Solving Multi-Choice Fractional Stochastic Transportation Problem involving Newton's Divided Difference Interpolation

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Abstract. There are limited numbers of methods to choose the optimal choice among the multiple choices. A numerical technique named Newton's Divided Difference Interpolation is used to find out the solution of multi-choice fractional stochastic transportation problem. Because of the uncertainty, the parameters of the problem supplies and demands are considered as multi-choice random parameters which are treated as independent random variables follows Logistic distribution. Also, the coefficients of the decision variables in the fractional objective function are taken as multi-choice type. To get the deterministic model, chance constrained programming is applied to the probabilistic constraints, and the transformed mathematical model is presented.

Keywords: Fractional Programming · Multi-choice random parameter · Stochastic Programming · Transportation problem.

1 Introduction

Business and industry are practically faced with economic optimization such as cost minimization, profit maximization. The transportation problem is concerned with the optimal way in which a homogeneous product is produced at different production houses (supplies locations) and distributed to a number of stores (destinations).

In day to day, the decision maker has to face many difficulties/challenges to find the optimal solution of the problem which occurs because of the vagueness and multiple choices. To solve the multi-choice programming, Chang [1] introduced the concept of binary variables and revised the model in which the binary variables has replaced by continuous variables. Mahapatra et al. [2] developed a model for multi-choice stochastic transportation problem involving extreme value distribution using binary variables. To optimize one or more ratios of the function, the fractional programming problem has been widely used. The extension of fractional programming in transportation problem is proposed by Swarup [3], which plays an important role in logistics and supply management.

Most of the studies considered with deterministic cases, in which the parameters are precisely known to the decision maker. But in real life applications, it is not always possible to find the exact value of the parameters. To tackle with the situation of uncertainty and impreciseness in making decisions, Zadeh [4] introduced the fuzzy theory. Liu [5] solved the fractional transportation problem in which all the parameters are considered as triangular fuzzy numbers. Also, fuzzy programming is used to convert the multi-objective function into a single objective function by Mahapatra et al. [6].

A computation method explored by Acharya et al. [7] for solving the fuzzy stochastic transportation problem. A technique which includes normal randomness is used to convert the stochastic transportation problem into a deterministic. To handle the situation of the multi-choice parameter in the transportation problem, Roy [8] proposed Lagrange's interpolating polynomial to convert the multi-choice parameter into a single choice. Pradhan et al. [9] presented a linear programming model in which each alternative of multi-choice parameters considered as a random variable and used the different type of optimization models (V-model, fractile criterion model, probability maximization model, and E-model).

In this paper, a fractional stochastic transportation problem (MCFSTP) is considered in which the objective function is in the ratio such that the cost and the corresponding profit are in multi-choice type. The supplies and demands are taken as multi-choice random parameters and each alternative of multi-choice parameter considered as an independent random variable which follows Logistic distribution. The detailed problem statement is presented in section 2 and the solution procedure of the presented problem is shown in section 3 and section 4 contains the concluding remarks.

2 Problem Statement

A transportation company wants to ship its product from different production houses to various retail stores. Assuming there are p number of production houses and q number of retail stores and a homogeneous product is transferred from k^{th} production house to t^{th} retail store. Let x_{kt} denotes the number of units of the product. Due to the uncertainty and the different choices in the environment, the values of the parameters are not always fixed therefore, the supplies and the demands are considered as multi-choice random parameters. Hence, the constraints of the formulated problem are in a probabilistic manner with their aspiration level. The objective function is in the ratio form, which represents the transportation cost associated with the obtained profit per unit of the product from k^{th} production house to t^{th} retail store. Because of the various choices for the transportation and other factors, the transportation cost and their profit are assumed as multi-choice type, so that the mathematical formulation of the said problem stated as

$$\min \ Z = \left\{ \frac{\sum_{k=1}^{p} \sum_{t=1}^{q} (c_{kt}^{1}, c_{kt}^{2}, ..., c_{kt}^{v}) \ x_{kt} + \xi}{\sum_{k=1}^{p} \sum_{t=1}^{q} (r_{kt}^{1}, r_{kt}^{2}, ..., r_{kt}^{v}) \ x_{kt} + \eta} \right\} \tag{1}$$

