Thermal Effects of a Fluid Flow in a Non-uniform inclined prone tube having multiple stenoses

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Abstract:

The effect of slip on Nano fluid in a circular tube of non-uniform cross section with multiple stenoses have been studied. By using Homotopy Perturbation Method, the coupled equations in temperature and nanoparticle phenomena are calculated. The expressions for the flow characteristics, like, pressure drop, resistance to the flow and shear stress at the wall have been derived and solutions have been obtained. It is observed that the resistance to the flow increases with the heights of the stenosis, Brownian motion number, Thermophoresis parameter, local temperature Grashof number, local nanoparticle Grashof number, inclination and permeability constant. It is also observed that the shear stress at the wall increases with Brownian motion parameter and with height of the stenosis, but decreases with local nanoparticle Grashof number, thermophoresis parameter and permeability constant. Also, it is found that the size of bolus decreases with the increase of permeability constant.

***Keywords*:** Stenoses, Nano Fluid, Permeability constant.

1. Introduction

The world mortality rate is increasing due to problems associated with cardiac. This is because of high grade of stenosis in arteries. Due to stenosis the effected arteries become hard. Stenosis is the main reason for clotting of blood, cerebral stokes and failure of the heart. Stenosis increases resistance of the flow in arteries. Due to this the pressure of the flow increases. This leads to substantial changes in pressure distribution and wall shear stress. A detailed understanding of the flow in the artery with stenosis gives knowledge of blood flow in the physiological systems.

In the past, many researchers studied blood flow in the artery systems by treating blood as Newtonian or non-Newtonian. (Young [1], Padmanabhan [2], Shukla et al., [3] and Ranadhir Roy et al., [4]). Most of these theoretical models studied the blood flow in a circular tube or channel having single stenosis. But in the reality, there is a possibility of forming multiple stenoses or over lapping Stenoses in the arteries. Notable researchers like Maruthi Prasad et al., [5], Gurju Awgichew et al., [6], Raja Agarwal et al., [7] investigated blood flow in arteries with multiple stenosis. In all these studies they considered the wall of the tube is not flexible.

Due to high thermal conductivity of nanofluids and its many applications in biomedical field, more number of researchers are making significant research in this field. (Choi [8], Sohail Nadeem et al., [9], Nabil et al., [10], Maruthi Prasad et al., [11], Rekha et al., [12]).

The present paper considered nanofluid in an inclined permeable tube having non-uniform cross section with two stenosis and investigated the effects of different parameters on pressure drop, resistance to the flow and wall shear stress.

2. Mathematical Formulation

The steady flow of Nano fluid through a circular tube of non-uniform cross with two stenosis is considered. Cylindrical polar coordinate system (r, θ, z) is taken so that the centre line of the tube coincides with z-axis. It is assumed that the tube is inclined at an angle $'\alpha'$ to the horizontal axis. It is also assumed that stenoses are mild and developed in an axially symmetric manner. The walls are permeable in nature. **[Fig. 1].**

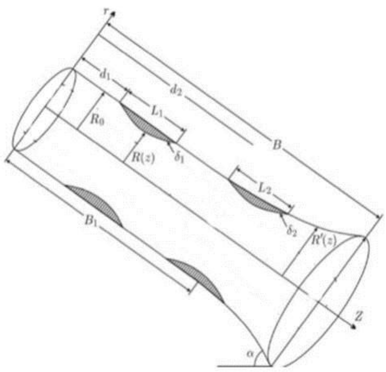

Fig.1. Geometry of an inclined tube with multiple stenoses.

