

## **Natural convection flow in a vertical channel with inclined magnetic field and Soret effects**

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The thermal diffusion and inclined magnetic field effects on free convection flow through a channel are investigated. This study includes the influence of hall current also. Homotopy Analysis Method (HAM) is applied to get solution of the dimensionless governing equations those were transformed by similarity transformations from the system of governing partial differential equations. Influence of all the emerging numbers of this study on all the profiles were presented through plots.

**Keywords:** Natural Convection, Soret Effect, Inclined Magnetic field.

### **1. Introduction**

Natural convection flow between vertical parallel plates with heat and mass transfer has a remarkable significance in various fields. The significance and developments of heat and mass transfer have been addressed many researchers [1-3]. In view of applications, Umavathi et al. [4] studied the variable viscosity and thermal conductivity effects on free convection flow of a viscous fluid between vertical parallel plates. Most recently, Kaladhar and Komuraiah [5] considered the free convection flow between vertical parallel plates with Navier slip, soret and Dufour effects.

Many researchers considered the magnetic field vertical to the plates. But, in many realistic circumstances such magnetic field may not suitable always. Also, the influence of Hall current becomes most significant mechanism for the electrical conduction in ionized gases and plasmas when the applied magnetic field is strong. The importance and past contributions can be seen in [6-9]. Recently, Ajaz and Elangovan [10] reported the impact of an inclined magnetic field in an inclined channel saturated with peristaltic flow of a couple stress fluids. Most recently, Mishra and Sharma [11] examined the effects of inclined magnetic field with the hall current on mixed convection flow in a revolving channel.

The Soret effect encountered in many areas for instance geosciences and chemical engineering, etc., For the significance and past literature one can refer in [12-14]. In view of the importance most recently, Motsa et al. [15] used new bivariate pseudo-spectral local linearization method to find the Soret effect on natural convection over the vertical frustum of a cone in a nano fluid.

In this paper, the impact of hall current, inclined magnetic field and diffusion thermo on natural convection flow through a vertical channel has been examined. The survey clearly shows that the present study has not been reported elsewhere. Due to the significance, the authors are enthused to come with this study. Homotopy analysis method (HAM) [16-17] has been applied to solve this problem. Then the nature of all the profiles against to the pertinent dimensionless numbers of this paper has been presented through graphs.

### **2. Mathematical modelling**

Three-dimensional free convection flow in a channel has been considered. Flow constitution is explained in Fig. 1. A constant external magnetic field  $B_0$  is applied in the direction which makes an

angle  $\alpha$  with the positive direction of  $x$ -axis. All the properties of the fluid are chosen to be fixed with the exclusion that the density in the buoyancy term. All the flow variables are functions of  $y$  only; this is because of the plates extended infinitely. The governing three-dimensional steady flow equations are in the form:

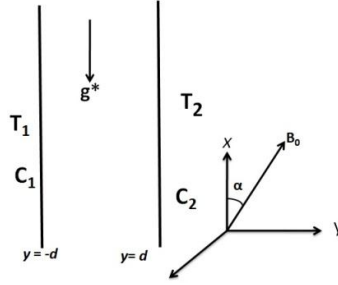


Figure 1: Geometry of the problem.

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} + \rho g^* [\beta_T (T - T_1) + \beta_C (C - C_1)] - \frac{\sigma B_0^2 \cos \alpha}{1 + m^2 \cos^2 \alpha} [u \cos \alpha - v_0 \sin \alpha + m w \cos^2 \alpha] \quad (1)$$

$$\rho v_0 \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2 \cos^2 \alpha}{1 + m^2 \cos^2 \alpha} [m u \cos \alpha - w - m v_0 \sin \alpha] \quad (2)$$

$$\rho C_p v_0 \frac{\partial T}{\partial y} = K_f \frac{\partial^2 T}{\partial y^2} + 2\mu \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \quad (3)$$

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

With

$$u(-d) = w(-d) = 0, T(-d) = T_1, C(-d) = C_1, u(d) = w(d) = 0, T(d) = T_2, C(d) = C_2 \quad (5)$$

where  $u, v, w$  are the velocities in  $x, y$  and  $z$  directions respectively.  $T_m, C_p, K_f, \rho, D, \mu, \beta_T, g^*, \beta_C$  are the mean fluid temperature, specific heat, thermal diffusion ratio, density, mass diffusivity, coefficient of viscosity, coefficient of thermal expansion, acceleration due to gravity and coefficient of solutal expansion respectively.

