Statistical model used for Performance of Raga Todi

1.Poonam Priyadarshini 2. Soubhik Chakraborty
1.Birla Institute of Technology Patna, Electronics and Communication Engg
2. Birla Institute of Technology Mesra, Ranchi, Applied Mathematics

Abstract. The backbone of statistics lies in modeling. Statistical models are not perfect but they are capable of matching, with fewer parameters, those true unknown complex models with multiple specifications an explicit idea of which or how they enter the true model we may not have. It is quite desirable to control and decrease the errors in these models as well as verify the integrity of fit of these models which are very complete in the scope of statistics. Modeling a music performance is a coveted field of research in computational musicology. Double exponential smoothing (DES) have been good fit for modeling raga Todi performance.

Keywords— statistical features, smoothing, inter-onset interval

Introduction: Indian Classical Music (ICM) has two distinct streams, Hindustani or North Indian and Carnatic or South Indian Classical Music. ICM concentrates on melody and rhythm. There is no harmony or counterpoint in ICM . ICM does not allow for harmony as it destroys the *shruti* concept of ragas. Western Classical Music has harmony and counterpart features. ICM is always monophonic (single melody line) unlike WCM which can be polyphonic (multiples melody lines), consistent or a combination of both. Each melody line is called a counterpoint . In Indian Classical Music (Sa), the first note is not correlated with any established values of pitch and other notes are realised accordingly with Sa as a reference point whereas in Western Classical Music the tonic note has a fixed value of the pitch. In Indian music, the notes are not on a complete scale but on a relative scale.

1.1 Single Exponential Smoothing: is used to statistically model time series data for smoothing purpose or for prediction. Although it was Holt [1] who proposed it first, it is Brown's single exponential smoothing that is commonly used nowadays [2].

Single exponential smoothing is achieved by the model $F_{t+1} = \alpha Y_t + (1-\alpha)$ Ft, $0<\alpha<1$, to the data (t, Y_t) where F_t is the predicted against Y_t and initially $F_o = Y_o$.

Here α is the smoothing factor. This is the only parameter in the model that needs to be determined from the data. The smoothed statistic F_{t+1} is a simple weighted average of the previous observation Y_t and the previous smoothed statistic F_t. The term *smoothing factor* applied to α here is something of a misnomer, as larger values of α actually reduce the level of smoothing, and in the limiting case with α = 1 the output series is just the same as the original series (with lag of one time unit). Simple exponential smoothing is easily applied and it produces a smoothed statistic as soon as two observations are available.

Values of α close to one have less of a smoothing effect and give greater weight to recent changes in the data, while values of α closer to zero have a greater smoothing effect and are less responsive to recent changes. There is no formally correct procedure for choosing α. Sometimes the statistician's judgment is used to choose an appropriate factor. Alternatively, a statistical technique may be used to *optimize* the value of α. For example, the method of least squares might be used to determine the value of α for which the sum of the quantities $(F_t - Y_t)^2$ is minimized[3-6]. This is the technique which has been employed here.

