

EFFECTS OF RADIATION AND RADIATION ABSORPTION ON UNSTEADY MHD FLOW PAST A VERTICAL POROUS FLAT PLATE IN A ROTATING SYSTEM WITH CHEMICAL REACTION IN A NANOFLUID

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ABSTRACT

The present work provides an analysis of the MHD Ag-nanofluid boundary flow in a rotating system in the presence of radiation, heat absorption and chemical reaction. The flow of a nano fluid is bounded by a semi-infinite flat plate in a rotating frame of reference. The plate is assumed to oscillate in time with steady frequency so that the solutions of the boundary layer are the similar oscillatory kind. The entire system rotates about the axes normal to the plate. The governing partial differential equations of system were converted to a system of ordinary differential equations, which are solved analytically using perturbation technique. The profiles of velocity, temperature and concentration of the boundary layer Ag-water based nanofluid are plotted and investigated in detail.

KEYWORDS

Radiation, Ag-water Nanofluid, sphere shape particles, heat transfer, porous.

INTRODUCTION

Choi [1] was the first to introduce the word nanofluid that represents the fluid in which nanoscale particles are suspended in the base fluid. The common nano particles that have been used are aluminum, copper, iron and titanium or their oxides. Nano fluids, due to high thermal conductivity have several useful applications. For example, they are used on a large scale in automobiles as a coolant, microelectronics, and microchips in computer, biomedicines, fuel cells, transportation, food processing, and solid-state lighting and manufacturing industries. Numerous methods have been developed to improve the thermal conductivity of these fluids by suspending nano particles in liquids. .

Khanafer et al. [2] analyzed the two dimensional natural convection flow of a nanofluid in an enclosure. Heat transfer problems for boundary layer flow concerning with a convective boundary conditions were investigated by Ishak [4] and Yacob et al. [5]. KhairyZaimi et al. [12] inspected the flow and heat transfer over a shrinking sheet in a nanofluid with suction at the

boundary. Kim et.al. [14] investigated the analysis of convective instability and heat transfer characteristics of the nanofluids.

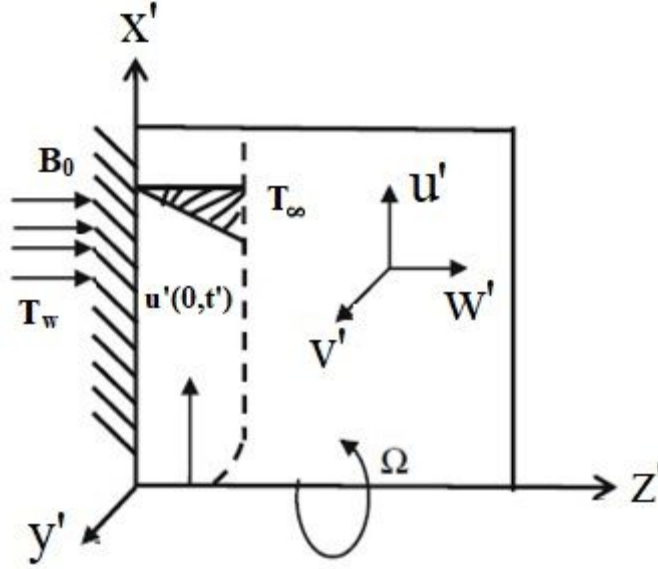
The study of MHD flow and heat transfer due to the effect of a magnetic field in a rotating frame of reference has attracted the interest of many investigators in view of its applications in many industrial, astrophysical, technological and engineering applications. Hamad and Pop [3] studied MHD free convection flow in a rotating frame of reference with constant heat source in a nanofluid. Prasad et al. [13] worked on MHD fluid flow over a stretching sheet.

The fluid flow over a rotating and stretching disk was first studied by Fang [6]. Fang and Zhang [7] investigated the flow between two stretching disks. Turkyilmazoglu [8] studied the MHD laminar flow of an electrically conducting fluid on a radially stretchable rotating disk in the presence of a uniform vertical magnetic field. Fang and Hua [9] investigated the laminar unsteady flow over a stretchable rotating disk with deceleration. Rashidi et al. [10] considered the entropy generation in MHD flow due to a rotating porous disk in a nanofluid. Bachok et al. [11] employed a numerical, Keller-Box technique for steady Nano fluid flow over a porous rotating disk.

