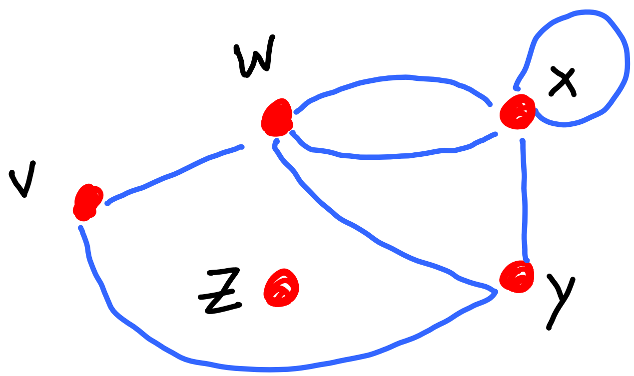
The **degree** of a vertex v in G, $d_G(v)$, is the number of edges of G that are incident with v (loops count twice)



G is **regular** if every vertex has **same degree** and **k-regular** if $d_G(v) = k$ for all v in G

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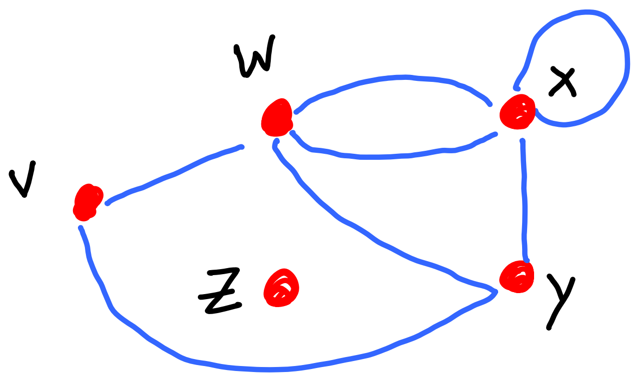


$d(v) = 2$

$d(w) = 4$

$d(x) = 5$

$d(y) = 3$

$d(z) = 0$

G is **regular** if every vertex has **same degree**

and **k-regular** if $d_G(v) = k$ for all v in G

Which are regular: K_n, $K_{n,n}$, Q_k, C_n, P_n, Petersen?

Proposition 1.3.3. For any graph, G,

$$\sum_{v \in V(G)} d_G(v) = 2e(G).$$

← Degree -sum formula

Proof: Each edge contributes 2 to the sum.

Corollary 1.3.5. For any graph G, the number of vertices of odd degree is even.

Ex. Apply the degree-sum formula to find the number of edges of: K_n, $K_{r,s}$, Q_k, P_n, C_n, Petersen

Ex. Can a graph have degree sequence $(8, 7, 6, 5, 4, 3)$?

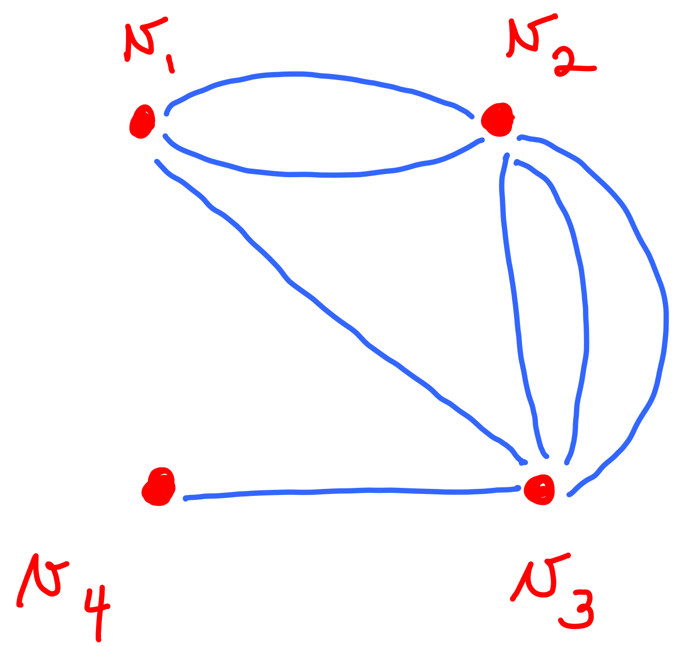
Ex. Show that in any simple graph with at least 2 vertices there must be (at least) 2 vertices of same degree.

