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Analysis of Hygrothermal Stresses on a Laminated Composite Plate Subjected to UDL using Classical Laminated Plate Theory (CLPT)

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Abstract

In this present work illustrates the evaluation/Simulation of Hygrothermal Stresses on a simply supported carbon laminated composite plate subjected to Uniformly Distributed Load(UDL) of Mechanical, Hygrothermal and both Mechanical and Hygrothermal Loads. The computed numerical results are compare with the published results the accuracy of procedure. Tsai-Wu failure criterion is employed to do study the first ply failure analysis. The developed procedure is implemented in MATLAB. The stresses and first ply failure loads are computed when a simply supported carbon composite plate is subjected to Mechanical, Hygrothermal, and both Mechanical and Hygrothermal Loads. The generated results are found to be in good agreement with the published results.

1. Introduction:

A composite is structural material consists of two or more combined constituents that are combined at macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. It is well known that composite materials are being widely used in recent years in such diverse industries as aerospace, chemical, automobile, ship building and construction. Very often structures made of composite materials are expected to perform under widely varying temperature and moisture loadings. This has led to an increased in the hygrothermoelastic response of composite structures in recent years.

- **CLASSICAL LAMINATE PLATE THEORY**

The classical laminated plate theory is an extension of the classical plate theory to composite laminates. In the classical laminated plate theory (CLPT) it is assumed that the Kirchhoff hypothesis holds:

An assumption is that which is necessary for the development of the mathematical model, whereas a restriction is not a necessary condition for the development of the theory.

(1) Straight lines perpendicular to the midsurface (i.e., transverse normals) before deformation remain straight after deformation.
(2) The transverse normals do not experience elongation (i.e., they are inextensible).
(3) The transverse normals rotate such that they remain perpendicular to the midsurface after deformation. The first two assumptions imply that the transverse displacement is independent of the transverse (or thickness) coordinate and the transverse normal strain ϵ_{zz} is zero. The third assumption results in zero transverse shear strains, $\epsilon_{xz} = 0$, $\epsilon_{yz} = 0$.

- **FIRST ORDER SHEAR DEFORMATION THEORY**

The classical laminated plate theory is used for relatively thin plates. As the plate is going to relatively very thick (i.e., $a/h < 20$), then CLPT does not give the accurate values. Then a method called as FSDT (First order Shear Deformation Theory) is used. In the first-order shear deformation theory (FSDT), a constant state of transverse shear stresses is accounted for, and often the transverse normal stress is neglected. The more significant difference between the classical and first-order theories is the effect of including transverse shear deformation on the predicted deflections, frequencies, and buckling loads. As noted in the classical laminate theory under predicts deflections and over predicts frequencies as well as buckling loads with plate side-to-thickness ratios of the order of 20 or less. For this reason alone it is necessary to use the first-order theory in the analysis of relatively thick laminated plates.

In CLPT the peel stresses are zero. To overcome this disadvantage First Order Shear Deformation theory is formulated.

2. Equations

Boundary Conditions:

The following are the different types of simply supported boundary conditions on all four edges of the plate.

$$\left.\begin{array}{l} \text{BC1a: } u=w=\Phi_y=0 \text{ at } x=0, a \text{ and } v=w=\Phi_x=0 \text{ at } y=0, b \\ \text{BC1b: } v=w=\Phi_y=0 \text{ at } x=0, a \text{ and } u=w=\Phi_x=0 \text{ at } y=0, b \\ \text{BC1c: } u=v=w=\Phi_y=0 \text{ at } x=0, a \text{ and } u=v=w=\Phi_x=0 \text{ at } y=0, b. \end{array}\right\} \quad (1)$$

Strength Ratio:

$$SR = \frac{Maximum\ Load\ Which\ Can\ Be\ Applied}{Load\ \ Applied} \longrightarrow (2)$$

According to Tsai-Wu criterion: $a\,S_{fk}^2 + bS_{fk} + c = 0 \longrightarrow (3)$

