Scattering of dust particles with and without magnetic field in the atmosphere

Meenapriya. P

Department of Mathematics, Government Arts and science college,Hosur-635110.

meenapriyapal@gmail.com

ABSTRACT

The simultaneous effect of chemical reaction with and without magnetic field on scattering of dust particles in the atmosphere is studied. The scattering coefficient of dust particles is evaluated analytically using Taylor’s method. The computed numerical results are depicted through graphs and conclusions are drawn.

Keywords: *Atmospheric fluid, electric number,Hartmann number.*

1. INTRODUCTION

Airborne dust are of particular concern because they are well known to be associated with many widespread lung diseases. Dust is a small, dry, solid particles projected into the air by natural forces or man-made process. Types of dust includes mineral dust, metallic dust, chemical dust, organic and vegetable dust and biohazards. Scattering is the redirection of electro magnetic energy by suspended particles in the atmosphere [1]. As scattering properties of dust particles play an important role, many researchers worked in this area. John Aitken [3] studied the methods for counting the dust particles in the air. Sunil et.al [4] dealt with the theoretical investigation of magnetic- field dependent viscosity in ferro-magnetic fluid saturating a porous medium in the presence of dust particles. Ulanowski et.al [5] concludes in his study that alignment of dust can significantly alter dust optical depth and the presence of electric field modify dust transport. Haijiao Liu et.al [2] presented the results of a comprehensive study of dust flux and magnetic signatures of atmospheric dust fall originating from east Asia and China.

The objective of this paper is to study the effects of scattering coefficient of dust particles in the atmosphere under the effect of electric field, both in the presence and absence of magnetic field.

2. MATHEMATICAL FORMULATION

Consider two-dimensional laminar incompressible viscous flow of dust particles as shown in figure 1. It consists of an infinite horizontal atmospheric fluid layer bounded on both sides by electro conducting impermeable rigid plates embedded with electrodes located at y = 0 and y = h with an applied magnetic field B_0. On the boundaries, the electric potentials $\phi = \frac{V}{h}x$ at y=0 and $\phi = \frac{V}{h}(x - x_0)$ at y=h are maintained where V is potential.

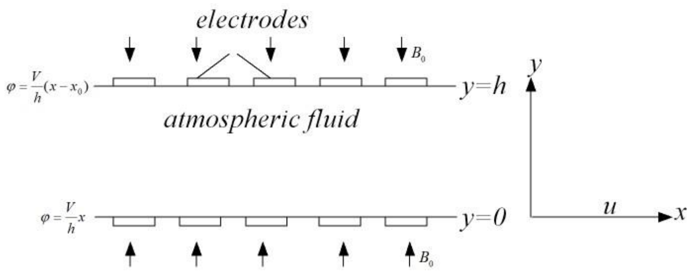

Figure 1. Physical configuration

Consider very small electrical conductivity (σ) , It makes the electric field $\vec{E}$, to be conservative.

$$\text{i.e. } \vec{E} = -\nabla\phi \tag{1}$$

The conservation of mass for an incompressible fluid

$$\nabla . \vec{q} = 0 \tag{2}$$

The conservation of momentum

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} + \rho_e\vec{E} + \mu(\vec{J}x\vec{B}) \tag{3}$$

where $\vec{q}$ the velocity, p the pressure, μ the viscosity, ρ_e the density of the charge distribution, $\vec{E}$ the electric field, $\vec{J}$ the current density and $\vec{B}$ the magnetic induction.

