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Nonlinear dynamic analysis of automotive turbocharger rotor system

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The present development on the automotive engines on reducing size and increasing greater fuel efficiency, and less emissions which took the researchers to several challenges in turbocharger rotor system. However the challenge towards the evolution of high efficiency bearing systems with reliability is still over the top. The paper shows the system with dynamic analysis of nonlinear turbocharger rotor system to interpret the unstable positions during running condition. The turbocharger in this study is examined as a uniform shaft in nature with varying lengths between two bearings which actually held between compressor end and the turbine disc. The two discs are having force unbalance and the compressor impeller exists with the seal forces. The rotor considered here is supported by bearings with the nonlinear time-varying forces as reactions at the supports. The results of nonlinear effects, time-histories, Campbell diagram and spectrograms are formulated and solved with analytical equations using finite element analysis. The impacts of different operating conditions at two different speeds are studied by phase plane diagrams to understand the stability of the system.

***Keywords*:** Dual disc Rotor; Ball bearing; Seal force; FEM; Stability analysis.

1. Introduction

In much kind of machineries nonlinear vibrations plays an important phenomenon in technical applications. Most of the practical cases, the system always dominated through the nonlinear effects. The system examined here in this study, called turbocharger rotor system supported by two bearings with the nonlinear time-varying forces.

In rocket engine generally the turbocharger is considered as a major part and it supplies the propellants with low temperature to the combustion chamber which operating at extreme high pressures. Turbochargers are regularly operates at very high speeds due to its size. The high speed of turbocharger will lead the system to a pressure drop region which below the vapour pressure causing the liquid propellant to cavitate the compressor blades. To avoid the cavitation, fuel pumps and oxidizer of engine rotates at different independent speeds of operations. In general, the speed of the oxidizer is high due to the greater density than the fuel. The main disadvantage of this provision is making the engine further complex and less reliable.

Several researchers made their extensive contribution of analysing the turbocharger rotor-bearing system. However, past one decade, experimental and theoretical studies were carried on turbochargers and reported the analysis. Ha *et al.*[1] presented a method for the dynamic analysis of rotor with a floating ring seal of turbo system unit of rocket engines. The study describes the leakage result of flow and the dynamic coefficients of running rotor. Fei *et al.*[2] discussed the rotor system behaviour with counter clockwise (CCW) rotating turbine. The analysis of a CCW rotating turbine pattern is presented with variable diameters of rotors and rotational velocity. Sadeghi *et al*.[3] considered the numerical finite element approach using experimental modal analysis to study the modal properties of a turbo-pump system. Nanri *et al*.[4] studied the turbo system model by assuming flow as incompressible to reveal the cavitation of rocket engine. They implemented the acoustic model with one-dimensional based analysis. Bai *et al.*[5], examined the consequences of nonlinear dynamic characteristics of a flexible support stiffness and system stability by using a dynamic modelling. The induced forces due to nonlinearity by seals and internal damping of rotor also included in the analysis. Son *et al*.[6] describes the theoretical design of a gas-generator of liquid rocket engine. The produced gases by the gas-generator, drives the turbo-pumps through turbine. An analysis based on chemical non-equilibrium and droplet vaporization was given to estimate the properties. Hu *et al.*[7], studied the methods for detection and health monitoring and a turbo-pump fault diagnosis with vector machines. Hong *et al.*[8] carried the test on hydraulic performance of a liquid rocket engine. Wang and Sun [9] extended the analysis of turbo-pump rotor with gyroscopic effects with one dimensional finite element model.

The present study shows the dynamic stability of a rotordynamic model of turbocharger system. The rotor system included the ball bearing with Hertzian contact forces, disk mass unbalances and forces of seal which is dynamic in nature based on the Muszynska relations. The equations which are dynamic in nature are solved using implicit scheme of Houbolt's time integration. Finally the stability of the system with different speeds of operation is carried using frequency response and phase-plane plots.