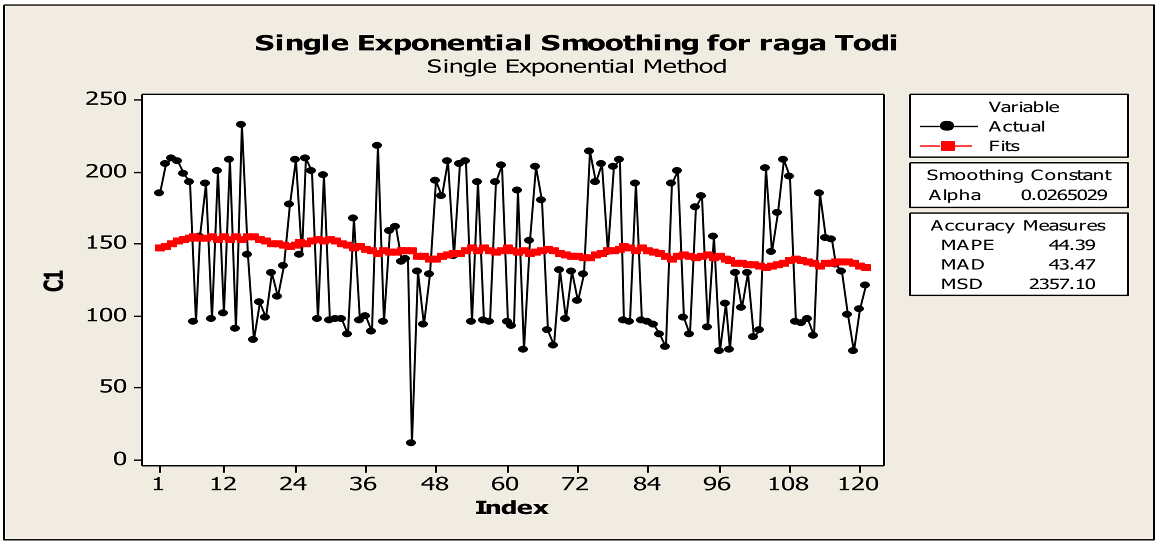

Figure 1.1: Single Exponential Smoothing for raga Todi

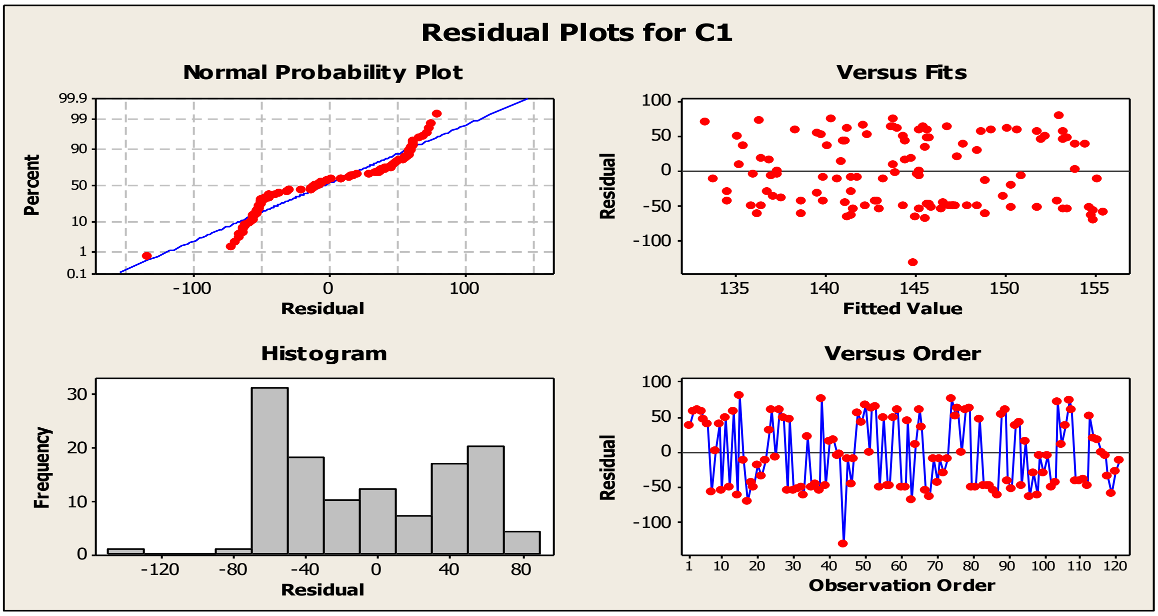

Figure 1.2: Residual plots for raga Todi (Single Exponential Smoothing)

Interpretations from figures 1.1 and 1.2:

1. From Figure 1.1 it is clear that the normal probability plot does not follow a straight line so it shows non-normality. There is change in slope so it also shows unidentified variable.
2. The graph of residual versus fitted values shows that residual are not scattered randomly about zero. This indicates that it have non-constant variance. It shows one outlier as a point far away from zero.
3. In histogram of the residual a bar at value between 0 to 120 is far away from the other bars so it indicates an outlier.
4. The graph residuals versus order plots the residuals should fluctuate in a random pattern around the center line but in our case residuals fluctuate between -50 to +50, without any correlation that is ascending or descending trends.

