Numerical study of second-grade fluid flow past a stretching sheet through a porous medium

by
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Presentation Overview

Introduction

Boundary Layer

- The thin fluid layer in the neighbourhood of the solid body where the viscous effects are predominant is termed as Boundary Layer.
- Flow exists within the boundary layer only.

Applications

- Mathematically, boundary-layer theory converts the character of governing Navier-Stokes equations from elliptic to parabolic.
- Boundary layer theory can be used to locate the point of seperation itself.

Introduction

Natural Convection

- Natural convection is a mechanism, or type of heat transport process, in which the fluid motion is not generated by any external source (like a pump, fan, suction device, etc.) but only by density differences in the fluid occurring due to temperature gradients.
- Occurs only in gases or liquids.
- Particles are moving without any external force.

Applications

- Spaceship, solar energy collectors, wind energy, geomechanics etc.

Introduction

Magnetohydrodynamics (MHD)

- The region around a magnetic material or a moving electric charge within which the force of magnetism acts is known as magnetic field.
- The study of motion of electrically conducting fluid in the presence of magnetic field is called magnetohydrodynamics. It is used in the design of heat exchangers, power generating systems, space vehicle propulsion, flow meters and Hall accelerators pumps.

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Introduction

Chemical Reaction

- Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, or the products. The substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.
- A reaction is said to be of first order, if the rate of the reaction is directly proportional to the concentration itself.

Examples

- Rusting of iron, Metabolism of food in the body,Digesting sugar with the amylase in saliva, Aerobic cellular respiration is a reaction with oxygen, Synthesis reactions etc.

Introduction

Second-grade Fluid

- A Second-grade fluid is a fluid where the stress tensor is the sum of all tensors that can be formed from the velocity field with up to two derivatives, much as a Newtonian fluid is formed from derivatives up to first order.
- Second-grade fluid is considered as one of the well known non-Newtonian fluid.

Examples

- Blood, Ketchup, Shampoo, Paints, Drilling mud, Soaps, Clay coatings, Certain oils and Greases, Elastomers, Suspensions and Emulsions etc.

Introduction

Aim and Scope

- Many of the studies were confined to Newtonian fluid flow problems.
- Very less attention has been paid for the boundary layer flow of second-grade fluid flow past a stretching sheet with MHD and heat source/sink effects in the presence of porous medium.
- Hence in the present paper an attempt has been made to study the boundary layer region developed by a stretching sheet which is kept in second-grade fluid.
- The objective of present study is to determine the influence of flow parameters on natural convection heat and mass transfer flow of second-grade fluid over a stretching sheet.

Problem Description

Schematic diagram of the investigated problem

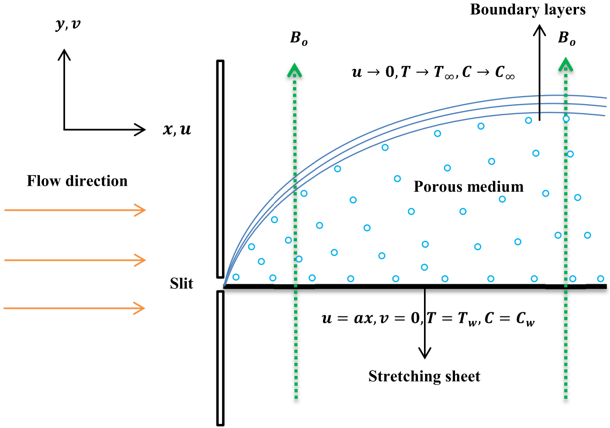

Fig. 1. Physical configuration of the problem

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Problem Description

Governing Equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \lambda_1\left(u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right)$$
$$-\frac{\sigma B_0^2}{\rho}u - \frac{\nu}{k_1}u + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty)$$

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Problem Description

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}(\frac{\partial u}{\partial y})^2 + \frac{\lambda_1}{c_p}\frac{\partial u}{\partial y}[\frac{\partial}{\partial y}(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y})]$$

$$+\frac{Q}{\rho c_p}(T - T_\infty) + \frac{\sigma B_o^2}{\rho c_p}u^2$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D(\frac{\partial^2 C}{\partial y^2}) - k_2(C - C_\infty)$$

