Bingham Plastic Fluid Film Lubrication between Two Parallel Plates with Temperature Effect

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ABSTRACT

Hydrodynamic lubrication of two parallel plates, both are at rest has been considered in this work. It describes the theoretical analysis of Bingham Plastic fluid model between the plates. The equation of motion, continuity equation and energy equations are solved simultaneously by assuming the plates are of same temperature. A semi analytical solution has been developed and the velocity and temperature profile has been discussed and represented with respect to different lubricant parameter.

Keywords: Parallel plates, Hydrodynamic lubrication, Bingham plastic, Temperature effect.

1. INTRODUCTION

Thermo hydrodynamic lubrication (THD) analyses describes the performance of journal bearings with thermal effects in the lubrication process. Since it can naturally yield the peak bearing temperature, then the bearing failure can be predicted at the design stage when the maximum temperature exceeds a certain limit. The research into THD lubrication has drawn research effort. For example, Ferron [1] solved Dowson's [2] generalized Reynolds equation simultaneously with the energy equation and reported excellent results. Khonsari and Beaman [3] obtained THD solutions under various boundary conditions considering the mixing of the recirculating fluid and the supply oil.

On the line of non-Newtonian fluid model, Power law lubricant model has got attention in the recent years because of its simplicity and potential to describe many lubricants such as silicon fluids, polymer solutions [4]. Dein and Elrod [5] examined the analysis of lubrication of journal bearing with the same non-Newtonian fluid model and developed a new numerical technique based on perturbation expansion for velocity under Couette dominated flow condition.

Since the use of ideal fluid to the present development of the science and technology known as rheology, the limitations forced on the quantity of factors influencing the dissemination of shear stress in a given fluid flow have been loose methodically. In this connection the traditional hydrodynamics have gotten new consideration.

These cases may not compare to a particular instance of commonsense intrigue. The chief idea behind these examinations is that the huge physical highlights of the stream will emerge obviously, unobscured by a numerical brush, and that the outcomes so gotten will fill in as a decent guess to the physical model of enthusiasm for anticipating the subjective liquid conduct, and will manage one in confronting more confounded circumstances [6].

The investigation of the movement of non-Newtonian fluids in the nonappearance and additionally within the sight of an attractive field has applications in numerous regions. In this problem Bingham plastic fluid model has been considered to analyze the velocity profile and temperature of the lubricant between the two parallel plates.

2. MATHEMATICAL FORMULATION

The governing equations of the fluid flow under some usual assumptions are [7, 8, 12]

$$\frac{dp}{dx} = \frac{d\tau}{dy} \tag{1}$$

$$\nabla . u = 0 \tag{2}$$

where $\tau = \tau_0 + \mu \dfrac{du}{dy}$

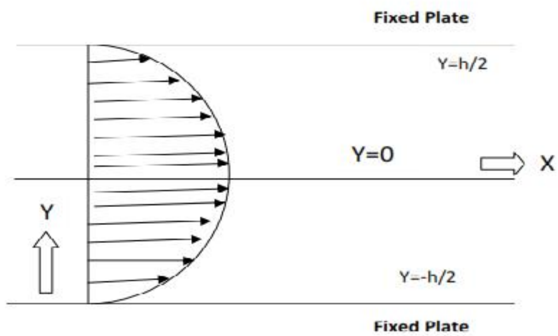

Fig.1: Velocity distribution of the fluid flow between two stationary plates.

The boundary conditions are

$$\left.\begin{aligned} & u = 0 \;\; at \;\; y = -h/2 \\ & u = 0 \;\; at \;\; y = h/2 \\ & \frac{du}{dy} = 0 \;\; at \;\; y = 0 \end{aligned}\right\} \tag{3}$$

The velocity profiles for the given region are

$$\frac{du}{dy} \geq 0, \quad -\frac{h}{2} < y < 0$$

$$\frac{du}{dy} \leq 0, \quad 0 < y < \frac{h}{2}$$

The velocity of the fluid can be obtained by solving equation (1) under the given boundary conditions

$$u_1 = \left(\frac{1}{\mu}\frac{dp}{dx}\right)\left(\frac{y^2}{2} - \frac{h^2}{8}\right), \quad -\frac{h}{2} < y < 0 \tag{4}$$

Similarly, 2

$$u_2 = \left(\frac{1}{\text{«}}\frac{dp}{dx}\right)\left(\frac{y^2}{2} - \frac{h^2}{8}\right) \;, \quad 0 < y < \frac{h}{2} \tag{5}$$

The velocity distribution across a section of the parallel plate is parabolic in nature as shown in Fig.1 and «, $\frac{dp}{dx}$, and 'h' are constants.

