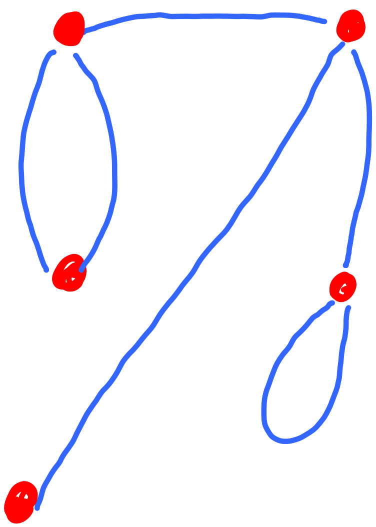
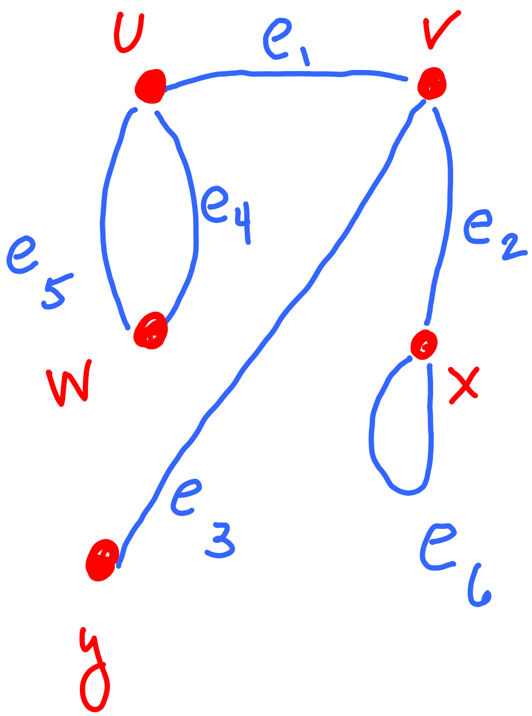
Graph G

- vertex set $V(G)$
- edge set $E(G)$
- relation associates to each edge a pair of vertices, not necessarily distinct

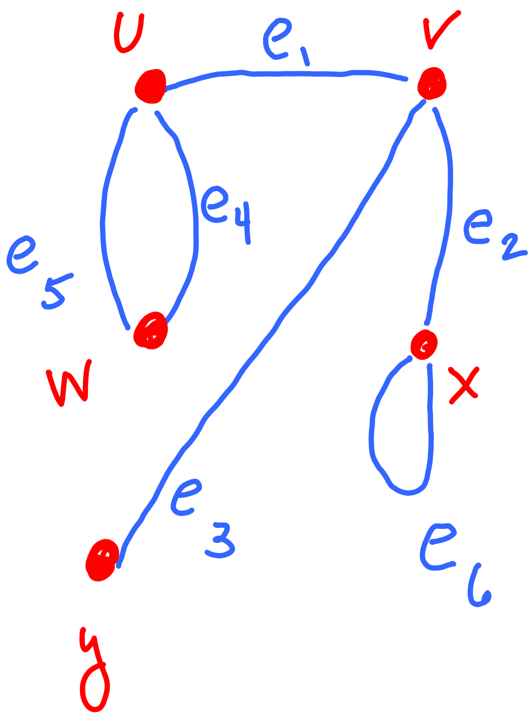


Graph G



- vertex set $V(G) = \{u, v, w, x, y\}$
- edge set $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$
- relation associates to each edge a pair of vertices, not necessarily distinct

Graph G

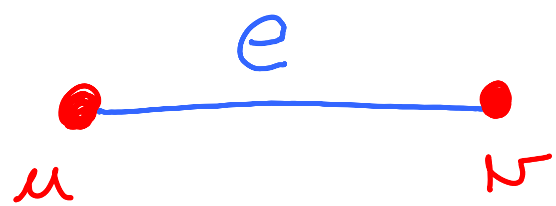


- vertex set $V(G) = \{u, v, w, x, y\}$
- edge set $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$
- relation: associates to each edge a pair of vertices, not necessarily distinct

