Natural convection in a heat generating non-Darcy porous enclosure with heat source

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Abstract
Two dimensional unsteady state natural convection in a closed square enclosure filled with porous medium, using a thermal non-equilibrium model for the heat transfer between the fluid and the solid phases has been studied numerically by adopting a two temperature model of heat transfer. The analysis assumes that the porous medium is homogeneous and isotropic. The present study also assumes the non-Darcy model of natural convection in porous media. It is assumed that the heat generation is only in solid phase. Such a model when subject to different sets of boundary conditions is found to modify the flow behaviour and heat transfer rates. The maximum heat transfer rate was obtained for large H and small γ in each Case considered. Knowledge of this behaviour is very important for the design of the many engineering applications.

Keywords: convection, porous cavity, nonequilibrium, Darcy-Brinkman-Forchheimer, heat generation

1. Introduction

The heat and fluid flow in porous media has gained considerable attention, and has been motivated by a broad range of engineering applications in particular where internal heat generation is involved. In case of post accident heat removal in nuclear reactors, storage of spent fuel and grain storage where heat is generated due to metabolism of agricultural products convection is driven by internal heat generation. Baytas and Pop (2002) studied free convection in a differentially heated square cavity using a thermal nonequilibrium model and found that such a model modifies the flow behaviour and heat transfer rate. Recently Pippal (2013) studied the influence of local thermal nonequilibrium state on a natural convective flow in a porous enclosure and concluded that in the equilibrium state the maximum heat transfer takes place at aspect ratio lying between 1 and 1.5. Besides only external sidewall heating, motion driven by internal heat generation arise in a variety of applications such as flows arising in the production of crystals, convection in the interior of the Earth and the dynamics of the atmosphere. Jue (2003) analyzed the transient and steady state flows in a fluid saturated porous cavity under the effects of external heating and internal heat generation and found that the permeability of porous medium seriously affects the final flow field.

From the glimpse of the literature given above the significant role played by clear understanding of LTNE effect and its importance in ongoing research is undeniable in the light of that, attempt is made to study the physics of natural convection in a porous enclosure with exponential heat generation when LTNE hypothesis is employed in energy equation.

2. Mathematical Formulation

A two dimensional square cavity filled with a fluid saturated porous medium, which is homogeneous and isotropic is considered in Fig. 1. It generates heat in its solid phase. All walls of the cavity are assumed to be rigid and impermeable. The flow is single phase, two dimensional and non-Darcian. The fluid is assumed to be a normal Boussinesq fluid. Under these assumptions the unsteady state governing equations based on the Darcy-BrinkmanForchheimer model with consideration of internal heat generation in the solid phase can be written in the following nondimensional form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\begin{aligned} \frac{\partial \bar{U}}{\partial \tau} + U\frac{\partial \bar{U}}{\partial X} + V\frac{\partial \bar{U}}{\partial Y} &= -\epsilon \nabla P + \bar{\mu} Pr \nabla^2 \bar{U} - \frac{\epsilon Pr}{Da}\bar{U} - \epsilon^2 F_0 (U^2 + V^2)^{1/2}\bar{U} \\ &\quad + \epsilon Ra Pr \theta_f \bar{j} \end{aligned} \tag{2}$$

$$\bar{\alpha}\frac{\partial \theta_s}{\partial \tau} = \nabla^2 \theta_s + \gamma H(\theta_f - \theta_s) + Q e^{c_0(\theta_s)} \tag{3}$$

$$\frac{\partial \theta_f}{\partial \tau} + \bar{U} \cdot \nabla \theta_f = \nabla^2 \theta_f + \gamma H(\theta_s - \theta_f) \tag{4}$$