Where

$$a=f_{11}(\sigma_1^2)_k+f_{22}(\sigma_2^2)_k+f_{66}(\tau_6^2)_k+2f_{12}(\sigma_1\sigma_2)_k;$$

$$b=f_1(\sigma_1)_k+f_2(\sigma_2)_k+2f_{11}(\sigma_1\,\sigma_{1e})_k+2f_{22}(\sigma_2\,\sigma_{2e})_k+2f_{66}(\tau_6\tau_{6e})_k+2f_{12}(\sigma_1\,\sigma_{1e}+\sigma_2\,\sigma_{2e})_k;$$

$$c=f_1(\sigma^{1e})_k+f_2(\sigma_{2e})_k+f_{11}(\sigma_{1e}^2)_k+f_{22}(\sigma_{2e}^2)_k+2f_{66}(\tau_{6e}^2)_k+2f_{12}(\sigma_{1e}\,\sigma_{2e})_k-1,$$

$$f_1=(1/(F_1^T)_{ult})-(1/F_1^C)_{ult}) \;,\; f_{11}=(1/(F_1^T)_{ult})(F_1^C)_{ult}) \,,\; f_2=(1/(F_1^T)_{ult})-(1/F_2^C)_{ult})$$

$$f_{22}=(1/(F_2^T)_{ult})-(F_2^C)_{ult}) \,,\; f_6=0 \,,\; f_{66}=(1/(F_{12}^T)_{ult}{}^2 \,,\; f_{12}=-\frac{1}{2}\sqrt{1/((F_1^T)_{ult}(F_1^C)_{ult}(F_2^T)_{ult}(\sigma_2^C)_{ult})} \,.$$

The deflection of a plate subjected to temperature:

The temperature variation in the shell structure is either uniform everywhere, i.e., $T(x,y,z)=T_0$, or linearly varying across the thickness, i.e., $T(x, y, z)=T_1 z/h$. The nondimensionalized deflection values are given as

$w=h\,w/\alpha_L T_1 a^2$ (or) $w^*=h\,w/\alpha_L T_0 a^2$.

where

α_L is the longitudinal coefficient of thermal expansion,

h is the laminate thickness, and a is the length of the plate.

3. RESULTS & CONCLUSIONS:

The following properties are used in the analysis [IV]:

$E_1/E_2=25$, $G_{12}=G_{13}=0.5E_2$, $\upsilon_{12}=0.25$, $\alpha_1=3\alpha_2$.

Table 3.1 Non dimensional transverse deflections and stresses of simply supported plate subjected to UDL

Laminate	Source	$\hat{w}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xz}}$
0°	Reference [IV] Present	0.6497 0.64973	0.7866 0.78663	0.0244 0.02441	0.0643 0.06434
(0°/90°/0°)	Reference [IV] Present	0.6660 0.66601	0.8075 0.80753	0.0306 0.03064	0.0425 0.04251

Table 3.2 Non dimensional transverse deflections and stresses of simply supported plate

a/h	Load	Source	$\hat{w}*10^2$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xy}}$	$\overline{\sigma_{xz}}$
20	UDL	Reference [IV] Present	1.7582 1.6835	1.0747 1.0998	0.0943 0.0828	0.5802 0.5307
100	UDL	Reference [IV] Present	1.6980 1.6835	1.0761 1.0998	0.0933 0.0828	0.5813 0.5307
CLPT (a/b = 1)	UDL	Reference [IV] Present	1.6955 1.6835	1.0761 1.0998	0.0933 0.0828	- -

The Hygrothermoelastic stresses of all the plies of [0/90] laminated plate subjected to a

temperature change.
The following properties are given for the analysis [II]:
E_1=181GPa, E_2=10.3GPa, G_{12}=7.17GPa, υ_{12}=0.28,α_1=0.02µm/m/˚c, α_2=22.5µm/m/˚c, α_{12}=0, β_1=0, β_2=0.6m/m/kg/kg, β_{12}=0, ΔT=45, Δc=0.01.
Table 3.3.1 The Hygrothermoelastic stresses at the Top of each ply of a simply supported plate.

Ply	σ_{1e}	σ_{2e}	σ_{12e}
0°	-1.1069e+004	-0.1802e+004	0.0000
90°	0.3704e+003	-8.8971e+003	0.0000

Table 3.3.2 The Hygrothermoelastic stresses at the Bottom of each ply of a simply supported Plate.

Ply	σ_{1e}	σ_{2e}	σ_{12e}
0°	8.8971e+003	-0.3704e+003	0.0000
90°	0.1802e+004	1.1069e+004	0.0000

The nondimensionalized central deflection (*w**) values of four-layer simply supported plates (BC 1b) with uniform temperature field are presented in table 3.4.
The following properties are given for the analysis [IV]:
$E_1 = 53.8GPa, E_2 = 17.9GPa, G_{12} = G_{23} = G_{13} = 8.62GPa, \upsilon_{12} = 0.25, \alpha_1 = 6.3*10^{-6}$ *m/m/°c*, $\alpha_2 = 22.5*10^{-6}$ *m/m/°c, a/b=1, a/h =10,* $T=T_0 =45$.
Table 3.4 the non dimensionalized central deflection (w*) of simply supported plate subjected to uniform temperature field.