Adjacency Matrix representation of loopless graph G with $V(G) = \{v_1, v_2, \ldots, v_n\}$:

$A(G)$: $\underline{n \times n \text{ matrix}}$, entries a_{ij} where

a_{ij} = # of edges joining v_i and v_j

$$A(G) = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 3 & 0 \\ 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Ex

How many simple undirected graphs are there with n vertices? i.e. with vertex set $\{v_1, v_2, \ldots, v_n\}$

(Think about how many ways there are to fill in the adjacency matrix.)

Simple graphs G and H are isomorphic

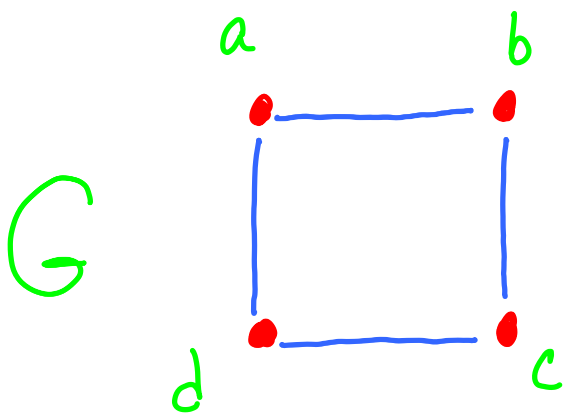
$$\boxed{G \cong H}$$

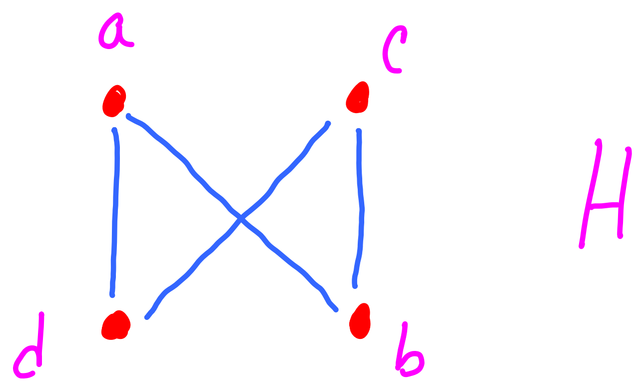
if there is a bijection f

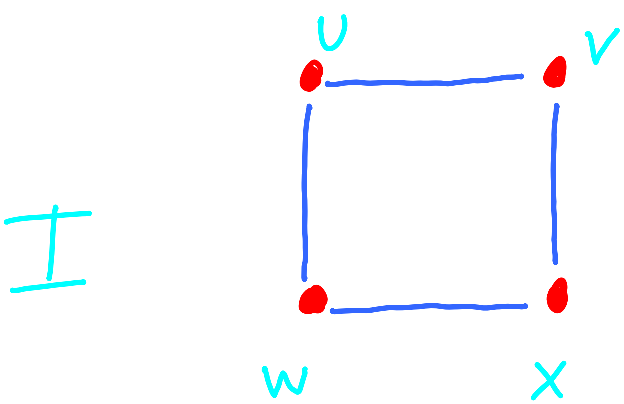
$$\boxed{f : V(G) \to V(H)}$$

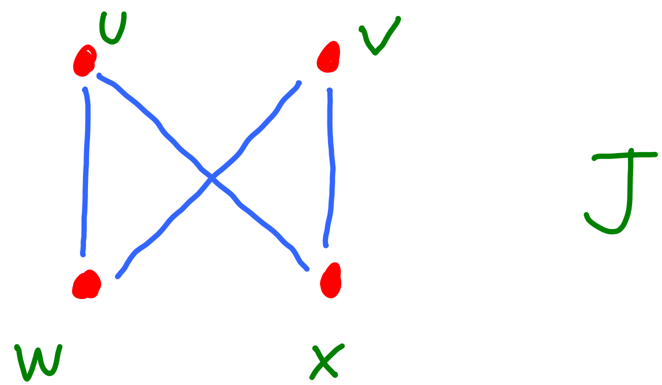
such that

$$\boxed{uv \in E(G)} \text{ iff } \boxed{f(u)f(v) \in E(H)}$$









Which are identical?

Which are isomorphic?

ex Are all graphs with the same degree sequence isomorphic?

Complexity of the problem:

Given G and H,

$G \cong H$?