The radius of the cylindrical tube is given as(Maruthi Prasad et al., [5])

$$h = R(z) = \begin{cases} R_0 & : \quad 0 \le z \le d_1, \\ R_0 - \frac{\delta_1}{2}\left(1 + \cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right) & : \quad d_1 \le z \le d_1 + L_1, \\ R_0 & : \quad d_1 + L_1 \le z \le B_1 - \frac{L_2}{2}, \\ R_0 - \frac{\delta_2}{2}\left(1 + \cos\frac{2\pi}{L_2}(z - B_1)\right) & : \quad B_1 - \frac{L_2}{2} \le z \le B_1, \\ R^*(z) - \frac{\delta_2}{2}\left(1 + \cos\frac{2\pi}{L_2}(z - B_1)\right) & : \quad B_1 \le z \le B_1 + \frac{L_2}{2}, \\ R^*(z) & : \quad B_1 + \frac{L_2}{2} \le z \le B\,. \end{cases} \tag{1}$$

The following restrictions for mild stenoses [7] are supposed to satisfy:

$$\delta_i \ll \min(R_0, R_{out}),$$

$$\delta_i \ll L_i \;\; where\; R_{out} = R(z) at\; z = B.$$

Here L_i are the lengths and δ_i $(i = 1,2)$are maximum heights of two stenoses (the suffixes 1 and 2 refer to the first and second stenosis respectively).

Equations for an incompressible fluid flow are given as (Maruthi Prasad [11])

$$\frac{1}{\bar r}\frac{\partial(\bar r\bar v)}{\partial \bar r}+\frac{\partial \bar w}{\partial \bar z}=0 \tag{2}$$

$$\rho\left[\bar v\frac{\partial \bar v}{\partial \bar r}+\bar u\frac{\partial \bar v}{\partial \bar z}\right]=-\frac{\partial \bar P}{\partial \bar r}+\mu\left[\frac{\partial^2 \bar v}{\partial \bar r^2}+\frac{1}{\bar r}\frac{\partial \bar v}{\partial \bar r}+\frac{\partial^2 \bar v}{\partial \bar z^2}-\frac{\bar v}{\bar r^2}\right]-\frac{\cos\alpha}{F}, \tag{3}$$

$$\rho\left[\bar v\frac{\partial \bar u}{\partial \bar r}+\bar w\frac{\partial \bar u}{\partial \bar z}\right]=-\frac{\partial \bar P}{\partial \bar z}+\mu\left[\frac{\partial^2 \bar w}{\partial \bar r^2}+\frac{1}{\bar r}\frac{\partial \bar w}{\partial \bar r}+\frac{\partial^2 \bar w}{\partial \bar z^2}\right]+\rho g\alpha(\bar T-\bar T_0)+\rho g\alpha(\bar C-\bar C_0)+\frac{\sin\alpha}{F} \tag{4}$$

$$\left[\bar v\frac{\partial \bar T}{\partial \bar r}+\bar w\frac{\partial \bar T}{\partial \bar z}\right]=\alpha\left[\frac{\partial^2 \bar T}{\partial \bar r^2}+\frac{1}{\bar r}\frac{\partial \bar T}{\partial \bar r}+\frac{\partial^2 \bar T}{\partial \bar z^2}\right]+\tau\left\{D_B\left[\frac{\partial \bar C}{\partial \bar r}\frac{\partial \bar T}{\partial \bar r}+\frac{\partial \bar C}{\partial \bar z}\frac{\partial \bar T}{\partial \bar z}\right]+\frac{D_{\bar T}}{\bar T_0}\left[\left(\frac{\partial \bar T}{\partial \bar r}\right)^2+\left(\frac{\partial \bar T}{\partial \bar z}\right)^2\right]\right\} \tag{5}$$

$$\left[\bar v\frac{\partial \bar C}{\partial \bar r}+\bar w\frac{\partial C}{\partial \bar z}\right]=D_B\left[\frac{\partial^2 \bar C}{\partial \bar r^2}+\frac{1}{\bar r}\frac{\partial \bar C}{\partial \bar r}+\frac{\partial^2 \bar C}{\partial \bar z^2}\right]+\frac{D_{\bar T}}{\bar T_0}\left[\frac{\partial^2 \bar T}{\partial \bar r^2}+\frac{1}{\bar r}\frac{\partial \bar T}{\partial \bar r}+\frac{\partial^2 \bar T}{\partial \bar z^2}\right] \tag{6}$$

Here$\tau=\frac{(\rho C)_P}{(\rho C)_f}$ being the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid.

