Effect of Axial Conduction in the Thermally Developing Region of the Channel Partially Filled with a Porous Medium: Constant Wall Heat Flux

J. Sharath Kumar Reddy [a] and D. Bhargavi[b]
[a,b]Dept. of Mathematics, NITW, India
*Corresponding author Email: jskreddy.amma@gmail.com
Received: Date? Accepted: Date?

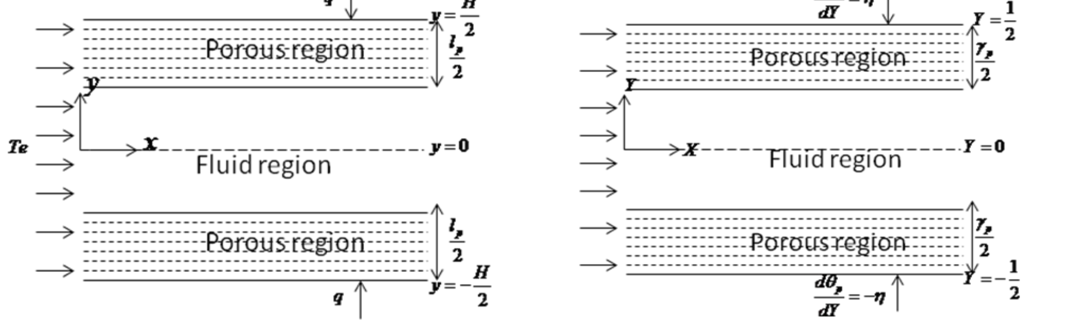
The effect of axial conduction on heat transfer characteristics in the entrance region of channel partially filled with a porous medium: constant wall heat flux is investigated. Porous insert attached to adjacent of both the walls of the channel. The flow in the fluid and porous region are governed by Poiseuille flow and Darcy-Brinkman model. The flow is assumed to be unidirectional. The effect of the various parameters such as Darcy number, Peclet number on the heat transfer coefficient has been studied. The local Nusselt number depends on the porous fraction. The effect of the axial conduction is high when Peclet number is small in the entrance region of the channel.
Keywords**:** Axial conduction; Porous medium; Thermal entrance region.

1. Introduction

Several studies (Agrawal [1]; Hennecke [2]; Vick and Ozisik [3]; Jagadeesh Kumar [4]) have shown that axial conduction term becomes significant in the equation of energy at low Peclet number in the case of forced convection in the ducts. Further, thermal field significantly gets altered because of axial conduction. Several researchers (Lundberg and Mccuen [5]; Worsoe-Schmidt [6]; Nguyen and Maclaine-cross [7]; Campo and Salazar [8]; Xiong [9]) studied the problem of forced convection considering axial conduction effect, under different conditions. In particular, Shah and London [10] studied the problem of heat transfer in the entrance region for a viscous incompressible fluid in both two dimensional channel and circular cylindrical tube taking into consideration axial conduction term. Nguyen [11] studied same problem under the boundary conditions of uniform wall temperature and uniform wall heat flux. Nield, Kuznetsov and Xiong [12] investigated the effects of viscous dissipation, axial conduction with uniform temperature at the walls, on thermally developing forced convection heat transfer in a parallel plate channel fully filled with a porous medium. Ramjee and Satyamurty [13] studied local and average heat transfer in thermally developing region of an asymmetrically heated channel.

In the present study, the effect of axial conduction in the entrance region of channel partially filled with a porous medium has been studied. It is assumed that the flow is unidirectional and thermal field is developing. Numerical solutions for the two dimensional energy equations in both the fluid and porous regions have been obtained using successive accelerated replacement (SAR) numerical scheme (Satyamurty and Bhargavi [14]; Bhargavi and Sharath Kumar Reddy [15]). The effect of relevant parameters on temperature, bulk mean temperature and local Nusselt number have been studied.

2. Mathematical Formulation



(a) Dimensional (b) Non-dimensional

Fig. 1: Physical Model and Coordinate System.

