Rheological analysis of CNT suspended Nanofluid with convective boundary condition using spectral method

S. Karmakar[1], V.M. Magagula[2], S. Mondal[1] *

[1]Department of Mathematics, Amity University, Kolkata, Newtown 700135, West Bengal, India

[2]Department of Mathematics, University of Eswantini M201 Kwaluseni, Swaziland

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Abstract

The rheological analysis on two-dimensional stagnation-point flow of carbon nano-tubes towards a stretching sheet with water as a base nano fluid with convective boundary condition based on the advantage of nanofluid flows in CNT is discussed here. After doing similarity transformation the resulting nonlinear coupled equations with the relevant boundary conditions are solved numerically using recently developed Spectral Quasilinearization Method (SQLM). The influence of the flow parameters on the dimensionless velocity and concentration profiles are depicted and described in forms of graphs here.

Keywords: Rheological analysis; Nanofluid flow; CNT; Convective boundary condition; Spectral quasilinearization method.

1 Introduction

Recently, many researchers have interest on heat and mass transfer on different types of nano fluid flow problems. Now a days, nanofluid is one of the emerging field of research in science and technology. In nanofluid small solid volume fraction of nanoparticles i.e., less than 5% is suspended in different type of base fluids like water, oil etc. In recent year, Nandy *et al.* (2014) examined forced convection in an unsteady nanofluid flow past a permeable shrinking sheet subject to heat loss due to thermal radiation. They studied the impacts of thermal radiation, magnetic field and unsteadiness on the flow properties of the fluid and heat transfer. Das *et al.* (2015) studied results from numerical simulation for heat and mass transfer in an electrically conducting incompressible nanofluid flow near a heated stretching sheet using convective boundary condition.

Rheology is the study of the flow of matter, primarily in a liquid state, but also as "soft solids" or solids under conditions in which they respond with plastic flow rather than deforming elastically

*Email id: smondal@kol.amity.edu

in response to an applied force. Rheology generally accounts for the behaviour of non-Newtonian fluids, by characterizing the minimum number of functions that are needed to relate stresses with rate of change of strain or strain rates. In practice, rheology is principally concerned with extending continuum mechanics to characterize flow of materials that exhibits a combination of elastic, viscous and plastic behaviour by properly combining fluid mechanics and elasticity. Rheology unites the seemingly unrelated fields of plasticity and non-Newtonian fluid dynamics by recognizing that materials undergoing these types of deformation are unable to support a stress in static equilibrium. One of the major tasks of rheology is to empirically establish the relationships between deformations (or rates of deformation) and stresses. Rheology has applications in materials science engineering, geophysics, physiology, human biology and pharmaceutics. Rheometers are instruments used to characterize the rheological properties of materials, typically fluids that are melts or solution. These instruments impose a specific stress field or deformation to the fluid, and monitor the resultant deformation or stress.

The spectral quasiliearization method is a recent spectral collocation based method. The techniques have been used successfully in a few prior studies in the literature, and for systems of varying complexities. The new approached of spectral method like spectral quasi-linearization method, successive linearization method, spectral perturbation method and the spectral homotopy analysis method have been used successfully to solve boundary value problems (Agbaje *et al.* (2018), Sithole *et al.* (2017), Goqo *et al.* (2016) and Motsa *et al.* (2014) etc).

The main objective of this study is to discuss the rheological analysis on two-dimensional stagnation-point flow through a carbon nanotubes towards a stretching sheet with water as a base fluid with convective boundary condition of nanofluid flows. The focus is on the study on stagnation point flow through CNT is used with water as a base fluid using convective boundary condition. The resulting nonlinear coupled equations with the relevant boundary conditions are solved numerically using recently developed spectral method. The influence of the flow parameters on the dimensionless velocity and concentration are depicted and described in forms of graphs.

2 Problem Formulation

Here, water based 2-dimensional stagnation-point flow over a stretching sheet with water as based fluids encompassing single-wall CNT's. The flow is presumed to be laminar, steady and incompressible. The base fluid and the CNT's are expected to be in updraft equilibrium. Sheet is assumed to be stretched with the different velocity u_w, v_w along the x-axis and y-axis, respectively. Further, we have taken the constant temperature T_w at wall and the free stream temperature T_∞. (see Fig. 1) With

