log, inv trig, alg, trig, exp)

$$\int \cos^j(x) \sin^k(x) dx$$

**If k odd** keep 1  $\sin(x)$ , convert rest using

$\sin^2 x = 1 - \cos^2 x$ . u-sub with  $u = \cos(x)$ .

**If j odd** keep 1  $\cos(x)$ , convert rest using

$\cos^2 x = 1 - \sin^2 x$ . u-sub with  $u = \sin(x)$ .

**If both even** use  $\sin^2 x = \frac{1 - \cos(2x)}{2}$ .

$$\int \tan^k(x) \sec^j(x) dx$$

**If j even and  $\geq 2$**  keep  $\sec^2(x)$ , convert

rest using  $\sec^2 x = \tan^2 x + 1$ . u-sub with

$u = \tan x$ .

**If k odd,  $j \geq 1$**  keep  $\sec(x)\tan(x)$ ,

convert rest using  $\tan^2 x = \sec^2 x$ . u-sub with

$u = \sec x$ .

**If k odd,  $k \geq 3$  and  $j = 0$**  turn one

$\tan^2 x$  into  $\sec^2 x - 1$ . Repeat process. If k

even, j odd, use  $\tan^2 x = \sec^2 x - 1$  to turn

$\tan^k x$  to  $\sec x$ .

## Reductions

$$\int \sec^n x dx =$$

$$\frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx$$

**Trig subs** Don't forget to change dx to dθ

$$a^2 - x^2 \quad \text{Use } x = a \sin \theta$$

$$a^2 + x^2 \quad \text{Use } x = a \tan \theta$$

$$x^2 - a^2 \quad \text{Use } x = a \sec \theta$$

### Arclength $ax^2$

$$\frac{x\sqrt{1+4a^2x^2}}{2} + \frac{\ln(|\sqrt{1+4a^2x^2}+2ax|)}{4a}$$

Note: if evaluating over interval [a,b], if a=0, just plug in b

**Reduction**  $\int \frac{1}{(ax^2+b)^n}$

$$\frac{2n-3}{2b(n-1)} \int \frac{1}{(ax^2+b)^{n-1}} dx + \frac{x}{2b(n-1)(ax^2+b)^{n-1}}$$

**Trapezoid rule**  $\frac{1}{2} \delta x (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n))$

### Simpson's rule

$$\int_b^a f(x) = \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)]$$