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Thermodynamics Analysis for a cross diffusive Magnetized couple stress fluid in Horizontal inner rotating cylinder

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The current study is analysed the entropy generation rate in the Mhd flow of couple stress fluid through the horizontal inner rotating cylinder is implemented by applying the second law of thermodynamics. An analytical solution for the governing equations is evaluated using the theory of Modified Bessel's functions. The influence of couple stress parameter, Magnetic parameter, Brinkmann number, Soret and Dufour number, Temperature and concentration difference parameters on the seepage velocity. The results are presented graphically. It is conclude that, the solutal diffusivity has a major effect on the entropy generation rate. It is irreversibility of heat in a couple stress fluid flow.

***Keywords*:** MHD flow**,** Thermodynamic analysis**,** Soret and Dufour effects, Couple stress fluid

1. Introduction

Throughout the years, the thermodynamics of flow heat exchange has been founded on the second law analysis and its structure related idea of entropy generation minimization. The entropy generation and proficiency estimation utilizing the second law of thermodynamics is additional dependable than first law-based counts. The study of entropy generation minimization in a warm framework was presented by Bejan [1] and clarified that this minimization enhances the effectiveness of a framework by enhancing the exergy. From that point, a few specialists have hypothetically considered entropy generation in heat and mass flow frameworks under numerous physical circumstances [2-10].

Most of the investigations revealed in the writing managed the conventional Newtonian fluids. A few fluids utilized in designing and modern procedures, for example, suspensions of arrangements and strands, slurries, polymer fluids and melts, surfactants, paints, sustenance and restorative items, body fluids, cleansers, inks, natural materials, cements, and so on., display stream properties that can't be clarified by Newtonian liquid stream demonstrate. Various numerical models have been projected to clarify the rheological conduct of these fluids. One among these models is couple stress fluid model presented by Stokes. The couple stress fluid classical has particular highlights, for example, body couples and non-symmetric stress tensor. This couple stress fluid hypothesis produces models for fluids those microstructure is of mechanical essentialness. These fluids display more confounded size-subordinate impact than those in Newtonian fluids. A few analysts have contemplated scientifically and for the most part numerically the convective stream of couple stress fluid through rotating cylinders. Kaladhar and Srinivasacharya[11] discussed the couple stress moves through the annulus in between two vertical cylinders with chemical reaction. They saw that focus ascends by the upgrade of the chemical reaction parameter. Dewakar et al. [12] inspected an analytical solutions of some fully developed flows of couple ctress fluid between concentric cylinders with slip boundary conditions. Nagaraju et al.[13] researched the dissipative, thermocouple stress courses through two vertical turning cylinders with the porous coating is lined at the external barrel and a uniform outspread attractive field. Adesanya et al. [14] inspected the entropy generation rate in the stream of a couple stress fluid with heat and mass transfer. Nagaraju et al. [15] explored the entropy generation in the stream in an annulus with porous coating between two pivoting cylinders under the nearness of an outspread attractive field, seeing that speed diminishes with expanding attractive field parameter. Second law examination of hydromagnetic couple stress fluid through a channel loaded up with non-Darcian porous medium have explored by Opanuga et al. [16].

Inspired by the examinations and applications referenced over, the point of this examination is to think about the entropy generation and irreversibility of heat and mass transfer dependent on the second law of thermodynamics in order to gauge the exergy of magnetized couple stress fluids. It might be commented that before studies, to the best of the creator's learning, did exclude the impact of mass transfer and cross-diffusion term on entropy generation rate in the stream couple stress liquid through horizontal cylinders. The subsequent common differential conditions are fathomed utilizing the investigative technique. The correct arrangements are utilized to process the entropy generation rate and the irreversibility investigation.