The non-dimensional variables are following

$$\left.\begin{aligned} &X = x/H\,,\ \ Y = y/H\,,\ \ U_f = u_f/u_{ref}\,,\ \ U_i = u_i/u_{ref}\,,\ U_p = u_p/u_{ref}\,,\ P = p/\rho\, u_{ref}^2\,, \\ &\theta_f = (T_f - T_e)/(qH/k_f)\,, \theta_p = (T_p - T_e)/(qH/k_f) \end{aligned}\right\} \tag{1}$$

In Eq.(1), u_f and u_p are the dimensional velocity in fluid and porous regions. X and Y are the dimensionless axial and normal coordinates. U_f and U_p are the dimensionless velocity in fluid and porous regions. p is the dimensional pressure and P is the dimensionless pressure. θ is the dimensionless temperature. μ_f is the viscosity of the fluid, μ_{fef} is effective viscosity of the porous. k_f is the thermal conductivity of the fluid, k_{eff} is the effective thermal conductivity of the porous. γ_p is the porous fraction defined as ratio of thickness of porous material and width of the channel(i.e., l_p/H)

The non-dimensional governing equations and boundary conditions become(using Eq. (1)),

Fluid Region

$$\frac{d^2U_f}{dY^2} = Re\ \frac{dP}{dX} \tag{2}$$

$$U_f\,\frac{\partial\theta_f}{\partial X^*} = A_c\,\frac{1}{Pe^2}\frac{\partial^2\theta_f}{\partial X^{*2}} + \frac{\partial^2\theta_f}{\partial Y^2} \tag{3}$$

Eq. (2), Re, the Reynolds number is defined by,

$$Re = \rho u_{ref} H\ /\ \mu_f \tag{4}$$

Eq. (3), Pe is the Peclet number and X^* is the normalized X, are defined

$$Pe = u_{ref} H\ /\ \alpha_f \ \text{ and } X^* = X\ /\ Pe \tag{5}$$

Porous Region

$$\frac{d^2U_p}{dY^2} - \frac{\varepsilon}{Da} U_p = \varepsilon\ Re\frac{dP}{dX} \tag{6}$$

$$U_p\,\frac{\partial\theta_p}{\partial X^*} = \frac{1}{\eta}\left(A_c\,\frac{1}{Pe^2}\frac{\partial^2\theta_p}{\partial X^{*2}} + \frac{\partial^2\theta_p}{\partial Y^2}\right) \tag{7}$$

Eq. (6), Da and ε are defined as,

$$Da = \frac{K}{H^2} \ \text{ and } \varepsilon = \mu_f\ /\ \mu_{eff} \tag{8}$$

Eq. (7) , η is defined as,

$$\eta = k_f\ /\ k_{eff} \tag{9}$$

When A_c = 1, in Eqs. (3) and (7) axial conduction is included, and when A_c = 0, axial conduction is neglected. When A_c = 0, the solutions to Eqs. (3) and (7) in terms of X^* do not depend on Pe.

Non-dimensional boundary conditions

$$\theta_{p,f}(0,Y) = 0 \quad \text{for} \quad -\frac{1}{2} \le Y \le 0 \ \ \text{\{inlet condition\}} \tag{10}$$

$$\frac{dU_f}{dY} = 0,\ \frac{\partial\theta_f}{\partial Y} = 0 \quad \text{at} \quad Y = 0 \ \ \text{\{symmetry condition\}} \tag{11}$$

$$U_f = U_p = U_i\,,\ \ \frac{dU_f}{dY} = \frac{1}{\varepsilon}\frac{dU_p}{dY} \qquad \text{at} \qquad Y = -\frac{1}{2} + \frac{\gamma_p}{2} \tag{12}$$

$$\theta_f = \theta_p = \theta_i\,,\ \ \frac{\partial\theta_f}{\partial Y} = \frac{1}{\eta}\frac{\partial\theta_p}{\partial Y} \qquad \text{at} \qquad Y = -\frac{1}{2} + \frac{\gamma_p}{2} \tag{13}$$

$$U_p = 0\,,\ \ \frac{\partial\theta_p}{\partial Y} = -\eta \quad \text{at} \quad Y = -1/2 \tag{14}$$

$$\frac{\partial\theta_b}{\partial X^*} = 0 \Rightarrow \frac{\partial\theta_{f,p}}{\partial X^*} = \frac{\theta_{f,p}}{\theta^*}\frac{\partial\theta^*}{\partial X^*} \text{ at } X^* \geq X^*_{fd} \text{ for } -1/2 \leq Y \leq 1/2 \qquad (15)$$

