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Heat and mass transfer by Natural convection in a doubly stratified porous medium saturated with Power-law fluid

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Abstract: This paper studies the effects of natural convection heat and mass transfer through a vertical plate embedded in a power-law fluid saturated Darcy's porous medium in addition to double stratification (stratification of medium with respect to thermal and concentration fields). By using similarity transformations, the governing partial differential equations are transformed into ordinary differential equations and consequently solved using shooting method. Taking into account the physical parameters involved in a problem, a parametric study is carried out and the representative set of numerical results are graphically illustrated.

1. Introduction

Free convection flow, heat and mass transfer in non-Newtonian fluids has gained much attention from the researchers because of its engineering and industrial applications such as the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Also, the non-linear behavior of non-Newtonian fluids in porous matrix is quite different from that of Newtonian fluids in porous media. The prediction of heat or mass transfer characteristics about natural convection of non-Newtonian fluids in porous media is very important due to its practical engineering applications, such as oil recovery and food processing. Several investigators have extended the convection of heat and mass transfer problems to fluids exhibiting non-Newtonian rheology. Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, the power law, the differential type, and the rate type model gained importance. Although this model is merely an empirical relationship between the stress and velocity gradients, it has been successfully applied to non-Newtonian fluids experimentally.

Free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical flat plate in a saturated porous media was studied by Rami and Arun [1]. Buoyant convection of power-law fluid in an enclosure filled with heat-generating porous media was considered by Kim and Hyun [2]. The flow of natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes was considered by Cheng [3]. Free convection heat transfer from a vertical flat plate embedded in a thermally stratified non-Newtonian fluid saturated non-Darcy porous medium is analyzed by Kairi and Murthy [4].

Stratification of fluid arises due to temperature variations, concentration differences or the presence of different fluids. The analysis of natural convection in a doubly stratified medium is a fundamentally interesting and important problem because of its broad range of engineering applications. The applications include heat rejection into the environment such as lakes, rivers and the seas; thermal energy storage systems such as solar ponds and heat transfer from thermal sources such as the condensers of power plants. Although the effect of stratification of the medium on the heat removal process in a fluid is important, very little work has been reported in the literature. Jumah and Mujumdar [5] studied the free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical flat plate in saturated porous media. Murthy, Srinivasacharya and Krishna [6] discussed the effect of double stratification on free convection heat and mass transfer in a Darcian fluid saturated porous medium using the similarity solution technique for the case of uniform wall heat and mass flux conditions. Cheng [7] discussed the combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification. Postelnicu, Lakshminarayana and Murthy [8] studied the free convection heat and mass transfer in a doubly stratified porous medium saturated with a power-law fluid. Recently, free convection boundary layer flow over a vertical surface in a doubly stratified fluid-saturated porous medium in the presence of constant suction/injection has been analyzed Srinivasacharya, Motsa and Surender [9].

Motivated by the investigations mentioned above, the purpose of the present work is to investigate the double stratification (stratification of medium with respect to thermal and concentration fields) on natural convection heat and mass transfer from vertical plate in Darcy porous media saturated with power-law fluid with variable surface temperature and concentration conditions.

2. Mathematical Formulation

Consider the Natural convection heat and mass transfer along a vertical plate in a non-Newtonian power-law fluid saturated Darcy porous medium. Choose the coordinate system such that *x*-axis is along the vertical plate and *y*-axis normal to the plate. The plate is maintained at variable surface heat flux $q_w(x)$ and mass flux $q_m(x)$. The temperature and concentration of the ambient medium are T_∞ (x) and C_∞ (x) respectively. Assume that the fluid and the porous medium have constant physical properties except for the density variation required by the Boussinesq approximation. The flow is steady, laminar, two dimensional. The porous medium is isotropic and homogeneous. The fluid and the porous medium are in local thermo dynamical equilibrium. In addition the thermal and solutal stratification effects are taken in to consideration. The ambient medium is assumed to be vertically non-linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0} + Gx^l$ and $C_\infty(x) = C_{\infty,0} + Hx^m$ respectively, where G and H are constants and varied to alter the intensity of stratification in the medium.

