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Hall and Ion-slip effects on MHD free convection flow through an oscillatory porous medium with constant suction velocity and radiation

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Abstract: In this paper influence of Hall and Ion-slip current on MHD free convection flow through an oscillatory porous medium with constant suction velocity and chemical reaction in presence of radiation has been investigated. An analytical solution of the governing equations illustrating the flow is attained by the Perturbation method. The effects of various physical parameters on velocity, temperature and concentration fields are presented graphically. In this paper it was conclude that as velocity, temperature and concentration declined with the rise in Hall, Ion-slip current, radiation, chemical reaction, Prandtl number and Schmidt number.

***Keywords*:** Prandtl number, Hall, Ion-slip current, Perturbation method.

1. Introduction

Due to versatile technological as well as manufacturing applications, it is great understand to examine the MHD flow. The foremost objective of MHD principles is to interrupt the flow field in a required direction by fluctuate the formation of the boundary layer. Thus, with the intention of modify the flow kinematics; the idea to execute MHD seems to be more flexible and reliable. In pharmaceutical as well as environmental science, MHD has been playing a essential role in the application of fluid dynamics and medical sciences, owing to its implications in chemical fluids as well as metallurgical fields. Rudraiah [1] analyzed hydro magnetic free convection flow through a porous medium between two parallel plates. In this paper, it was found that velocity as well as temperature diminished with rise in Darcy dissipations and porous parameter. Abdul Hakeem [2] considered analytical solutions for two-dimensional oscillatory flow on free convective radiation through a highly porous medium bounded by an infinite perpendicular plate. Hamza et al. [3] in paper it was observed that the velocity declined with the reduced in magnetic field, porous parameter and Grashof number but reverse effect was shown in case of slip parameter. In this investigation, the governing equations are solved analytically by using Perturbation method. M.A. El-Hakiem et al. [4] and Makinde [5] examined constant suction velocity on MHD free convection oscillatory flow on radiation through a porous medium.. Raptis [6] analyzed effect of non-constant 2D free convective flow throughout the motion of a viscous incompressible fluid through a very much porous medium. Gholizadeh [7] and Swati [8] analyzed the thermal and mass diffusion effects on MHD oscillatory flow past a vertical porous plate through a porous medium in the presence of heat source**.** In the above all investigation Hall and Ion lip current was not taken into considered.

2. Mathematical formulation and solution of the problem:

Consider two-dimensional, unsteady, free convection with thermal radiation flow of a viscous, incompressible and electrically conducting fluid through a highly porous medium which is bounded by a vertical infinite plane surface under the influence of a transverse magnetic field. The fluid is supposed to be a gray, absorbing emitting but non-scattering medium. The x^*-axis is taken along the plane surface with a direction opposite to the direction of gravity and y^* -axis is taken to be normal to the surface. The physical variables are functions of y^* and the time t^* only. The radiate heat flux in the x^*- direction is considered negligible in comparison with that in y^* - direction. Hall and Ion slip current and heat source parameter has been taken into consideration. The magnetic Reynolds number of the flow is taken to be small enough, so that the induced magnetic field can be neglected. The homogeneous chemical reaction is of first order with rate constant K_r between the diffusing species and the fluid is considered. Therefore, the equation expressing the conservation of mass, momentum and energy within a concentration boundary layer are given by

Equation of continuity:

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$\Rightarrow v^* = -v_0 \quad (2)$$

Equation of Momentum:

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{-1}{\rho}\frac{\partial p^*}{\partial y^*} + \vartheta \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T^*_\infty) + g\beta^*\left(C^* - C^*_\infty\right) - \frac{\vartheta}{k^*}\left[u^*\right] - \frac{B_0^2\sigma_e\left[\alpha_e u^* + \beta_e w^*\right]}{\rho\left[\alpha_e^2 + \beta_e^2\right]} \quad (3)$$

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} = \vartheta\left[\frac{\partial^2 w^*}{\partial y^{*2}}\right] - \frac{\vartheta}{k} w^* - \left[\frac{B_0^2\sigma_e}{\rho\left(\alpha_e^2 + \beta_e^2\right)}\right]\left(\beta_e\left(v_1^* - u^*\right) - \alpha_e w^*\right) \quad (4)$$

