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Heat and mass transfer of a MHD nanofluid over a stretching sheet with viscous dissipation effect

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Abstract
A study of viscous dissipation effect of magnetohydrodynamicnanofluid flow passing over a stretched surface with the convective boundary condition has been analyzed numerically. The constitutive equations of the flow model are solved numerically and the impact of physical parameters concerning the flow model on dimensionless temperature and concentration are presented through graphs. Also, a comparison of the obtained numerical results with the published results of Aminreza Noghrehabadi has been made and found that both are in excellent agreement.

Keyword: Adam's – Moultanmethod; Thermophoresis; Brownian motion; viscous dissipationnanofluids.

1. Introduction

The transfer of heat by the movement of fluids from one place to another is called convective heat transfer. Convective heat transfer is combination of heat diffusion and bulk fluid flow that are called conduction and advection simultaneously. In engineering problems convective heat transfer has wide applications. Large number of investigations on nanofluids (i.e. mixture of fluid and nanoparticles) show that canbe improved thermal conductivity in fluids. Nanofluid is a fluid containing nanometer-sized particles called nanoparticles. These nanoparticles are made of metals, oxides, carbides etc.Nanofluids have properties that make them potentially useful in many heat transfer applications. They exhibit enhanced thermal conductivity and convective heat transfer coefficient.

Choi [1] introduced the idea of nanofluids for improving the heat transfer potential of the conventional fluids. He experimentally concluded with an evidence that injection of these particles helps in improving the fluid's thermal conductivity. This conclusion opened the best approach to utilize such fluids in mechanical engineering, chemical engineering, pharmaceuticals and numerous different fields. Buongiorno [2], Kuznetsov and Nield [3] followed him and extended the investigation. They worked on the effects of Brownion motion in convective transport of nanofluids and the investigation of natural convective transport of nanofluids passing over a vertical surface in a situation when nanoparticles are dynamically controlled at the boundary. Khan and Pop [4] used this concept to evaluate the laminar boundary layer flow, nanoparticles fraction and heat transfer for nanofluids passing over a stretching surface.

In the process of heat transfer, viscous dissipations mean heating up the fluid via different source. In short, in this mechanism the viscosity of the fluid will absorb heat from the kinetic energy and transform it into internal energy of the system. Moreover, the process in which the electric current through a conductor produce heat is known as Joule heating. Eldahab et al. [5] studied the viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate. Viscous dissipations play an important role in the natural convection in various devices.Viscous dissipation is quite often a negligible effect, but its contribution might become important when the fluid viscosity is very high. Rate of heat transfer is affected by the variation in temperature distribution. Anjali Devi and Ganga [6] studied the MHD flow over a porous stretching sheet under the influence of Joule heating and viscous dissipation. Makinde and Mutuku [7] investigated the thermal boundary layer of hydromagnetic nanofluids over a heated plate under the impact of Ohmic heating and viscous dissipation.Naramgari and Sulochana [8] outlined the mass and heat transfer of

the thermophoretic fluid flow past an exponentially stretched surface inserted in porous media in the presence of internal heat generation/absorption, infusion and viscous dissemination. Afify [9] examined the MHD free convective heat and fluid flow passing over the stretched surface with chemical reaction. A numerical analysis of insecure MHD boundary layer flow of a nanofluid past a stretched surface in a porous media was carried out by Anwar et al. [10]. Nadeem and Haq [11] studied the magnetohydrodynamic boundary layer flow with the effect of thermal radiation over a stretching surface with the convective boundary conditions.
Hence the objective of this study is to review study of Aminrezan Noghrehabadi et al. [12] and extend the flow analysis by considering the additional effect of viscous dissipation with the assumptions of laminar, steady, incompressible, two-dimensional, isothermal stretching sheet, nanofluid with elector-hydrodynamic, convective boundary condition.

2. Mathematical Formulations

A two-dimensional boundary layer flow of a study, viscous and incompressible nanofluid flow through a at plate in a porous medium has been considered with focus on the heat and mass transfer.
From the slot at the origin thin solid surface is extruded which is being stretched in x-direction. The stretching velocity $u_w(x) = cx$ is assumed to vary linearly from the origin, where c is a positive constant ($c > 0$) as displayed in figure 1.