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Problem Description

The initial and boundary conditions

$$u = ax, v = 0, T = T_w, C = C_w \qquad \text{at } \; y = 0$$

$$u \to 0, \frac{\partial u}{\partial y} \to 0, T \to T_\infty, C \to C_\infty \qquad \text{as } \; y \to \infty$$

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Problem Description

By introducing the similarity transformations

$$u = axf'(\eta)$$

$$v = -f(\eta)\sqrt{a\nu}$$

$$\eta = \sqrt{\frac{a}{\nu}}y$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Problem Description

The governing equations in non-dimensional form are

$$f'''(\eta)+f(\eta)f''(\eta)-f'^2(\eta)+\lambda\left(2f'''(\eta)f'(\eta)-f''^2(\eta)-f(\eta)f^{iv}(\eta)\right)$$
$$-Mf'(\eta)-Pf'(\eta)+Gr\theta(\eta)+Gr_c\phi(\eta)=0$$

$$\theta''(\eta)+Prf(\eta)\theta'(\eta)-Pr(2f'(\eta)-\alpha)\theta(\eta)$$
$$+PrEc[f''^2(\eta)+\lambda f''(\eta)\{f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\}+Mf'^2(\eta)]=0$$

$$\phi''(\eta)+Sc[f(\eta)\phi\prime(\eta)-Kr\phi(\eta)]=0$$

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Problem Description

The non-dimensional initial and boundary conditions are

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad at \quad \eta = 0$$

$$f'(\infty) \to 0, \quad f''(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0 \quad as \quad \eta \to \infty$$

Numerical Procedure

$$\frac{df_3}{d\eta} = \frac{1}{\lambda f(\eta)}[f_3(\eta) + f_0(\eta)f_2(\eta) - f_1^2(\eta) + \lambda(2f_1(\eta)f_3(\eta) - f_2^2(\eta))$$

$$-Mf_1(\eta) - Pf_1(\eta) + Gr\theta_0(\eta) + Gr_c\phi_0(\eta)]$$

$$\frac{d\theta_0}{d\eta} = \theta_1$$

$$\frac{d\theta_1}{d\eta} = -Prf_0(\eta)\theta_1(\eta) + Pr(2f_1(\eta) - \alpha)\theta_0(\eta)$$

$$-PrEc[f_2^2(\eta) + \lambda f_2(\eta)\{f_1(\eta)f_2(\eta) - f_0(\eta)f_3(\eta)\} + Mf_1^2(\eta)]$$

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Numerical Procedure

$$\frac{d\phi_0}{d\eta} = \phi_1$$

$$\frac{d\phi_1}{d\eta} = -Sc[f_0(\eta)\phi_1(\eta) - Kr\phi_0(\eta)]$$

Numerical Procedure

- BVP4C Matlab Solver

Results and Discussion

Results and Discussion

- Flow variables
  - Velocity
  - Temperature
  - Concentration
- Average momentum, heat transport and mass transfor coefficients
- Validation