$$s.t.\quad P\left(\sum_{t=1}^{q} x_{kt} \leq (S_k^1, S_k^2, ..., S_k^m)\right) \geq 1-\theta_k \quad k=1,2,...,p \tag{2}$$

$$P\left(\sum_{k=1}^{p} x_{kt} \geq (R_t^1, R_t^2, ..., R_t^n)\right) \geq 1-\sigma_t \quad t=1,2,...,q \tag{3}$$

$$x_{kt} \geq 0 \quad \text{for every } k,\ t \tag{4}$$

where, $(S_k^1, S_k^2, ..., S_k^m)$ are the multi-choice random parameters for the total availability S_k at the k^{th} production house, considered as independent random variable. $(R_t^1, R_t^2, ..., R_t^n)$ are the multi-choice random parameters for the total requirement R_t of the product at the t^{th} retail store which considered as independent random variable. θ_k and σ_t represent the probability of meeting the constraints and η and ξ are the fixed costs in the objective function.

3 Solution Methodology

3.1 Newton's Divided Difference Interpolating Polynomial for Multi-Choice Parameters

To transform the multi-choice parameter into its optimal choice, the numerical approximation technique Newton's Divided Difference Interpolation is used. For each alternative of multi-choice parameter, introducing an integer variable such that the interpolating polynomial is formulated. Since there are v number of alternatives to the cost and profit parameters in the above problem, and the integer variables $w^i_{c_{kt}}$ $(i=0,1,...,v-1)$, $w^i_{r_{kt}}$ $(i=0,1,...,v-1)$ are introduced respectively.

Being supplies and demands are as multi-choice random parameters, the integer variables $w^j_{S_k}$ $(j=0,1,...,m-1)$, $w^u_{R_t}$ $(u=0,1,...,n-1)$, to each alternative is introduced. Different order of divided difference calculated according to alternatives for each multi-choice parameter, which is shown in Table 1.

Table 1. Divided Difference (DD) table

$w^i_{c_{kt}}$	$F_{c_{kt}}(w^i_{c_{kt}})$	First DD	Second DD	Third DD
0	c^1_{kt}			
		$f[w^0_{c_{kt}}, w^1_{c_{kt}}]$		
1	c^2_{kt}		$f[w^0_{c_{kt}}, w^1_{c_{kt}}, w^2_{c_{kt}}]$	
		$f[w^1_{c_{kt}}, w^2_{c_{kt}}]$		$f[w^0_{c_{kt}}, w^1_{c_{kt}}, w^2_{c_{kt}}, w^3_{c_{kt}}]$
2	c^3_{kt}		$f[w^1_{c_{kt}}, w^2_{c_{kt}}, w^3_{c_{kt}}]$	
		$f[w^2_{c_{kt}}, w^3_{c_{kt}}]$		
3	c^4_{kt}			

Using Table 1, the Newton's Divided Difference interpolating polynomial is formulated for the cost parameter.

$$\begin{aligned} F_{c_{kt}}(w_{c_{kt}}) = f[w^0_{c_{kt}}] + (w_{c_{kt}} - w^0_{c_{kt}})\ f[w^0_{c_{kt}}, w^1_{c_{kt}}] + (w_{c_{kt}} - w^0_{c_{kt}}) \\ (w_{c_{kt}} - w^1_{c_{kt}})\ f[w^0_{c_{kt}}, w^1_{c_{kt}}, w^2_{c_{kt}}] + \ldots + (w_{c_{kt}} - w^0_{c_{kt}}) \\ (w_{c_{kt}} - w^1_{c_{kt}}) \ldots (w_{c_{kt}} - w^{v-1}_{c_{kt}})\ f[w^0_{c_{kt}}, w^1_{c_{kt}}, \ldots, w^{v-1}_{c_{kt}}] \end{aligned} \tag{5}$$

$$\begin{aligned} F_{c_{kt}}(w_{c_{kt}}) = c^1_{kt} + (w_{c_{kt}} - w^0_{c_{kt}})\ (c^2_{kt} - c^1_{kt}) + (w_{c_{kt}} - w^0_{c_{kt}})(w_{c_{kt}} - w^1_{c_{kt}}) \\ \left(\frac{c^3_{kt} - 2c^2_{kt} + c^1_{kt}}{w^2_{c_{kt}} - w^0_{c_{kt}}} \right) + \ldots + \left(\sum_{i=1}^{v} \frac{c^{(i)}_{kt}}{\prod\limits_{i \neq j+1, j=0}^{v-1} (w^{i-1}_{c_{kt}} - w^j_{c_{kt}})} \right) \end{aligned} \tag{6}$$

