NONLINEAR BEHAVIOR OF FIXED-FIXED BEAM WITH A MOVING MASS

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This study addresses the coupled nonlinear behaviour of fixed-fixed beam under travelling mass. Because of the beam and mass interaction phenomenon, coupling terms are more likely to arise which results kinematic nonlinearities in the system. The major focus of this paper is to develop a theoretical model by introducing nonlinearities in the system. Later analysis of modal amplitude, mass position and tip deflection are done. For the beam modelling Euler-Bernoulli beam assumptions are taken for consideration. Initially a coupled mathematical model of mentioned system is derived by using Hamilton's principle. Afterward Galerkin discretization technique followed by perturbation method is implemented in the mathematical system to analyse dynamic characteristics of the desired system. Then Matlab ODE solver is used to plot various graphs for variation of amplitude and deflection with respect to time in case of both beam and mass. Under the internal resonance condition the time response curves are plotted to analyze the beating phenomenon for the beam and mass.

Keywords: Fixed-Fixed beam, Nonlinear analysis, Matlab

1. Introduction

The research concerning the traversing load and mass problem is voluminous. The dynamic analysis of moving object acquires an important place in the field of research due to its practical implementation in today's world. Applications of moving mass problems are generally manifested in the area of transportation. Rails, bridges, runways, overhead cranes are some examples of structural elements to carry the moving mass. Since the lateral stage of last decades the challenge to design these systems has got the attention of many researchers. A small contribution towards the solution of the problem was made by different investigators. A broad discussion constituting various types of problem with moving mass is presented in the book by Fryba[1]. Yang presented a dynamic study of Vehicle and bridge interaction to analyze the railway bridge behavior due to passing of high speed train[2]. Dynamic analysis of simply supported beam (SSB) containing moving load using finite element method was done by Olsson[3]. Further Foda and Abduljabbar[4] used Green function approach to determine the behaviour of SSB subjected to moving mass. Ye and Chen[5] investigated the effect of moving load on dynamic behavior of SSB by changing various parameters. Abdelghyany et al.[6] analyzed the dynamic characteristics of SSB under the action of moving load having linear viscoelastic foundation.

According to the various recent studies the structures are mostly affected by abrupt changes in masses. Hence inertia effect of mass is unavoidable during the study of dynamic behaviour problem. Influence of speed of moving mass on the structures with different boundary conditions was analysed by Dehestani et al.[7]. Simsenk[8] studied the vibration response of functionally graded beam with travelling mass implementing different theories of beam. A theoretical model of single span beam with suspended moving mass was designed and dynamic analysis of that system was presented by He[9]. In recent year zupan and zupan[10] presented a numerical analysis of three dimensional beam under moving mass considering geometrical non-linearity. From the previous studies it is realised that there is further scope to analyse the dynamic behaviour of moving beam-mass system. Present paper is based on the dynamical characteristics of beam with fixed-fixed boundary condition having spring mass system.

2. Problem Formulation

Euler-Bernoulli beam theory is considered in the present analysis. It is presumed that there is always contact between moving mass and beam. Fig.1 shows the uniform beam-mass system with fixed-fixed condition.

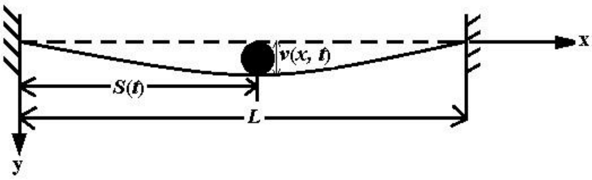

Fig.1. Schematic diagram of fixed-fixed beam with moving mass.

The length of beam is L, cross-sectional area is a, mass density is ρ **,**and flexural rigidity is EI. The mass of moving body is M which moves with a uniform velocity(v). Mass is connected with a linear spring of stiffness 'k' with one fixed end. Here all these dimenssionalized parameters are normalized by dividing required parameters for the proper calculation. Therefore, $s = S/L$ **,** $A = a/L^2$**,** $m = M/\rho AL$ and $\omega^2 = k\rho Al^4 / mEI$.

