

Topology Optimization of Concrete Dapped Beams Under Multiple Constraints

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Abstract. Topology optimization is now becoming the effective method for solving various problems related to engineering. Optimization is a mathematical method to find the optimum solution by satisfying all the constraints associated with that problem while topology optimization is a branch of structural optimization as it finds optimum material layout within the given boundary. This study focuses on the topology optimization of concrete dapped beams with various constraints to ensure the applicability of topology optimization during the design phase of structures. Compliance minimization with three different constraints along with volume constraint has been selected to derive the truss like pattern for beams. To derive a light weight structure with stress constraint, volume based topology optimization has adopted. Strut and Tie modelling (STM) of concrete members has been identified as a powerful method for modeling discontinuity regions within the structural member. Topology optimization can be used as a supporting method for developing more reliable Strut and tie models.

Keywords: Topology optimization, SIMP, Strut and Tie model, Compliance

1 Introduction

Topology optimization is intended to find the optimum load path associated with a particular load and boundary conditions by satisfying various requirements in the form of objective and constraints. The history of structural optimization have traced from the work of Michell [1] in which he derived light weight structure for material economy with stress constraint. Vanderplaats[2] identified Schmit's work[3] as the turning point in modern structural optimization as he combines the finite element analysis with nonlinear structural optimization. Topology optimization methods can be divided based on the domain and search method for solution. Starting from the work of Prager[4], Rozvany[5] had adopted optimality criteria methods for different optimization problems. Homogenization method has been widely used in topology optimization [6] in which a microstructure is created in material resulting in a composite structure. Bendsoe [7] introduced a method to derive a non-discrete solution by introducing a density function that allows the discrete variables to vary continuously. This approach is termed as Solid Isotropic material with Penalization (SIMP). Buhl *et al.* [8] used the SIMP approach along with the Method of Moving Asymptotes (MMA) to minimize various objective functions of geometrically nonlinear structures subject to volume constraints.

Discontinuity regions (D-regions) are the portions of structural member where nonlinear strain distribution occurs as a result of geometry or loading. Strut and tie modeling is an accepted method for designing D-regions where Bernoulli's hypothesis is not valid. German civil engineer Ritter [9] designed concrete beams using truss analogy method where concrete beams could carry compressive forces and reinforcing bars could carry tensile forces. The STM method has become popular for designing D-regions after the landmark paper of Schlaich[10]. Conventional methods of STM involves lot of trial and error

procedure which can be overcome by iterative computer programs [11]. Bruggi [12] proposed a methodology deals with the generation of truss-like designs to derive preliminary strut-and-tie models not only in the established bi-dimensional context but also in a 3D environment. This paper focuses on the topology optimization of Concrete Dapped beams with multiple constraints in ABAQUS finite element software.

2 Modeling of Dapped end beams

Reinforced concrete dapped end beams (RC-DEB) are commonly used in concrete bridge girders and prepared concrete buildings. The use of dapped beams enables the erection of precast members due to its lateral stability of an isolated dapped end beam than that of an isolated beam. In some cases the nib of a dapped end beam is similar to an inverted corbel. Due to the geometrical nonlinearity, high stress concentration arises at reentrant corners of dapped beams which should be reinforced properly to avoid failure [13]. Aim of the present study is to evolve truss like pattern of RC-DEBs with two daps under different constraints using topology optimization. The model dimensions and boundary conditions are shown in Fig.1. Symmetrical loading and boundary conditions are given. Concrete grade of M30 has selected with Poisson's ratio 0.15. The beam is modeled in ABAQUS with a mesh size of 40mm. ABAQUS software adopts SIMP material interpolation scheme and the penalty factor entered as 3. Initial design will converge to an optimum topology after finite element analysis, sensitivity analysis and design variable updating with optimality criteria method. The finite element model with load and boundary condition is given in Fig.2.

