The only way to learn mathematics is to do mathematics.
— Paul Halmos —

ELEMENTARY PROBABILITY THEORY

I hear and I forget.
I see and I remember.
I do and I understand.
— Confucious —

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BASICS

- Experiment
- Sample Space
- Event
  - Simple or Elementary Event
  - Composite or Compound Event
- Event Space
- Probability of an Event
- Mutually Exclusive Events
- Mutually Exhaustive Events
- Equally Likely Events

BASICS

Note:

- The probability of an event lies between 0 and 1, both inclusive.
  i.e. $0 \leq P(A) \leq 1$.
- Non-occurrence of event A is denoted by A' or A^c or $\bar{A}$ and it is known as complimentary event of A.
- The event A along with its complimentary A' forms a set of mutually exclusive and exhaustive events.
  i.e. $P(A) + P(A') = 1 \quad \Rightarrow P(A) = 1 - P(A')$

EXAMPLES

Example 1: Determine the probability p, or an estimate of it, for each of the following events:

i. An odd number appears in a single toss of a fair die.
ii. At least one head appears in two tosses of a fair coin.
iii. At least one head appears in three tosses of a fair coin.
iv. An ace, 10 of diamonds, or 2 of spades appears in drawing a single card from a well-shuffled ordinary deck of 52 cards.
v. The sum 7 appears in a single toss of a pair of fair dice.
vi. A tail appears in the next toss of a coin if out of 100 tosses 56 were heads.

EXAMPLES

Example 2: A coin is tossed three times. What is probability of getting

a) 2 heads
b) At least 2 heads
c) At most 1 head

EXAMPLES

Example 3: A dice is rolled twice. What is a probability of getting a difference of 2 points?

EXAMPLES

Example 4: Two dice are thrown simultaneously. Find the probability that the sum of points on the two dice would be 7 or more.

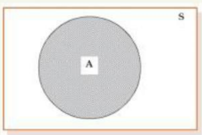
EXAMPLES

Example 5: A committee of 4 members is to be formed from a group comprising 6 gentlemen and 4 ladies. What is the probability that the committee would comprise:

i. 2 ladies

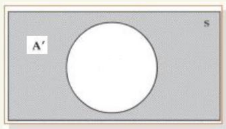
ii. at least 1 ladies.

SET-THEORETIC APPROACH



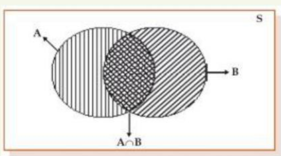
Example: A fair die is rolled. Let A be an event that even number appears.
Then $S = \{1,2,3,4,5,6\}$ *and* $A = \{2,4,6\}$.

SET-THEORETIC APPROACH



Example: A fair die is rolled. Let A be an event that even number appears.
Then $S = \{1,2,3,4,5,6\}, A = \{2,4,6\}$ *and* $A' = \{1,3,5\}$.

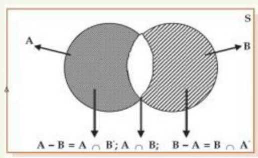
SET-THEORETIC APPROACH



Example: A fair die is rolled. Let A be an event that even number appears and B be an event that prime number appears.
Then $S = \{1,2,3,4,5,6\}, A = \{2,4,6\}$ *and* $B = \{2,3,5\}$

$$A \text{ or } B = A + B = A \cup B = \{2,3,4,5,6\}$$
$$A \text{ and } B = AB = A \cap B = \{2\}$$

SET-THEORETIC APPROACH



Example: A fair die is rolled. Let A be an event that even number appears and B be an event that prime number appears.
Then $S = \{1,2,3,4,5,6\}, A = \{2,4,6\}$ *and* $B = \{2,3,5\}$

$$A \text{ but not } B = A - B = A/B = \{4,6\}$$
$$B \text{ but not } A = B - A = B/A = \{3,5\}$$

ADDITION THEOREMS

Theorem 1: For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B

i.e. $P(A\,or\,B) = P(A+B) = P(A \cup B) = P(A) + P(B)$.

Example 1: A single card is drawn from an ordinary deck of cards. What is the probability that the selected card is an ace or a king?

Example 2: A pair of dice rolled. What is the probability that the sum of outcomes is 7 or 11?

Example 3: A number is selected from the first 25 natural numbers. What is the probability that it would be divisible by 4 or 7?

ADDITION THEOREMS

Theorem 2: For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B

i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 1: A single card is drawn from an ordinary deck of cards. What is the probability that the selected card is an ace or a spade?

Example 2: A fair die is rolled. What is the probability that the outcome is odd or prime?

Example 3: A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 5 or 9?

ADDITION THEOREMS

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ADDITION THEOREMS

Example 4: The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?

Example 5: If $P(A-B) = \frac{1}{5}, P(A) = \frac{1}{3}\ and\ P(B) = \frac{1}{2}$, what is the probability that out of the two events A and B, only B would occur?

