PERISTALTIC PUMPING OF FOURTH GRADE FLUID THROUGH POROUS MEDIUM IN AN INCLINED CHANNEL WITH VARIABLE VISCOSITY UNDER THE INFLUENCE OF SLIP EFFECTS

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Abstract: This article deals with the peristaltic motion of fourth grade fluid through porous medium in an inclined channel with variable viscosity under the effect of slip condition. A long wave length approximation is used, the system of the governing non-linear partial differential equation has been solved by using perturbation method. In the obtained solution expressions the long wavelength and low Reynolds number assumptions are utilized. Effects of various parameters of interest on pumping characteristics is discussed through graphs, streamlines pattern and trapping of peristaltic flow pattern are studied with particular emphasis.

***Keywords*:** Fourth grade fluid, porous medium, inclined, variable viscosity, slip condition, peristaltic transport, non-Newtonian fluid.

1. Introduction

Latham [1] who reported the mechanism of peristalsis with examples. Shapiro et al. [2] studied pumping of viscous fluid. Peristaltic pumping of a HB fluid in an inclined tube is studied by Vajravelu, Sreenadh et al. [3]. Peristaltic transport of a Jeffrey fluid under the influence of a magnetic field in an inclined passage studied by Ramana Murthy, Ravi Kumar et al. [4]. Krishna Kumari, Ravi Kumar et al. [5] investigated peristaltic transport of MHD Casson fluid with kept an angle.

Shehawes [6] studied effects on porous boundaries on peristatic transport through a porous medium and interaction of peristaltic flow with pulsatile Magneto fluid with a porous medium results are obtained by Afif et al. [7]. Mekheimer et al. [8] discussed nonlinear peristaltic pumping through a porous medium. Chaube et al. [9] studied slip influence on peristatic transport of micropolor fluid.

Misery et al. [10] studied influence of a fluid with variable viscosity. Ali [11] investigated slip influence on the peristatic pumping of MHD fluid with variable viscosity. Ravi Kumar [12] studied pulsatile pumping of a viscous strafed fluid of variable viscosity. Abbasi et al. [13] investigated Soret and Dufour effect on flow MHD fluid with variable viscosity. Siddiqui [14] studied flow of a second-order fluid in a planar passage & corresponding results in tubes are obtained by Siddiqui, Schwarz [15]. Tan et al. [16] investigated 1st problem for a second grade fluid in a porous half space. Hayat [17] studied peristaltically induced motion of a MHD third grade fluid. Harojun [18] studied effect of peristatic transport of a 3rd order fluid in an asymmetric passage. Hayat [19] investigated the influence of slip on the peristaltic pumping of a 3rd order fluid. Sreenadh [20] studied peristaltic flow of a 4th grade fluid between porous walls. Bhikshu et al. [21] studies the effects of MHD on the peristaltic flow of 4th grade fluid .

In view of these works done by various researchers, we propose to study peristaltic flow of 4th grade fluid through porous medium in an inclined passage with variable viscosity under influence of slip effects. A regular perturbation technique is employed in solving the non – linear governing equations.

2. Mathematical Modelling

Equation of flow of 4th grade fluid passing through porous medium

$$\rho\frac{d\bar{V}}{dt}=-grad\ p+div\,\bar{S}-\frac{\mu}{\bar{k}}\bar{V} \tag{1}$$

where

ρ : Density

$\bar{V}$: Velocity vector

$\bar{k}$: Permeability of the porous medium

p : Pressure and

$\frac{d}{dt}$: Material derivative

The law of conservation of mass is defined by

$$div\,\bar{V}=0 \tag{2}$$

4th grade fluid stress tensor $\bar{S}$ is

$$\bar{S}=\mu\bar{A}_1+\alpha_1\bar{A}_2+\alpha_2\bar{A}_1^2+\beta_1\bar{A}_3+\beta_2\left(\bar{A}_2\bar{A}_1+\bar{A}_1\bar{A}_2\right)+\beta_3\left(tr\,\bar{A}_1^2\right)\bar{A}_1+\gamma_1\bar{A}_4+\gamma_2(\bar{A}_3\bar{A}_1+\bar{A}_1\bar{A}_3)+\gamma_3\bar{A}_2^2+\gamma_4(\bar{A}_2\bar{A}_1^2+\bar{A}_1^2\bar{A}_2)+\gamma_5(tr\bar{A}_2)\bar{A}_2+\bar{A}_2+\gamma_6(tr\bar{A}_2)\bar{A}_1^2+[\gamma_7(tr\bar{A}_3+\gamma_8 tr(\bar{A}_2\bar{A}_1))]\bar{A}_1 \tag{3}$$

