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Thermally Developing Region of a Parallel Plate Channel Partially Filled with a Porous Material with the Effect of Axial Conduction and Viscous Dissipation: Uniform Wall Heat Flux

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The present investigation has been undertaken to assess the effect of axial conduction and viscous dissipation on heat transfer characteristics in the thermally developing region of a parallel plate channel with porous insert attached to both the walls of an channel. Both the walls are taken uniform heat flux. The fully developed flow field in porous region corresponds to Darcy-Brinkman equation and the clear fluid region to that of plane Poiseuille flow. The effect of parameters, Brinkman number, *Br*, Darcy number, *Da*, Peclet number, *Pe* and a porous fraction, γ_p have been studied. The numerical solutions have been obtained for, $0.005 \le Da \le 1.0$, $0 \le \gamma_p \le 1.0$. and $-1.0 \le Br \le 1.0$ and Pe = 5, 25, 50, 100 and neglecting axial conduction (designated by $A_c = 0$) by using the numerical scheme successive accelerated replacement(SAR). There is a unbounded swing in the local Nusselt number because of viscous dissipation.

***Keywords*:** Viscous dissipation; Axial conduction; Parallel plate channel partially filled with a porous material.

1. Introduction

Several studies (Agrawal [1], Hennecke [2], Ramjee and Satyamurty [3] and Jagadeesh Kumar [4]) have shown that axial conduction term becomes significant in the equation of energy at low Peclet number in the case of forced convection in the ducts. In particular, Shah and London [5] studied the problem of heat transfer in the entrance region for a viscous incompressible fluid in both two dimensional channel and circular cylindrical tube taking into consideration axial conduction term. Ramjee and Satyamurty [3] studied local and average heat transfer in thermally developing region of an asymmetrically heated channel.

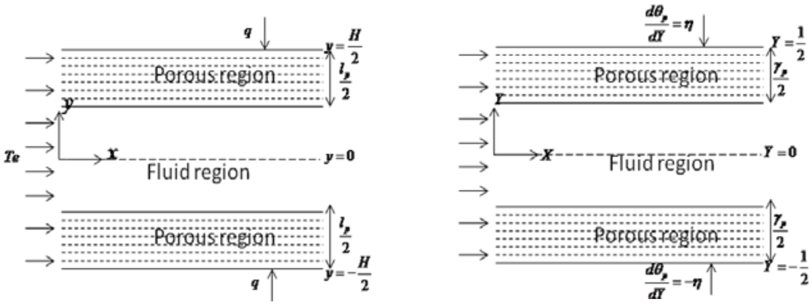
Hooman, Haji-Sheikh and Nield [6] have studied thermally developing forced convection in rectangular ducts with subjected to uniform wall temperature. Thermally developing forced convection in circular duct filled with porous medium with longitudinal conduction and viscous dissipation effects subjected to uniform wall temperature studied by Kuznetsov, Xiong and Nield[7]. Nield, Kuznetsov and Xiong [8] investigated the effects of viscous dissipation, axial conduction with uniform temperature at the walls, on thermally developing forced convection heat transfer in a parallel plate channel fully filled with a porous medium.

In the present paper, thermally developing region of a parallel plate channel partially filled with a porous material with the effect of axial conduction and viscous dissipation with wall boundary condition uniform heat flux, has been studied. Numerical solutions for the two dimensional energy equations in both the fluid and porous regions have been obtained using numerical scheme Successive Accelerated Replacement(Ramjee and Satyamurty [3], Satyamurty and Bhargavi [9] and Bhargavi and Sharath Kumar Reddy [10]). The effects of important relevant parameters on local Nusselt number has been studied.

