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A Novel Numerical Technique for Free Convective Heat and Mass Transfer under Mixed Thermal Boundary Condition

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This work deals with a novel numerical technique for similarity analysis of the effect of magnetic field on free convection in micropolar fluid about a vertical plate in the presence of mixed thermal boundary conditions and non-uniform wall concentration boundary conditions. The governing boundary layer equations of the physics of the present problem are cast into coupled nonlinear ordinary differential equations using the Lie Group point transformations. These equations involving in a non-negative parameter m which assumes the values $0, 1$and ∞ for the cases of prescribed wall temperature, prescribed heat flux, and radiation boundary condition, has been solved using the novel numerical technique, paired quasi-linearisation method (PQLM). The results are validated and are found to be in good agreement with previously published results as special cases of the present investigation.

***Keywords*:** Mixed Thermal Boundary Conditions, Micropolar Fluid, MHD, Paired QLM

1. Introduction

Sparrow and Gregg[1] have pioneered the problem of the convective heat transfer from surface of a heated vertical plate. Later a great deal of attention has been drawn by many researchers on the problem of free convection next to heated vertical surface. These earlier studies of the free-convection boundary–layer flow over a heated surface have dealt with numerical solutions associated with either prescribed surface temperature or heat flux. The problem of free convection has not received enough attention when the plate subjected to a radiation boundary condition (RBC) or when it is subject to a mixed thermal boundary condition. The mixed thermal boundary condition associated with the case $m \to \infty$ is known as radiation boundary condition and corresponds to the leading order approximation to the heat transfer by radiation from the surface to the medium when the temperature difference is relatively small which in turn leads to the definition of heat transfer coefficient for the fluid. A similarity analysis of free convection about a wedge and a cone which are subjected to mixed thermal boundary conditions have been studied by Ramanaiah and Malarvizhi[2]. Ece[3] analyzed a similarity analysis to investigate the laminar free-convection boundary–layer flow in the presence of a transverse magnetic field over a wedge with mixed thermal boundary condition. Cheng [4] studied the steady natural convection boundary layer flow over a downward pointing vertical cone in porous media saturated with non-Newtonian power–law fluids under mixed thermal boundary condition. The aim of the present study is to obtain similarity solutions for the velocity, microrotation, temperature and concentration profiles under the mixed thermal boundary condition.

2. Mathematical Formulation

Consider a steady, laminar, incompressible, two-dimensional free convective heat and mass transfer along a heated semi infinite vertical plate embedded in a micropolar fluid. Choose the co-ordinate system such that $\bar{x}-$axis is along the vertical plate and $\bar{y}-$axis normal to the plate. T_∞ and C_∞ are the ambient temperature and ambient concentration, respectively. By employing laminar boundary layer flow assumptions and Boussinesq approximation the governing equations are:

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}=\frac{\mu+\kappa}{\rho}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\kappa}{\rho}\frac{\partial \bar{\omega}}{\partial \bar{y}}-\frac{\sigma B_0{}^2}{\rho}\bar{u}+g^*\left(\beta_T\left(\bar{T}-T_\infty\right)+\beta_C\left(\bar{C}-C_\infty\right)\right) \tag{2}$$

$$\bar{u}\frac{\partial \bar{\omega}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\omega}}{\partial \bar{y}}=\frac{\gamma}{j\rho}\frac{\partial^2 \bar{\omega}}{\partial \bar{y}^2}-\frac{\kappa}{j\rho}\left(2\bar{\omega}+\frac{\partial \bar{u}}{\partial \bar{y}}\right) \tag{3}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}=\alpha\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{4}$$

