Proceedings of the International Conference on Applied Mechanical Engineering Research (IC-AMER2019)
NIT Warangal, India – May 02-05, 2019
Paper No. XXX (The number assigned when the papaer is accepted)

NATURAL CONVECTION ON A VERTICAL STRETCHING SURFACE WITH SUCTION AND BLOWING

Shankar Goud B[a], Ratna Kumari Jilugu[b,*], Gayathri Bharadwaj[c]
[a] *Department of Mathematics, JNTUH College of Engineering Hyderabad, Kukatpally-85, Telangana state, India*
[b] *02 Electrical Machines, BHEL Ramachandrapuram, Hyderabad-32, Telangana state, India*
[c] *Department of Mechanical Engineering, JNTUH College of Engineering Hyderabad, Kukatpally-85, Telangana state, India*
*Email: bsgoud.mtech@gmail.com, ratna.227@gmail.com, gayathribharadwaj.b@gmail.com

---

In this paper, the natural convective heat transfer from a stretching sheet oriented vertically involving surface mass transfer is of primary focus. A similarity solution in three dimensions is described for energy and momentum. The transformed equations are solved by using MATLAB in-built numerical solver bvp4c. For a range of Prandtl numbers and surface mass transfer rates, friction factor and Nusselt numbers are tabulated. Heat transfer mechanism is observed to have an influence by surface mass transfer.

***Keywords*:** Stretching sheet, Heat transfer, Natural convention, steady flow, friction factor.

1. Introduction

There are many practical applications of the viscous fluid flow study involving heat transfer due to stretching surface in the fields of manufacturing where plastic and rubber sheets are blown a gaseous medium through the unsolidified material. Other notable applications of the study are spinning of fibres, Extrusion processes, continuous coating and glass blowing. This was noted by some of the researchers viz., Surma Devi et.al [1] studied an Unsteady, three-dimensional, boundary-layer flow due to a stretching surface. Heat and mass transfer on a stretching sheet with suction or blowing was studied by Gupta PS and Gupta AS [2]. Bhattacharyya [3] shows the effects of heat source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction. Jat and Chaudhary [4] investigated on MHD flow and heat transfer over a stretching Sheet. Vajravelu K [5] presented the convection heat transfer at a stretching Sheet with Suction or Blowing. Fathizadeh et.al [6] has studied an effective modification of the homotopy perturbation method for MHD viscous flow over a stretching sheet. Heat transfer over a steady stretching surface in the presence of suction was analysed by Zailan et.al [7]. Chen and Char [9] have studied the heat transfer of a continuous, stretching surface with suction or blowing. J. E. Daskalakis[10] then studied free convection effects in the boundary layer along a vertically stretching flat surface. Similarity Solution on MHD boundary layer over stretching surface considering heat flux was studied by Ferdows et.al[11]. B.S.Goud. and MNR[12] nhave studied the Finite element solution of viscous dissipative effects on unsteady MHD flow past a parabolic started vertical plate with mass diffussion and variable temperature .Wang[13] present a free convection on a vertical stretching surface. Bestman,[14] investigated on natural convection boundary layer with suction and mass transfer in a porous medium. Acharya et.al[15] have studied on heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing. Finite element solution on effects of viscous dissipation & diffusion thermo on unsteady MHD flow past an impulsively started inclined oscillating plate with mass diffusion &variable temperature was studied by B.S.Goud and MNR [16]. Ali [17] showed on thermal boundary layer on a power-law stretched surface with suction or injection. Free convection boundary-layer over a vertical permeable flat plate in a porous medium with internal heat generation analysed by Postelnicu et.al [18]. Shateyi

[19] presented the thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. Chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet with suction or blowing has been studied by Bhattacharyya and G.C. Layek[20]. B.SGoud and MNR [21] presented the finite element method application of effects on an unsteady MHD convective heat and mass transfer flow in a semi-infinite vertical moving in a porous medium with heat source and suction.

The current work is about natural convective heat transfer in an incompressible, three-dimensional steady flow past a stretching sheet involving blowing and suction. Using Boussinesq assumptions, a three-dimensional similarity solution to the Navier-Stokes equations are described. The governing equations are transformed into a system of nonlinear ordinary differential equations by using the MATLAB in-built numerical solver bvp4c.