The conservation of species

$$\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = D\nabla^2 C \tag{4}$$

The conservation of charges

$$\frac{\partial \rho_e}{\partial t} + (\vec{q}.\nabla)\rho_e + \nabla.\vec{J} = 0 \tag{5}$$

The Maxwell's equations

$$\nabla.\vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{6}$$

$$\nabla x\vec{E} = \frac{-\partial\vec{B}}{\partial t} \tag{7}$$

$$\vec{J} = \sigma\vec{E} \tag{8}$$

where ε_0 the dielectric constant and σ the electrical conductivity. The above equations are solved using the following boundary conditions on velocity are,

$$\left.\begin{array}{l} u = 0 \text{ at } y = 0 \\ u = 0 \text{ at } y = h \end{array}\right\} \tag{9}$$

The boundary conditions on electric potential are,

$$\left.\begin{array}{ll} \phi = \dfrac{V}{h}x \text{ at} & y = 0 \\ \\ \phi = \dfrac{V}{h}(x - x_0) \text{ at} & y = h \end{array}\right\} \tag{10}$$

In cartesian form, using the above approximation equation (3) becomes

$$0 = -\frac{\partial p}{\partial x} + \mu\nabla^2 u + We\rho_e E_x - M^2 u, \nabla^2 = \frac{\partial^2}{\partial y^2}$$

where $M^2 = \dfrac{\sigma B_0^2 h^2}{\mu}$ (M is the Hartmann number), B_0 the uniform applied magnetic field, σ the electrical conductivity and μ the coefficient of viscosity. In a poorly conducting fluid, the electrical conductivity is assumed to vary linearly with temperature and increases with temperature in the form

$$\sigma = \sigma_0 [1+\alpha_h(T_b-T_0)] \tag{11}$$

where α_h is the volumetric coefficient of expansion.

After making dimensionless, using

$$y^* = \frac{y}{h};\ u^* = \frac{u}{\frac{\nu}{h}};\ E_x^* = \frac{E_x}{\frac{\nu}{h}};\ \rho_e^* = \frac{\rho_e}{\frac{\varepsilon_0 \nu}{h^2}};\quad P^* = \frac{P}{\rho\left(\frac{\nu}{h}\right)^2}; x^* = \frac{x}{h};$$

where V is electric potential, we get electric potential through electrodes.

Equations (3) to (11) becomes,

$$\frac{d^2u}{dy^2} - M^2 u = W_e \rho_e E_x + P \tag{12}$$

where $W_e = \dfrac{\varepsilon_0 V^2}{\mu}$, $P = \dfrac{-\partial p}{\partial x}$,

Equation (5) becomes, $\nabla.\vec{J} = 0$

Using equation (1) we get,

$$\sigma(\nabla^2\phi) + \nabla\phi.\nabla\sigma = 0 \quad (13)$$

After dimensionless the boundary conditions on velocity and electric potential are

$$\left.\begin{array}{l} u = 0 \text{ at } \ y = 0 \\ u = 0 \text{ at } \ \ y = 1 \end{array}\right\} \quad (14)$$

$$\left.\begin{array}{ll} \phi = x \text{ at} & y = 0 \\ \phi = x - x_0 \text{ at} & y = 1 \end{array}\right\} \quad (15)$$

The electrical conductivity σ is negligibly small, $\sigma << 1$ and it depends on the conduction temperature T_b namely,

$$\frac{d^2T_b}{dy^2} = 0 \quad (16)$$

using the boundary conditions

$$\left.\begin{array}{ll} T_b = T_0 \text{ at} & y = 0 \\ T_b = T_1 \text{ at} & y = h \end{array}\right\} \quad (17)$$

$$\text{is } T_b - T_0 = \Delta T_y \quad (18)$$

equation (11) becomes

$$\sigma = \sigma_0[1+\alpha_h \Delta T_y] = \sigma_0(1+\alpha y) = \sigma_0 e^{\alpha y}$$

$$\sigma \approx e^{\alpha y} \quad (19)$$

where $\alpha = \alpha_h \nabla T$.