The boundary conditions are

$$\frac{\partial \bar w}{\partial \bar r}=0,\frac{\partial \bar T}{\partial \bar r}=0,\frac{\partial \bar C}{\partial \bar r}=0\ at\ \bar r=0 \tag{7}$$

$$\bar w=-k\frac{\partial \bar w}{\partial r}\ ,\bar T=\bar T_0\ ,\bar C=\bar C_0\ at\ \bar r=R(z) \tag{8}$$

Using the following non-dimensional quantities

$$\bar z=\frac{z}{B}\ ,\bar d_1=\frac{d_1}{B}\ ,\bar L_1=\frac{L_1}{B}\ ,\bar L_2=\frac{L_2}{B}\ ,\bar B_1=\frac{B_1}{B}\ ,\bar v=\frac{B}{\delta W}v\ ,\bar w=\frac{w}{W},$$

$$\bar R(z)=\frac{R(z)}{R_0}\ ,\bar\delta_i=\frac{\delta_i}{R_0}\ ,\bar P=\frac{P}{\mu WL/R_0^2}\ ,\bar q=\frac{q}{\pi R_0^2 W}\ ,R_e=\frac{2\rho c_1 R_0}{\mu},$$

$$N_b=\frac{(\rho C)_P D_{\bar B}\bar C_0}{(\rho C)_f}\ ,N_t=\frac{(\rho C)_P D_T\bar T_0}{(\rho C)_f\beta}\ ,G_r=\frac{g\beta\bar T_0 R_0^3}{\gamma^2}\ ,B_r=\frac{g\beta\bar C_0 R_0^3}{\gamma^2} \tag{9}$$

and apply the mild stenosis approximation, the Eq. (6) to Eq. (12) and reduces to

$$\frac{\partial w}{\partial r}+\frac{w}{r}+\frac{\partial w}{\partial z}=0 \tag{10}$$

$$\frac{\partial P}{\partial r}=-\frac{\cos\alpha}{F} \tag{11}$$

$$\frac{\partial P}{\partial z}-\frac{\sin\alpha}{F}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+G_r\theta_t+B_r\sigma\ , \tag{12}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_t}{\partial r}\right)+N_b\frac{\partial \sigma}{\partial r}\frac{\partial \theta_t}{\partial r}+N_t\left(\frac{\partial \theta_t}{\partial r}\right)^2=0, \tag{13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \sigma}{\partial r}\right)+\frac{N_t}{N_b}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_t}{\partial r}\right)\right)=0\ , \tag{14}$$

where w being the velocity averaged over section of the tube with radius $R_0, \theta_t\ ,\sigma\ ,N_t\ ,G_r$ and B_r are temperature profile, nanoparticle phenomena, Brownian motion number, thermophoresis parameter, local temperature Grashof number and local nanoparticle Grashof number.

The non-dimensional boundary conditions are

$$\frac{\partial w}{\partial r}=0\ ,\frac{\partial \theta_t}{\partial r}=0\ ,\frac{\partial \sigma}{\partial r}=0\ at\ r=0,$$

$$w = -k\frac{\partial w}{\partial r}, \theta_t = 0, \sigma = 0 \; at \; r = h(z). \tag{15}$$

3. Solution

The coupled equations Eq. (13) and Eq. (14) are solved using Homotopy Perturbation Method (HPM) with the initial guesses

$$\theta_{10}(r,z) = \left(\frac{r^2-h^2}{4}\right), \; \sigma_{10}(r,z) = -\left(\frac{r^2-h^2}{4}\right) \tag{16}$$

Then the solution for temperature profile and nanoparticle phenomena can be written as

$$\theta_t(r,z) = \left(\frac{r^2-h^2}{64}\right)(N_b - N_t) \tag{17}$$

$$\sigma(r,z) = -\left(\frac{r^2-h^2}{4}\right)\frac{N_t}{N_b}. \tag{18}$$

Substituting Eq. (17) and Eq. (18) in Eq. (12) and applying boundary conditions, then solution for the velocity can be written as

$$w(r,z) = \left(\frac{r^2-h^2}{4} - \frac{kr}{2}\right)\left(-\frac{\sin\alpha}{F} + \frac{dP}{dz}\right) + B_r\frac{N_t}{N_b}\left(\frac{r^4}{64} - \frac{r^2h^2}{16} + \frac{3h^4}{64} - \frac{kr^3}{16} + \frac{krh^2}{8}\right) - G_r(N_b - N_t)\left(\frac{r^6}{2304} - \frac{r^2h^4}{256} + \frac{h^6}{288} - \frac{kr^5}{384} + \frac{krh^4}{128}\right) \tag{23}$$