(no poly. time alg. known, but not known to be NP-complete)

NEW RESULT HERE:

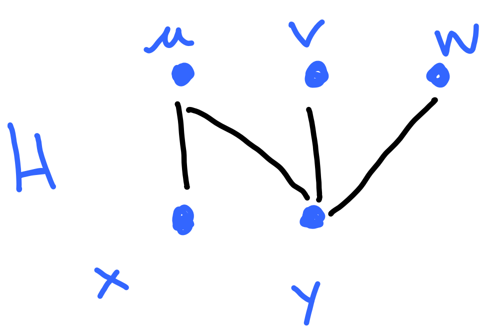
Graph H is a subgraph of graph G if

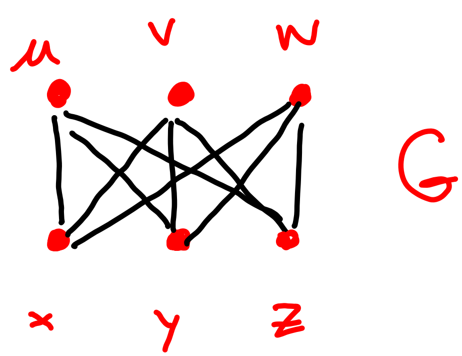
$$V(H) \subseteq V(G)$$

$$E(H) \subseteq E(G)$$

and the assignment of endpoints to edges in H is the same as in G

(denoted $H \subseteq G$)





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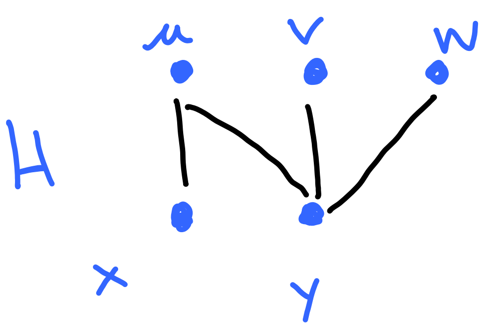
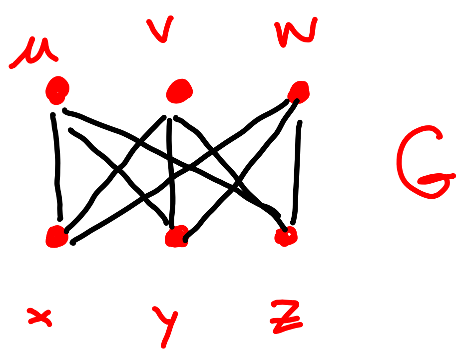




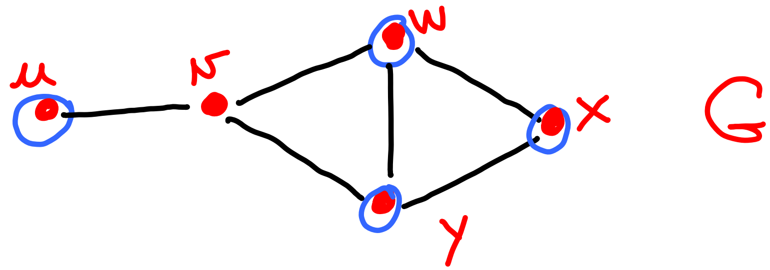
proper if $H \neq G$

spanning if $V(H) = V(G)$

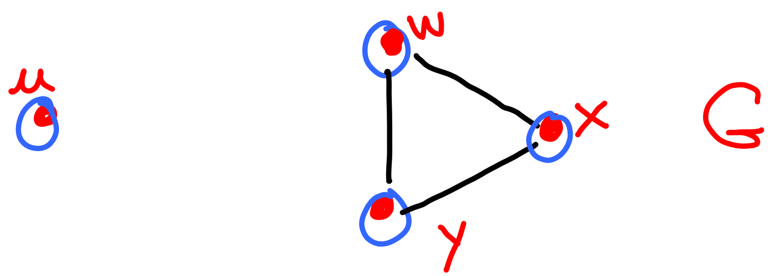
Induced Subgraphs

Given $S \subseteq V(G)$

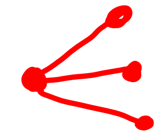
$G[S]$: subgraph of G induced by S is the subgraph of G with vertex set S, containing every edge of G with endpoints in S.



$G[\{u, w, x, y\}]$



Ex A graph is claw-free if it has no induced subgraph isomorphic to $K_{1,3}$.


the "claw"

Which of these are claw-free?