Then (13) using (19) we get

$$\frac{d^2\phi}{dy^2} + \alpha \frac{d\phi}{dy} = 0 \quad (20)$$

The solution for ϕ satisfying the boundary equation (15) is

$$\phi = x - \frac{x_0}{1-e^{-\alpha}}[1-e^{-\alpha y}] \quad (21)$$

Using the non-dimensional quantities and equation (21), equations (6), (7) and (8) reduce to

$$\rho_e = \nabla.\vec{E} = -\nabla^2\phi = -\frac{x_0\alpha^2 e^{-\alpha y}}{1-e^{-\alpha}}\ ;\ E_x = -1$$

Then $\rho_e E_x = \dfrac{x_0^2\alpha^2 e^{-\alpha y}}{1-e^{-\alpha}}$ (22)

3. SCATTERING OF AEROSOLS

We consider two cases. First in the presence of magnetic field and the second without magnetic field

Case 1: M ≠ 0 (with magnetic field)

The solution of equation (12) satisfying the condition (14) is

$$u = A\cosh My + B\sinh My + \frac{W_e a_0 e^{-\alpha y}}{(\alpha^2 - M^2)} - \frac{P}{M^2} \tag{23}$$

where $a_0 = \frac{W_e x_0 \alpha^2}{e^{-\alpha} - 1}$; $A = \frac{P}{M^2} - \frac{W_e a_0 e^{-\alpha y}}{(\alpha^2 - M^2)}$;

$$B = \frac{1}{\mathrm{Sinh}M}\left(\frac{P}{M^2}(1 - \mathrm{Cosh}M) - \frac{W_e a_0 e^{-\alpha y}}{(\alpha^2 - M^2)}(e^{-\alpha} - \mathrm{Cosh}M)\right)$$

The average velocity is given by,

$$\bar{u} = \int_0^1 u\,dy = \frac{ASinhM}{M} - \frac{BCoshM - 1}{M} + \frac{W_e a_0}{\alpha(\alpha^2 - M^2)}(e^{-\alpha} - 1) - \frac{P}{M^2} \tag{24}$$

The concentration of aerosol C with chemical reaction K in the atmosphere which diffuse in a fully developed flow can be written as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - KC \tag{25}$$

The longitudinal diffusion is very much less than the transverse diffusion which implies $\frac{\partial^2 C}{\partial x^2} << \frac{\partial^2 C}{\partial y^2}$

Equation (25) becomes

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial y^2} - KC \tag{26}$$

The dimensionless boundary conditions on concentrations are

$$\left.\begin{aligned} \frac{\partial C}{\partial y} = 0 \text{ at } \quad & y = 0 \\ \text{and } C = 1 \text{ at } \quad & y = 1 \end{aligned}\right\} \tag{27}$$

Now introduce the following non-dimensional variables,

$$y^* = \frac{y}{h}; C^* = \frac{C}{C_0}; t^* = \frac{t}{\bar{t}}; \xi = \frac{x - \bar{u}t}{L}; \beta^2 = \frac{h^2}{D}K$$

where L is the characteristic length along the flow direction and β is the dimensionless reaction rate parameter. Equation (26) in dimensionless form can be written as

$$\frac{1}{\bar{t}}\cdot\frac{\partial C^*}{\partial t^*}+\frac{(u-\bar{u})}{L}\cdot\frac{\partial C^*}{\partial \xi}=\frac{D}{h^2}\cdot\frac{\partial^2 C^*}{\partial y^{*2}}-\frac{\beta^2 D}{h^2}C$$

For simplicity, neglecting the asterisks (*) we get

$$\frac{1}{\bar{t}}\cdot\frac{\partial C}{\partial t}+\frac{V}{L}\cdot\frac{\partial C}{\partial \xi}=\frac{D}{h^2}\cdot\frac{\partial^2 C}{\partial y^2}-\frac{\beta^2 D}{h^2}C \tag{28}$$

where $V=u-\bar{u}$ = AcoshMy + BsinhMy + $\frac{W_e a_0 e^{-\alpha y}}{(\alpha^2-M^2)}$ + f (29)

where $f=-\frac{A\mathrm{Sinh}M}{M}-\frac{B\mathrm{Cosh}M-1}{M}+\frac{W_e a_0}{\alpha(\alpha^2-M^2)}(e^{-\alpha}-1)$

To obtain C as the variation of y by approximating equation (28) in the form

$$\frac{\partial^2 C}{\partial y^2}-\beta^2 C=QV \tag{30}$$

where Q = $\frac{h^2}{DL}\frac{\partial C}{\partial \xi}$ and β the reaction rate parameter.