The dimension less flux q can be calculated as

$$q = \int_0^h 2rw\,dr. \tag{24}$$

By substituting the Eq. (23) in Eq. (24), then the flux is

$$q = -\left(\frac{h^4}{8} + \frac{kh^3}{3}\right)\left(-\frac{\sin\alpha}{F} + \frac{dP}{dz}\right) + B_r\frac{N_t}{N_b}\left(h^6(0.02083) + kh^5(0.05833)\right) - G_r(N_b - N_t)\left(h^8(0.001627) + kh^7(0.004464)\right) \tag{25}$$

From Eq. (29), $\frac{dP}{dz}$ can be given as

$$\frac{dP}{dz} = \frac{1}{\left(\frac{h^4}{8}+\frac{kh^3}{3}\right)}\left[-q + \left(\frac{h^4}{8} + \frac{kh^3}{3}\right)\frac{\sin\alpha}{F} - G_r(N_b - N_t)\left(h^8(0.001627) + kh^7(0.004464)\right) + B_r\frac{N_t}{N_b}\left(h^6(0.02083) + kh^5(0.05833)\right)\right] \tag{26}$$

The pressure drop over one wave length is given by

$\Delta p = p(0) - p(\lambda)$ is

$$\Delta p = -\int_0^1 \frac{dP}{dz}\,dz$$

$$\Delta p = \int_0^1 \frac{1}{\left(\frac{h^4}{8}+\frac{kh^3}{3}\right)}\left[q - \left(\frac{h^4}{8} + \frac{kh^3}{3}\right)\frac{\sin\alpha}{F} + G_r(N_b - N_t)\left(h^8(0.001627) + kh^7(0.004464)\right) - B_r\frac{N_t}{N_b}\left(h^6(0.02083) + kh^5(0.05833)\right)\right]dz \tag{27}$$

The resistance to the flow λ is

$$\lambda = \frac{\Delta p}{q} = \frac{1}{q}\int_0^1 \frac{1}{\left(\frac{h^4}{8}+\frac{kh^3}{3}\right)}\left[q - \left(\frac{h^4}{8}+\frac{kh^3}{3}\right)\frac{\sin\alpha}{F} + G_r(N_b - N_t)\left(h^8(0.001627) + kh^7(0.004464)\right) - B_r\frac{N_t}{N_b}\left(h^6(0.02083) + kh^5(0.05833)\right)\right]dz \quad (28)$$

The pressure drop in the absence of stenosis $h = 1$ is denoted by Δp_n and is obtained from Eq. (27) as

$$\Delta p_n = \int_0^1 \frac{1}{\left(\frac{1}{8}+\frac{k}{3}\right)}\left[q - \left(\frac{1}{8}+\frac{k}{3}\right)\frac{\sin\alpha}{F} + G_r(N_b - N_t)\left((0.001627) + k(0.004464)\right) - B_r\frac{N_t}{N_b}\left((0.02083) + k(0.05833)\right)\right]dz \quad (29)$$

The resistance to the flow in the normal artery is

$$\lambda_n = \frac{\Delta p_n}{q} = \frac{1}{q}\int_0^1 \frac{1}{\left(\frac{1}{8}+\frac{k}{3}\right)}\left[q - \left(\frac{1}{8}+\frac{k}{3}\right)\frac{\sin\alpha}{F} + G_r(N_b - N_t)\left((0.001627) + k(0.004464)\right) - B_r\frac{N_t}{N_b}\left((0.02083) + k(0.05833)\right)\right]dz \quad (30)$$

