Proceedings of the International Conference on Applied Mechanical Engineering Research (IC-AMER2019)
NIT Warangal, India – May 02-05, 2019
Paper No. XXX (The number assigned when the papaer is accepted)

Dynamic Response of a Beam Structure to a Moving Mass Using Green's Function

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The dynamic responses of a beam subjected to a moving load or moving masses have been of importance in the design of railway tracks and bridges and machining processes. The importance of this problem find uses in many applications in the field of transportation, Bridges, guide ways, overhead cranes, cableways, rails, roadways, runways, tunnels and pipelines. These structural elements are designed to support moving masses. The inertial effect of the moving mass cannot be ignored in comparison with the gravitational effect even if the velocity of the moving mass is comparatively little. In the present study the equation of motion in matrix form for an Euler beam subjected to a concentrated mass moving at a steady speed is formulated by using the Green's function approach. The solutions of the declared problems for the case of cantilever beams are evaluated by using Dynamic Green's function. In the present work, the effect of the moving mass and its speed on the dynamic response of cantilever beam have been investigated. The beam is divided into twenty divisions and the deflection of beam at end is recorded while a mass moves at constant velocity over the beam through different stations. The deflection at end is recorded when the mass is at different stations with the help of a oscilloscope. The result obtained is plotted in the form of graphs for different velocities of mass.

***Keywords*:** Euler Beam, Moving Mass, Green's Function.

1. Introduction

The moving load problem is a fundamental problem in structural dynamics. Engineers have been investing the potential hazard produced by the moving masses on structures for many several years. The dynamic response of structures carrying moving masses is a problem of widespread practical significance. A lot of hard work has been accounted during the last ten decades relating with the dynamic response of railway bridges and later on highway bridges under the effect of moving loads. Beam type structures are widely used in many branches of civil, mechanical and aerospace engineering[3]. The importance of moving mass is found in several applications in the field of transportation. Railway and highway bridges, suspension bridges, guide ways, crane runways, cableways, rails, roadways, runways, tunnels and pipelines are example of structural elements to be designed to support moving masses. Also, in the design of machining processes, many members can be modeled as beams acted upon by moving loads. Moving loads have a great effect on the bodies or structures over which it travels. It causes them to vibrate intensively, especially at high velocities. The peculiar features of moving loads are they are variable in both space and time. Modern means of transport are ever faster and heavier, while the structures over which they move are ever more slender and lighter. That is why the dynamic stresses they produce are larger by far than the static ones. The majority of the engineering structures are subjected to time and space varying loads. Moving loads have substantial effects on the dynamic behavior of the engineering structures.

2. Dynamic Response of a Beam Subjected to a Moving Mass

The In the present research work, dynamic response of beams such as cantilever beams subjected to moving mass under various conditions has been calculated by using theoretical analysis. The differential equation of a beam, which is assumed as a Euler-Bernoulli beam subjected to a point force is given by[1]

$$EI\,\frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2 w(x,t)}{\partial t^2} = F\delta(x-u) \tag{1}$$

Where E is Young's modulus, I is moment of area of the beam cross-section, E is Young's modulus, m is the mass per unit length of the beam, x is the axial co-ordinate, t is the time, $w(x,t)$ is the transverse displacement of the beam, F is the applied point force and $\delta(x-u)$ is the Dirac delta function.

