FINITE ELEMENT ANALYSIS OF INCLINED BEAM SUBJECTED TO MOVING POINT LOAD

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In this paper, dynamic response of the inclined beam with rectangular cross-section is formulated, considering moving concentrated load with constant velocity. Finite element method is suitably used for formulation of the problem. Equation of motion of beam is based on Euler-Bernoulli beam theory. Convergence study is made for natural frequencies of pinned-pinned beam. The case is extended for horizontal and inclined beam as well. Newmark integration method is implemented for forced vibration of the structure. The results are obtained using MATLAB code. Influence of dynamic magnification factor on varying speed of load is investigated when load is applied at mid-point of simply supported beam. Effect of velocity parameter of moving load is studied. Results are presented in graphical form with respect to vertical dynamic displacement of beam for different angle of inclination.

***Keywords*:** Finite element method, Newmark's Integration method, Inclined beam, Moving load

1. Introduction

The dynamic response for forced vibrations of beam type structure under the act of concentrated loads is widely used in aerospace and mechanical applications. Many studies have been done in past few years to observe and predict behaviour of isotropic structure as well as FGM made system. Little works have been done to study finite element analysis of inclined beam subjected to point load. Bridges under influence of vehicles, and trolleys used in the runways of bridge cranes can be simplified as moving loads on simply supported beams. Using perturbation and finite difference method , motion and impact of mass and velocity of pinned-pinned beam has been studied subjected to moving load [1],[2],[3].To study infinite beam with multiple span, modal analysis is done to understand the problem[4] and for finite beam, green function approach is preferred [5]. To understand simply supported[6]–[8] and cantilever beam [9], finite element method is preferably used for Euler–Bernoulli and Timoshenko beam [10], [11]. and is compared with Galerkin's [12] and finite difference technique[13] [14]for verification. Work has been done for multi-span Euler–Bernoulli beam [15] and Timoshenko beams studied [11], [16] subjected to load moving at constant velocity using Eigen function expansion. In some work of beam structure, soil foundation are considered [17]–[19] for dynamic analysis.

From above work, it is limited to horizontal beam. But no work has been done to demonstrate effect of dynamic magnificent factor with respect to various parameters of beam, considering influence of angle of beam due to point load condition. Dynamic magnification factor "D" is the ratio of the highest values of dynamic deflection to the static deflection at the mid-point of the beam. Before understanding of above title, Convergence is made for natural frequencies of pinned-pinned beam. The case is extended for horizontal and inclined beam as well. Newmark integration method is implemented for forced vibration of the structure.

2. Modelling and formulation

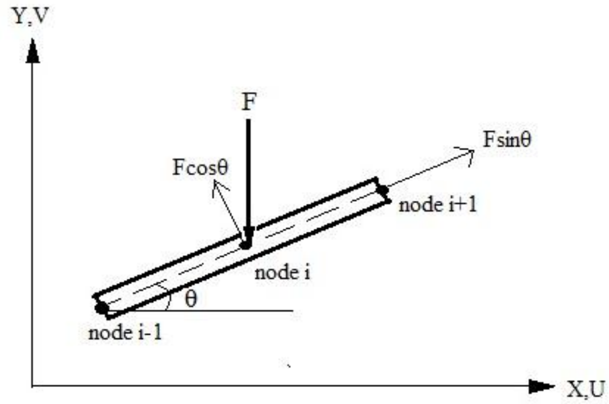


Fig. 1. Moving load analysis at i[th] node

An inclined beam of length L with angle of inclination of θ , subjected to vertical force of F with constant velocity, $v=L/\tau$, where τ represent time taken by vertical load to cover the beam. As it is an inclined beam, axial deformation is also considered along with vertical displacement and rotation for stiffness matrix of beam element as shown in Eq. (1) .

$$K = \begin{bmatrix} \frac{A_e E}{L_e} & 0 & 0 & -\frac{A_e E}{L_e} & 0 & 0 \\ 0 & \frac{12EI_e}{L_e^{3}} & \frac{6EI_e}{L_e^{2}} & 0 & -\frac{12EI_e}{L_e^{3}} & \frac{6EI_e}{L_e^{2}} \\ 0 & \frac{6EI_e}{L_e^{2}} & \frac{4EI_e}{L_e} & 0 & -\frac{6EI_e}{L_e^{2}} & \frac{2EI_e}{L_e} \\ -\frac{A_e E}{L_e} & 0 & 0 & \frac{A_e E}{L_e} & 0 & 0 \\ 0 & -\frac{12EI_e}{L_e^{3}} & -\frac{6EI_e}{L_e^{2}} & 0 & \frac{12EI_e}{L_e^{3}} & -\frac{6EI_e}{L_e^{2}} \\ 0 & \frac{6EI_e}{L_e^{2}} & \frac{2EI_e}{L_e} & 0 & -\frac{6EI_e}{L_e^{2}} & \frac{4EI_e}{L_e} \end{bmatrix} \quad (1)$$

