Useful Stuff

Completing the square

$ax^2 + bx + c = 0$
$a(x+d)^2 + e = 0$
$d = \frac{b}{2a}$
$e = c - \frac{b^2}{4a}$

Trig Functions

$sin^2(x) = \frac{1-cos(2x)}{2}$
$cos^2(x) = \frac{1+cos(2x)}{2}$
$\frac{d}{dx}tan(x) = sec^2(x)$
$\frac{d}{dx}sec(x) = sec(x)tan(x)$
$\frac{d}{dx}tan^{-1}(x) = \frac{1}{1+x^2}$
$\int sec(x) = ln|sec(x) + tan(x)|$

Area & Volume

Volume by slicing A(x) = area of cross section at x,
$V(S) = \int_a^b A(x)dx$

Volume by disk If f(x) rotated around x-axis,
$V(S) = \int_a^b \pi[f(x)]^2 dx$

Volume by cylindrical shells If f(x) rotated around y-axis,
$V(S) = \int_a^b (2\pi x f(x))dx$

Applications

Arc Length $\int_a^b \sqrt{1 + [f'(x)]^2}dx$
Note use of derivative!

Surface Area of Revolution
Revolve f(x) around x axis,
$SA(x) = \int_a^b (2\pi f(x)\sqrt{1 + [f'(x)]^2})dx$

Mass-density for 1-d object If p(x) linear density for given x,
$m = \int_a^b p(x)dx$

Mass-density for circular object If p(x) radial density for given x, and radius = r,
$m = \int_0^r 2\pi x p(x)dx$

Work done If F(X) = force at point x,
$W = \int_a^b F(x)dx$
*Recall constant force yields $F * d$

Hyperbolic Functions

$f(x)$	$\frac{d}{dx}f(x)$
$sinh(x)$	$cosh(x)$
$cosh(x)$	$sinh(x)$
$tanh(x)$	$sec^2(x)$
$coth(x)$	$-csch^2(x)$
$sech(x)$	$-sech(x)tanh(x)$
$csch(x)$	$-csch(x)coth(x)$

Integration Techniques

Int by parts $\int udv = uv - \int vdu$
Pick u using LIATE (log, inv trig, alg, trig, exp)

$\int cos^j(x)sin^k(x)dx$

If k odd keep 1 $sin(x)$, convert rest using $sin^2x = 1 - cos^2x$. u-sub with $u = cos(x)$.

If j odd keep 1 $cos(x)$, convert rest using $cos^2x = 1 - sin^2x$. u-sub with $u = sin(x)$.

If both even use $sin^2x = \frac{1-cos(2x)}{2}$.

$\int tan^k(x)sec^j(x)dx$

If j even and ≥ 2 keep $sec^2(x)$, convert rest using $sec^2x = tan^2x + 1$. u-sub with $u = tanx$.

If k odd, $j \geq 1$ keep $sec(x)tan(x)$, convert rest using $tan^2x = sec^2x$. u-sub with $u = secx$.

If k odd, $k \geq 3$ and $j = 0$ turn one tan^2x into $sec^2x - 1$. Repeat process. If k even, j odd, use $tan^2x = sec^2x - 1$ to turn tan^kx to $secx$.

Reductions

$$\int sec^n xdx = \frac{1}{n-1}sec^{n-2}xtanx + \frac{n-2}{n-1}\int sec^{n-2}xdx$$

$$\int tan^n xdx = \frac{1}{n-1}tan^{n-1}x - \int tan^{n-2}xdx$$

Trig subs Don't forget to change dx to dθ

$a^2 - x^2$	Use $x = asin\theta$
$a^2 + x^2$	Use $x = atan\theta$
$x^2 - a^2$	Use $x = asec\theta$

Arclength ax^2
$\frac{x\sqrt{1+4a^2x^2}}{2} + \frac{ln(|\sqrt{1+4a^2x^2}+2ax|)}{4a}$

Note: if evaluating over interval [a,b], if a=0, just plug in b

Reduction $\int \frac{1}{(ax^2+b)^n}$
$\frac{2n-3}{2b(n-1)}\int \frac{1}{(ax^2+b)^{n-1}}dx + \frac{x}{2b(n-1)(ax^2+b)^{n-1}}$

Trapezoid rule $\frac{1}{2}\Delta x(f(x_0) + 2f(x_1) + 2f(x_2) + \ell + 2f(x_{n-1}) + f(x_n))$

Simpson's rule
$\int_b^a f(x) = \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n)]$

Simpson's error bound Let M = max value of $|f^4(x)|$ over [a,b]. Error is:
$E \leq \frac{M(b-a)^5}{180n^4}$

Sequences/Series

Geometric Convergence

For $\sum_{n=1}^{\infty} ar^{n-1}$, if $|r| < 1$, it converges to $\frac{a}{1-r}$

Divergence Test

For $\sum_{n=1}^{\infty} a_n$ to converge, nth term a_n must satisfy $a_n \to 0$ as $n \to \infty$. Converse is not true (if limit = 0, does not mean it converges)

Integral test

For $\sum_{n=1}^{\infty} a_n$, if a_n is all positive terms, and there exists a function f such that:

- f is continuous
- f is decreasing
- $f(n) = a_n$ for all integers $n \geq N$

Then $\sum_{n=1}^{\infty} a_n$ and $\int_N^{\infty} f(x)dx$ both converge or both diverge.

P-Series

For $\sum_{n=1}^{\infty} \frac{1}{n^p}$, converges if $p > 1$, diverges if $p \leq 1$

Remainder estimate

Same rules as integral test for $f(n)$.
$\int_{N+1}^{\infty} f(x)dx < R_N < \int_N^{\infty} f(x)dx$

Comparison test

If you have sum of a_n, find a similar sum of b_n (only works if a_n and $b_n \geq 0$). If $a_n \leq b_n$ and b_n's sum converges, a_n's sum converges. If $a_n \geq b_n$ and b_n's sum diverges, a_n's sum diverges.

Root test

For $\sum_{n=1}^{\infty} a_n$ such that $\lim_{n\to\infty} \sqrt[n]{|a_n|} = p$ for some p. If $0 \leq p < 1$, the sum converges absolutely. If $p > 1$ or $p = \infty$, it diverges. If $p = 1$, test provides no information.

Ratio Test

For $\sum_{n=1}^{\infty} a_n$ with positive terms, and $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L$, if $L < 1$, sum converges. If $L > 1$, sum diverges

Parametric/Polar

Derivative of parametric

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)}$

2nd Derivative of parametric

$\frac{d^2y}{dx^2} = \frac{(d/dt)(dy/dx)}{dx/dt}$

Area under parametric

$A = \int_a^b y(t)x'(t)dt$

Arc length of parametric

$s = \int_{t1}^{t2} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}$

Polar conversions

$r^2 = x^2 + y^2$
$tan(\theta) = \frac{y}{x}$
$x = r * cos(\theta)$
$y = r * sin(\theta)$

Polar symmetry

About polar axis: $(r, \theta) = (r, -\theta)$
About pole: $(r, \theta) = (r, \pi + \theta)$
About vertical line: $(\frac{\pi}{2})$: $(r, \theta) = (r, \pi - \theta)$