In Eq. (15), θ_b is defined by

$$\theta_b = \frac{T - T_e}{T_b - T_e} = \frac{\theta}{\theta^*} \qquad (16)$$

3.1 Local Nusselt number:

The local Nusselt number at $Y = -1/2$ (using Eq. (1)), Nu_{px} is given by

$$Nu_{px} = \frac{h_{px}(2H)}{k_f} = \frac{2}{\theta_w - \theta^*} \qquad (17)$$

3. Numerical Scheme: Successive Accelerated Replacement (SAR)

Solutions to non-dimensional energy Eqs. (3) and (7) along with the non-dimensional boundary conditions on θ given in Eqs. (10) to (15) have been obtained using the numerical scheme SAR given in [13, 14 and 15]. The scheme is basically the Gauss Siedel Successive Over-relaxation scheme, see, [16]. The terminology of SAR has been used by Dellinger [17].

Expressions for U_p and U_f have been taken from Bhargavi and Sharath Kumar Reddy [15 and 18]. Also, U_p and U_f can be easily obtained as analytical solutions to Eqs. (2) and (6), applying the non-dimensional boundary conditions given in Eqs. (11) and (14) along with the interface condition at the porous-fluid region given by Eq. (12).

4. Result and Discussion

We have assumed that $\varepsilon = \mu_f/\mu_{eff} = 1$ and $\eta = k_f/k_{eff} = 1$.

4.1 Thermal field:

Variation of θ profiles with Peclet number, Pe :

Non-dimensional temperature in excess of wall temperature $\theta_w - \theta_p, \theta_w - \theta_f$ profiles at different axial locations for Da = 0.005 and γ_p = 0.4 are shown in Fig. 2(a) to 2(f) respectively, for Peclet numbers, Pe = 5, 10, 25, 50, 100 and A_c = 0, i.e., when axial conduction is neglected. From Fig. 2, as X^* increases, $\theta_w - \theta_p$, $\theta_w - \theta_f$ increases in both porous and fluid regions for all Peclet numbers. Fig. 2(e) {Pe = 100} and 2(f) { A_c = 0} are almost identical except for very small X^* values, indicating that the effect of axial conduction is negligible when $Pe \geq 100$. That is, if X^* is larger (say, = 0.4), $\theta_w - \theta_p$, $\theta_w - \theta_f$ reaches to fully developed profiles which is available in [18] when $Pe \geq 100$.

4.2 Bulk mean temperature :

Variation of dimensionless bulk mean temperature in excess of wall temperature, $\theta_w - \theta^*$ with X^*, for different Peclet numbers, Pe = 5, 10, 25, 50 and 100 for Da = 0.05 for γ_p = 0, 0.2, 0.4, 0.6, 0.8 and 1.0 presented in Fig. 3(a) to 3(f). From Fig. 3, effect of the Peclet number can be accessed. For all X^*, $\theta_w - \theta^*$ is lower for lower Pe. The effect of axial conduction thus results in the fluid getting less heated or less cooled. From Fig. 3(a) to 3(f), as X^* increases, $\theta_w - \theta^*$ increases for all Peclet numbers and porous fractions. As Peclet number increases, $\theta_w - \theta^*$ increases with X^* values for all porous fractions.