Using the Boussinesq and boundary layer approximations, the governing equations for the power law fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u^n = \frac{gK}{\upsilon}\left(\beta_T\left(T - T_\infty\right) + \beta_C\left(C - C_\infty\right)\right) \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} \qquad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} \qquad (4)$$

where u and v are the Darcian velocity components along x and y directions, T is the temperature, C is the concentration, k_T is the thermal diffusion ratio, υ is the kinematic viscosity, K is the permeability, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_C is the coefficient of concentration expansion, α_m is the thermal diffusivity, D_m is the mass diffusivity of the porous medium, and n is the power-law index. When $n = 1$, the

Eq. (2) represents a Newtonian fluid. Therefore, deviation of n from a unity indicates the degree of deviation from Newtonian behavior. For $n< 1$, the fluid is shear thinning and for $n> 1$, the fluid is shear thickening.

The boundary conditions are

$$v=0,\ -k\frac{\partial T}{\partial y}=q_w(x),\ -D_m\frac{\partial c}{\partial y}=q_m(x) \text{ at } y=0 \text{ and } u\rightarrow 0,\ T\rightarrow T_\infty(x),\ C\rightarrow C_\infty(x) \text{ as } y\rightarrow\infty \quad (5)$$

2. Solution of the problem

In view of the continuity eq. (1), we introduce the stream function ψ by

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x} \quad (6)$$

substituting (6) in (2),(3) and (4)and then using the following similarity transformations

$$\left.\begin{aligned}&\psi = A x^{2/3} f(\eta),\quad \eta = B y x^{-1/3}\quad T = T_\infty(x)+\frac{q_w(x)}{k}\theta(\eta)\\ &\frac{q_w(x)}{k} = E x^{\frac{n}{3}},\ C = C_\infty(x)+\frac{q_m(x)}{D}\phi(\eta),\ \frac{q_m(x)}{D} = F x^{\frac{n}{3}}\end{aligned}\right\} \quad (7)$$

We get the following nonlinear system of differential equations.

$$(f')^n = (\theta + N\phi) \quad (8)$$

$$\theta'' = \frac{1}{3}\left(n f'\theta - 2 f\theta' + \varepsilon_1 f'\right) \quad (9)$$

$$\phi'' = \frac{Le}{3}\left(n f'\phi - 2 f\phi' + \varepsilon_2 f'\right) \quad (10)$$

Where primes denote differentiation with respect to η alone, $\varepsilon_1 = \frac{nG}{E}$ is the thermal stratification parameter, $\varepsilon_2 = \frac{nH}{F}$ is the solutal stratification parameter, $N = \frac{\beta_C F}{\beta_T E}$ is the buoyancy ratio, is the $Le = \frac{\alpha_m}{D_m}$ is the Lewis number. $A = \left[\frac{EgK\beta_T\alpha_m^n}{\nu}\right]^{\frac{1}{2n}}$ and $B = \left[\frac{EgK\beta_T}{\nu\alpha_m^n}\right]^{\frac{1}{2n}}$

