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Simple Optimization Algorithm for Design of a Uniform Column

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Traditional methods for solving constrained optimization problems are not robust enough to get the solution in reasonable computational time. They have drawback of get stuck in local optima. To overcome these problems meta-heuristic techniques are now widely used. This paper introduces simple optimization algorithm (SOPT), a meta-heuristic technique for solving constrained optimization problems. To handle the constraints, a constraint fitness priority based ranking method is included in the algorithm. SOPT algorithm for constraint optimization is coded in MATLAB and applied to design a uniform column for minimum design cost. Result obtained is compared with the result obtained by another important meta-heuristic algorithm called cuckoo search (CS) algorithm.

***Keywords*:** Simple optimization; Cuckoo search; Constraint handling; Design optimization;

1. Introduction

Engineering design attempts to formulate a plan which results in the creation of some real physical thing which having qualities of proper functionality, reliability, usability manufacturability and marketability. A generalized process of design is outlined in the literature [1]. The process begins with recognizing the need and ends with presenting the plans. Several iterations are made in between to reach the final plans for satisfying the need. One of the important phases in design process is analysis and optimization. This phase requires constructing or devising abstract models of the system which are called mathematical models. These models are created in the hope that they will simulate the real physical system very well. Modelling of an entire mechanical system may result in complex mathematical equations with large number of variables. To overcome the difficulty of analysis and optimization of such large system, it is wise to design individual elements of a system. For example, in an automobile power transmission system, design of a gearbox is computationally and mathematically simpler than the design of complete system.

Once the mathematical model is formed, it should be solved for optimizing the criterion selected. Optimization is the process of finding a solution to a problem in which a single or set of objective functions are to be maximized or minimized within a domain which contains suitable values of variables with restrictions or constraints to be satisfied. Those sets of variables which satisfy the given restrictions or constraint in the domain are feasible solutions.

The best among all the feasible solutions is known as optimum solution of the problem. Models formed are mostly non-linear in nature for engineering design problems. A non-linear problem can be represented as

maximize or minimize $y = f(\bar{x})$ where $\bar{x}$ is a column vector of n design variables i.e.

$\bar{x} = [x_1, x_2, x_3, \ldots\ldots, x_n]^T$ Subjected to the m inequality and p equality constraints i.e.

$g_i(\bar{x}) \le 0,\ i = 1,2,\ldots.,m \quad h_j(\bar{x}) \le 0,\ j = 1,2,\ldots.,p$

with variable bounds

$x_k^{(L)} \le x_k \le x_k^{(U)},\quad k = 1,2,\ldots\ldots.,n$ $x_k^{(L)}$ and $x_k^{(U)}$ are the upper and lower limit of values which are permissible in the variables.

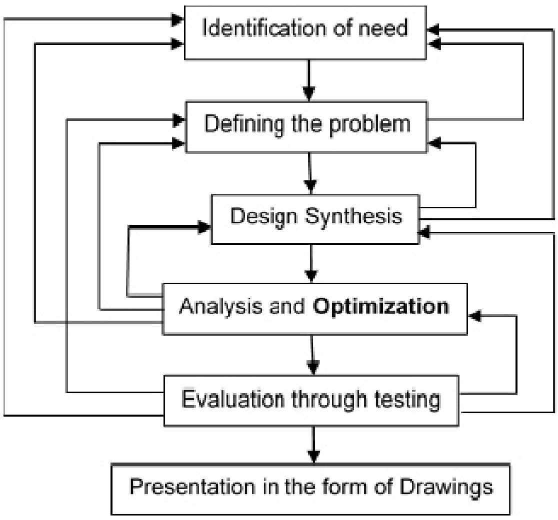


Fig. 1. Iterative phases in a design process

There are large numbers of techniques available for solving optimization problem. These are basically divided into two groups such as 1) Traditional methods 2) Non-traditional methods. Two distinct types of algorithms are used in traditional approach. Direct search methods use only objective function values to locate the optimum point. Indirect search or gradient based methods use the first and/or the second-order derivatives of the objective function to locate the optimum point. There is another class of optimization techniques which is based on the stochastic search and do not require the objective function to be continuous or differentiable. These algorithms are general purpose algorithms and can be applied to solve a wide variety of optimization problems. Another reason for using such type of algorithms lies in ability of getting global solution while traditional methods get stuck to the local optimum solution many times.