The main aim of this study is an analysis of the magneto hydro dynamics Ag-nanofluid boundary flow in a rotating system in the presence of radiation, heat absorption and chemical reaction. The governing partial differential equations converted to a system of ordinary differential equations, which are solved analytically using perturbation technique. Graphs illustrate the significance of key parameters on the nanofluid velocity, temperature and concentration distributions.

MATHEMATICAL MODEL OF THE FLOW

We consider an unsteady three-dimensional flow of an electrically conducting incompressible nanofluid past a semi-infinite vertical permeable plate with radiation and heat absorption. A uniform external magnetic field B_0 is taken to be acting along the z' axis. It is assumed that there is no applied voltage which implies the absence of an electric field. The flow is assumed to be in the x' – direction which is taken along the plate in the upward direction and z' axis is normal to the plate. Also it is assumed the whole system is rotated with a constant vector Ω about the z' -axis. The fluid is grey, absorbing emitting but not scattering medium. The radiation heat flux in x' -direction is considered negligible in comparison that in the z' - direction. Due to semi-infinite plate surface assumption the flow variables are functions of z and time t only.



Physical Model and Co-ordinate system

Assumed that the regular fluid and the suspended nano particles are in thermal equilibrium and no slip occurs between them. Boundary layer approximations, the boundary layer equations governing the flow, temperature and concentration along with the Boussinesq are:

$$\frac{\partial w'}{\partial z'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + w' \frac{\partial u'}{\partial z'} - 2\Omega v' = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u'}{\partial z'^2} + \frac{[\rho\beta]_{nf}}{\rho_{nf}} g(T - T_\infty) - \frac{1}{\rho_{nf}} \sigma B_0^2 u' - \frac{1}{\rho_{nf}} \frac{v_f u'}{K'} \quad (2)$$

$$\frac{\partial v'}{\partial t'} + w' \frac{\partial v'}{\partial z'} + 2\Omega u' = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v'}{\partial z'^2} - \frac{1}{\rho_{nf}} \sigma B_0^2 v' - \frac{1}{\rho_{nf}} \frac{v_f v'}{K'} \quad (3)$$

$$\frac{\partial T}{\partial t'} + w' \frac{\partial T}{\partial z'} = \alpha_{nf} \frac{\partial^2 T}{\partial z'^2} - \frac{1}{[\rho C p]_{nf}} \left(Q'(T - T_\infty) + \frac{\partial q_r}{\partial z'} \right) + Q_1^*(C - C_\infty) \quad (4)$$

$$\frac{\partial C}{\partial t'} + w' \frac{\partial C}{\partial z'} = D^* \frac{\partial^2 C}{\partial z'^2} - K_1^*(C - C_\infty) \quad (5)$$

The boundary conditions for the problem are given by

$$u'(z', t') = 0, v'(z', t') = 0, T = T_\infty, C = C_\infty \quad \text{for } t' \leq 0 \text{ and any } z' \quad (6)$$

$$u'(\infty, t') = U_0 \left[1 + \frac{\varepsilon}{2} (e^{in't'} + e^{-in't'}) \right], T(\infty, t') \rightarrow T_\infty, C(\infty, t') \rightarrow C_\infty \quad \text{for } t' \geq 0 \quad (7)$$

Here u' , v' and w' are the velocity components along the x' , y' and z' axis respectively. The properties of nanofluids are defined as [cf. 16]

$$\begin{aligned}
\rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s, \\
(\rho Cp)_{nf} &= (1 - \phi)(\rho Cp)_f + \phi(\rho Cp)_s, \\
(\rho\beta)_{nf} &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s, \\
\mu_{nf} &= \frac{\mu_f}{[1 - \phi]^{2.5}}, \\
\alpha_{nf} &= \frac{K_{nf}}{(\rho Cp)_{nf}}, \\
\frac{K_{nf}}{K_f} &= \left[\frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)} \right]
\end{aligned} \tag{8}$$

The thermo physical properties of the base fluid(water) and silver are given below.