2. Mathematical Formulation

For this study, consider an incompressible, steady fluid flowing with a velocity bx past a vertical plane stretching in the x −direction. Also assume that y-direction makes an angle α with the horizontal line and z −direction normal to the sheet. Velocity components are given by (u, v, w) in (x, y, z) directions respectively. Figure 1 can be used as a reference for flow model and coordinate system. Let us make an assumption that the edge effects are negligible, hence all variables will be independent of the y −direction. Therefore the Boussinesq approximation governing equations are:

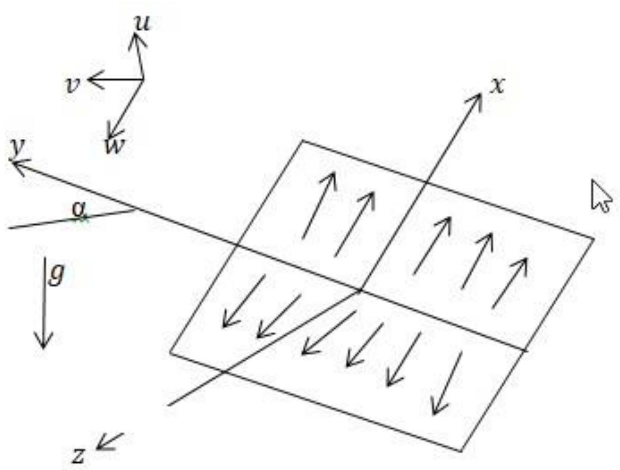

Figure:1. Flow model and coordinate system.

Momentum:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad \text{... (1)}$$

$$u\frac{\partial u}{\partial x} + w\frac{\partial w}{\partial z} = v\frac{\partial^2 u}{\partial x^2} + g\beta(T - T_\infty)\cos\alpha \qquad \text{... (2)}$$

$$w\frac{\partial \upsilon}{\partial z} = v\frac{\partial^2 \upsilon}{\partial z^2} + g\beta(T - T_\infty)\sin\alpha \qquad \text{... (3)}$$

$$w\frac{\partial w}{\partial z} = \upsilon\frac{\partial^2 w}{\partial z^2} - \frac{1}{\rho}\frac{\partial p}{\partial z} \qquad \text{... (4)}$$

Energy:

$$w\frac{\partial T}{\partial z} = \frac{v}{\Pr}\frac{\partial^2 T}{\partial z^2} \qquad \text{... (5)}$$

The boundary conditions are given by

$$z = 0: \quad u = bx, \; v = 0, \; w = W, \; T = T_w$$

$$z \to 0: \quad u = 0, \; v = 0, \; \frac{\partial w}{\partial z} = 0, \; T = T_\infty$$

Coordinate Transform

The number of independent variables is reduced by one or more if similarity solutions are used. This is the majority of existing exact solutions in fluid mechanics. Ames [8] describes the methods for equations of physical interest for generating similarity transformations. In the area of limiting solutions there may be a utility to a given problem for similarity solutions, which are usually asymptotic solutions. A physical insight into these details of complex fluid flows may be obtained by using similarity solutions for complex fluid flows. Most of the characteristics exhibited by these solutions describe the actual problem for the physical, dynamic and thermal parameter.

$$u = bxf'(\eta) + A\cos\alpha.\psi(\eta),\; v = A\sin\alpha\phi(\eta),\; w = -\sqrt{b\upsilon\varphi}\;, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \eta = \sqrt{\frac{b}{\upsilon}}\;, A = \frac{g\beta(T_w - T_\infty)}{b} \quad \text{... (6)}$$

Upon substituting the expressions in (6) into equations (1)-(5), we have

$$f''' + ff'' - (f')^2 = 0 \quad \text{... (7)}$$

$$\theta'' + \Pr f\theta' = 0 \quad \text{... (8)}$$

$$\phi'' + f\phi' + \theta = 0 \quad \text{... (9)}$$

$$\psi'' + f\psi' - \psi f' + \theta = 0 \quad \text{... (10)}$$

In the above equations, prime denotes differentiation with respect to η only. The transformed boundary conditions for the velocity and temperature fields are given by

$$\begin{aligned} &f(0) = f_w,\; f'(0) = 1,\; \theta(0) = 1,\; \phi(0) = 0,\; \psi(0) = 0,\\ &f'(\infty) = 0,\;\; \theta(\infty) = 0,\quad \phi(\infty) = 0,\quad \psi(\infty) = 0 \end{aligned} \quad \text{... (11)}$$

The mass transfer parameter $f_w = \frac{-W}{\sqrt{bv}}$ is negative for injection and positive for surjection.

