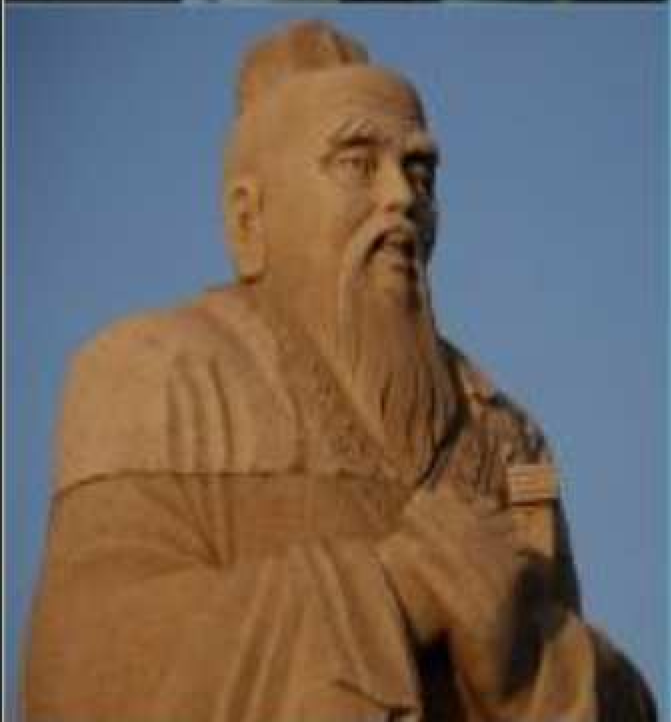

The only way to learn mathematics is to do mathematics.
— Paul Halmos —

MEASURES OF CENTRAL TENDANCY

I hear and I forget.
I see and I remember.
I do and I understand.
— Confucious —

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BASICS

- Types of Data
  - Non-Frequency Data
    - Raw Data
  - Frequency Data
    - Discrete Frequency Data
    - Continuous Frequency Data

BASICS

1. Non-Frequency Data or Raw Data:

$$x_1, x_2, x_3, \dots \dots \dots, x_n$$

Example:

1. $2, 5, 26, 13, 11, 15, 10$

2. The grades of a student on six examinations were 84, 91, 72, 68, 87, and 78.

3. A review of the first 30 pages of a book reveals the following printing mistakes :

0	1	3	3	2	5	6	0	1	0
4	1	1	0	2	3	2	5	0	4
2	3	2	2	3	3	4	6	1	4

BASICS

2. Frequency Data:

i. Discrete Frequency Data:

OR

Ungrouped Frequency Data:

Class	Frequency
x_1	f_1
x_2	f_2
x_3	f_3
.	.
.	.
.	.
x_n	f_n

Example:

1.

Size	Frequency
8	14
9	21
10	17
11	12
12	6

BASICS

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2. Frequency Data:

i. Discrete Frequency Data:

OR

Ungrouped Frequency Data:

Class	Frequency
x_1	f_1
x_2	f_2
x_3	f_3
.	.
.	.
.	.
x_n	f_n

Example:

2. A review of the first 30 pages of a book reveals the following printing mistakes :

0	1	3	3	2	5	6	0	1	0
4	1	1	0	2	3	2	5	0	4
2	3	2	2	3	3	4	6	1	4

Printing Mistake	Tally marks	Frequency (No. of Pages)
0	卌	5
1	卌	5
2	卌 I	6
3	卌 I	6
4	IIII	4
5	II	2
6	II	2
Total	–	30

BASICS

2. Frequency Data:

ii. Continuous Frequency Data:

OR

Grouped Frequency Data:

a. Inclusive Type Class Interval Grouped Frequency Data:

b. Exclusive Type Class Interval Grouped Frequency Data:

Height (in)	Frequency
60 - 62	5
63 - 65	18
66 - 68	42
69 - 71	27
72 - 74	8
	100

CI	Frequency
0 - 10	5
10 - 20	10
20 - 30	12
30 - 40	18
40 - 50	9
50 - 60	6

BASICS

Continuous Frequency Data:

Following are the weights in kgs. of 36 students of a class:

70	73	49	61	61	47	57	50	59
59	68	45	55	65	68	56	68	55
70	70	57	44	69	73	64	49	63
65	70	65	62	64	73	67	60	50