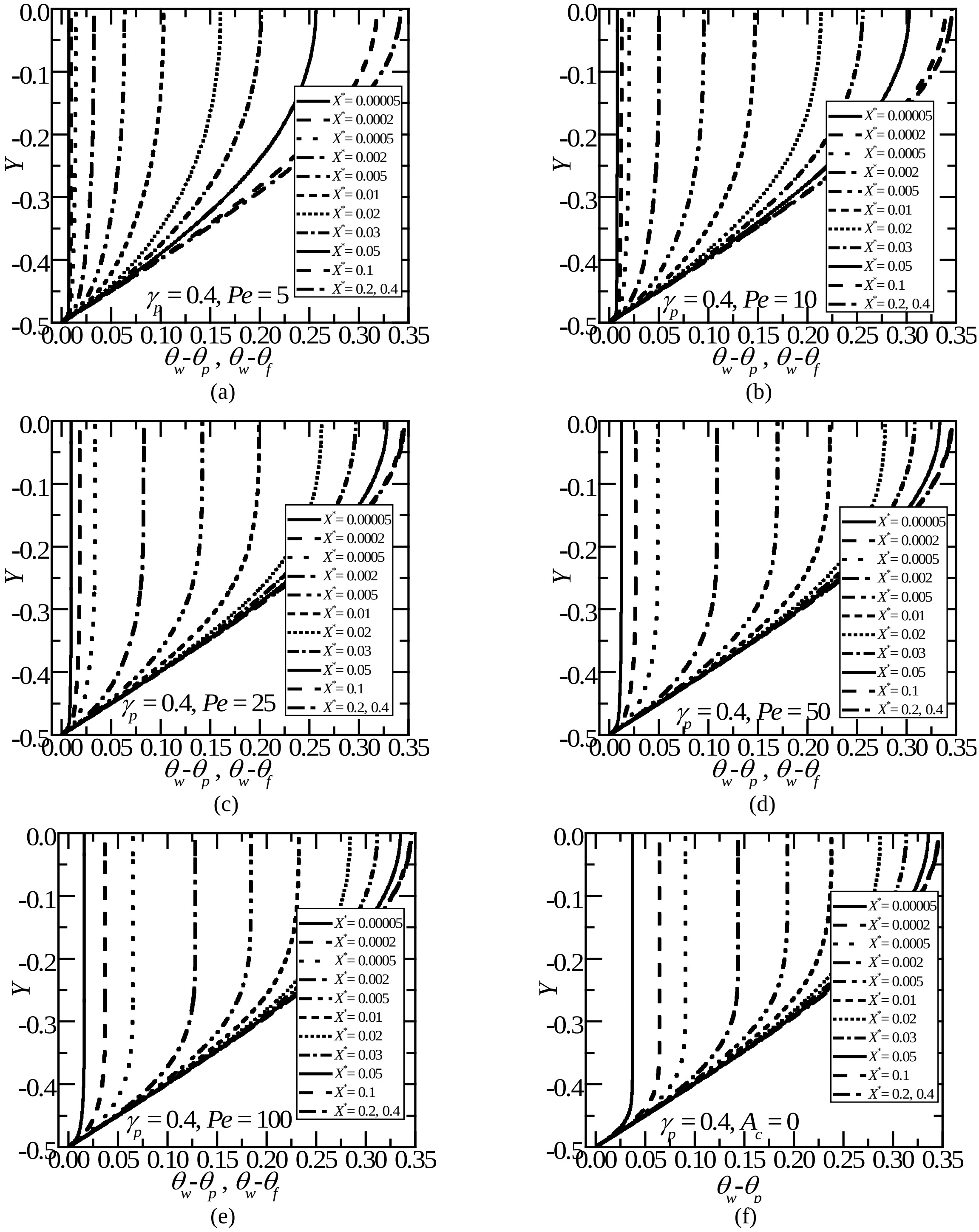


Fig. 2: Variation of $\theta_w - \theta_p$, $\theta_w - \theta_f$ profiles for different X^* values for Da = 0.005 and $\gamma_p = 0.4$ for (a) Pe = 5 (b) Pe = 10 (c) Pe = 25 (d) Pe = 50 (e) Pe = 100 and (f) A_c = 0, i.e., when axial conduction is neglected.