In the similar way, the interpolating polynomials formulated for the profit, supply and demand which denoted by $F_{r_{kt}}(w_{r_{kt}}),\ F_{S_k}(w_{S_k}), F_{R_t}(w_{R_t})$ and the multi-choice parameters replaced by its interpolating polynomial in the problem.

3.2 Conversion of Probabilistic Constraints

The multi-choice parameters converted into their interpolating polynomial so that the obtained probabilistic constraints will convert into its deterministic form.

Consider the constraint (2) for every $k = 1, 2, ..., p$,

$$P\left(\sum_{t=1}^{q} x_{kt} \leq F_{S_k}(w_{S_k}) \right) \geq 1 - \theta_k \tag{7}$$

$$\implies 1 - P\left(F_{S_k}(w_{S_k}) \leq \sum_{t=1}^{q} x_{kt} \right) \geq 1 - \theta_k \tag{8}$$

$$\implies P\left(\frac{F_{S_k}(w_{S_k}) - E(F_{S_k}(w_{S_k}))}{\sqrt{V(F_{S_k}(w_{S_k}))}} \leq \frac{\sum_{t=1}^{q} x_{kt} - E(F_{S_k}(w_{S_k}))}{\sqrt{V(F_{S_k}(w_{S_k}))}} \right) \leq \theta_k \tag{9}$$

$$\implies P\left(\zeta_{S_k} \leq \frac{\sum_{t=1}^{q} x_{kt} - E(F_{S_k}(w_{S_k}))}{\sqrt{V(F_{S_k}(w_{S_k}))}} \right) \leq \theta_k \tag{10}$$

$$\implies \phi\left(\frac{\sum_{t=1}^{q} x_{kt} - E(F_{S_k}(w_{S_k}))}{\sqrt{V(F_{S_k}(w_{S_k}))}} \right) \leq \phi(-g_{\theta_k}) \tag{11}$$

$$\implies \frac{\sum_{t=1}^{q} x_{kt} - E(F_{S_k}(w_{S_k}))}{\sqrt{V(F_{S_k}(w_{S_k}))}} \leq -g_{\theta_k} \tag{12}$$

$$\implies \sum_{t=1}^{q} x_{kt}\ \leq\ E(F_{S_k}(w_{S_k})) - g_{\theta_k}\ \sqrt{V(F_{S_k}(w_{S_k}))} \tag{13}$$

where, $E(F_{S_k}(w_{S_k})), V(F_{S_k}(w_{S_k}))$ denote the mean and the variance of $F_{S_k}(w_{S_k})$ respectively. Also, ϕ be the cumulative distribution function of Standard Normal distribution and g_{θ_k} denotes the value of standard normal variable. Hence Equation (13) represents the deterministic constraint of the probabilistic constraint (7).

Likewise, applying the same procedure to the demands constraints (3), the equivalent deterministic constraint for every $t = 1, 2, ..., q$ are

$$\sum_{k=1}^{p} x_{kt} \;\leq\; E(F_{R_t}(w_{R_t})) + g_{\sigma_t}\, \sqrt{V(F_{R_t}(w_{R_t}))} \tag{14}$$

where, $E(F_{R_t}(w_{R_t}))$, $V(F_{R_t}(w_{R_t}))$ denote the mean and the variance of the $F_{R_t}(w_{R_t})$ respectively and g_{σ_t} denotes the value of standard normal variable.