Using the equation (29) and satisfying the boundary condition (27), the solution of equation (30) we get

$$C = Q\,(C1\,\mathrm{Cosh}\beta y + C2\,\mathrm{Sinh}\beta y + \frac{A\mathrm{Cosh}My}{M^2-\beta^2}+\frac{B\mathrm{Sinh}My}{M^2-\beta^2}+\frac{W_e a_0}{(\alpha^2-M^2)(\alpha^2-\beta^2)}e^{-\alpha y}-\frac{f}{\beta^2}) \tag{31}$$

where

$$C1 = \frac{1}{\mathrm{Cosh}\beta}(1-\frac{B}{M^2-\beta^2}(\frac{-M\mathrm{Sinh}\beta}{\beta}+\mathrm{Sinh}M)-\frac{W_e a_0}{(\alpha^2-M^2)(\alpha^2-\beta^2)}(\frac{\alpha\mathrm{Sinh}\beta}{\beta}+e^{-\alpha}+\frac{f}{\beta^2});$$

$$C2 = \frac{1}{\beta}\left(-\frac{BM}{M^2-\beta^2}+\frac{\alpha W_e a_0}{(\alpha^2-M^2)(\alpha^2-\beta^2)}\right)$$

C is the concentration of aerosols in the presence of chemical reaction.The fluid is transported across the section of layer per unit breadth then the volumetric rate of the fluid is given by,

$$M = h\int_0^1 CV dy \tag{32}$$

Using equations (31) and (29), performing the integration and after simplification we get,

$$M = \frac{h^3}{DL}G\frac{\partial C}{\partial \xi} \tag{33}$$

We assume that the variation of C with ξ are small compared to the longitudinal direction and if C_m is the mean concentration over a section, then $\frac{\partial C}{\partial \xi}$ is indistinguishable from $\frac{\partial C_m}{\partial \xi}$ (Taylor, 1953) so that (33) may be written as

$$M = \frac{h^3}{DL} G \frac{\partial C_m}{\partial \xi} \tag{34}$$

No material is lost in the process which is expressed by the continuity equation for C_m namely

$$\frac{\partial M}{\partial \xi} = -\frac{2}{L} \frac{\partial C_m}{\partial t} \tag{35}$$

where $\frac{\partial}{\partial t}$ represents differentiation with respect to time at point where ξ is constant.

Equation (34) using (35)

$$-\frac{2}{L} \frac{\partial C_m}{\partial t} = \frac{h^3}{DL} G \frac{\partial^2 C_m}{\partial \xi^2}$$

$$\frac{\partial C_m}{\partial t} = D^* \frac{\partial^2 C_m}{\partial \xi^2} \tag{36}$$

where $D^* = -\frac{h^3}{2D} G$ (36a)

Case 2: M= 0 (without magnetic field)

When M = 0, Following the same procedure explained in case1,from equation (12)

$$\frac{\partial C_m}{\partial t} = D1^* \frac{\partial^2 C_m}{\partial \xi^2} \tag{37}$$

where, $D1^* = \frac{h^3}{2D} G_0$ (37a)

which is the equation of the longitudinal dispersion. From equations (36a) and (37a), the lengthy expression of scattering coefficients D^* and $D1^*$ are computed and the results obtained from the study are discussed in the next section.

