A State of Review on Digital Conjugate Surfaces

Pagidi Madhukar[1], N Selvaraj[2], and C S P Rao[3]

[1,2,3] Department of Mechanical Engineering, National Institute of Technology, Warangal, Telangana, India-506004
E-mail: pmadhu88@gmail.com

Abstract: Complex surface methods are widely used in various industrial tools such as cutting tools, shell of television, turbine blades etc. Researchers tend to concentrate on conjugate surfaces, which can be produced by digital method. Conjugate surface deals with relative motion. Most of the cases pertain to power transmitting devices such as gears. Providing conjugate surface is very difficult, and to overcome this problem, digital conjugate method has been introduced. It produces event complex conjugate surfaces. In such cases, researchers have focused on Digital Gear Tooth Surface (DGTS). It causes conjugate motion between the gear teeth, which is represented by discrete points, and with these discrete points, the digital surface of conjugate moments is solved. Computer simulated examples generate and machine the non-standard and complex shapes of digital conjugate tooth surfaces. This method is not only applicable for machining discrete digital gear tooth surfaces and gear tooth surface with complex design but also other 3D digital surfaces.

Keywords: Conjugate surface, Digital gear tooth surface, complex surface

Introduction: Technology continues to improve every day according to the demand for modern precision machinery. In such machineries, gear components play a crucial role as power transmitting mechanism. Therefore to meet the demand and manufacturing of these components requires complicated analysis and treatment of surface engagement. Computer technology helps to develop discrete treatment method.

General purpose CAD/CAM system are relatively mature in modeling various complex surface shapes but they are quite weak in further treatment of complex surface models. Nowadays, direct machining is being generated using algorithms via two methods. One of them is Area by Area Machining [1, 2]. According to this method, very complex surface portions are separated from not so complex portions and a larger or smaller tool is selected respectively. The second method involves intersection of planes on the surface to achieve as above purpose. For machining of parallel plane strategy, Z-map model is used [3]. Drive planes have also been used to intersect the mean least square (MLS) surface [4].

Modern technology is encouraging machine-building aligning towards synthesis, digitalization and intelligence. This accelerates the research on generating method of digitized surface. The generating digitized surface problem solving depends on the theory of digitized conjugate surface [5] Theory of Conjugate surface is a new technology which examines mutual conversions and the relationships between paired motions and paired geometric graphs under conditions of mechanical transmission and machinery processing. It involves mechanics, mechanism, differential geometry, and can be used for the design of cutting tool contour, gear tooth face and cam contour. It can be also applied to the synthesis of mechanism, motion analyses, processing simulation and so on.

Concept of Conjugate Surface:

For both a digitized surface and an analysis surface, conjugate relationship and conditions play primary roles, and these are important for conjugate theory study.

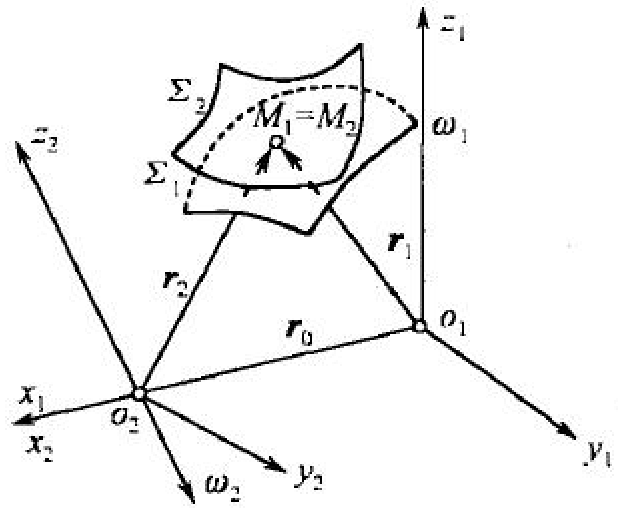

Fig.1 Conjugate motion of two surfaces

∑1 and ∑2 are two arbitrary conjugate surfaces, If ∑1 is considered as mother surface, S1 (Ol X1 Yl Zl) and S2 (O2 X5 Y2 Z2) are the two coordinates.

Surfaces 1 and 2 are associated at point M where M1=M2

Position vectors are r1 and r2

Normal vectors are n1 and, n2

Surface S1 is connected with ∑1, and surface S2 with ∑2 . Similarly, position vectors r1 point to ∑1 and r2 point to ∑2. Unit normal vectors n1 have point rl on surface ∑1 and n1 point r2 on surface ∑2. If the mother surface ∑1 may be expressed by an equation with u and v as two parameters: then r1 = r1(u,v)

Surface ∑1 changes according to the law depending on Θ parameters, and it can form a series of surfaces (surface cluster). The equation describing these surfaces is:

r1 = r1(u v Θ)

u and v are the mother surface geometrical parameters, and Θ is the motion parameter. Fig.2 shows the conjugate profiles and Fig.3 shown the various applications of conjugate surfaces.