$$\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}=D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the components of velocity along $\bar{x}$ and $\bar{y}$ directions respectively, $\bar{\omega}$ is the component of microrotation whose direction of rotation lies normal to the $\bar{x}\,\bar{y}-$plane, g^* is the gravitational acceleration, $\bar{T}$ is the temperature, $\bar{C}$ is the concentration, β_T is the coefficient of thermal expansions, β_C is the coefficient of solutal expansions, μ is the dynamic coefficient of viscosity of the fluid, κ is the vortex viscosity, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, ν is the kinematic viscosity, α is the thermal diffusivity, D is the molecular diffusivity. We follow the work of many recent authors by assuming that $\gamma=(\mu+\kappa/2)j$. This assumption is invoked to allow the field of equations predicts the correct behaviour in the limiting case when the microstructure effects become negligible and the total spin $\bar{\omega}$ reduces to the angular velocity.

The boundary conditions are:

$$\left.\begin{array}{ll}\bar{u}=0,\bar{v}=0,\bar{\omega}=0,\bar{C}=\bar{C}_w(\bar{x}) & at\ \bar{y}=0\\ \bar{u}\to 0,\bar{\omega}\to 0,\ \bar{T}\to T_\infty,\ \bar{C}\to C_\infty(\bar{x})\ as\,\bar{y}\to\infty & \end{array}\right\} \tag{6}$$

The generalized form of the mixed thermal boundary condition on the surface of the vertical plate $\bar{y}=0$ is assumed to be:

$$a_0(\bar{x})(\bar{T}-T_\infty)_{\bar{y}=0}-a_1(\bar{x})\frac{\partial \bar{T}}{\partial \bar{y}}=a_2(\bar{x}) \tag{7}$$

where the subscripts w and ∞ indicates the conditions at wall and at the outer edge of the boundary layer, respectively.

3. Method of solution

Introducing the following non-dimensional variables

$$\left.\begin{array}{l}x=\dfrac{\bar{x}}{L},\ y=\dfrac{\bar{y}}{L}Gr^{\frac{1}{4}},\ u=\dfrac{\bar{u}L}{\nu\, Gr^{\frac{1}{2}}},\ v=\dfrac{\bar{u}L}{\nu\, Gr^{\frac{1}{4}}},\ \omega=\dfrac{\bar{\omega}L^2}{\nu\, Gr^{\frac{3}{4}}},\ Gr=\dfrac{g^*\beta_T\Delta T L^3}{\nu^2},\\ \theta=\dfrac{\bar{T}-T_\infty}{\bar{T}_w(\bar{x})-T_\infty},\ x\Delta T=\bar{T}_w(\bar{x})-T_\infty,\ \phi=\dfrac{\bar{C}-C_\infty}{\bar{C}_w(\bar{x})-C_\infty},\ x\Delta C=\bar{C}_w(\bar{x})-C_\infty\end{array}\right\} \tag{8}$$

and the stream function ψ through $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ in to the Eqs. (1) - (5) and (6) - (7) we get

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=(1+K)\frac{\partial^3\psi}{\partial y^3}+K\frac{\partial\omega}{\partial y}+x(\theta+B\emptyset)-M\frac{\partial\psi}{\partial y} \tag{9}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}=(1+K/2)\frac{\partial^2\omega}{\partial y^2}-K\left(2\omega+\frac{\partial^2\psi}{\partial y^2}\right) \tag{10}$$

$$x\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-x\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}+\theta\frac{\partial\psi}{\partial y}=\frac{1}{Pr}x\frac{\partial^2\theta}{\partial y^2} \tag{11}$$

$$x\frac{\partial\psi}{\partial y}\frac{\partial\emptyset}{\partial x}-x\frac{\partial\psi}{\partial x}\frac{\partial\emptyset}{\partial y}+\emptyset\frac{\partial\psi}{\partial y}=\frac{1}{Sc}x\frac{\partial^2\emptyset}{\partial y^2} \tag{12}$$

where $K=\kappa/\mu$ is the Coupling number, $B=(\beta_C\Delta C)/(\beta_T\Delta T)$ is the buoyancy parameter, $M=(\sigma B_0^2L^3)/(\rho\nu^2 Gr)$ is the Magnetic parameter $Pr=\nu/\alpha$ is the Prandtl number and $Sc=\nu/D$ is the Schmidt number.

