

EFFECTS OF MASS SUCTION ON MHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A POROUS SHRINKING SHEET WITH HEAT SOURCE/SINK

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An attempt has been made to understand the influence of the mass suction on the steady two dimensional magnetohydrodynamic (MHD) boundary layer flow and also the transfer of heat across a shrinking sheet with source or sink as a supplement. A dynamic framework with plane of flow and uniform magnetic field acting perpendicular to each other is considered. The main catch is transforming the governing non- dimensional PDEs into the ordinary differential equations with the help of similarity transformations. By making the best use of the in- built MATLAB solver bvp5c, the so derived ordinary differential equations are numerically solved. From the keen examinations it is found out that, with an increment of wall mass suction, the velocity inside the boundary layer rises and consequently, the thickness of momentum layer goes down. It is also observed that there is a fall in temperature as the Prandtl constant rises. With heat source specifications; the Hartmann constant, heat sink specifications and the temperature are seen to go up. Besides, absorption of heat at the sheet surface is observed to occur for a strong heat source.

Keywords: Transfer of heat, A source of heat/sink, MHD boundary layer, shrinking sheet, mass suction, bvp5c.

1. Introduction

In the fabrication and technological areas of applications, metals and polymer processing industries, the production of paper, wire drawing and many more, the analysis of mixed convection boundary layer flow of an non- compressible fluids with higher viscosity across a shrinking sheet has a major role to play. Several authors have examined the complications in boundary layer flow pertaining to the electrically conducting fluid with higher viscosity about the stretching sheet such as With the help of Soret and Dufour impacts, Postelnicu[1]used chemical reactions as the base to examine the impact on heat and mass transfer using the method of natural convection from vertical surfaces in permeable mediums. Using a similar base, Bhattacharyya and Layek [3] examined using suction or blowing. Shankar Goud and Raja Shekar[2]inspected the impacts of thermal radiation and heat source on MHD free convection over a vertical plate with thermal diffusion and diffusion thermo. Taking a prescribed skin friction for a stretched continuous surface into account, Mohamed Ali and Khaled Al-Salem[4] reviewed the impact of suction or injection on the boundary layer flows. Similar effects between perpendicular porous plates with thermal diffusion were analyzed by Uwanta and Hamza[5]on unsteady hydromagnetic convective flow of reactive fluids with higher viscosity. For the MHD flow of micropolar fluid across a stretching sheet and MHD flow across perpendicular oscillating plate with chemical and radiation based reactions in a porous medium Shankar Goud et.al[6,13] considered the implicit finite difference method. With chemical reactions base and concentration in the presence of suction or injection, Kandasamy et.al[7,8] examined the heat and mass transfer on boundary layer folw along a wedge with heat source and also a porous wedge with heat radiations.. Shankar Goud and Dharmendar Reddy[9] studied the chemical reaction effect on heat and mass transfer of MHD fluid flow over a moving perpendicular plate

with heat source and convective boundary constraint. Effects of heat source/ sink on MHD flow and heat transfer over a shrinking sheet with mass suction was analysed by Krishnendu Battacharya [10]. Muhaimin et.al[11] examined the effects of heat and mass transfer on MHD boundary layer flow over a shrinking sheet in the presence of suction. G. Bal Reddy et.al [12] studied the Keller box solution of magnetohydrodynamic boundary layer flow of nanofluid over an exponentially stretching permeable sheet. Finite element method application of effects on an unsteady MHD convective heat and mass transfer flow in a semi-infinite vertical moving in a porous medium with heat source and suction was studied by Shankar Goud, and MN Rajashekar[14].

In the current analysis, with the help of heat source or sink, the impact of suction on MHD boundary layer flow and heat transfer over a porous stretching sheet are investigated. The prime equations are transformed into a set of non-linear ordinary differential equations using the similarity transformation. MATLAB in built solver is used to solved these numerically. Results are plotted in figures for different physical specifications involved in the equations and are discussed in detail.