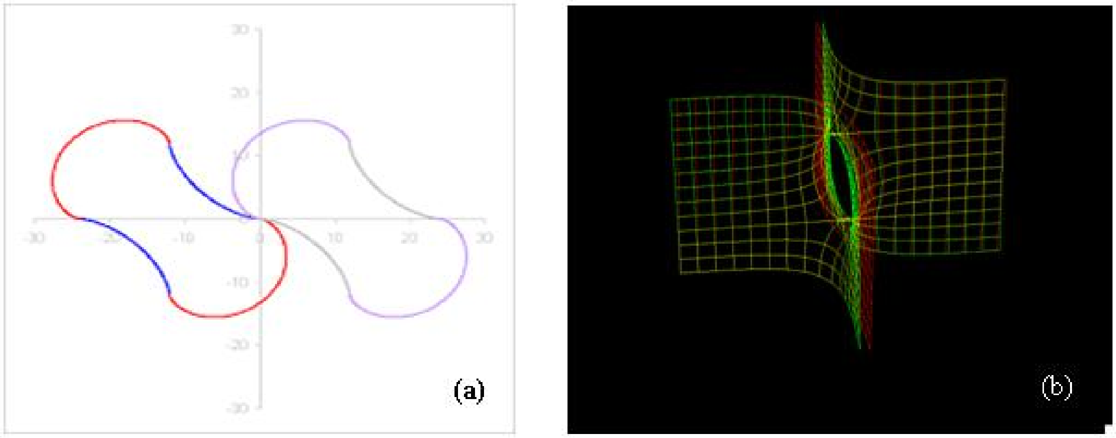

Fig.2: (a) Conjugate motion of a gear profile, (b) Conjugate profile at contact surfaces.

Various conjugate surfaces applications such as Paradaxical gear set, turbine blade, gerotor, dental crowne figures are shown in below.

Fig.3 various applications of conjugate surfaces

Researchers have tended to focus on various tooth profiles for improvement of performance from a few decades. Litvin et al [6] investigated a parabolic tooth profile to minimize transmission errors. Tsay et al [7,8] proposed a modified shape for helical gears with circular tooth and involute tooth. Zhang et al. [9] studied double involute profiles with a transition between them. The scope may be extended to multiple involute profiles with different pressure angles. Ariga et al [10] presented a tooth profile combined by circular arc and involute. And the combinations are closer to the idea of applying splines. Komori et al. [11] developed a spur gear with zero relative curvature at many contact points. Luo et al. [12] creatively designed cosine tooth profile gears which have better load capacity. These tooth profiles can satisfy the conjugate condition with relatively enhanced performances. More flexible tooth profiles with property improvement can be found through conjugation equations expressed in certain polynomial forms [13, 14] unfortunately, with tooth profile selection restricted to a very small pool of traditional curves, the potential improvement is too restricted. On the other hand, if a combination of such curves is used, continuity between sections would become an issue.

To avoid manufacturing difficulties in conjugate surfaces, the digital surface approach was introduced and is widely used. A digital surface implies that, the digitalized points of a continuous surface can be stored as a three dimensional matrix on a computer. Digital surface is also defined as a point set of $\sum_m$ which looks like a surface rather than a curve or solid object. This method describes the surfaces in terms of discrete points. Digital surface method not only meets the demand of complex shape representation and geometric design, but also helps in product data exchange and contributes to the form information transmission. Therefore it's widely used in manufacturing and mechanical designs. There are two methods to obtain digital surfaces. One is "Modern Design" method including boundary element method and finite element method...etc. The second one is 3D-digital measurement of manufactured part or real objective models.

Literature Review:

Qiu Ming[15] discussed digital conjugate surfaces for solving computationally intensive problems by dimension reduction interpolation method and also alternative surface interpolation using the interpolation curve to achieve a conjugate surface normal vector as well as relative velocity of the system. Dong Lijun[16], Conjugate surfaces theoretical analysis is based on the principle of rotation, its beginning being envelope surface of derived surface, forming the cylindrical surface of the rotating conjugate surface equation and contact line equation. It explained how the line of contact is drawn by computer and how round steel leveler rolls and round conjugate motion may be achieved by computer simulation. Yanchang Gang et al[17] proposed modern computing technology and mathematical programming based on the principle of conjugating surfaces. They provided a direct description of the conjugate process mathematical model and this model is used for both theoretical and digital research. A result obtained by digital simulation method covers the traditional conjugate surface theory of the basic content, adapted to solve interference and non-deformation problem under complex conditions.