$e_1 \to u, v$ $\quad e_2 \to v, x$ $\quad e_3 \to v, y$

$e_4 \to u, w$ $\quad e_5 \to u, w$ $\quad e_6 \to x, x$

Edge terminology



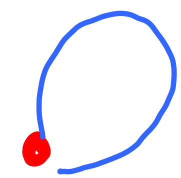
e **joins** u and v

u and v are **endpoints** of e

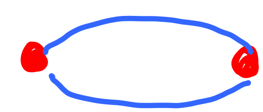
u and v are **adjacent**.

u and v are **neighbors**

u and e are **incident**

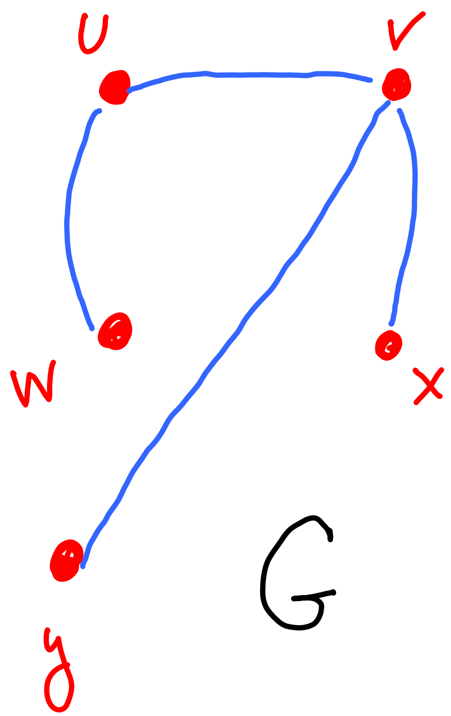


loop



multiple edges

Simple graph: no loops or multiple edges (so, can identify each edge with its end points)



$V(G) = \{u, v, w, x, y\}$

$E(G) = \{uv, vx, uw, vy\}$

More notation

$n(G)$ – number of vertices of G (order of G)

$e(G)$ – number of edges of G

G is trivial if $e(G) = 0$

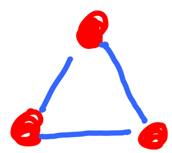
G is null if $n(G) = e(G) = 0$

(See remark 1.1.6)

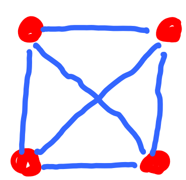
Complete graph : simple graph in which every pair of vertices is joined by an edge.