Solution 1 with Inclusive type Class Interval

Weight in kg (Class Interval)	Tally marks	No. of Students (Frequency)
44-48	III	3
49-53	IIII	4
54-58	~~IIII~~	5
59-63	~~IIII~~ II	7
64-68	~~IIII~~ IIII	9
69-73	~~IIII~~ III	8
Total	–	36

Solution 2 with Exclusive type Class Interval

Weight in kg (Class interval)	No. of Students (Frequency)
44 – 49	3
49 – 54	4
54 – 59	5
59 – 64	7
64 – 69	9
69 – 74	8
Total	36

BASICS

Continuous Frequency Data:

Some Important Terms:

i. Class Limits (CL)

ii. Class Boundaries (CB)

iii. Mid-Point or Class Mark (CM)

iv. Class Width or Size of a Class Interval

v. Cumulative Frequency

Weight in kg (Class Interval)	No. of Students (Frequency)
44-48	3
49-53	4
54-58	5
59-63	7
64-68	9
69-73	8
Total	36

Weight in kg (Class interval)	No. of Students (Frequency)
43.5 – 48.5	3
48.5 – 53.5	4

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BASICS

Continuous Frequency Data:

Conversion of Class Intervals From Inclusive Type to Exclusive Type:

Step 1: Compute D = LLCI – ULPCI

Step 2: Compute D/2

Step 3: Subtract D/2 from each LL
and add D/2 to each UL.

Resultant class intervals will be exclusive type.

Weight in kg (Class Interval)	No. of Students (Frequency)
44-48	3
49-53	4
54-58	5
59-63	7
64-68	9
69-73	8
Total	36

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INTRODUCTION

In many a case, like the distributions of height, weight, marks, profit, wage and so on, it has been noted that starting with rather low frequency, the class frequency gradually increases till it reaches its maximum somewhere near the central part of the distribution and after which the class frequency steadily falls to its minimum value towards the end.

Thus, central tendency may be defined as the tendency of a given set of observations to cluster around a single central or middle value and the single value that represents the given set of observations is described as a measure of central tendency or, location, or average.

INTRODUCTION

Hence, it is possible to condense a vast mass of data by a single representative value. The computation of a measure of central tendency plays a very important part in many a sphere.

Furthermore, the central tendency also facilitates us in providing a basis for comparison between different distribution. Following are the different measures of central tendency:

i. Arithmetic Mean (AM)

ii. Median (Me)

iii. Mode (Mo)

ARITHMETIC MEAN

- For Raw Data: $\bar{x} = \frac{\sum x_i}{n}$

- For Frequency Data: $\bar{x} = \frac{\sum f_i x_i}{N}$ *where* $N = \sum f_i$

  In case of grouped data x_i is mid point of each class interval.

- Coding Method: $\bar{x} = A + \frac{\sum f_i u_i}{N} \times C$

  where, $u_i = \frac{x_i - A}{C}$

ARITHMETIC MEAN

Example 1: The grades of a student on six examinations were 84, 91, 72, 68, 87, and 78. Find the arithmetic mean of the grades.

Solution:

Formula: $\bar{x} = \frac{\sum x_i}{n}$

$$\therefore AM\ of\ the\ grades = \frac{84 + 91 + 72 + 68 + 87 + 78}{6}$$

$$= \frac{480}{6} = 80$$

ARITHMETIC MEAN

Example 2: Find the arithmetic mean of the numbers

5, 3, 6, 5, 4, 5, 2, 8, 6, 5, 4, 8, 3, 4, 5, 4, 8, 2, 5, and 4.

Solution:

Formula: $\bar{x} = \frac{\sum f_i x_i}{N}$

$\therefore AM = \frac{96}{20} = 4.8$

x	f	fx
2	2	4
3	2	6
4	5	20
5	6	30
6	2	12
8	3	24
	20	96

ARITHMETIC MEAN

Example 3: Find the mean wage for the following:

x ↓ Daily Wage	f ↓ No. of Employees	fx
255	8	2040
265	10	2650
275	16	4400
285	15	4275
295	10	2950
305	8	2440
315	3	945
	70	19700
	↑ N	↑ $\sum fx$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{19700}{70}$$
$$= 281.43$$

ARITHMETIC MEAN

Solution by Coding Method:

Daily Wage	u = (x - 285)/10	No. of Employees	fu
255	-3	8	-24
265	-2	10	-20
275	-1	16	-16
285	0	15	0
295	1	10	10
305	2	8	16
315	3	3	9
		70	-25

$$\bar{x} = A + \frac{\sum f_i u_i}{N} \times C$$
$$= 285 + \frac{(-25)}{70} \times 10$$
$$= 285 - 3.57$$
$$= 281.43$$