The normalized resistance to the flow denoted by

$$\bar{\lambda} = \frac{\lambda}{\lambda_n} \quad (31)$$

and the wall shear stress

$$\tau_h = -\frac{h}{2}\frac{dP}{dz} = -\frac{h}{2}\left[\frac{1}{\left(\frac{h^4}{8}+\frac{kh^3}{3}\right)}\left[-q + \left(\frac{h^4}{8}+\frac{kh^3}{3}\right)\frac{\sin\alpha}{F} - G_r(N_b - N_t)\left(h^8(0.001627) + kh^7(0.004464)\right) + B_r\frac{N_t}{N_b}\left(h^6(0.02083) + kh^5(0.05833)\right)\right]\right] \quad (32)$$

4. Results and Analysis

The effects of different parameters on pressure drop (Δp), resistance to the flow$(\bar{\lambda})$ and wall shear stress (τ_h) have been computed numerically using Mathematica 9.1, by taking $\frac{R^*(z)}{R_0} = \exp[\beta B^2(z - B_1)^2]$, where $d_1 = 0.2,\ L_1 = L_2 = 0.2,\ B_1 = 0.7, B = 1$ and $\beta = 0.01$.

The effects of various parameters on the resistance to the flow $(\bar{\lambda})$ are shown in figures (2-8) for different values of various parameters. It is observed that, the resistance to the flow $(\bar{\lambda})$ increases with the increase of heights of the stenosis (δ_1 *and* δ_2), Brownian motion number (N_b), thermophoresis parameter (N_t), local temperature Grashof number (G_r), local nanoparticle Grashof number (B_r), inclination (α) and permeability constant (k).

The shear stress acting on the wall (τ_h) over the height of stenosis has shown in the figs. (9-13).

It can be shown that the shear stress at the wall increases with the increase of N_b and height of the stenosis. It is also observed that the shear stress at the wall decreases with the increase of B_r, N_t and k.

Fig.14 illustrates the stream line patterns. It is observed that the size of trapped bolus decreases with the increase of permeability constant (k).

5. Graphs

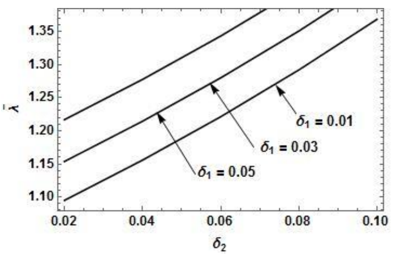

Fig. 2: Effect of δ_2 and δ_1 on $\bar{\lambda}$
$\left(\begin{matrix} q = 0.3, F = 0.3, B_r = 0.3, G_r = 0.2, \\ N_b = 0.3, N_t = 0.8, \alpha = \frac{\pi}{6}, k = 0.05 \end{matrix}\right)$

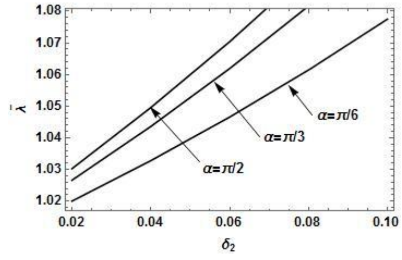

Fig. 3: Effect of δ_2 and α on $\bar{\lambda}$
$\left(\begin{matrix} q = 0.3, F = 0.3, B_r = 0.3, G_r = 0.2, \\ N_b = 0.3, N_t = 0.8, \delta_1 = 0.01, k = 0.0 \end{matrix}\right.$

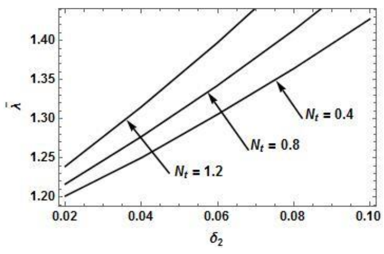

Fig. 4: Effect of δ_2 and N_t on $\bar{\lambda}$
$\left(\begin{matrix} q = 0.3, \delta_1 = 0.05, F = 0.3, B_r = 0.3, \\ G_r = 0.2, N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05 \end{matrix}\right)$