3. Theoretical Analysis and Results

3.1. Displacement Field

The Euler-Bernoulli beam model is assumed for the theoretical study. Here, $v(x, t)$ measures the vertical displacement of the reference axis. U, V, W denote the displacements in the undeformed x, y and z direction respectively. The following displacement field is assumed to be:

$$\begin{aligned} U &= -y\sin(\theta), \sin(\theta) = \left(\frac{\partial}{\partial x} v(x,t)\right) \\ V &= v(x,t), \cos(\theta) = 1 \\ W &= 0 \end{aligned} \tag{1}$$

3.2. Equation of Motion

The equations of motion of the system can be derived by using Hamilton's principle as

$$s_{tt} + \omega^2 (s - s_e) + \left[\phi_i \phi_j'\right]_{x=s} \left\{\ddot{\alpha}_i \alpha_j\right\} = 0 \tag{2}$$

$$\begin{aligned} &m\left\{ \left[\phi_i \phi_j''\right]_{x=s} \left\{\ddot{\alpha}_j\right\} + \ddot{s} \left[\phi_i \phi_j''\right]_{x=s} \left\{\alpha_j\right\} + 2\dot{s} \left[\phi_i \phi_j'\right]_{x=s} \left\{\dot{\alpha}_j\right\} + \dot{s}^2 \left[\phi_i \phi_j''\right]_{x=s} \left\{\alpha_j\right\} \right\} \\ &+ \left[\int_0^l \phi_i \phi_j dx\right] \left\{\ddot{\alpha}_j\right\} + \left[\int_0^l \phi_i'' \phi_j'' dx\right] \left\{\alpha_j\right\} = 0 \end{aligned} \tag{3}$$

Here '$\dot{\alpha}$' denotes the derivative with respect to 't' and 'ϕ'' denotes the derivative with respect to x. Later boundary condition for fixed-fixed beam is used to get the eigen function which is as follows

$$\phi_i = (\sinh(k_i x) - \sin(k_i x)) + \frac{\sinh(k_i l) - \sin(k_i l)}{\cos(k_i l) - \cosh(k_i l)} (\cosh(k_i x) - \cos(k_i x)) \tag{4}$$

For the analysis purpose 1st mode of fixed-fixed beam is taken into consideration. Considering 1st mode (ϕ_1) of desired beam as basis function, equations (2) and (3) reduce to:

$$\ddot{s}+\omega^2 s+c_1\ddot{\alpha}_1\alpha_1=0 \tag{5}$$

$$\ddot{\alpha}_1+\omega_1^2\alpha_1+mc_2\ddot{s}\alpha_1+2mc_2\dot{s}\dot{\alpha}_1+2mc_2 s\ddot{\alpha}_1=0 \tag{6}$$

$$\text{where, } c_1=\phi_1\phi_1'\Big|_{x=s_e},\ c_2=\frac{\phi_1\phi_1'\Big|_{x=s_e}}{\int_0^l(\phi_1)^2\,dx+m(\phi_1)^2\Big|_{x=s_e}},\ (\omega_1)^2=\frac{\int_0^l(\phi_1'')^2\,dx}{\int_0^l(\phi_1)^2\,dx+m(\phi_1)^2\Big|_{x=s_e}} \tag{7}$$

Here ω is the frequency of moving mass and ω_1 is the 1st frequency of beam. Relationship between them is: $\omega=2\omega_1+\varepsilon\sigma$ (8)

where σ is a small detuning parameter. While σ is zero we get perfect internal resonance 1:2. To obtain the analytical solution for the derived mathematical model, method of multiple scale (MMS) is used having time scale $T_n=\varepsilon^n t$. Where ε is scaling parameter and n=0,1,2… Here two time scale is considered i.e, T_0 and T_1 as the nonlinearities have a very small effect. After simplifying equation(5) and (6) implementing this MMS technique, the solution for the required equation can be written as;