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} = \frac{dv_1^*}{dt^*} + \frac{\vartheta}{k} v_1^* + \frac{\sigma_e}{\rho\left(\alpha_e^2 + \beta_e^2\right)} B_0^2 v_1^* \quad (5)$$

Equation of Energy:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \alpha\left(\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{k_0}\frac{\partial q_r^*}{\partial y^*}\right) + \frac{Q_0}{\alpha}(T^* - T^*_\infty) \quad (6)$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_r(C^* - C^*_\infty) \quad (7)$$

The related boundary conditions are

$$\left.\begin{array}{l} at\ \ y^* = 0: \quad u^* = 0, \quad w^* = 0, \quad T^* = T^*_\infty, \quad C^* = C^*_\infty \\ As\ \ y^* \to \infty: \ u^* \to v_1^* = U_0(1 + e^{i\omega t^*}),\ w^* \to 0\ T^* = T^*_\infty, C^* = C^*_\infty, \end{array}\right\} \quad (8)$$

From the equation (2) and (4) then we get

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{dv_1^*}{dt^*} + \vartheta\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T^*_\infty\right) + \frac{\vartheta}{K^*}\left(v_1^* - u^*\right) - \frac{B_0^2\sigma_e}{\rho\left(\left(1+\beta_i\beta_e\right)^2 + \beta_e^2\right)}\left(\left(1+\beta_i\beta_e\right)\left(v_1^* - u^*\right) + \beta_e w^*\right) \quad (9)$$

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} = \vartheta\left[\frac{\partial^2 w^*}{\partial y^{*2}}\right] - \frac{\vartheta}{k} w^* - \frac{B_0^2\sigma_e}{\rho\left(\left(1+\beta_i\beta_e\right)^2 + \beta_e^2\right)}\left(\beta_e\left(v_1^* - u^*\right) - \left(1+\beta_i\beta_e\right)w^*\right) \quad (10)$$

$$q_r^* = -\frac{4\sigma}{3k_1}\frac{\partial T^{*4}}{\partial y^*} \quad (11)$$

Now Non-dimensional quantities are defined as

$$\left.\begin{array}{l} f = \dfrac{u^*}{U_0}, g = \dfrac{w^*}{U_0}\ y = \dfrac{V_0 y^*}{\vartheta}, V_1 = \dfrac{v_1^*}{U_0}, t = \dfrac{V_0^2 t^*}{\vartheta}, U = \dfrac{U^*}{U_0} \\ K = \dfrac{K^*\vartheta^2}{V_0^2}, T^* = T^*_\infty + \theta\left(T^*_w - T^*_\infty\right), C^* = C^*_\infty + C\left(C^*_w - C^*_\infty\right) \end{array}\right\} \quad (12)$$

After substituting the boundary conditions and non-dimensional variables in the governing equations (3), (4), (6) & (7) then we get,

$$\frac{\partial f}{\partial t}-\frac{\partial f}{\partial y}=\frac{dv_1}{dt}+\frac{\partial^2 f}{\partial y^2}-\frac{B_0^2\sigma_e\vartheta}{V_0^2\rho\left((1+\beta_i\beta_e)^2+\beta_e^2\right)}\left((1+\beta_i\beta_e)(v_1-f)+\beta_e g\right)+G_r\theta+G_m C+\frac{1}{k}(v_1-f) \quad (13)$$

$$\frac{\partial g}{\partial t}-\frac{\partial g}{\partial y}=\frac{\partial^2 g}{\partial y^2}-\frac{1}{k}g-\frac{B_0^2\sigma_e\vartheta}{V_0^2\rho\left((1+\beta_i\beta_e)^2+\beta_e^2\right)}\left(\beta_e(v_1-f)-(1+\beta_i\beta_e)g\right) \quad (14)$$

$$\frac{\partial\theta}{\partial t}-\frac{\partial\theta}{\partial y}=(\Pr)^{-1}\left(\left(1+\frac{4R}{3}(\theta+\phi)^3\right)\frac{\partial^2\theta}{\partial y^2}+4R(\theta+\phi)^2\left(\frac{\partial\theta}{\partial y}\right)^2\right)+\eta\theta \quad (15)$$

$$\frac{\partial C}{\partial t}=(Sc)^{-1}\frac{\partial^2 C}{\partial y^2}-K_r C \quad (16)$$