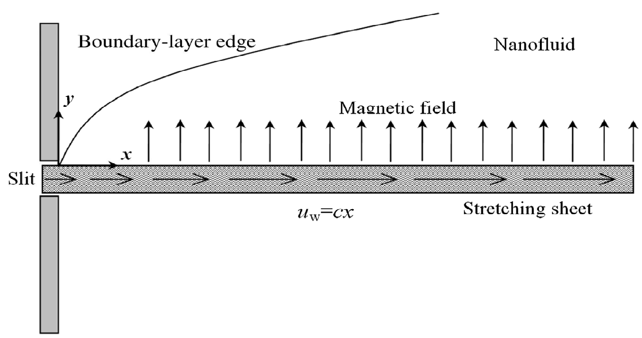

Figure 1: Geometry

In the direction of the flow, normal to the surface, it is directed towards the magnetic field of strength B_0 which is supposed to be applied in the direction of $+ve\ y-$axis. It is also assumed that the temperature and nanofluid volume fraction at the surface of the sheet are respectively T_w andC_w, while the uniform temperature and the nanofluid volume fraction far from the surface of the sheet are T_∞ andC_∞, respectively as y tends to ∞. The flow is described by the equation of continuity, equation of momentum and energy equation as

Continuity equation

Physical principal: Mass is conserved

$$\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \frac{\partial \mathrm{v}}{\partial \mathrm{x}} = 0, \tag{1}$$

Momentum equation

Physical principle: F = ma

$$u\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial \mathrm{y}} = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 u}{\eth x^2} + \frac{\partial^2 u}{\eth y^2}\right) - \frac{\sigma B^2}{\rho_f}u, \tag{2}$$

In equation 2. u and v are the components of velocity respectively in the x and y directions, p is the fluid pressure, $\rho_f$is the density of base fluid, ρ_p is the density of the particles, ν is the kinematic viscosity of the base fluid.

Energy equation

Physical principle: Energy is conserved

By using the boundary layer approximations, the boundary layer equation of energy for fluid temperature$T$ is

$$u\frac{\partial \mathrm{T}}{\partial \mathrm{x}}+\mathrm{v}\frac{\partial \mathrm{T}}{\partial \mathrm{y}}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left(D_B\left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\frac{\partial \mathrm{C}}{\partial \mathrm{y}}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)^2\right)+\frac{\mu}{\rho_p}\left[\left(\frac{\partial u}{\partial y}\right)^2\right], \tag{3}$$

Mass transfer equation

$$u\frac{\partial \mathrm{C}}{\partial \mathrm{x}}+\mathrm{v}\frac{\partial \mathrm{C}}{\partial \mathrm{y}}=D_B\left(\frac{\partial \mathrm{C}}{\partial \mathrm{y}}\right)^2+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

The associated boundary conditions for the above system of equations are,

$$\left.\begin{array}{l} u=U_w(x), v=0, T=T_w, C=C_w, \text{at } y=0 \\ \quad u=v=0, T=T_\infty, C=C_\infty, \text{as } y\to\infty \end{array}\right\} \tag{5}$$

3. Non-dimensional Form of the Governing Equations:

In order to obtain the solution of the problem, first of all the system of Eqs. (1), (2), (3) and (4) together with the boundary conditions (5) is converted into the dimensionless form by using suitable similarity transformation.The following similarity transformation as defined in [12] has been used.

$$\eta=y\sqrt{\frac{c}{\nu}}, \qquad \psi=\sqrt{c\nu}\,xf(\eta), \qquad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, \qquad \beta(\eta)=\frac{C-C_\infty}{C_w-C_\infty}. \tag{6}$$

In above, $\psi(x,y)$ denotes stream function obeying

$$u=\frac{\partial\psi}{\partial y},\; v=-\frac{\partial\psi}{\partial x} \tag{7}$$

Using the similarity transformation from Eq. (6) in momentum Eq. (2), energy Eq. (3) and concentration Eq. (4) along the boundary conditions (5) we get the following system of ODEs:

$$f'''+ff''-f'^2-Mf'=0 \tag{8}$$

$$\theta''+Pr\left(f\theta'+Nb\theta'\beta'+Nt\theta'^2+Ecf''^2\right) \tag{9}$$

$$\beta''+Lef\beta'+\frac{Nt}{Nb}\theta''=0 \tag{10}$$

Here f,θ and β are function of η and prime denotes derivative w.r.t η. The transformed BCs in the modeled problem are:

$$\left.\begin{array}{l} \eta=0, f(\eta)=0, f'(\eta)=1,\; \theta(\eta)=1, \beta(\eta)=1 \text{ at } \eta=0, \\ \qquad \eta\longrightarrow\infty,\; f'(\infty)\to 0, \theta(\infty)\to 0,\; \beta(\infty)\to 0\,. \end{array}\right\} \tag{11}$$