2. Mathematical Formulation

Governing equations and the boundary conditions are non-dimensionalizing by introducing the following non-dimensional variables.

$$\left.\begin{aligned} &X = x/H,\ Y = y/H,\ U_f = u_f/u_{ref},\ U_i = u_i/u_{ref},\ U_p = u_p/u_{ref},\ P = p/\rho\, u_{ref}^2, \\ &\theta_f = (T_f - T_e)/(qH/k_f),\ \theta_p = (T_p - T_e)/(qH/k_f) \end{aligned}\right\} \quad (1)$$



(a) Dimensional (b) Non-Dimensional

Fig. 1: Geometry of physical model of the problem.

The non-dimensional governing equations and boundary conditions for momentum and energy equations applicable in the fluid and porous regions become, (using non-dimensional variables given in Eq. (1))

Fluid Region:

$$\frac{d^2 U_f}{dY^2} = Re\ \frac{dP}{dX} \tag{2}$$

$$U_f \frac{\partial \theta_f}{\partial X^*} = A_c \frac{1}{Pe^2} \frac{\partial^2 \theta_f}{\partial X^{*2}} + \frac{\partial^2 \theta_f}{\partial Y^2} + Br\left(\frac{dU_f}{dY}\right)^2 \tag{3}$$

In Eq. (2), *Re*, the Reynolds number is defined by

$$Re = \rho u_{ref} H / \mu_f \tag{4}$$

In Eq. (3), *Pe*, Peclet number and *Br*, Brinkman number and X^* are defined by,

$$Pe = u_{ref} H / \alpha_f,\ Br = \mu_f u_{ref}^{\ 2} / qH\ \ and\ \ X^* = X / Pe \tag{5}$$

when *Br* < 0 represents the fluid is getting heated. *Br* > 0 represents, the fluid is getting cooled.

Porous Region

$$\frac{d^2 U_p}{dY^2} - \frac{\varepsilon}{Da} U_p = \varepsilon\ Re \frac{dP}{dX} \tag{6}$$

$$U_p \frac{\partial \theta_p}{\partial X^*} = \frac{1}{\eta}\left(A_c \frac{1}{Pe^2} \frac{\partial^2 \theta_p}{\partial X^{*2}} + \frac{\partial^2 \theta_p}{\partial Y^2}\right) + \frac{Br}{Da} U_p^{\ 2} \tag{7}$$

In Eqs. (6) and (7) , *Da*, ε and η are defined as,

$$Da = K / H^2, \varepsilon = \mu_f / \mu_{eff}\ \text{ and }\ \eta = k_f / k_{eff} \tag{8}$$

When A_c = 1 in Eqs. (3) and (7) means that axial conduction is included, and when A_c = 0, axial conduction is neglected. When A_c = 0, the solutions to Eqs. (3) and (7) in terms of X^* do not depend on *Pe*.

Non-dimensional Boundary Conditions

$$\frac{dU_f}{dY} = 0,\ \frac{\partial \theta_f}{\partial Y} = 0 \qquad \text{at} \qquad Y = 0 \tag{9}$$

$$U_f = U_p = U_i,\ \frac{dU_f}{dY} = \frac{1}{\varepsilon}\frac{dU_p}{dY} \qquad \text{at} \qquad Y = -\frac{1}{2} + \frac{\gamma_p}{2} \tag{10}$$

$$\theta_f = \theta_p = \theta_i,\ \frac{\partial \theta_f}{\partial Y} = \frac{1}{\eta}\frac{\partial \theta_p}{\partial Y} \qquad \text{at} \qquad Y = -\frac{1}{2} + \frac{\gamma_p}{2} \tag{11}$$

$$U_p = 0,\ \frac{\partial \theta_p}{\partial Y} = -\eta \qquad \text{at} \qquad Y = -1/2 \tag{12}$$