The boundary conditions are

$$\left.\begin{array}{ll} At\ y=0 & f=0,\ g=0,\ \theta=1, C=1 \\ As\ y\to\infty & f=\left(1+\varepsilon e^{i\omega t}\right),\ g\to 0, \theta\to 0, C\to 0 \end{array}\right\} \quad (17)$$

Take $F=f+i\,g$ (18)

$$\frac{\partial F}{\partial t}-\frac{\partial F}{\partial y}=\frac{dv_1}{dt}+\frac{\partial^2 F}{\partial y^2}+G_r\theta+N(v_1-F) \quad (19)$$

$$\left.\begin{array}{l} G_r=\frac{\vartheta\beta g\left(T_w^*-T_\infty^*\right)}{V_0^2 U_0}, M=\frac{\sigma B_0^2}{\rho V_0^2}, \eta=\frac{\vartheta Q_0}{V_0^2\alpha}, K_r=\frac{k_1\vartheta}{V_0^2}, \Pr=\frac{\vartheta}{\alpha}, R=\frac{4\sigma\left(T_w^*-T_\infty^*\right)^3}{k_0 k_1}, \\ \phi=\frac{T_\infty^*}{\left(T_w^*-T_\infty^*\right)} N=\left[\frac{1}{k}+\frac{M}{\left((1+\beta_i\beta_e)^2+\beta_e^2\right)}\left((1+\beta_i\beta_e)+i\beta_e\right)\right], Sc=\frac{\vartheta}{D} \end{array}\right\} \quad (20)$$

So as to solve the deferential equation (15), (16) & (19) it was suppose that

$$\left.F=F_0(y)+\varepsilon e^{i\omega t}F_1(y)......\theta=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y).........C=C_0(y)+\varepsilon e^{i\omega t}C_1(y)\right\} \quad (21)$$

From equation (15), (16), (19) & (21) then we get;

$$F_0''+F_0'-NF_0=-G_r\theta_0-G_m C_0-N \quad (22)$$

$$F_1''+F_1'-(N+i\,\omega)F_1=-(N+i\,\omega)-G_r\theta_1-G_m C_1 \quad (23)$$

$$\left[1+\frac{4}{3}R(\theta_0+\phi)^3\right]\theta_0''+4R(\theta_0+\phi)^2(\theta_0')^2+\Pr\theta_0'+\Pr\eta\,\theta_0=0 \quad (24)$$

$$\left.\begin{array}{l} \left[1+\frac{4}{3}R(\theta_0+\phi)^3\right]\theta_1''+8R(\theta_0+\phi)^2(\theta_0')^2\theta_1+8R(\theta_0+\phi)^2\theta_0'\theta_1'+\Pr\theta_1' \\ \qquad +4R(\theta_0+\phi)^2\theta_0''\theta_1-i\omega\Pr\theta_1+\Pr\eta\theta_1=0 \end{array}\right\} \quad (25)$$

$$C_0''-S_C K_r C_0=0 \quad (26)$$

$$C_1''-Sc(K_r+n)C_1=0 \quad (27)$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} F_0=0, F_1=0, \theta_0=1, \theta_1=0, C_0=1, C_1=0, \qquad at\ y=0 \\ F_0=1, F_1=1, \theta_0\to 0, \theta_1\to 0, C_0\to 0, C_1\to 0\ as\ y\to\infty \end{array}\right\} \quad (28)$$

Initially solve equation (25) &equation (26) then we obtain,

$$C_0=e^{-\left(\sqrt{ScKr}\right)y} \quad \& \quad C_1=0 \quad (29)$$

If we suppose that the radiation parameter R to be small, we expand the velocity and temperature as

$$\left.\begin{array}{l} F_0=F_{01}(y)+R\ F_{02}(y)........F_1=F_{11}(y)+R\ F_{12}(y) \\ \theta_0=\theta_{01}(y)+R\ \theta_{02}(y)..........\theta_1=\theta_{11}(y)+R\ \theta_{12}(y) \end{array}\right\} \quad (30)$$