$$\frac{\partial p_1}{\partial T_1}=\frac{1}{4}\frac{c_1 p_2^2\omega_1^2}{\omega}\sin(2\varphi_2-\varphi_1-\sigma T_1),\ p_1\frac{\partial\varphi_1}{\partial T_1}=-\frac{1}{4}\frac{c_1 p_2^2\omega_1^2}{\omega}\cos(2\varphi_2-\varphi_1-\sigma T_1)$$

$$\frac{\partial p_2}{\partial T_1}=-\frac{1}{4}\frac{mc_2\left(\omega^2-2\omega\omega_1+2\omega_1^2\right)}{\omega_1}p_1p_2\sin(2\varphi_2-\varphi_1-\sigma T_1)$$

$$p_2\frac{\partial\varphi_2}{\partial T_1}=-\frac{1}{4}\frac{mc_2\left(\omega^2-2\omega\omega_1+2\omega_1^2\right)}{\omega_1}p_1p_2\cos(2\varphi_2-\varphi_1-\sigma T_1) \tag{9}$$

where, p_1,ϕ_1 are the modal amplitudes and phase angle for moving mass, p_2,φ_2 are the amplitudes and phases for the beam deflection.

4. Results

Analysis of beam under fixed-fixed condition is done by using perturbation method. Only 1st mode for the mentioned boundary condition is taken for the analysis purpose. Natural frequencies are calculated by using equation(8). The initial value for mass displacement (s_0) and tip deflection (v_{t0}) are taken as 0.00001 and 0.1 respectively. p_{20} is taken as $0.5v_{t0}$. To maximize accuracy and efficiency, the final ordinary differential Equations **Error! Reference source not found.** are solved using Jacobi iteration method. Stiff ODE solver is used to plot the modal amplitude and the variation of mass and beam deflection. The time response in Figures, shows the beating phenomenon for the mass and the beam by changing different parameter under internal resonance case. As mentioned in Nonlinear Oscillation by Nayfeh [11] the detuning parameter is calculated. Various graphs are plotted for different detuning parameter σ and with $\varepsilon=1$.