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Results and Discussion

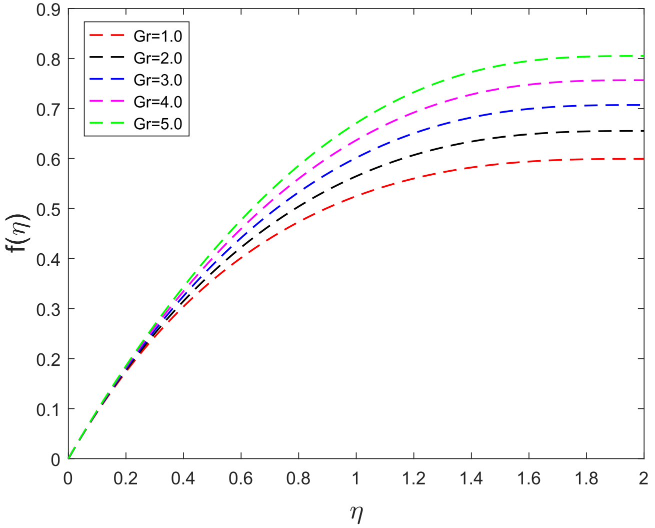

Fig. 2. Effect of *Gr* on *f*

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Results and Discussion

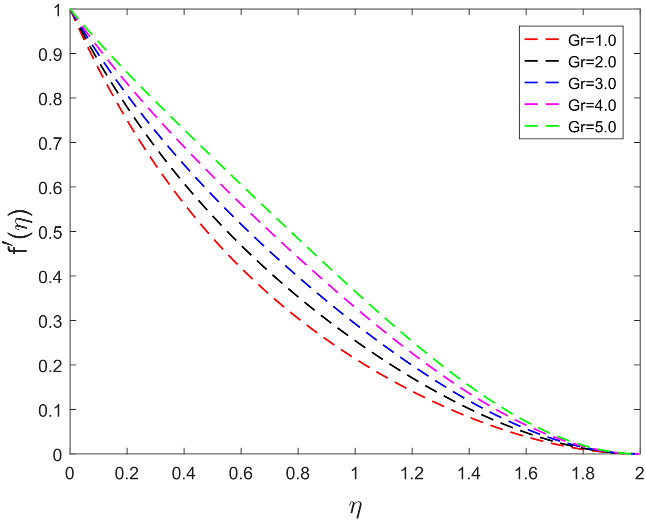

Fig. 3. Effect of Gr on f'

Results and Discussion

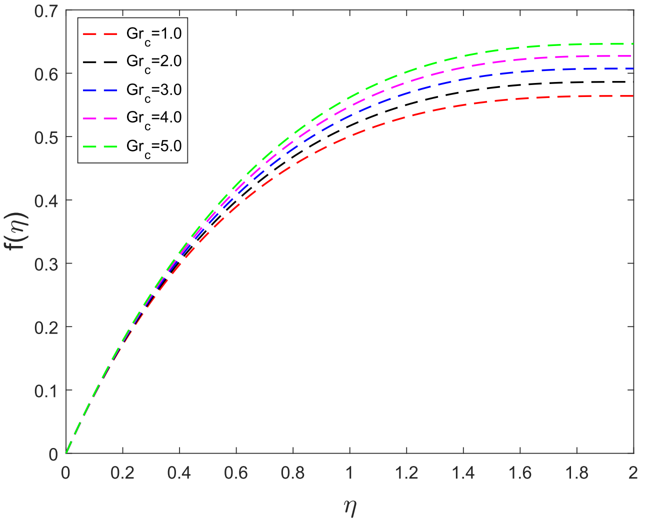

Fig. 4. Effect of Gr_c on f

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Results and Discussion

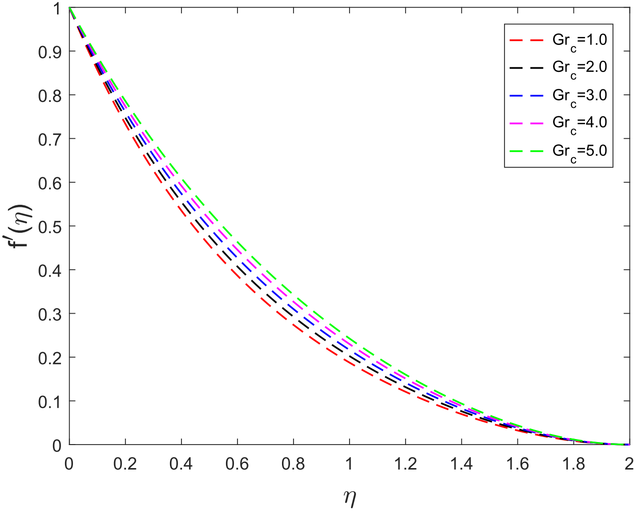

Fig. 5. Effect of Gr_c on f'