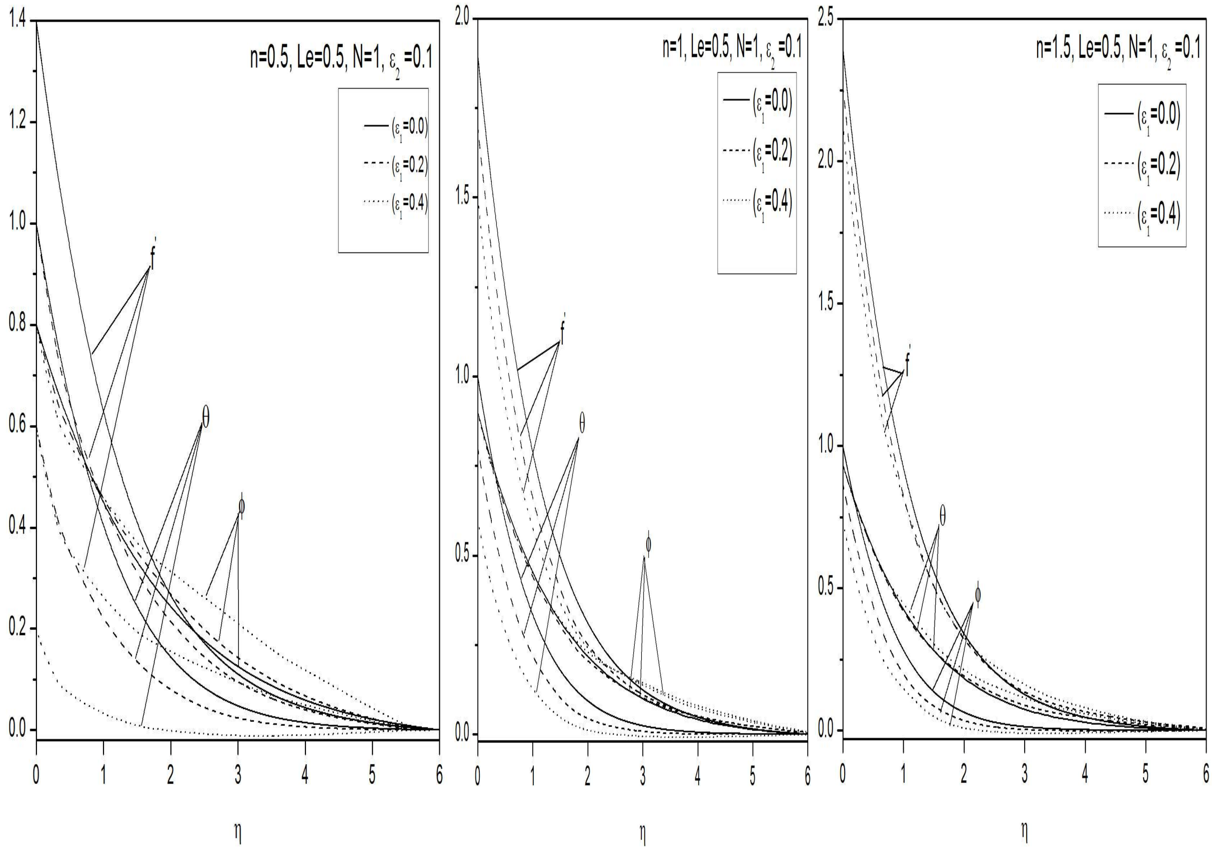
The boundary conditions (5) in terms of f, θ, and ϕ become

$f(0) = 0, \quad \theta'(0) = -1, \quad \phi'(0) = -1, \quad f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0$

The parameters of engineering interest for the present problem are the Nusselt and Sherwood numbers, which are given by the expressions

$$\frac{Nu_x}{Bx^{2/3}} = \frac{1}{\theta\ (0)} \quad \text{and} \quad \frac{Sh_x}{Bx^{2/3}} = \frac{1}{\phi\ (0)}$$