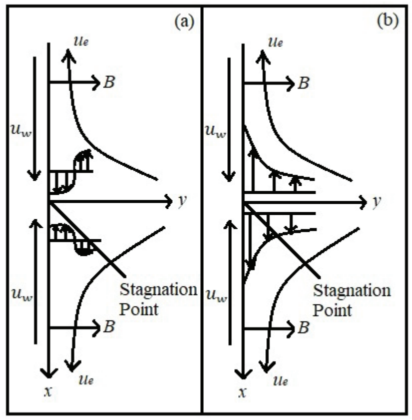

Figure 1: Geometry of the problem (a) Shrinking case (b) Stretching case.

the above analysis the boundary layer equations for the proposed model can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{1}{\rho_{nf}}\frac{\partial}{\partial y}(\mu_{nf}(T)\frac{\partial u}{\partial y})-\frac{\sigma B_0^2}{\rho_{nf}}(u-u_e)\pm g\beta(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}-k_1(C-C_\infty). \tag{4}$$

The relevant boundary conditions are of the form:

$$u=u_w(x)=cx,\;\; v=v_w,\;\; -k_0\frac{\partial T}{\partial y}=h_f(T_w-T),\;\; C=C_w,\;\; \text{at}\;\; y=0, \tag{5}$$

$$u\to u_e(x)=ax,\;\; v\to 0,\;\; T\to T_\infty,\;\; C\to C_\infty,\;\; \text{as}\;\; y\to\infty. \tag{6}$$

In above equations u and v are the velocity components along the $x-$ and $y-$axes respectively; $a,c>0$ the constants; u_w is velocity at wall; T is the temperature; C is the concentration; ρ_{nf} is the nanofluid density; μ_{nf} is the viscosity of nanofluid and α_{nf} is the thermal diffusivity of nanofluid. These are defined as

$$\mu_{nf}(\theta)=\frac{\mu_f(\theta)}{(1-\phi)^{2.5}},\;\; (\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_{CNT},$$
$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_{CNT},\;\; \alpha_{nf}=\frac{k_{nf}}{\rho_{nf}(C_p)_{nf}},$$
$$\frac{k_{nf}}{k_f}=\frac{(1-\phi)+2\phi k_{CNT}/(k_{CNT}-k_f)\ln(k_{CNT}+k_f)/2k_f}{(1-\phi)+2\phi k_f/(k_{CNT}-k_f)\ln(k_{CNT}+k_f)/2k_f}, \tag{7}$$

where $(\rho C_p)_{nf}$ is the effective heat capacity of a nanoparticle, μ_f is the viscosity of base fluid, ϕ is the nanoparticle fraction, k_{nf} is the thermal conductivity of nanofluid, ρ_f and ρ_{CNT} are the densities of the base fluid and carbon nanotubes respectively, k_f and k_{CNT} are the thermal conductivities of the base fluid and carbon nano tubes respectively,.

Reynolds model of viscosity expression can be taken as

$$\mu_f(\theta)=e^{-\alpha\theta}=1-(\alpha\theta)+O(\alpha\theta)^2, \tag{8}$$

where α is the viscosity parameter.

Introducing the following similarity transformations

$$\eta = \sqrt{\frac{c}{\nu_f}}y, \quad u = cxf'(\eta), \quad v = -\sqrt{c\nu_f}f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}. \tag{9}$$

Making use of Eqs. (7) − (9) in Eq. (1) to Eq. (6), we have

$$\frac{(1-\alpha\theta)}{(1-\phi)^{2.5}}f''' + \frac{(-\alpha\theta')}{(1-\phi)^{2.5}}f'' + [1-\phi+\phi\frac{\rho_{CNT}}{\rho_f}][ff'' - f'^2 + S^2] + M^2(S - f') \pm Gr\theta = 0, \tag{10}$$

$$(\frac{k_{nf}}{k_f})\theta'' + P_r[(1-\phi) + \frac{(\rho C_p)_{CNT}}{(\rho C_p)_f}]f\theta' = 0, \tag{11}$$

$$\phi'' + (Sc)f\phi' - (Sc)\gamma\phi = 0. \tag{12}$$

The non-dimensional boundary conditions are of the form

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -B_i(-\theta(0) + 1), \quad \phi(0) = 1,$$
$$f'(\infty) = S, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \tag{13}$$

where $S = \frac{a}{c}$ is the stagnation parameter, $Sc = \frac{\nu_f}{D_B}$ is the Schmidt number, $P_r = \frac{(\mu C_p)_f}{k_f}$ is the Prandtl number, $B_i = (\frac{h_f}{k_0})\frac{\nu_f}{c}$ is the Biot number, $\gamma = \frac{k_1}{c}$ is the chemical reaction parameter, $M = \frac{\sigma B_0^2 \nu_f}{c}$ is the magnetic field parameter and $Gr_x = \frac{g\beta\nu_f(T_w - T\infty)}{c^2 x}$ is the local Grashof number.