Results and Discussion

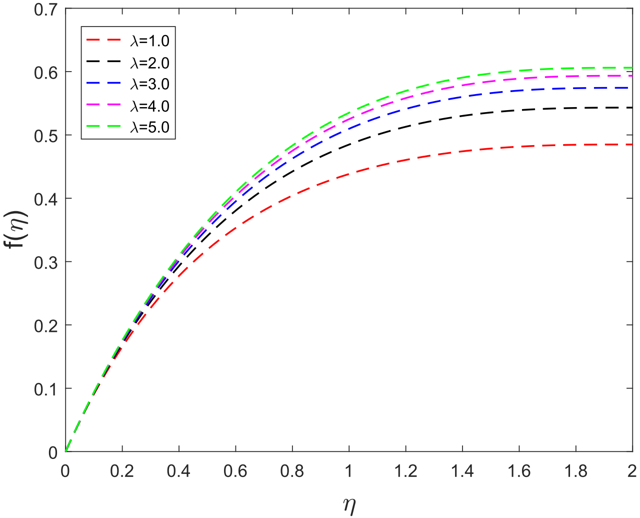

Fig. 6. Effect of λ on f

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Results and Discussion

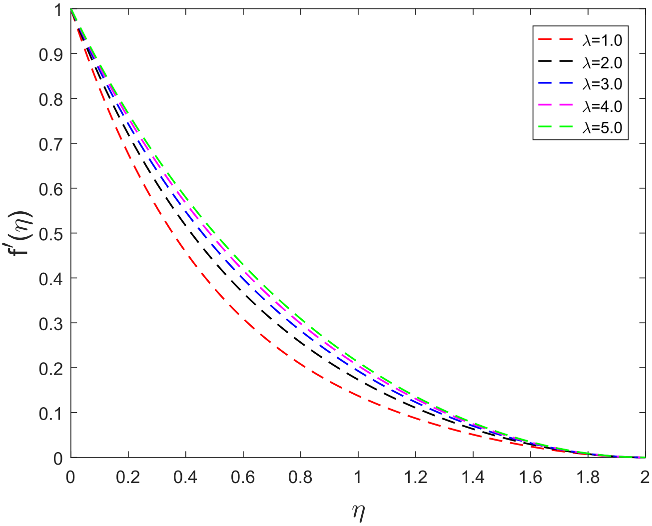

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Fig. 7. Effect of λ on f'