The non-dimensional variables are:

$$(X, Y) = (x, y)/D, \quad (U, V) = (u, v)D/\alpha_f, \quad P = pD^2/\rho\alpha_f^2 \tag{5}$$

$$\tau = k_f t/(\rho c_p)_f D^2, \qquad \theta = (T - T_c)/\Delta T, \quad \Delta T = T_h - T_c$$

where D is the length of porous cavity and k_f, α_f are thermal conductivity and thermal diffusivity of the fluid respectively and the parameters, $\bar{\mu} = \frac{\mu_e}{\mu}$ dynamic viscosity ratio , $Pr = \nu/\alpha$ Prandtl Number , $Ra_I = \frac{g\beta D^5 q_s^{'''}}{\nu\alpha_f k_s}$ internal Rayleigh Number , $F_0 = \frac{C_E D}{\sqrt{K}}$ form coefficient , $Ra_E = \frac{g\beta D^3(T_h - T_c)}{\nu\alpha_f}$ external Rayleigh Number ,

$\bar{\alpha} = \dfrac{\alpha_f}{\alpha_s}$ thermal diffusivity ratio , $\gamma = \dfrac{\epsilon k_f}{(1-\epsilon)k_s}$ modified conductivity ratio ,
$H = \dfrac{hD^2}{\epsilon k_f}$ heat transfer coefficient , $c_0 = c_1 \Delta T,$ heat generation parameter ,
$Da = \dfrac{K}{D^2}$ Darcy Number , $Q = \dfrac{Ra_I}{Ra_E}$

The nondimensional initial conditions are:

$$\tau = 0 : U = V = 0,\ \theta_f = \theta_s = 0 \text{ for } 0 \leq X, Y \leq 1 \tag{6}$$

The nondimensional temperature boundary conditions:

$$\tau > 0 : \theta_f = \theta_s = 1 \text{ on } X = 0 \text{ and } 0 \leq Y \leq 1$$
$$\tau > 0 : \theta_f = \theta_s = 0 \text{ on } X = 1 \text{ and } 0 \leq Y \leq 1$$
$$\frac{\partial \theta}{\partial Y} = 0 \text{ on } Y = 0, 1 \text{ and } 0 \leq X \leq 1$$

The nondimensional velocity boundary conditions are:

$$\tau > 0 : U, V = 0, \text{ on } X, Y = 0, 1 \text{ and } 0 \leq X, Y \leq 1$$

The nondimensional wall Nusselt numbers are defined for fluid and solid phases separately at hot wall. We denote

$$Nu_f = -\int_0^1 \left.\frac{\partial \theta_f}{\partial X}\right|_{X=0} dY, \quad Nu_s = -\int_0^1 \left.\frac{\partial \theta_s}{\partial X}\right|_{X=0} dY \tag{7}$$

as the fluid and solid phase Nusselt numbers respectively for hot wall.

3. Numerical procedure

The governing equations, (7) to (10) were discretized by the finite volume method on a uniform staggered grid system using the SIMPLE algorithm of Patankar (1980). The grid points were at the center of the control volumes which replaced the entire enclosure. The power law and the second order central difference scheme were, respectively, used for the convection and diffusion terms. The set of discretized equations for each variable were then solved by a line-by-line procedure of the tri-diagonal matrix algorithm (TDMA). The steady state result alone was considered when the convergence criteria

$$\frac{\sum_{i,j} |\eta_{i,j}^m - \eta_{i,j}^{m-1}|}{\sum_{i,j} |\eta_{i,j}^m|} \leq 10^{-5} \tag{8}$$

was achieved. Here η represents the variables U, V, θ_s or θ_f, the superscript m refers to the iteration number and (i, j) refers to the space coordinates.

To test the grid independency of the solution, a grid refinement study was performed for $H = 1000$, $\gamma = .001$, $Pr = .05$ and $Ra = 10^7$(see Table 1). Five different grid systems $20 \times 20, 40 \times 40, 60 \times 60, 80 \times 80$ and 100×100 were chosen. The difference between the Nu of the 100×100 grid and the 120×120 grid is less than 2%. The grid was selected as a trade off between numerical accuracy, stability and computation time thus 100×100 grid was used in all calculations.