K_n - complete graph of order n

K_1 

K_3 

K_2 

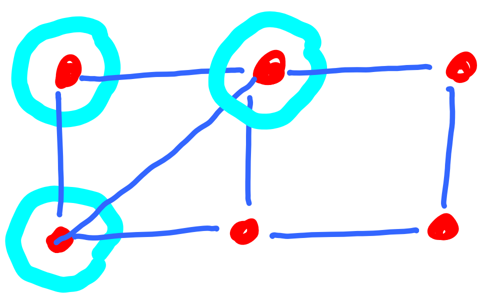
K_4 

$e(K_n) = \,?$

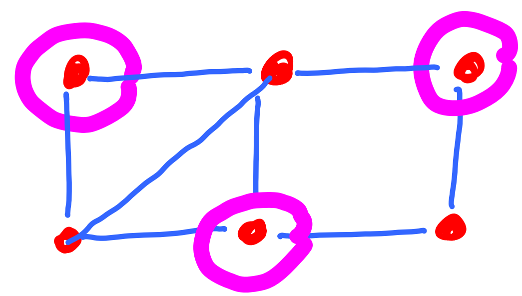
In a graph G,

a clique is a set of pairwise adjacent vertices

an independent set is a set of pairwise nonadjacent vertices



clique



independent set

In a graph G,

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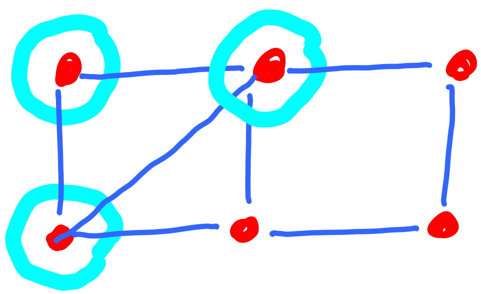
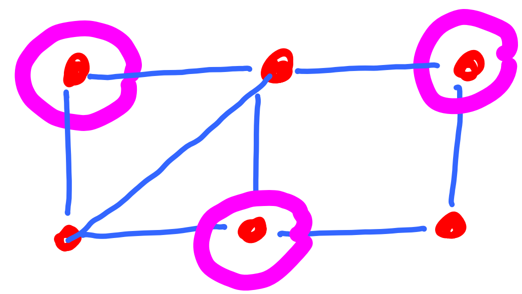
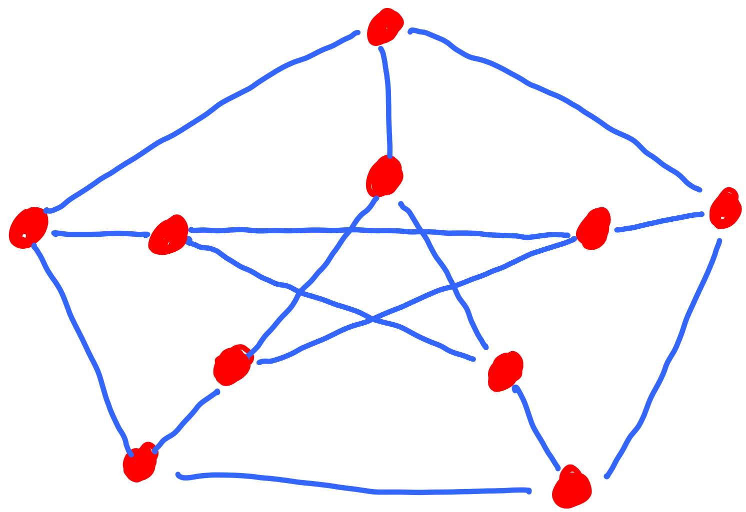


clique



independent set

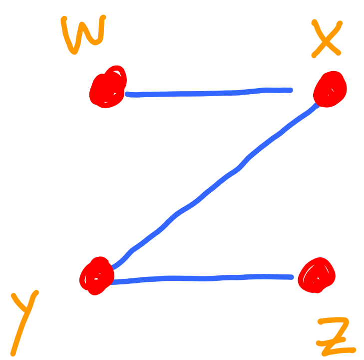
Petersen Graph



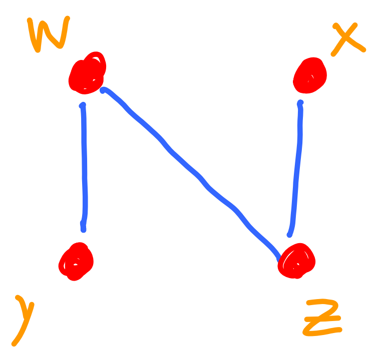
Size of largest clique?

largest independent set?

The complement of a simple graph G, denoted $\overline{G}$, is the graph with $V(\overline{G}) = V(G)$ and with edge set defined by $uv \in E(\overline{G})$ iff $uv \notin E(G)$



G



$\overline{G}$

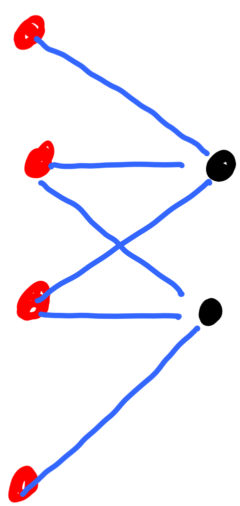
The complement of a simple graph G, denoted $\overline{G}$, is the graph with $V(\overline{G}) = V(G)$ and with edge set defined by $uv \in E(\overline{G})$ iff $uv \notin E(G)$

If S is a clique in G,

what is S in $\overline{G}$?

