# Effects of radiation and radiation absorption on unsteady MHD flow past a vertical porous flat plate in a rotating system with chemical reaction in a Nanofluid

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**Abstract:** In this paper unsteady MHD Ag-nanofluid based boundary flow in a rotating system with chemical reaction and radiation in presence of heat absorption has been investigated. The partial differential equations of system were solved using perturbation technique. The influence of diverse physical parameters on concentration, velocity as well as temperature and fields are illustrated graphically. In this examination it was established that the Nano fluid velocity rises with the enhancement of porous medium and thermal radiation. However, concentration, velocity as well as temperature declined with amplification in chemical reaction.

**Keywords:** Radiation, Ag-water Nanofluid, sphere shape particles, heat transfer, porous.

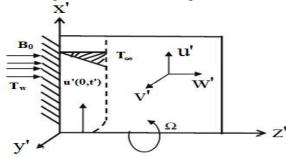
### 1.Introduction

Choi [1] was the first one who reported regarding the fluids, renowned as Nano fluids in which particles of Nano scale are suspended in the base fluid. Aluminium, copper, iron and titanium or their oxides are some of the common Nano particles which are used widely. Owing to high thermal conductivity, Nano fluids have several useful applications and they are used on a large scale in manufacturing industries, fuel cells, solid-state lighting, and as coolants in auto mobiles etc. Numerous methods have been developed to improve the thermal conductivity of these fluids by suspending Nano particles in liquids. Khanafer et al. [2] analyzed buoyancy driven heat transfer enhancement in a two dimensional enclosure utilizing Nano fluids. Ishak [3] examined analytically, similarity solutions for flow as well as heat transfer over a permeable surface by means of convective boundary condition. KhairyZaimi et al. [4] reported the flow as well as heat transfer over a shrinking sheet in a nanofluid with suction at the boundary. Kim et.al. [5] investigated convective instability and heat transfer characteristics of the nanofluids. Hamad and Pop [6] examined unsteady MHD natural convective flow past a vertical permeable flat plate in a rotating frame of reference with constant heat source in a nanofluid. The fluid flow over a rotating and stretching disk and the flow between two stretching disks was studied by Fang ([7] & [8]). Prasad et al. [9], Turkyilmazoglu [10] discussed on MHD Nano fluid flows over a stretching sheet in dissimilar environments. Fang and Hua [11], Rashidi et al [12] investigated the fluid flows over stretchable rotating disks. Bachok et al. [13] used Keller-Box technique for steady Nano fluid flow over a porous rotating disk.

The main aim is to examine the unsteady MHD Ag-nanofluid based boundary flow in a rotating system with chemical reaction and radiation in presence of heat absorption. The partial differential equations of system were solved using perturbation technique. The influence of diverse physical parameters on concentration, velocity as well as temperature and fields are illustrated graphically.

## 2. Mathematical formulation and solution of the problem

Consider unsteady MHD three dimensional incompressible, electrically conducting nanofluid flows which passes through a semi-infinite vertical permeable plate which is grey emitting or absorbing but not scattering medium in the influence of heat absorption and radiation by nonappearance of an electric field.  $B_0$  is exterior magnetic field parameter. Here the fluid flow is in the x' – direction and z' axis is normal to the plate, whole system is rotates about the z'-axis with a constant vector  $\Omega$ . The radiation heat flux in x'-direction is insignificant in comparison to that in the z'- direction and the flow variables are functions of z & time t only. Supposed that the regular fluid as well as the suspended Nano particles is in thermal equilibrium and no slip takes place among them. Boundary layer approximations, the boundary layer equations governing the flow, concentration and temperature along with the Boussinesq are:



Physical Model and Co-ordinate system

$$\frac{\partial w'}{\partial z'} = 0 \quad \Rightarrow w' = -w_0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + w' \frac{\partial u'}{\partial z'} - 2\Omega v' = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u'}{\partial z'^2} + \frac{[\rho \beta]_{nf}}{\rho_{nf}} g(T - T_{\infty}) - \frac{1}{\rho_{nf}} \sigma B_0^2 u' - \frac{1}{\rho_{nf}} \frac{v_f u'}{K'}$$

$$\tag{2}$$

$$\frac{\partial v'}{\partial t'} + w' \frac{\partial v'}{\partial z'} + 2\Omega u' = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v'}{\partial z'^2} - \frac{1}{\rho_{nf}} \sigma B_0^2 v' - \frac{1}{\rho_{nf}} \frac{v_f u'}{K'}$$
(3)

$$\frac{\partial T}{\partial t'} + w' \frac{\partial T}{\partial t'} = \alpha_{nf} \frac{\partial^2 T}{\partial z'^2} - \frac{1}{[\rho C_p]_c} \left( Q'(T - T_{\infty}) + \frac{\partial q_r}{\partial z'} \right) + Q_1^* (C - C_{\infty})$$
(4)

$$\frac{\partial C}{\partial t'} + w' \frac{\partial C}{\partial z'} = D^* \frac{\partial^2 C}{\partial z'^2} - K_1^* (C - C_{\infty})$$
(5)