2. Mathematical Formulation

Take into account the MHD flow of an incompressible viscous electrically conducting fluid, where heat transfer takes place over a permeable shrinking sheet which coincides with the plane $y = 0$. The flow is restricted to $y > 0$ in the presence of heat generation and absorption. The x – axis is taken along sheet and y – axis is perpendicular to sheet respectively and variable magnetic field B_0 is applied to normal to the plate. Consider the geometry of a physical complication as in figure A:

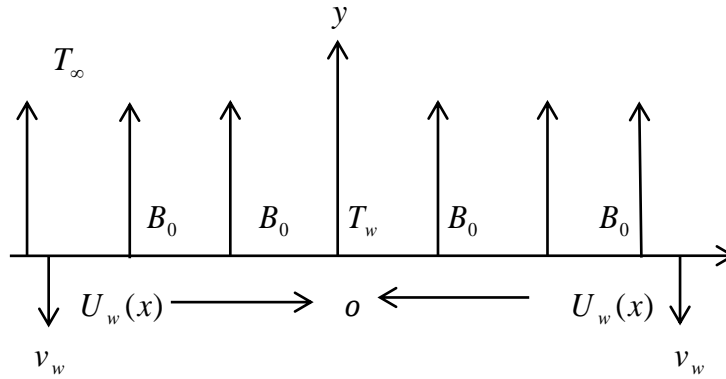


Figure A: An outline of the physical problem

According to the boundary layer conditions the prime equations are given by

$$\text{Continuity equation: } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots (1)$$

$$\text{Momentum equation: } u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad \dots (2)$$

$$\text{Energy equations: } u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad \dots (3)$$

The appropriate boundary condition are given by

$$\begin{aligned} u = U_w(x) = -cx, \quad v = v_w, \quad T = T_w \quad \text{at } y \rightarrow 0 \\ u \rightarrow 0, \quad T \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad \dots (4)$$

Now bringing in the stream function ψ , the velocity components u and v can be written as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad \dots (5)$$

Equation (1) satisfied using the eqn.(5) and momentum and temperature equations take the following forms:

$$\frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B_0^2}{\rho} \frac{\partial \psi}{\partial y} \quad \dots (6)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) \dots (7)$$

Also the boundary conditions is suited to

$$\begin{aligned} \frac{\partial \psi}{\partial y} &= -cx, \frac{\partial \psi}{\partial x} = v_w \text{ at } y \rightarrow 0 \\ \frac{\partial \psi}{\partial y} &\rightarrow 0, \frac{\partial \psi}{\partial x} \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \quad \dots (8)$$

Next, we bring in the non-dimensional variables for ψ and T as $\psi = \sqrt{c\nu}xf(\eta)$ and $T = T_\infty + (T_w - T_\infty)\theta(\eta)$ where $\eta = y\sqrt{\left(\frac{c}{\nu}\right)}$ is a similarity variable. Using the non-dimensional and similarity variables, eqns. (6) and (7) reduces to the following form:

$$f''' + ff'' - f'(f' - M^2) = 0 \quad \dots (9)$$

$$\theta'' + \text{Pr}(f\theta' - \lambda\theta) = 0 \quad \dots (10)$$

Here $\lambda = Q_0/\rho C_p$ is a heat source ($\lambda < 0$) or sink ($\lambda > 0$) parameter.

The boundary conditions (8) and (4) becomes in the following form:

$$\begin{aligned} f &= S, f' = -1, \theta = 1 \text{ at } \eta \rightarrow 0 \\ f' &\rightarrow 0, \theta \rightarrow 0, \text{ as } \eta \rightarrow \infty \end{aligned} \quad \dots (11)$$

Where $S = v_w/\sqrt{c\nu} > 0$ is the mass suction parameter.

3. Numerical procedure for solution

With the above described numerical procedures, Numerical solutions have been attained for the governing equations (9) and (10) with the assorted boundary condition (11) for some of the non-dimensional parameters, namely Magnetic parameter (M), Prandtl number (Pr), mass suction parameter(S), heat source parameter(λ). The Effects of M, Pr, S and λ on the fluid velocity and temperature flow over shrinking sheet are discussed in detail. The numerical computations are done by using the MATLAB in-built numerical solver bvp5c.

4. Results and Findings

Numerical computations are used to carry out results using the MATLAB in built solver for various values of non-dimensional parameters. For explaining the results, numerical values are located in figures from B to E. Also, in the table below, the comparison between the current and past results of skin friction coefficient $f''(0)$ for different values of mass suction parameter(S) with $M^2 = 2$ are shown. In this study a focus is laid on the numerical values of physical parameters namely, the mass suction parameter S , the Hartmann number M , the heat source/sink parameter λ and the Prandtl number Pr . In order to enforce a steady flow near the sheet by restricting the initiated vorticity inside the boundary layer, large values are assigned to S and M indicating a strong magnetic field and wall mass suction. Figures B to H depict the flow characteristics and