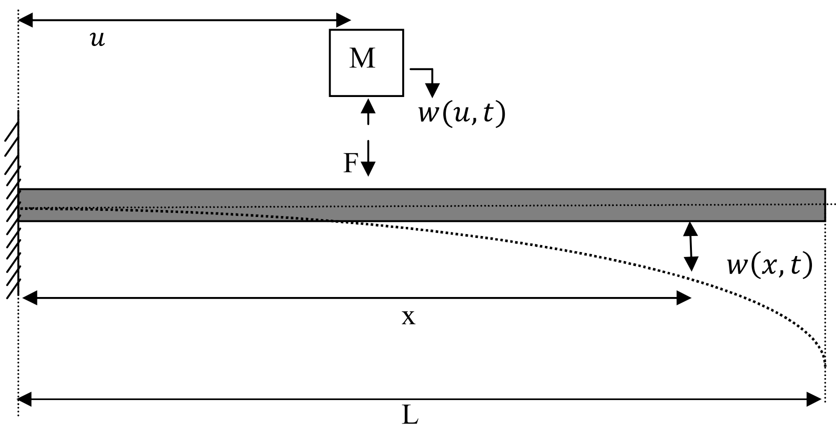

Fig.1 A mass traversing on a cantilever beam with constant velocity

Referring to above Figures, F is the reaction force exerted by the mass M on the beam. Newton's second law when applied to the mass M we find,

$$F = M\left(\mathrm{g} - \frac{\partial^2\beta}{\partial t^2}\right), \tag{2}$$

Here β is the transverse displacement of the mass and g is the acceleration due to gravity. Therefore, in the above and here in after we use the notation

$$\beta(t) = w(x,t)\big|_{x=u.} \tag{3}$$

Here, the solution of the differential equation is going to be obtained by using dynamic Green's function. Hence, if $G(x,u)$ is the dynamic Green function[4], then the solution of equation (1) is of the form

$$w(x,t) = G(x,u)F \tag{4}$$

Where $G(x,u)$ is the solution of the differential equation

$$\frac{\partial^4 w(x)}{\partial x^4} - q^4 w(x) = \delta(x-u), \tag{5}$$

Where q is the frequency parameter and it is given by

$$q^4 = \frac{\omega^2 \mathrm{m}}{\mathrm{EI}} \tag{6}$$

In which ω is the circular frequency that gives the motion of the mass and is equal to $\pi v/L$.

The solution of equation (5) is assumed in the form

$$G(x,u)=\begin{cases} A_1 cos(qx) + A_2 \sin(qx) + A_3 \cosh(qx) + A_4 \sinh(qx) & 0 \le x \le u \\ B_1 cos(qx) + B_2 \sin(qx) + B_3 \cosh(qx) + B_4 \sinh(qx) & u \le x \le L \end{cases} \quad (7)$$

With proper boundary conditions the Green function determined for cantilever beam is given by[1]

$$G(x,u) = \frac{1}{2EIq^3\Delta}\begin{cases} g(x,u) & 0 \le x \le u \\ g(u,x) & u \le x \le L \end{cases} \quad (8)$$

Where

$$g(x,u) = D_1(\cos \mathrm{qx} - \cosh \mathrm{qx}) + D_2(\sin \mathrm{qx} - \sinh \mathrm{qx}) \quad (9)$$

Here

$$\Delta = 2(1 + \cos \mathrm{qL} \cosh \mathrm{qL})$$

$$\mathrm{D}_1 = (\cos \mathrm{qL} + \cosh \mathrm{qL})(\sin \mathrm{z} + \sinh \mathrm{z}) - (\sin \mathrm{qL} + \sinh \mathrm{qL})(\cos \mathrm{z} + \cosh \mathrm{z})$$

$$\mathrm{D}_2 = (\sin \mathrm{qL} - \sinh \mathrm{qL})(\sin \mathrm{z} + \sinh \mathrm{z}) + (\cos \mathrm{qL} + \cosh \mathrm{qL})(\cos \mathrm{z} + \cosh \mathrm{z})$$

And $g(u,x)$ is obtained by switching x and u in $g(x,u)$ and $z = q(L-u)$.

Eliminating F in equation (4)

$$w(x,u) = G(x,u)\, M\left[g - v^2 \frac{d^2\beta}{du^2}\right], \quad (10)$$

Equation (10) specifies the beam deflection at position x caused by the load at position u.