2. Dynamic model of Turbocharger unit

Turbochargers are principal parts of rocket engine unit. A schematic view of turbocharger is shown in Fig.1. The turbo-pump rotor is supported on two bearings and exists liquid seals at pump disc. Turbo-pump is considered as a simple single disc in the present task, and has an inducer and diffuser in it. The turbine produces power at the other end is driving unit of turbocharger system.

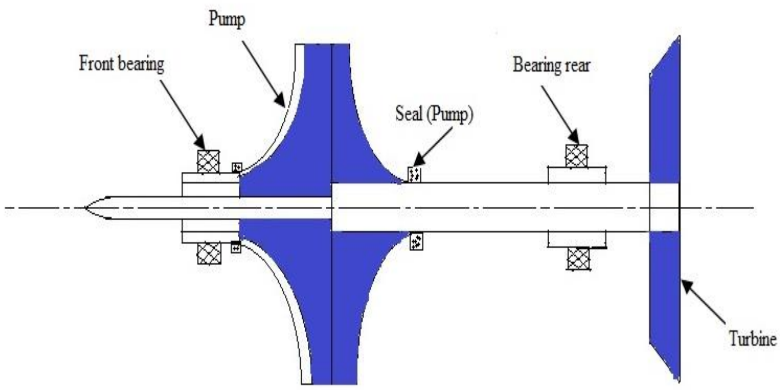

Fig.1. Turbocharger schematic diagram

2. 1. Dynamic formulations of system

By using Timoshenko beam theory, a simplified rotor model of the system is considered with finite elements. The shaft elements are divided by four and five nodes thoroughly. Every node of the shaft element has the degrees of freedom (DOF) four, which includes rotational (θ_y, θ_z) and translational displacements (v, w)

For the near in-depth to the dynamic effects of the system the turbocharger parts are taken as rigid bodies which lies at two nodes 3 and 5 respectively. The nodes 2 and 4 are represents theball bearings. The point of seals with dynamic forces, which have an extremely nonlinear in nature. The seals are placed at the discharge side of the impeller and inlet end which is equal to the element forces concerned at node 3. Moreover, the component force of mass unbalance and the forces due to gravity are considered at turbine and pump discs. Now, the kinetic and the potential energy of spinning shaft elements can be written by with the effects of bending and shearing effects as:

$$T_s = \frac{1}{2}\int_0^{\ell} \rho\left\{A(\dot{v}^2+\dot{w}^2) + I_D(\dot{\theta}_y^2+\dot{\theta}_z^2) + I_P\left[\Omega^2 + \Omega(\dot{\theta}_z\theta_y - \dot{\theta}_y\theta_z)\right]\right\}ds \tag{1}$$

$$U_s = \frac{1}{2}\int_0^{\ell} \left\{EI(\theta_y'^2+\theta_z'^2) + kGA\left[(\theta_y - w')^2 + (\theta_z + v')^2\right]\right\}ds \tag{2}$$

The disk kinetic energy can be given as:

$$T_d = \frac{1}{2}m_d\left(\dot{v}^2+\dot{w}^2\right) + \frac{1}{2}J_d\left(\dot{\theta}_y^2+\dot{\theta}_z^2\right) + \frac{1}{2}J_p\left[\Omega^2 + \Omega(\dot{\theta}_z\theta_y - \dot{\theta}_y\theta_z)\right] \tag{3}$$

The mass eccentricity of the disk taken as:

$$W_d = m_d r_d \Omega^2 (w\cos\Omega t + v\sin\Omega t) \tag{4}$$

The translation (v, w) and rotational displacements (θ_y,θ_y) of the shaft element can be approximated by finite element method as:

$$\begin{Bmatrix} v \\ w \end{Bmatrix} = [N_t(s)]\{q\} \qquad \begin{Bmatrix} \theta_y \\ \theta_z \end{Bmatrix} = [N_r(s)]\{q\} \tag{5}$$