2. Mathematical formulation of the Problem

Consider a steady, incompressible flow of a couple stress fluid between horizontal inner rotating infinite cylinders and magnetic field applied Z-direction as shown in Figure 1. Additionally, the effects of Diffusion-themo(Dufour) and thermal-diffusion(Soret) is considered. The viscous heating effects in the thermal equation are maintained. Here in cylindrical coordinate geometry (r, θ, z) here $\frac{\partial}{\partial\theta} = 0$ since the axial symmetry of the flow. The governing flow equations of MHD, mass and heat transfer of couple stress fluid as follows

$$\frac{dP}{dR} = \rho\frac{V^2}{R} \tag{1}$$

$$\mu D^2V + \eta_0 D^4V - \sigma B_0^2\, V = 0 \tag{2}$$

$$K_T\left(\frac{d^2T}{dR^2} + \frac{1}{R}\frac{dT}{dR}\right) + \mu\left(\frac{dV}{dR} - \frac{V}{R}\right)^2 + \eta_0(D^2V)^2 + \frac{DK}{C_sC_p}\left(\frac{d^2C}{dR^2} + \frac{1}{R}\frac{dC}{dR}\right) = 0 \tag{3}$$

$$D\left(\frac{d^2C}{dR^2} + \frac{1}{R}\frac{dC}{dR}\right) + \frac{DK}{T_m}\left(\frac{d^2T}{dR^2} + \frac{1}{R}\frac{dT}{dR}\right) = 0 \tag{4}$$

Where $D^2V = \frac{d^2V}{dR^2} + \frac{1}{R}\frac{dV}{dR} - \frac{V}{R^2}$

The boundary conditions considering for inner and outer cylinders are taken as pursues

(i) $V = R\Omega,\ D^2V = 0, T = T_1, C = C_1 at R = R_1$

(ii) $V = D^2V = \frac{dT}{dR} = \frac{dC}{dR} = 0 \quad at \quad R = R_2$ (5)

The basic equation together with boundary conditions, Eq. (2) to (4), which are currently ends up dimensional less form:

$$D^2v + SD^4v - Ha^2(1-\eta)^{-2}\, v = 0 \tag{6}$$

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} + Br\left(\frac{dv}{dr} - \frac{v}{r}\right)^2 + SBr\left(D^2v\right)^2 + Pr\, D_f\left(\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr}\right) = 0 \tag{7}$$

$$\left(\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr}\right) + Sc\, Sr\left(\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr}\right) = 0 \tag{8}$$

(i) $v = \theta = \phi = 1\ and\ D^2v = 0$ at $r = \eta$

(ii) and(ii) $v = D^2v = \theta' = \phi' = 0$ at $r = 1$ (9)

Where $= \frac{V}{R_1\Omega}$, $r = \frac{R}{R_2}, \eta = \frac{R_1}{R_2}, p = \frac{P}{\rho\Omega^2 R_1^2}, \theta = \frac{T-T_2}{T_1-T_2}, \phi = \frac{C-C_2}{C_1-C_2}, Ha = B_0(R_2-R_1)\sqrt{\frac{\sigma}{\mu}},$ $S = \frac{\eta_0}{\mu R_2^2}, Br = \frac{\mu\Omega^2 R_1^2}{K_T(T_1-T_2)}, D_f = \frac{DK(C_1-C_2)}{\vartheta C_s C_p(T_1-T_2)}, Pr = \frac{\mu C_p}{K_T}, Sc = \frac{v}{D}$, and $Sr = \frac{DK(T_1-T_2)}{v T_m(C_1-C_2)}$.

From equation (6), find the following equation for v:

$$D^4 v + \frac{1}{S} D^2 v - \frac{Ha^2(1-\eta)^{-2}}{S} v = 0 \tag{10}$$

The above equation can be expressed as follows:

$$(D^2 - \lambda_1^2)(D^2 - \lambda_2^2)v = 0 \tag{11}$$

Where $\lambda_1^2 + \lambda_2^2 = \frac{-1}{S}$ and $\lambda_1^2 \lambda_2^2 = -\frac{Ha^2(1-\eta)^{-2}}{S}$. Since velocity v is finite in the interval $\eta < r < 1$, the solution of Eq. (10) can be written as follows:

$$v = a_1 I_1(\lambda_1 r) + a_2 K_1(\lambda_1 r) + a_3 I_1(\lambda_2 r) + a_4 K_1(\lambda_2 r) \tag{12}$$

The constants a_1, a_2, a_3, a_4 can be found by using the no-slip boundary condition and vanishing of couple stresses on the boundary(Type A condition) on azimuthal velocity v.