The transformed boundary conditions are

$$\frac{\partial\psi}{\partial y}=0,\frac{\partial\psi}{\partial x}=0,\omega=0,b_0\Delta T\theta(x,0)-b_1(\Delta T)^{\frac{5}{4}}\theta'^{(x,0)}=1,\emptyset(x,0)=1\ at\ y=0 \tag{13}$$

$$\frac{\partial\psi}{\partial y}\to 0,\omega\to 0,\theta\to 0,\emptyset\to 0\ \ as\ y\to 0 \tag{14}$$

The functions $b_0(x)$ and$b_1(x)$ of Eq.(13) are given by$b_0(x) = \left(a_0(\bar{x})\right)/\left(a_2(\bar{x})\right)$ and $b_1(x) = \left(a_1(\bar{x})\right)/\left(a_2(\bar{x})\right)$. Each of these functions must be equal to a constant to enable a similarity solution and they are referred to as b_0 and $b_1$hereon. For given values of the constants b_0, b_1 and T_∞ the reference temperature T may be chosen to satisfy the following equation without any loss of generality, $b_1(\Delta T)^{\frac{5}{4}} + b_0\Delta T - 1 = 0$. By defining$m = b_1(\Delta T)^{\frac{5}{4}}$, the thermal boundary condition at $y = 0$ can be written as

$$(1-m)\theta(x,0) - m\theta'(x,0) = 1 \tag{15}$$

4. Similarity solutions via Lie group analysis

We now introduce the one-parameter scaling group of transformations which is a simplified form of Lie group transformation

$$\tau: \hat{x} = xe^{\varepsilon\alpha_1}, \hat{y} = ye^{\varepsilon\alpha_2}, \hat{\psi} = \psi e^{\varepsilon\alpha_3}, \hat{\omega} = \omega e^{\varepsilon\alpha_4}, \hat{\theta} = \theta e^{\varepsilon\alpha_5}, \hat{\emptyset} = \emptyset e^{\varepsilon\alpha_6} \tag{16}$$

where$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are transformation parameters and ε is a small parameter. This scaling group of transformations transform coordinates $(x, y, \psi, \omega, \theta, \emptyset)$ to $(\hat{x}, \hat{y}, \hat{\psi}, \hat{\omega}, \hat{\theta}, \hat{\emptyset})$. Eqs. (9) – (12) and boundary conditions (13) – (15) are invariant under the point transformations (16), and reduces to

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial^2\hat{\psi}}{\partial\hat{x}\partial\hat{y}} - \frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial^2\hat{\psi}}{\partial\hat{y}^2}\right) = (1+K)e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\hat{\psi}}{\partial\hat{y}^3} + Ke^{\varepsilon(\alpha_2-\alpha_4)}\frac{\partial\hat{\omega}}{\partial\hat{y}} + \hat{x}\left(e^{\varepsilon(\alpha_1-\alpha_5)}\hat{\theta} + Be^{\varepsilon(\alpha_1-\alpha_6)}\hat{\emptyset}\right) - e^{\varepsilon(\alpha_2-\alpha_3)}M\frac{\partial\hat{\psi}}{\partial\hat{y}} \tag{17}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_4)}\left(\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial\hat{\omega}}{\partial\hat{x}} - \frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial\hat{\omega}}{\partial\hat{y}}\right) = \left(1+\frac{K}{2}\right)e^{\varepsilon(2\alpha_2-\alpha_4)}\frac{\partial^2\hat{\omega}}{\partial\hat{y}^2} - K\left(2e^{-\varepsilon\alpha_4}\hat{\omega} + e^{\varepsilon(2\alpha_2-\alpha_3)}\frac{\partial^2\hat{\psi}}{\partial y^2}\right) \tag{18}$$

