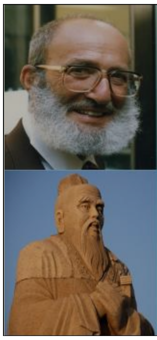

The only way to learn mathematics is to do mathematics.
— Paul Halmos —

MOMENTS, SKEWNESS and KURTOSIS

I hear and I forget.
I see and I remember.
I do and I understand.
— Confucious —

MOMENTS

For Raw Data:

r^{th} moments about A is defined as,

$$m'_r = \overline{(X-A)^r} = \frac{\sum (X-A)^r}{N}$$

MOMENTS

For Raw Data:

r^{th} moments about $A = 0$ is defined as,

$$m'_r = \overline{X^r} = \frac{\sum X^r}{N}$$

First four moments about $A = 0$,

$$m'_1 = \bar{X} = \frac{\sum X}{N}$$
$$m'_2 = \overline{X^2} = \frac{\sum X^2}{N}$$
$$m'_3 = \overline{X^3} = \frac{\sum X^3}{N}$$
$$m'_4 = \overline{X^4} = \frac{\sum X^4}{N}$$

MOMENTS

For Raw Data:

r^{th} moments about $A = \bar{X}$ is defined as,

$$m_r = \overline{(X-\bar{X})^r} = \frac{\sum (X-\bar{X})^r}{N}$$

First four moments about $A = \bar{X}$,

$$m_1 = \overline{(X-\bar{X})} = \frac{\sum (X-\bar{X})}{N}$$
$$m_2 = \overline{(X-\bar{X})^2} = \frac{\sum (X-\bar{X})^2}{N}$$
$$m_3 = \overline{(X-\bar{X})^3} = \frac{\sum (X-\bar{X})^3}{N}$$
$$m_4 = \overline{(X-\bar{X})^4} = \frac{\sum (X-\bar{X})^4}{N}$$

MOMENTS

For Frequency Data:

r^{th} moments about A is defined as,

$$m'_r = \overline{(X-A)^r} = \frac{\sum f(X-A)^r}{N}$$

MOMENTS

For Frequency Data:

r^{th} moments about $A = 0$ is defined as,

$$m'_r = \overline{X^r} = \frac{\sum fX^r}{N}$$

First four moments about $A = 0$,

$$m'_1 = \bar{X} = \frac{\sum fX}{N}$$
$$m'_2 = \overline{X^2} = \frac{\sum fX^2}{N}$$
$$m'_3 = \overline{X^3} = \frac{\sum fX^3}{N}$$
$$m'_4 = \overline{X^4} = \frac{\sum fX^4}{N}$$

MOMENTS

For Frequency Data:

r^{th} moments about $A = \bar{X}$ is defined as,

$$m_r = \overline{(X-\bar{X})^r} = \frac{\sum f(X-\bar{X})^r}{N}$$

First four moments about $A = \bar{X}$,

$$m_1 = \overline{(X-\bar{X})} = \frac{\sum f(X-\bar{X})}{N}$$

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$$m_3 = \overline{(X-\bar{X})^3} = \frac{\sum f(X-\bar{X})^3}{N}$$

$$m_4 = \overline{(X-\bar{X})^4} = \frac{\sum f(X-\bar{X})^4}{N}$$

MOMENTS

Note:

1. First moment about $A = 0$, $m_1' = \bar{X} = AM$.
2. First moment about $A = \bar{X}$, $m_1 = \overline{(X-\bar{X})} = 0$.
3. Second moment about $A = \bar{X}$, $m_2 = \overline{(X-\bar{X})^2} = Variance$.

RELATION BETWEEN MOMENTS

Let m_r denotes moments about the mean and m_r' denotes moments about an arbitrary origin.

$$m_2 = m_2' - m_1'^2$$

$$m_3 = m_3' - 3m_1'm_2' + 2m_1'^3$$

$$m_4 = m_4' - 4m_1'm_3' + 6m_1'^2m_2' - 3m_1'^4$$

Note that $m_1' = \bar{X} - A$.

MOMENTS

Find the first four moments for the set 4, 7, 5, 9, 8, 3, 6.

X	X^2	X^3	X^4
4	16	64	256
7	49	343	2401
5	25	125	625
9	81	729	6561
8	64	512	4096
3	9	27	81
6	36	216	1296
42	280	2016	15316

$$First\ Moment = m_1' = \frac{\sum X}{N} = \frac{42}{7} = 6$$

$$Second\ Moment = m_2' = \frac{\sum X^2}{N} = \frac{280}{7} = 40$$

$$Third\ Moment = m_3' = \frac{\sum X^3}{N} = \frac{2016}{7} = 288$$

$$Fourth\ Moment = m_4' = \frac{\sum X^4}{N} = \frac{15316}{7} = 2188$$

MOMENTS

Find the first four moments about mean for the set 4, 7, 5, 9, 8, 3, 6.