From the equation (22)-(25) & equation (30) then we obtain:

$$F_{01}'' + F_{01}' - NF_{01} = -Gr\theta_{01} - G_m C_0 - N \quad (31)$$

$$F_{02}'' + F_{02}' - NF_{02} = -Gr\theta_{02} \quad (32)$$

$$F_{11}'' + F_{11}' - (N + i\omega)F_{11} = -(N + i\omega) - Gr\theta_{11} - G_m C_1 \quad (33)$$

$$F_{12}'' + F_{12}' - (N + i\omega)F_{12} = -Gr\theta_{12} \quad (34)$$

$$\theta_{11}'' + \Pr\theta_{11}' + \left(\eta - i\omega\right)\Pr\theta_{11} = 0 \quad (35)$$

$$\theta_{01}'' + \Pr\theta_{01}' + \Pr\eta\theta_{01} = 0 \quad (36)$$

$$\theta_{02}'' + \frac{4}{3}(\theta_{01} + \phi)^3\theta_{01}'' + 4(\theta_{01} + \phi)^2(\theta_{01}')^2 + \Pr\theta_{02}^1 + \Pr\eta\ \theta_{02} = 0 \quad (37)$$

$$\theta_{12}'' + \frac{4}{3}(\theta_{01} + \phi)^3\theta_{11}'' + 8(\theta_{01} + \phi)^2(\theta_{01}')^2\theta_{11} + 8(\theta_{01} + \phi)^2\theta_0^1\theta_{11}^1 + \Pr\theta_{12}' + 4(\theta_{01} + \phi)^2\theta_0''\theta_{11} + \Pr\left(\eta - i\omega\right)\theta_{12} = 0 \quad (38)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &F_{01} = 0, F_{02} = 0, \theta_{01} = 1, \theta_{02} = 0, F_{11} = 0, F_{12} = 0\,\theta_{11} = 0, \theta_{12} = 0 \quad at\ y = 0 \\ &F_{01} = 1, F_{02} = 1, \theta_{01} = 0, \theta_{02} = 0, F_{11} = 1, F_{12} = 0\,\theta_{11} = 0, \theta_{12} = 0 \quad at\ y \to \infty \end{aligned}\right\} \quad (39)$$

Solve (31) – (38) subject to boundary condition (39), & (29) we get velocity, temperature and concentration

$$F = \left(A_1\ e^{-R_3 y} + N_1 e^{-R_2 y} + N_{10} e^{-l\ y} + 1\right) + R\begin{pmatrix} A_3\ e^{-R_3 y} + N_6 e^{-R_2 y} + N_7 e^{-4R_2 y} \\ + N_8 e^{-3R_2 y} + N_9 e^{-2R_2 y} \end{pmatrix} + \varepsilon e^{i\omega t}\left(e^{-R_4 y}\right) \quad (40)$$

$$\theta = e^{-R_2 y} + R\left(A_2\ e^{-R_2 y} + N_2 e^{-4R_2 y} + + N_3 e^{-R_2 y} + N_4 e^{-3R_2 y} + N_5 e^{-2R_2 y}\right) \quad (41)$$

$$C = e^{-\left(\sqrt{ScKr}\right)y} \quad (42)$$

2. Results and Discussion

Fig 1: Represented that the behaviour of velocity for dissimilar estimators of Hall parameter β_e. From this figure it was found that the enhancement of various values of hall parameter it leads to reduced in velocity and it is very near to the plate. Due to the production of an extra prospective dissimilarity transverse to the direction of accumulate free charge and applied magnetic field among the opposite surfaces induces an electric current vertical to both the fields, magnetic as well as electric. Fig 2: Reflects that the velocity diminished owing to enhancement of diverse values of