$$e^{\varepsilon(\alpha_2-\alpha_3-\alpha_5)}\left(\hat{x}\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial\hat{\theta}}{\partial\hat{x}} - \hat{x}\frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial\hat{\theta}}{\partial\hat{y}} + \hat{\theta}\frac{\partial\hat{\psi}}{\partial\hat{y}}\right) = \frac{1}{Pr}e^{\varepsilon(-\alpha_1+2\alpha_2-\alpha_5)}\hat{x}\frac{\partial^2\hat{\theta}}{\partial\hat{y}^2} \tag{19}$$

$$e^{\varepsilon(\alpha_2-\alpha_3-\alpha_6)}\left(\hat{x}\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial\hat{\emptyset}}{\partial\hat{x}} - \hat{x}\frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial\hat{\emptyset}}{\partial\hat{y}} + \hat{\emptyset}\frac{\partial\hat{\psi}}{\partial\hat{y}}\right) = \frac{1}{Sc}e^{\varepsilon(-\alpha_1+2\alpha_2-\alpha_6)}\hat{x}\frac{\partial^2\hat{\emptyset}}{\partial\hat{y}^2} \tag{20}$$

and the associated boundary conditions become

$$\left.\begin{aligned} &\frac{\partial\hat{\psi}}{\partial\hat{y}} = 0, \frac{\partial\hat{\psi}}{\partial\hat{x}} = 0, \hat{\omega} = 0, \hat{\emptyset}e^{-\varepsilon\alpha_6} = 1 \ at\ \hat{y}e^{-\varepsilon\alpha_2} = 0 \\ &\frac{\partial\hat{\psi}}{\partial\hat{y}} \to 0, \hat{\omega} \to 0, \hat{\theta} \to 0, \hat{\emptyset} \to 0 \quad as \quad \hat{y}e^{-\varepsilon\alpha_2} \to \infty \end{aligned}\right\} \tag{21}$$

and the mixed thermal boundary condition reduces to

$$(1-m)e^{-\varepsilon\alpha_5}\hat{\theta} - me^{\varepsilon(\alpha_2-\alpha_5)}\hat{\theta}' = 1 \tag{22}$$

Since the group transformations (16) keep the system invariant, hence the relations among the parameters from the Equations (17) – (20) is as below

$$\left.\begin{aligned} \alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 = \alpha_2 - \alpha_4 = \alpha_1 - \alpha_5 = \alpha_1 - \alpha_6 = \alpha_2 - \alpha_3, \\ \alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 = 2\alpha_2 - \alpha_4 = -\alpha_4 = 2\alpha_2 - \alpha_3 \\ \alpha_2 - \alpha_3 - \alpha_5 = -\alpha_1 + 2\alpha_2 - \alpha_5 \\ \alpha_2 - \alpha_3 - \alpha_6 = -\alpha_1 + 2\alpha_2 - \alpha_6 \end{aligned}\right\} \tag{23}$$

These relations give $\alpha_1 = \alpha_3 = \alpha_4$ & $\alpha_2 = 0 = \alpha_5 = \alpha_6$, so the infinitesimal transformations reduces to point transformations in one parameter as follows

$$\tau: \hat{x} = xe^{\varepsilon\alpha_1}, \hat{y} = y, \hat{\psi} = \psi e^{\varepsilon\alpha_1}, \hat{\omega} = \omega e^{\varepsilon\alpha_1}, \hat{\theta} = \theta, \hat{\emptyset} = \emptyset \tag{24}$$

After expanding the Lie group of point transformations in one parameter using the Taylor's series in powers of ε by considering the terms up to $\boldsymbol{O}(\varepsilon)$, we get

$$\hat{x} - x = x\varepsilon\alpha_1, \hat{y} - y = 0, \hat{\psi} - \psi = \psi\varepsilon\alpha_1, \hat{\omega} - \omega = \omega\varepsilon\alpha_1, \hat{\theta} - \theta = 0, \hat{\emptyset} - \emptyset = 0 \tag{25}$$