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ARITHMETIC MEAN

Example 4: Find the mean height for the following:

		x	
		↓	
Height (in)	Frequency	Class Mark	fx
60 - 62	5	61	305
63 - 65	18	64	1152
66 - 68	42	67	2814
69 - 71	27	70	1890
72 - 74	8	73	584
	100		6745

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{6745}{100}$$
$$= 67.45 \text{ in}$$

ARITHMETIC MEAN

Example 5: Find the mean weight for the following:

Weight	No. of Children
0 – 20	3
20 – 40	10
40 – 60	15
60 – 80	12
80 – 100	8
100 – 120	12

PROPERTIES OF ARITHMETIC MEAN

i. If all the observations assumed by a variable are constants, say k, then the AM is also k.

Example: If the height of every student in a group of 10 students is 170 cm, then the mean height is, of course, 170 cm.

ii. The algebraic sum of deviations of a set of observations from their arithmetic-mean is zero.

Example: If a variable "x" assumes five observations, say 5, 7, 12, 15, 26, then $\bar{x} = 13$.

The deviations of the observations from arithmetic mean $(x - \bar{x})$ are -8, -6, -1, 2, 13.

Now, $\sum(x - \bar{x}) = (-8) + (-6) + (-1) + 2 + 13 = 0$.

PROPERTIES OF ARITHMETIC MEAN

iii. Arithmetic-mean is affected due to a change of origin and/or scale which implies that if the original variable "x" is changed to another variable "y" effecting a change of origin, say "a" and scale, say "b", of "x".

i.e. if $y = a + bx$

Then we have,

$$Arithmetic - mean\ of\ y = \bar{y} = a + b\bar{x}$$

COMBINED ARITHMETIC MEAN

iv. If there are two groups containing n_1 and n_2 observations $\overline{x_1}$ and $\overline{x_2}$ are the respective arithmetic means, then the combined arithmetic-mean is given by

$$\bar{x} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2}$$

This property could be extended to more than two groups.

Example: Four groups of students, consisting of 15, 20, 10, and 18 individuals, reported mean weights of 162, 148, 153, and 140 pounds (lb), respectively. Find the mean weight of all the students.

$$\bar{x} = \frac{15 \times 162 + 20 \times 148 + 10 \times 153 + 18 \times 140}{15 + 20 + 10 + 18} = 150$$

WEIGHTED ARITHMETIC MEAN

If $x_1, x_2, x_3, \ldots \ldots \ldots, x_n$ have weights $w_1, w_2, w_3, \ldots \ldots \ldots, w_n$ then $weighted\ average = \bar{x} = \frac{\sum wx}{\sum w}$

ARITHMETIC MEAN

Q1. A student received grades of 85, 76, 93, 82, and 96 in five subjects. Determine the arithmetic mean of the grades.

Q.2 The reaction times of an individual to certain stimuli were measured by a psychologist to be 0.53, 0.46, 0.50, 0.49, 0.52, 0.53, 0.44, and 0.55 seconds respectively. Determine the mean reaction time of the individual to the stimuli.

Q.3 A student's grades in the laboratory, lecture, and recitation parts of a physics course were 71, 78, and 89, respectively. If the weights accorded these grades are 2, 4, and 5, respectively, what is an appropriate average grade?

ARITHMETIC MEAN

Q.4 What is arithmetic mean of $11.8, 12.6, 10.9, 8.8$ *and* 9.6?

Q.5 If there are two groups containing 60 and 40 members and having mean values 30 and 25 respectively, then what is the combined arithmetic mean?

Q.6 If the two variables are given by $2y = 3x - 10$ and mean of x is 6, then what is mean of y?

Q.7 If a group of 5 students have same height 153 cm then what is the mean height?

Q.8 What is the sum of deviations of $10, 18, 12, 15$ *and* 9 from arithmetic mean?

ARITHMETIC MEAN

Q.9 Find AM for the following:

Maximum Load (short tons)	Number of Cables
9.3–9.7	2
9.8–10.2	5
10.3–10.7	12
10.8–11.2	17
11.3–11.7	14
11.8–12.2	6
12.3–12.7	3
12.8–13.2	1
	Total 60

Weight (lb)	Frequency
118–126	3
127–135	5
136–144	9
145–153	12
154–162	5
163–171	4
172–180	2

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ONLINE MATH EDUCATION

MEDIAN

- The median of a set of numbers arranged in order of magnitude (i.e., in an array) is either the middle value or the arithmetic mean of the two middle values.

