Homework 3
Graph Theory CSC/MA/OR 565
Due Tuesday, February 9, 2016

Instructions

- For this assignment, once again the class has been divided into teams by the instructor. Each team will work together, via the team's message board forum, to evolve a "perfect" solution to each of the problems.
- When your team has arrived at the perfect solution to a problem, typeset it and leave a link to it as the last entry in the "topic" for that problem in your team's message board forum. Alternatively, if, after collaborating on the problems via the message board, a team prefers to produce a single pdf file with the team's final solutions to all the problems, that would be appreciated by the TA. Make it clear to us if you are doing that and leave a link to it within your message board so it is accessible to me, Zexi, and all team members.
- Find your team at the end of this document. Teams have been slightly rearranged, since several students have dropped since the HW2 teams were formed. Each team should now have five members. Make sure you are on exactly one team and that you have the required access. If not, let us know.
- Discussion within teams is *encouraged*, with the goal of evolving solutions that are (i) mathematically correct, (ii) clear and concise, and (iii) well-presented. Each team homework submission will be graded by the TA and evaluated according to these three criteria. Some teams did a terrific job of collaborating on HW2.

Academic Integrity Guidelines (this is very serious):

- Be honest and ethical - *do* cite all sources used, if any, but *do not plagiarize*. Do not cut and paste material from the web and present it as your team's solution. That is plagiarizing.
- Since this is to be a group project, every member is expected to contribute. If you do not contribute, inform the instructor and do not take credit for the solutions. This seemed to be only a slight problem on HW2. When I went through the message boards, most non-responders had dropped the class.

Message Board Guidelines:

- A forum will be set up on the message board for each team.
- No team will have access to another team's message board.

- Teams should be viewed as"in competetion", so don't share secrets.
- You will be able to edit your posts.
- Your number of posts is not restricted.
- Be polite and considerate.
- The TA will monitor the forums.
- Contact the TA if there are any concerns or questions about the use of the message boards.
- Unfortunately, I found out that it is no longer possible to post html on Wolfware Classic message boards.

Goals of this assignment:

- To master fundamental concepts in graph theory.
- To practice proof techniques.
- To get more experience using a graph drawing package.
- To work on learning from the text.
- To begin using graph theory "tools" (i.e. results proved in theorems, lemmas, corollaries in the text) to solve problems. If may be helpful to use a computational tool for one or more of the problems (e.g. Maple, Matlab).
- To work on formulating and articulating solutions that involve an explanation or a justification. To work on using the proper "graph theory" terminology.
- To learn to think critically about proposed solutions (especially your own).

The Problems

The first two problems use terms in Definitions 1.1.39, 2.1.9, and 2.1.12 in the text and in Remark 7.2.20.

1. In the graph shown on the next page, find the eccentricity of each vertex. Find the diameter and radius of the graph.

2. In the graph shown on the next page, find the girth and the circumference of the graph. What is the length of the longest path? Find the center of the graph.

3. Problem 1.4.15 in the text. There is a typo in the first line: "$0 \leq n$" should be "$0 \leq j \leq n$".

4. Problem 1.4.36 in the text.

5. Problem 2.1.27 in the text.

6. Problem 2.1.37 in the text. (How does this compare to Propositions 2.16 and 2.17?)

7. Problem 2.2.10 in the text. For the first part, use Proposition 2.2.8 to get a recurrence for $\tau(K_{2.m})$. Then solve it.

8. In Lecture 6, we showed that for the n-ribbed fan graph G_n,

$$\tau(G_n) = \tau(G_{n-1}) + \tau(H_{n-1})$$

(this corrects the typo in the slides) and

$$\tau(H_n) = \tau(G_n) + \tau(H_{n-1}),$$

where H_n is the graph obtained from G_n by "doubling" the first rib. Show how to use this to complete the work of finding a formula for $\tau(G_n)$. Can you also find $\tau(H_n)$?