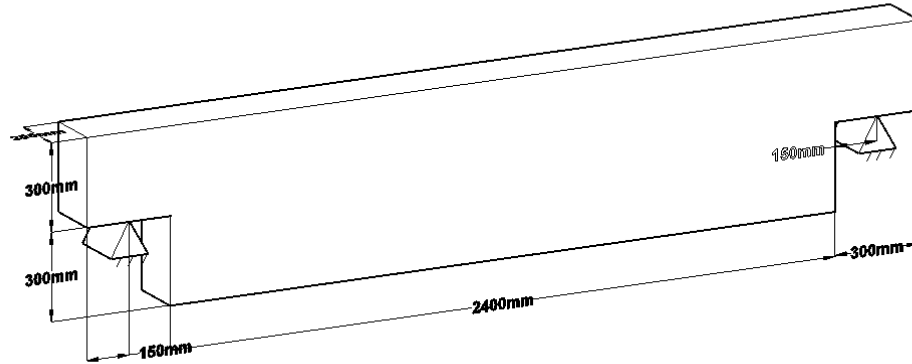


Fig.1. Dimensions and boundary conditions of Dapped beam

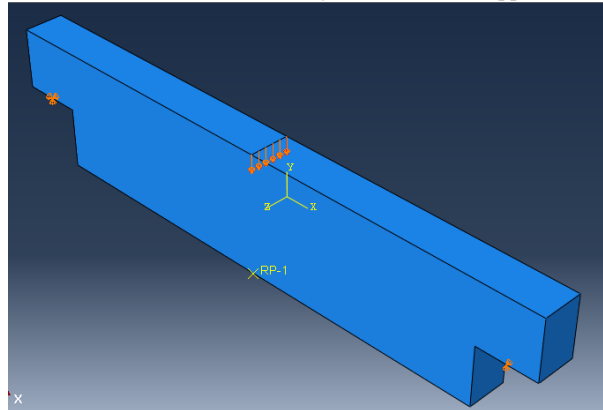


Fig.2. Finite element model of Dapped beam

3. Formulation of Topology optimization problems

Four different optimization problems adopted for simulation are stated below. Compliance is an inverse indicator of stiffness. To derive the stiffest layout of a structure for a given set of loadings and boundary conditions, minimization of compliance can be selected as an objective function.

$$\begin{aligned} \text{Problem 1: Minimize} \quad & \mathbf{C} = \mathbf{C}(x) = \frac{1}{2} \mathbf{u}^T \mathbf{K} \mathbf{u} \\ \text{Subject to:} \quad & \mathbf{d}_j(x) - \mathbf{d}^* \leq 0 \\ & \frac{V(x)}{V^*} = f \end{aligned} \quad (1)$$

$$\begin{aligned} \text{Problem 2: Minimize} \quad & \mathbf{C} = \mathbf{C}(x) = \frac{1}{2} \mathbf{u}^T \mathbf{K} \mathbf{u} \\ \text{Subject to:} \quad & \omega^* - \omega_n(x) \leq 0 \\ & \frac{V(x)}{V^*} = f \end{aligned} \quad (2)$$

$$\begin{aligned} \text{Problem 3: Minimize} \quad & \mathbf{C} = \mathbf{C}(x) = \frac{1}{2} \mathbf{u}^T \mathbf{K} \mathbf{u} \\ \text{Subject to:} \quad & \sigma_n(x) - \sigma^* \leq 0 \\ & \frac{V(x)}{V^*} = f \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Problem 4: Minimize} \quad & \mathbf{Volume} \\ \text{Subject to:} \quad & \sigma_n(x) - \sigma^* \leq 0 \end{aligned} \quad (4)$$

where x is the vector of design variables, u is the displacement vector, K is the global stiffness matrix, C is the mean compliance, d_j is the magnitude of displacement vector of j th node, ω_n is the n th mode natural frequency, σ_n is the stress of n th element, V is the material volume, V^* is the design domain volume, f is the prescribed volume fraction with d^*, ω^* and σ^* being the imposed constraint values.

First three problems adopts minimization of compliance as an objective function along with volume constraint. Equation 1 represents optimization formulation with compliance minimization and displacement constraint as an additional constraint while second (equation 2) and third problem (equation 3) adopts Eigen frequency and stress as a constraint. Fourth problem derives a light weight structure with stress as a constraint.

