A study of thermal convection in a horizontal porous layer with effect of variable gravity and internal heat source

Anjanna Matta

Department of Mathematics, Faculty of Science and Technology,
The ICFAI Foundation for Higher Education, Hyderabad Telangana - 501203

Abstract

In this study analyze the thermal convection with the effect of heat source and variable gravity in a infinite length of horizontal porous layer. Both the boundaries are considered to be inclined temperature gradients with bottom wall heating and top wall cooling. For a better understanding the combined influence of variable gravity and heat source is investigated by using linear stability analysis. This thermal study is evaluated by applying shooting and Runga-Kutta methods for the velocity, temperature and vertical thermal Rayleigh number (R_z) corresponding to different flow parameters.

Key words: Thermal convection, variable gravity, heat source, horizontal porous layer.

1 Introduction

The theoretical study of thermal convection induced by an internal heat source and mass flow is a source of interest for many researchers, in the last few decades, due to its important applications in daily existence. In this analysis the additional effect of gravity in a horizontal porous layer is investigated. It has many real life practical applications are geophysical problems, underground transport of pollutants, gas reservoirs, crystal growth, environmental problems and cooling of nuclear reactors [1,2] etc.

A detailed survey of theoretical analysis and experimental investigations on thermal convection in porous medium can be found in the recent book of Nield and Bejan [3]. Nowadays, studying the effect of gravity on fluid flow in the presence of porous medium has brought attention by many

Email address: `anjireddyiitm@gmail.com` (Anjanna Matta).

researchers. The influence of gravity modulation in a fluid medium was first analyzed by Gresho and Sani [4]. Different problems of gravity-driven flows in a porous layer are described by Bear [5]. Due to the horizontal temperature gradient a thermogravitational flow occurs which is known as Hadley circulation. For the first time an overview of the Hadley circulations has been discussed by Hart ([6], [7]). Thermal convection in a horizontal porous layer with the inclined temperature gradient has been investigated by Nield [8,9]. Further, Manole and Lage [10] have been studied both the Hadely-flow and evolution of heat transfer beyond supercritical regimes. The influence of variable gravity field on the Hadley-flow induced by inclined thermal gradient is investigated by Alex and Patil [11].

Several authors written report on convection as a result of internal heat sources. Few papers of Schwiderski et al. [12], tveitereid [13], Yoo and Schultz [14] and Tritton et al. [15], are pertaining to the experimental investigations. Roberts [16] and Thirlby [17] did theoretical analysis on the results of Schwiderski et al. [12] and Tritton et al. [15]. Parthiban and Patil [18] studied the thermal convection because of nonuniform heating along the boundaries in addition to inclined temperature gradients, after that, they study of extension of anisotropic porous bed in their article Parthiban and Patil [19]. Articles of Alex and Patil [20] are to be referred for general theory and daily life applications. Borujerdi et al. [21,22] investigated the steady state heat conduction with uniform heat source and fluid and solid phases are at dissimilar temperatures. Later, they are extended the effect of Darcy number on the vertical thermal Rayleigh number in onset of convection with unvarying internal heat source.

The main aim of current theoretical examination is to analyze the effects of heat source, variable gravity , mass flow and inclined temperature gradient in a horizontal porous bed. Equations of the flow are converted into an eigenvalue problem. Later, it's solved, to determine the various effects of thermal instability, by using shooting and Runga-Kutta method.

2 Mathematical analysis

Let us consider the horizontal fluid-saturated porous layer with height d, where the $z^{'}$ axis is vertical with heat source $Q^{'}$ and $\nabla\theta$ is vertical thermal differences along the boundaries. Consider M is the net mass flow along the $x^{'}$ axis shown in Fig. 1. The imposed thermal gradient (β_{θ_x}) along the $x^{'}$ axis shown in Fig. 1. Since flow passes through porous layer and fluid varies in temperature from place to place. It follows the Darcy law and linear Boussinesq approximation. Hence, governing equations of the flow field in non-dimensional form are

