Non-Linear Regression for Identifying the Optimal Soil Hydraulic Model Parameters

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Abstract. Study is focussed on determination of soil hydraulic parameters and analysis of the moisture retention function for different soil textures. Experiments utilizing pressure plate apparatus were conducted to estimate actual soil moisture characteristics. Optimal values of soil hydraulic parameters of Brooks-Corey (1964), Van-Genuchten (1980) and Modified Van Genuchten (2005) models were estimated using nonlinear regression in SPSS. These parameters were used as input in HYDRUS-1D forward simulation to yield analytical soil moisture retention curve. The analytically obtained moisture retention curve is compared with actual SMC curve to assess the performance of nonlinear regression based approach for identifying optimal soil hydraulic parameter values.

Keywords: Soil moisture; Matric potential; Iteration; Unsaturated zone; Optimization.

1 Introduction

Water movement in soil occurs as liquid flow in saturated soils and as liquid-vapour flow in unsaturated soils [1]. The soil water flow is generally specified by solving the proper governing partial differential equations for different initial and boundary conditions [2,3]. The soil hydraulic parameters at the soil surface and in the unsaturated zone are key variables, because they form essential inputs for large scale hydrologic and hydro-climatic processes [4]. An important problem commonly encountered in computer based numerical solutions is obtaining the primary input parameters, that is, the relationship between soil matric potential and hydraulic conductivity as a function of soil water content [5]. Soil water retention functions describes its ability of storing and releasing water and forms an essential parameter in the unsaturated flow and infiltration models [6,7]. Water retention function is reliant on particle-size distribution, mineralogy of clay, organic matter content and hysteresis.

The concept of SMC was initially proposed by [8]. For many years, SMC has been considered as the subject for considerable research and several field, laboratory and theoretical techniques have been developed for its estimation [9-11]. The soil moisture characteristic (SMC) expresses the functional relationship between the volumetric moisture content (θ) and the suction head (ψ) in an unsaturated porous medium [12,13].

Modelling SMC is generally based on solution of mathematical equations containing various parameters characterizing the system. The most popular analytical models available in literature are Brooks-Corey [14], Campbell [15], Van-Genuchten [16], Kosugi [17] and Modified Van-Genuchten [18]. In order to have a reliable model, the value of the parameters should fit their actual ones. The parameters are obtained either experimentally or analytically. In present study, optimal values of parameters of three analytical models [14,16,18] are estimated using nonlinear regression. Following objectives have been set:

i. Estimation of field parameters of SMC through laboratory experiments and development of experimental SMC using pressure plate apparatus.
ii. Calculation of optimal soil hydraulic model parameters through nonlinear regression using SPSS.
iii. Comparison between experimental SMC and model predicted SMC using HYDRUS-1D.

2 Material and Methods

2.1 Analytical Models

Three analytical models are considered in the study as given below,
Brooks and Corey (BC):

$$\frac{\theta-\theta_r}{\theta_S-\theta_r}=\left(\frac{\psi_b}{\psi}\right)^{\lambda} \qquad \text{for } \psi \le \psi_b \qquad (1)$$

Van Genuchten (VG):

$$\frac{\theta-\theta_r}{\theta_S-\theta_r}=\left[\frac{1}{1+\left\|\alpha_v\psi\right\|^{n_v}}\right]^m \qquad \text{for } \psi\le 0 \qquad (2)$$

Modified Van Genuchten (MVG)

$$\frac{\theta_m-\theta_r}{\theta_S-\theta_r}=\left[\frac{1}{1+\left\|\alpha_{mv}\psi\right\|^{n_{mv}}}\right]^m \qquad \text{for } \psi\le 0 \qquad (3)$$

Where, $\theta_m=\theta_r+(\theta_s-\theta_r)(1+\left\|\alpha h_s\right\|^n)^m$

In above models, ψ is the soil matric potential, ψ_b is the bubbling pressure, λ is the pore size index, α_v and n_v are unsaturated soil parameters with $m = 1-(1/n_v)$, θ is the actual volumetric moisture content ($cm^3\ cm^{-3}$) and the subscripts s and r represent saturation and residual values of the moisture content.

2.2 Experimental Data

Laboratory experiments were utilized to estimate relevant soil textural and hydraulic parameters. Three different sites were selected and 30 samples from each site were subjected to sieve and hydrometer analysis [19]. The soil texture comes out to be loam, sandy loam and loamy sand from site 1, 2 and 3 respectively. Figure 1 shows the particle size distribution and soil textural classification chart (USDA) for loam soil. Experimental SMC was estimated using pressure plate apparatus, which involved concurrent measurements of soil suction and soil moisture. The duration of experiments varied from sample to sample and for different pressure readings. θ_s is assumed to be equal to soil porosity and θ_r is estimated from pressure plate apparatus.