where

$\mu,\alpha_1,\alpha_2,\beta_1,\beta_2,\beta_3,\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5,\gamma_6,\gamma_7,\gamma_8$ being material constants and A_n is the Rivlin – Ericksen tensors defined

$$\bar{A}_{n+1}=\frac{d\bar{A}_n}{dt}+\bar{A}_n(grad\bar{V})+(grad\bar{V})^T\bar{A}_n,\quad n>1 \tag{4}$$

$$\bar{A}_1=(grad\bar{V})+(grad\bar{V})^T \tag{5}$$

Flow of 4th grade fluid is consider in a uniform passage with porous medium of width 2d, inclined at an angle γ with variable viscosity

$$H(\bar{X},\bar{t})=d+b\,\mathrm{Cos}\left(\frac{2\pi}{\lambda}\left(\bar{X}-c\bar{t}\right)\right) \tag{6}$$

where

b : Wave amplitude

λ : Wave length

$\bar{t}$: Time

$\left(\bar{U},\bar{V}\right)$, $\left(\bar{u},\bar{v}\right)$: Velocity components in fixed & wave frame respectively and the relation between fixed and wave frames as

$$\bar{x}=\bar{X}-c\bar{t},\quad \bar{y}=\bar{Y},\quad \bar{u}=\bar{U}-\bar{c},\quad \bar{v}=\bar{V} \tag{7}$$

when we non dimensionalize equations (3), (6) and (7) we have

$$S=A_1+\lambda_1A_2+\lambda_2A_1^2+\xi_1A_3+\xi_2\left(A_2A_1+A_1A_2\right)+\xi_3\left(tr\,A_1^2\right)A_1+\eta_1A_4+\eta_2(A_3A_1+A_1A_3)+\eta_3A_2^2+\eta_4(A_2A_1^2+A_1^2A_2)+\eta_5(trA_2)A_2+A_2+\eta_6(trA_2)A_1^2+[\eta_7(trA_3+\eta_8tr(A_2A_1))]A_1 \tag{8}$$

$$h(x)=1+\phi\cos2\pi x \tag{9}$$

$$y=\frac{\bar{y}}{d},\quad u=\frac{\bar{u}}{c},\quad v=\frac{\bar{v}}{\delta c},\quad h=\frac{\bar{H}}{d},$$

$$S=\frac{d}{\mu_0 c}\bar{S}(\bar{x}),\quad \mu(y)=\frac{\mu(\bar{y})}{\mu_0},\quad \delta=\frac{d}{\lambda},$$

$$\mathrm{Re}=\frac{\rho cd}{\mu_0},\quad Fr=\frac{c^2}{gd},\quad \sigma^2=\frac{d^2}{k},$$

$$\lambda_1=\frac{\alpha_1c}{\mu_0d},\quad \lambda_2=\frac{\alpha_2c}{\mu_0d},\quad \xi_1=\frac{\beta_1c^2}{\mu_0d^2},$$

$$\xi_2=\frac{\beta_2c^2}{\mu_0d^2},\quad \xi_3=\frac{\beta_3c^2}{\mu_0d^2},\quad \phi=\frac{b}{d},$$

$$\eta_1=\frac{\gamma_1c^3}{\mu a^3},\quad \eta_2=\frac{\gamma_2c^3}{\mu a^3},\quad \eta_3=\frac{\gamma_3c^3}{\mu a^3},\quad \eta_4=\frac{\gamma_4c^3}{\mu a^3},$$

$$\eta_5=\frac{\gamma_5c^3}{\mu a^3},\quad \eta_6=\frac{\gamma_6c^3}{\mu a^3},\quad \eta_7=\frac{\gamma_7c^3}{\mu a^3},$$

$$\eta_8=\frac{\gamma_8c^3}{\mu a^3}. \tag{10}$$

where

δ : Wave number

Re: Reynolds number

ϕ : Amplitude ratio

σ^2 : Porosity parameter

$\lambda_i\left(i=1,2\right)$: Non-Newtonian parameters and

$\xi_i\left(i=1,2,3\right)$: Non-Newtonian parameters.