$$\nabla\cdot\mathbf{q} = 0, \tag{1}$$

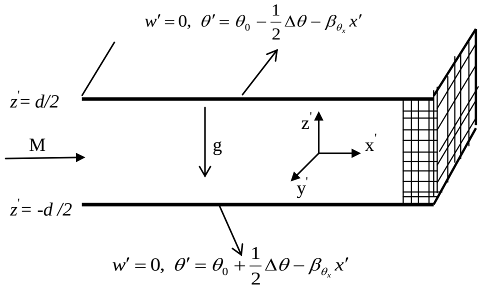

Fig. 1. Physical system

$$\mathbf{q} + \nabla P - \theta\left(1 + \gamma z\right)\mathbf{k} = 0, \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{q}\cdot \nabla\theta = \nabla^2\theta + QR_z\ , \tag{3}$$

boundary conditions at walls:

$$w = 0, \quad \theta = -\frac{1}{2}\left(\pm R_z\right) - R_x x \quad \texttt{at} \quad z = \pm\frac{1}{2}\ . \tag{4}$$

Dimensionless variables are given below

$$(x, y, z) = \frac{1}{d}\left(x', y', z'\right), \quad t = \frac{\alpha_m t'}{ad^2}, \quad \mathbf{q} = \frac{d\mathbf{q}'}{\alpha_m}, \tag{5}$$

$$P = \frac{K\left(P' - \rho_0 \int g dz'\right)}{\mu\alpha_m}, \quad \theta = \frac{R_z\left(\theta' - \theta_0\right)}{\triangle\theta}, \quad Q = \frac{d^2 Q'}{k_m \triangle\theta}. \tag{6}$$

Where

$$\gamma = \frac{\alpha_m \gamma'}{ad^2}, \qquad \alpha_m = \frac{k_m}{\left(\rho c_p\right)_f}, \qquad a = \frac{\left(\rho c\right)_m}{\left(\rho c_p\right)_f}. \tag{7}$$

Here, R_x denotes thermal Rayleigh number along the x' axis and R_z denotes vertical thermal Rayleigh number, it is defined as follows

$$R_x = \frac{\rho_0 g \gamma_\theta K d^2 \beta_{\theta_x}}{\mu\alpha_m}, \qquad R_z = \frac{\rho_0 g \gamma_\theta K d \triangle\theta}{\mu\alpha_m}. \tag{8}$$

2.1 Steady-State Solution

Basic state solution of the equations (1) - (3), with the conditions (4) is as follows

$$\theta_s = \widetilde{\theta}\left(z\right) - R_x x\ , \tag{9}$$

$$\left(u_s, v_s, w_s\right) = \left(u\left(z\right), 0, 0\right), \quad P_s = P\left(x, y, z\right)\ , \tag{10}$$

with

$$u_s = -\frac{\partial P}{\partial x}\ , \quad v_s = -\frac{\partial P}{\partial y}\ , \tag{11}$$

$$0 = -\frac{\partial P}{\partial z} + (1+\gamma z)\left(\widetilde{\theta}\,(z) - R_x x\right)\ , \tag{12}$$

$$D^2\widetilde{\theta} = -u_s R_x - QR_z\ . \tag{13}$$

Where $D = \frac{d}{dz}$, here, the net flow is $\int_{-1/2}^{1/2} u(z)dz = M$ along the $x-$ axis and $\int_{-1/2}^{1/2} v(z)dz = 0$ along the $y-$ axis. Steady state solution appeared in the form of flow velocity and temperature is obtained.

$$u_s = R_x(z + \gamma\frac{z^2}{2} - \gamma/24) + M\ ,\quad v_s = 0\ , \tag{14}$$

$$\widetilde{\theta} = -R_z z + \frac{{R_x}^2}{24}\left(z - 4z^3 + \frac{\gamma}{16}(4z^2-1)^2\right) + \frac{QR_z + MR_x}{8}\left(1 - 4z^2\right)\ , \tag{15}$$

2.2 Linear Stability Analysis

Let us consider the following perturbed solution $\mathbf{q} = \mathbf{q}_s + \overline{q}$, $\theta = \theta_s + \overline{\theta}$ and $P = P_s + \overline{P}$. Following linear system is obtained from the equations (1) - (3) after substituting the above perturbed solution and after neglecting the nonlinear terms.