The corresponding characteristic equations are given by

$$\frac{dx}{x\alpha_1} = \frac{dx}{0} = \frac{d\psi}{\psi\alpha_1} = \frac{d\omega}{\omega\alpha_1} = \frac{d\theta}{0} = \frac{d\emptyset}{0} \tag{26}$$

The self similar solutions of characteristic equations (26) give the similarity transformations as $\hat{y} = \eta, \hat{\psi} = \hat{x}f(\eta), \hat{\omega} = \hat{x}\,g(\eta), \hat{\theta} = \theta(\eta), \hat{\emptyset} = \emptyset(\eta)$, these reduce the Eqs. (17) - (20) into

$$(1+K)f''' - (f')^2 + ff'' + Kg' + \theta + B\emptyset - Mf' = 0 \tag{27}$$

$$\left(1+\frac{K}{2}\right)g'' + fg' - gf' - K(2g + f'') = 0 \tag{28}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta = 0 \quad (29)$$
$$\frac{1}{Sc}\emptyset'' + f\emptyset' - f'\emptyset = 0 \quad (30)$$
The boundary conditions (21) and (22) in terms of f, g, θ and $\emptyset$ now get transformed into
$$\left.\begin{aligned} f' = 0, f = 0, g = 0, \emptyset = 1 \quad at\ \eta = 0 \\ f' \to 0, g \to 0, \theta \to 0, \emptyset \to 0\ \ as\ \eta \to \infty \end{aligned}\right\} \quad (31)$$
and the mixed thermal condition at $\eta = 0$ is given by
$$(1 - m)\theta(0) - m\theta'(0) = 1 \quad (32)$$
The following cases of mixed thermal boundary condition are of special interest:

Case(a): Variable Wall Temperature(VWT): If $T = T_w(x)$ on the wall (i.e., at y = 0), where $T_w(x) = T_\infty + \Delta T x^m$ gives $m = 0$ thus mixed thermal boundary condition reduces to the form $\theta(0) = 1$

Case(b): Variable Wall Heat Flux(VWHF):

In the case of variable wall heat flux, if the thermal boundary condition on the wall is $-\frac{\partial T}{\partial y} = \Delta T x^m$, then this gives $m = 1$, hence, the mixed thermal boundary condition reduces to $\theta'(0) = -1$

Case(c): Radiation Boundary Condition (RBC): At the wall i.e., at *y = 0*, radiative boundary condition $-\frac{\partial T}{\partial y} = \Delta T\, x^m(T_w(x) - T_\infty)$ gives $m \to \infty$, hence the mixed thermal boundary condition reduces to $\theta(0) + \theta'(0) = 0$.

5. Skin-Friction and Wall Couple Stress

The non-dimensional skin friction C_f and wall couple stress M_w are given by

$\frac{Re^2}{Gr^{\frac{3}{4}}}C_f = 2(1 + K)f''(0)$ and $\frac{Re^2}{Gr}M_w = \left(1 + \frac{K}{2}\right)g'(0)$.