Constrained optimization is the process of optimizing an objective function with respect to some variables in the presence of constraints. Coello [2] has discussed constraint handling techniques adopted in popular evolutionary algorithms. The constraint handling techniques are well suited for all types of constraints (linear, nonlinear, equality, non-equality) mostly used in engineering applications. Wang and Fang [3] have used an effective shuffled frog-leaping algorithm (SFLA) for solving multi-modal resource constrained project scheduling problem. Few benchmark problems have been solved by SFLA and compared with existing algorithms like simulated annealing, genetic algorithm, hybrid scatter search, hybrid rank-based evolutionary algorithm, truncated branch and bound and differential evolution (DE) algorithm. Coello and Montes [4] have proposed a dominance-based selection process for constrained handing in genetic algorithm for global optimization. This approach has been compared with several evolutionary optimization algorithms and traditional mathematical programming for problems like welded beam design, design of a 10-bar plane truss, design of a pressure vessel, disjoint feasible region, minimization of the weight of a tension/compression string. The method performs well in several test problems both in form of the number of fitness function evaluations required and in terms of the quality of the solution obtained. Michalewicz [5] has reviewed the methods for handling constraints in genetic algorithm for numerical optimization problems and test them on selected problems highlighting the strength and weakness of each method.

2. Simple optimization (SOPT) algorithm

Simple Optimization (SOPT) is also a population based metaheuristic algorithm where a random population of solutions is generated. Each iteration consists of two stages - exploration and exploitation. In exploration stage, a new solution is generated based on the best solution of the population as given by the equation 1. A similar equation is used in exploitation stage with only difference of constant c_2, which is half of constant c_1 used in exploration stage. In each iteration, the generated new solution is compared with the worst solution of the population and if newly generated solution is better than the worst solution then it is included in the population removing the worst solution. Iterations will be continued till the termination criterion is not reached reached which may be the best solution achieved or maximum number of iterations is reached.

$$x_{i,new} = x_{i,best} + c_1 \times R_i \tag{1}$$

$$x_{i,new} = x_{i,best} + c_2 \times R_i \tag{2}$$

In equations 1 and 2, $x_{i,new}$ is the ith parameter of the new solution and $x_{i,best}$ is the ith parameter of the best solution in the same iteration. c_1, c_2 are the positive constants and c_2 is taken as half of c_1. R_i is the normally distributed random number with a mean zero and standard deviation σ_i, which is the standard deviation of ith parameter of all the members in the population. Calculation of standard deviation for the normal distribution is shown in Fig. 2. Here, N is population size and M is number of optimization parameters.

	variable 1	variable 2	variable 3		variable M
Candidate solution 1	X11	X12	X13	…	X1M
Candidate solution 2	X21	X22	X23	…	X2M
Candidate solution 3	X31	X32	X33	…	X3M
…	…	…	…	…	…
Candidate solution N	XN1	XN2	XN3	…	XNM

σ_1 σ_M

Fig. 2. Schematic representation of calculating standard deviation

For handling constraints SOPT algorithm incorporated a constraint priority-based ranking method. This method consists of calculating constraint fitness function $F_i(\bar{x})$ as given by the equations 3 and 4. Equation 4 is used for m number of inequality constraints $g_i(\bar{x}) \le 0$.

$$F_i(\bar{x}) = \begin{cases} 1, & g_i(\bar{x}) \le 0 \\ 1 - \dfrac{g_i(\bar{x})}{g_{max}(\bar{x})}, & g_i(\bar{x}) > 0 \end{cases} \tag{3}$$

Where $g_{max}(\bar{x}) = \max\{g_i(\bar{x})\},\ i = 1, 2, \ldots\ldots m$

For p number of equality constraints $h_i(\bar{x}) = 0$ equation (4) is used.