Where A_e= cross section area of beam element,
E= Young's modulus of element,
I_e=Beam element's moment of inertia about Z-axis
L_e=element length of beam

Similarly, consistent mass matrix for 3-DOF beam element is obtained as:

$$M = \begin{bmatrix} 2a & 0 & 0 & a & 0 & 0 \\ 0 & 156b & 22L_e^{2}b & 0 & 54b & -13L_e b \\ 0 & 22L_e^{2}b & 4L_e^{2}b & 0 & 13L_e b & -3L_e^{2}b \\ a & 0 & 0 & 2a & 0 & 0 \\ 0 & 54b & -13L_e b & 0 & 156a & -22L_e^{2}b \\ 0 & 13L_e b & -3L_e^{2}b & 0 & -22L_e^{2}b & 4L_e^{2}b \end{bmatrix}, \text{ where } a = \frac{\rho A_e L_e}{6} \text{ and } b = \frac{\rho A_e L_e}{420} \quad (2)$$

Using Eqs. (1)-(2), overall matrices of the beam [M] & [K] are formed by assembling mass and stiffness matrices e.g K and M matrices of "n" number of elements, showing connectivity of it. Equation of motion for the beam is represented as

$$[M]\{\ddot{U}\}+[K]\{U\}=\{F(t)\} \tag{3}$$

Where $\ddot{U}$ and U are acceleration and velocity vectors of the beam and F(t) is time dependent load vector. Natural frequencies of the beam are obtained under free vibration condition. Due to inclination, vertical load F acting on the i^{th} node of the beam element is considered in the x and y direction as Fsinθ and Fcosθ respectively as shown in Fig. (1). For analysis, Newmark integration method is being used, considering integration parameters of constant average acceleration, $\beta = 1/4$ and $\gamma = 1/2$. Velocity parameter is considered, $\alpha = T/\tau$ where T is time period of first frequency and τ is considered as time taken by point load to cover the beam.

3. Numerical results and discussions

For numerical results, a simply supported rectangular beam of length 1m is considered with width 0.05m and height 0.01m, mass density 7860kg/m^3 and modulus of elasticity 206 x 10^9 kg/m^3. Fig. (1) shows graphical representation of frequencies vs. number of elements in mess for pinned-pinned conditions of beam respectively under free vibration. Convergence study of first six frequencies is done. The maximum number of elements is taken as 120 in order to get convergent and accurate values of first six frequencies. In order to validate dynamic analysis of inclined beam subjected to point load, angle of inclination is taken as zero and compared with existing literature [20]. Fig. (2) shows effect of velocity parameter "α" on dynamic behaviour when load is applied on the midpoint of beam, considering angle of inclination as 0^o, 15^o, 30^o and 45^o. With increasing angle of inclination of beam, vertical displacement decreases resulting in decrease of dynamic magnification factor of mid-point of simply supported beam w.r.t velocity parameter. Similarly in Fig. (3), it shows effect of load variation on dynamic behaviour on midpoint of beam when angle of inclination is 15^o. With increasing value of point load, dynamic magnification factor increases significantly. In Fig. (4), Time histories for dynamic vertical displacements of midpoint of beam due to varying load, when α=1.6 and θ=15^o is shown. One can visualised that value of load is directly proportional to vertical displacement caused by moving load. Increment of loads, result in decrement of vertical displacement caused by moving load, which is presented in Fig. (5)

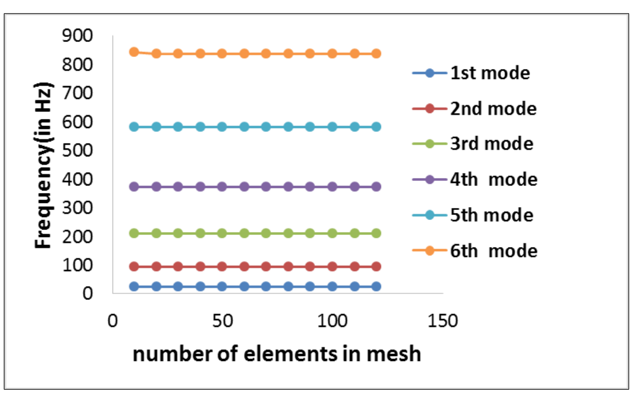


Fig. 1. convergence of frequencies with no of elements in mesh for the P-P beam.