6. Results and Discussion

Similarity analysis of free-convection boundary-layer flow about a vertical plate embedded in a micropolar fluid under mixed thermal boundary conditions in the presence of a transverse magnetic field using Lie group point transformations is studied. Using the Lie group point transformations on the governing equations result in a set of coupled second-order non-linear differential equations (27) – (30) for $f(\eta), g(\eta), \theta(\eta)$ and $\emptyset(\eta)$ along the boundary conditions Eqs. (31) and (32). These equations (27) – (30) were solved numerically for the radiation boundary condition (RBC) case $(m \to \infty)$, by employing the local linearisation method which linearise the equations then apply the spectral collocation method for discretization and subsequent solution. We remark that the choice of linearisation method is influenced by the wall temperature conditions under investigation. It was observed that the local linearization method (LLM) [5] is applicable for the variable wall temperature and variable wall heat flux condition. Applying the LLM on the radiation boundary condition gives only the trivial solution $\theta = 0$ for temperature which is not meaningful physically. The non-trivial solution can be obtained by linearising the momentum and energy equations as a coupled pair of equations with unknown functions f and θ at any level of iteration. Below, we give the development of the iteration scheme used for the generation of results under the different wall temperature conditions. The derivation of the LLM scheme starts with writing the system of ODEs in compact operator form as
$$\Omega_1[F(\eta)] = 0, \Omega_2[G(\eta)] = 0, \Omega_3[T(\eta)] = 0, \Omega_4[P(\eta)] = 0 \quad (33)$$
where $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 are non-linear operators that denote equation (27 – 30) respectively, and F, G, T, P are defined as
$$F = \left\{f, \frac{df}{d\eta}, \frac{d^2f}{d\eta^2}, \frac{d^3f}{d\eta^3}\right\}, G = \left\{g, \frac{dg}{d\eta}, \frac{d^2g}{d\eta^2}\right\}, T = \left\{\theta, \frac{d\theta}{d\eta}, \frac{d^2\theta}{d\eta^2}\right\}, P = \left\{\emptyset, \frac{d\emptyset}{d\eta}, \frac{d^2\emptyset}{d\eta^2}\right\}$$
The assumption made in presenting the governing equations in the form (33) is that at any iteration level the dominant unknown functions are the ones corresponding to the term with the highest derivative in each equation. Linearising the first equation by applying the Taylor series expansion of Ω_1 about some previous approximation of the solution denoted by f_r, gives
$$(1 + K)f'''_{r+1} + a_{11}^{(2)}f''_{r+1} + a_{11}^{(1)}f'_{r+1} + a_{11}^{(0)}f_{r+1} = R_1 \quad (34)$$

$f_{r+1} = 0, f'_{r+1} = 0, f_{r+1}(\eta) = 0$ as $\eta \to \infty$ (35)
where $a_{11}^{(2)} = f_r$, $a_{11}^{(1)} = -2f'_r - M$, $a_{11}^{(0)} = f''_r$, $R_1 = -Kg'_r - \theta_r - B\emptyset_r + f_r f''_r - (f'_r)^2$

In the above equation $r+1$ and r denote the current and previous iteration levels. It is worth noting that in view of the assumption that the solutions for g, θ and $\emptyset$ are known at the previous iteration level the solution for f can be obtained by solving (34) and (35) independently. The rest iteration schemes derived from the others equations are given by

$\left(1+\frac{K}{2}\right) g''_{r+1} + a_{22}^{(1)} g'_{r+1} + a_{22}^{(0)} g_{r+1} = R_2, g_{r+1} = 0, g_{r+1}(\eta) = 0$ as $\eta \to \infty$ (36)

$\frac{1}{Pr}\theta''_{r+1} + a_{33}^{(1)}\theta'_{r+1} + a_{33}^{(0)}\theta_{r+1} = R_3, (1-m)\theta_{r+1}(0) - m\theta'_{r+1}(0) = 1, \theta_{r+1}(\eta) = 0$ as $\eta \to \infty$ (37)

$\frac{1}{Sc}\emptyset''_{r+1} + a_{44}^{(1)}\emptyset'_{r+1} + a_{44}^{(0)}\emptyset_{r+1} = R_4, \emptyset_{r+1}(0) = 1, \emptyset_{r+1}(\eta) = 0$ as $\eta \to \infty$ (38)

The coefficients are defined as

$$a_{22}^{(1)} = f_{r+1}, a_{22}^{(0)} = -f'_{r+1} - 2K, a_{33}^{(1)} = f_{r+1}, a_{33}^{(0)} = -f'_{r+1}$$
$$a_{44}^{(1)} = a_{33}^{(1)}, a_{44}^{(0)} = a_{33}^{(0)}, \quad R_2 = 2Kf''_{r+1}, R_3 = R_4 = 0$$