1. 2 Double Exponential Smoothing

Double exponential smoothing employs a level component and a trend component at each period. It uses two weights, or smoothing parameters, to update the components at each period [7]. The double exponential smoothing equations are:

$$L_t = \alpha\, Y_t + (1 - \alpha)\,[L_{t-1} + T_{t-1}]$$

$$T_t = \gamma\,[L_t - L_{t-1}] + (1 - \gamma)\,T_{t-1}$$

$$Y_t \text{ predicted} = L_{t-1} + T_{t-1} \quad \text{where}$$

- L_t is the level at time t, α which ranges from 0 to 1 is the weight for the level
- T_t is the trend at time t, γ which also ranges from 0 to 1 is the weight for the trend
- Y_t is the data value at time t, and Y_t predicted is the fitted value, or one-step-ahead

forecast, at time t .If the first observation is numbered one, then level and trend estimates at time zero must be initialized in order to proceed. The initialization method used to determine how the smoothed

1.3. Experimental Results

Using the music software Solo Explorer 1.0 we obtained the fundamental frequencies Y_t against onset time t in sec of raga Todi performances of 15 artists (8 instrumentalists; 7 vocalists) for 3 minutes recording for each. Figure 1.3 gives the DES fit to the first of the 15 recordings while figure. 1.4 gives the corresponding goodness of fit plots. These graphs are obtained using MINITAB statistical package version 16. In figure 1.3, the variable name C1 along y axis corresponds the fundamental frequencies that characterize the notes. The term index in X axis refers to the instance of realization of the notes, i.e. 1, 2, 3........etc (in a stochastic process, time need not be the time of clock; it could simply be the instance at which a phenomenon, in this case the note, occurs.