Nanofluid	ρ	Cp	K	$\beta \times 10^5$
Silver(Ag)	10,500	235	429	1.89
Pure water	997.1	4179	0.613	21

By using Rosseland approximation, the radiative flux vector q_r can be written as:

$$q'_r = -\frac{4\sigma^*}{3k'_1} \frac{\partial T'^4}{\partial z} \tag{9}$$

where, σ^* and k'_1 are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that T'^4 may be expressed as a linear function of the temperature. So,

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{10}$$

$$\text{The solution of Eq. (1) is considered as } w' = -w_0 \tag{11}$$

Introducing dimensionless variables

$$\begin{aligned}
u &= \frac{u'}{U_0}, v = \frac{v'}{U_0}, z = \frac{z'U_0}{v_f}, t = \frac{t'U_0^2}{v_f}, n = \frac{v_f n'}{U_0^2}, \\
K &= \frac{K' \rho_f U_0^2}{v_f^2}, S = \frac{w_0}{U_0}, R = \frac{2\Omega v_f}{U_0^2}, Q_H = \frac{Q' v_f^2}{K_f U_0^2}, \\
Q_1 &= \frac{Q_1^* (C_w - C_\infty)}{(T_w - T_\infty) U_0^2}, F = \frac{4\sigma^* T_\infty^3}{k k'_1}, \text{Pr} = \frac{v_f (\rho Cp)_f}{K_f}
\end{aligned} \tag{12}$$

Substituting Eq. (12) into Eqs. (2) – (5) yields the following dimensionless equations:

$$\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial z} - Rv = B_1 \frac{\partial^2 u}{\partial z^2} + B_2 \theta - B_3 u \left[M + \frac{1}{K} \right] \quad (13)$$

$$\frac{\partial v}{\partial t} - S \frac{\partial v}{\partial z} + Ru = B_1 \frac{\partial^2 v}{\partial z^2} - B_3 v \left[M + \frac{1}{K} \right] \quad (14)$$

$$\frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial z} = \frac{B_4}{Pr} \frac{\partial^2 \theta}{\partial z^2} - \frac{B_5 Q_H}{Pr} \theta + Q_1 \psi \quad (15)$$

$$\frac{\partial \psi}{\partial t} - S \frac{\partial \psi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \psi}{\partial z^2} - Kr \psi \quad (16)$$

where the corresponding boundary conditions (5) can be written in the dimensionless form as:

$$\left. \begin{aligned} u(z, t) = 0, v(z, t) = 0, \theta(z, t) = 0, \psi(z, t) = 0 \quad \text{for } t \leq 0 \text{ and any } z \\ u(0, t) = 1 + \frac{\varepsilon}{2} [e^{\text{int}} + e^{-\text{int}}], v(0, t) = 0, \theta(z, t) = 1, \psi(z, t) = 1 \\ u(\infty, t) \rightarrow 0, v(\infty, t) \rightarrow 0, \theta(\infty, t) \rightarrow 0, \psi(\infty, t) \rightarrow 0 \end{aligned} \right\} \text{for } t \geq 0 \quad (17)$$

Using Eq. (13) the velocity characteristic U_0 is defined as

$$U_0 = \sqrt[3]{g \beta_f \nu_f (T_w - T_\infty)}$$

In order to obtain the desired solution, we assume that the fluid velocity in the complex form as:

$$\chi(z, t) = u(z, t) + iv(z, t) \quad (18)$$

By using Eq. (18) we can simplify Eqs. (13) and (14) to the following equation

$$\frac{\partial \chi}{\partial t} - S \frac{\partial \chi}{\partial z} - Rv = B_1 \frac{\partial^2 \chi}{\partial z^2} + B_2 \theta - B_3 \chi \left[M + \frac{1}{K} \right] \quad (19)$$

The boundary conditions (17) becomes

$$\left. \begin{aligned} \chi(z, t) = 0, \theta(z, t) = 0, \psi(z, t) = 0 \quad \text{for } t \leq 0 \text{ and any } z \\ \chi(0, t) = 1 + \frac{\varepsilon}{2} [e^{\text{int}} + e^{-\text{int}}], \theta(z, t) = 1, \psi(z, t) = 1 \\ \chi(\infty, t) \rightarrow 0, \theta(\infty, t) \rightarrow 0, \psi(\infty, t) \rightarrow 0 \end{aligned} \right\} \text{for } t \geq 0 \quad (20)$$