Introducing the following dimensionless variables

$$y = \eta d, f = \frac{d}{\gamma Gr} u, g = \frac{d}{\gamma Gr} w, T - T_1 = (T_2 - T_1) \theta, C - C_1 = (C_2 - C_1) \phi \quad (6)$$

in Eqs. (1) - (5), dimensionless equations of the governing system obtained as

$$f'' - Re f' + \theta + N \phi - \frac{Ha^2 \cos \alpha}{1 + m^2 \cos^2 \alpha} \left[ f \cos \alpha - \frac{Re}{Gr} \sin \alpha + m g \cos^2 \alpha \right] = 0 \quad (7)$$

$$g'' - Re g' + \frac{Ha^2 \cos \alpha}{1 + m^2 \cos^2 \alpha} \left[ m f \cos \alpha - g - \frac{m Re}{Gr} \sin \alpha \right] = 0 \quad (8)$$

$$\theta'' - Re Pr \theta' + 2 Br Gr^2 [(f')^2 + (g')^2] = 0 \quad (9)$$

$$\phi'' - Re Sc \phi' + Sr Sc \theta'' = 0 \quad (10)$$

With

$$f(-1) = g(-1) = \theta(-1) = \phi(-1) = 0, f(1) = g(1) = 0, \theta(1) = \phi(1) = 1 \quad (11)$$

where the primes represents differentiation with respect to  $\eta$ ,  $Re = \frac{v_0 d}{\nu}$  is the Reynolds

number,  $Gr = \frac{g^* \beta_T (T_2 - T_1) d^3}{\nu^2}$  is the temperature Grashof number,  $Pr = \frac{\mu C_p}{K_f}$  is the Prandtl number,

$Sc = \frac{\nu}{D}$  is the Schmidt number,  $Br = \frac{\mu \nu^2}{K_f d^2 (T_2 - T_1)}$  is the Brinkman number,  $Sr = \frac{DK_T (T_2 - T_1)}{\nu T_m (C_2 - C_1)}$

the Soret number,  $N = \frac{\beta_C (C_2 - C_1)}{\beta_T (T_2 - T_1)}$  is the buoyancy parameter,  $Ha = dB_0 \sqrt{\frac{\sigma}{\nu}}$  is the Hartmann number and  $m$  is the hall number.

Effects of the different emerging parameters present in this study on physical quantities are presented in the following section.

### 3. Solution of the problem

To obtain HAM solutions, the initial guess and the auxiliary linear operators for  $f(\eta)$ ,  $g(\eta)$ ,  $\theta(\eta)$  and  $\phi(\eta)$  are considered as

$$f_0(\eta) = 0, g_0 = 0, \theta_0(\eta) = \frac{1+\eta}{2}, \phi_0(\eta) = \frac{1+\eta}{2} \quad L = \frac{\partial^2}{\partial \eta^2} \text{ such that } L(c_1 + c_2\eta) = 0 \quad (12)$$

Zeroth-order deformations are initiated and in which  $h_1, h_2, h_3$  and  $h_4$  (convergence control parameters) were introduced.

Homotopy solutions were obtained by considering  $N_1, N_2, N_3$  and  $N_4$  (non-linear operators); total average residual errors for  $f, g, \theta$  and  $\phi$  are considered as presented in [18].  $h$  - Curves are plotted to decide the suitable (Optimal) values of  $h_1, h_2, h_3$  and  $h_4$  by taking  $Re=2.0, m=2, Pr=0.71, Ha=2, Sc=0.22, N=2.0, Br=0.1, Gr=0.5, Sr=2.0$ . From the  $h$ -curves and the average residual errors, the suitable values for  $h_1, h_2, h_3$  and  $h_4$  are fixed as  $-0.79, -1.08, -0.81, -1.38$  respectively.