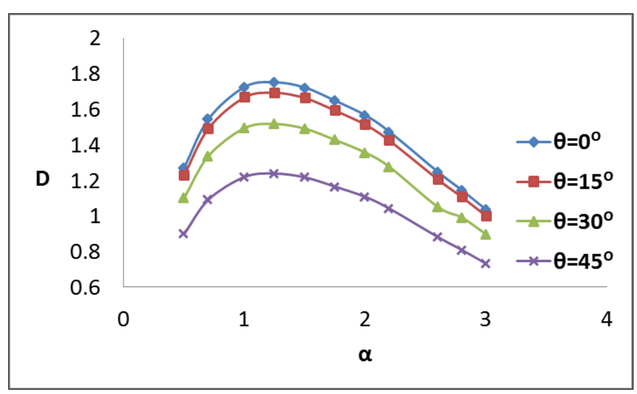
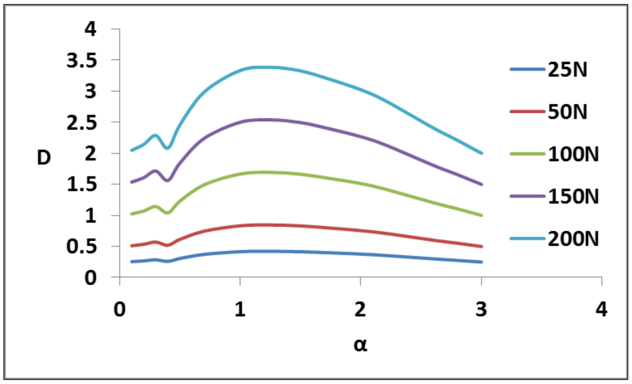


Fig. 2. Effect of α on dynamic magnification factor on midpoint of beam at various angle of inclination.



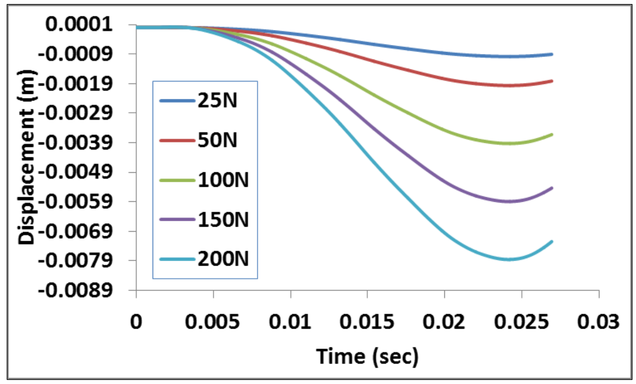


Fig. 3. Effect of α on dynamic magnification factor on midpoint of beam due to varying load.

Fig. 4. Time histories for dynamic vertical displacements of midpoint of P-P beam due to varying load, when α=1.6 and θ=15°

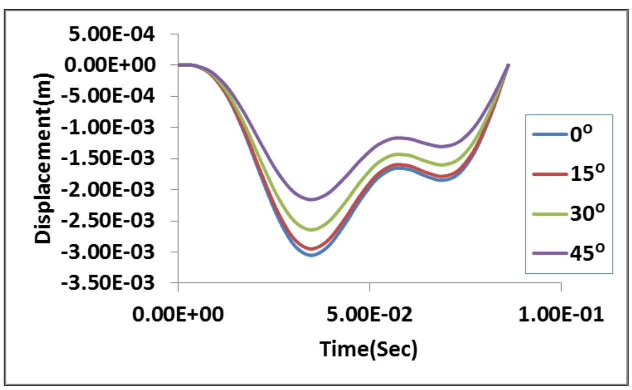


Fig. 5. Time histories for dynamic vertical displacements of midpoint of P-P beam due to varying angle of inclination, when α=0.5

4. Conclusion

This study comprises of dynamic analysis of inclined beam subjected to point load, moving at constant velocity. For this purpose, pinned-pinned conditions of the beam are taken. To validate the results, convergence studies are done and comparative studies is done with existing literature. Various graphs are successfully presented to study the effect of dynamic characterises due to angle of inclination of beam.

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