To solve Eqs. (13)- (16) under the boundary conditions (17) in the neighborhood of the plate, we assume that (see Ganapathy [15]).

$$\chi(z,t) = \chi_0(z,t) + \frac{\varepsilon}{2} \{ e^{\text{int}} \chi_1 + e^{\text{int}} \chi_2 \} \quad (21)$$

$$\theta(z,t) = \theta_0(z,t) + \frac{\varepsilon}{2} \{ e^{\text{int}} \theta_1 + e^{\text{int}} \theta_2 \} \quad (22)$$

$$\psi(z,t) = \psi_0(z,t) + \frac{\varepsilon}{2} \{ e^{\text{int}} \psi_1 + e^{\text{int}} \psi_2 \} \quad (23)$$

Substituting the above equations (21) - (23) into the Eqs. (13) - (16) with boundary conditions (17), we obtain

$$\chi(z,t) = P_3 e^{-\xi_1 z} + P_4 e^{-\xi_2 z} + P_5 e^{-\xi_3 z} + \frac{\varepsilon}{2} \{ e^{-\xi_4 z} e^{\text{int}} + e^{-\xi_5 z} e^{-\text{int}} \} \quad (24)$$

$$\theta(z,t) = P_1 e^{-\xi_1 z} + P_2 e^{-\xi_2 z} \quad (25)$$

$$\psi(z,t) = e^{-\xi_1 z} \quad (26)$$

The skin-friction coefficient, Nusselt number and Sherwood number are given by:

$$C_f = \left(\frac{\partial \chi}{\partial z} \right)_{z=0} = (-1) \left\{ (P_3 \xi_1 + P_4 \xi_2 + P_5 \xi_3) + \frac{\varepsilon}{2} (\xi_4 \exp(\text{int}) + \xi_5 \exp(-\text{int})) \right\} \quad (27)$$

$$Nu = - \left(\frac{\partial \theta}{\partial z} \right)_{z=0} = P_1 \xi_1 + P_2 \xi_2 \quad (28)$$

$$Sh = - \left(\frac{\partial \psi}{\partial z} \right)_{z=0} = \xi_1 \quad (29)$$

RESULTS AND DISCUSSION

The effects of Ag-water spherical shape nanoparticles on the velocity, the temperature and the concentration profiles are discussed through graphs. In the present study the following default parameter values are adopted. $n=10$, $nt = \pi/2$, $Pr = 6.72$, $\varepsilon = 0.001$, $F = 1$, $K = 0.5$, $Kr = 0.5$, $M = 1$, $Q_H = 10$, $Q_1 = 1$, $R = 0.02$, $S = 1$, $Sc = 0.16$, $\phi = 0.10$.