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial y}+\frac{\partial S_{xy}}{\partial y}$$
$$-\mu(y)\,\sigma^2(u+1)+\frac{\mathrm{Re}}{Fr}\mathrm{Sin}\,\gamma \tag{11}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}$$
$$+\delta\frac{\partial S_{yy}}{\partial y}+\delta^2\,\mu(y)\,\sigma^2\,v-\delta\frac{\mathrm{Re}}{Fr}\mathrm{Cos}\,\gamma \tag{12}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)=\left(\frac{\partial^2}{\partial y^2}-\delta^2\frac{\partial^2}{\partial y^2}\right)S_{xy}$$
$$+\delta\frac{\partial^2}{\partial x\partial y}\left(S_{xx}+S_{yy}\right)-\sigma^2\left(\frac{\partial}{\partial y}\left(\mu(y)\,(u+1)\right)+\delta^2\frac{\partial v}{\partial x}\right) \tag{13}$$

using the long wavelength and less Re no. approximations we obtain

$$\frac{\partial p}{\partial x}=\frac{\partial S_{xy}}{\partial y}-\mu(y)\,\sigma^2(u+1)+\frac{\mathrm{Re}}{Fr}\mathrm{Sin}\,\gamma \tag{14}$$

$$\frac{\partial p}{\partial y}=0 \tag{15}$$

$$S_{xy}=\mu(y)\frac{\partial u}{\partial y}+2\Gamma\left(\frac{\partial u}{\partial y}\right)^3 \tag{16}$$

where $\Gamma=\xi_2+\xi_3$ is the Deborah number.

Eq. (15) indicates that $p\neq p(y)$.

From equation (13) we have

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial u}{\partial x}+2\Gamma\left(\frac{\partial u}{\partial y}\right)^3\right)$$
$$-\sigma^2\left(\frac{\partial}{\partial y}\left(\mu(y)\,(u+1)\right)+\delta^2\frac{\partial v}{\partial x}\right)=0 \tag{17}$$

The boundary conditions are,

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \tag{18}$$

$$u+\beta\left(\mu(y)\frac{\partial u}{\partial y}+2\Gamma\left(\frac{\partial u}{\partial y}\right)^3\right)=-1 \quad \text{at}$$
$$y=h \tag{19}$$

where $\beta=\dfrac{l}{d}$ is a slip parameter

In order to check the influence of variable viscosity on peristaltic flow

$$\mu(y)=e^{-\alpha y},\quad \alpha<<1 \tag{20}$$

i.e. $\mu(y)=1-\alpha\,y+\mathrm{O}(\alpha^2)$ (21)

The volume flow rate q is

$$q=\int_0^h u\,dy \tag{22}$$

The instantaneous flow Q (X ,t) is

$$Q(X,t)=\int_0^h U\,dY=\int_0^h (u+1)\,dy=q+h \tag{23}$$

The time averaged volume flow rate $\bar{Q}$ over is

$$\bar{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1 \tag{24}$$

The pressure rise Δp

$$\Delta p=\int_0^1\frac{dp}{dx}dx \tag{25}$$

3. Solution of the Problem

Perturbation Solution

We use the following expansions in order to obtain perturbation solution

$$u=u_0+\Gamma u_1+.....$$
$$p=p_0+\Gamma\,p_1+..... \tag{26}$$

where

$$u_0=u_{00}+\alpha\,u_{01}+.....$$
$$u_1=u_{10}+\alpha\,u_{11}+.....$$
$$p_0=p_{00}+\alpha\,p_{01}+..... \tag{27}$$
$$p_1=p_{10}+\alpha\,p_{11}+.....$$

when we substitute Eq. (27) into Eqs. (17) to (25) and separate the terms of differential order in Γ and α we obtain

Zeroth-order Equation and Boundary Conditions:

$$\frac{\partial p_{00}}{\partial x}=\frac{\partial^2 u_{00}}{\partial y^2}-\sigma^2(u_{00}+1)+\frac{\mathrm{Re}}{Fr}\mathrm{Sin}\,\gamma \tag{28}$$

$$\frac{\partial u_{00}}{\partial y}=0 \quad \text{at} \quad y=0 \tag{29}$$

$$u_{00}+\beta\frac{\partial u_{00}}{\partial y}=-1 \quad \text{at} \quad y=h \tag{30}$$