4. Results and Discussion

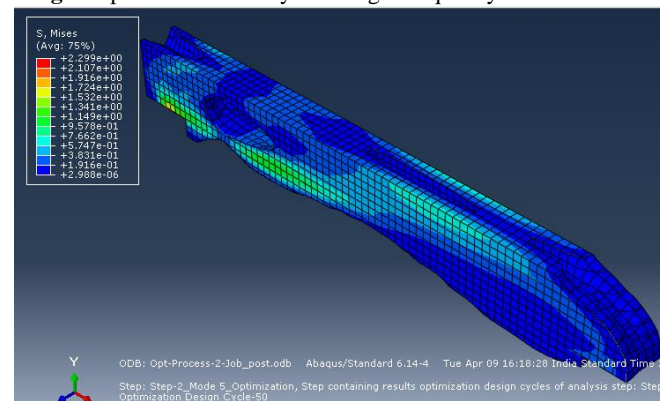
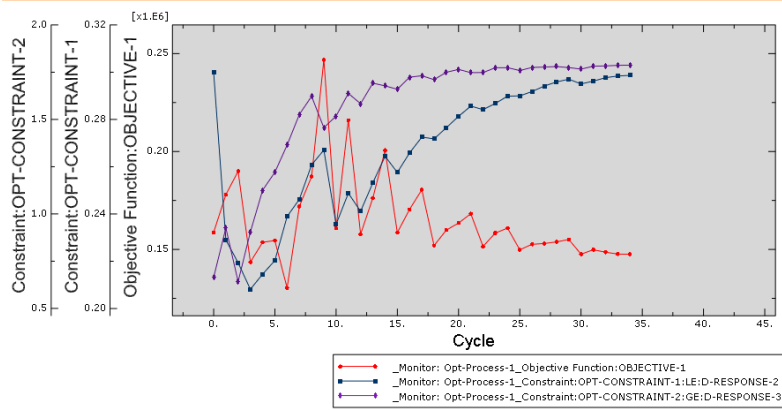
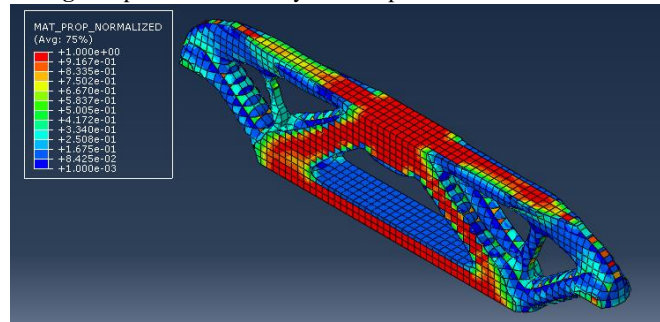
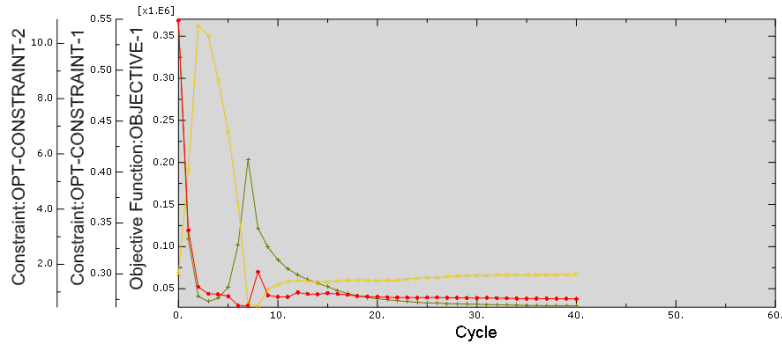
In all the compliance minimization problems volume fraction of 30% selected as volume constraint. A central load of 60KN has applied for simply supported concrete dapped beam.

4.1 Problem 1

Displacement constraint $\leq 1\text{mm}$ at the centre of the bottom surface including all nodes had applied. The optimization history of objective and constraints is shown in Fig.3. Final layout of beam is shown in Fig.4.