Inlet conditions

$$\theta_p(0, Y) = 0 \quad \text{for} \quad -\frac{1}{2} \le Y \le -\frac{1}{2} + \frac{\gamma_p}{2} \tag{13}$$

$$\theta_f(0,Y)=0 \quad \text{for} \quad -\frac{1}{2}+\frac{\gamma_p}{2}\le Y\le 0 \tag{14}$$

$$\frac{\partial\theta_b}{\partial X^*}=0 \Rightarrow \frac{\partial\theta_{f,p}}{\partial X^*}=\frac{\theta_{f,p}}{\theta^*}\frac{\partial\theta^*}{\partial X^*} \text{ at } X^*\ge X^*_{fd} \text{ for } -1/2\le Y\le 0 \text{ \{downstream condition\}} \tag{15}$$

In Eq. (15), θ_b is the non-dimensional temperature based on the bulk mean temperature defined by

$$\theta_b=\frac{T-T_e}{T_b-T_e}=\frac{\theta}{\theta^*} \tag{16}$$

The velocity expressions in fluid and porous regions satisfying the interfacial conditions are available in Bhargavi and Sharath Kumar Reddy [10].

3. Numerical Scheme: Successive Accelerated Replacement (SAR)

Numerical solutions to non-dimensional energy equations Eqs. (3) and (7) along with the non-dimensional boundary conditions on θ given in Eqs. (9) to (16) have been obtained using the numerical scheme Successive Accelerated Replacement([3], [9] and [10]).

3.1 Local Nusselt number

After non-dimensionalizing (using Eq. (1)), Nu_{px}, the local Nusselt number at the lower plate $Y=-1/2$, is given by

$$Nu_{px}=\frac{h_{px}(2H)}{k_f}=\frac{2}{\theta_w-\theta^*} \tag{17}$$

4. Results and Discussion

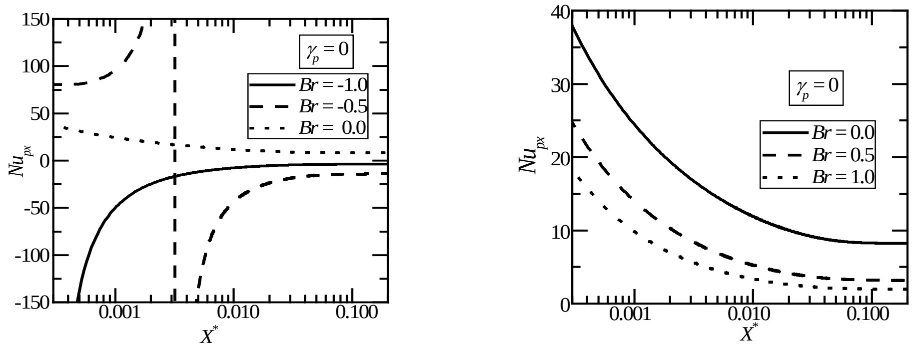
Assumed that $\varepsilon=\mu_f/\mu_{eff}=1$ and $\eta=k_f/k_{eff}=1$. The channel referred as clear fluid channel when porous fraction, $\gamma_p=0$. The channel referred as fully filled with a porous medium, when porous fraction, $\gamma_p=1.0$. The channel referred as partially filled with a porous medium, when porous fraction, $0<\gamma_p<1.0$.

4.1 Local Nusselt number

Variation of the local Nusselt number, Nu_{px} against X^* for the Darcy number, Da = 0.005 is shown in Figs. 2(a) and 2(b) for $Br\le 0$ and $Br\ge 0$ respectively, for porous fractions, $\gamma_p=0$ when axial conduction is neglected. Similarly, for porous fractions γ_p = 0.2, 0.8 and 1.0 are shown in Figs. 3 to 5 respectively. Clearly, Nu_{px} displays an unbounded swing for $Br<0$ at say, X^*_{sw} in all Figs. 2 to 4. This unbounded swing X^*_{sw} occurs for $\gamma_p\le 0.8$. But when $Br>0$, Nu_{px} displays an unbounded swing X^*_{sw} for $\gamma_p>0.8$ see in Fig. 5. The value of X^*_{sw} (which occurs for $Br<0$) increases as γ_p increases from 0 to 0.8. The Nusselt number values as well as the limits differ if Da is larger. The local Nusselt number, Nu_{px} displays an unbounded swing for $Br<0$ since the bulk mean temperature reaches wall temperature and exceeds because of viscous dissipation for $Br<0$. Beyond X^*_{sw}, Nu_{px} starts decreasing to reach the limiting value.