The shear stresses at the stretching surface are given by

$$\tau_{zx} = \mu \left.\frac{\partial u}{\partial z}\right|_{z=0} = \rho\sqrt{b\nu}\left[bxf''(0) + A\cos\alpha.\psi'(0)\right] \quad \text{... (12)}$$

$$\tau_{zx} = \mu \left.\frac{\partial v}{\partial z}\right|_{z=0} = \rho\sqrt{b\nu}A\sin\alpha.\phi'(0) \quad \text{... (13)}$$

The surface skin friction coefficients in x and y directions may be written as

$$Cfx = \frac{2\tau_{zx}(0)}{\rho(bx)^2} = \frac{2}{\sqrt{\mathrm{Re}_x^2}}\left(-1 + \left(\frac{Gr_x}{\mathrm{Re}_x^2}\right).Cos\alpha.\psi'(0)\right) \quad \text{... (14)}$$

$$Cfy = \frac{2\tau_{zy}(0)}{\rho(bx)^2} = \frac{2}{\sqrt{\mathrm{Re}_x^2}}\left(\frac{Gr_x}{\mathrm{Re}_x^2}\right).Sin\alpha.\phi'(0) \quad \text{... (15)}$$

Where $Gr_x = \frac{x^3 g\beta(T_w - T_\infty)}{\nu^2}$, $\mathrm{Re}_x = \frac{(bx)x}{\nu}$, the local heat flux may be written by Fourier's Law as

$$q_w = -K_f \left.\frac{\partial T}{\partial z}\right|_{z=0} = -K_f\sqrt{\frac{b}{\nu}}(T_w - T_\infty)\theta'(0). \quad \text{... (16)}$$

The local heat transfer coefficient is given by $h = \frac{q_w}{(T_w - T_\infty)}$ ... (17)

The local Nusselt number may be written may be written as

$$Nu_L = \frac{hL}{K_f} == -\sqrt{\text{Re}_L}\,\theta'(0) \quad \text{... (18)}$$

Numerical Solution and Discussion

In this section the numerical solutions of the coupled non-linear differential equation (7)-(10) under the boundary conditions (10) have been performed by MATLAB in-built numerical solver bvp5c for various values of the parameters.

Wall friction and rate of heat transfer due to the varied mass transfer parameter f_w is numerically shown in Table 1. Injection increases friction factor and decreases the rate of heat transfer. Alternatively, suction decreases friction factor and increases the rate of heat transfer.

Increased Prandtl number leads to an increased rate of heat transfer, decreased induced buoyancy flow and shear stress. This behaviour is clearly observed from the data in table 2, where there are no conditions of surface mass transfer.