3 Method of Solution

The governing equations are solved by using Spectral Quasilinearization Method (SQLM). In our case the non-dimensional equations with its boundary conditions will express into the following the form:

$$\frac{(1-\alpha f_2)}{(1-f_3)^{2.5}}f_1''' - \frac{\alpha f_2 f_1''}{(1-f_3)^{2.5}} + \left[1 - f_3 + f_3\frac{\rho_{CNT}}{\rho_f}\right]\left[f_1 f_1'' - f_1'^2 + S^2\right] + M^2(S - f_1') \pm Gr f_2 = 0, \tag{14}$$

$$\left(\frac{k_{nf}}{k_f}\right)f_2'' + P_r\left[(1-f_3) + \frac{(\rho C_p)_{CNT}}{(\rho C_p)_f}\right]f_1 f_2' = 0, \tag{15}$$

$$f_3'' + (Sc)f_1 f_3' - (Sc)\gamma f_3 = 0. \tag{16}$$

In this case, we have $n = 3$, so we have

$$\Gamma_1 = \frac{(1-\alpha f_2)}{(1-f_3)^{2.5}}f_1''' - \frac{\alpha f_2 f_1''}{(1-f_3)^{2.5}} + \left[1 - f_3 + f_3\frac{\rho_{CNT}}{\rho_f}\right]\left[f_1 f_1'' - f_1'^2 + S^2\right] + M^2(S - f_1') \pm Gr f_2, \tag{17}$$

$$\Gamma_2 = \left(\frac{k_{nf}}{k_f}\right)f_2'' + P_r\left[(1-f_3) + \frac{(\rho C_p)_{CNT}}{(\rho C_p)_f}\right]f_1 f_2', \tag{18}$$

$$\Gamma_3 = f_3'' + (Sc)f_1 f_3' - (Sc)\gamma f_3 \tag{19}$$

The variable coefficients are then determined using equation and we leave them as they can be easily computed once you know the number of equations a system has. The η domain was truncated to $\eta_\infty = 10$ for all computations. Accurate results for all the quantities of physical interest were obtained using this value of η. Graphs displaying physical quantities of interest are presented to general performance of the method. All the results in this section were obtained using MATLAB 2015, and with varying constants.

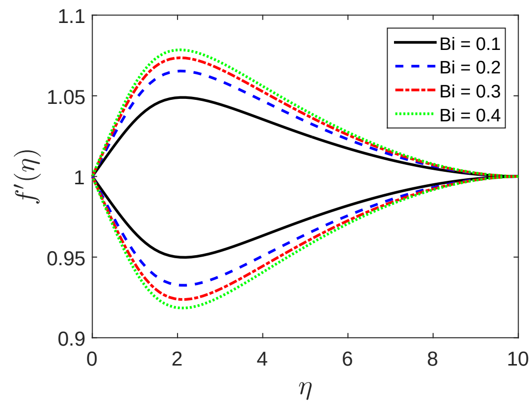

Figure 2: Velocity profile for different values of Bi

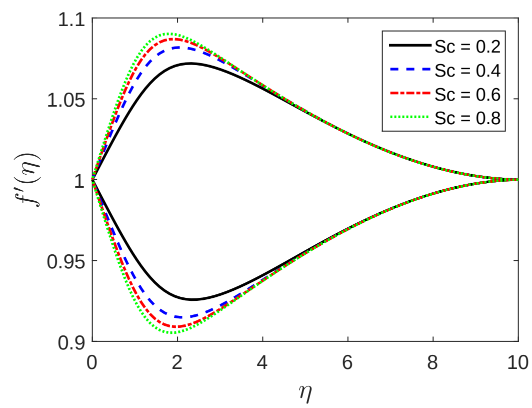

Figure 3: Velocity profile for different values of Sc

4 Results and Discussion

After doing the numerical solutions of the system of governing equations, the outcomes are depict and describe by graphs here. The influence of the flow parameters (i.e., effect of the Biot number, Schmidt number and the chemical reaction parameter) on the dimensionless velocity, temperature, concentration are presented in Figures 2 - 6. The velocity profiles are displayed for both assisting and opposing flows.