4. RESULTS AND DISCUSSIONS

Tthe analytical results reported in the previous section is performed using MATHEMATICA 8.0 and a representative set of results are depicted graphically .

The scattering coeficient given in equation (36a) and (37a) is computed for different values of electric number and reaction rate for both presence and absence of Hartmann number which are graphically represented in figures 2 to 5.Figures 2 and 3 represent the dispersion coefficient D^* with reaction rate β for different values of electric number We and scattering coefficient D^* with electric number We for different values of reaction rate β . It is observed that D^* decreases with an increase in reaction rate β in the presence of Hartmann number M and increases with increase in electric number We. Similarly in the absence of Hartmann number the scattering coefficient against reaction rate β and versus electric number We are represented through figures 4 and 5 .It is observed that D^* increases with an increase in electric number We and decreases with increases in reaction rate β and Hartmann number M.

5. CONCLUSIONS

From the figures, it is concluded that for both with and without magnetic field the electric field enhances the scattering of dust particles and the reaction rate decreases the transport of particles. The mathematical model presented here is the representative of distribution of ultrafine dust particles in the atmosphere and it suggests that the scattering of particles would depend upon the chemical reaction, electric field, magnetic field and other parameters in the atmosphere.

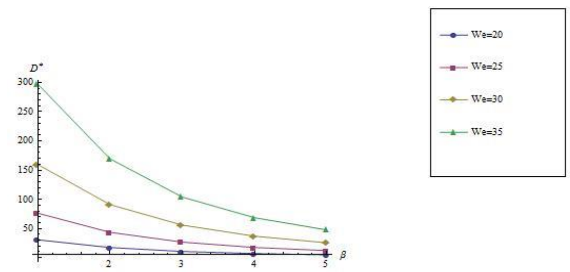

Figure 2. Scattering coefficient D* versus reaction rate β for different values of electric number We and for fixed value of Hartmann number M=0.2

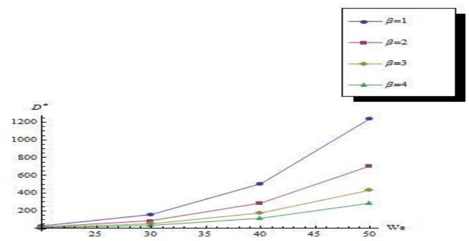

Figure 3. Scattering coefficient D* versus electric number We for different values of reaction rate β and for fixed value of Hartmann number M=0.2

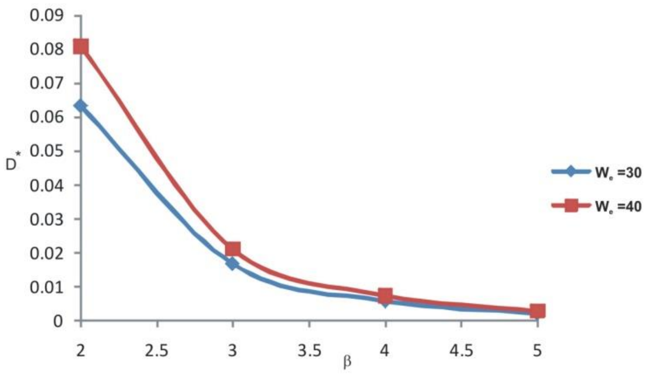

Figure 4. Scattering coefficient D1* versus reaction rate β for different values of electric number We with Hartmann number M=0

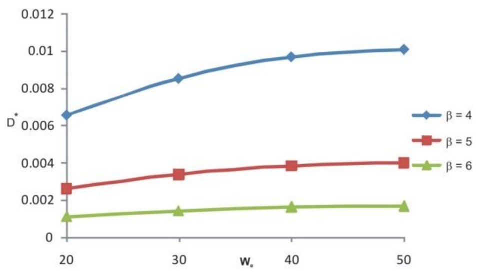

Figure 5. Scattering coefficient D1* versus electric number We for different values of reaction rate β with Hartmann number M=0

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