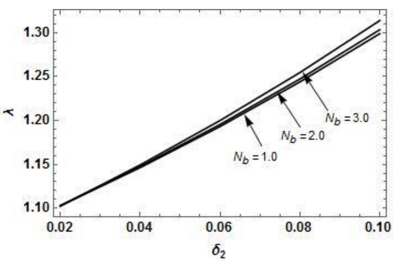

Fig. 5: Effect of δ_2 and N_b on $\bar{\lambda}$
$\left(\begin{matrix} q = 0.3, \delta_1 = 0.05, F = 0.3, B_r = 0.3 \\ G_r = 0.2, N_t = 0.8, \alpha = \frac{\pi}{6}, k = 0.05 \end{matrix}\right.$

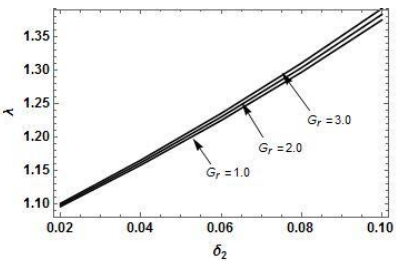

Fig. 6: Effect of δ_2 and G_r on $\bar{\lambda}$
$\left(\begin{matrix} q = 0.3, \delta_1 = 0.05, F = 0.3, B_r = 0.3 \\ N_b = 0.3, N_t = 0.8, \alpha = \frac{\pi}{6}, k = 0.05 \end{matrix}\right.$

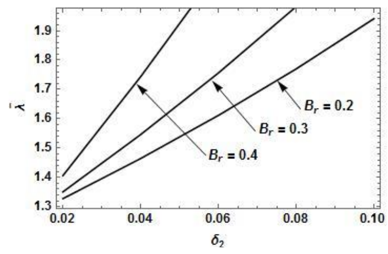

Fig. 7: Effect of δ_2 and B_r on $\bar{\lambda}$
$\left(\begin{matrix} q = 0.3, \delta_1 = 0.05, F = 0.3, G_r = 0.2, \\ N_b = 0.3, N_t = 0.8, \alpha = \frac{\pi}{6}, k = 0.05 \end{matrix}\right)$

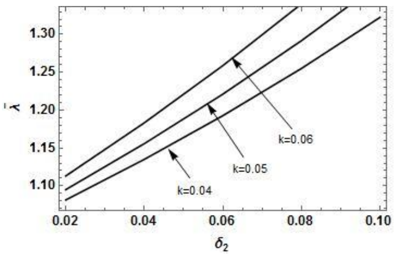

Fig. 8: Effect of δ_2 and k on $\bar{\lambda}$
$\left(\begin{matrix} q = 0.3, \delta_1 = 0.05, F = 0.3, G_r = 0.2, \\ N_b = 0.3, N_t = 0.8, \alpha = \frac{\pi}{6}, B_r = 0.3 \end{matrix}\right.$

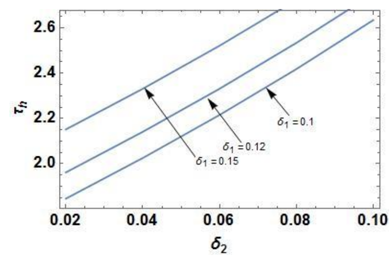

Fig. 9: Effect of δ_2 and δ_1 on τ_h
$\left(\begin{matrix} q = 0.3, k = 0.05, F = 0.3, G_r = 0.2, \\ N_b = 0.1, N_t = 0.3, \alpha = \frac{\pi}{6}, B_r = 0.3 \end{matrix}\right)$

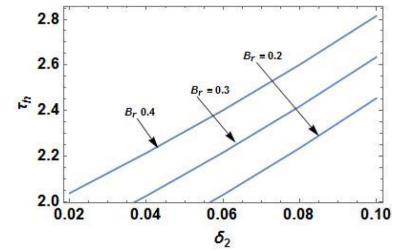

Fig. 10: Effect of δ_2 and B_r on τ_h
$\left(\begin{matrix} q = 0.3, k = 0.05, F = 0.3, G_r = 0.2, \\ N_b = 0.1, N_t = 0.3, \alpha = \frac{\pi}{6}, \delta_1 = 0.1 \end{matrix}\right)$