$$F_i(\bar{x}) = \begin{cases} 1, & h_i(\bar{x}) = 0 \\ 1 - \dfrac{|h_i(\bar{x})|}{h_{max}(\bar{x})}, & h_i(\bar{x}) \ne 0 \end{cases} \tag{4}$$

Where $h_{max}(\bar{x}) = \max\{h_i(\bar{x})\},\ i = m+1, m+2, \ldots\ldots\ldots, m+p$

Thereafter a total constraint fitness function $F_{con}(\bar{x})$ is calculated using the equation (5).

$$F_{con}(\bar{x}) = \sum_{i=1}^{m+p} w_i F_i(\bar{x}) \tag{5}$$

w_i is weight for ith constraint and is randomly selected in the range [0,1] such that $\sum_{i=1}^{m+p} w_i = 1$. Weight associated with constraint maintains the diversity of the solutions. Value of $F_{con}(\bar{x})$ varies in between 0 to 1. When no constraint is violated by a solution, the value of $F_{con}(\bar{x})$ will be 1 and if all constraints are violated by the solution then its value will be 0. To rank the solutions from best solution to worst solution, they are sorted on the basis of decreasing value of $F_{con}(\bar{x})$. For same values of $F_{con}(\bar{x})$, the solutions should be sorted in ascending order of objective function values.

2. 1. SOPT algorithm for constrained optimization

The various steps involved in the proposed SOPT algorithm for constrained problems are as below:

1: Initialize population
2: Calculate fitness function of the population
3: Calculate constraint fitness function of the population as explained in the section 2.
4: Repeat
5: Sort population in decreasing order of constraint fitness function and increasing order of fitness value, giving priority to constraint fitness function.
6:Generate new solution by equation $x_{i,new} = x_{i,best} + c_1 \times R_i$
7: Compare worst solution of population with new one and replace it if new solution is better.
8:Generate new solution by equation $x_{i,new} = x_{i,best} + c_2 \times R_i$
9: Compare worst solution of population with new one and replace it if new solution is better.
10: Store best solution found so far.
11: Replace best solution of the population, if it remains best for limiting number of iterations (Replacement counter).
12: Until maximum number of iterations is not reached.

In this method, all solutions are ranked based on the constraint fitness function and the solution with highest constraint fitness value will be in first position. If two different solutions have same value of constraint fitness then the one with lower value of objective function is prioritize over other. The advantage of this method is that feasible solutions, which always have constraint fitness value higher than the infeasible solutions will get higher ranking in the list. Therefore, during the iterations best feasible optimal solution is obtained by maintaining both feasible solutions as well as better infeasible solutions.

3. Design of a uniform column for minimum cost

To analyze the SOPT algorithm for its application in solving nonlinear optimization problems, it was coded in MATLAB 8.5(R2015a) and is used to optimize design of a uniform column for minimum cost.

It is an example of designing a uniform column of tubular section hinge supported at both ends (Fig. 3), for minimum cost [6] the problem is defined as:

Compressive load to be carried by the column F=2500kgf
Yield stress of the column material σy= 500kgf /cm2
Modulus of elasticity of the column material E= 0.85×106 kgf/cm2
Weight density of material ρ = 0.0025 kgf /cm3
Length of column l =250 cm
Range of mean diameter of the column d in cm = [2, 14]
Range of available column thickness t in cm = [0.2, 0.8]

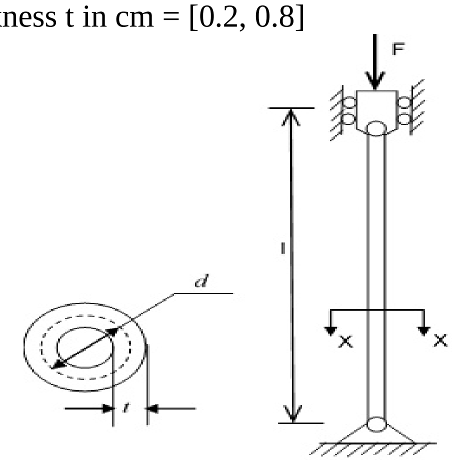


Fig. 3. Tubular column hinged at both ends under compression

The purpose is to design the column which should be able to withstand the load so that induced stress is less than yield stress of the material (constraint $g_1(\bar{x})$) and less than the Euler's buckling stress for the given configuration (constraint $g_2(\bar{x})$), with minimum cost. The cost of the column consists of weight and construction cost and in the example it is taken as 5 × (weight of the column) + 2 × (mean diameter of the column)