	[0/90/90/0]	[0/90/0/90]	[45/-45/-45/45]	[45/-45/45/-45]	[30/50/30/50]
Reference[IV] Present	0.0 $7.5363*10^{-8}$	0.0 $7.5363*10^{-8}$	0.0 $6.0129*10^{-8}$	0.0 $6.0129*10^{-8}$	0.0 $6.2961*10^{-8}$

The Strength Ratio's of [0/90/0] laminate subjected to tensile normal load in the X-direction are presented in Table 3.5.1, 2, 3.
The following properties are given for the analysis [II]:
E_1=181GPa, E_2=10.3*GPa*, G_{12}=7.17*GPa*, υ_{12}=0.28 and thickness of each ply is 5mm, f_1=0 Pa^{-1}, f_2=2.093*10^{-8} Pa^{-1}, f_{11}=4.4444*10^{-19} Pa^{-2}, f_{22}=1.0162*10^{-16} Pa^{-2}, f_6=0 Pa^{-1}, f_{66}=2.1626*10^{-16} Pa^{-2}, f_{12}=3.360*10^{-18} Pa^{-2}.
Table 3.5.1 The Local Stresses (Pa) of each layer of the laminate [0/90/0]

Ply No.	Position	Source	σ_1	σ_2	τ_{12}
1 (0˚)	Top	Reference[II] Present	$9.726*10^1$ 97.2639	$1.313*10^0$ 1.3131	0.0 0.0
	Middle	Reference[II] Present	$9.726*10^1$ 97.2639	$1.313*10^0$ 1.3131	0.0 0.0
	Bottom	Reference[II] Present	$9.726*10^1$ 97.2639	$1.313*10^0$ 1.3131	0.0 0.0
2 (90˚)	Top	Reference[II] Present	$-2.626*10^0$ -2.6263	$5.472*10^0$ 5.4721	0.0 0.0
	Middle	Reference[II] Present	$-2.626*10^0$ -2.6263	$5.472*10^0$ 5.4721	0.0 0.0
	Bottom	Reference[II] Present	$-2.626*10^0$ -2.6263	$5.472*10^0$ 5.4721	0.0 0.0
3(0˚)	Top	Reference[II] Present	$9.726*10^1$ 97.2639	$1.313*10^0$ 1.3131	0.0 0.0
	Middle	Reference[II] Present	$9.726*10^1$ 97.2639	$1.313*10^0$ 1.3131	0.0 0.0
	Bottom	Reference[II] Present	$9.726*10^1$ 97.2639	$1.313*10^0$ 1.3131	0.0 0.0

Table 3.5.2 The Local Strains of each layer of the laminate [0/90/0]

Ply No.	Position	Source	ε_1	ε_2	γ_{12}
1(0˚)	Top	Reference[18] Present	$5.353*10^{-10}$ $0.5353*10^{-9}$	$-2.297*10^{-11}$ $-0.023*10^{-9}$	0.0 0.0
	Middle	Reference[18] Present	$5.353*10^{-10}$ $0.5353*10^{-9}$	$-2.297*10^{-11}$ $-0.023*10^{-9}$	0.0 0.0
	Bottom	Reference[18] Present	$5.353*10^{-10}$ $0.5353*10^{-9}$	$-2.297*10^{-11}$ $-0.023*10^{-9}$	0.0 0.0
2 (90˚)	Top	Reference[18] Present	$-2.297*10^{-11}$ $-0.023*10^{-9}$	$5.353*10^{-10}$ $0.5353*10^{-9}$	0.0 0.0
	Middle	Reference[18] Present	$-2.297*10^{-11}$ $-0.023*10^{-9}$	$5.353*10^{-10}$ $0.5353*10^{-9}$	0.0 0.0
	Bottom	Reference[18] Present	$-2.297*10^{-11}$ $-0.023*10^{-9}$	$5.353*10^{-10}$ $0.5353*10^{-9}$	0.0 0.0
3(0˚)	Top	Reference[18] Present	$5.353*10^{-10}$ $0.5353*10^{-9}$	$-2.297*10^{-11}$ $-0.023*10^{-9}$	0.0 0.0
	Middle	Reference[18] Present	$5.353*10^{-10}$ $0.5353*10^{-9}$	$-2.297*10^{-11}$ $-0.023*10^{-9}$	0.0 0.0
	Bottom	Reference[18] Present	$5.353*10^{-10}$ $0.5353*10^{-9}$	$-2.297*10^{-11}$ $-0.023*10^{-9}$	0.0 0.0