First-order Equation and Boundary Conditions:

$$\frac{\partial p_{01}}{\partial x}=\frac{\partial^2 u_{01}}{\partial y^2}-\sigma^2 u_{01}+\sigma^2\, y\,(u_{00}+1)-\frac{\partial}{\partial y}\left(y\frac{\partial u_{00}}{\partial y}\right) \qquad (31)$$

$$\frac{\partial u_{01}}{\partial y}=0 \text{ at } \quad y=0 \qquad (32)$$

$$u_{01}-\beta y\frac{\partial u_{00}}{\partial y}+\beta\frac{\partial u_{01}}{\partial y}=0 \quad \text{at} \quad y=h \qquad (33)$$

Zeroth-order Solution

Solving equation (28) we get,

$$u_{00}=c_1\,Cosh\,(\sigma h)-\frac{G_0}{\sigma^2} \qquad (34)$$

where

$$c_1=\frac{1}{L_1}\left(\frac{G_0}{\sigma^2}-1\right)$$

$$L_1=Cosh\,(\sigma h)+\beta\,\sigma\,Sinh\,(\sigma h)$$

$$G_0=\frac{\partial p_0}{\partial x}+\sigma^2-\frac{Re}{Fr}Sin\,(\gamma)$$

First-order Solution

Sub. zeroth order soln. Eq. (34) in Eq. (31) and then solving the
Eq. (31), we get

$$u_{01}=c_3\,Cosh\,(\sigma h)+c_4\,Sinh\,(\sigma h)$$
$$-\frac{(G_1+y)}{\sigma^2}+\frac{c_1\,y}{2}Cosh\,(\sigma h) \qquad (35)$$

$$G_1=\frac{\partial p_{01}}{\partial x}$$

$$L_2=Sinh\,(\sigma h)+\beta\,\sigma\,Cosh\,(\sigma h)$$

$$L_3=\frac{c_1}{2}\big((h+\beta)\,Cosh\,(\sigma h)-\beta\,\sigma\,h\,Sinh\,(\sigma h)\big)$$

$$L_4=\frac{G_1+h+\beta}{\sigma^2}$$

$$c_3=\frac{1}{L_1}\left(L_4-\frac{c_1 L_3}{2}-c_4\,L_2\right)$$

$$c_4=\frac{1}{\sigma^3}-\frac{c_1}{2\sigma}$$

using equations (34) and (35) along with the relation $u=u_{00}+\Gamma\,u_{01}$ we get

$$u=\frac{1}{\sigma^2}\left(\frac{\partial p}{\partial x}+\sigma^2+\frac{Re}{Fr}Sin(\gamma)\right)\left(\frac{Cosh\,(\sigma h)}{L_1}-1\right)-\frac{Cosh\,(\sigma h)}{L_1}$$
$$+\Gamma\left(\begin{array}{l}\dfrac{Cosh\,(\sigma h)}{L_1}\left(\dfrac{h+\beta}{\sigma^2}+\dfrac{L_3}{2}-\dfrac{L_2}{2\sigma^3}-\dfrac{G_0}{2\sigma^2}\left(L_3+\dfrac{L_2}{\sigma}\right)\right)\\ +\dfrac{1}{2L_1}\left(\dfrac{G_0}{\sigma^2}-1\right)\left(\dfrac{Sinh\,(\sigma h)}{\sigma}+y\,Cosh\,(\sigma h)\right)\\ -\dfrac{Sinh\,(\sigma h)}{\sigma^3}-\dfrac{y}{\sigma^2}\end{array}\right) \qquad (36)$$

The volume flow rate is

$$q=\int_0^h u\;dy$$

$$q=\frac{1}{\sigma^2}\left(\frac{\partial p}{\partial x}+\sigma^2+\frac{Re}{Fr}Sin(\gamma)\right)\left(\frac{Sinh(\sigma h)}{\sigma L_1}-h\right)-\frac{Sinh(\sigma h)}{\sigma L_1}$$
$$+\Gamma\left(\begin{array}{l}\dfrac{Sinh(\sigma h)}{\sigma L_1}\left(\dfrac{h+\beta}{\sigma^2}+\dfrac{L_3}{2}-\dfrac{L_2}{2\sigma^3}-\dfrac{G_0}{2\sigma^2}\left(L_3+\dfrac{L_2}{\sigma}\right)\right)\\ +\left(\dfrac{G_0}{\sigma^2}-1\right)\dfrac{h\,Sinh(\sigma h)}{2\sigma L_1}-\dfrac{Cosh(\sigma h)}{\sigma^4}-\dfrac{h^2}{2\sigma^2}+\dfrac{1}{\sigma^4}\end{array}\right) \qquad (37)$$

