

Homework 2
Graph Theory CSC/MA/OR 565
Due Tuesday, January 26, 2016

Instructions

- For this assignment, students will be divided into teams by the instructor. Each team will work together, via the team's message board forum, to evolve a "perfect" solution to each of the problems.
- When your team has arrived at the perfect solution to a problem, typeset it and leave it (or a link to it) as the last entry in the "topic" for that problem in your team's message board forum.
- Find your team at the bottom of this file. Make sure you are on exactly one team and that you have the required access. (If not, post to "General Questions" message board and someone will help.)
- Discussion within teams is *encouraged*, with the goal of evolving solutions that are (i) mathematically correct, (ii) clear and concise, and (iii) well-presented. Each individual homework submission will be graded by the TA and evaluated according to these three criteria.

Academic Integrity Guidelines (this is very serious):

- Be honest and ethical - *do* cite all sources used, if any, but *do not plagiarize*.
- Since this is to be a group project, every member is expected to contribute. If you do not contribute, inform the instructor and do not take credit for the solutions.

Message Board Guidelines:

- A forum will be set up on the message board for each team.
- No team will have access to another team's message board.
- Teams should be viewed as "in competition", so don't share secrets.
- You will be able to edit your posts.
- Your number of posts is not restricted.
- Be polite and considerate.
- The TA will monitor the forums.
- Contact the TA if there are any concerns or questions about the use of the message boards.

Goals of this assignment:

- To master fundamental concepts in graph theory
- To practice proof techniques.
- To learn to use a graph drawing package.
- To work on learning from the text
- To begin using graph theory "tools" (i.e. results proved in theorems, lemmas, corollaries in the text) to solve problems
- To work on formulating and articulating solutions that involve an explanation or a justification.
- To learn to think critically about proposed solutions (especially your own)

The Problems

For 1 - 4, look up the Johnson graphs $J_{n,k}$ on Wikipedia.

1. Draw $J(5, k)$ for $k = 1, \dots, 5$. Draw the complement of $J(5, 2)$. Show that it is isomorphic to the Petersen graph.
2. $J_{n,k}$ is regular. Show how to use the degree-sum formula to find its number of edges. Use one of our results to show that $J(9, 5)$ is not bipartite.
3. Prove that $J(n, k)$ is isomorphic to $J(n, n - k)$.
4. For which pairs (n, k) is $J(n, k)$ claw free?
5. Let G_k be the graph whose vertices are (all of) the subsets of $\{1, 2, \dots, k\}$. Vertices A and B of G_k are joined by an edge if either $A \subseteq B$ or $B \subseteq A$ and $|A| - |B| = \pm 1$. Find the number of vertices and edges of G_k . Prove that G_k is bipartite. Prove that G_k is isomorphic to Q_k .
6. Problem 1.1.30 in the text. (We talked about the adjacency matrix already, but you'll need to look up the definition of incidence matrix in the text.)
7. Problem 1.3.14 in the text.
8. Find, if possible, a simple graph with 8 vertices that has neither a clique of size 3 nor an independent set of size 4. What about the other way around: neither a clique of size 4 nor an independent set of size 3?
9. Problem 1.2.20 in the text.
10. Problem 1.3.47 in the text.

HW 2 Teams

TEAM A: aagrawa6 aawellin anthorn apatel10 arrao bcdutton
 TEAM B: bcpilche bzhong2 cghobbs cncody csdabral djzager
 TEAM C: drwiner efarhan glingna hguan2 hguo5
 TEAM D: jchen37 jduan3 jjiang13 jrmeredi jsduvall
 TEAM E: lan4 mbushou nshivra rabrown7 rkrissh11 rrsizemo
 TEAM F: rshah6 rshu rssawyer rzou schinch2 sjain9
 TEAM G: sju2 skukret spshriva ssingh28 stanvir thultum
 TEAM H: tpande vsharma5 yho yhuang26 ymao4 zbcleghe