Table 3.5.3 The Strength Ratios of each layer of the laminate [0/90/0]

Ply No.	Source	S.R. at Top	S.R. at Middle	S.R. at Bottom
1(0˚)	Reference[II] Present	$1.339*10^7$ $1.3395*10^7$	$1.339*10^7$ $1.3395*10^7$	$1.339*10^7$ $1.3395*10^7$
2(90˚)	Reference[II] Present	$7.2799*10^6$ $7.2773*10^6$	$7.2799*10^6$ $7.2773*10^6$	$7.2799*10^6$ $7.2773*10^6$
3 (0˚)	Reference[II] Present	$1.339*10^7$ $1.3395*10^7$	$1.339*10^7$ $1.3395*10^7$	$1.339*10^7$ $1.3395*10^7$

The global strains and stresses of all the plies of [0/90] graphite/epoxy laminate subjected to a temperature change are presented in Table 3.6.1, 2.

The following properties are given for the analysis [II]:

E_1=181GPa, E_2=10.3GPa, G_{12}=7.17GPa, υ_{12}=0.28,α_1=0.02m/m/ ˚c, α_2=22.5m/m/ ˚c, ΔT= -75˚C.

Table 3.6.1 The Global Strains of each layer of a laminate.

Ply No.	Position	Source	ε_x	ε_y	γ_{xy}
1(0˚)	Top	Reference[II] Present	$2.475*10^{-4}$ 0.0002	$-1.029*10^{-3}$ -0.0010	0.0 0
	Middle	Reference[II] Present	$-7.160*10^{-5}$ $-0.0716*10^{-3}$	$-7.098*10^{-4}$ $-0.7098*10^{-3}$	0.0 0
	Bottom	Reference[II] Present	$-3.907*10^{-4}$ $-0.3907*10^{-3}$	$-3.907*10^{-4}$ $-0.3907*10^{-3}$	0.0 0
2(90˚)	Top	Reference[II] Present	$-3.907*10^{-4}$ $-0.3907*10^{-3}$	$-3.907*10^{-4}$ $-0.3907*10^{-3}$	0.0 0
	Middle	Reference[II] Present	$-7.098*10^{-4}$ $-0.7098*10^{-3}$	$-7.160*10^{-5}$ $-0.0716*10^{-3}$	0.0 0
	Bottom	Reference[II] Present	$-1.029*10^{-3}$ -0.0010	$2.475*10^{-4}$ 0.0002	0.0 0

Table 3.6.2 The Global Stresses (Pa) of each layer of a laminate.

Ply No.	Position	Source	σ_x	σ_y	τ_{xy}

1(0˚)	Top	Reference[II] Present	$4.718*10^7$ $4.7183*10^7$	$7.535*10^6$ $0.7535*10^7$	0.0 0
	Middle	Reference[II] Present	$-9.912*10^6$ $-9.9120*10^6$	$9.912*10^6$ $9.9120*10^6$	0.0 0
	Bottom	Reference[II] Present	$-6.701*10^7$ $-6.7007*10^7$	$1.229*10^7$ $1.2289*10^7$	0.0 0
2(90˚)	Top	Reference[II] Present	$1.229*10^7$ $1.2289*10^7$	$-6.701*10^7$ $-6.7007*10^7$	0.0 0
	Middle	Reference[II] Present	$9.912*10^6$ $9.9120*10^6$	$-9.912*10^6$ $-9.9120*10^6$	0.0 0
	Bottom	Reference[II] Present	$7.535*10^6$ $0.7535*10^7$	$4.718*10^7$ $4.7183*10^7$	0.0 0

4. Conclusion

Graphite composites have exceptional mechanical properties. These materials are strong, stiff, and lightweight. Graphite composite is the material of choice for applications where lightweight & superior performance is important, such as components for spacecraft, fighter aircrafts, and racecars.