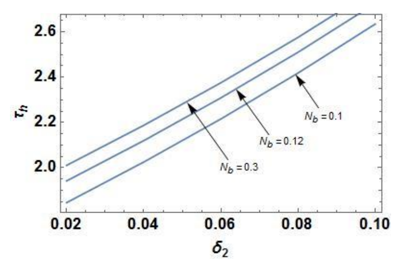

Fig. 11: Effect of δ_2 and N_B on τ_h
$\left(\begin{matrix} q = 0.3, k = 0.05, F = 0.3, G_r = 0.2, \\ B_r = 0.3, N_t = 0.3, \alpha = \frac{\pi}{6}, \delta_1 = 0.1 \end{matrix}\right)$

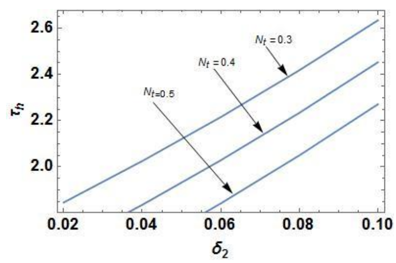

Fig. 12: Effect of δ_2 and N_t on τ_h
$\left(\begin{matrix} q = 0.3, k = 0.05, F = 0.3, G_r = 0.2, \\ B_r = 0.3, N_B = 0.1, \alpha = \frac{\pi}{6}, \delta_1 = 0.1 \end{matrix}\right)$

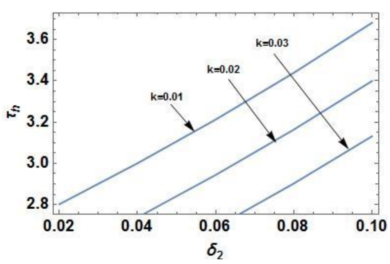

Fig. 13: Effect of δ_2 and kon τ_h
$\left(\begin{matrix} q = 0.3, N_t = 0.3, F = 0.3, G_r = 0.2, \\ B_r = 0.3, N_B = 0.1, \alpha = \frac{\pi}{6}, \delta_1 = 0.1 \end{matrix}\right)$

Streamlines

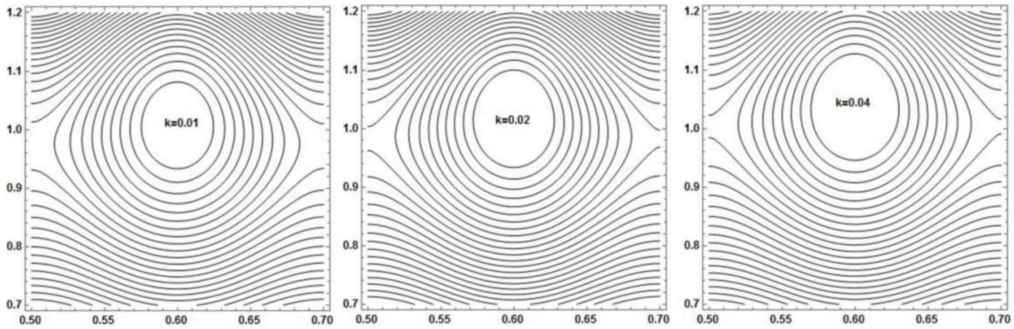

Fig. 14 Stream Lines patterns for different values of k

6. CONCLUSIONS

A theoretical approachment of a nanofluid flowing through a permeable tube having two Stenoses is presented. The cross section of the tube is varying its length. The solutions for impedance and shear stress of the wall are obtained.

The conclusions of this model are

1. The resistance to the flow increases with the increase of δ_1, N_b, N_t, G_r,B_r and k.
2. As heights of the stenosis increases and as N_b increases there is an increase in the wall shear stress.
3. The wall shear stress decreases with the increase of G_r,B_r, and k.
4. The volume of the bolus increases with the increase of permeability constant.

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