To find the solution of equation (10) the derivatives are replaced by their finite difference approximation. If the beam is divided into $(N-1)$ intervals each of length h. The discretized equation (10) is

$$w(x_i,u) = G(x_i,u)\, M\left[g - v^2 \frac{d^2\beta}{du^2}\right], \quad (11)$$

Here the subscript i refers to any one of the N discrete station points. The Houbolt method is chosen, since u is equivalent to the time variable. $f(u)$ is approximated as [2]

$$\frac{d^2 f}{du^2} = \frac{1}{h^2}\sum_{k=0}^{3} a_k\, f(u_{j-k}) + O(h^2) \quad (12)$$

Where u_j indicates that the mass is at the jth station point. The co-efficient a_k are $a_0 = 2$, $a_1 - 5$, $a_2 = 4, a_3 = -1$.

Application of equation (12) to equation (11) results in the following set of algebraic equations:

$$h^2 w(x_i,u_j) = G(x_i,u_j) M\left[h^2 g - v^2 \sum_{k=0}^{3} a_k\, w(x_{j-k},u_j)\right], \qquad i,j = 1,2, \dots \dots, N \quad (13)$$

Equation (13) can be expressed in matrix form[1]:

$$[h^2[I] + a_0 v^2 [G] M [P_{ij}]]\{w_{u_j}\} = M[G]\ [h^2 g\{\Delta_{u_j}\} - v^2 \sum_{k=1}^{3} a_k\, [P_{j,j-k}]\{w_{u_{j-k}}\}], \quad (14)$$

Equation (14) describes the transverse displacement of the beam when the mass is at any station j. In order to include the initial conditions, which describes the displacement and slope of beam while in motion; one looks up to the change of variables introduced earlier. Accordingly, the initial conditions correspond to the mass being at the initial station point $j = 0$. For this station, equation (14) requires a value of $\{w_{u_j}\}$, where $k \le 0$. These values may be set equal to zero.

The speed ratio α is defined as

$$\alpha = \frac{vL}{\pi}\sqrt{\frac{M}{EI}} \tag{15}$$

And the critical speed v_{cr} is defined as

$$v_{cr} = \frac{2L}{T} = \frac{\pi}{L}\sqrt{\frac{EI}{m}} \tag{16}$$

The case $\frac{v}{v_{cr}} = 1$ corresponds to resonance with the fundamental mode when the load is a constant force.

3. Numerical Analysis

The displacement of the beam at any time can be obtained from equation (14).The values of the dynamic response have been calculated for cantilever beams. The materials considered is steel, which are mostly used in the construction of the bridges. For these beams the displacements at the end or middle points on the beam are found for different values of mass and its speed. In equation (14) In order to evaluate approximate response of beams the derivatives are replaced by finite difference approximation.

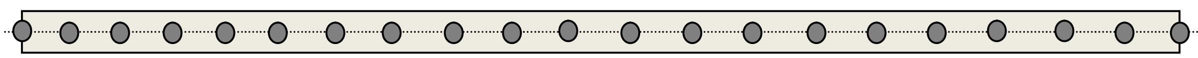

Fig 2. Beam with 21 stations

Here the beam is divided into 20 intervals of length 2.5 m for the total span of 50 m. So the number of discrete station points located on the beam are 21 stations, therefore displacement of the beams can be found at 21 points while mass is moving on the span.

1.Beam type: Cantilever Beam
2. Material: Steel (E=200 GPa)
3. Weight of the moving mass: 25000 kg

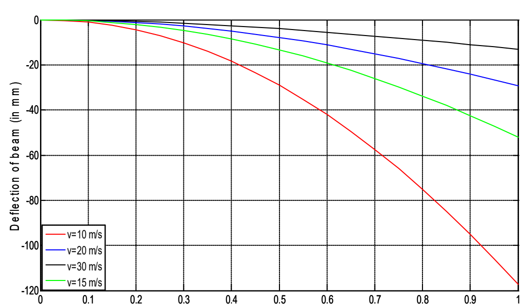

Fig.3 Deflections of Cantilever Beam at the end point for different Velocities as shown