X	$(X-\bar{X})$	$(X-\bar{X})^2$	$(X-\bar{X})^3$	$(X-\bar{X})^4$
4	-2	4	-8	16
7	1	1	1	1
5	-1	1	-1	1
9	3	9	27	81
8	2	4	8	16
3	-3	9	-27	81
6	0	0	0	0
42	0	28	0	196

$$\bar{X} = 6$$

$$First\ Moment = m_1 = \frac{\sum(X-\bar{X})}{N} = 0$$

$$Second\ Moment = m_2 = \frac{\sum(X-\bar{X})^2}{N} = \frac{28}{7} = 4$$

$$Third\ Moment = m_3 = \frac{\sum(X-\bar{X})^3}{N} = 0$$

$$Fourth\ Moment = m_4 = \frac{\sum(X-\bar{X})^4}{N} = \frac{196}{7} = 28$$

MOMENTS

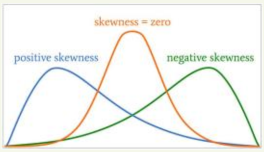
Find the first four moments about 7 for the set 4, 7, 5, 9, 8, 3, 6.

X	X - 7	$(X - 7)^2$	$(X - 7)^3$	$(X - 7)^4$
4	-3	9	-27	81
7	0	0	0	0
5	-2	4	-8	16
9	2	4	8	16
8	1	1	1	1
3	-4	16	-64	256
6	-1	1	-1	1
	-7	35	-91	371

SKEWNESS

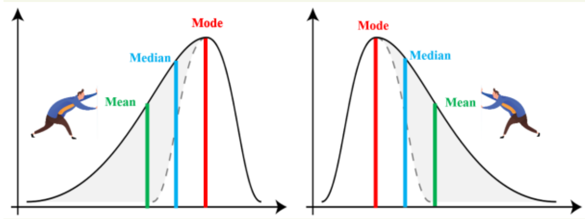
Skewness is the degree of asymmetry, or departure from asymmetry, of a distribution.

If the frequency curve of a distribution has a longer tail to the right of the central maximum than to the left, the distribution is said to be skewed to the right, or to have positive skewness. If the reverse is true, it is said to be skewed to the left, or to have negative skewness.



SKEWNESS

For skewed distributions, the mean tends to lie on the same side of the mode as the longer tail.



SKEWNESS

Pearson's First and Second Coefficient of Skewness:

$$Skewness = \frac{mean - mode}{Standard\ Deviation}$$

$$Skewness = \frac{3(mean - median)}{Standard\ Deviation}$$

Other measures of Skewness,

$$Quartile\ Coefficient\ of\ Skewness = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$$

$$10 - 90\ Percentile\ Coefficient\ of\ Skewness = \frac{P_{90} - 2P_{50} + P_{10}}{P_{90} - P_{10}}$$

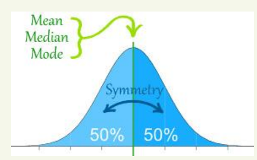
$$Moment\ Coefficient\ of\ Skewness = a_3 = \frac{m_3}{\left(\sqrt{m_2}\right)^3}$$

SKEWNESS

Another measure of skewness is given by

$$b_1 = \beta_1 = a_3^2 = \frac{m_3^2}{m_2^3}$$

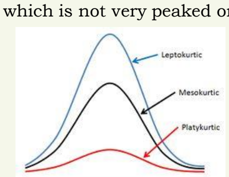
For perfectly symmetric curves, such as normal curve, a_3 and β_1 are zero.



KURTOSIS

Kurtosis is the degree of peakedness of a distribution, usually taken relative to a normal distribution.

A distribution having a relatively high peak is called leptokurtic, while one which is flat-topped is called platykurtic. A normal distribution, which is not very peaked or very flat-topped, is called mesokurtic.



KURTOSIS

Measures of Kurtosis,

$$Moment\ Coefficient\ of\ Kurtosis = a_4 = \beta_2 = \frac{m_4}{m_2^2}$$

For the normal distribution, $Moment\ Coefficient\ of\ Kurtosis = 3$.

So $Moment\ Coefficient\ of\ Kurtosis > 3$, for leptokurtic distributions and $Moment\ Coefficient\ of\ Kurtosis < 3$, for platykurtic distributions.

Another measure of kurtosis is given by,

$$\kappa = \frac{Semi - Interquartile\ Range}{10 - 90\ Percentile\ Range} = \frac{(Q_3 - Q_1)/2}{P_{90} - P_{10}}$$

For the normal distribution, $\kappa = 0.263$.

So $\kappa > 0.263$, for leptokurtic distributions and $\kappa < 0.263$, for platykurtic distributions.

KURTOSIS

The Kurtosis is sometimes defined by

$$\gamma_2 = \beta_2 - 3$$

Which is positive for leptokurtic distribution, negative for platykurtic distribution and zero for normal distribution.