Assignment-9

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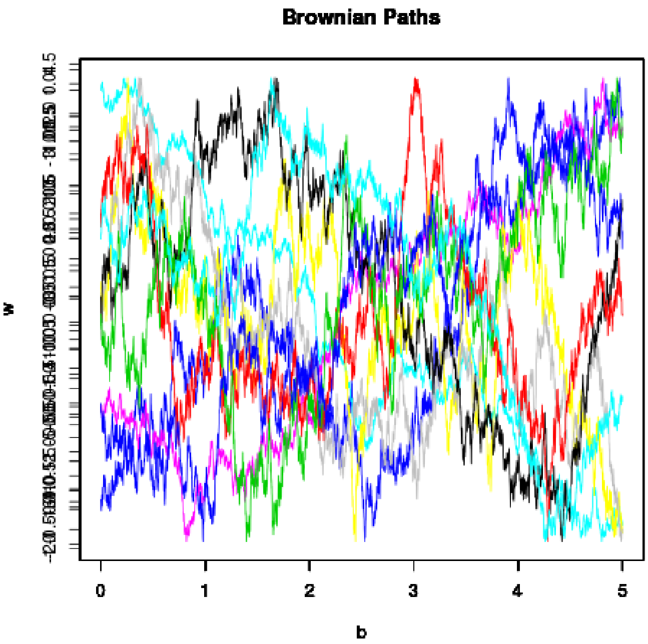
April 12th, 2016

Q 1 Generate 10 sample paths for the standard Brownian Motion in the time interval [0, 5] using the recursion.
$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i}.Z_{i+1}$
with 5000 generated values for each of the paths where $Z_{i+1} \backsim N(0,1)$ .Plot all the sample paths in a single figure. Also estimate $E[W(2)]$ and $E[W(5)]$ from the 10 paths that you have generated.

Solution:
Code for R

```
x=NULL;
y=NULL;
png("1.png");
for ( i in 1:10)
{
    n=rnorm(5000);
    a=5/5000;
    b=seq(from=0,to=5,by=0.001);
    w=NULL;
    w[1]=0;
    for( k in 1:5000)
    {
        w[k+1]=w[k]+sqrt(a)*n[k];
    }
    plot(b,w, col=i+3, blim=c(-5,5),type='l', main="Brownian Paths");
    par(new=TRUE);
    x[i]=w[2000];
    y[i]=w[5000];
}
print( cat("E[W[2]] = ",mean(x)));
print( cat("E[W[5]] = ",mean(y)));
```

OUTPUT: E[W[2]] = -0.1799204
E[W[5]] = -0.2217755



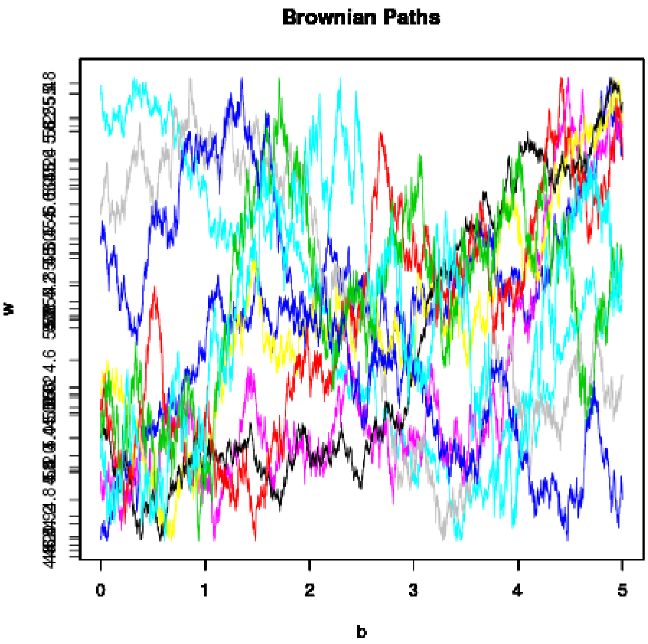
(a) Brownian motion

Q 2 Repeat the above exercise with the following Brownian motion $(BM(\mu, \sigma, 2))$ discretization $X(t_{i=1} = X(t_i) = \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}.Z_{i+1}$. Take $X(0) = 5, \mu = 0.06$ and $\sigma = 0.3$.