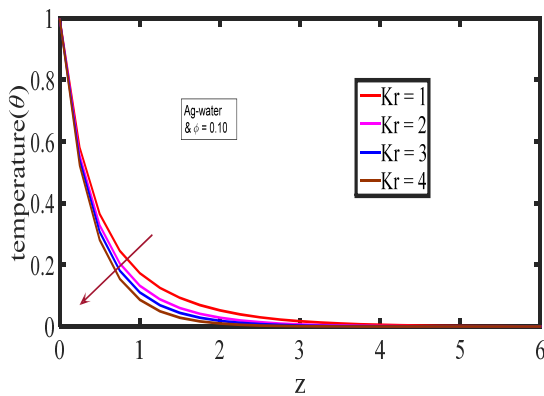


Fig. 1

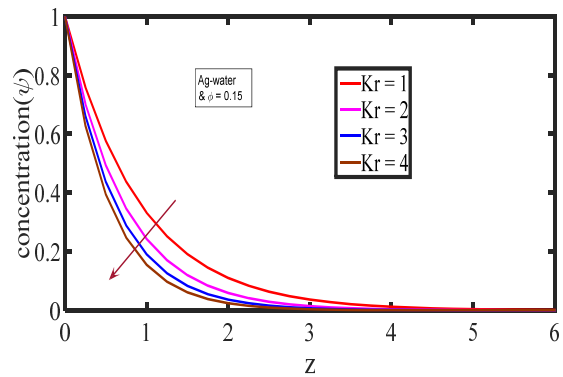


Fig. 2

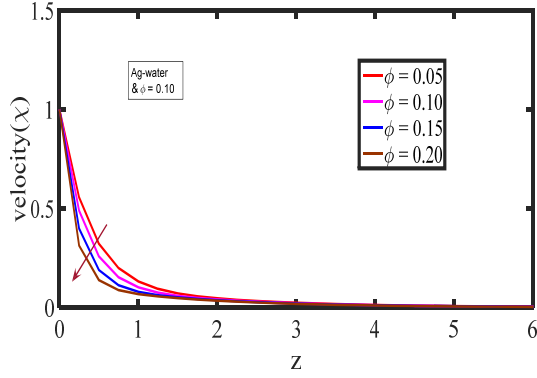


Fig. 3

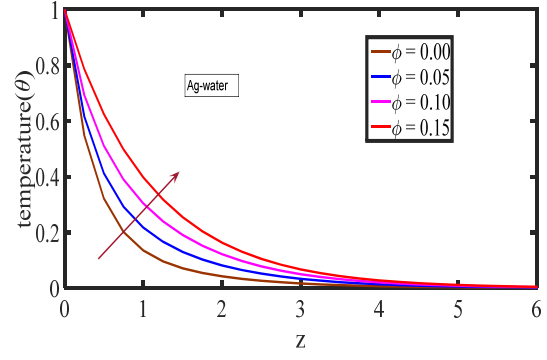


Fig. 4

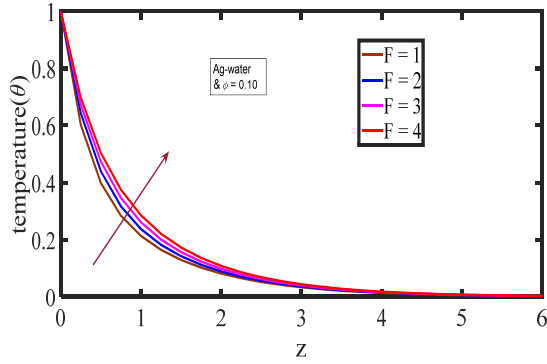


Fig. 5

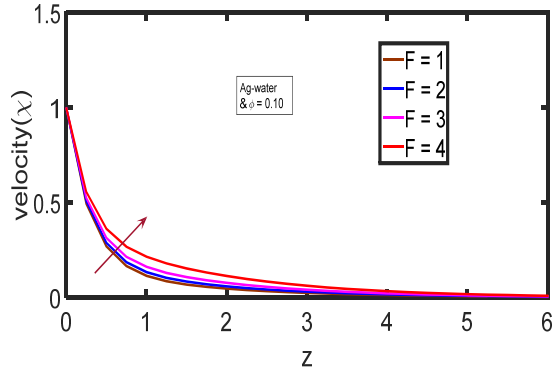


Fig. 6

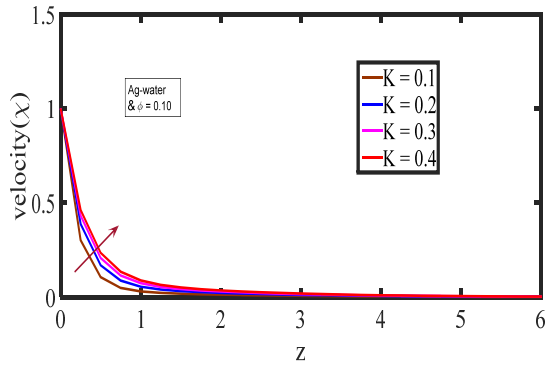


Fig. 7

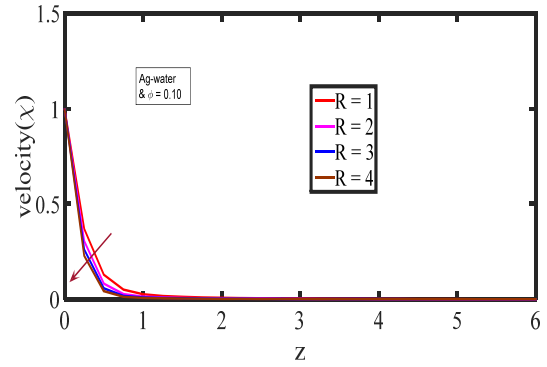


Fig. 8

The influence of the chemical reaction parameter K_r on the nanofluid temperature and concentration profiles for the Ag-water nano fluid particles are illustrated in Figs. 1 and 2. It is observed that an increase in K_r contributes to the decrease in the nanofluid temperature and concentration distributions. Figures 3 and 4 respectively, illustrates the effect of the nanoparticle volume fraction on the velocity and temperature profiles. It is clear that as the nanoparticle volume fraction increases, the nanofluid velocity decreases and temperature distributions increases. This agrees with the physical behavior that, when the volume fraction of silver increases, the thermal conductivity increases, and then the thermal boundary-layer thickness increases.