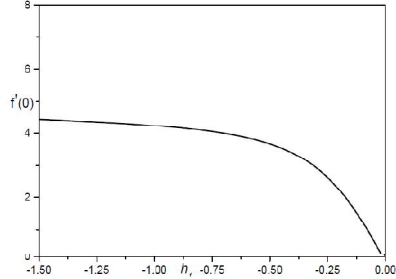


Fig. 2:  $h$  curves for  $f(\eta)$

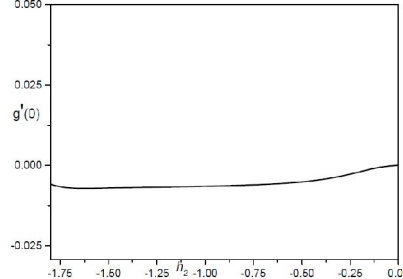


Fig. 3:  $h$  curves for  $g(\eta)$

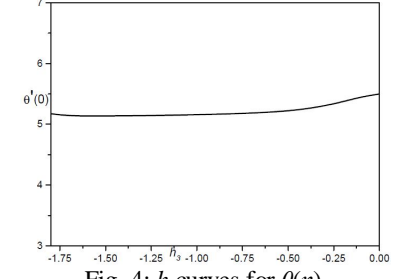


Fig. 4:  $h$  curves for  $\theta(\eta)$

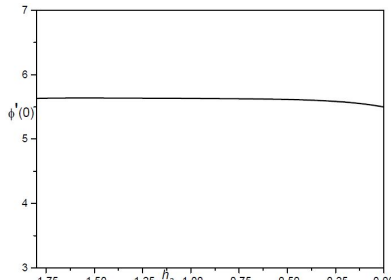


Fig. 5:  $h$  curves for  $\phi(\eta)$

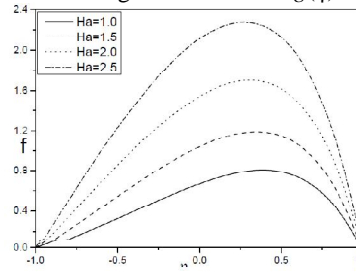


Fig. 6: Magnetic parameter effect on  $f(\eta)$

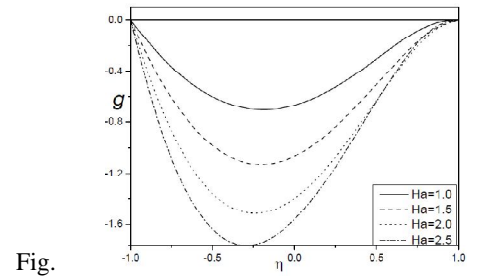


Fig. 7:  $Ha$  effect on  $g(\eta)$

### 4. Discussion of Results

The influence of  $Ha, m, \alpha, Sr$  on velocities ( $f(\eta), g(\eta)$ ), temperature ( $\theta(\eta)$ ) and concentration ( $\phi(\eta)$ ) are calculated and are explained Figs. 6 to 21 by fixing  $Pr, Br, Re, Sc, Gr, N$  at 0.71, 0.5, 2, 0.22, 0.5, 2 respectively.

Figures 6-9 shows the impact of Hartmann number ( $Ha$ ) on  $f, g, \theta$  and  $\phi$  when  $\alpha=\pi/3, Sr=2, m=2$ . It is seen from Fig.6 that as  $Ha$  increases, the flow velocity increases. It is noted that the applied magnetic field has an inclination angle  $\alpha>0$  with that the drag force cannot be generated. It is identified from Fig. 7 that the cross flow velocity increases as  $Ha$  increases. It can depict from Figs. 8-9 that the dimensionless temperature enhances and concentration diminishes with the increase of magnetic parameter.

The influence of  $m$  on  $f, g, \theta$  and  $\phi$  can be found in Figs. 10 to 13 at  $Ha=2, \alpha=\pi/3, Sr=2$ . It is noticed from Figs. 10-11 that the flow velocity decreases and the cross velocity increases as  $m$  magnifies. It is noted from Figs. 12-13 that the temperature of the fluid decreases and the concentration of the fluid increases as an increase in  $m$ .

The effect of magnetic inclination angle  $\alpha$  on  $f, g, \theta$  and  $\phi$  can be noted in Figs. 14-17 by fixing the other parameters at  $Ha=2, m=2, Sr=2$ . It is noticed from Fig. 14 and 15 that the flow velocity and cross flow velocity increases as  $\alpha$  increases. This is due to the reason that as an inclination angle increases the direction of the applied magnetic field changes and the drag force will reduce on the net flow. It is observed from Fig. 16-17 that the dimensionless temperature increases and concentration decreases as  $\alpha$  increases.