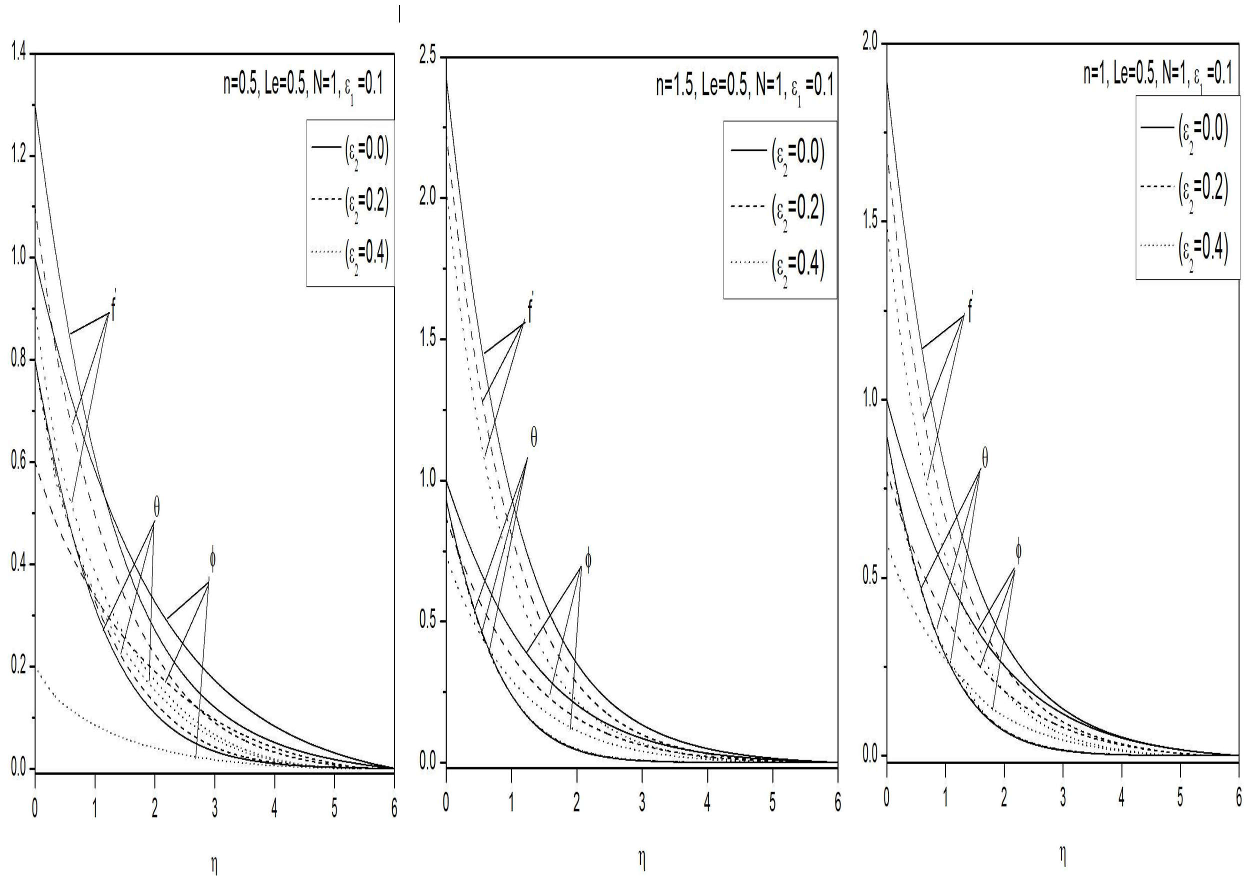
3.Results and Discussion



(a) (b) (c)

Figure 1: Velocity, Temperature and Concentration profiles for various values of ε_1 for (a) Pseudo-plastic fluids, (b) Newtonian and (c) dilatant fluids.

The non-dimensional velocity, temperature and concentration are plotted for N = 1, Le = 0.5, ε_2 = 0.1 in Figs1(a)-1(c) with varying thermal stratification parameter by considering pseudo-plastic(n=0.5), Newtonian(n=1) and dilatant fluids. Figs1(a)-1(c) demonstrates that the velocity and temperature of the fluid decreased with increasing the value of thermal stratification parameter where as the concentration of the fluid is increased.



(a) (b) (c)

Figure 2: Velocity, Temperature and Concentration profiles for various values of ε_2 for (a) Pseudo-plastic fluids, (b) Newtonian and (c) dilatant fluids.

The non-dimensional velocity, temperature and concentration are plotted for N = 1, Le = 0.5, ε_1 = 0.1 in Figs2(a)-2(c) with varying solutal stratification parameter by considering pseudo-plastic(n=0.5), Newtonian(n=1) and dilatant fluids. It can be observed from Figs2(a)-2(c) that the velocity and concentration of the fluid decreased where as the temperature of fluid is increased with increasing the value of solutal stratification parameter.

4. Conclusions:

In this paper, Natural convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated Darcy porous medium in presence of thermal and solutal stratification parameters has been considered. The wall is maintained at variable temperature and concentration $q_w(x)$ and $q_m(x)$ respectively. It can be concluded from the present analysis that the higher values of thermal and solutal stratification parameter result in lower velocity. An increase in the thermal stratification parameter, decrease temperature but increase the concentration distribution. The reverse trend is observed for temperature and concentration distributions in case of solutal stratification parameter increases.

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