Figures 5 and 6 respectively, shows the effect of thermal radiation parameter F on the nano fluid temperature and velocity profiles. It is clear that, as the thermal radiation parameter F increases, the nano fluid velocity and temperature distributions across the boundary layer increases. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature x increases as the thermal boundary layer thickness become thinner.

Figure 7 illustrates the effect of permeability parameter K on the nano fluid velocity distribution. It is clear from the figure that as K increases, the nano fluid velocity also increases. The influence of rotation parameter R on the nano fluid velocity profiles for different values of Ag nano particles is shown in Fig. 8. From the figure, we see that the nano fluid velocity distribution across the boundary layer decreases with an increase of R .

Table 1					
S	ϕ	Q_H	F	Q_1	Nu
2	0.10	0.1	1.0	0.1	4.2597
3	0.10	0.1	1.0	0.1	6.6137
4	0.10	0.1	1.0	0.1	9.0643
5	0.10	0.1	1.0	0.1	11.5333
1	0.05	0.1	1.0	0.1	2.0994
1	0.10	0.1	1.0	0.1	2.2352
1	0.15	0.1	1.0	0.1	2.4240
1	0.20	0.1	1.0	0.1	2.6763
1	0.10	2	1.0	0.1	0.5746
1	0.10	4	1.0	0.1	1.1705
1	0.10	6	1.0	0.1	1.6034
1	0.10	8	1.0	0.1	1.9472
1	0.10	0.1	0.1	0.1	2.8532
1	0.10	0.1	0.2	0.1	2.7496
1	0.10	0.1	0.3	0.1	2.6589
1	0.10	0.1	0.4	0.1	2.5786
1	0.10	0.1	1.0	0.1	3.3450
1	0.10	0.1	1.0	0.2	3.2217
1	0.10	0.1	1.0	0.3	3.0984
1	0.10	0.1	1.0	0.4	2.9751

Table 2			
S	Sc	Kr	Sh
2	0.30	0.5	1.2928
3	0.30	0.5	1.7396
4	0.30	0.5	2.2100
5	0.30	0.5	2.6937
1	0.45	0.5	1.1925
1	0.60	0.5	1.4832
1	0.78	0.5	1.8212
1	0.96	0.5	2.1520
1	0.30	0.1	0.6240
1	0.30	0.2	0.7062
1	0.30	0.3	0.7743
1	0.30	0.4	0.8339

From Table 1, it is clear that with the increasing values of S , ϕ and Q_H , Nusselt number increases and with the increasing values of F and Q_1 , Nusselt number decreases. From Table 2, it is observed that Sherwood number increases with the increasing values of S , Sc and Kr .

CONCLUSIONS

In this work, we have theoretically studied the effect of chemical reaction and radiation absorption on unsteady MHD free convection heat and mass transfer flow of an incompressible, Ag-water based nano fluid along a semi-infinite vertical flat plate in a rotating frame of reference. The effects of various parameters on velocity, temperature and concentration profiles are discussed through graphs. The following are the conclusions derived from the present investigation.

1. As K increases, the nano fluid velocity also increases, actually this means that the porous medium impact on the boundary layer growth is significant due to the increase in the thickness of the thermal boundary layer.
2. As the thermal radiation parameter F increases, the nanofluid velocity and temperature distributions across the boundary layer increases.
3. An increase in K_r contributes to the decrease in the nanofluid velocity, temperature and concentration distributions.
4. It is clear from the graphs that all the nanofluid velocity and temperature profiles increases with increase of Q_1 .
5. It is clear that as the nanoparticle volume fraction increases, the nanofluid velocity decreases and temperature distributions increases.

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