$$\nabla\cdot\overline{\mathbf{q}} = 0, \tag{16}$$

$$\overline{\mathbf{q}} = -\nabla\overline{P} + \overline{\theta}\,(1+\gamma z)\,\mathbf{k}\ , \tag{17}$$

$$\frac{\partial\overline{\theta}}{\partial t} + \mathbf{q}_s\cdot\nabla\overline{\theta} + \overline{\mathbf{q}}\cdot\nabla\theta_s = \nabla^2\overline{\theta}\ , \tag{18}$$

where

$$\nabla\theta_s = -\left(R_x, R_y, R_z - \frac{{R_x}^2}{24}\left[1 - 12z^2 - \gamma(4z^2-1)\right] - (QR_z + MR_x)\,z\right). \tag{19}$$

boundary conditions at walls:

$$\overline{w} = 0\ \texttt{ and }\ \overline{\theta} = 0\ \texttt{ at }\ z = \pm 0.5\ . \tag{20}$$

These conditions in Eq. (20) say that perturbed velocity and perturbed temperature vanished at the plates. There after, solution of Eqs. (16) - (18) is found in the form of normal modes using the following

$$\left[\overline{\mathbf{q}}, \overline{\theta}, \overline{P}\right] = \left[\mathbf{q}\,(z)\,, \theta\,(z)\,, P\,(z)\right]\exp\left\{i\left[kx + ly - \sigma t\right]\right\}\ , \tag{21}$$

Later, we get the following equations after elimination of P from the equation (17).

$$\left(D^2 - \alpha^2\right)w + \alpha^2\,(1+\gamma z)\,\theta = 0, \tag{22}$$

$$\left(D^2 - \alpha^2 + i(\sigma - ku_s - lv_s)\right)\theta + \frac{i}{\alpha^2}\,(kR_x)\,Dw - \left(D\widetilde{\theta}\right)w = 0, \tag{23}$$

The above Eqs. (22) - (23) together with conditions $w = \theta = 0$ at upper plate $z = \frac{1}{2}$ and lower plate $z = -\frac{1}{2}$, where γ, M, Q, R_x, k and l are parameters, it is an eigenvalue problem in terms of vertical thermal Rayleigh number R_z. Here, the overall wave number $\alpha = \sqrt{k^2 + l^2}$.

	R_x	0	10	20	30	40	50	60
$\gamma = 0$	R_z	39.4784	42.0076	49.5486	61.9566	78.9663	100.1163	124.4724
	α	3.1399	3.1399	3.1499	3.1599	3.2199	3.3399	3.6699
$\gamma = 1$	R_z	39.2368	41.8353	49.5842	62.3395	79.8415	101.6537	126.9460
	α	3.1599	3.1499	3.1499	3.1599	3.1999	3.30999	3.56499
$\gamma = 2$	R_z	38.5398	41.3396	49.6931	63.4610	82.40460	106.1582	134.1289
	α	3.1999	3.1899	3.1799	3.1599	3.1699	3.2099	3.3499

Table 1
Vertical thermal Rayleigh number at $Q = 0$ and $M = 0$

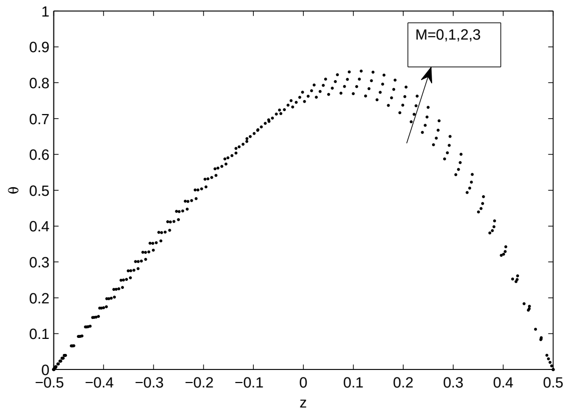

Fig. 2. Temperature contours (θ) with z by varying mass flow (M).

3 Results and Discussion

In this study analyzed the thermal convection in a horizontal fluid-saturated porous bed due to an internal heated and variable gravity. Classical normal mode technique is used to study the thermal stability of Hadley-Prats flow. Here, minimum of all R_z values is defined as the vertical thermal Rayleigh number (R_z) as the wave number (α) varies. Stationary convection is achieved by putting $\sigma = 0$ as in Nield [8]. The term longitudinal disturbance is characterized by $k = 0$. From the Table 1, it is noticed that the results are in very good comparison with the past results of the article Nield [23], when $Q = 0$, $M = 0$ and $\gamma = 0$. Also, the critical values of R_z increased if the value of γ increases from 0 to 2, in Table 1. Thus, strong stabilization is the result of the flow field observed in the medium.