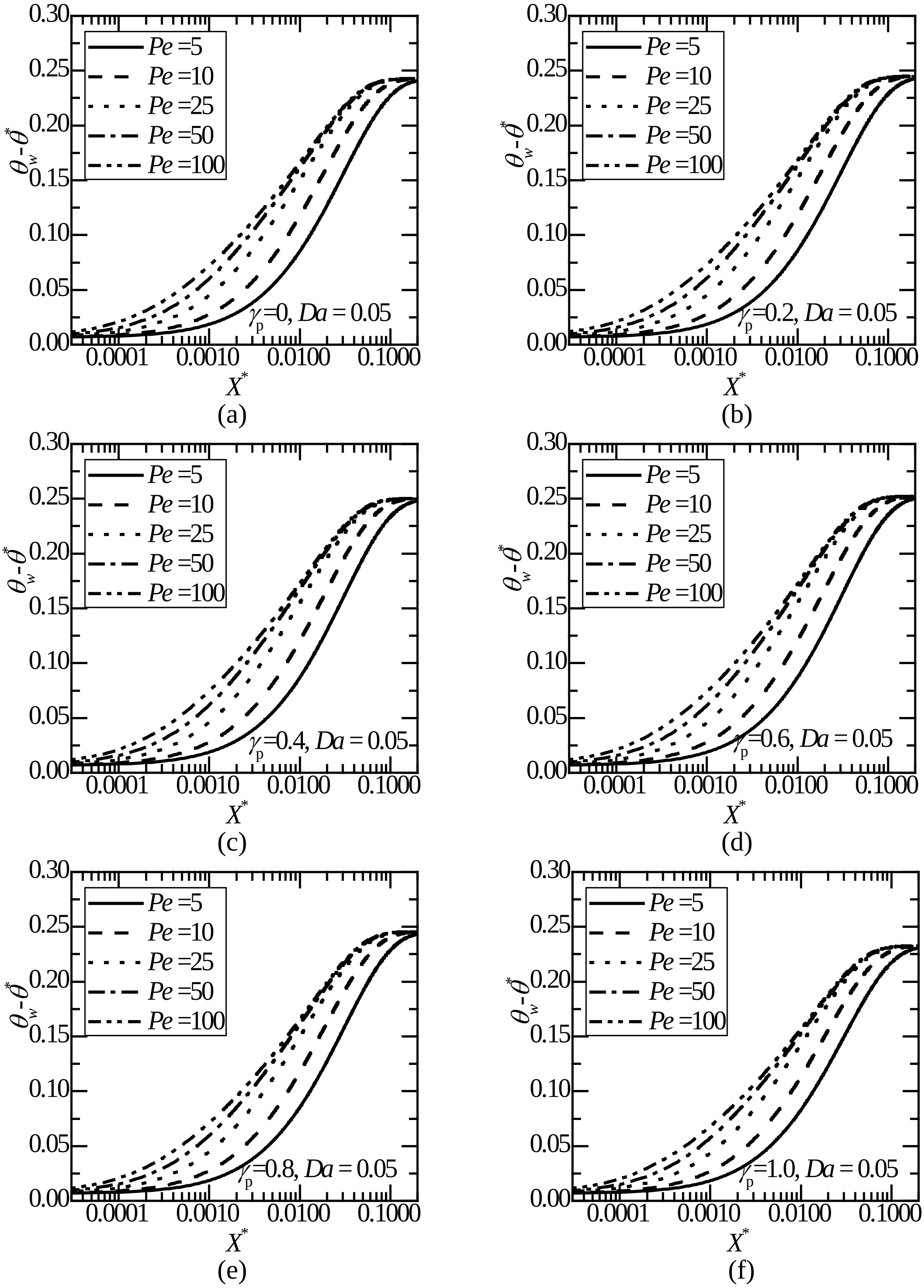
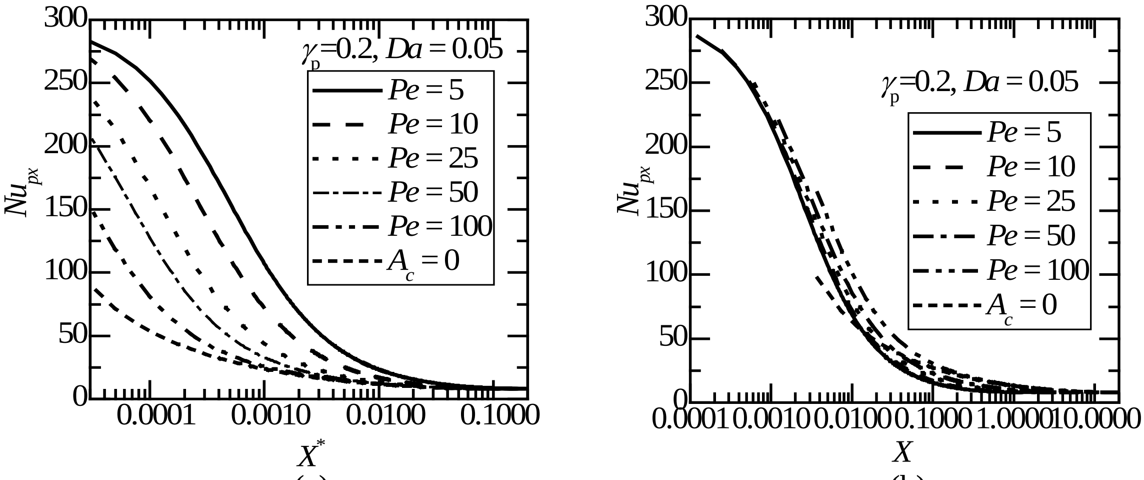