To solve the linearised system of equations (34 – 38) we use the standard spectral collocation method that transforms the continuous derivatives to the matrix vector products at N selected collocation points $\eta_j = cos\left(\frac{\pi j}{N}\right)$, according to the definition

$f'(\eta_j) = \sum_k^N D_{j,k} f(\eta_k), \ j = 0,1,2,\dots,N.$ (39)

The use of (39) leads to

$\boldsymbol{F}' = \boldsymbol{D}\boldsymbol{F}, \boldsymbol{F}'' = \boldsymbol{D}^2\boldsymbol{F}, \boldsymbol{F}''' = \boldsymbol{D}^3\boldsymbol{F}$ (40)

Where $\boldsymbol{D}$ is the scaled differentiation matrix whose entries are defined in [6] and $\boldsymbol{F} = [f(\eta_0), f(\eta_1), \dots, f(\eta_N)]^{\boldsymbol{T}}$. Using equation (40) on the linearized equations (34 – 38) gives

$$\boldsymbol{A_1 F} = \boldsymbol{R_1}, \boldsymbol{A_2 G} = \boldsymbol{R_2}, \boldsymbol{A_3 T} = \boldsymbol{R_3}, \boldsymbol{A_4 P} = \boldsymbol{R_4}$$

where $\boldsymbol{A_1} = (1+K)\boldsymbol{D^3} + a_{11}^{(2)}\boldsymbol{D^2} + a_{11}^{(1)}\boldsymbol{D} + a_{11}^{(0)}, \boldsymbol{A_2} = \left(1+\frac{K}{2}\right)\boldsymbol{D^2} + a_{22}^{(1)}\boldsymbol{D} + a_{22}^{(0)}$

$$\boldsymbol{A_3} = \frac{1}{Pr}\boldsymbol{D}^2 + a_{33}^{(1)}\boldsymbol{D} + a_{33}^{(0)}, \boldsymbol{A_4} = \frac{1}{Sc}\boldsymbol{D}^2 + a_{44}^{(1)}\boldsymbol{D} + a_{44}^{(0)}$$

To analyze the results for the present investigation under the radiation boundary condition (case(c)) $m \to \infty$ case, the values of $f''(0)$ and $\theta(0)$ were given in the Table 1 for $K = 0, M = 0, B = 0$ and $Pr = 10.0$. These values compared with the results given by Buyuk and Ece \cite{Ece1} for free convection in the absence of magnetic field were found to be in good agreement.

For the case (c), i.e., radiation boundary condition (RBC) case the Figs.1(a) – 1(d) depict the variation of coupling number (K) on the profiles of velocity, microrotation, temperature and concentration with η. As for the consideration of the fluid model, the coupling number K characterises as a result of the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, $K = \kappa/\mu$ signifies the coupling between the Newtonian and rotational viscosities. The effect of microstructure becomes significant for the increasing values of K while the individuality of the substructure is much less pronounced for the decreasing values of K. We can see that in the limiting case of $K \to 0$, i.e., $\kappa \to 0$, the micropolarity behav ior of the fluid vanishes and the fluid behaves as nonpolar fluid, and this leads to the case of viscous fluid. It is observed from Fig.1(a) that as the value of K increases there is a asymptotical decrease in the velocity. The maximum of velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall with an increase of K. From the viscous case $(K \to 0)$ to the non-viscous case the velocity is decreasing. It is seen from Fig.1(b) that the microrotation component decreases near the vertical plate and increases far away from the plate with increasing K. As $\kappa \to 0$, i.e., $K \to 0$, the Eqs. (1) and (2) are uncoupled with Eq. (3) and they reduce to viscous fluid flow equations. It is very interesting to notice from Fig.1(c) that the temperature increases wit-