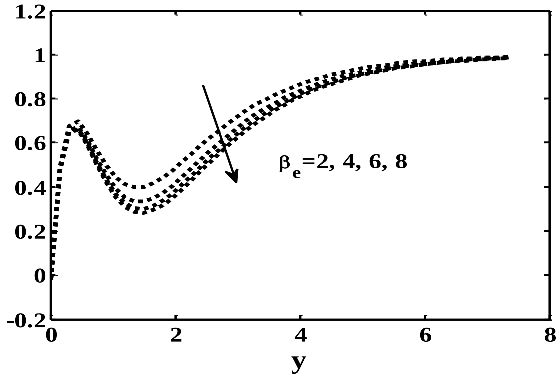

Fig 1: Effect of β_e on Velocity

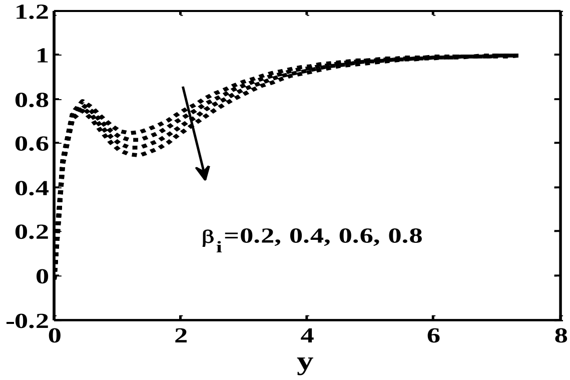

Fig 2: Effect of β_i on Velocity

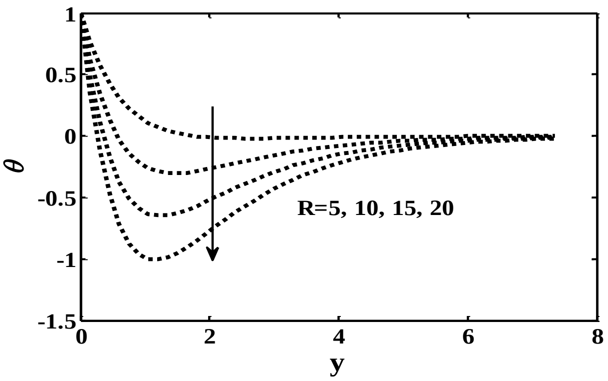

Fig 3: Effect of *R* on Temperature

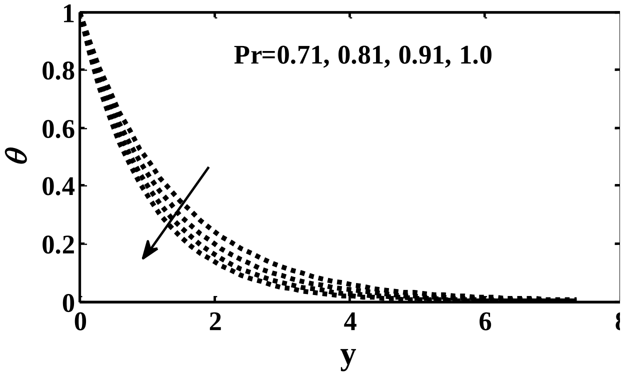

Fig 4: Effect of *Pr* on Temperature

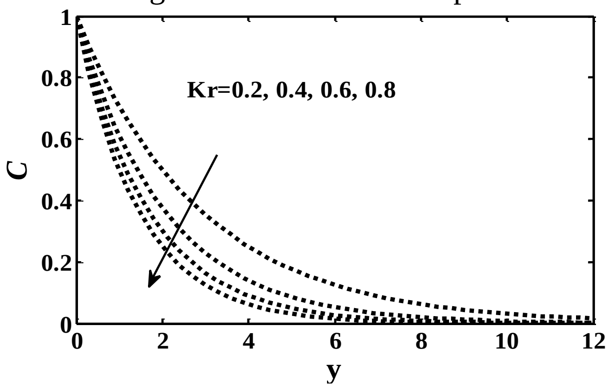

Fig 5: Effect of *Kr* on Concentration

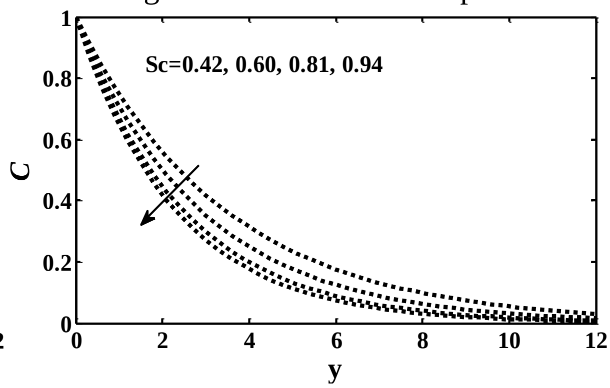

Fig 6: Effect of *Sc* on Concentration

Ion-slip parameter β_i owing to the fact that the β_i reduced the resistive force inflict by magnetic field. Fig 3: Illustrated that for dissimilar values of radiation parameter rises that it leads to declined in temperature. Fig 4: Established that for dissimilar values of Prandtl number (*Pr*) rises then it leads to reduced in temperature. This is owing to the reality that a higher Prandtl number fluid has comparatively low thermal conductivity, which diminishes the conduction as an outcome temperature reduces. As a result higher Prandtl number *Pr* show the way to more rapidly cooling of the plate. Fig 5: Demonstrate the performance of concentration for disparate estimators of chemical reaction (*Kr)*. The results obtain from this figure it was perceived that the concentration reduced due to rise in chemical reaction parameter. For the reason that the chemical reaction reinforce momentum transfer and consequently accelerates the flow. For incongruent estimators of the Schmidt number on the fluid concentration is exposed in the Fig 6: from this figure the outcomes indicates that the enhancement of *Sc* leads to diminished in concentration. This causes the influence of concentration buoyancy to diminished, yielding a decline in the velocity. The depletion in the concentration is accompanied by instantaneous depletion in the concentration boundary layers, which is perceptible from the Fig 6.

4. Conclusion:

- The velocity distribution declined with the enhancement of Hall and Ion-slip parameter.
- The temperature profile is reduced with the rise in Radiation parameter and Prandtl number
- The concentration diminished with amplifies in chemical reaction parameter and Schmidt number.

1. References:

[1] N.Rudraiah, S.T. Nagaraj, (1977)Natural convection through vertical porous stratum, International Journal of Engineering Science, 15(3), 589-600.

[2] A.K. Abdul Hakeem., K. Sathiyanathan, (2009). An analytic solution of an oscillatory flow through a porous medium with radiation effect. Nonlinear Analysis: Hybrid Systems, 3 (1), 288-295.

[3] M. M. Hamza., B.Y Isah., H. Usman., (2011). Unsteady Heat Transfer to MHD Oscillatory Flow through a Porous Medium under Slip Condition. International Journal of Computer Applications. 33 (4), 12-17.

[4] M.A. El-Hakiem., (2000) MHD oscillatory flow on free convection-radiation through a porous medium with constant suction velocity. Journal of Magnetism and Magnetic Materials, 220 (2), 271-276.

[5] O.D. Makinde, (2005) Free-convection flow with thermal radiation and mass transfer past a moving porous plate, International. Communication Heat Mass Transfer, 32 (1), 1411-1419.

[6] Raptis A. C.P. Perdikis (1985), Oscillatory flow through a porous medium by the presence of free convective flow, International Journal of Engineering Science, 23(2), 51-55.

[7] Gholizadeh, (1990), MHD oscillatory flow past a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source, Astrophysics Space Science, 174, 303-310.

[8] Swati Mukhopadhyay (2013), MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. Alexandria Engineering Journal, 52(1), 259–265.

Nomenclature

x^*, y^*	Coordinate axis along the plate*(m)*, Co-ordinate axis normal to the plate*(m)*
B_0	Magnetic induction *(A.m^{-1})*
T	Temperature of the fluid *(K)*
T_w	Fluid temperature at walls *(K)*
T_w^*	Fluid temperature at the wall *(K)*
T_∞	Dimensional free stream temperature *(K)*
u	components of velocity vector in x direction(m.S^{-1})
v	Velocity component in y direction *(m.S^{-1})*
g^*	acceleration due to gravity *(m.S^{-2})*
η	Heat generation parameter
Pr	Prandtl number
q_r^*	Radiation heat flux density *(W.m^{-2})*
ω	Frequency of vibration of the fluid
t^*	Dimensional time *(S)*
M	Magnetic parameter
K	Permeability of porous medium
Gr	local temperature Grashof number
β	Spin gradient viscosity (K^{-1})
μ	Fluid dynamic viscosity
ρ	Density of the fluid *(kg.m^{-3})*
R	The radiative parameter
ϕ	Temperature difference parameter
β_i,	Ion-slip parameter
β_e	Hall parameter