Where, [$N_t(s)$] and [$N_r(s)$] are translation and rotational shape functions. By substituting the above expressions into the equation no (1) and (2) and integrating over the length of the element with Hamilton's principle, the following equation of motion for the shaft and disk units are originated:

$$[M_s] = \int_0^{\ell} \rho A[N_t]^T[N_t]ds + \int_0^{\ell} \rho I_d[N_r]^T[N_r]ds \tag{6}$$

$$[G_s] = \int_0^{\ell} \rho I_p[N_r]^T\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}[N_r]ds \tag{7}$$

$$[K_s] = \int_0^{\ell} EI[N_r']^T[N_r']ds + \kappa GA\int_0^{\ell}\left\{[N_t]^T[N_t] + [N_r]^T[N_r] + 2[N_t]^T\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}[N_r]\right\}ds \tag{8}$$

$$[M_d] = \begin{bmatrix} m_d & & & \\ & m_d & & \\ & & J_d & \\ & & & J_d \end{bmatrix}, \qquad [G_d] = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & J_p \\ & & -J_p & 0 \end{bmatrix} \tag{9}$$

By including the effects of damping, the system the equation of motion can be given as [10]

$$[M]\{\ddot{q}\} + [[C] + \Omega[G]]\{\dot{q}\} + [K]\{q\} = \{F\} \tag{10}$$

2. 2. Seal force (nonlinear)

The force model of the seal [11] can be taken as follows:

$$\begin{Bmatrix} F_{sz} \\ F_{sy} \end{Bmatrix} = -\begin{bmatrix} K_g - m_g \tau_g^2 \Omega^2 & \tau_g \Omega D_g \\ -\tau_g \Omega D_g & K_g - m_g \tau_g^2 \Omega^2 \end{bmatrix} \begin{Bmatrix} w \\ v \end{Bmatrix} - \begin{bmatrix} D_g & 2\tau_g \Omega m_g \\ -2\tau_g \Omega m_g & D_g \end{bmatrix} \begin{Bmatrix} \dot{w} \\ \dot{v} \end{Bmatrix} - \begin{bmatrix} m_g & 0 \\ 0 & m_g \end{bmatrix} \begin{Bmatrix} \ddot{w} \\ \ddot{v} \end{Bmatrix} \tag{11}$$

where K_g, m_g, D_g and τ_g are equivalent stiffness, mass, damping and the ratios of fluid circumferential velocity respectively.

2. 3. Bearing forces

The turbo-pump rotor system is held on ball bearings. The reaction forces at the bearing nodes are in the nature of Hertzian nonlinear radial contact force. The Hertzian contact theory due to nonlinear says that, due to the rolling contact, the force of contact between ball and race is given in terms of Hertzian contact stiffness C_b as:

$$F_{zb} = \sum_{j=1}^{N_b} \left(-C_b (w\cos\theta_j + v\sin\theta_j - r_0)^{3/2} H \right) \cos\theta_j \tag{12}$$

$$F_{yb} = \sum_{j=1}^{N_b} \left(-C_b (w\cos\theta_j + v\sin\theta_j - r_0)^{3/2} H \right) \sin\theta_j \tag{13}$$

3. Numerical simulation and discussion

Finite element equations are solved by an interactive method in MATLAB. The physical and mechanical properties of rotor bearing systems with the dynamic data are listed in Table 1.

Table. 1. Caption for table goes at the top

Properties	**Value**
Density of shaft material (kg/m^3)	7810
Young's modulus, E (GPa)	197
Shear modulus, G (GPa)	80
Radius of shaft (m)	0.012
Length of shaft (m)	0.600
Radius of disc, (m)	0.15
Thickness of disc, (m)	20
Moment of inertia of disc, (kg-mm^2)	41750 & 168100 (I_{yy}=I_{zz});
	83500 & 336200 (I_{xx})
Bearing inner radii, r (m)	0.031
Bearing outer radii, R (m)	0.049
Bearing radial clearance (micr.)	20
Bearing stiffness coefficients (N/m)	4×10^7 (k_{zz}=k_{yy});
	1×10^8 (k_{zz}=k_{yy});
No. of balls (bearing)	8