Fig.3: Variation of $\theta_w - \theta^*$ with X^* values for different Peclet numbers, *Pe* and Darcy number, *Da* = 0.05 for (a) $\gamma_p = 0$ (b) $\gamma_p = 0.2$ (c) $\gamma_p = 0.4$ (d) $\gamma_p = 0.6$ (e) $\gamma_p = 0.8$ and (f) $\gamma_p = 1.0$.

4.3 Local Nusselt number:

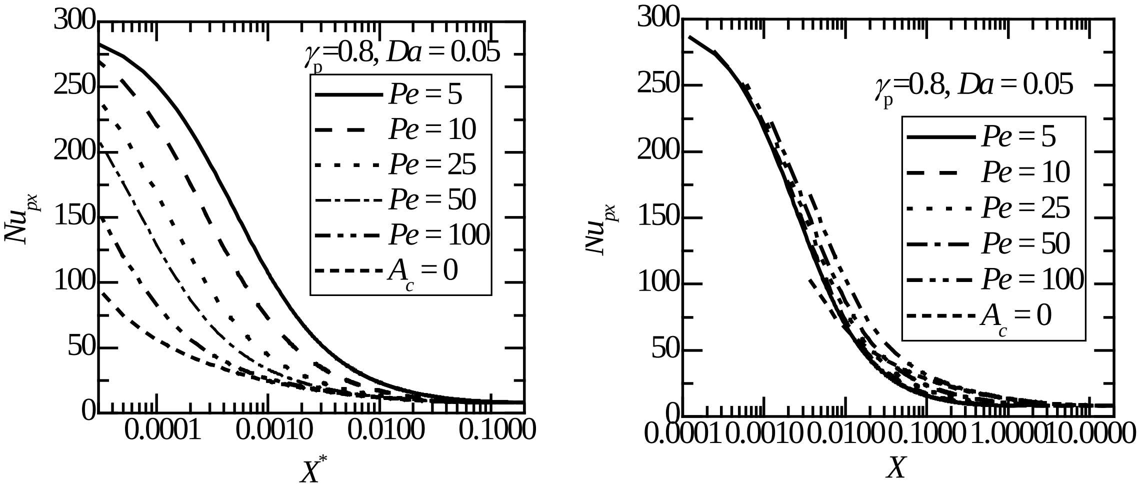
Effect of Axial Conduction

Variation of Nu_{px} vs. X^* and Nu_{px} vs. X are shown in Fig. 4(a) and 4(b) for different Peclet numbers Pe = 5, 10, 25, 50, 100 and A_c = 0 for Da = 0.05 and γ_p = 0.2. Variation of Nu_{px} vs. X^* and Nu_{px} vs. X are shown in Fig. 5(a) and 5(b) for different Peclet numbers Pe = 5, 10, 25, 50, 100 and A_c = 0 for Da = 0.05 and γ_p = 0.8. From Figs. 4 and 5, Nu_{px}, increases as Pe decreases at a fixed X^*, whereas, Nu_{px} decreases as Pe decreases at a fixed $X = X^*.Pe$. This feature is similar to that followed by clear fluid channel.