Table 1. Grid independence test with $Ra = 10^7$, $H = 1000$, $\gamma = .001$ and $Pr = .05$

Grid	Nu_s	Nu_f
20	45.848	45.683
40	39.247	35.079
60	36.651	33.856
80	35.524	32.343
100	35.020	32.231
120	35.011	32.193

A comparison of the present results and those of Nithiarasu (1997) and Lauriat (1989) for the Forchheimer model with $\epsilon = 0.4$, $F_0 = .55$, $Pr = 1$, $Da = 10^{-6}$ and $Ra = 10^7 - 10^9$, are presented in Table 2. The agreement is found to be good. To validate LTNE model a comparison of the present result with the results of Baytas (2002) has been displayed in Table 3. with $Ra = 10^3$, $Da = 10^{-6}$, $F_0 = 0$, $c_0 = 0$ and $Pr = 7.1$ for different values of H and γ.

Table 2. Comparison of the average Nusselt numbers of present study with Forchheimer model of Nithiarasu (1997) and Lauriat (1989) in thermal equilibrium state

Ra	$\overline{Nu}$ of Present	$\overline{Nu}$ of Nithiarasu (1997)	$\overline{Nu}$ of Lauriat (1989)
10^7	1.08	1.08	1.07
10^8	3.08	3.00	3.09
10^9	12.66	11.57	12.80

Table 3. Comparison of the average Nusselt numbers of present study with those of Baytas (2002) in thermal nonequilibrium state for different values of H and γ when $Ra = 10^3$

H	γ	Baytas	(2002)	Present	
		Nu_f	Nu_s	Nu_f	Nu_s
1	1	14.61	1.077	14.20	1.056
1	.001	14.61	1.035	14.18	1.001
1000	1	11.79	5.985	11.69	5.899
1000	10	14.08	10.80	13.94	10.31

4. Results and Discussion

Results in this section illustrate the effect of local thermal nonequilibrium (LTNE) state between the porous matrix and fluid on natural convection occurring within the enclosure. The evolution of flow with the effect of internal heat generation for different H and γ is depicted in Fig. 2 for $Ra_E = 10^5$ with $Pr = .05$ and $c_0 = 0$. For $Ra_I = 10^6$ the relative impact of internal heat generation is minor and hence the flow is attributed by the presence of a single clockwise circulation cell for chosen H and γ. The thermal equilibrium is attained for high H which is evident from the isotherms of solid and fluid phases which are identical, whereas for low H the solid and fluid phase isotherms vary. The flow and thermal fields experience a strong influence of internal heat generation for $Ra_I > 10^6$ and $Ra_E = 10^5$ for different H and γ. When Ra_I is increased to 10^8 (see Fig. 3) for each H and γ, the cavity is occupied by two counter rotating cells. This is due to the fact that dominance of internal heat generation renders a favouring (or opposing) buoyancy effect to the fluid in the vicinity of the cold (or hot) wall. At higher value of Ra_I, the hindrance of the flow is so strong and hence a sinking motion is established near the hot wall. Thus two irregular counter rotating cells of differential strength and opposite direction are introduced.

For each Ra_I it is noted that at $\gamma = .001$ the $|\theta_f|_{max}$ increases with the increase in H whereas the temperature of the solid phase seems to be less affected. However at $\gamma = 10$ the $|\theta_f|_{max}$ increases and the $|\theta_s|_{max}$ decreases with increase in H. Since large value of γ corresponds to fluids having high thermal conductivity relative to that of the solid, the fluid properties dominate the development of flow.