The simulated frequencies for the system have been found with prompt code as 8.3756 Hz, 36.249 Hz, 52.887 Hz, and 89.683 Hz. Above results are simulated with ANSYS program and compared with the frequencies from numerical solutions by taking into the effect of rotary inertia (Rayleigh's beam). The finite element coupled nonlinear differential equations are solved with Houbolt's implicit time integration scheme. This method carries last two time steps to calculate the present displacement. To solve the time steps in the equations, central difference method was employed. The second module of numerical simulation was achieved in the program for obtaining frequency response and phase plane diagrams.

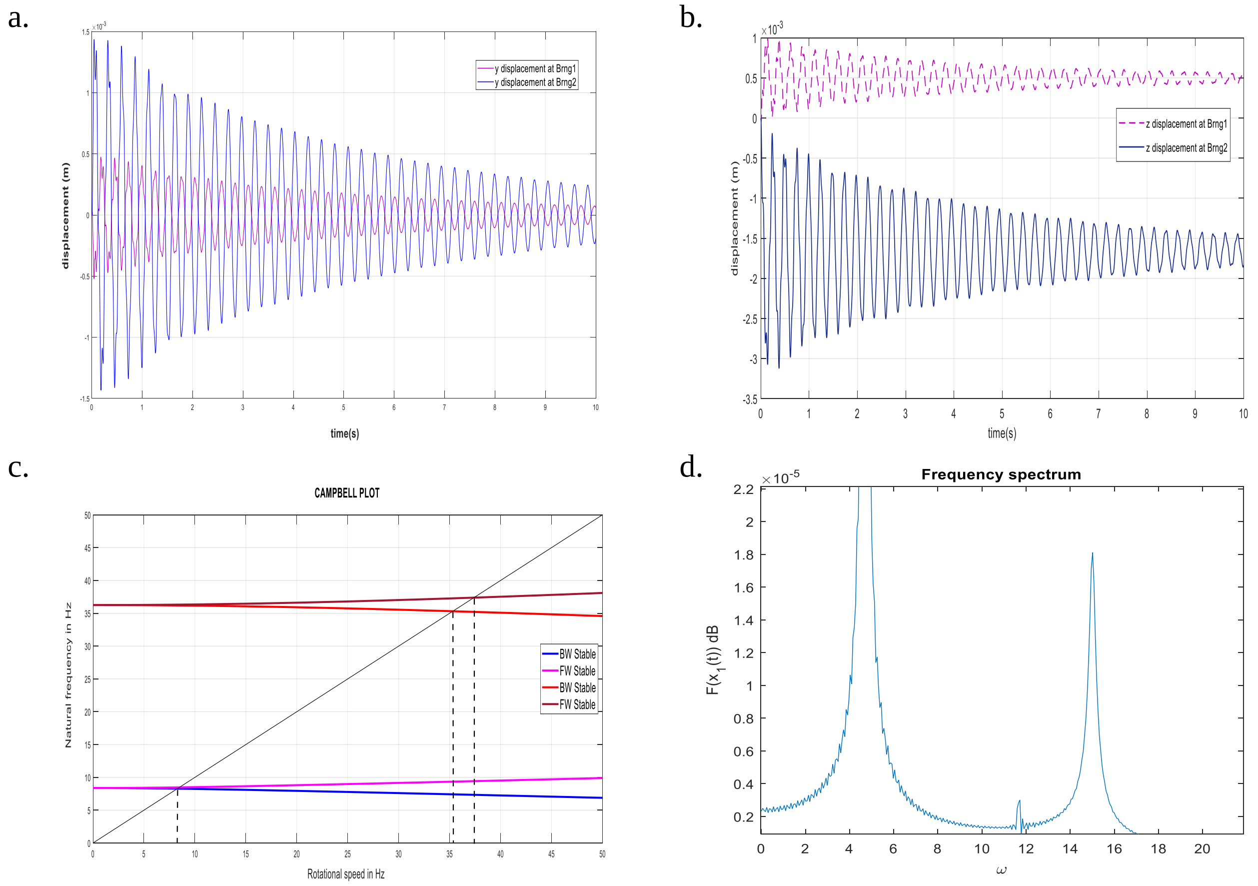

Fig. 2. Dynamic simulation of a turbocharger held in bearings with nonlinear radial contact force; (a). Time histories, Ω =1000 rpm (bearings); (b). bearing(s) z-displacements, Ω =1000 rpm; (c). Campbell plot; (d). frequency spectrum of rotor at Ω =1000 rpm (y-displacements);