2. 1. Second Law Analysis

In existence of Joule Heating, the volumetric entropy generation number can be expressed as

$$S_G = \frac{K_T}{T_1^2}(\nabla T)^2 + \frac{\mu}{T_1}\varphi + \frac{J^2}{\sigma T_1} + \frac{RD}{C_1}(\nabla C)^2 \tag{13}$$

In Eq.(13), φ is the viscous heating, J is current density, ΔT is temperature difference, ΔC is Concentration difference, R is ideal gas constant.

$$S_G = \frac{K_T}{T_1^2}\left(\frac{\Delta T}{R_2}\right)^2\left(\frac{d\theta}{dr}\right)^2 + \frac{\mu R_1^2 \Omega^2}{R_2^2 T_1}\left(\frac{dv}{dr} - \frac{v}{r}\right)^2 + \frac{\eta_0 R_1^2 \Omega^2}{R_2^4}(D^2 v)^2 + \frac{\sigma R_1^2 \Omega^2 B_0^2}{T_1} v^2 + \frac{RD}{C_1}\left(\frac{\Delta c}{R_2}\right)^2\left(\frac{d\emptyset}{dr}\right)^2 \tag{14}$$

The non-dimensional form of Entropy generation $Ns$the equation (14) can be given as

$$Ns = \frac{R_2^2 T_1^2}{K_T(\Delta T)^2} S_G$$

$$Ns = \left(\frac{d\theta}{dr}\right)^2 + \frac{Br}{T_d}\left[\left(\frac{dv}{dr} - \frac{v}{r}\right)^2 + S(D^2 v)^2\right] + Ha^2(1-\eta)^{-2}\frac{Br}{T_d} v^2 + \lambda\left(\frac{C_d}{T_d}\right)^2\left(\frac{d\emptyset}{dr}\right)^2$$

$$Ns = N_H + N_F + N_M + N_D \tag{15}$$

Where $T_d = \frac{\Delta T}{T_1}$, $C_d = \frac{\Delta C}{C_1}$ and $\lambda = \frac{RDC_1}{K_T}$.

2. 2. Bejan Number

Bejan number is described to be fraction of entropy generation due to heat transfer.

$$Be = \frac{1}{1 + \frac{N_F + N_M + N_D}{N_H}} = \frac{1}{1+\emptyset} \tag{16}$$

3. Results and discussions

A logical model for a cross diffusive Magnetized couple stress fluid in Horizontal inner rotating cylinder has been produced. To comprehend the conduct of the fluid qualities, Velocity(v), Temperature (θ), Concentration(ϕ), Entropy age number(Ns), Bejan number(Be), for various parameters like Couple stress parameter (S), Hartmann number(Ha), Soret number(Sr), Dufour number(Df), Concentration difference number(Cd), Brinkmann number(Br), Temperature distinction number(Td) are determined and are portrayed through diagrams.