Figures 2 to 6 focus on the temperature and velocity distributions for the effect of surface mass transfer. Velocity distribution component in $z-$direction is depicted in figure 2 where $\text{Pr} = 7.0$. There is a lot of influence of surface mass transfer value. A constant value $f(0)$ is approached by $f(\eta)$ monotonically such that $f'(\infty) = 0$. The numerical values of $f(\infty)$ are a measure of entrainment velocity. At the surface, higher entrainment velocity is due to higher suction velocity. Within the boundary layer, $f'(\infty)$ distribution is shown in figure 3. It can be clearly understood that more linear shape for $f'(\infty)$ is obtained by higher values of injection. Distribution of normalized temperature profiles for $Pr = 7.0$ is shown in figure 4 with varying surface mass parameter. Higher injection values thicken the thermal boundary layer. Alternative behaviour of thermal boundary layer is observed in case of higher suction. Exponential decay of thermal boundary layer is evident in all specified cases. Location of maximum values of $\psi(\eta)$ (normalized free convection velocity in $z-$direction) and $\phi(\eta)$ (normalized free convection velocity in $y-$direction) is moved away from the surface on increased injection rates. Figures 5 and 6 clearly depict the exponential decay of the velocity boundary layers. With an increased velocity level, suction is increased. Temperature gradient at the surface is steep on higher values of injection. This is amalgamated with a decrease in temperature of the outer boundary region. Outer region of a downstream location has a temperature approximately equal to the local ambient temperature. In case of rapid increase in ambient temperature with height, the physical temperature increase due to heat transfer mechanisms may not be able to attain the local ambient temperature.When $\alpha = 0$, on a vertically stretched surface, equation 14 describes a zero shear stress $\left(\frac{Gr_x}{\text{Re}_x^2}\right)\psi'(0) = 1$.Critical value of the boundary parameter $\left(\frac{Gr_x}{\text{Re}_x^2}\right)$representing zero shear stress is smaller for lower Prandtl numbers than the ones with higher Prandtl numbers. For example, $\left(\frac{Gr_x}{\text{Re}_x^2}\right)_{crit}$ at $\text{Pr} = 0.07$ is given by 0.1740 while its value at $\text{Pr} = 70$ is given by 10.79. For $\alpha \neq 0$, velocity is present in all the directions hence, in a rigid surface, induced natural convection will not arise vertically. Alternatively, induced natural convection is slanted in the positive direction of $y-$axis, since $\phi(\eta)$ is larger than $\psi(\eta)$.

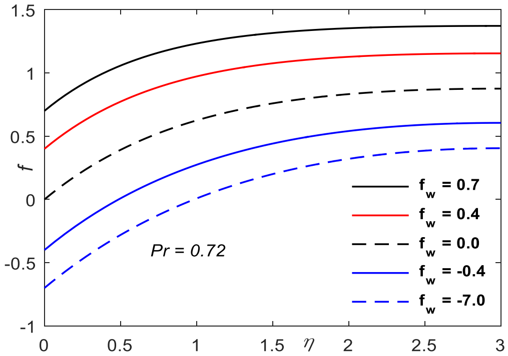

Figure 2: Stream function

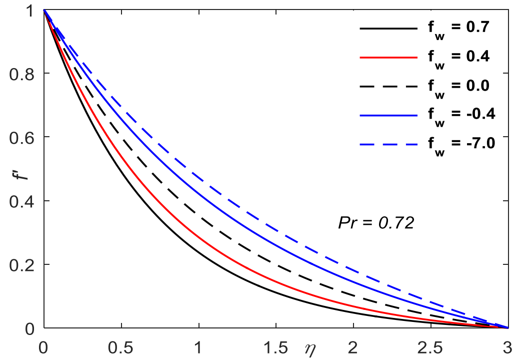

Figure 3. Distribution of $f'(\eta)$ within the boundary layer (velocity in z −direction).

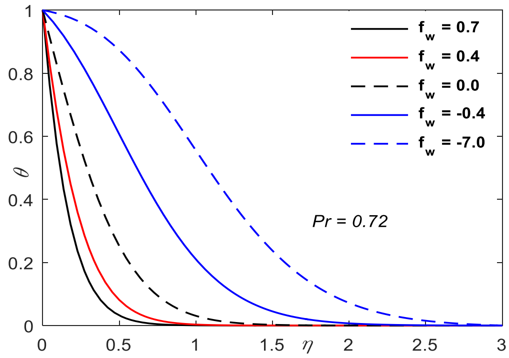

Figure 4. Temperature profiles.

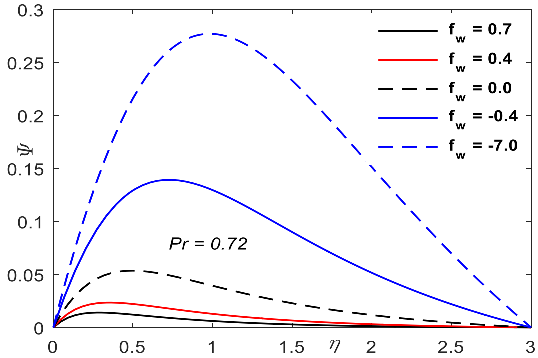

Figure 5. Decay of velocity boundary layers (velocity in z-direction).