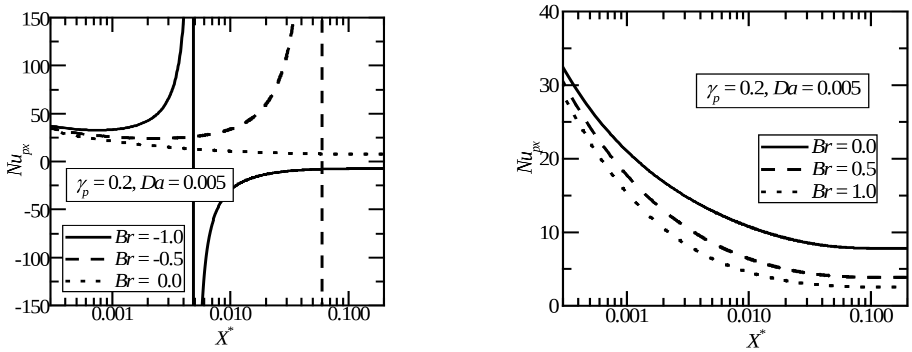
4.2 Local Nusselt number with the effect viscous dissipation and axial conduction

Variation of local Nusselt number, Nu_{px} against X^* for the Darcy number, Da = 0.005 and at Brinkman numbers, (a) $Br=-0.5$ and (b) $Br=0.5$ for different Peclet numbers, Pe = 5 and 25 respectively are shown in Fig. 6 to Fig. 9 for porous fractions, γ_p = 0, 0.2, 0.8 and 1.0 respectively. In parallel plate channel partially filled with a porous medium also, Nu_{px} displays an unbounded swing, X^*_{sw} for $Br<0$ in all Figs. 6 to 8. This unbounded swing depends on porous fractions, γ_p. At low Peclet number the value of the X^*_{sw} is more for all porous fractions. This unbounded swing X^*_{sw} occurs for $\gamma_p\le 0.8$. But when $Br>0$, Nu_{px} displays an unbounded swing X^*_{sw} for $\gamma_p>0.8$ see in Fig. 9. As, Darcy number increases, there is no unbounded swing in the local Nusselt number for all porous fractions.



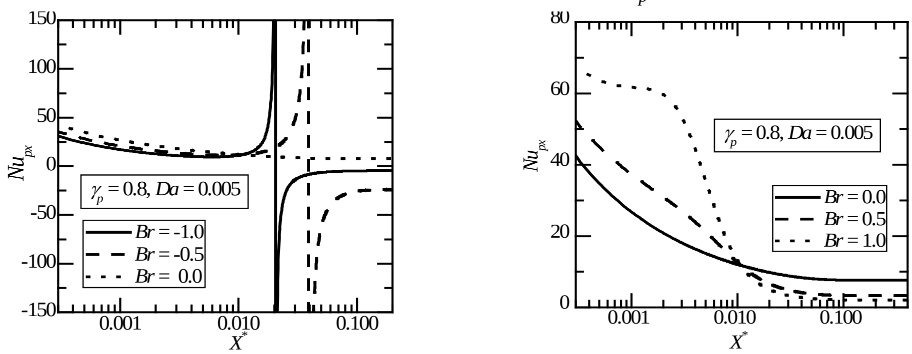
(a) (b)

Fig. 2: Variation of local Nusselt number against X^* for (a) $Br \leq 0$ and (b) $Br \geq 0$ for axial conduction neglected(A_c = 0) for $\gamma_p = 0$.



