Proceedings of the National Conference on Computational Modeling of Fluid Dynamics Problems (CMFDP-2019)
NIT Warangal, India – Jan 18-20, 2019
Paper No. XXX (The number assigned when the papaer is accepted)

Effect of Viscous dissipation on natural convection flow past an impulsively moving vertical plate with ramped temperature

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This article illustrates the influence of viscous dissipation on free convective flow past an impulsively moving vertical plate with ramped temperature. In order to analyze all the essential features, the flow governing partial differential equations are solved numerically by utilizing Finite element technique, consequently obtained results are exhibited with the aid of graphs. The effects of skin friction, Nusselt number and Sherwood number are exhibited in a tabular form. Comparison is made with previously published literature and a great coordination between the results exists.

***Keywords*:** Finite Element Method**,** Viscous dissipation**,** Natural convection, Ramped temperature.

1. Introduction

Free convection is as frequently as conceivable experienced in our environment and engineering devices. Free convection flow occurs by the temperature as well as concentration gradient. Frequently one can see that both heat and mass exchange at once in free convection. This investigation of flow has a wide extent of usages in the field of science and technology, for example, toxic waste in water bodies, vaporization of mist and fog, photosynthesis, drying of porous solids, transpiration and sea-wind formation. Because of this one a few specialists have examined the issue of natural convection flow, couple of commitments are Gebhart and Pera (1971) observed the steady state natural convection on a vertical plate with variable surface temperature and variable mass diffusion. Callahan and Marner (1976) solved the problem of transient free convection with mass transfer on an isothermal vertical plate. Soundalgekar and Ganesan (1981) have explained the problem for transient free convection with mass transfer on a vertical plate with constant heat flux. Ekambavannan and Ganesan (1995) studied the problem of transient natural convection flows over an inclined plate with variable surface temperature and mass diffusion. Ganesan and Palani (2000) have analyzed numerically for transient free convection flow past a semi-infinite inclined plate with variable surface heat and mass flux.

Viscous dissipation occurs in natural convection in various devices. Such dissipation effects may also be present in stronger gravitational fields and in process wherein the scale of the process is very large, e.g., on larger planets, in large masses of gas in space, and in geological processes in fluids internal to various bodies. With viscous dissipative heat included in the energy equation, Gebhart (1962) studied the affect of viscous dissipation in natural convection. Gebhart and Mollendorf (1969) analyzed viscous dissipation in external natural convection. Lekas and Georgantopoulos (1992) have illustrated the influence of viscous dissipation on a hydromagnetic field.

Numerous authors examined for analytical or numerical solutions which are gotten by considering conditions at the plate to be constant and well defined. However many practical problems required non-uniform thermal conditions. Narahari (2012) reviewed the Transient free convection flow between long vertical parallel plates with ramped wall temperature at one boundary in the presence of thermal radiation and constant mass diffusion. Seth et.al (2013) investigated numerical solution of unsteady hydromagnetic natural convection flow of heat absorbing fluid past an impulsively moving vertical plate with ramped temperature. Barik (2014) discussed the chemical reaction and radiation effects of MHD free convective flow past an impulsively moving vertical plate with ramped wall temperature and concentration. Siva reddy Sheri et.al (2015) have found Transient approach to heat absorption and radiative heat transfer past an impulsively moving plate with ramped temperature. Vijay Bhaskar et.al (2018) have studied MHD natural convection flow past a moving vertical plate with ramped temperature.

The present numerical investigation deals with the effect of heat and mass transfer on natural convection flow past an impulsively moving vertical plate with ramped temperature by introducing Eckert number. The governing equations are solved using finite element method. The numerical result is validated by comparing the present scheme with the earlier published results.

2. Mathematical formulation

Assume a stream of viscous incompressible electrically conducting fluid past an infinite vertical plate embedded in a porous medium. The physical model and facilitate framework are appeared in Fig.1. Choose the Coordinate system in such a way that the x'-axis is taken along the plate in the upward way, y'-axis normal to the plane of the plate in the fluid. The liquid is pervaded by a uniform transverse magnetic field B_0 applied parallel to y'-axis. Initially at time $t' \leq 0$, both the fluid and plate are at rest with a consistent temperature T'_∞ and species concentration C'_∞. At time $t' > 0$, the plate begins moving in x'-direction against gravitational field with time dependent velocity $U(t')$. Temperature and concentration surface of plate is raised or lowered to $T'_\infty + (T'_w - T'_\infty)t'/t_0$ and $C'_\infty + (C'_w - C'_\infty)t'/t_0$ when $t' \leq t_0$, and thereafter, for $t' > t_0$, temperature and concentration are constantly maintained at T'_w and C'_w respectively. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate Kr' between the diffusing species and the fluid. Since the plate is infinite in x' and z' directions and is electrically non-conducting all physical quantities, except pressure, will be functions of y' and t' only.