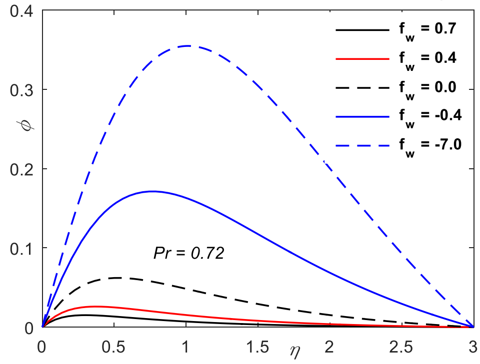

Figure .6. Decay of velocity boundary layers (velocity in y-direction).

Table 1: The values of $f''(0),\theta'(0),\phi'(0),\psi'(0)$ for varous values of mass transfer parameter fw for $Pr = 7.0$

fw	$f''(0)$	$\theta'(0)$	$\phi'(0)$	$\psi'(0)$
0.7	-1.418835	-5.799705	0.147772	0.142456
0.4	-1.231802	-3.989315	0.194900	0.185540
0.0	-1.014349	-1.890462	0.305539	0.281972
-0.4	-0.83420	-0.500116	0.507098	0.447013
-0.7	-0.722423	-0.087148	0.697118	0.589307

Table 2: The values of $f''(0),\theta'(0),\phi'(0),\psi'(0)$ for varous values of Prandtl number for $fw = 0$.

Pr	$f''(0)$	$\theta'(0)$	$\phi'(0)$	$\psi'(0)$

0.07	-1.014349	-0.352182	0.976804	0.822892
0.2	-1.014349	-0.388200	0.941902	0.795626
0.7	-1.014349	-0.534879	0.815210	0.696205
2.0	-1.014349	-0.911669	0.577967	0.507509
3.0	-1.014349	-1.159692	0.474313	0.423375
7.0	-1.014349	-1.890462	0.305539	0.281972
10	-1.014349	-2.303501	0.254389	0.237519
20	-1.014350	-3.349977	0.178888	0.170144
50	-1.014350	-5.424807	0.112816	0.109193
70	-1.014350	-6.458883	0.095310	0.092695

3. Conclusions

Free convection from a vertical stretching surface is analysed for the effects of surface mas transfer. Fluid flow and heat transfer characteristics are numerically computed for its dependence on Prandtl number and surface mass transfer. Tables are constructed for the missing values of thermal functions and velocity. Predictions on rate of heat transfer can be performed using this information. Also, surface friction factor can be estimated. Temperature field and flow field experience a higher influence by surface mass transfer. Higher entrainment velocity is observed at the surface due to suction. Linearity temperature profile and velocity profile is higher with injection. Thermal boundary layer thickness is increased by injection and is alternatively reduced by surface suction as well as temperature and velocity profiles decay exponentially. The rate of heat transfer increases with an increase in Prandtl number. With smaller Prandtl number, thermal boundary layer's thickness is found to reduce leading to reduced resistance to heat transfer.

Nomenclature

Cf	= friction factor
Gr	= Grashoff number
g	= acceleration due to gravity
h	= heat transfer coefficient
H	= dimensionless temperature
k	= thermal conductivity
M	= dimensionless velocity in y-direction
N	= dimensionless velocity in x-direction
Nu	= Nusselt number
$\Pr$	= Prandtl number
Re	= Reynolds number
T	= temperature
u, v, w	= velocity components in x, y, z directions respectively
x, y, z	= coordinate directions
α	= angle
β	= coefficient of thermal expansion
μ	= viscosity
ρ	= density
ϕ	= dimensionless velocity
η	= dimensionless distance
A	= free convection parameter
τ	= shear stress