Example 6: In a survey of a group of people it was found that 25% were smokers and drinkers, 10% were smokers but not drinkers, and 35% were drinkers but not smokers. What percent in the survey were either smokers or drinkers or both?

ADDITION THEOREMS

Theorem 3: For any three events A, B and C, the probability that at least one of the events occurs is given by

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Example : A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above. Find the probability that the randomly selected employee

i. is proficient in at least one language.

ii. is not proficient in any of these three languages.

CONDITIONAL PROBABILITY

If A and B are two events, and the occurrence of one event, say B, is influenced by the occurrence of another event A, then the two events A and B are known as dependent events.

The probability that B occurs given that A has occurred is denoted by $P(B|A)$, and is called the conditional probability of B given that A has occurred. This is given by

$$P(A \cap B) = P(A)P(B|A)$$

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}, \quad \text{provided } P(A) \neq 0.$$

Example 1: Two cards are drawn from a well-shuffled ordinary deck of 52 cards without replacement. Find the probability that they are both aces.

Example 2: A purse contains 2 silver coins and 4 copper coins, and a second purse contains 4 silver coins and 3 copper coins. If a coin is selected at random from one of the two purses, what is the probability that it is a silver coin?

INDEPENDENT EVENTS

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If the occurrence of the second event B is not influenced by the occurrence of the first event A, then B is known to be independent of A.

It also follows that in this case, A is also independent of B and A and B are known as mutually independent.

In this case, we get $P(B|A) = P(B)$ *and also* $P(A|B) = P(A)$

$$\Rightarrow P(A \cap B) = P(A)P(B)$$

Also note that if the two events A and B are independent then the following pairs of events are also independent:

i. A *and* B'

ii. A' *and* B

iii. A' *and* B'

INDEPENDENT EVENTS

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Example 1: Two cards are drawn from a well-shuffled ordinary deck of 52 cards with replacement. Find the probability that they are both aces.

Example 2: The odds in favour of A winning a game of chess against B are 3 : 2. If three games are to be played, what are the odds

i. in favour of A's winning at least two games out of the three and

ii. against A losing the first two games to B?

THEOREMS OF COMPOUND PROBABILITY

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Theorem 4: For an two events A and B, the probability that A and B occur simultaneously is given by $P(A \cap B) = P(A)P(B|A)$, provided $P(A) > 0$.

Theorem 5: For any three event A, B and C, the probability that they occur jointly is given by $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$, provided $P(A \cap B) > 0$.

In case events A, B and C are mutually independent,

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

THEOREMS OF COMPOUND PROBABILITY

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Example 1: Three balls are drawn successively from a box containing 6 red balls, 4 white balls, and 5 blue balls. Find the probability that they are drawn in the order red, white, and blue if each ball is

i. replaced and

ii. not replaced

THEOREMS OF COMPOUND PROBABILITY

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Example 2: The probability of A hitting a target is 2/3 and that of B is 4/5. they both fire at the target. What is the probability that target would be hit? What is the probability that only one of them will hit the target?

THEOREMS OF COMPOUND PROBABILITY

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Example 3: A fair die is rolled twice. Find the probability of getting a 4, 5 or 6 on the first toss and a 1, 2, 3 or 4 on the second toss.

THEOREMS OF COMPOUND PROBABILITY

Example 4: In connection with a random experiment, it is found that

$$P(A) = \frac{2}{3}, P(B) = \frac{3}{5} \text{ and } P(A \cup B) = \frac{5}{6}.$$

Evaluate (i) $P(A|B)$ (ii) $P(B|A)$ (iii) $P(A'|B)$ (iv) $P(A|B')$ (v) $P(A'|B')$.

BAYES' THEOREM

THEOREM: Let $E_1, E_2, E_3, \ldots\ldots\ldots, E_n$ be mutually exclusive and exhaustive events. Let A be any event. Then

$$P[E_k/A] = \frac{P[E_k]P[A/E_k]}{\sum_{i=1}^{n} P[A/E_i]}$$

BAYES' THEOREM

Example 1: Three identical urns contains red and blue marbles. The first urn contains 4 red and 5 blue marbles, the second urn contains 2 red and 4 blue marbles and the third urn contains 5 red and 3 blue marbles. An urn is chosen randomly and a marble is drawn from it. If the marble drawn is red, what will be the probability that the second urn is chosen?

BAYES' THEOREM

Example 2: In a city, 54% of the adults are males. Also 20% of males are smokers and 12% of females are smokers. One adult is randomly selected from the population. If the selected person is a smoker then what is the probability that he is male?

BAYES' THEOREM

Example 3: In a survey, it was found that out of 100 people 4 has disease. If test has 2% false negative rate and 5% false positive rate. Find

i) the probability that a person who has disease, tests positive
ii) the probability that a person who has disease, tests negative
iii) the probability that a person who is healthy, tests positive
iv) the probability that a person who is healthy, tests negative
v) the probability that a person tests positive actually has disease
vi) the probability that a person tests negative is actually healthy