4.2 Problem 2

Eigen frequency of first mode $\geq 12.57 \text{ rad/time}$ had applied as a constraint. Fig.5 and Fig.6 represents the optimization history and optimum material layout.



4.3 Problem 3

Material strength of concrete is 35N/mm^2 . A stress criteria of von Mises stress should not exceed material strength had adopted. Optimization history and material layout is shown in Fig.7 and Fig.8.

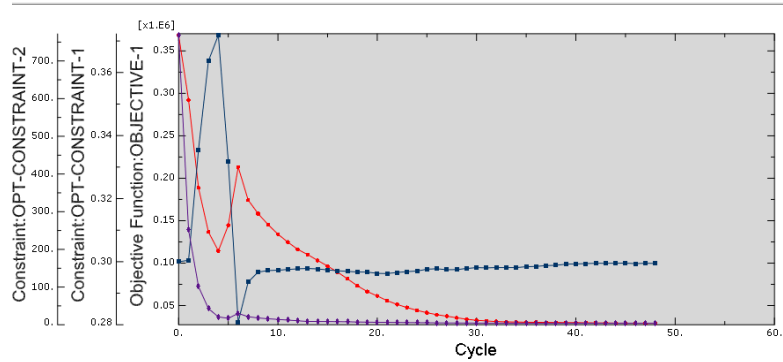


Fig.7. Optimization history with Stress as Constraint-2

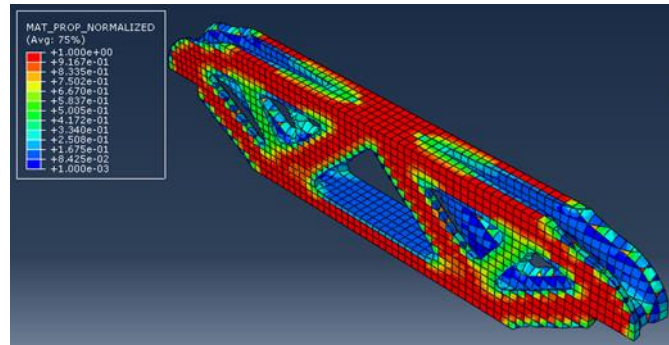


Fig.8. Optimum material layout

4.4 Problem4

Some convergence issues had occurred while selecting volume as an objective function with material strength as constraint. However a reasonable optimum material layout without violating constraint is shown in Fig.9.

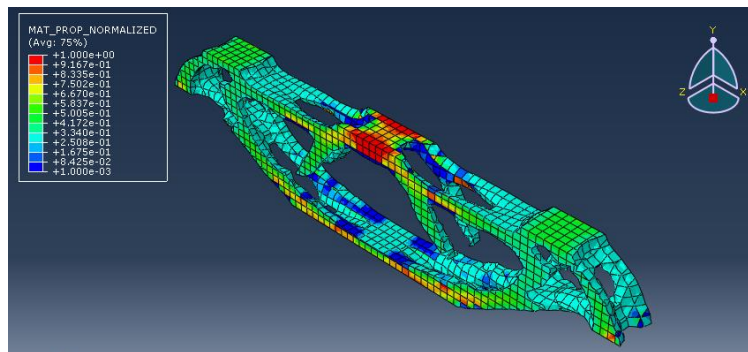


Fig.9. Optimum material layout

5. Conclusions

Present study focuses on the topology optimization of concrete dapped beams with different constraints. As Strut and Tie modeling is an accepted method for modeling discontinuity regions, all inaccuracies related to STM can be avoided with the help of topology optimization which relies on structural mechanics. Different constraints based on the design requirements had adopted in this study to ensure the applicability of topology optimization during the design phase. The constraints selected evaluates the structural performance which help to save material while satisfying functional constraints in optimization problems. As this method results in more accurate results based on load path method which can be utilized for solving various civil engineering problems. As the dimensioning of Strut and Tie model is beyond the scope of this study, it has not presented.

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