temperature field with varying parameter values. The current results can be assured the accuracy of applied numerical scheme as the results of Muhaimin et al[11]. in the above table and the computed skin friction coefficients are in perfect agreement. Practically, a lot of importance is given to the impact of Hartmann number M on temperature and velocity profiles. This is studied in Figures B and C, as M goes up, the value of f' increases indicating the rise in dimensionless velocity. Reasoning to this behaviour is that with a velocity of electrically conducting fluid, there arises a Lorentz force. The thus generated Lorentz force aids to the velocity of fluid in the boundary layer region thereby reducing momentum boundary layer thickness. It can be observed from Figure C that temperature value at a point decreases with M . Considering the impact of mass suction specification S on velocity and temperature profiles, we understand the following.

S	Current study	Muhaimin <i>et.al</i> [11]	Battacharryya[10]
2	2.414476	2.414214	2.41300
3	3.302816	3.302776	3.30275
4	4.236072	4.236068	4.23609

From figure D, it is clear that for a constant value of η , higher the value of applied suction, higher is the velocity profile leading to a thinner momentum boundary layer. Figure E illustrates wall mass suction has an effect on not only on velocity field but also on the dimensionless temperature distribution. Clearly it is evident that for fixed η , the temperature θ decreases on increasing the value of suction. This reduces the thickness of thermal boundary layer as a consequence. With increase in Prandtl number Pr , Figure F depicts the reduce in thermal boundary layer thickness and the dimensionless temperature profile. Also, it is understood that increased value of Pr means a reduced thermal fluid conductivity. This is the reason for reduced temperature. Also, as momentum equation is the same with or without θ , velocity of fluid field is not affected by Prandtl number Pr . Figure G indicates that on increasing heat sink strength, the dimensionless temperature θ decreases while on increasing heat source strength temperature increases. Hence it can be said that increased heat sink parameter reduces the thickness of thermal boundary layer but alternate behaviour can be observed with heat source parameter. Figure H depicts the varying temperature gradient at the sheet $\theta'(0)$ for varying magnitude of Pr and λ , which is considered important in calculating rate of heat transfer. The sign of $\theta'(0)$ indicates the heat transfer type, absorption is positive value and negative is transfer. On increasing Prandtl number, the rate of heat transfer increases an observation is made that, for small values of Prandtl number, Pr with higher magnitudes of heat source specification ($\lambda < 0$) there is heat absorption at the sheet. However, heat transfer increases with increased heat sink parameter ($\lambda > 0$).