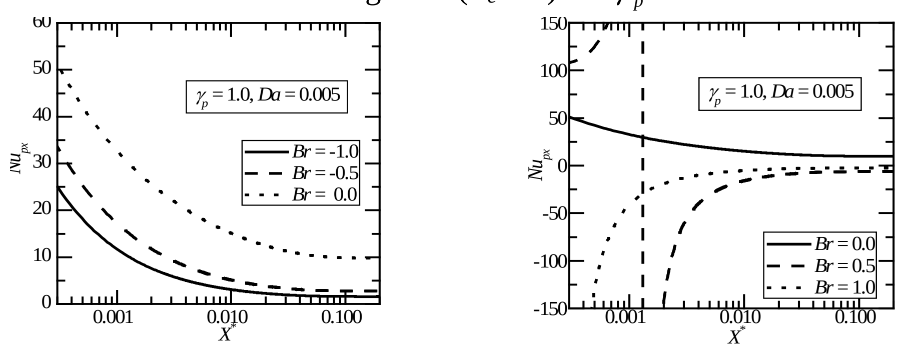
(a) (b)

Fig. 3: Variation of local Nusselt number against X^* for (a) $Br \leq 0$ and (b) $Br \geq 0$ for axial conduction neglected (A_c = 0) for $\gamma_p = 0.2$.



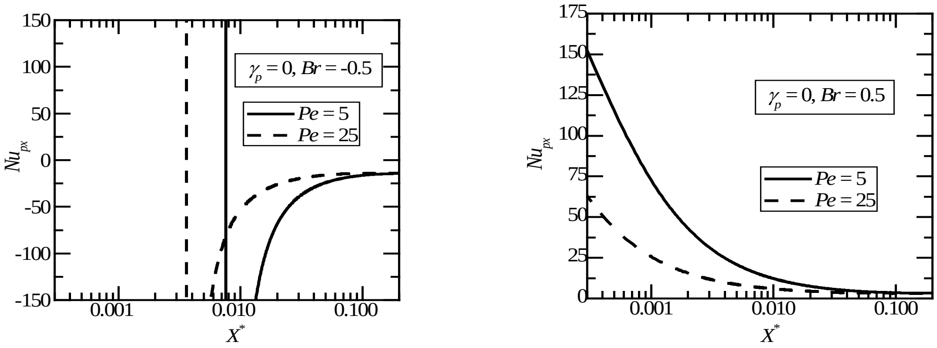
(a) (b)

Fig. 4: Variation of local Nusselt number against X^* for (a) $Br \leq 0$ and (b) $Br \geq 0$ for axial conduction neglected(A_c = 0) for $\gamma_p = 0.8$.



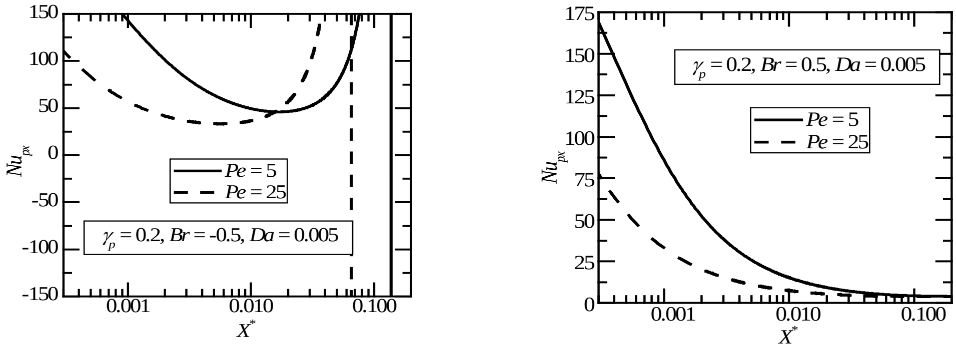
(a) (b)

Fig. 5: Variation of local Nusselt number against X^* for (a) $Br \leq 0$ and (b) $Br \geq 0$ for axial conduction neglected(A_c = 0) for $\gamma_p = 1.0$.