The above Fig.2 (a) & (b) indicates the simulated dynamic systems time histories at bearing nodes with, Ω =1000 rev/min, Fig.2 (c) gives the Campbell plot of the system which shows the first three critical speeds at 420 rpm, 1320 rpm and 2430 rpm respectively. The algorithm converges very closely irrespective of the time histories to Newmark's method.

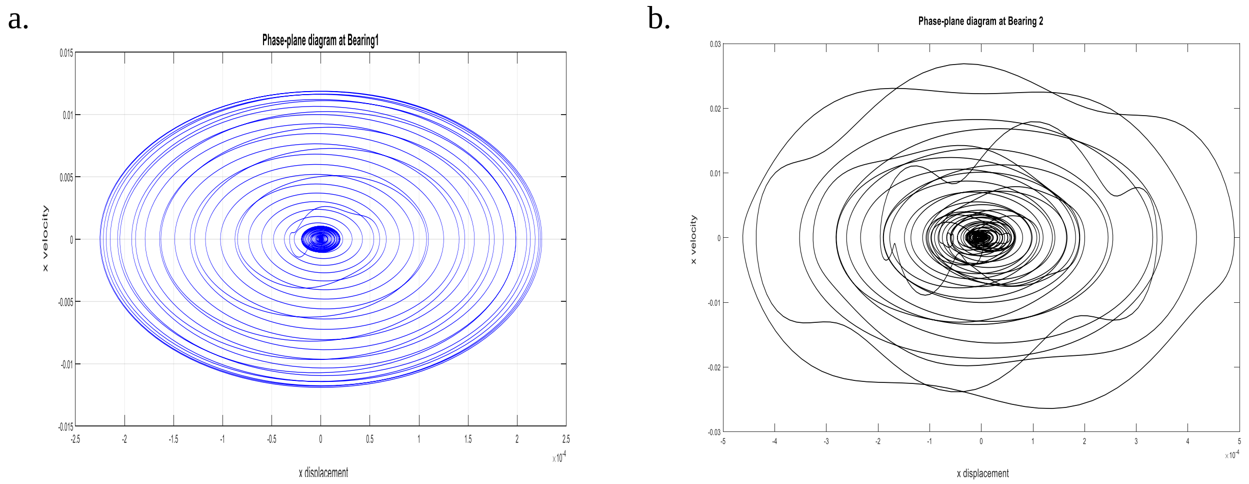

Fig. 3. (a). Phase-plane diagram of bearing1 at Ω =1000 rpm; (b). Phase-plane diagram of bearing 2 at Ω =1000 rpm;

Fig.3 (a) and (b) depicts the orbit plots of the two bearing nodes at Ω =1000 rpm. The system becomes stable (periodic) after 15000 rpm and

4. Conclusions

Simulated vibrations of automotive turbocharger rotor supported in bearings were studied and presented with nonlinear hydrodynamic seal forces. The system modelled with Timoshenko beam theory and analysis was carried by using finite element method. At the bearing supports which mounted on the base is experiencing the excitation and the same is transferred to the rotor. The analysis of frequency response of the rotor was calculated with time integration scheme. The systems includes the force due to imbalance, gyroscopic, bearing reactions and the effect of gravity also into account for the specific operational speed level in the dynamic study and have been presented. The results are shown in Fig.2 and Fig.3, and the results are simulated with ANSYS program and compared with the frequencies from numerical solutions.

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