Figs. 2-3 shows the temperature and velocity contours by varied the mass flow (M) at $R_x = 10$ and $Q = 1$. It is noted that as increase in the mass flow (M) from 0 to 3, the temperature and velocity profiles increased middle of the porous plates are observed. Hence, the temperature and velocity of fluid is increased.

Figs. 4-5 shows the velocity and temperature profiles by varied the heat source (Q) at $R_x = 10$

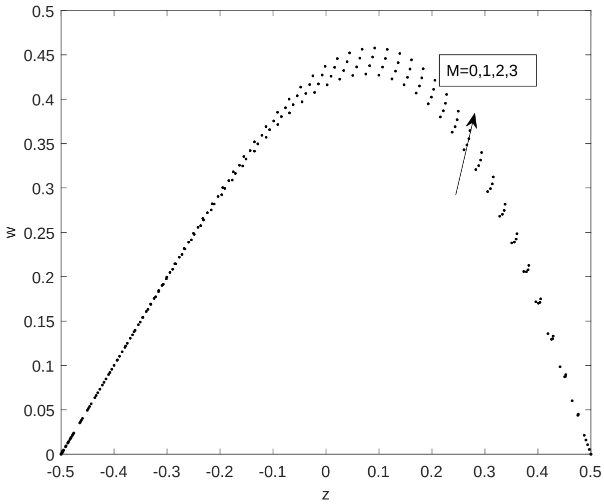

Fig. 3. Velocity profiles (w) with z by varying mass flow (M).

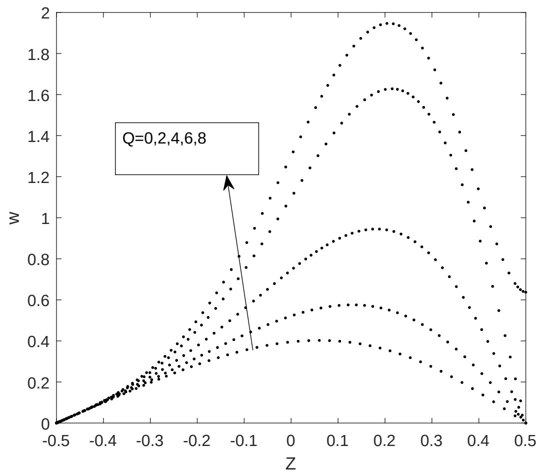

Fig. 4. Velocity profiles w with z by varying heat source (Q).

and $Q = 1$. It is noted that as increase in the heat source (Q) from 0 to 8, the temperature value also increased middle of the porous bed is observed. It is indicates that, flow moves from bottom top layer as increased the heat source. Hence, the velocity and temperature of fluid is increased, due to enhance the global temperature of the system. The velocity and temperature profiles symmetric structure also distributed.

4 Conclusion

The present study used the stability analysis to analyze the thermal convention of a Hadley-Prats flow in a horizontal porous bed. It also study the effects of internal heat generation and variable gravity effects on velocity and temperature fields. The vertical thermal Rayleigh number (R_z) computed for different combinations of parameters governing the flow. It have been concluded as follows.

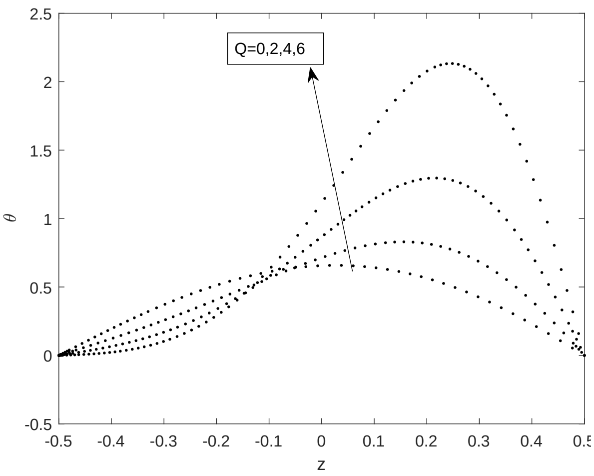

Fig. 5. Temperature contours θ with z by varying heat source (Q).