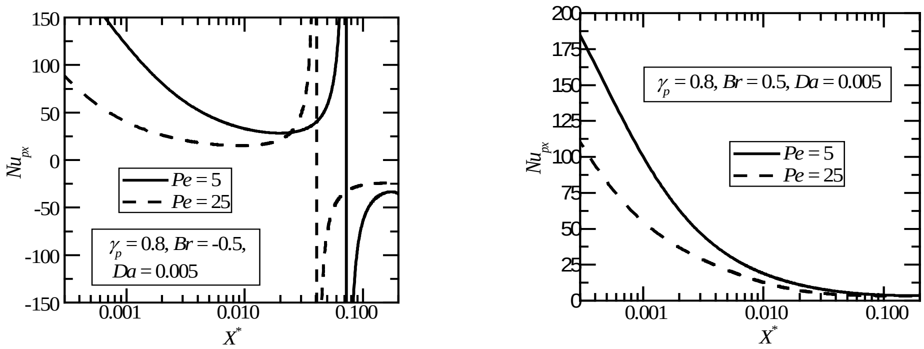
(a) (b)

Fig. 6: Variation of local Nusselt number against X^* for different Peclet numbers, Pe for (a) (a) $Br = -0.5$ and (b) $Br = 0.5$ for $\gamma_p = 0$.



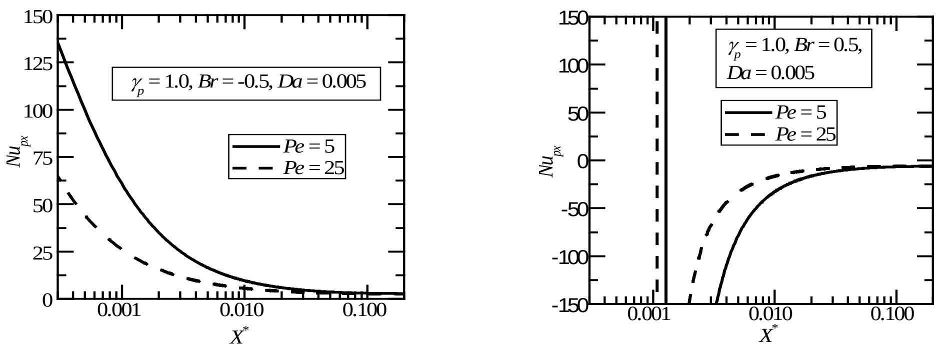
(a) (b)

Fig. 7: Variation of local Nusselt number against X^* for different Peclet numbers, Pe for (a) $Br = -0.5$ and (b) $Br = 0.5$ for $\gamma_p = 0.2$.



(a) (b)

Fig. 8: Variation of local Nusselt number against X^* for different Peclet numbers, Pe for (a) $Br = -0.5$ and (b) $Br = 0.5$ for $\gamma_p = 0.8$.



(a) (b)

Fig. 9: Variation of local Nusselt number against X^* for different Peclet numbers, Pe for (a) $Br = -0.5$ and (b) $Br = 0.5$ for $\gamma_p = 1.0$.

5. Conclusions

Numerical solutions have been obtained for $0 \le \gamma_p \le 1.0$, $5 \le Pe \le 100$, $-1.0 \le Br \le 1.0$ and Da = 0.005, 0.01 and 0.1, using the numerical scheme Successive Accelerated Replacement ([3], [9] and [10]). There is a unbounded swing in the local Nusselt number, since the bulk mean temperature reaches wall temperature and exceeds because of viscous dissipation, $Br < 0$ for porous fraction, $\gamma_p \le 0.8$. In case of porous fraction, $\gamma_p > 0.8$, unbounded swing occurs at $Br > 0$. As, Darcy number increases, there is no unbounded swing in the local Nusselt number.

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