The boundary conditions are

$$u'(z',t') = 0, v'(z',t') = 0, T = T_{\infty}, C = C_{\infty} \quad \text{for } t' \le 0 \text{ & any } z'$$
 (6)

$$u'(\infty,t') = U_0 \left[ 1 + \frac{\varepsilon}{2} (e^{in't'} + e^{-in't'}) \right], T(\infty,t') \to T_{\infty}, C(\infty,t') \to C_{\infty} \text{ for } t' \ge 0$$
 (7)

Here u', v' and w' are the velocity components along the x', y' and z' axis respectively. The properties of Nano fluids are defined

$$\rho_{nf} = (1 - \phi)\rho_{f} + \phi\rho_{S}, \quad (\rho Cp)_{nf} = (1 - \phi)(\rho Cp)_{f} + \phi(\rho Cp)_{s}, \alpha_{nf} = \frac{K_{nf}}{(\rho Cp)_{nf}}$$

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_{f} + \phi(\rho\beta)_{s}, \mu_{nf} = \frac{\mu_{f}}{[1 - \phi]^{2.5}}, \frac{K_{nf}}{K_{f}} = \left[\frac{K_{S} + 2K_{f} - 2\phi(K_{f} - K_{s})}{K_{S} + 2K_{f} + 2\phi(K_{f} - K_{s})}\right]$$
(8)

The thermo physical properties of the base fluid (water) &silver are given below.

Nanofluid	ρ	Ср	K	βX10 <sup>5</sup>
Silver(Ag)	10,500	235	429	1.89
Pure water	997.1	4179	0.613	21

$$q_r' = -\frac{4\sigma^*}{3k_1'} \frac{\partial T'}{\partial z} \tag{9}$$

Where,  $q_r$  is the radiative flux vector  $\sigma^*$  &  $k'_1$  are respectively the Stefan-Boltzmann constant and the mean absorption coefficient.

Suppose that 
$$T^{\prime 4} \cong 4T_{\infty}^{\prime 3}T^{\prime} - 3T_{\infty}^{\prime 4}$$
 (10)

due difference of temperature within the flow is adequately small.

Introducing dimensionless variables

$$u = \frac{u'}{U_{0}}, v = \frac{v'}{U_{0}}, z = \frac{z'U_{0}}{v_{f}}, t = \frac{t'U_{0}^{2}}{v_{f}}, n = \frac{v_{f}n'}{U_{0}^{2}}, K = \frac{K'\rho_{f}U_{0}^{2}}{v_{f}^{2}}, S = \frac{w_{0}}{U_{0}}$$

$$R = \frac{2\Omega v_{f}}{U_{0}^{2}}, Q_{H} = \frac{Q'v_{f}^{2}}{K_{f}U_{0}^{2}}, Q_{1} = \frac{Q_{1}^{*}(C_{w} - C_{\infty})}{(T_{w} - T_{\infty})U_{0}^{2}}, F = \frac{4\sigma * T_{\infty}^{3}}{kk_{1}^{'}}, \Pr = \frac{v_{f}(\rho Cp)_{f}}{K_{f}}$$
(11)

From the Eqs (11), & Eqs. (2) – (5) we obtain:

$$\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial z} - Rv = B_1 \frac{\partial^2 u}{\partial z^2} + B_2 \theta - B_3 u \left[ M + \frac{1}{K} \right]$$
(12)

$$\frac{\partial v}{\partial t} - S \frac{\partial v}{\partial z} + Ru = B_1 \frac{\partial^2 v}{\partial z^2} - B_3 v \left[ M + \frac{1}{K} \right]$$
(13)

$$\frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial z} = \frac{B_4}{\text{Pr}} \frac{\partial^2 \theta}{\partial z^2} - \frac{B_5 Q_H}{\text{Pr}} \theta + Q_1 \psi \tag{14}$$

$$\frac{\partial \psi}{\partial t} - S \frac{\partial \psi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \psi}{\partial z^2} - Kr \quad \psi \tag{15}$$

The relevant boundary conditions are

$$u(z,t) = 0, v(z,t) = 0, \theta(z,t) = 0, \psi(z,t) = 0 \quad \text{for } t \le 0 \text{ and any } z$$

$$u(0,t) = 1 + \frac{\mathcal{E}}{2} [e^{\text{int}} + e^{-\text{int}}], v(0,t) = 0, \theta(z,t) = 1, \psi(z,t) = 1$$

$$u(\infty,t) \to 0, v(\infty,t) \to 0, \theta(\infty,t) \to 0, \psi(\infty,t) \to 0$$

$$\{ for \ t \ge 0 \}$$

$$\{ for \ t \ge 0 \}$$

Obviously, the velocity characteristic  $U_0$  is defined as  $U_0 = \sqrt[3]{g\beta_f v_f (T_w - T_\infty)}$  (17)

$$\frac{\partial \chi}{\partial t} - S \frac{\partial \chi}{\partial z} - Rv = B_1 \frac{\partial^2 \chi}{\partial z^2} + B_2 \theta - B_3 \chi \left[ M + \frac{1}{K} \right]$$
(18)