Figure 2 shows the effect of the Biot number on the velocity profile. We note that increasing the Biot number, results in an increase of the peak of the velocity profile of the assisting flow while the opposite is observed in the opposing flow. That is, increasing the Biot number, results in decreasing of the peak of the velocity profile of the opposing flow while due to the increasing value of Biot number increase the boundary layer thickness for assisting flow which causes the increasing nature of velocity but decreases velocity profile for opposing flow.

Figure 3 displays the effect of Schmidt number (Sc) on the velocity profile and similar, nature of velocity profile can be found for the increasing value of Sc as Figure 2. Figure 4 shows the effect of chemical reaction parameter on velocity profiles. It can be observed that similar, nature of velocity profile can be found for the increasing value of γ as Figure 2. From these above three figures it can

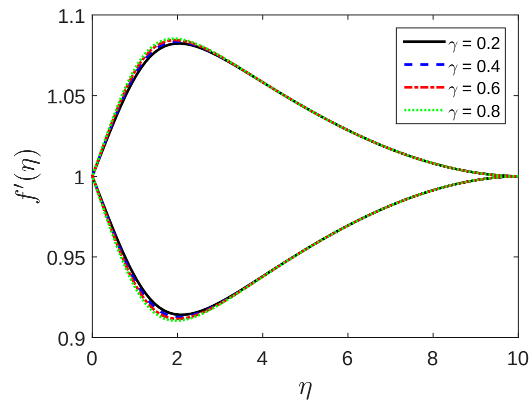

Figure 4: Velocity profile for different values of γ

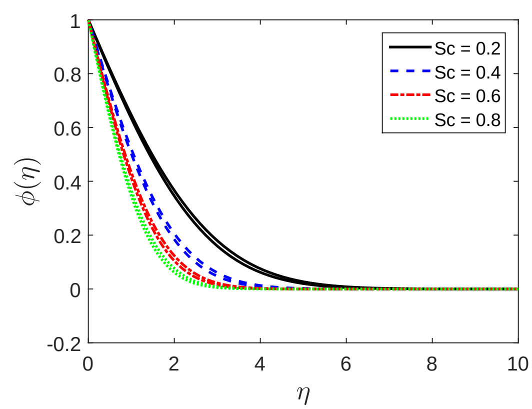

Figure 5: Concentration profile for different values of Sc

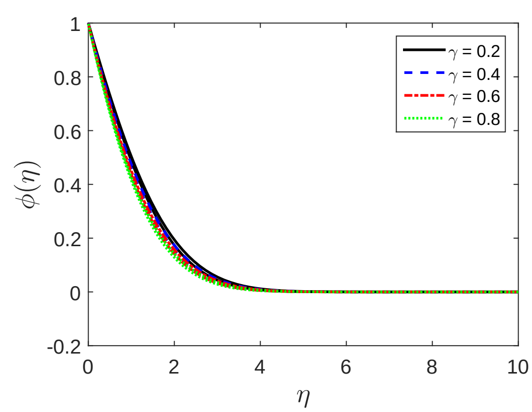

Figure 6: Concentration profile for different values of γ

be also seen that increasing or deceasing nature of velocity profiles are less in Figure 4 than Figures $2-3$ due to the increasing or decreasing nature of boundary layer thickness of γ, Bi and Sc.

Figures 5 - 6 display the effect of Sc and γ on concentration profiles. Figure 5 shows the effect of Sc on concentration profiles when the other papers are fixed. It can be seen that concentration profiles decrease with increase in the value of Sc. Because due to the increasing value of Sc decrease the thickness of thermal boundary layer which causes the increasing nature of concentration profiles. Figure 6 shows the effect of chemical reaction parameter γ on concentration profile. Chemical reaction parameter γ displays the same behavior on concentration profile as we are consider in Figure 5. Because, volume fraction of nanoparticles in concentration profile decreases with increase in Sc and γ.

5 Conclusions

In this paper, the problem related to rheological analysis of CNT suspended nanofluid flow, heat and mass transfer on a sheet in the presence of magnetic field, Biot number, Schmidt number and chemical reaction parameter etc with convective boundary condition is studied in detail. In this chapter, the final conclusions drawn from the problems investigated in the present project are as follows:
The following conclusions are drawn from this problem:

- Increasing value of Biot number, Schmidt number and chemical reaction parameter increase the boundary layer thickness for assisting flow which causes the increasing nature of velocity but decreases velocity profile for opposing flow.

- Concentration profiles decrease with increase in the value of Sc and γ due to the increasing value of Sc and γ decrease the thickness of thermal boundary layer which causes the increasing nature of concentration profiles.

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