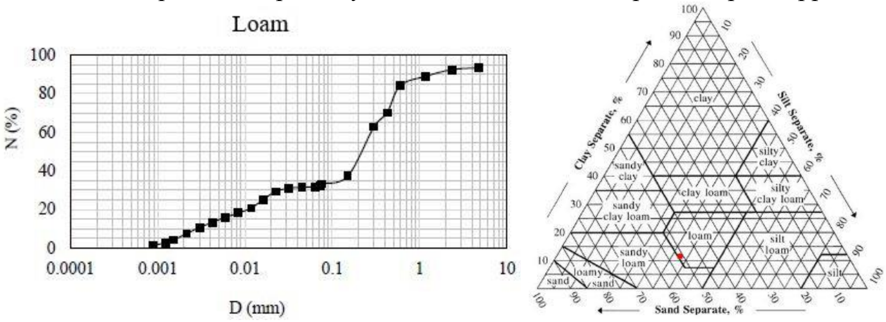

Fig. 1. (a) Particle size distribution (b) Soil textural classification chart for Loam soil

2.3 Nonlinear Regression

The optimal values of model parameters are determined through nonlinear regression using IBM SPSS (Statistical Package for Social Sciences) software. In nonlinear regression, a function is used for modelling the data, which is nonlinear combination of model parameters and is dependent on one or more independent variables. The experimentally obtained input and output variables are used. An initial guess of parameters is provided, and the iterations are performed. After a certain number of model and derivative assessments, the iteration stops resulting in model fit parameter values.

2.4 HYDRUS

The obtained parameters of the analytical models from SPSS and those from laboratory experiments are used as the input data for forward simulation in HYDRUS-1D software. HYDRUS is based on soil moisture flow equation [20]. The simulated results in development of analytical SMC from each model considered in the study.

3 Results and Discussion

3.1 Soil Moisture Characteristics

A series of experiments were conducted using pressure plate apparatus for different soil textures. Thirty set of sample observations were taken for each soil texture to accurately determine its SMC curve. The SMC curves obtained for different soil textures considered in the study are shown in Fig. 2. There is no clear difference between the moisture retention near the saturation end, however at the drier end, the loam exhibits higher moisture retention in comparison to sandy loam and loamy sand soil.

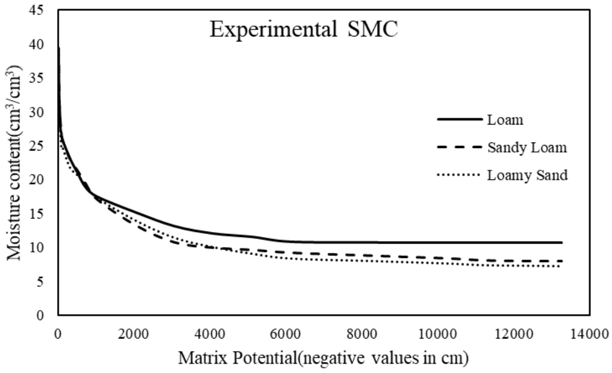

Fig. 2. Experimental soil moisture characteristics curve for different soils

3.2 Soil Hydraulic Parameter Estimation

The soil hydraulic parameters of three analytical models is estimated through iterative optimization using nonlinear regression. The optimal values of these parameters for loam, sandy loam and loamy sand is given in Table 1. The parameters are found to be different for each soil texture. The statistical results for nonlinear regression are shown in Table 2. For each model, a number of model assessments (MA) and derivative assessments (DA) are run which converges to a final value as mentioned in Table 2. The R-squared (R^2) value of analysis of variance is also given in Table 2. R^2 is found to be above 0.95 for each model in case of all soil textures considered in the study which suggests that the estimated values are reliable.

Table 1. Values of soil hydraulic model parameters

	SMC models					
Soil Texture	Brooks and Corey		Van Genuchten		Modified Van Genuchten	
	ψ_b (cm)	λ	α_v (cm^{-1})	n_v	α_{mv} (cm^{-1})	n_{mv}
Loam	8.952	0.28	0.032	1.405	0.053	1.402
Sandy Loam	16.522	0.467	0.048	1.682	0.051	1.663
Loamy Sand	11.831	0.278	0.017	1.457	0.024	1.362