The associated parameters appearing in the modeled problem are:

$$Pr=\frac{\nu}{\alpha}, Ec=\frac{u_w^2}{\left[(C_p)_f(T_w-T_\infty)\right]}, Le=\frac{\nu}{D_B}, Nb=\frac{(\rho c)_p D_B(C_w-C_\infty)}{(\rho c)_f\nu}, Nt=\frac{(\rho c)_p D_T(T_w-CT_\infty)}{(\rho c)_f\nu T_\infty}, M=\frac{\sigma B^2}{(\rho c)_f}.$$

where Pr denotes the Prandtl number,Ec is the Eckert number, Le is the Lewis number, Nb the Brownian motion parameter, Nt the thermophoresis parameter,and$M$ is the magnetic parameter.

In this problem, the desired physical quantities are the local Nusselt number Nu_x, and reduced Sherwood number Sh_x and the skin-friction coefficientC_f . These quantities are defined as

$$C_f=\frac{\tau_w}{\rho u_w^2}, \qquad Nu_x=\frac{xq_w}{k((T_w-T_\infty)}, \qquad Sh_x=\frac{xh_m}{D_B((\varphi_w-\varphi_\infty)} \tag{12}$$

Here, $\tau_w$is the shear stress along the stretching surface, $q_w$is the heat flux from the stretching surface and h_w is the wall mass flux, are given as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, h_m = -D_B\left(\frac{\partial \varphi}{\partial y}\right)_{y=0} \tag{13}$$

With the help of above equations, we get

$$C_f\sqrt{R_x} = -f''(0),\ \frac{Nu_x}{\sqrt{R_x}} = -\theta'(0), \frac{Sh_x}{\sqrt{R_x}} = -\beta'(0) \tag{14}$$

where $R_x = ax^2$ is the local Reynolds number.

4. Numerical Solution

The set of non-linear coupled differential equations (8) - (10) with the conditions (11) is solved numerically in the following manner. Firstly, it is noticed that heurist infinity for the independent variable chosen as η_{max}. The comment on the choice on the η_{max}, for solving is presented at the end of the section.

Equation (13) is solved with $f''(0) = r$, assumed number using the initial conditions

$$f(0) = 0, f'(0) = 1, f''(0) = r \tag{15}$$

r is iteratively found using Newton's method using $F'(\eta_{max}) = \frac{\partial f'(max)}{\partial \alpha}$ which is obtain by solving,

$$F''' = 2f'F' - fF'' - f''F' + Mf' \tag{16}$$

with $F(0) = 0, F'(0) = 0$ and $F''(0) = 1$ (17)

After finding $f(\eta)$ we solved the equations 15&16 with the initial conditions.

$$\left.\begin{array}{ll}
\mathrm{y}_1' = \mathrm{y}_2, & \mathrm{y}_1(0) = 1 \\
\mathrm{y}_2' = -\mathrm{Pr}(\mathrm{FF(I)y}_2 + \mathrm{Nby}_2\mathrm{y}_4 + \mathrm{Nty}_2^2 + \mathrm{Ec\ D2FF(I)} ** 2), & \mathrm{y}_2(0) = \mathrm{p}_1 \\
\mathrm{y}_3' = \mathrm{y}_4, & \mathrm{y}_3(0) = 1 \\
\mathrm{y}_4' = -\mathrm{LeFF(I)y}_4 - \frac{\mathrm{Nb}}{\mathrm{Nt}}\mathrm{y}_2^1, & \mathrm{y}_4(0) = \mathrm{p}_2
\end{array}\right\} \tag{18}$$

Here $p_1 = \theta'(0) and\ p_2(0) = \beta'(0)$.

p_1, p_2 are to be found satisfying end conditions $y_1 \to 0, y_3 \to 0$ as $\eta \to \infty$. Adams Moultan fourth order method (with the corresponding predictor) is used to solve the initial value problem. Assumed values of $p_1$and p_2 are corrected using Newton method.

Derivatives of $\theta(\infty, p_1, p_2)$ and $\beta(\infty, p_1, p_2)$ with respect to any parameter$p(p_1 or\ p_2)$ are found by solving the equation which are obtained by differentiating system (21).

$Y_i = \frac{\partial y_i}{\partial p}$for all $i = 1,2,3,4$.