The three greatest advantages of graphite composites are:

- High specific stiffness (stiffness divided by density)
- High specific strength (strength divided by density)
- Extremely low coefficient of thermal expansion (CTE)

1. The nondimensionalized transverse deflections and stresses in specially orthotropic square plates subjected to various types of mechanical loadings under Classical Lamination Plate Theory are obtained with negligible errors.
2. The nondimensionalized maximum transverse deflections and stresses of simply supported antisymmetric cross-ply square plates using First Order Shear Deformation Theory are obtained with in 0 and 2% errors
3. The failure of multidirectional laminated plates subjected to purely mechanical and combined mechanical and hygrothermal loadings are presented in this work are validated.
4. The nondimensionalized central deflection values of four layer plates with uniform temperature field for one boundary condition are obtained with negligible error.
5. The residual stresses at the top, middle, bottom surfaces of all the plies of [0/90] laminate subjected to temperature change are obtained with negligible error.
6. The hygrothermoelastic stresses are calculated.
7. The strength ratios for all the plies of [0/90/0] laminate using Tsai-Wu failure theory are obtained with negligible errors.
8. The variation of deflection along the length of the plate under mechanical loading and subjected to moisture are plotted.

Tsai-Wu failure criterion is achieved in association with CLPT to predict the failure of a simply supported plate subjected to both mechanical and hygrothermal loads.

References

1. Journal article

1) S.W. Tsai, Composite Design, fourth ed., Think Composites, Dayton, 1988.
2) H.T. Hahn, R.Y. Kim, Swelling of composite laminates, Advanced Composite Materials-Environmental Effects, ASTM STP 658, 1978, pp. 98–120.
3) G.C. Sih, J.G. Michopoulos, S.C. Chou, Hygrothermoelasticity, Marinus Nijhoff Publishers, 1986.

4) G. Springer, Numerical procedures for the solutions of onedimensional Fickian diffusion problems, Environmental Effects on Composite Materials, Technomic Publishing Co. Westport, CT06880, USA, 1981, pp. 166–199.
5) R.B. Pipes, J.R. Vinson, T.W. Chou, On the hygrothermal response of laminated composite systems, Journal of Composite Materials 10 (1976) 129–148.
6) Y.Stavsky, Thermoelasticity of heterogeneous aelotropic plates. J.Engng Mech. Div., Proc.ASCE 89,89-105 (1963)
7) C.H.Wu and T .R.Tauchert, Thermoelastic analysis of laminated plates. 1: Symmetric specially orthotropic laminates, J. Therm. Stresses 3, 247-259 (1980).
8) C.H.Wu and T .R.Tauchert, Thermoelastic analysis of laminated plates. 2: Antisymmetric cross-ply and angle-ply laminates, J. Therm. Stresses 3, 247-259 (1980).
9) J.N.Reddy and Y.S. Hsu,Effects of shear deformation and anisotropy on the thermal bending of layered composite plates. J. Therm. Stresses 3, 475-493 (1980).
10) L.W.Chen,Thermal deformation and Stress analysis of composite laminated plates by finite element method. Comput. Struct. 35,41-49 (1990).
11) K.S.Sai Ram, & P.K.Sinha, Hygrothermal effects on the bending characteristics of laminated composite plates. Int.j.Computers & Structures, 40 (1991) 1009-15.
12) H.T. Hahn, Residual stresses in polymer matrix composite laminates, Journal of Composite Materials 10 (1976) 266– 278.
13) Pagano, N.J. and Pipes, R.B., “inter laminar stress in composite laminates under uniform axial extension”. journal of composite Materials 4,538-548, 1972.
14) K. Chandrashekara and A. Bhimaraddi “Thermal Stress Analysis of Laminated Doubly Curved Shells Using a Shear Flexible Finite Element”, 40 (1994) 1023-1030.
15) S. Timoshenko and W.Woinowsky-Krieger, Theory of Plates and Shells. McGraw-Hill, New York (1959).

2. Book

I. Issac M. Daniel and Ori Ishai “Engineering Mechanics of Composite Materials”. OXFORD UNIVERSITY PRESS, 1994.
II. Autar K. Kaw “Mechanics of Composite Materials”, 2nd Ed.v.29, CRC Press, Taylor & Francis Group (2006).
III. S. Timoshenko and W.Woinowsky-Krieger, Theory of Plates and Shells. McGraw-Hill, New York (1959).
IV. J.N.Reddy. “Mechanics of composite plates theory and analysis”. CRC-Press, 1997.