Useful Stuff

Trig Functions

$sin^2(x) = \frac{1-cos(2x)}{2}$

$cos^2(x) = \frac{1+cos(2x)}{2}$

Area & Volume

Volume by slicing A(x) = area of cross section at x,

$V(S) = \int_a^b A(x)dx$

Volume by disk If f(x) rotated around x-axis,

$V(S) = \int_a^b \pi[f(x)]^2 dx$

Volume by cylindrical shells If f(x) rotated around y-axis,

$V(S) = \int_a^b (2\pi x f(x))dx$

Applications

Arc Length $\int_a^b \sqrt{1+[f'(x)]^2}dx$

Note use of derivative!

Surface Area of Revolution

Revolve f(x) around x axis,

$SA(x) = \int_a^b (2\pi f(x)\sqrt{1+[f'(x)]^2})dx$

Mass-density for 1-d object If p(x) linear density for given x,

$m = \int_a^b p(x)dx$

Mass-density for circular object If p(x) radial density for given x, and radius = r,

$m = \int_0^r 2\pi x p(x)dx$

Work done If F(X) = force at point x,

$W = \int_a^b F(x)dx$

*Recall constant force yields $F*d$

Hyperbolic Functions

$f(x)$	$\frac{d}{dx}f(x)$
$sinh(x)$	$cosh(x)$
$cosh(x)$	$sinh(x)$
$tanh(x)$	$sec^2(x)$
$coth(x)$	$-csch^2(x)$
$sech(x)$	$-sech(x)tanh(x)$
$csch(x)$	$-csch(x)coth(x)$

Integration Techniques

Int by parts $\int u\,dv = uv - \int v\,du$

Pick u using LIATE (log, inv trig, alg, trig, exp)

$\int cos^j(x)sin^k(x)dx$

If k odd keep 1 $sin(x)$, convert rest using $sin^2 x = 1 - cos^2 x$. u-sub with $u = cos(x)$.

If j odd keep 1 $cos(x)$, convert rest using $cos^2 x = 1 - sin^2 x$. u-sub with $u = sin(x)$.

If both even use $sin^2 x = \frac{1-cos(2x)}{2}$.

$\int tan^k(x)sec^j(x)dx$

If j even and ≥ 2 keep $sec^2(x)$, convert rest using $sec^2 x = tan^2 x + 1$. u-sub with $u = tanx$.

If k odd, $j \geq 1$ keep $sec(x)tan(x)$, convert rest using $tan^2 x = sec^2 x$. u-sub with $u = secx$.

If k odd, $k \geq 3$ and $j = 0$ turn one $tan^2 x$ into $sec^2 x - 1$. Repeat process. If k even, j odd, use $tan^2 x = sec^2 x - 1$ to turn $tan^k x$ to $secx$.

Reductions

$\int sec^n x\,dx = \frac{1}{n-1}sec^{n-2}x tanx + \frac{n-2}{n-1}\int sec^{n-2}x\,dx$

$\int tan^n x\,dx = \frac{1}{n-1}tan^{n-1}x - \int tan^{n-2}x\,dx$

Trig subs Don't forget to change dx to dθ

$a^2 - x^2$	Use $x = asin\theta$
$a^2 + x^2$	Use $x = atan\theta$
$x^2 - a^2$	Use $x = asec\theta$

Arclength ax^2

$\frac{x\sqrt{1+4a^2x^2}}{2} + \frac{ln(|\sqrt{1+4a^2x^2}+2ax|)}{4a}$

Note: if evaluating over interval [a,b], if a=0, just plug in b