The objective function is to minimize the cost given by [7]

$$f(\bar{x}) = 5W + 2d = 5\rho l \pi dt + 2d = 9.82dt + 2d$$

where, $\bar{x} = \begin{bmatrix} d \\ t \end{bmatrix}$

Objective function has two decision variables mean diameter of column and thickness of the column.

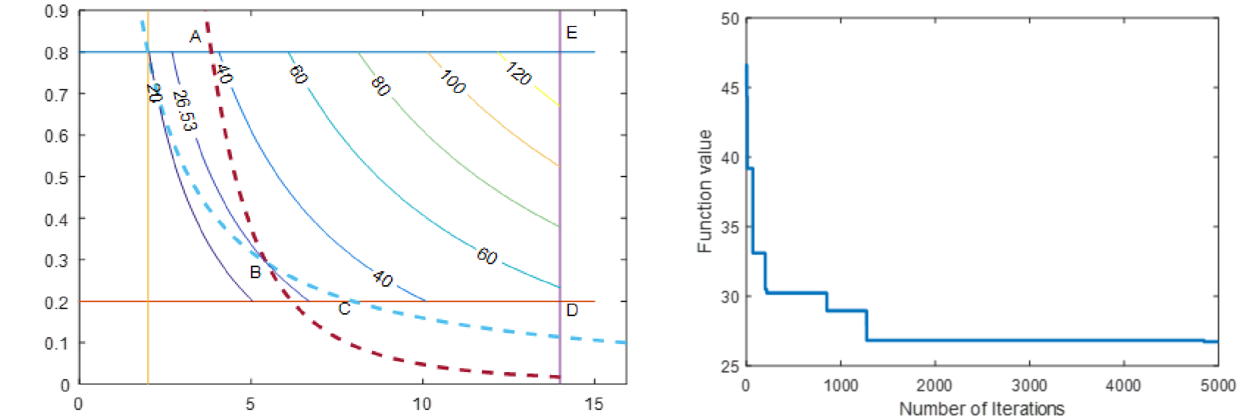
Subject to the constraint

$$g_1(\bar{x}) = \frac{2500}{\pi dt} \leq 500$$

$$g_2(\bar{x}) = \frac{2500}{\pi dt} \leq \frac{\pi^2 (0.85 \times 10^6)(d^2 + t^2)}{8(250^2)}$$

4. Result and discussion

Contour curves of the objective function along with constraints curves are shown in Fig. 4(a). This problem is solved by Gandomi et al [8] using cuckoo search algorithm the best result obtained is 26.5317 in 15000 function evaluations and in 2.56919 seconds. While SOPT algorithm is able to give better result of 26.52751 at (5.442073, 0.292721) in less than 10000 function (Fig. 4(b)) evaluations and in 2.37915 seconds.



(a) Contour plot (b) Convergence curve

Fig. 4. Contour plot and convergence curve for column design problem

5. Conclusion

In this work, a simple and efficient SOPT algorithm to solve constrained optimization problem is proposed. To handle the constraint, a constraint fitness priority based ranking method is used.

SOPT algorithm consists of two simple equations with only two function evaluations in one iteration. To check the effectiveness of SOPT to get optimum solution it is coded in MATLAB and applied to solve a mechanical engineering design problem. Result of optimization is compared with the another important non-traditional algorithm cuckoo search and result is observed better in SOPT algorithm in reaching better optimized value in less computational time. This makes SOPT algorithm a good choice to attempt solving engineering optimization problems.

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