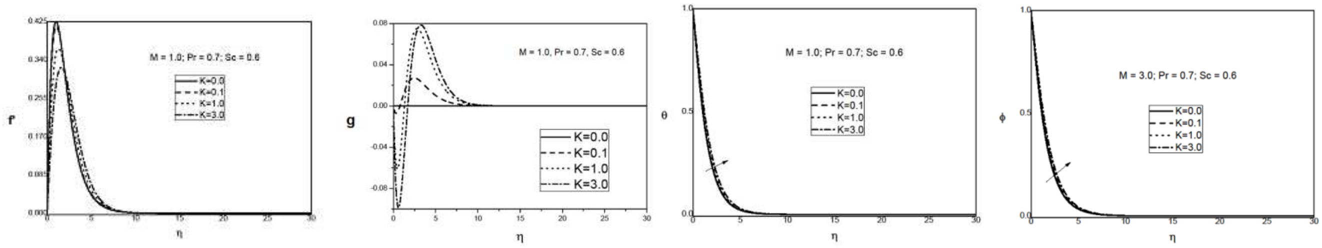

Fig. 1. Velocity, microrotaton, temper ature and concentration profiles for different values of K.

h the increasing values of coupling number. This is because of the decrease in fluid velocity, which causes to decrease the replacement of hot fluid chunks near the plate by cold fluid chunks. Same trend can be observe in Fig.1(d), that the non-dimensional concentration increases from Newtonian case to non-Newtonian case (with increasing values of K).

Figs.2(a) and 2(b) depict that the variation of heat and mass transfer rates (Nusselt number (Nu) and Sherwood number(Sh)) with coupling number$(N)$for different values of magnetic parameter(M). From the Figs. 2(a) and 2(b) it is seen that both the Nusselt number and Sherwood number are decreasing as the magnetic parameter is increasing. This is due to the Lorentz force created by transverse magnetic field, which tends to resist the flow. This leads to slow down the mechanism of replacement of hot fluid at the surface of the plate by the cold fluid chunks away from the plate. Hence the heat and mass transfer rates decrease as M increases.

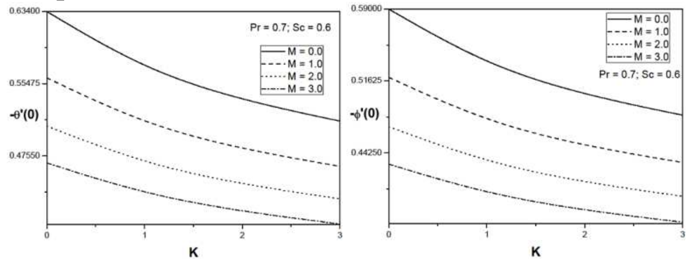

Table 1

	[7]	Present
$f''(0)$	0.283227	0.283227
$\theta'(0)$	0.496337	0.496337

Fig. 2. Effect of magnetic parameter (*M*) on (a) heat transfer rate (b) mass transfer rate for different values of K.

References

1. E. M. Sparrow and J. L. Gregg, ASME Trans. 78, 435-440, (1956)
2. G. Ramanaiah and G. Malarvizhi, Acta Mechanica, 93, 119-123, (1992)
3. M. C. Ece, Heat and Mass Transfer, 41(4), 291-297, (2005)
4. Ching-Yang Cheng, International Communications in Heat and Mass Transfer, 36, 693-697, (2009)
5. S. S. Motsa, Journal of Applied Mathematics, 2013, 189 – 201 (2013)
6. C. Canuto, M. Y. Hussaini, A. Quarteroni and T.A. Zang, Spectral Methods in Fluid Dynamics, Springer-Verlag, Berlin (1988)
7. Buyuk and M. C. Ece, Proceedings of the international mechanical engineering congress and exposition (IMECE), (1999), November; Nashville