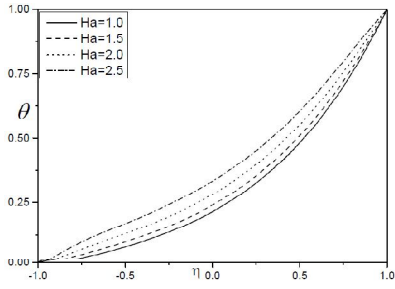


Fig.8. Magnetic parameter effect on  $\theta(\eta)$

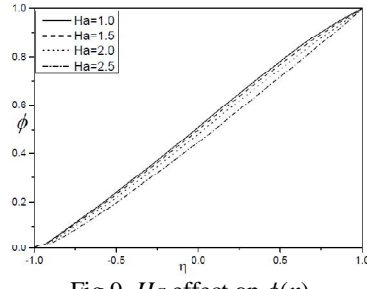


Fig.9.  $Ha$  effect on  $\phi(\eta)$

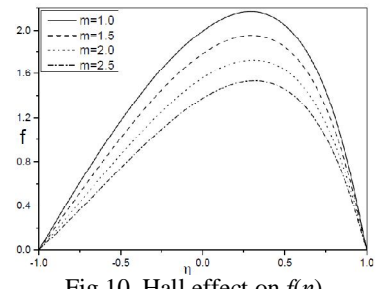


Fig.10. Hall effect on  $f(\eta)$

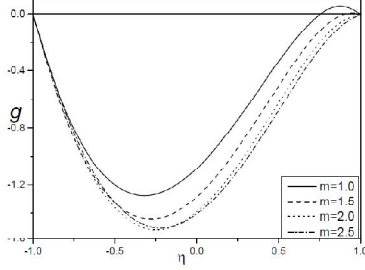


Fig.11. Hall effect on  $g(\eta)$

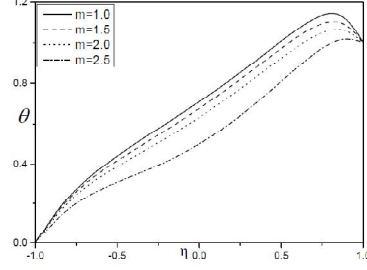


Fig.12. Hall effect on  $\theta(\eta)$

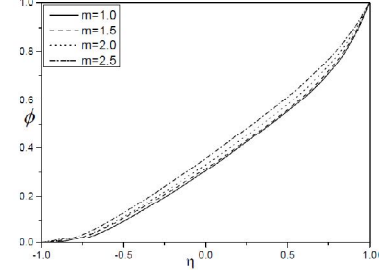


Fig.13. Hall effect on  $\phi(\eta)$

Figure 18 presents the influence of sorlet parameter on the flow velocity  $f(\eta)$  at  $Ha=2$ ,  $m=2$ ,  $\alpha=\pi/3$ . It is noted from Fig.18 that the flow velocity and the concentration enhances as the sorlet parameter increases. These results clearly disclose that the flow field is appreciably influenced by the sorlet parameter.

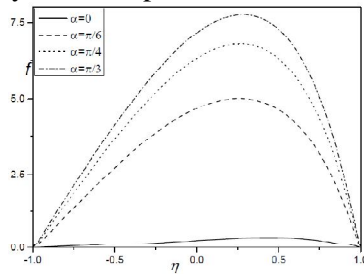


Fig.14: Influence of  $\alpha$  on  $f(\eta)$

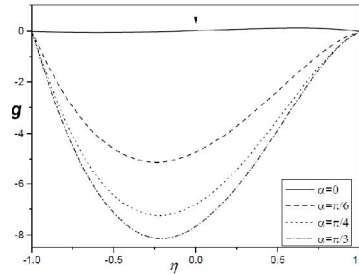


Fig.15: Influence of  $\alpha$  on  $g(\eta)$

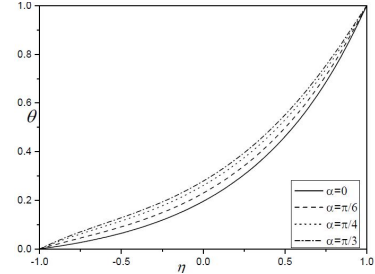


Fig.16: Influence of  $\alpha$  on  $\theta(\eta)$

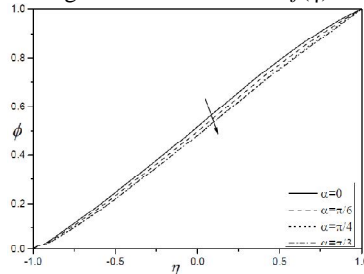


Fig.17: Influence of  $\alpha$  on  $\phi(\eta)$

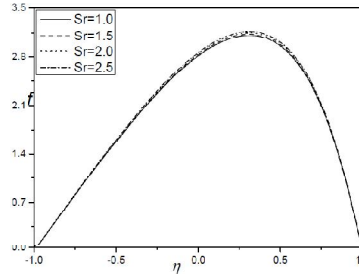


Fig.18: Influence of  $Sr$  on  $f(\eta)$

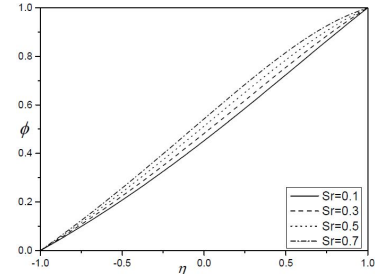


Fig.19: Influence of  $Sr$  on  $\phi(\eta)$

## 5. Conclusions

The present investigation presents the steady inclined magnetohydrodynamic fluid flow in a vertical channel with Hall and diffusion thermo effect. Homotopy method is applied to solve the final system. It is noted from this present analysis that the velocities and the temperature of the fluid amplifies but the concentration of the fluid flow reduces with the increase of  $Ha$  and  $\alpha$ . As  $m$  enhances,  $f$  and  $\theta$  decreases but reverse trend is observed in case  $g$  and  $\phi$ . It is found that the flow velocity and the concentration magnifies as  $Sr$  increases.

## 7. References

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