  Let $x_1, x_2, x_3, \dots\dots\dots, x_n$ be the data arranged in order.

  - If n is odd, $Median = \left(\frac{n+1}{2}\right)^{th}$ $observation.$
  - If n is even, $Median = AM\ of\ \frac{n}{2}^{th}\ and \left(\frac{n}{2}+1\right)^{th}\ observation$

MEDIAN

Example:

1. If there are 13 data points in order then median = 7^{th} Observation.
2. If there are 18 data points in order then median = AM of 9^{th} Observation and 10^{th} Observation.

$$= \frac{9^{th} Observation + 10^{th} Observation}{2}$$

MEDIAN

Example 1: Find median of $5, 4, 8, 3, 7, 2, 9$.

Solution: Data in ascending order: $2, 3, 4, 5, 7, 8, 9$

Here we have 7 values (odd),

therefore $Median = 4^{th}\ observation = 5$.

Example 2: Find median of $18.3, 20.6, 19.3, 22.4, 20.2, 18.8, 19.7, 20.0$.

Solution:

Data in descending order: $22.4, 20.6, 20.2, 20.0, 19.7, 19.3, 18.8, 18.3$

Here we have 8 values (even),

therefore $Median = \frac{4^{th}\ obs + 5^{th}\ obs}{2} = \frac{20.0+19.7}{2} = 19.85$

FIND MEDIAN

Size	Frequency	cf	
8	14	14	1-14
9	21	35	15-35
10	17	52	36-52
11	12	64	53-64
12	6	70	65-70

MEDIAN

- For grouped data, the median, obtained by interpolation, is given by

$$Median = l_1 + \left[\frac{(l_2 - l_1)\left(\frac{N}{2} - pcf\right)}{f} \right]$$

where, $l_1 = LL\ of\ median\ class$

$l_2 = UL\ of\ median\ class$

$f = frequency\ of\ median\ class$

$pcf = cf\ of\ pre-median\ class$

$N = total\ number\ of\ observations$

Note: Class intervals has to be exclusive type.

MEDIAN

Example 1: Find median rainfall for the following:

Rainfall in cms	No. of Cities	CF
0 - 10	10	10
10 - 20	15	25
20 - 30	20	45
30 - 40	10	55
40 - 50	5	60
	60	

$$M = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - pcf\right)}{f}$$

$$= 20 + \frac{10 \times 5}{20}$$

$$= 22.5\ cms$$

MEDIAN

Example 2: Find median height for the following:

Height (in)	Frequency	cf
59.5 - 62.5	5	5
62.5 - 65.5	18	23
65.5 - 68.5	42	65
68.5 - 71.5	27	92
71.5 - 74.5	8	100
	100	

$$M = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - pcf\right)}{f}$$

$$= 65.5 + \frac{3\times 27}{42}$$

$$= 67.43\ in$$

MEDIAN

Example 3: Find median for the following

Maximum Load (short tons)	Number of Cables
9.3–9.7	2
9.8–10.2	5
10.3–10.7	12
10.8–11.2	17
11.3–11.7	14
11.8–12.2	6
12.3–12.7	3
12.8–13.2	1
	Total 60

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MEDIAN

Example 4: Find median for the following

Weight (lb)	Frequency
118–126	3
127–135	5
136–144	9
145–153	12
154–162	5
163–171	4
172–180	2

MEDIAN

Example 5: If $2y - 3x = 6$ and median of x = 6 then what is median of y?

QUARTILE

Example: Find first, second and third quartile for the following:

$$5, 15, 12, 7, 30, 25, 22, 14, 36, 53, 42$$

Solution:

Data in ascending order:

$$5, 7, 12, 14, 15, 22, 25, 30, 36, 42, 53$$

$$5, 7, 12, 14, 15, 22, 25, 30, 36, 42, 53$$

$$Q_1 = 12$$

$$Q_2 = 22 = M$$

$$Q_3 = 36$$

QUARTILE

- For grouped data,
  - $Q_1 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{4} - pcf\right)}{f}$
  - $Q_2 = l_1 + \frac{(l_2 - l_1)\left(\frac{2N}{4} - pcf\right)}{f}$
  - $Q_3 = l_1 + \frac{(l_2 - l_1)\left(\frac{3N}{4} - pcf\right)}{f}$