Table 2. Statistical results for nonlinear regression

	SMC models								
Soil Texture	Brooks and Corey			Van Genuchten			Modified Van Genuchten		
	MA	DA	R^2	MA	DA	R^2	MA	DA	R^2
Loam	12	6	0.97	16	8	0.95	16	8	0.95
Sandy Loam	12	6	0.96	16	8	0.95	14	7	0.96
Loamy Sand	14	7	0.96	16	8	0.96	14	7	0.96

3.3 SMC Comparison

The analytical SMC is computed using HYDRUS-1D employing the equations mentioned in 2.1. Each soil texture exhibited its peculiar SMC curve as shown in Figs. 3-5. The graphical comparison between experimental SMC and analytical SMC obtained from three models is shown in Figs. 3-5. From graphical comparison it is found that Van Genuchten presented precise SMC to experimental one. In case of sandy loam soil, all the three models were found to present close agreement with experimental SMC curve. These observations are further substantiated by results of statistical analysis using coefficient of determination (COD) and coefficient of variation (COV). The numerical formulation of COD and COV can be referred in [Shankar et al. 2012].

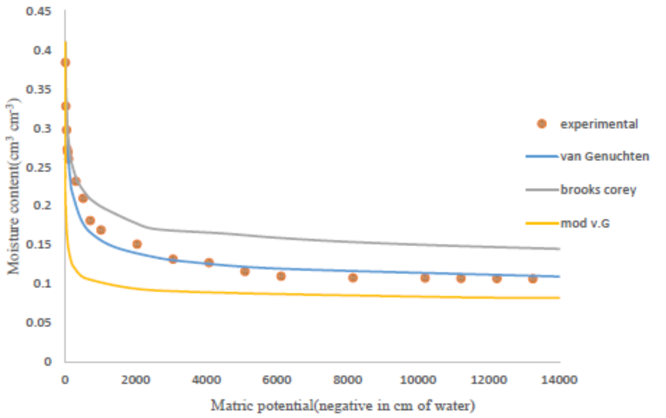

Fig. 3. Comparison of experimental and analytical SMC for loam soil

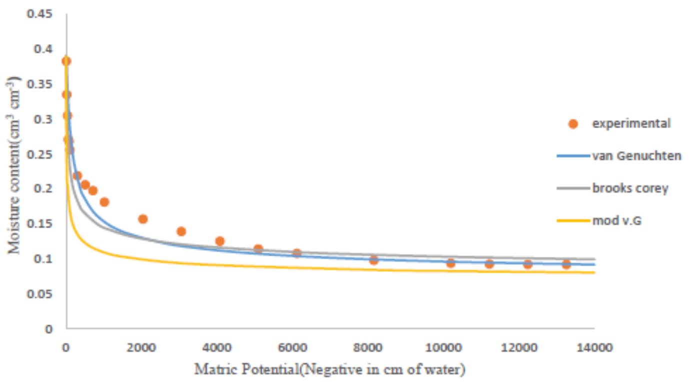

Fig. 4. Comparison of experimental and analytical SMC for sandy loam soil

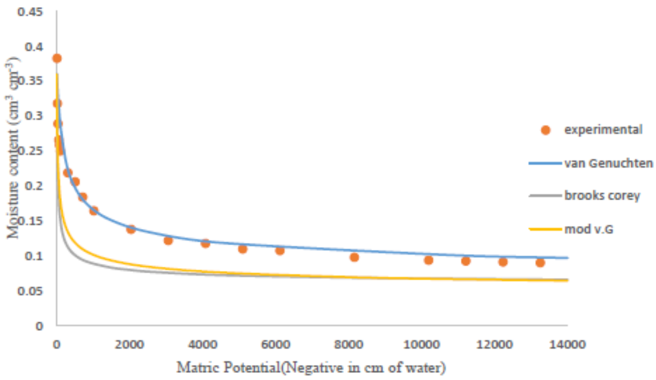

Fig. 5. Comparison of experimental and analytical SMC for loamy sand soil

Table 3. Results of statistical comparison between experimental and analytical SMC

Soil Texture	SMC models					
	Brooks and Corey		Van Genuchten		Modified Van Genuchten	
	COD	COV	COD	COV	COD	COV
Loam	0.812	0.19	0.924	0.06	0.824	0.22
Sandy Loam	0.884	0.05	0.902	0.04	0.801	0.12
Loamy Sand	0.786	0.21	0.967	0.02	0.795	0.20

4 Conclusion

Following conclusions are drawn from the study:

i. Soil hydraulic parameter estimation using nonlinear regression is a reliable method for obtaining optimal values.
ii. Van Genuchten model simulated SMC was closest to experimental SMC indicating the accuracy of model among other models considered in the study.
iii. In case of sandy loam soil, all the models presented satisfactory results.

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