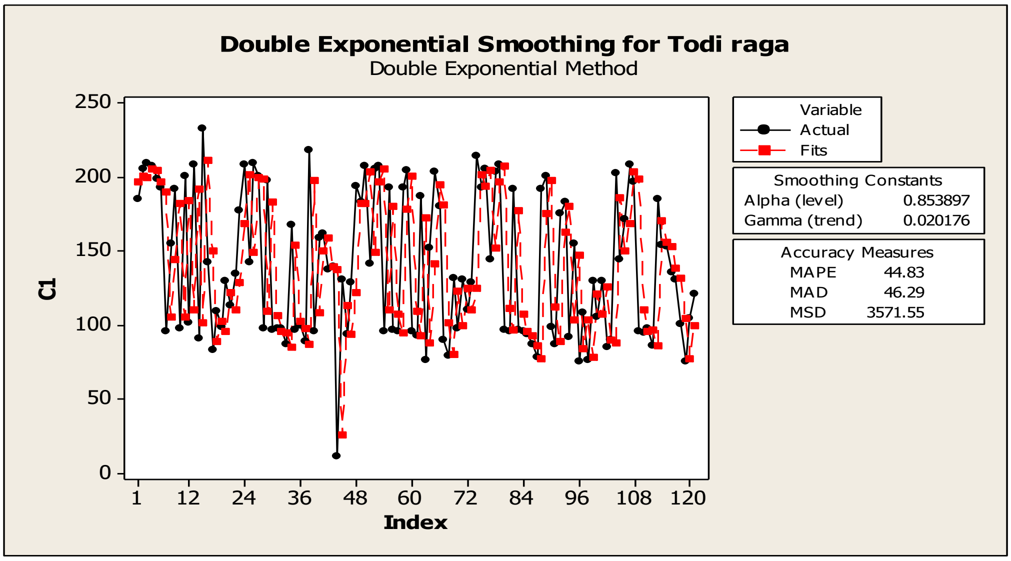

Figure 1.3: Double Exponential Smoothing for raga Todi

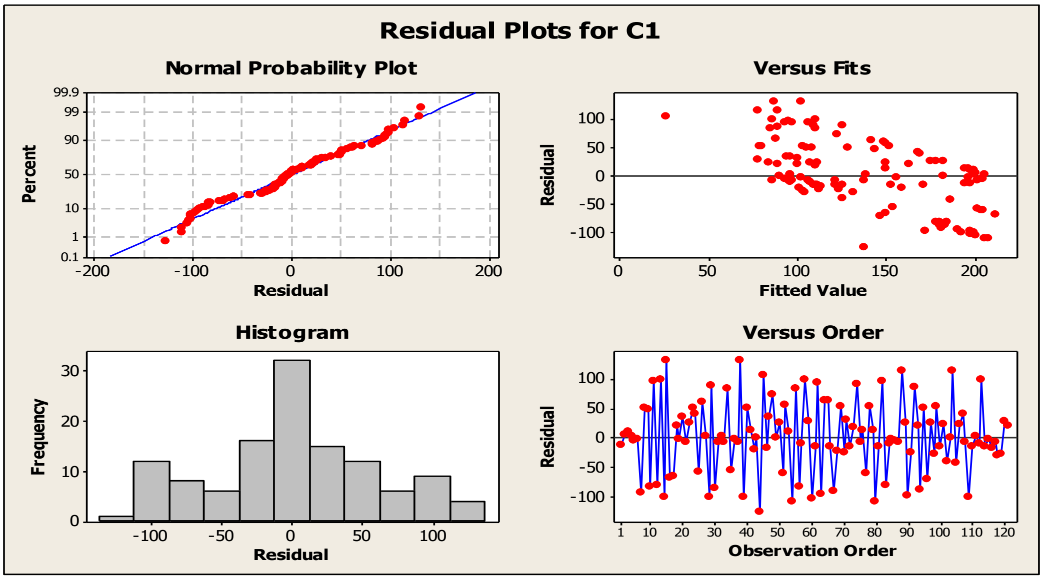

Figure 1.4: Residual plots for raga Todi (Double Exponential Smoothing)

Interpretations from figures 1.3 and 1.4: The random pattern of the residuals (figure-1.3) together with the closeness of smoothed data with the observed one (figure-1.4) justifies the Double Exponential Smoothing. A detailed discussion of the findings is given next.

MAPE (Mean Absolute Percent Error) - measures the accuracy of fitted time series values. It expresses accuracy as a percentage. MAD (Mean Absolute Deviation) - measures the accuracy of fitted time series values. It expresses accuracy in the same units as the data, which helps conceptualize the amount of error. MSD (Mean Squared Deviation) - measures the accuracy of fitted time series values. MSD is always computed using the same denominator (the number of forecasts) regardless of the model, so one can compare MSD values across models and hence compare the accuracy of two different models. For all three measures, smaller values generally indicate a better fitting model. In case we fit other models to the same data, it is of interest to compare the corresponding MAPE, MAD and MSD values. This is reserved as a rewarding future work.

1. From Figure 1.4 it is clear that the normal probability plot does follow a straight line so it shows good approximation to normality for the residuals.

2. The graph of "residual versus fitted values" shows that residual are scattered randomly about zero. This indicates that residuals have a constant variance.

3. A histogram of the residuals shows the distribution of residuals for all observations and it should be bell shaped which is so in our case. one can use this histogram as an exploratory tool to learn about data features such as typical values, spread, shape and outliers. For example, long tails imply skewness and a bar away from the other bars implies an outlier.

4. The graph 'residuals versus order" plots the residuals in the order of the corresponding observations and they should fluctuate in a random pattern around the center line which is so in our case

Results

Single Exponential Smoothing for C1 for Todi Raga (Zero values of Y_t exist; MAPE calculated only for non-zero Y_t) Data C1 (C1 in Minitab represents pitch Y_t), Length 54, Smoothing Constant, Alpha (α) = 0.0265029

Accuracy Measures: MAPE (Mean Absolute Percent Error) =44.39, MAD (Mean Absolute Deviation) =43.47, MSD (Mean Squared Deviation) = 2357.10

Double Exponential Smoothing for Todi raga

Smoothing Constant, Alpha (level) 0.853897 , Gamma (trend) 0.020176

Accuracy Measures: MAPE 44.83, MAD 46.29, MSD 3571.55

Conclusion:

Double Exponential Smoothing shows better fits for Todi raga as there is closeness of smoothed data with the observed one. Normal probability plot does follow a straight line showing normality for the residuals. The graph of "residual versus fitted values" shows that residual are scattered randomly about zero showing that residuals have a constant variance. The residuals should fluctuate in a random pattern around the centre line in graph of 'residuals versus order" plot. This shows that correlation exists among residuals that are near to each other.

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