The pressure rise is $\Delta p=\int_0^1\frac{dp}{dx}dx$

where

$$\frac{dp}{dx}=\frac{\sigma^3 L_1}{Sinh(\sigma h)-h\sigma L_1}\left(q+\frac{Sinh(\sigma h)}{\sigma L_1}-\Gamma\left(\begin{array}{l}\dfrac{Sinh(\sigma h)}{\sigma L_1}\left(\begin{array}{l}\dfrac{h+\beta}{\sigma^2}+\dfrac{L_3}{2}-\dfrac{L_2}{2\sigma^3}\\ -\dfrac{G_0}{2\sigma^2}\left(L_3+\dfrac{L_2}{\sigma}\right)\end{array}\right)\\ +\dfrac{h\,Sinh\,(\sigma h)}{2\sigma L_1}\left(\dfrac{G_0}{\sigma^2}-1\right)\\ -\dfrac{Cosh\,(\sigma h)}{\sigma^4}\\ -\dfrac{h^2}{2\sigma^2}+\dfrac{1}{\sigma^4}\end{array}\right)\right)$$
$$-\sigma^2-\frac{Re}{Fr}Sin(\gamma) \qquad (38)$$

4. Results and Discussions

Fig. 1 to 5. shows the changes in pressure rise due to different parameters: In Fig. 1 to Fig. 5, it is noticed that with the rise in γ, Γ there is a rise in Δp. In Fig. 2 and Fig. 4 it is noticed that with a fall in values of ϕ, σ there is a rise in Δp. In Fig. 3 it is noticed that with a fall in values of β $(\Delta p < 0)$ there can be seen a rise in Δp and the graph also shows that there is a rise in Δp with a rise in values of β $(\Delta p > 0)$.

Fig. 6 to Fig. 10 shows the changes on pressure gradient due to different parameters.

Fig. 7 to 8 shows graphs for pressure gradient vs x. It can be noticed that there is a rise in pressure gradient with a rise in σ, β in first half wave length of the passage and the effect reverses in second half wave length of the passage. Fig. 6, Fig. 9 and Fig. 10 shows graphs for pressure gradient vs x. We can notice that there is a rise in pressure gradient with lowering in values of Γ, ϕ, γ in first half wave length of the passage and the effect reverses in second half wave length of the passage. Fig. 11 to 15 shows the effects of on axial velocity due to various parameters.

Fig. 11, 13 and 14 shows for velocity vs y. It is noticed that there is a fall in velocity with a rise in values of Γ, β, ϕ for a stable y value. In Fig. 15 it is noticed that velocity rises with rise in γ for stable y value. In Fig. 12 it is noticed that there is a fall in velocity with rise in the porosity parameter σ.

Trapping:

Fig. 16 shows the stream line patterns and trapping for various values of the β and here we noticed that the size of the bolus decreases with rise in β. Fig. 17 shows the stream line patterns and trapping for various values of Γ here we observed that the size of bolus falls with rise in Γ. Fig. 18 shows the stream line patterns and trapping for various values of γ, here we observed that the size of bolus increases with rise in γ. Fig. 19 shows the stream line patterns and trapping for various values of σ, here we noticed that the size of the bolus increases with increase of σ.

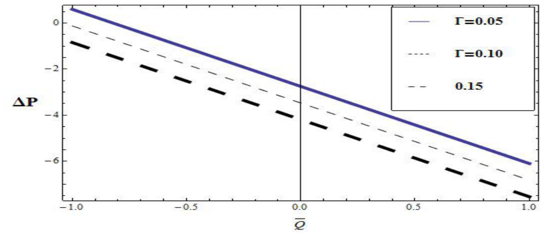


Fig. 1 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'Γ'.

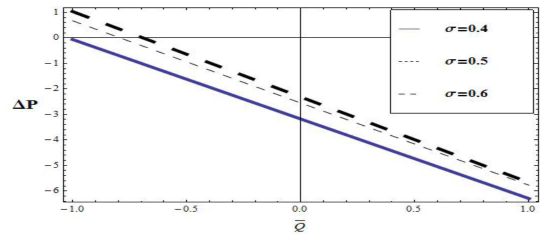


Fig. 2 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'σ'.

