

Homework 6  
Graph Theory CSC/MA/OR 565  
Due 10:15 a.m., Thursday, April 21, 2016

1. Prove that if  $n$  is odd and  $n > 1$ , then  $\chi'(K_n) > n - 1$ .

The next few questions refer to the *line graph*  $L(G)$  of a graph  $G$ , defined in the text, Definition 7.1.1.

2. Problem 7.1.11, text.
3. Problem 7.1.33, text
4. A simple graph  $G$  is said to be of class 1 if  $\chi'(G) = \Delta(G)$  and of class 2 otherwise. Determine (and prove) the class number of each of the following Harary graphs:  $H_{6,3}$ ,  $H_{6,4}$ ,  $H_{6,5}$ ,  $H_{7,3}$ ,  $H_{7,4}$ .
5. a. Prove or disprove: If  $G$  is Eulerian then  $L(G)$  is Hamiltonian.  
b. Find a 2-connected non-Eulerian graph whose line graph is Hamiltonian. (Hint: Read exercise 7.2.10.)
6. Problem 7.2.2, text. Determine any of our necessary or sufficient conditions apply.
7. Problem 7.2.8, text.
8. Answer both of these questions for the graph below:
  - a. Is the Chvatal condition satisfied?
  - b. Is the closure complete?