These equations are

$$\left.\begin{array}{c}
Y_1' = Y(2) \\
Y_2' = -\mathrm{Pr}\ [FF(I)Y(2) + Nb(y(2)Y(4) - Y(2)y(4)) + 2Nty(2)Y(2)] \\
Y_3' = Y(4) \\
Y_4' = -LeFF(I)y(4) - \frac{Nt}{Nb}Y_2'
\end{array}\right\} \tag{19}$$

This system is solved with three different sets of initial conditions $y_i(0) = 0$ for all $i = 1,2,3,4$.

Newton's method is

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}^{New} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}^{Old} - \begin{bmatrix} \frac{\partial y_1}{\partial p_1} & \frac{\partial y_1}{\partial p_2} \\ \frac{\partial y_3}{\partial p_1} & \frac{\partial y_3}{\partial p_2} \end{bmatrix}^{-1}_{\eta=\infty} \begin{pmatrix} y_1 \\ y_3 \end{pmatrix} \tag{20}$$

It may be noticed that the choice of initial guess of p_1, p_2 is very crucial. Once we obtain solution for a set of physical parameters, a single parameter changed slightly to achieve convergence of newton's method. The choice of $\eta_{max} = 4$ was more than enough for end condition. The convergence criteria is chosen to be successive value agree up to 3 significant digits.

5. Results and discussion

In order to validate the code adopted for the numerical solution of equations governing the natural convective flow, the comparison of current results with some of the earlier published work on free convection [12] are displayed in Table 1. Results obtained from the current code are in good agreement with the published results [12]. The comparison of present results with those obtained by AminrezaNoghrehabadi et al. [12] corresponding to $-\theta'(0)$and $-\beta'(0)$ is presented in Table 1.
Increase of the magnetic parameter increases the drag force and consequently decreases the magnitude of the temperature and concentration profile as well as thermal and nanoparticle boundary layer thickness, which are shown in Figs. 2 and 3. Figure 2 shows that the increase of both the thermophoresis and Brownian parameters increases the temperature and thermal boundary-layer thickness. The Brownian motion tends to uniform the concentration of nanoparticles, but, by contrast, the thermophoresis force tends to move nanoparticles from hot to cold areas. These two different effects, i.e., Brownian motion and thermophoresis, on the nanoparticle concentration profiles are shown in Fig. 3.Figure 4 shows the influence of Ec on dimensionless temperature profile. Temperatureprofile increases with an increase in the values of Eckert number Ec.As Eckert number is the ratio of the kinetic energy dissipated in the flow to thethermal energy conducted into or away from the fluid. This figure shows thatwhen we increase the values of Eckert number for temperature profile, the profileof temperature distribution also increases.Figure. 5 display the

Nt	Nb	$-\theta'(0)$				$-\phi'(0)$			
		[4]	**[9]**	**[12]**	**Present Result**	**[4]**	**[9]**	**[12]**	**Present Result**
0.1	0.1	0.9524	0.9523768	0.9523767	0.952376800	2.1294	2.1293938	2.1293938	2.129389000
	0.2	0.6932	0.6931743	0.6931743	0.693174600	2.2740	2.2740215	2.2740215	2.274012000
	0.3	0.5201	0.5200790	0.5200790	0.520079700	2.5286	2.5286382	2.5286381	2.528624000
	0.4	0.4026	0.4025808	0.4025810	0.402581500	2.7952	2.7951701	2.7951703	2.795155000
	0.5	0.3211	0.3210543	0.3210544	0.321055100	3.0351	3.0351425	3.0351425	3.035121000
0.2		0.5056	0.5055814	0.5055814	0.505581300	2.3819	2.3818706	2.3818707	2.381866000
0.3		0.2522	0.2521560	0.2521560	0.252155500	2.4100	2.4100188	2.4100188	2.410016000
0.4		0.1194	0.1194059	0.1194059	0.119405400	2.3997	2.3996502	2.3996502	2.399647000
0.5		0.0543	0.0542534	0.0542534	0.054252970	2.3836	2.3835712	2.3835712	2.383568000

influence of Eckert number Ec on concentration profile.It is observed that the concentration of the fluid decreases near the plate. However,it rises away from the surface as the value of Eckert number is enhanced.