Fig. 2(a)- 2(b) exhibit the impact of azimuthal velocity (v) concerning Hartmann number (Ha) and couple pressure parameter (S). It is seen that the velocity increments with Ha and diminishes with S. Fig. 3(a) depict the impact of temperature by changing the Dufour number. It clears that, there is a steady decline in the temperature with increment in Dufour number. Fig. 3(b) demonstrates the variety of concentration on Soret number (Sr). It is seen that there is an expansion in concentration with increment in Sr. Fig 4(a)- 4(b) gives the impact of temperature difference number(Td) on entropy generation number and the Bejan number. It is seen that as the temperature difference number builds the entropy generation number increments close to the internal cylinder; Bejan number abatements close to the axis of the cylinder (i.e., Be increment as r increment up to r = 0.2) as Td increment after that Be stays constant. Fig. 5(a)- 5(b) demonstrates the response of concentration difference number (Cd) on entropy generation number and Bejan number. The impact of Cd is irrelevant on Ns(fig.5a), additionally it is seen that as Cd expands Bejan number. Fig. 6(a)- 6(c) demonstrate the impact of Brinkmann number on temperature, entropy generation number and Bejan number. It is cleared that as Brinkmann number builds entropy generation number and Bejan number increment, and temperature diminishes concerning r.

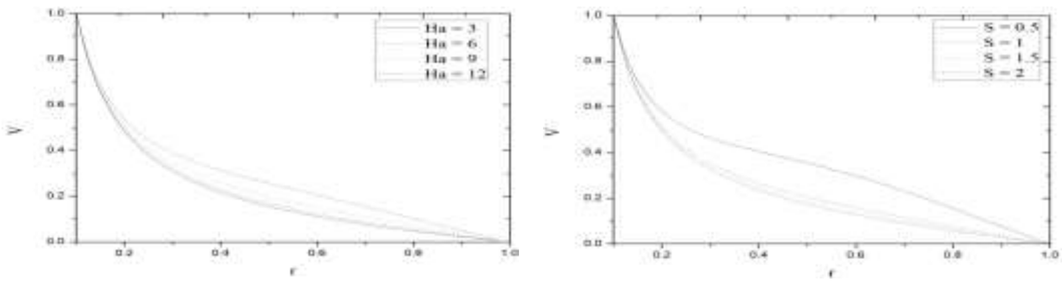

Fig. 2(a)-2(b) Variation of Hartmann number (*Ha*) and couple stress parameters (*S*) on velocity (**V**) with respect *r*

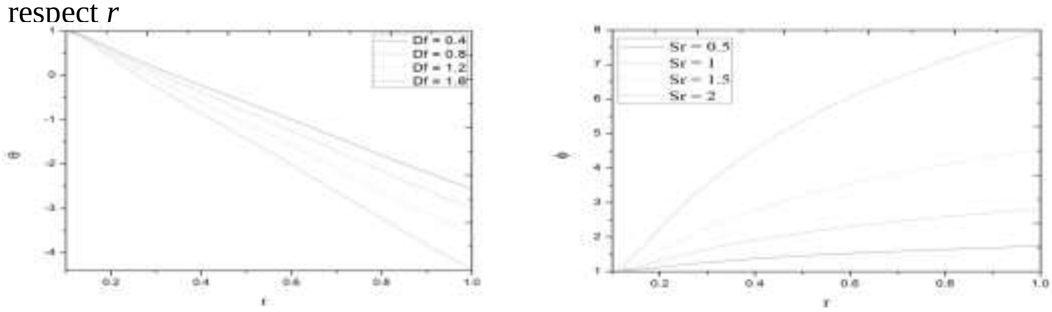

Fig. 3(a)-3(b) Variation of Dufour parameter on Temperature and effect of Soret parameter on concentration

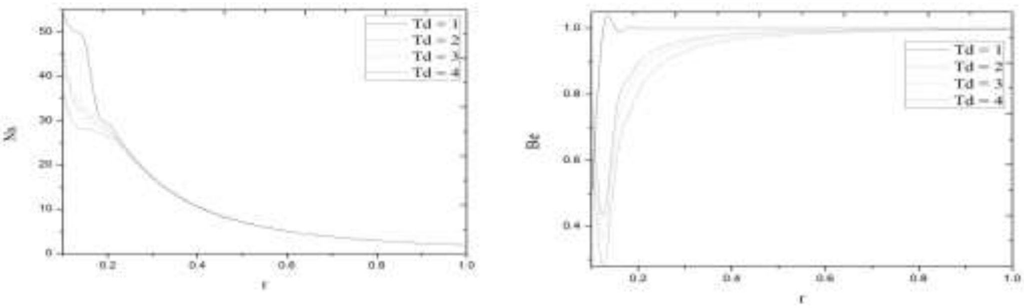

Fig. 4(a)-4(b) Variation of temperature difference number on entropy generation number and Bejan number