ω	= conditions at the surface
∞	= conditions faraway from the surface

References

1. C. D. Surma Devi, H S. Takhar and G. Nath[1] "Unsteady, three-dimensional, boundary-layer flow due to a stretching surface", Int. J. Heat mass transfer,.29(2), 1996-1999,1986.
2. Gupta PS and Gupta AS [2], "Heat and mass transfer on a stretching sheet with suction or blowing", The Canadian Journal of Chemical Engineering, 55(6):pp. 744–746, 1977.
3. Krishnendu Bhattacharyya [3] Effects of heat source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction", Chemical Engineering Research Bulletin, 15, pp: 12-17,2011. DOI:10.3329/cerb.v15i1.6524.
4. R. N. Jat and Santosh Chaudhary[4] "MHD Flow and Heat Transfer over a Stretching Sheet , Applied Mathematical Sciences, 3(26), pp:1285 - 1294, 2009.
5. Vajravelu K [5]Convection Heat Transfer at a Stretching Sheet with Suction or Blowing Journal of Mathematical Analysis and Applications, 188(3),pp: 1002-1011,1994.
6. M. Fathizadeh, M. Madani, Yasir Khan, Naeem Faraz, Ahmet Yldırım, Serap Tutkun[6] An effective modification of the homotopy perturbation method for MHD viscous flow over a stretching sheet", Journal of King Saud University - Science, 25(2), pp. 107-113, 2013.
7. Zailan Siri , Nor Artisham Che Ghani and Ruhaila Md. Kasmani [7]"Heat transfer over a steady stretching surface in the presence of suction" . Boundary Value Problems),126, pp.1-16,2018.
8. Ames, W.F.,[8]Nonlinear Partial Differential Equations. New York: Academic Press (1965).
9. J. E. Daskalakis, "Free convection effects in the boundary layer along a vertically stretching flat surface," Canadian Journal of Physics, 70, pp. 1253–1260, 1993.
10. C. K. Chen and M. I. Char[10]Heat Transfer of a Continuous Stretching Surface with Suction or Blowing," Journal of Mathematical Analysis and Applications,135(2),pp. 568-580,1988.
11. M. Ferdows, Tania S. Khaleque and M.Z.I. Bangalee[11]Similarity Solution on MHD Boundary Layer over Stretching Surface Considering Heat Flux", Int.j. of heat and technology , 34(3), pp. 521-526, 2016.
12. B.Shankar Goud. and M.N. Raja Shekar[12]Finite element solution of viscous dissipative effects on unsteady MHD flow past a parabolic started vertical plate with mass diffussion and variable temperature", *I-manager's Journal on Mathematics*, 7(1), pp.20-27, 2018.
13. Wang, C. Y[13]. Free Convection on a Vertical Stretching Surface. ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift Für Angewandte Mathematik Und Mechanik, 69(11), pp. 418–420,1989.
14. A. R. Bestman,[14]"Natural convection boundary layer with suction and mass transfer in a porous medium," International Journal of Energy Research, 14(4), pp. 389–396, 1990.
15. M. Acharya, L. P. Singh, and G. C. Dash[15]"Heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing," International Journal of Engineering Science, 37(2), pp. 189–211, 1999.
16. B. Shankar Goud., M.N Rajashekar[16] "Finite element solution on effects of viscous dissipation & diffusion thermo on unsteady MHD flow past an impulsively started inclined oscillating plate with mass diffusion &variable temperature", *Int. Research Journal of Engineering and Technology*, 4(2), pp.471-477,2017.
17. Ali, M. E[17]. "On thermal boundary layer on a power-law stretched surface with suction or injection", International Journal of Heat and Fluid Flow, 16(4), 280–290,1995.
18. A. Postelnicu, T. Grosan, I. Pop[18], Free convection boundary-layer over a vertical permeable flat plate in a porous medium with internal heat generation, Int. Comm. Heat Mass Transfer, 27 (5) 729–738,2000.

19. S. Shateyi [19]"Thermal Radiation and Buoyancy Effects on Heat and Mass Transfer over a Semi-Infinite Stretching Surface with Suction and Blowing", Journal of Applied Mathematics Volume 2008, Article ID 414830, 12 pages.
20. K. Bhattacharyya and G.C. Layek[20]Chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet with suction or blowing", Chemical Engineering Communications, Vol. 197, Issue 12, pp.1527-1540,2010.
21. B. Shankar Goud, MN Rajashekar[21] "Finite element method application of effects on an unsteady MHD convective heat and mass transfer flow in a semi-infinite vertical moving in a porous medium with heat source and suction*", IOSR Journal of Mathematics (IOSR-JM),* 12(6) Ver. IV, pp.55-64, 2016.