Also the velocity is same in both the region as it is symmetric. Also the velocity of the fluid is maximum at y=0. [ ]

$$u_{\max} = \frac{1}{8}\left(-\frac{h^2}{\text{«}}\frac{dp}{dx}\right) \tag{5.1}$$

Now, the total discharge of the fluid is given by $Q = \int_{-h/2}^{h/2} u \, dy$

$$Q = \left(-\frac{1}{\text{«}}\frac{dp}{dx}\right)\left(\frac{h^3}{12}\right) \tag{6}$$

From the continuity equation one can get $\frac{\partial Q}{\partial x} = 0$

$$\frac{dp}{dx} = \text{«}\left(c + \frac{h^3}{12}\right) \tag{7}$$

3. THE ENERGY EQUATION

$$k\left(\frac{d^2T}{dy^2}\right) + {}^{2}\left(\frac{du}{dy}\right) = 0 \tag{8}$$

Solving above equation with the following boundary conditions

$T_1 = T_l \; at \; y = -h/2 \; and \; T_2 = T_l \; at \; y = h/2$, this shows that both plates are with the same temperature.

Also by using interface temperature condition $T_1 = T_2 \; at \; y = 0$ and interface temperature gradient $\frac{dT_1}{dy} = \frac{dT_2}{dy} \; at \; y = 0$.

$$T = \frac{{}^{2}{}_{0}}{\text{«}}\frac{y^2}{2} + \frac{\text{«}}{k}u + cy + d \tag{9}$$

Let T_m denotes the temperature at the middle of the channel, i.e. $T_1 = T_2 = T_m \; at \; y = 0$ at y=0

$$T_m - T_l = \frac{{}^{2}{}_{0}h^2}{8\text{«}} + \left(\frac{1}{k}\frac{dp}{dx}\right)\left(\frac{y^2}{2} - \frac{h^2}{8}\right) \tag{10}$$

This result is same for both the regions $-h/2 < y < 0$ and $0 < y < h/2$.

4. RESULT AND DISCUSSION

The following values are assumed for numerical computation [13].

${}^{2}{}_{0} = 0.1, \; y/h = 1, \; \text{«} = 0.001, \; T_l = 0.4, \; p = 1, 2$

The velocity of the fluid 'u' increases with y/h in the lower region, and decreases with y/h in the upper region for different values of P as shown in Fig. 2 and Fig.3. Further, the temperature T_m of the fluid increases with y/h in the lower region, and decreases with y/h in the upper region as shown in Fig. 4. This result is in good agreement with the previous findings of [9-11]. The temperature profile presented in Fig. 5 is taken when lower plate is adiabatic and upper plate is at constant temperature.