Table 1. Comparison of the results of $-\theta'(0)\& -\phi'(0)$ when $Pr = Le = 10, Ec = 0, and\ M = 0.$

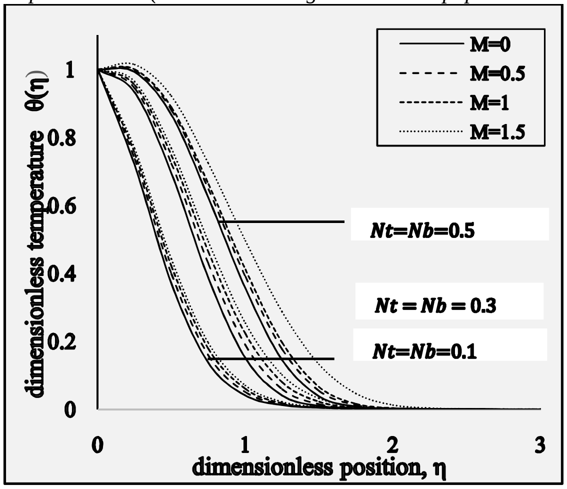

Figure 2. Temperature profiles for selected values of M, Nb, and Nt when $Pr = Le = 10$,and $Ec = 0.1$

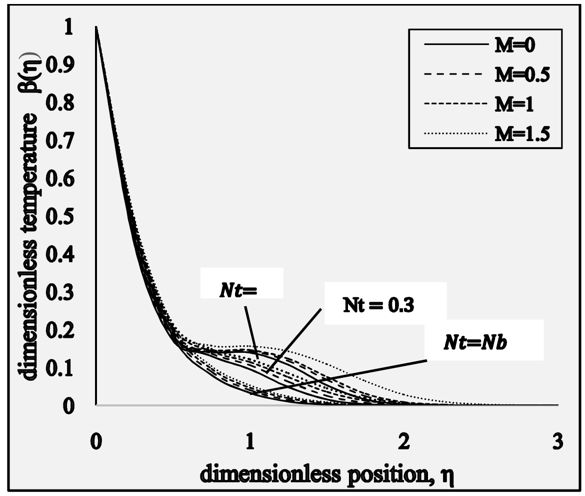

Figure 3. Profiles of nanoparticle volume fraction for selected values of M, and Nt, when $Pr = Le = 10, Nb = 0.5$ and $Ec = 0.1$

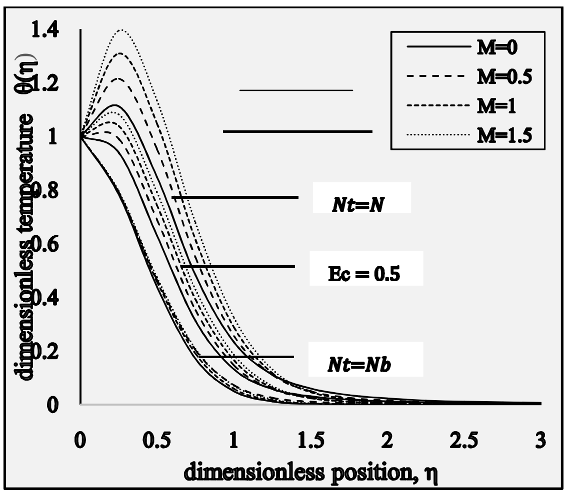

Figure 4. Temperature profiles for selected values of M, Ec, and Nt when $Pr = Le = 10, Nb = 0.2$ and $Nt = 0.1$

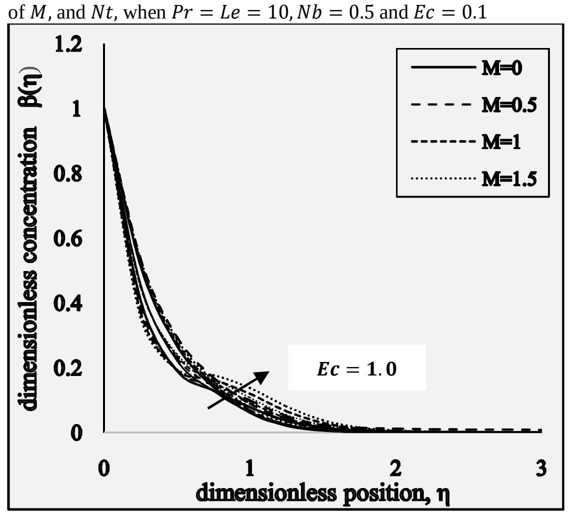

Figure 5. Profiles of nanoparticle volume fraction for selected values of M when $Pr = Le = 10, Nb = 0.2, Ec = 0.1$ and $Nt = 0.1$

6. Conclusion

From the present study the important findings are listed below.

- The thermal boundary-layer thickness is increased by the increase in magnetic parameter.
- Temperature and concentration increase by enlarging thermophoresis parameter Nt.
- Increase in viscous dissipation increases temperature and concentration profile.

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