(a) (b)

Fig. 4: Variation of (a) Nu_{px} vs. X^* (b) Nu_{px} vs. X for different Peclet numbers, Pe and $\gamma_p = 0.2$ for Da = 0.05.



(a) (b)

Fig. 5: Variation of (a) Nu_{px} vs. X^* (b) Nu_{px} vs. X for different Peclet numbers, Pe and $\gamma_p = 0.8$ for Da = 0.05.

Comparison and Experimental Validation:

Table 1: Comparing present values of Nu_{px} for Peclet number, Pe =100 for clear fluid channel (γ_p = 0) with the literature values[10].

X^*	0.002	0.008	0.02	0.04	0.125	0.2	0.3	0.4
Present	20.732	12.859	10.063	8.832	8.249	8.236	8.235	8.235
Shah and London[10]	19.113	12.604	9.988	8.803	8.246	8.235	8.235	8.235

From the Table 1, the present values are found to be good agreement with literature values for γ_p = 0. Comparing for γ_p = 1.0 (fully filled with porous medium) with experimental values available in the literature [15 and 19].

To examine further, a plot of Nu_{px} with γ_p for different Da values at (a) X^* = 0.005, (b) X^* = 0.01, (c) X^* = 0.05 and (d) X^* = 0.1 for Pe = 5 is shown in Fig. 6. It is clear from Fig. 6, that the variation of Nu_{px} with Da depends on γ_p. Nu_{px} clearly increases as Darcy number increases when γ_p < 0.8, whereas, for γ_p > 0.8, Nu_{px} decreases as Darcy number increases. As Darcy number increases, Nu_{px}, decreases for γ_p = 1.0, becoming equal to the clear fluid channel value for large Darcy number, Da. This fact is observed in thesis of Bhargavi [20] for different channel geometry in Chapter 3. Also, Nu_{px} decreases as Pe increases with p γ_p, for all Da. Minimum value of Nu_{px} depends on Da but is independent of Pe and X^*.