Results and Discussion

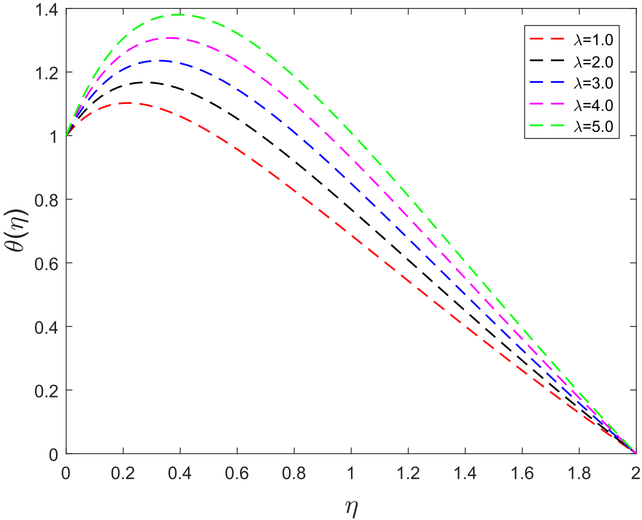

Fig. 8. Effect of λ on θ

Results and Discussion

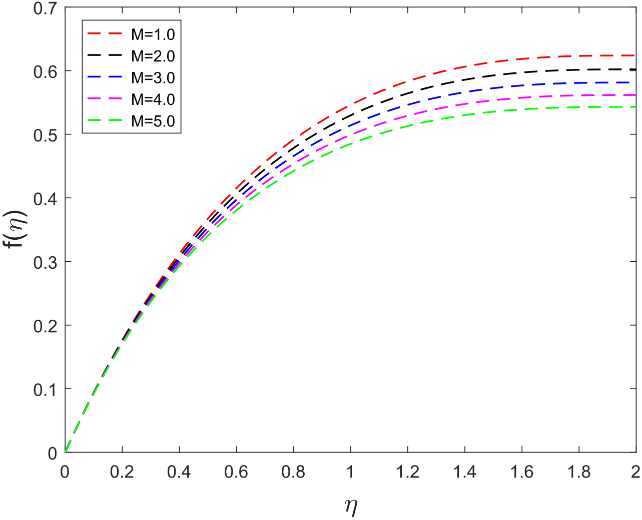

Fig. 9. Effect of M on f

Results and Discussion

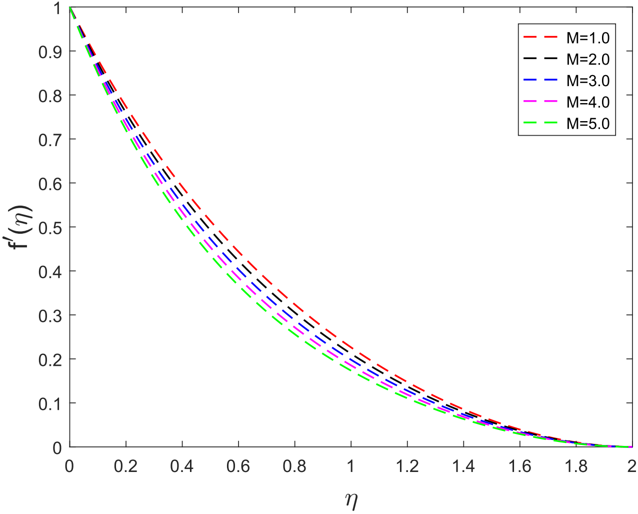

Fig. 10. Effect of M on f'