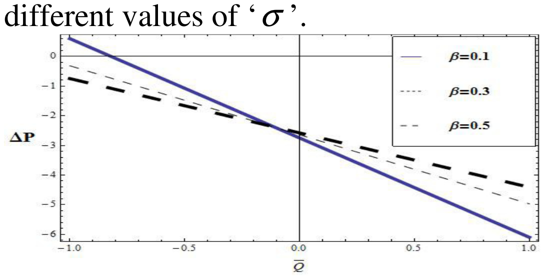


Fig. 3 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'β'.

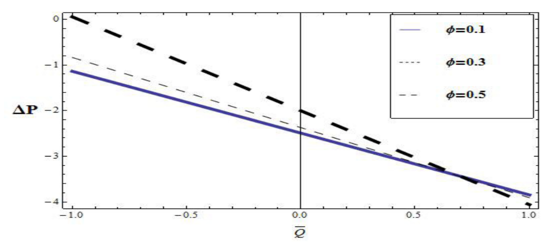


Fig. 4 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'ϕ'.

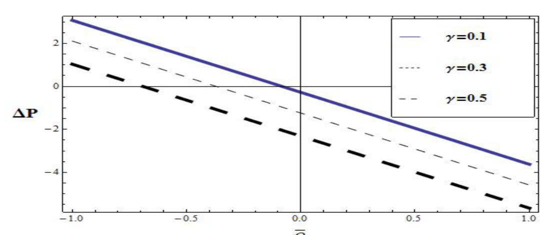


Fig. 5 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'γ'.

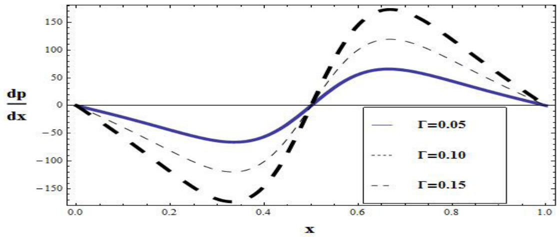


Fig. 6 Pressure gradiant $\frac{dp}{dx}$ vs x for different values of 'Γ'.

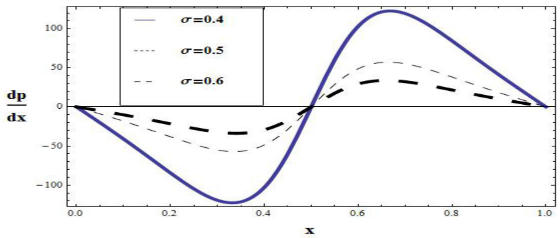


Fig. 7 Pressure gradiant $\frac{dp}{dx}$ vs x for different values of 'σ'.

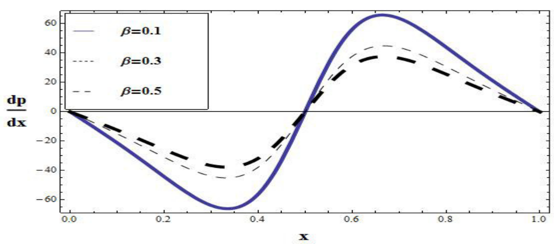


Fig. 8 Pressure gradiant $\frac{dp}{dx}$ vs for different values of 'β'.

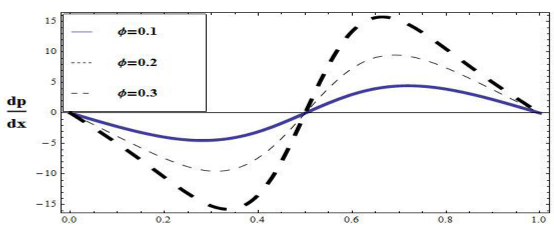


Fig. 9 Pressure gradiant $\frac{dp}{dx}$ vs x for different values of 'ϕ'.

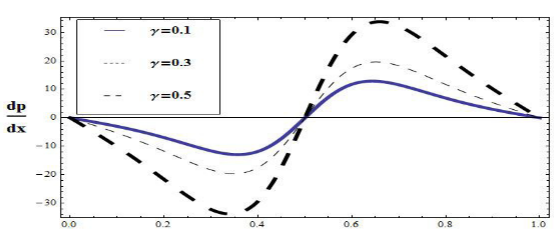


Fig. 10 Pressure gradiant $\frac{dp}{dx}$ vs x for different values of 'γ'.