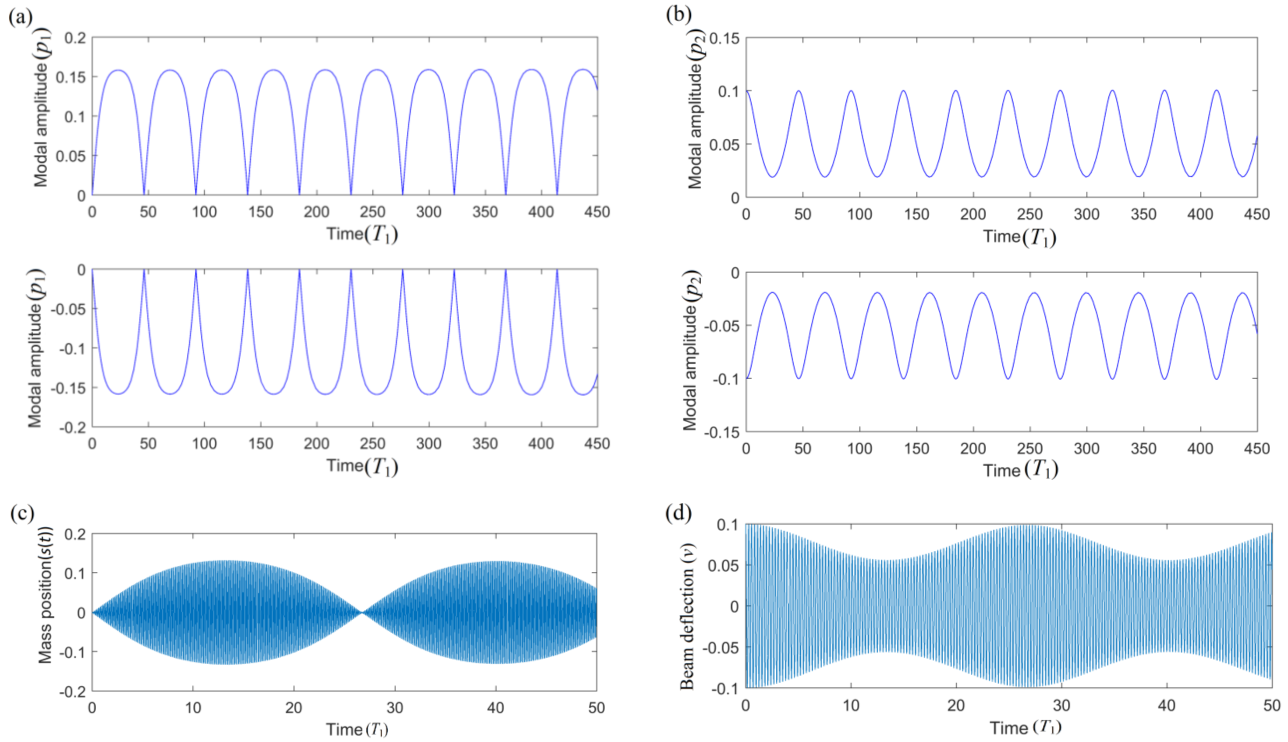

Fig.2. m=0.1, S_e =0.9, S_{10} =0.00001, v_{t0} =0.1, (a)and (b)perturbation solution σ =-0.0085 for mass and beam respectively, (c) mass position and (d) beam deflection

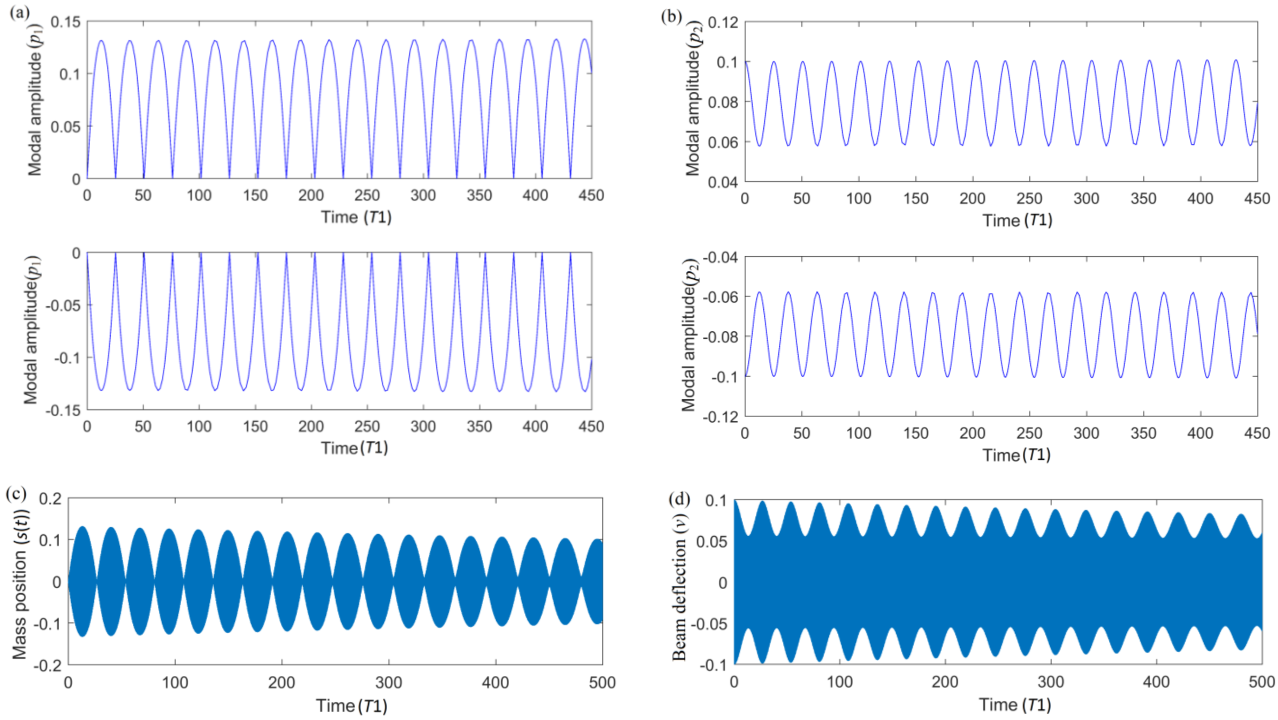

Fig.3. m=0.1, S_e =0.9, S_{10} =0.00001, v_{t0} =0.1, σ =0(a) perturbation solution for mass and (b) perturbation solution for beam (c)mass position and (d) Beam deflection