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Results and Discussion

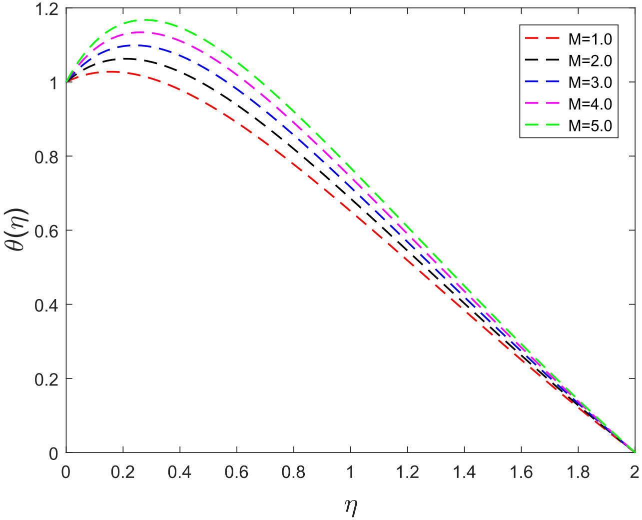

Fig. 11. Effect of M on θ

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Results and Discussion

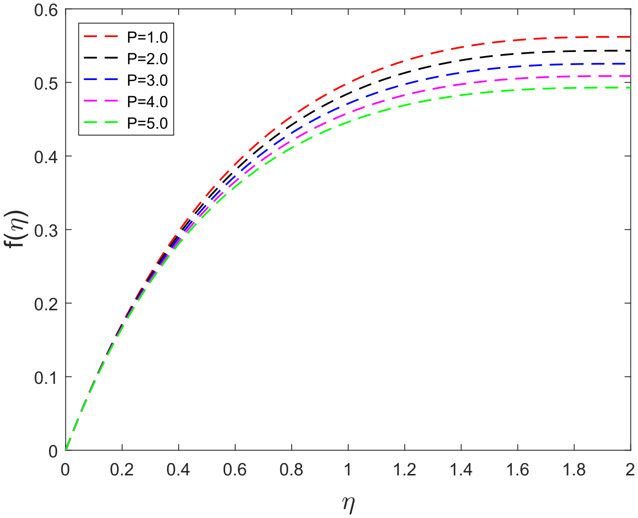

Fig. 12. Effect of P on f

Results and Discussion

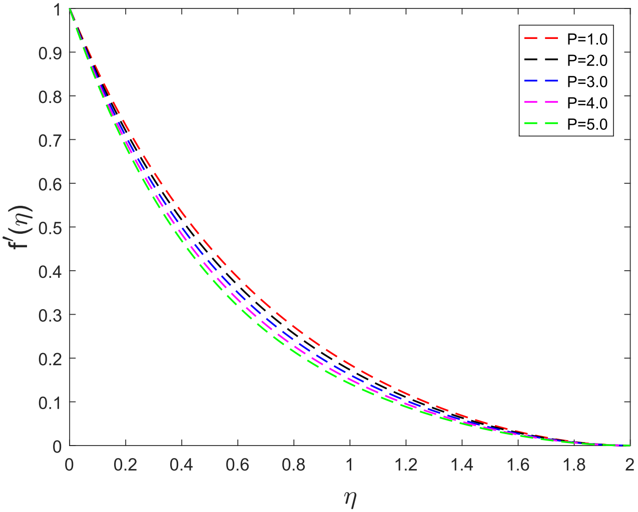

Fig. 13. Effect of P on f'