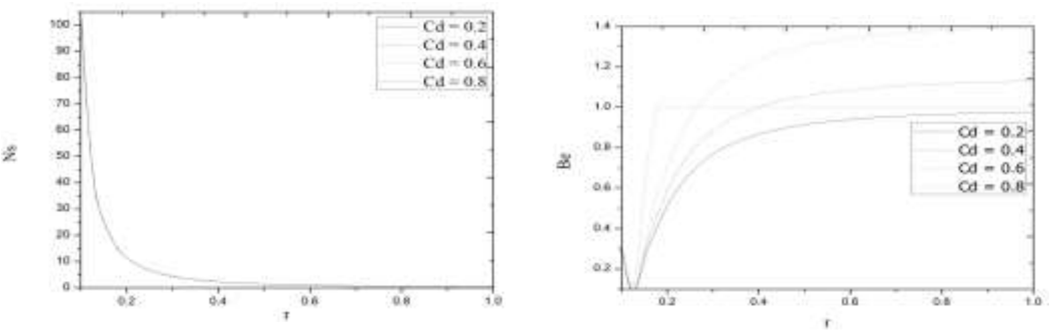

Fig. 5(a)-5(b) Variation of concentration difference number on entropy generation number and Bejan number

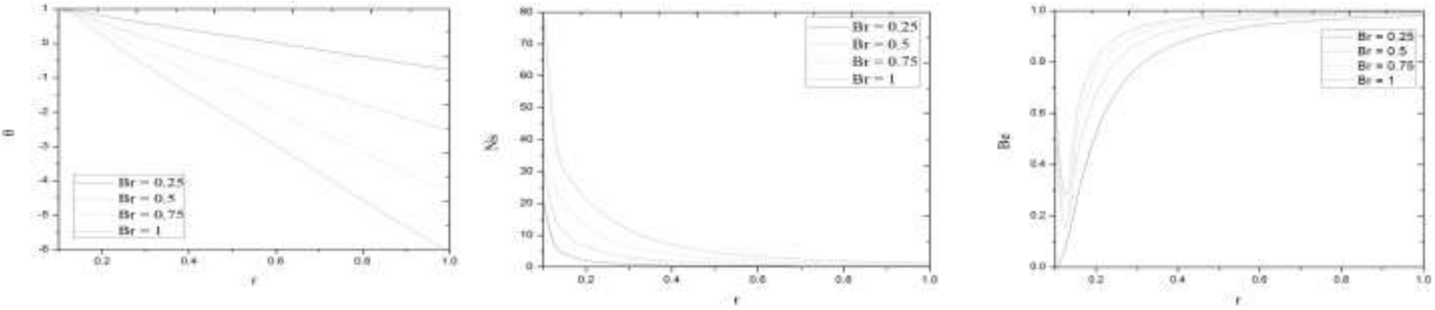

Fig. 6(a)-6(c) Variation of Brinkman number on temperature, entropy generation number and Bejan number

4. Conclusion

In this present work, the investigative arrangement acquired by utilizing adjusted Bessel's elements of a magnetized couple stress fluid through the horizontal inner rotating cylinders has been talked about.

1. There is a consistent decline in the temperature as Dufour and Brinkmann number increment.

2. Entropy number has a decrement and stays consistent with the climb of the Temperature distinction parameter, no critical change as there is an addition in Concentration contrast number.

3. Bejan number reductions with the expansion of Concentration contrast number, Brinkmann number

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