Fig. 2 shows the response curve for moving mass at equilibrium position and beam response due to the effect of moving mass for the required system. Here the equilibrium position (s_e) of moving mass is 0.9 and non-dimenssionalized mass ratio m=0.1 is taken. Fig.2 (a) and Fig.2 (b) presents the fluctuation of modal amplitude (p_1) in case of mass and in case of beam (p_2) respectively. The value of σ is used to compare various solutions. In this case it is considered as -0.0085. Figure 2 (c) and 2 (d) are the numerical solution obtained from Equations (5) and (6) by using stiff ODE23s solver. In Fig.2 maximum deflection of mass is 0.16 and minimum is 0 whereas in case of beam maximum deflection is 0.1 and minimum deflection is 0.04. Fig.3 shows the similar response curve by taking the mass ratio as 0.1 and $\sigma = 0$. It is found that for same initial value by changing the parameters like mass ratio and σ, value of amplitude changes. It is found that after time period 250, amplitude decreases gradually. In this case maximum deflection obtained is 0.125 and minimum deflection is 0 for the mass and in case of beam maximum deflection is 0.1 and minimum is 0.06. It divulges from the comparison of Fig. 2 and Fig.3 that time period decreases by reducing non-dimenssionalized mass ratio and detuning parameter σ.

5. Conclusion

1. Perturbation method can be successfully used for the dynamic analysis of the moving mass system.
2. The numerical results reveal that for small value of σ, system works near the resonance condition.
3. The minimum amplitude variation of the beam deflection never goes to zero. However in case of moving mass minimum variation of amplitude tends to zero.
4. Both mass position and beam deflection appear dark due to small time steps and very high frequency variation.

References

1. L. Fryba. Vibration of Solids and Structures under Moving Loads,Thomas Telford Publishing. (1999)

2. Y. B. Yang, J. D. Yau. Vehicle-Bridge Interaction Element for Dynamic Analysis. J. Struct. Eng. **123**, 1512–1518 (1997)

3. M. Olsson. On the fundamental moving load problem. *J. Sound Vib.* **145**, 299–307 (1991)

4. M. A. Foda, Z. Abduljabbar. A Dynamic Green Function Formulation For The Response Of A Beam Structure To A Moving Mass. J. Sound Vib. **210**, 295–306 (1998)

5. Z. Ye, H. Chen. Vibration analysis of a simply supported beam under moving mass based on moving finite element method. *Front. Mech. Eng. China* **4**, 397–400 (2009)

6. S. M. Abdelghany, K. M. Ewis, A. A. Mahmoud, M. M. Nassar. Dynamic response of non-uniform beam subjected to moving load and resting on non-linear viscoelastic foundation. *Beni-Suef Univ. J. Basic Appl. Sci.* **4**, 192–199 (2015)

7. M. Dehestani, M. Mofid, A. Vafai. Investigation of critical influential speed for moving mass problems on beams. *Appl. Math. Model.* **33**, 3885–3895 (2009)

8. M. Şimşek. Vibration analysis of a functionally graded beam under a moving mass by using different beam theories. *Compos. Struct.* **92**, 904–917 (2010)

9. W. He. Vertical dynamics of a single-span beam subjected to moving mass-suspended payload system with variable speeds. *J. Sound Vib.* **418**, 36–54 (2018)

10. E. Zupan, D. Zupan. Dynamic analysis of geometrically non-linear three-dimensional beams under moving mass. *J. Sound Vib.* **413**, 354–367 (2018)

11. A. H. Nayfeh, D. T. Mook. *Nonlinear Oscillations*. John Wiley & Sons. (2008)