Solution:
Code for R

```
x=NULL;
y=NULL;
png("2.png");
for ( i in 1:10)
{
    n=rnorm(5000);
    a=5/5000;
    b=seq(from=0,to=5,by=0.001);
    w=NULL;
    w[1]=5;
    mu=0.06;
    rho=0.3;
    for( k in 1:5000)
    {
        w[k+1]=w[k]+mu*(a)+rho*sqrt(a)*n[k];
    }
    plot(b,w, col=i+3, blim=c(4,6),type='l', main="Brownian Paths");
    par(new=TRUE);
    x[i]=w[2000];
    y[i]=w[5000];
}
print( cat("E[W[2]] = ",mean(x)));
print( cat("E[W[5]] = ",mean(y)));
```

The output of the code is:
E[W[2]] = 5.085238
E[W[5]] = 5.322875



(b) Brownian motion

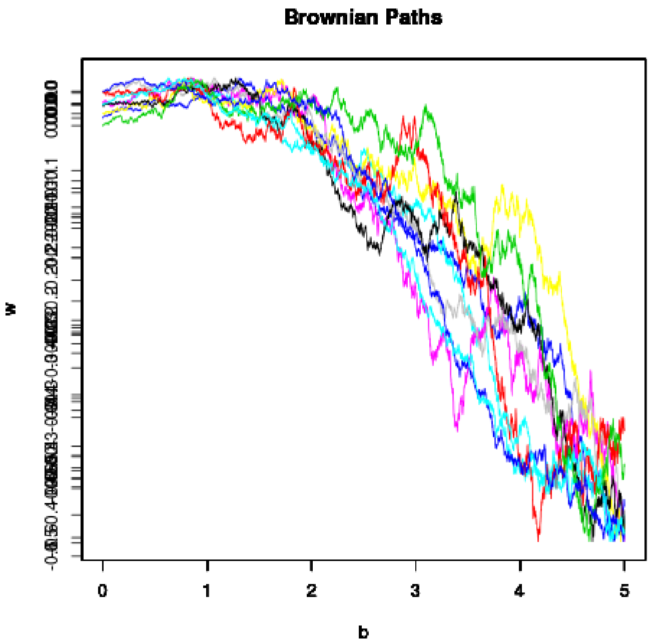
Q 3 The Euler approximated recursion with time dependent μ and σ is given by
$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i}.Z_{i+1}$
Repeat the above exercise by taking
$Y(0) = 5, \mu(t) = 0.0325 - 0.05t, \sigma(t) = 0.012 + 0.0138t + 0.00125t^2$

Solution:
Code for R

```
x=NULL;
y=NULL;
png("3.png");
for ( i in 1:10)
{
    n=rnorm(5000);
    a=5/5000;
    b=seq(from=0,to=5,by=0.001);
    w=NULL;
    w[1]=0;
    for( k in 1:5000)
    {

        rho=(0.012 + 0.0138*b[k]+0.00125*b[k]*b[k]);
        mu=(0.0325-0.05*b[k]);
        w[k+1]=w[k]+mu*(a)+rho*sqrt(a)*n[k];
    }
    plot(b,w,col=i+3, blim=c(-0.7,0.1),type='l', main="Brownian Paths");
    par(new=TRUE);
    x[i]=w[2000];
    y[i]=w[5000];
}
print( cat("E[W[2]] = ",mean(x)));
print( cat("E[W[5]] = ",mean(y)));
```

The output of the code is:
E[W[2]] = -0.02562826
E[W[5]] = -0.4975501



(c) Brownian motion