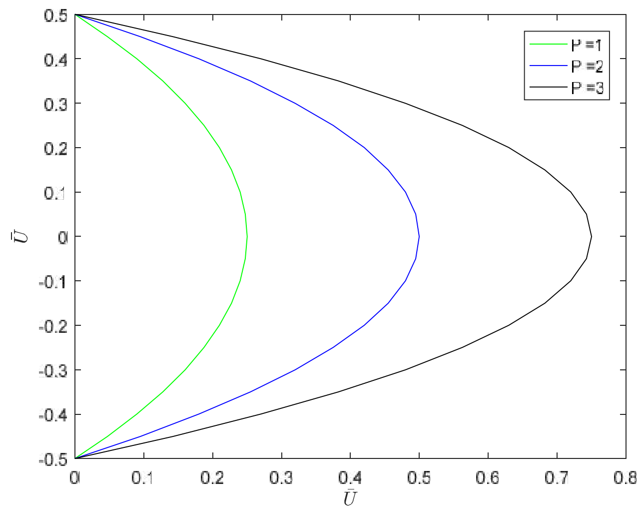

Fig. 2: Velocity profile for Newtonian case

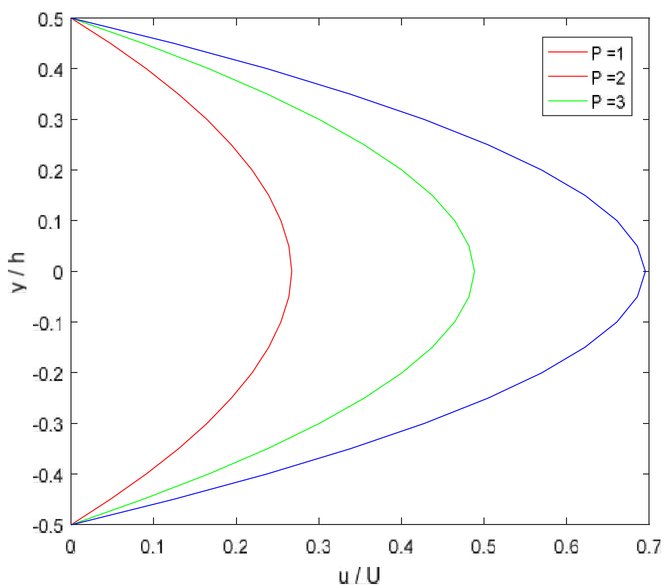

Fig.3: Velocity profile for non- Newtonian case

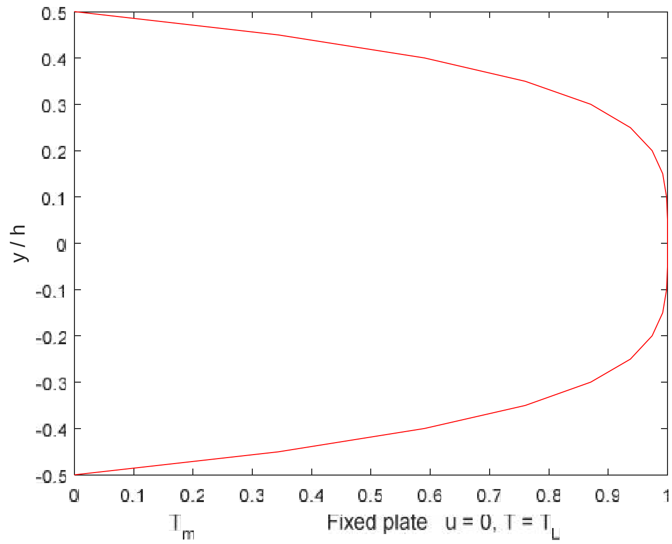

Fig.4: Temperature profile when two plates having same temperature

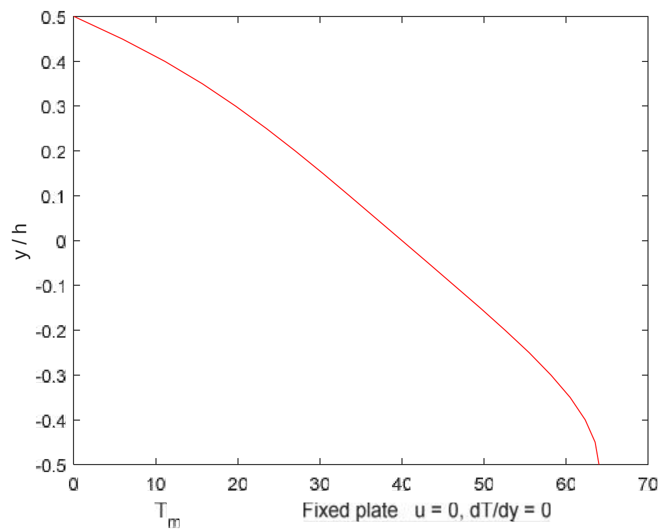

Fig. 5: Temperature profile when lower plate is adiabatic and upper plate is at constant temperature

5. CONCLUSION

The steady state Bingham plastic fluids flow between two parallel plates is considered throughout the problem with incompressibility, total discharge of fluid, average velocity, shear stress and pressure head loss, velocity distributions of fluids. This problem deals with the Poiseuille flow between two plates at rest mentioning maximum velocity and ratio of maximum velocity to average velocity. The velocity distributions of fluids as well as temperature distribution are calculated mathematically and are plotted using MATLAB under constant and zero pressure gradients.

6. REFERENCES

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