$$E(F_{S_k}(w_{S_k})) = E\Bigg\{S_k^1 + (w_{S_k} - w_{S_k}^0)\ (S_k^2 - S_k^1) + (w_{S_k} - w_{S_k}^0)(w_{S_k} - w_{S_k}^1) \left(\frac{S_k^3 - 2S_k^2 + S_k^1}{w_{S_k}^2 - w_{S_k}^0}\right) + \ldots + \left(\sum_{i=1}^{v} \frac{S_k^{(i)}}{\prod\limits_{i\neq j+1, j=0}^{v-1} (w_{S_k}^{i-1} - w_{S_k}^{j})}\right)\Bigg\} \tag{15}$$

$$= E(S_k^1) + (w_{S_k} - w_{S_k}^0)\ (E(S_k^2) - E(S_k^1)) + (w_{S_k} - w_{S_k}^0)(w_{S_k} - w_{S_k}^1) \left(\frac{E(S_k^3) - 2E(S_k^2) + E(S_k^1)}{w_{S_k}^2 - w_{S_k}^0}\right) + \ldots + \left(\sum_{i=1}^{v} \frac{E(S_k^{(i)})}{\prod\limits_{i\neq j+1, j=0}^{v-1} (w_{S_k}^{i-1} - w_{S_k}^{j})}\right)\Bigg\} \tag{16}$$

$$V(F_{S_k}(w_{S_k})) = V\Bigg\{S_k^1 + (w_{S_k} - w_{S_k}^0)\ (S_k^2 - S_k^1) + (w_{S_k} - w_{S_k}^0)(w_{S_k} - w_{S_k}^1) \left(\frac{S_k^3 - 2S_k^2 + S_k^1}{w_{S_k}^2 - w_{S_k}^0}\right) + \ldots + \left(\sum_{i=1}^{v} \frac{S_k^{(i)}}{\prod\limits_{i\neq j+1, j=0}^{v-1} (w_{S_k}^{i-1} - w_{S_k}^{j})}\right)\Bigg\} \tag{17}$$

$$= V(S_k^1) + (w_{S_k} - w_{S_k}^0)^2\ (V(S_k^2) + V(S_k^1)) + (w_{S_k} - w_{S_k}^0)^2(w_{S_k} - w_{S_k}^1)^2 \left(\frac{V(S_k^3) + 4E(S_k^2) + V(S_k^1)}{(w_{S_k}^2 - w_{S_k}^0)^2}\right) + \ldots + \left(\sum_{i=1}^{v} \frac{V(S_k^{(i)})}{\prod\limits_{i\neq j+1, j=0}^{v-1} (w_{S_k}^{i-1} - w_{S_k}^{j})^2}\right)\Bigg\} \tag{18}$$

Deterministic Model:

$$\min\ Z = \left\{\frac{\sum_{k=1}^{p}\sum_{t=1}^{q} F_{c_{kt}}(w_{c_{kt}})\ x_{kt} + \xi}{\sum_{k=1}^{p}\sum_{t=1}^{q} F_{r_{kt}}(w_{r_{kt}})\ x_{kt} + \eta}\right\} \tag{19}$$

$$s.t.\quad \sum_{t=1}^{q} x_{kt} \;\leq\; E(F_{S_k}(w_{S_k})) - g_{\theta_k}\; \sqrt{V(F_{S_k}(w_{S_k}))} \quad k=1,2,...,p \tag{20}$$

$$\sum_{k=1}^{p} x_{kt} \;\leq\; E(F_{R_t}(w_{R_t})) + g_{\sigma_t}\; \sqrt{V(F_{R_t}(w_{R_t}))} \quad t=1,2,...,q \tag{21}$$

$$x_{kt} \geq 0 \quad \text{for every } k,\ t \tag{22}$$

The mean and the variance of the interpolating polynomial for the demands can be calculated using Equation (16) and (18).

4 Conclusion

In this paper, the solution methodology of fractional stochastic transportation problem with multi-choice parameters is presented. With the help of Newton's Divided Difference Interpolation, the multi-choice parameters converted into its single choice so that the obtained solution should be optimal. Also, in the real-life situations, the decision maker may not get the exact information about the supplies and demands, therefore, it considered as a multi-choice random parameter in which each alternative is treated as independent random variables, which follow Logistic distribution. The chance constrained programming is applied to the probabilistic constraints to get the deterministic constraints. This fractional stochastic transportation problem plays an important role in various fields such as logistics and supply management to reduce cost and improving service.

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