

Useful Stuff

Completing the square

$$ax^2 + bx + c = 0$$

$$a(x + d)^2 + e = 0$$

$$d = \frac{b}{2a}$$

$$e = c - \frac{b^2}{4a}$$

Trig Functions

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\int \sec(x) = \ln|\sec(x) + \tan(x)|$$

Area & Volume

Volume by slicing A(x) = area of cross section at x,

$$V(S) = \int_a^b A(x) dx$$

Volume by disk If f(x) rotated around x-axis,

$$V(S) = \int_a^b \pi[f(x)]^2 dx$$

Volume by cylindrical shells If f(x) rotated around y-axis,

$$V(S) = \int_a^b (2\pi x f(x)) dx$$

Applications

Arc Length $\int_a^b \sqrt{1 + [f'(x)]^2} dx$

Note use of derivative!

Surface Area of Revolution

Revolve f(x) around x axis,

$$SA(x) = \int_a^b (2\pi f(x) \sqrt{1 + [f'(x)]^2}) dx$$

Mass-density for 1-d object If p(x) linear density for given x,

$$m = \int_a^b p(x) dx$$

Mass-density for circular object If p(x) radial density for given x, and radius = r,

$$m = \int_0^r 2\pi x p(x) dx$$

Work done If F(X) = force at point x,

$$W = \int_a^b F(x) dx$$

*Recall constant force yields $F * d$

Hyperbolic Functions

$f(x)$	$\frac{d}{dx} f(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\sec^2(x)$
$\coth(x)$	$-\operatorname{csch}^2(x)$
$\operatorname{sech}(x)$	$-\operatorname{sech}(x) \tanh(x)$
$\operatorname{csch}(x)$	$-\operatorname{csch}(x) \coth(x)$

Integration Techniques

Int by parts $\int u dv = uv - \int v du$

Pick u using LIATE (log, inv trig, alg, trig, exp)

$$\int \cos^j(x) \sin^k(x) dx$$

If k odd keep 1 $\sin(x)$, convert rest using $\sin^2 x = 1 - \cos^2 x$. u-sub with $u = \cos(x)$.

If j odd keep 1 $\cos(x)$, convert rest using $\cos^2 x = 1 - \sin^2 x$. u-sub with $u = \sin(x)$.

If both even use $\sin^2 x = \frac{1 - \cos(2x)}{2}$.

$$\int \tan^k(x) \sec^j(x) dx$$

If j even and ≥ 2 keep $\sec^2(x)$, convert rest using $\sec^2 x = \tan^2 x + 1$. u-sub with $u = \tan x$.

If k odd, $j \geq 1$ keep $\sec(x) \tan(x)$, convert rest using $\tan^2 x = \sec^2 x$. u-sub with $u = \sec x$.

If k odd, $k \geq 3$ and $j = 0$ turn one $\tan^2 x$ into $\sec^2 x - 1$. Repeat process. If k even, j odd, use $\tan^2 x = \sec^2 x - 1$ to turn $\tan^k x$ to $\sec x$.

Reductions

$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx$$

Trig subs Don't forget to change dx to dθ

$$a^2 - x^2 \quad \text{Use } x = a \sin \theta$$

$$a^2 + x^2 \quad \text{Use } x = a \tan \theta$$

$$x^2 - a^2 \quad \text{Use } x = a \sec \theta$$

Arclength ax^2

$$\frac{x\sqrt{1+4a^2x^2}}{2} + \frac{\ln(|\sqrt{1+4a^2x^2}+2ax|)}{4a}$$

Note: if evaluating over interval [a,b], if a=0, just plug in b

Reduction $\int \frac{1}{(ax^2+b)^n}$
 $\frac{2n-3}{2b(n-1)} \int \frac{1}{(ax^2+b)^{n-1}} dx + \frac{x}{2b(n-1)(ax^2+b)^{n-1}}$

Trapezoid rule $\frac{1}{2} \Delta x (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n))$

Simpson's rule

$$\int_b^a f(x) = \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)]$$

Simpson's error bound Let M = max value of $|f^4(x)|$ over [a,b]. Error is:

$$E \leq \frac{M(b-a)^5}{180n^4}$$

Sequences/Series

Geometric Convergence For $\sum_{n=1}^{\infty} ar^{n-1}$, if $|r| < 1$, it converges to $\frac{a}{1-r}$

Divergence Test For $\sum_{n=1}^{\infty} a_n$ to converge, nth term a_n must satisfy $a_n \rightarrow 0$ as $n \rightarrow \infty$. Converse is not true (if limit = 0, does not mean it converges)

Integral test For $\sum_{n=1}^{\infty} a_n$, if a_n is all positive terms, and there exists a function f such that:

- f is continuous

- f is decreasing
- $f(n) = a_n$ for all integers $n \geq N$

Then $\sum_{n=1}^{\infty} a_n$ and $\int_N^{\infty} f(x)dx$ both converge or both diverge.

P-Series For $\sum_{n=1}^{\infty} \frac{1}{n^p}$, converges if $p > 1$, diverges if $p \leq 1$

Remainder estimate Same rules as integral test for $f(n)$.

$$\int_{N+1}^{\infty} f(x)dx < R_N < \int_N^{\infty} f(x)dx$$

Comparison test If you have sum of a_n , find a similar sum of b_n (only works if a_n

and $b_n \geq 0$). If $a_n \leq b_n$ and b_n 's sum converges, a_n 's sum converges. If $a_n \geq b_n$ and b_n 's sum diverges, a_n 's sum diverges.

Root test For $\sum_{n=1}^{\infty} a_n$ such that $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = p$ for some p . If $0 \leq p < 1$, the sum converges absolutely. If $p > 1$ or $p = \infty$, it diverges. If $p = 1$, test provides no information.

Ratio Test For $\sum_{n=1}^{\infty} a_n$ with positive terms, and $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$, if $L < 1$, sum converges. If $L > 1$, sum diverges

Parametric/Polar

Derivative of parametric
 $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)}$

2nd Derivative of parametric
 $\frac{d^2y}{dx^2} = \frac{(d/dt)(dy/dx)}{dx/dt}$

Area under parametric
 $A = \int_a^b y(t)x'(t)dt$

Arc length of parametric
 $s = \int_{t1}^{t2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$