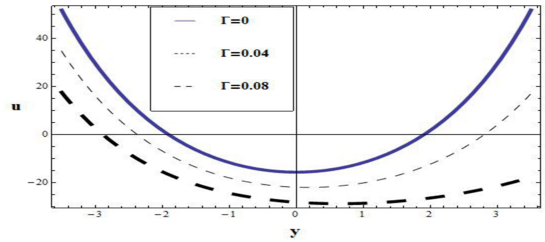


Fig. 11 Axial velocity u vs y for different values of 'Γ'.

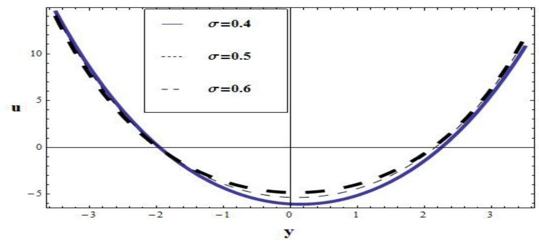


Fig. 12 Axial velocity u vs y for different alues of 'σ'.

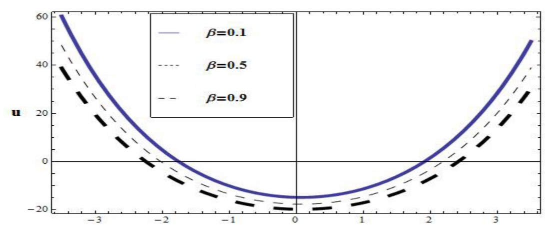


Fig. 13 Axial velocity u vs y for different values of 'β'.

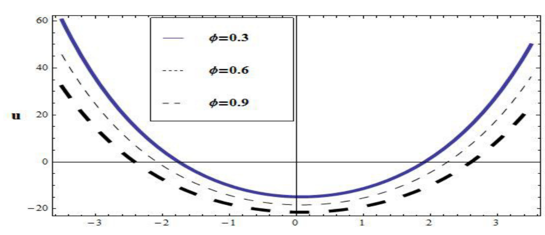


Fig. 14 Axial velocity u vs y for different values of 'ϕ'.

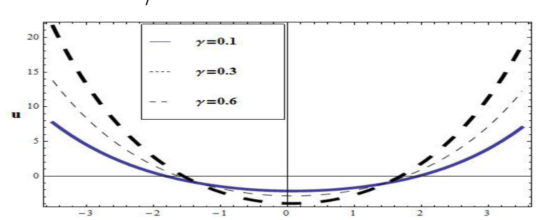
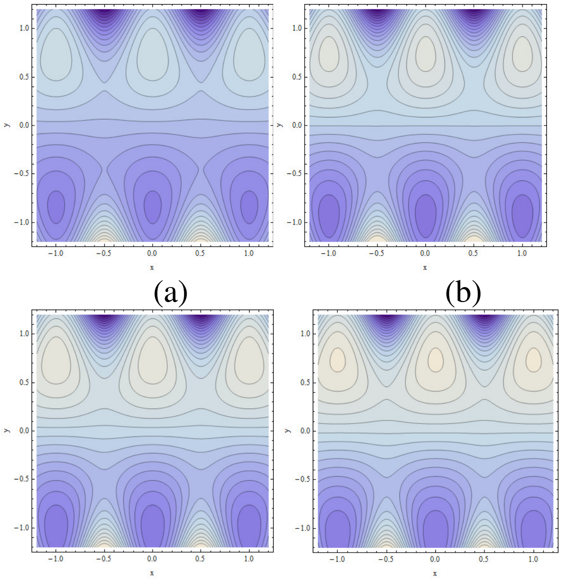