Liu Xin et al[18], proposed a new method of numerical simulation to solve special problems of conjugate.surface in the form of discrete numerical data. The conjugating surface problems are changed into mathematical programs under minimum conditions. In this paper Guo Wei Zhong et al[19] proposed the discretization and analysis principle of conjugating surfaces for adaptive synthesis in order to construct computer software for conjugating surfaces by taking the generating process of conjugating surfaces into account. QIU Ming[20] used MATLAB to solve digitized conjugating surface problems. And he showed that surface interpolation was substituted by curve interpolation, to get the vectors and relative rate of digitized conjugating surfaces.By the lower dimension interpolation method, the mathematical model of digitized conjugating surfaces problems were solved and a solution program was developed in MATLAB while the interface was achieved by Visual C++

XIAO Lai yuan et al[21], presents a design method for digitized conjugating surface by using MATLAB. The researchers proposed a mathematical model for conjugating surface and solving it according to conjugating principle and computing feature of MATLAB. He invented a method to extract 1 dimensional arrays from a 3 dimensional array, and its results were optimized by a simple MATLAB function called the design process. Qiu Ming[22] has shown how the conjugating surfaces are solved by a mathematical model based on the digital dimension reduction interpolation method. The author used computing software MATLAB for powerful numerical and graphical display capabilities to develop the digital conjugate surface algorithm, with visual C ++ to achieve a human-machine interface and processing of data. Anadil Masood et al [23] introduced Direct machining, which is done by generating efficient tool paths directly from point cloud data, stored in STL format. The primary objective is to achieve high efficiency in the machining of free-form surface geometries, having complex machining areas.

Yanzhong Wang et al [24] used machining method for complex surface face gear with hob cutter and CNC machine tools, based on gear hob principle and surface characteristics. Five axes CNC was used for machining complex parts. A CNC hob method was proposed about the surface orthogonal gear. **Kwun-lon Ting [25]** presented the first theory and practice of free-form conjugation modeling. They proposed a new concept of master-slave that broadens the traditional computer aided single geometry modeling to dual geometry modeling. They later applied gear geometry to establish the first free-form conjugation modeling technique. F.L. Litvin [26] developed a simplified approach to determine normal and principal curvatures for complex surfaces where the representation is in three-parameter form. Da Zhun Xiao [27] proposed average comprehensive radius of curvature (ACRC) and a new quality index for characterize the leakage path of the geometrical shape.

Stephen P [28] rectified the problems while machining sculptured surfaces on the multi-axis NC machine. The author developed six necessary and proper conditions for sculptured surface machining (SSM). Chih-Hsin Chen [29] proposed a pair of line conjugation ruled surfaces which produce instantaneous conjugate motion to one degree of freedom for various conciseness. The base conjugate complex surface is stipulated to be a minimal surface, a surface with only one parameter for specification of surface curvatures. Fulin Wang et al [30] introduced a generating method for digital gear tooth surfaces, and focused on relative conjugate motion of digital gear tooth surfaces and solved non-standard shapes of digital gear tooth surface profiles shapes. Ta-Shi Lai [31] presented a design procedure and mathematical model for designing the epicycloid planet gear of cycloid drives. It is based on envelop theory, transformation of coordinate, and theories of conjugate surfaces, from where the equation of meshing is derived. F. Di Puccio et al[32] proposed an approach that always deals with vectors to avoid the need of a chain reference systems, where components are used in the traditional method. And the results are shown to be more compact as well as clearer. The second approach proposed is applied to get the geodesic torsions and normal curvatures of two conjugate surfaces of relative motion gear generation. The author used CNC face-milling machine to improve the effectiveness of spiral bevel pinion gear.

Peng Wang et al[33] proposed an invariant approach for the manufacture, design, and simulation of complex surfaces. The conjugate surfaces Curvature analysis is a difficult and important concept in gear theory. But introducing a surface curvature tensor for the curvatures of surface and a rotation tensor for rigid-body motion, tensor expressions of curvature relations between conjugate surfaces of point contact and line contact are directly obtained based on tensor analysis. Yueping Chena et al[34] suggested an on-line inspection system with error compensation in order to get high machining accuracy for free-form surface components. The author used the compensation of machining errors and a combination of on-line inspection to achieve the problem successfully. To Verify the effectiveness of the proposed method a, number of experiments were conducted.

Conclusions:

The researchers have been trying to solve conjugate surface using digitized surface method, for which one must construct analysis equation for digitized surface. To achieve this reduced dimension and decomposition techniques and optimization algorithms can largely reduce the complexity of simplifying conjugate surface. Invariant approach, and rigid-body motions are represented by the curvatures of the surface and rotation in terms of a single surface curvature tensor. All the surface quantities are represented by invariants (tensors as well as vectors) and the final results of line contact and point contact are both directly obtained based on tensor algebra and analysis. This approach will be helpful for theoretical development in the theory of gearing system without reference to coordinate systems, the components can be expressed in the form of a matrix, carried out by clearer and easier processing calculations. With this other complex conjugate shapes can be easily designed, fabricated and simulated.

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