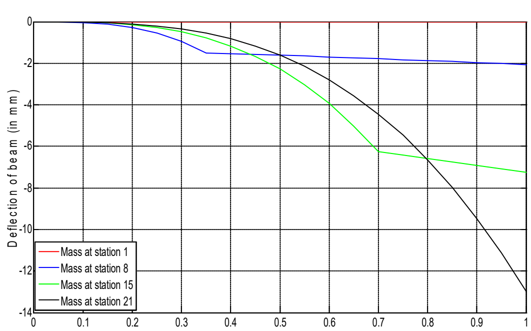

Fig.4 Shape of the beam when the mass is moving through different stations at velocity of 30 m/s

1.Beam type: Cantilever Beam
2. Material: Steel (E=200 GPa)
3. Weight of the moving mass: 50000 kg

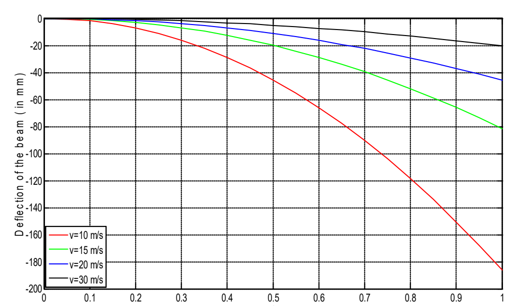

Fig.6 Deflections of Cantilever Beam at the end point for different Velocities as shown

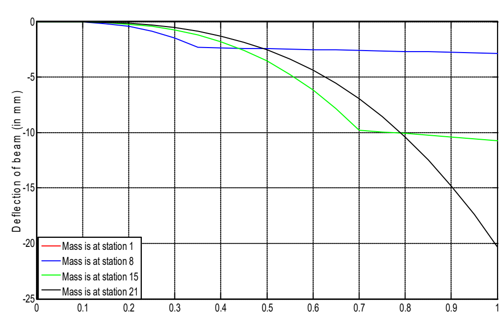

Fig.7 Shape of the beam when the mass is moving through different stations at velocity of 30 m/s

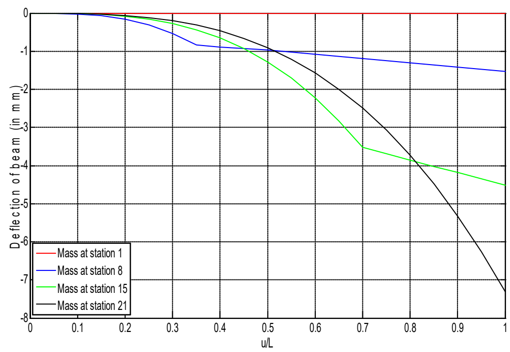

Fig.5 Shape of the beam when the mass is moving through different stations at velocity of 40 m/s

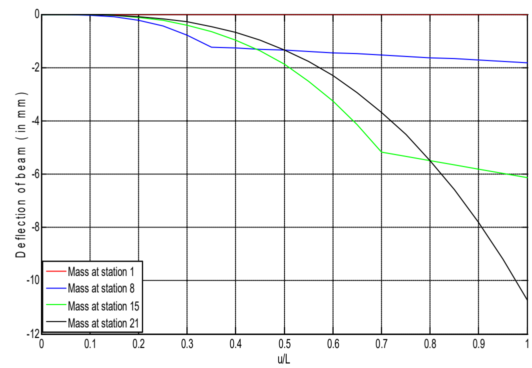

Fig.8 Shape of the beam when the mass is moving through different stations at velocity of 40 m/s

4. Conclusion

Numerical results have been presented for the dynamic response of cantilever beam with a moving mass. Results have been shown for Deflection Vs Position of the beam. The following observations are made from the numerical Analysis:

1. In case of Cantilever Beam, it is observed that as the velocity of the moving mass increases, the deflection at the free end point of the beam decreases. This is because at higher velocity of the moving mass, lower modes are not excited, which mainly contribute for larger dynamic deflection of the beam. As the lower modes of vibration contribute larger amplitude as compared to that of higher modes, dynamic deflection of the beam decreases.

2. It is also observed for a cantilever beam that as the mass of the moving body increases, the end deflection also increases. This is due to increase in the inertia of the moving mass.

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