Fig. 15 Axial velocity u vs y for different values of 'γ'with

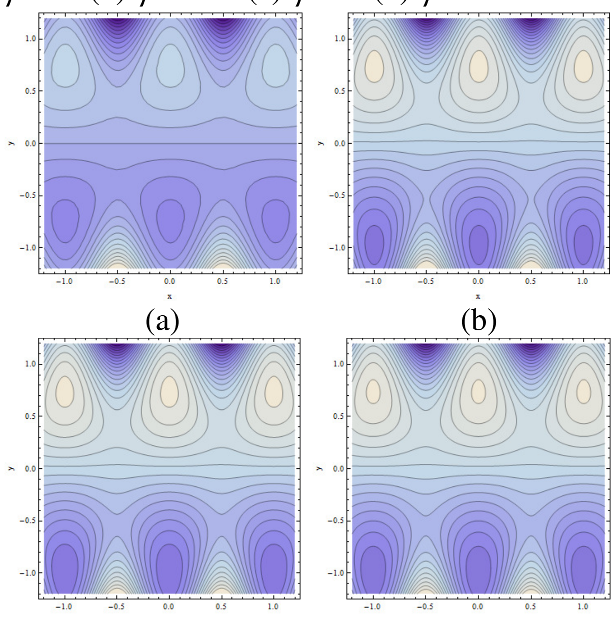
$\phi = 0.3,\ \sigma = 0.6,\ \alpha = 0.04,\ \Gamma = 0.02,\ \beta = 0.1.$



(a) (b)

(c) (d)

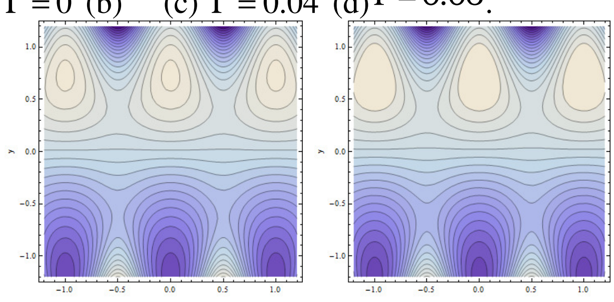
Fig. 16 Stream lines for different values of (a) $\beta = 0$ (b) $\beta = 0.5$ (c) $\beta = 1$ (d) $\beta = 1.5$.



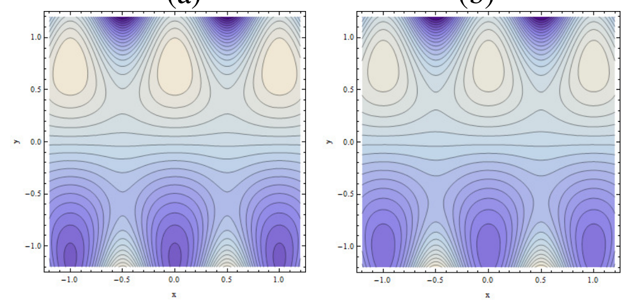
(a) (b)

(c) (d)

Fig. 17 Stream lines for different values of (a) $\Gamma = 0$ (b) (c) $\Gamma = 0.04$ (d) $\Gamma = 0.06$.

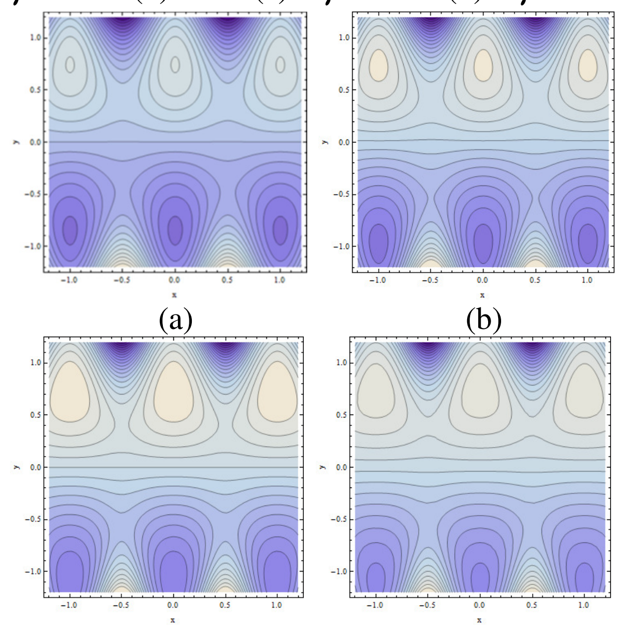


(a) (b)



. (c) (d)

Fig. 18 Stream lines for different values of (a) $\gamma = 0.1$ (b) (c) $\gamma = 0.3$ (d) $\gamma = 0.4$.



(a) (b)

(c) (d)

Fig. 19 Stream lines for different values of (a) $\sigma = 0.4$ (b) $\sigma = 0.6$ (c) $\sigma = 0.8$ (d) $\sigma = 1$.

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