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Results and Discussion

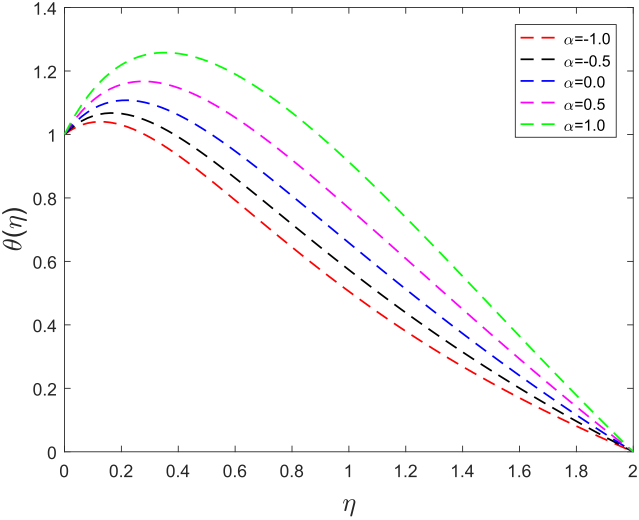

Fig. 14. Effect of α on θ

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Results and Discussion

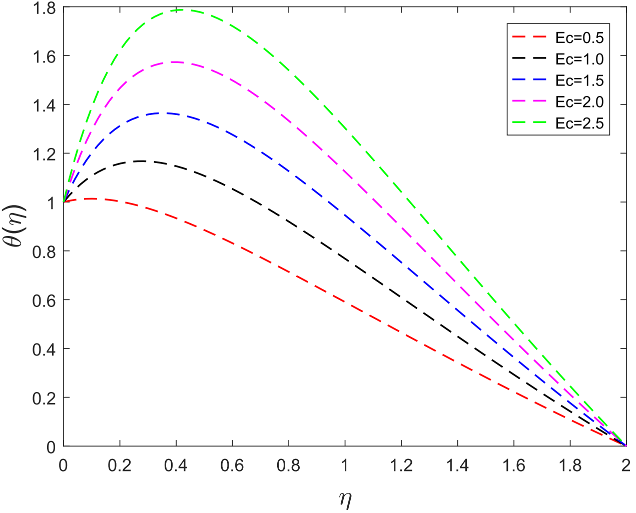

Fig. 15. Effect of Ec on θ

Results and Discussion

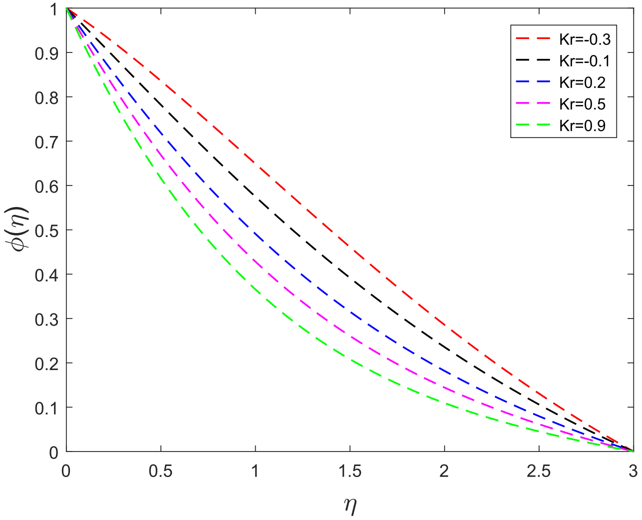

Fig. 16. Effect of Kr on ϕ

Results and Discussion

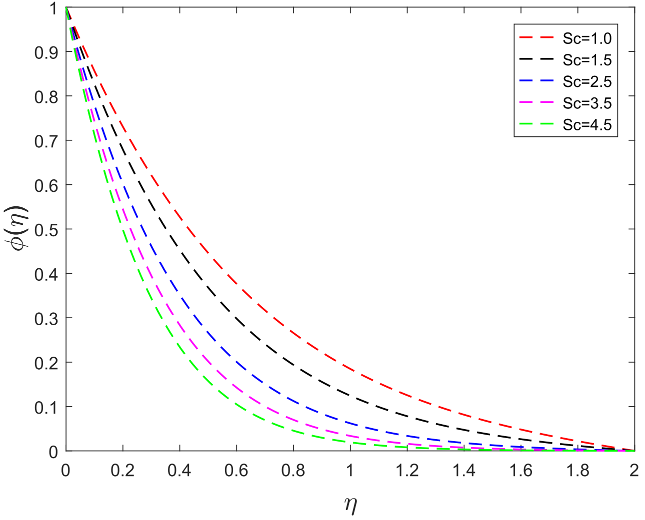

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Fig. 17. Effect of Sc on ϕ

Results and Discussion

Validation of present results with existing results

Table 1. Validation of present results with those of Vajravelu and Roper [47].

Parametric Values	Pr	Results of Vajravelu and Roper [47] (Ec=0.0)	Present results (Ec=0.0)	Results of Vajravelu and Roper [47] (Ec=0.02)	Present results (Ec=0.02)
$\lambda = 1$, $\alpha = -1$	0	-0.010000	-0.010000000000000	-0.010000	-0.010000000000000
	1	-1.757867	-1.757841442124818	-1.750994	-1.750989674737535
	2	-2.535115	-2.535104840234069	-2.523906	-2.523898937347685
	3	-3.132089	-3.132071535126984	-3.117364	-3.117346570130741
	4	-3.635523	-3.635514407065811	-3.617752	-3.617742893137518
	5	-4.079128	-4.079118335901348	-4.058629	-4.058617870318757
$\lambda = 1$, $\alpha = 0$	0	-0.010000	-0.010000000000000	-0.010000	-0.010000000000000
	1	-1.414214	-1.414210574298190	-1.406186	-1.406182354163806
	2	-2.078035	-2.078039592657075	-2.064758	-2.064762299262886
	3	-2.586440	-2.586440151594636	-2.568855	-2.568855962959885
	4	-3.014721	-3.014724793533113	-2.993381	-2.993383687603092
	5	-3.391900	-3.391907818788401	-3.367179	-3.367185517991996

Conclusions

Conclusions

- The velocity field increases for the increasing values of buoyancy parameter.
- The temperature and velocity profile shows the increasing trend for increasing value of second-grade fluid parameter in the flow region.
- The velocity field decreases for increasing values of magnetic parameter.

Conclusions

- The velocity field decreases as the porous parameter increases.
- The temperature field is enhanced for the increasing values of Eckert number and heat source or sink parameter.
- The concentration field is suppressed for the increasing values of chemical reaction parameter and Schmidt number.

References

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Thank You