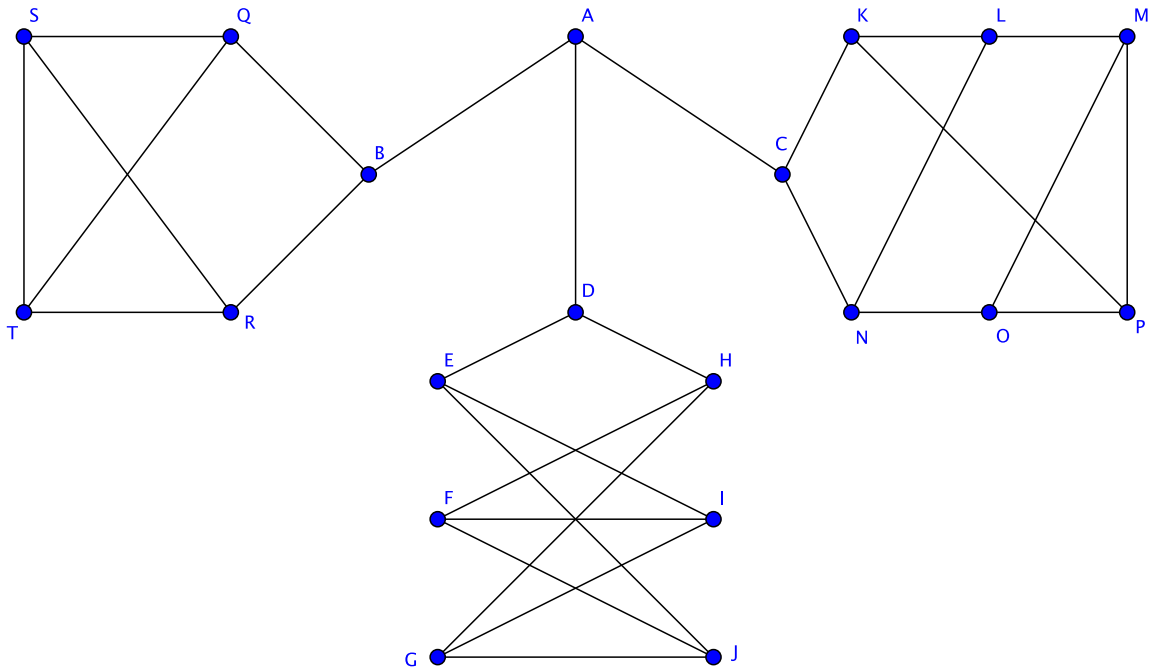
9. Show how to use one of our tools to compute the number of spanning trees of the Petersen graph.

10. a. Show that Dijkstra's shortest path algorithm can fail in the presence of negative weight edges, even if there are no negative cycles. Specifically, find a (small) weighted graph G that has some negative weight edges, but no negative cycles, and vertices $u, v \in V(G)$ such that Dijkstra's algorithm incorrectly computes $d(u, v)$.

b. The following is sometimes suggested to fix the problem in part (a): Let $-m$ be the most negative weight in the graph. Add m to the weight of every edge. Now all weights are nonnegative, so run Dijkstra's algorithm. When shortest paths are found, restore edge weights to their original values.

Show that this will not work. Specifically, find a (small) weighted graph G which has some negative weight edges, but no negative cycles, and vertices $u, v \in V(G)$ such that this algorithm incorrectly computes $d(u, v)$.

Graph for Problem 1:



HW 3 Teams

TEAM A: aagrawa6 aawellin amthorn arrao bcdutton
TEAM B: bzhong2 cncody csdabral jchen37 jjiang13
TEAM C: drwiner efarhan glingna hguan2 hguo5
TEAM D: bcpilche djzager cghobbs jsduvall rkrish11
TEAM E: lan4 mbushou nshivra rabrown7 rrsizemo
TEAM F: rshah6 rshu rssawyer rzou schinch2
TEAM G: sju2 skukret spshriva thultum tpande
TEAM H: vsharma5 yho yhuang26 ymao4 zbcleghe