Computations were carried out by increasing heat generation parameter to $c_0 = .01$ for $Pr = .05$ (see Figs. 4 & 5). The heat and flow lines are similar to those obtained earlier where it is characterized by the occurrence of single closed cell as in case of $Ra_I = 10^6$. For further increase in $Ra_I = 10^8$, as expected the flow transforms from single cell to a two cell pattern since the internal heat generation enhances the flow near the cold wall. On the other hand the opposing temperature gradient caused by c_0 produces a vertical downward flow motion in the vicinity of hot walls. These interactions result in a two cell pattern. However for $H = 1$ and $\gamma = .001$ the single cell pattern continues in spite of increase in Ra_I.]

In the presence of internal heat sources, the values of Nu_s and Nu_f along the hot side wall are governed by the direction and strength of the flow adjacent to the hot wall. At each Ra_I depending on Ra_E, a part of the interior hot fluid flows downwards along the hot surface forming a counter direction circulation near the hot wall. Thus the Nu_f and Nu_s along hot wall becomes negative which means that the hot wall absorbs the heat from the interior higher temperature fluid. Increasing the internal heat generation parameter causes a rapid rise in the Nu_f and Nu_s. Since the order of magnitude of external heating is comparable to internal heat generation, the positive values of Nu_s and Nu_f indicates that there is a rising motion near the hot wall though the circulation feels retardation due to the buoyancy effect, influenced by the magnitude of internal heat generation. Therefore as Ra_I increases in magnitude the average Nu decreases upto certain value of Ra_I. After that it increases in negative direction indicating the sinking motion near the hot side wall.

Ra_I against Nu_s, Nu_f has been plotted for different Pr and c_0 in Fig. 6. For the chosen Pr and c_0 values it is observed that maximum heat transfer was obtained for $Pr = 100$ with $c_0 = 0$. In general an increase in c_0 results in the decrease of heat transfer rate. Since higher value of c_0 results in the increase of heat generation in the solid phase which has an adverse effect on the temperature gradient thereby deteriorating the heat transfer rate. For $c_0 = .01$ the higher Nusselt number is obtained for Nu_s of low H and high γ when $Pr = .05$. One can also observe that for $c_0 = 0.01$ thermal equilibrium is attained for high H irrespective of the value of γ considered.

In case of $c_0 = 0$ for each Ra_I maximum heat transfer rate is obtained in case of high H and low γ for fluid phase. It is noticed that the values of $|\psi|_{max}$ & $|\theta_f|_{max}$ are high in this case. For an increase in Pr there is a slight increase in Nu_f, Nu_s when $Ra_I = 10^8$ whereas for lower Ra_I there is no influence of Pr on heat transfer rate. However for $c_0 = 0.01$ at higher $Ra_I (= 10^8)$ maximum heat transfer rate is noticed in case of high H and low γ for solid phase. For lower $Ra_I (< 10^8)$ maximum heat transfer rate is seen in case of high H and low γ with high $|\psi|_{max}$ & $|\theta_f|_{max}$ values. For an increase in Pr there is an increase in heat transfer rate for low Ra_I whereas for high $Ra_I = 10^8$, an increase in Pr results in a decrease of Nu_f, Nu_s when $\gamma = .001$ and an increase of Nu_f, Nu_s when $\gamma = 10$

For an increase in c_0 there is an increase in heat transfer rate for each H, γ and $Pr = 100$ considered when $Ra_I = 10^7$. At $Ra_I = 10^8$ for an increase in c_0 there is an increase in Nu_f, Nu_s for high γ , but for low γ it decreases when $Pr = 100$.

6. Conclusion

We have attempted to understand the influence of local thermal non-equilibrium (LTNE) state on heat transfer rate. The following conclusions have been made. Thermal nonequilibrium effects are significant for small H regardless of value of γ. Small H and large γ characterizes weak convective flow. Whereas small γ characterizes conduction dominance in the flow. The heat transfer rate is always higher when $H = 1000$ and $\gamma = .001$. For large values of H and γ the thermal equilibrium case has been recovered where the fluid and solid temperatures fields are identical. For sufficiently large H in spite of small γ thermal equilibrium may be recovered. Increase in c_0 results in retardation of occurrence of two cell pattern for small H and small γ.

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