- Enhance in the heat source results the strong destabilization irrespective of gravity and mass flow.
- Strong stabilization of the flow field is observed at higher horizontal Rayleigh number and in the existence of gravity.
- The effect in vertical Rayleigh number is seen subject to mass flow, variable gravity and heat source .

References

[1] Xi Chen, Angui Li, An experimental study on particle deposition above near-wall heat source, Building and Environment 81 (2014) 139-149.

[2] O. Kwon, K. Bae, C. Park, Cooling characteristics of ground source heat pump with heat exchange methods, Renewable Energy 71 (2014) 651-657.

[3] D.A. Nield, A. Bejan, Convection in Porous Media, forth ed., Springer 2013.

[4] Gresho, P.M. and Sani, R.L., The effect of gravity modulation on the stability of a heated fluid layer, J. Fluid Mech., vol. 40, pp. 783-806, 1970.

[5] Bear, J., *Dynamics of Fluids in Porous Media*, Dover, 1988.

[6] Hart, J.E., Stability of thin non-rotating Hadley circulations, J. Atmos. Sci., 29 (1972) 687-697.

[7] Hart, J.E., A note on the stability of low-Prandtl-number Hadley circulations, J. Fluid Mech., 132 (1983) 271-281.

[8] D.A. Nield, Convection in a porous medium with inclined temperature gradient, Int. J. Heat Mass Transfer 34 (1991) 87-92.

[9] Nield, D.A., Convection induced by an inclined temperature gradient in a shallow horizontal layer, Int. J. Heat and Fluid Flow, 15 (1994a) 157-162.

[10] Manole, D.M. and Lage, J.L., Numerical simulation of supercritical Hadley circulation within a porous layer induced by inclined temperature gradients, Int. J. Heat Mass Transfer, vol. 38, pp. 2583-2593, 1995.

[11] Alex, S.M. and Patil, P.R., Effect of a variable gravity field on thermal instability in a porous medium with inclined temperature gradient and vertical through flow, J. Porous Media, vol. **5**, pp. 137-147, 2002.

[12] Schwiderskei,. W., Schwabh . J.A. Convection experiments with electrolytically heated fluid layers 1971 J. Fluid Mech. 48, 703-719.

[13] M. tveitereid, Thermal convection in a horizontal porous layer with internal heat sources, Int. J. Heat Mass Transfer. Vol. 20, pp. 1045-1050. 1977.

[14] Joo-Sik Yoo, William W. Schultz, Thermal convection in a horizontal porous layer with spatially periodic boundary temperatures: small Ra flow, Int. J. of Heat and Mass Transfer, vol.46, pp. 47474750, 2003.

[15] Trittond, . J. Zarraua,M N. Convection in horizontal layers with internal heat generation (1967) J. Fluid Mech. 30, 21-31.

[16] Roberts P. H. Convection in horizontal layers with internal heat generation: Theory 1967 J. Fluid Mech. 30, 33-49.

[17] R. Thirlby, Convection in an internally heated layer. J. Fluid Mech. 44 (1970) 673-693.

[18] C. Parthiban, P.R. Patil, Effect of non-uniform boundary temperatures on thermal instability in a porous medium with internal heat source, Int. Commun. Heat Mass Transfer 22 (1995) 683-692.

[19] C. Parthiban, P.R. Patil, Thermal instability in an anisotropic porous medium with internal heat source and inclined temperature gradient, Int. Comm. Heat Mass Transfer 24 (1997) 1049-1058.

[20] S.M. Alex, P.R. Patil, Effect of a variable gravity field on convection in an anisotropic porous medium with internal heat source and inclined temperature gradient, Transactions of the ASME (2002) 144-150.

[21] A.N.Borujerdi, A.R. Noghrehabadi, D.A.S. Rees, Onset of convection in a horizontal porous channel with uniform heat generation using a thermal nonequilibrium model, Transp. Porous Med. 69 (2007) 343-357.

[22] A.N. Borujerdi, A.R. Noghrehabadi, D.A.S. Rees, Influence of Darcy number on the onset of convection in a porous layer with a uniform heat source, Int. J. of Thermal Sciences 47 (2008) 1020-1025.

[23] D.A. Nield, Convection in a porous medium with inclined temperature gradient: additional results, Int J. Heat Mass Transfer 37 (1994) 3021-3025.