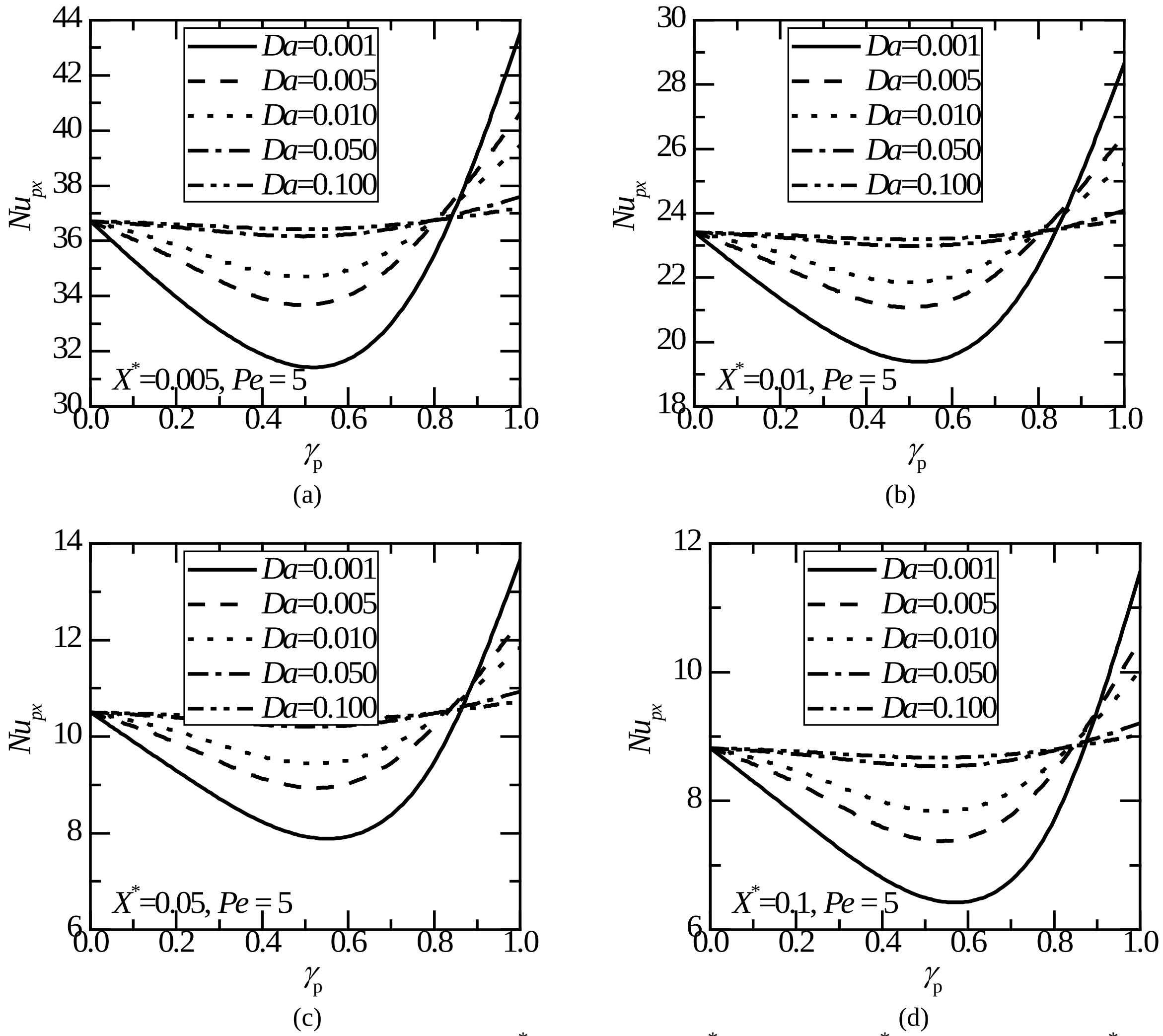


Fig. 6: Variation of Nu_{px} with γ_p at (a) $X^* = 0.005$ (b) $X^* = 0.01$ (c) $X^* = 0.05$ and (d) $X^* = 0.1$ for $Pe = 5$ at different Darcy numbers.

5. Conclusions:

Numerical solutions have been obtained for wide range of parameters, using SAR scheme ([13], [14] and [15]). It has been concluded that the non-dimensional temperature profiles become independent of Peclet number for $Pe \geq 100$ indicates that the effect of axial conduction has become negligible. The downstream condition satisfied, by the clear fluid ducts, $\partial\theta_b / \partial X^* \rightarrow 0$, has been found to be valid for channels partially filled with a porous material also. This feature assumes importance since the flow and thermal fields are not symmetric when channel is partially filled with a porous material. Dimensionless bulk mean temperature excess of wall temperature, $\theta_w - \theta^*$, increases as X^* increases. $\theta_w - \theta^*$ decreases as Peclet number decreases. This indicates that a stronger axial conduction effect present at lower Peclet numbers makes the fluid get less heated or less cooled compared to when neglecting axial conduction.

The values of the local Nusselt number when $\gamma_p = 0$ are found to be good agreement with the values available in [10]. Nu_{px} decrease as X^* increases for all γ_p and reach the fully developed values for $X^* \geq 0.4$. Similarly, Nu_{px} increases as Pe decreases for a given X^*. However, at a given X, Nu_{px} decreases as Pe decreases. For $Pe \geq 100$, the axial conduction effect becomes negligible except very near the entry. Nu_{px} attains a minimum almost independent of Peclet number and X^*. There exists an minimum porous fraction to attain low Nusselt number.

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