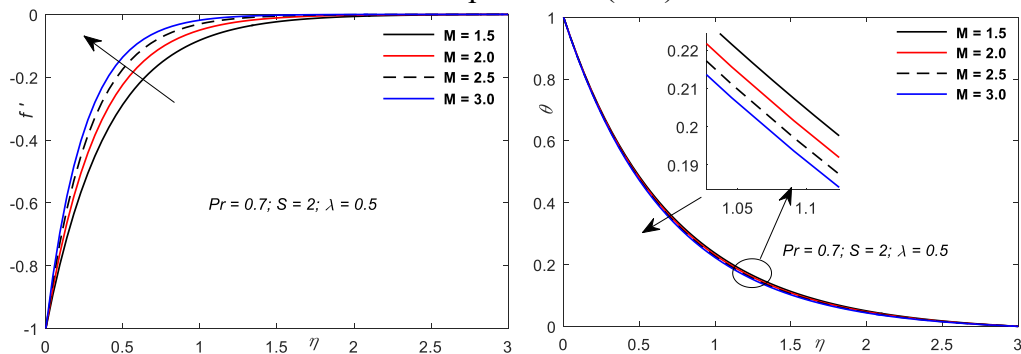
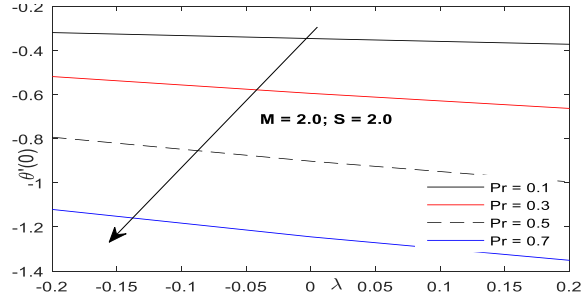
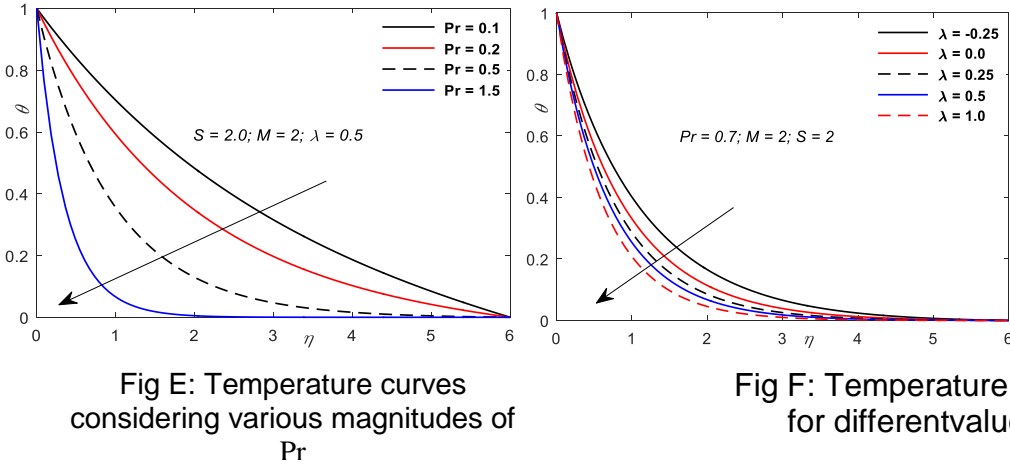
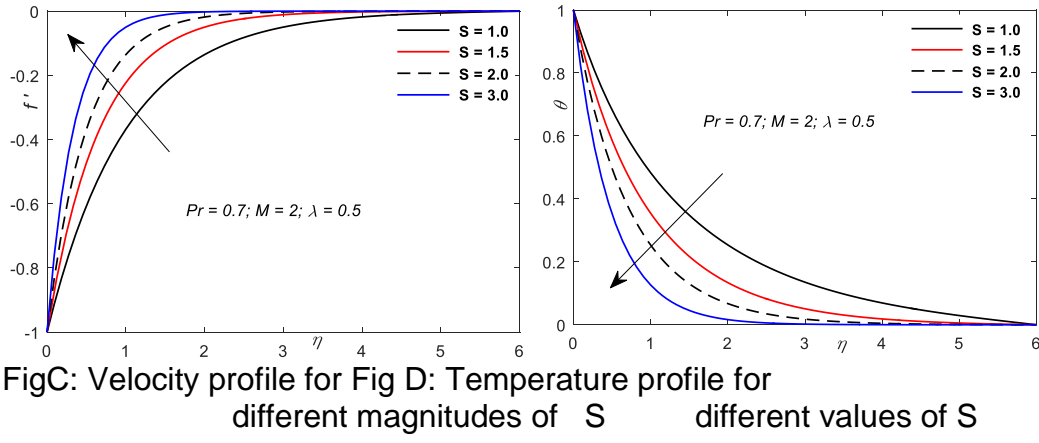


Fig A: Velocity curve for different magnitudes of M Fig B: Temperature profile for different magnitudes of M



5. Conclusions

Subjected to strong suction, a study is made on effect of heat source/sink on MHD boundary layer flow and heat transfer over a shrinking sheet. Similarity transformations helped to obtain the self-similar connections. The obtained equations are solved using MATLAB in built solver bvp5. It can be said that with an increase in Hartmann number and mass suction specifications, temperature at a point and thickness of momentum boundary layer reduces. On increasing Prandtl number, the thermal boundary layer thickness and temperature go down.

At the sheet, heat absorption is observed for higher values of heat source parameter. The transfer of heat is considerably high for higher values of Prandtl constant and heat sink specifications which is a critical aspect of production engineering useful to yield a good quality final product.

Nomenclature

u, v	Velocity components in x & y directions
$\nu = \mu/\rho$	Kinematic fluid viscosity
μ	Coefficient of fluid viscosity
ρ	Fluid density
σ	Electrical conductivity of the fluid
B_0	Applied uniform magnetic field
T, T_∞ & T_w	Temperature, Free stream Temperature and Sheet temperature.
C_p	Specific heat of the considered fluid.
k	Thermal conductivity of the considered fluid.
$c > 0$	Shrinking constant
$v_w > 0$	Wall mass suction parameter
$M = \sqrt{\sigma B_0^2 / c \rho}$	Hartmann constant
Pr	Prandtl constant

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