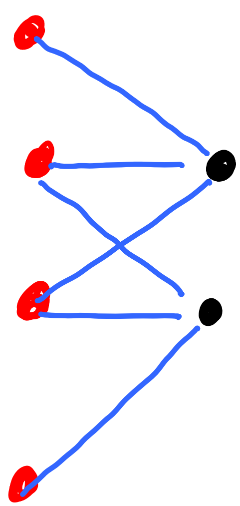
G is bipartite if V(G) is the union of two disjoint independent sets (called partite sets)

Examples

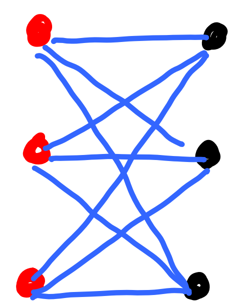


G is bipartite if $V(G)$ is the union of two disjoint independent sets (called partite sets)

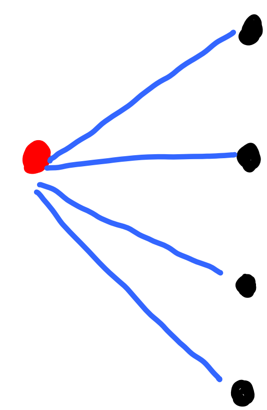
Examples



Complete bipartite graphs $K_{r,s}$



$K_{3,3}$

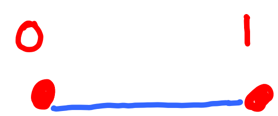


$K_{1,4}$

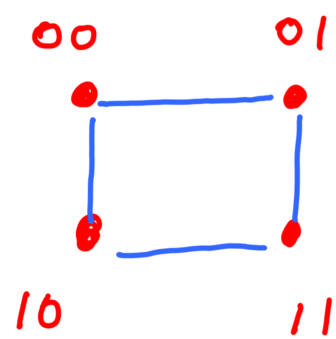
Q_k - the k-dimensional cube

is the graph whose vertices are the k-bit binary strings, with edges joining vertices that differ in exactly 1 bit position

Q_1



Q_2



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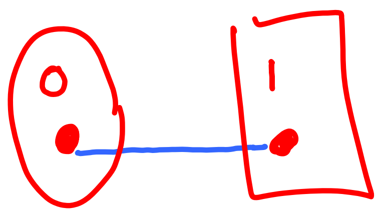
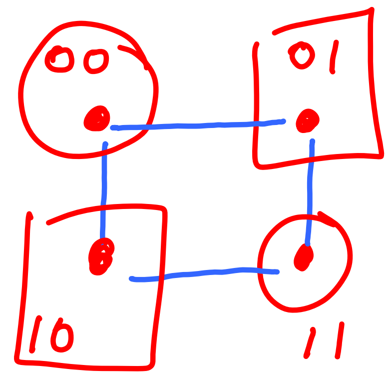
Q_1



Q_2



- Draw Q_3

- Is Q_n bipartite?

Is Q_k bipartite? <u>yes</u>

<u>Proof</u>.

Let X be the set of vertices $\{b_1 \dots b_k \mid \sum_{i=1}^{k} b_i \text{ is even}\}$

Let Y be the set of vertices $\{b_1 \dots b_k \mid \sum_{i=1}^{k} b_i \text{ is odd}\}$

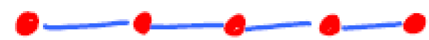
No edge of Q_k joins 2 vertices of X or 2 vertices of Y. So X and Y are independent sets $\blacksquare$

G is bipartite if $V(G)$ is the union of two disjoint independent sets (called partite sets)

Examples

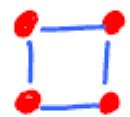
P_n - path with n vertices

P_5 :

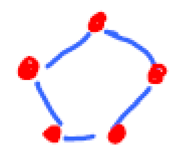


C_n - cycle with n vertices

C_4 :



C_5 :



Are these bipartite?