Homework 4
Graph Theory CSC/MA/OR 565
Due 5 pm, Friday, March 4, 2016

Some of these are very easy (but which ones?)

1. Problem 4.3.2, text.

2. Problem 4.3.5, text.

3. Find the chromatic number of the graphs in exercise 8.1 of these notes of Frédéric Havet:

   `http://www-sop.inria.fr/members/Frederic.Havet/Cours/coloration.pdf`

Are either of the graphs critical?

4. Show that in every k-chromatic graph there are at least k vertices of degree at least $k-1$. (Hint: "critical")

5.

   a. Show $\chi(G) \leq \chi(G-v)+1$ for any vertex v of G.

   b. Show that if $\chi(G) = \chi(G-v)+1$ then $d_G(v) \geq \chi(G-v)$

   c. Use (a) and (b) to do 5.1.41 in text.

6. Look up on the internet the definition of the *binomial tree* B_k, which has 2^k vertices. (Draw $B_1, \ldots, B_4$.) Show by induction that for every k there is an ordering of the vertices of B_k for which the greedy coloring uses k colors.

7. Without using Brooks' theorem, prove that if G is a simple connected graph which is not regular, then $\chi(G) \leq \Delta(G)$.

8. Find a 4-chromatic graph in which the largest clique has size 2.

9. Find a critical graph which is not regular.

10. Do 5.2.16 in the text using Turán's Theorem.