QUARTILE

Example 1: Find first and third quartile for the following

CI	Frequency	cf
0 - 10	5	5
10 - 20	10	15
20 - 30	12	27
30 - 40	18	45
40 - 50	9	54
50 - 60	6	60
	60	

QUARTILE

Example 2: Find first and third quartile for the following

Rainfall in cms	No. of Cities	CF
0 - 10	10	10
10 - 20	15	25
20 - 30	20	45
30 - 40	10	55
40 - 50	5	60
	60	

DECILE AND PERCENTILE

- For grouped data,
  - $D_k = l_1 + \frac{(l_2 - l_1)\left(\frac{kN}{10} - pcf\right)}{f}, \qquad k = 1, 2, 3, \dots\dots\dots, 9$
  - $P_k = l_1 + \frac{(l_2 - l_1)\left(\frac{kN}{100} - pcf\right)}{f}, \qquad k = 1, 2, 3, \dots\dots\dots, 99$

DECILE AND PERCENTILE

Example: Find D_4 and P_{65} for the following

Rainfall in cms	No. of Cities	CF
0 - 10	10	10
10 - 20	15	25
20 - 30	20	45
30 - 40	10	55
40 - 50	5	60
	60	

QUARTILES, DECILE AND PERCENTILE

Example: Find Q_3, D_4 and P_{65} for the following:

$$20, 15, 18, 5, 10, 17, 21, 19, 25, 28$$

Solution: Data in ascending order:

$$5, 10, 15, 17, 18, 19, 20, 21, 25, 28$$

$$Q_3 = \left(\frac{3(n+1)}{4}\right)^{th} term = 8.25^{th} term$$

$$\therefore Q_3 = 8^{th} term + 0.25 \times \left(9^{th} term - 8^{th} term\right)$$

$$\therefore Q_3 = 21 + 0.25 \times 4 = 22$$

$$D_4 = \left(\frac{4(n+1)}{10}\right)^{th} term = 4.4^{th} term$$

$$\therefore D_4 = 4^{th} term + 0.4 \times \left(5^{th} term - 4^{th} term\right) = 17.4$$

$$P_{65} = \left(\frac{65(n+1)}{100}\right)^{th} term = 7.15^{th} term$$

$$\therefore P_{65} = 20.15$$

MODE

The mode of a set of numbers is that value which occurs with the greatest frequency; that is, it is the most common value.

The mode may not exist, and even if it does exist it may not be unique. A distribution having only one mode is called unimodal.

Example:

1. 2, 2, 5, 7, 9, 9, 9, 10, 10, 11, 12, 18
   is unimodal with mode = 9.
2. 2, 3, 4, 4, 4, 5, 5, 7, 7, 7, 9
   is bimodal with two modes, 4 and 7.
3. 3, 5, 8, 10, 12, 15, 16
   has no mode.
4. 2, 2, 2, 3, 3, 3
   has no mode.

MODE

Example: Find modal size.

Size	Frequency
8	14
9	21
10	17
11	12
12	6

MODE

For grouped data:

$$Mode = Z = l_1 + \frac{(l_2 - l_1)(f_1 - f_0)}{(2f_1 - f_0 - f_2)}$$

$$where, f_1 = frequency\ of\ modal\ class$$

$$f_0 = frequency\ of\ pre - modal\ class$$

$$f_2 = frequency\ of\ post - modal\ class$$

MODE

Example 1: Find modal rainfall.

Rainfall in cms	No. of Cities
0 - 10	10
10 - 20	15
20 - 30	20
30 - 40	10
40 - 50	5
	60

MODE

Example 2: Find modal weight.

Weight (lb)	Frequency
118–126	3
127–135	5
136–144	9
145–153	12
154–162	5
163–171	4
172–180	2

MODE

Example 3: If $3x - y = 6$ and mode of x = 9 then what is mode of y?

RELATION between Mean, Median and Mode

For moderately skewed distribution, we may consider the following empirical relationship between mean, median and mode,

$$Mean - Mode = 3(Mean - Median)$$

Example: For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean and median marks were found to be 55.60 and 52.40. What is the modal marks?

Solution: WKT, $Mean - Mode = 3(Mean - Median)$

$$\Longrightarrow Mode = 3\,Median\ - 2\,Mean$$

$$= 3\,(52.40) - 2\,(55.60) = 46$$

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