The corresponding boundary conditions are

$$\chi(z,t) = 0, \theta(z,t) = 0, \psi(z,t) = 0 \quad \text{for } t \le 0 \text{ and any } z$$

$$\chi(0,t) = 1 + \frac{\varepsilon}{2} [e^{\text{int}} + e^{-\text{int}}], \theta(z,t) = 1, \psi(z,t) = 1$$

$$\chi(\infty,t) \to 0, \theta(\infty,t) \to 0, \psi(\infty,t) \to 0$$

$$for \ t \ge 0$$

$$(19)$$

Here  $\chi(z,t) = u(z,t) + iv(z,t)$  is fluid velocity in the complex form To solve Eqs. (12)- (15) using Eqs. (17), we suppose that

$$\chi(z,t) = \chi_0(z,t) + \frac{\varepsilon}{2} \left\{ e^{int} \chi_1 + e^{int} \chi_2 \right\} \dots, \theta(z,t) = \theta_0(z,t) + \frac{\varepsilon}{2} \left\{ e^{int} \theta_1 + e^{int} \theta_2 \right\} \dots$$

$$\psi(z,t) = \psi_0(z,t) + \frac{\varepsilon}{2} \left\{ e^{int} \psi_1 + e^{int} \psi_2 \right\} \dots$$
(20)

Solve Eqs.(14), Eqs (15) & Eqs.(18) the we obtained;

$$\chi(z,t) = P_3 e^{-\xi_1 z} + P_4 e^{-\xi_2 z} + P_5 e^{-\xi_3 z} + \frac{\varepsilon}{2} \left\{ e^{-\xi_4 z} e^{int} + e^{-\xi_5 z} e^{-int} \right\}$$
(21)

$$\theta(z,t) = P_1 e^{-\xi_1 z} + P_2 e^{-\xi_2 z}$$
(22)

$$\psi(z,t) = e^{-\xi_1 Z} \tag{23}$$

## 3. Results and discussion:

The effects on the velocity, the temperature as well as the concentration are discussed through graphs.

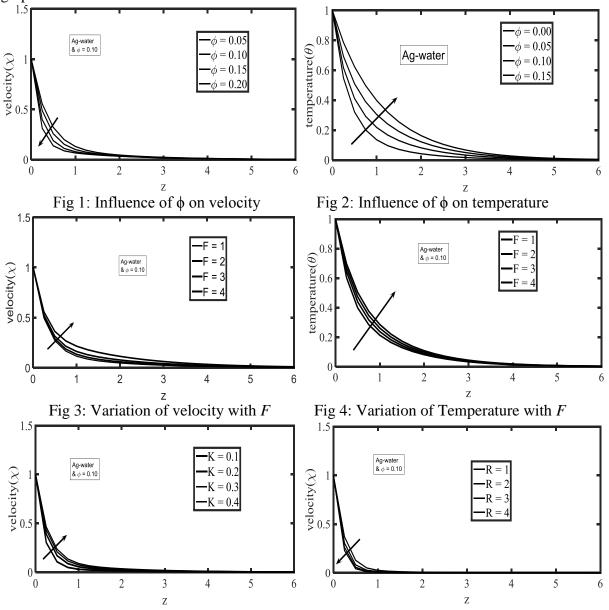


Fig 5: Influence of *K* on Velocity

Fig 6: Influence of R on Velocity

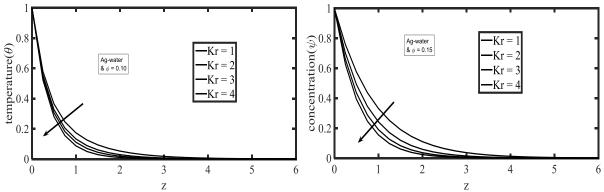


Fig 7: Variation of Temperature with *Kr* 

Fig 8: Variation of concentration with Kr

Fig 1 & Fig 2: Represents the influence of the Nano particle volume fraction  $\phi$  on the velocity as well as temperature. From this figure it was obvious that different enhancement values of  $\phi$  lead to decline the velocity but reverse effect was occurred in case of temperature. Fig 3 & Fig 4: illustrated that the effect of thermal radiation (F) in the velocity as well as temperature. From these figures the outcomes indicates that the velocity and temperature rises with the enhancement of thermal radiation (F). Owing to the enhancement of F results to discharge of heat energy as of the flow region and hence temperature rises as the thermal boundary layer thickness turn out to be thin. Fig 5: Reflects that the Nano fluid velocity rises with the enhancement of porous parameter (K) Rotation parameter R. Fig 6: Demonstrated that for dissimilar values of rotation parameter R rises then it leads to diminished in velocity across the boundary layer. The influence of chemical reaction Kr on temperature as well concentration is illustrated in the Fig 7 and Fig 8: In this figure the results reflect that concentration as well as temperature declined with the enhancement of Kr.

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