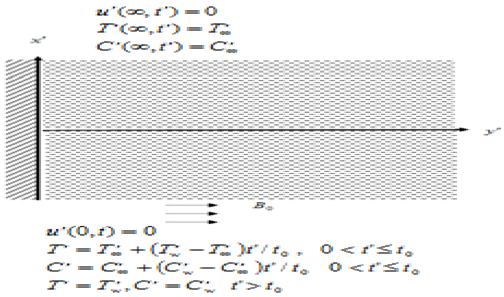

Fig. 1. Physical model and coordinate system of the Problem

Governing equations for above assumptions under Boussinesq approximation, are given by

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0{}^2}{\rho} u' - \frac{\upsilon}{K'} u' + g\beta'(T' - T'_\infty) + g\beta^*(C' - C'_\infty) \tag{2.1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q'_r}{\partial y'} - \frac{Q_0}{\rho c_p}(T' - T'_\infty) + \frac{v}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_r(C' - C'_\infty) \tag{2.3}$$

Where $u', T', C', g, \beta', \beta^*, \upsilon, \sigma, \rho, k, K', c_p, q'_r \quad Q_0, D$ and K'_r are fluid velocity, temperature, species concentration, acceleration due to gravity, volumetric coefficient of thermal expansion for species concentration, volumetric coefficient of expansion, kinematic coefficient of viscosity, electrical conductivity, fluid density, thermal conductivity, permeability of porous medium, specific heat at constant pressure, radiative flux vector, heat absorption coefficient, chemical molecular diffusivity and chemical reaction parameter respectively.

Initial and boundary conditions for the problem are specified as

$$\left.\begin{array}{l} u'=0,\ T'=T'_\infty,\ C'=C'_\infty \ \ for \ \ y'\ge 0 \ \ and \ t'\le 0 \\ u'=U(t') \ at \ y'=0 \ \ for \ t'>0 \\ T'=T'_\infty+\left(T'_w-T'_\infty\right)t'/t_0, C'=C'_\infty+\left(C'_w-C'_\infty\right)t'/t_0 \ at \ \ y'=0 \ \ for \ \ 0<t'\le t_0 \\ T'=T'_w, C'=C'_w \ \ at \ \ y'=0 \ \ \ for \ t'>t_0 \\ u'\to 0,\ T'\to T'_\infty,\ \ C'\to C'_\infty \ as \ \ y'\to\infty \ \ for \ t'>0 \end{array}\right\} \quad (2.4)$$

By using Azzam (2002) and Bestman (1985) The radiative flux vector q'_r under Rosseland approximation becomes

$$q'_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \quad (2.5)$$

Where k^* is mean absorption co-efficient and σ^* is Stefan-Boltzmann constant.

T'^4 is expanded in Taylor series about a free stream temperature T'_∞ and neglecting higher order terms we obtain

$$T'^4 \approx 4T_\infty'^3 T' - 3T_\infty'^4 \quad (2.6)$$

Making use of Equations (2.5) and (2.6) in Equation (2.2), we Obtain

$$\frac{\partial T'}{\partial t'}=\frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2}+\frac{1}{\rho c_p}\frac{16\sigma^* T_\infty'^3}{3k^*}\frac{\partial T'^2}{\partial y'^2}-\frac{Q_0}{\rho c_p}\left(T'-T'_\infty\right) \quad (2.7)$$

Introducing following non-dimensional quantities and parameters

$$\left.\begin{array}{l} y=\dfrac{y'}{U_0 t_0},\ u=\dfrac{u'}{U_0},\ t=\dfrac{t'}{t_0}, T=\dfrac{(T'-T'_\infty)}{(T'_w-T'_\infty)}, C=\dfrac{(C'-C'_\infty)}{(C'_w-C'_\infty)}, M=\dfrac{\sigma B_0^2\nu}{\rho U_0^2}, \\ K=\dfrac{K' U_0^2}{\nu^2}, Gr=\dfrac{g\beta'\nu(T'_w-T'_\infty)}{U_0^3}, Gc=\dfrac{g\beta^*\nu(C'_w-C'_\infty)}{U_0^3}, \Pr=\dfrac{\nu\rho c_p}{k}, \\ R=\dfrac{16\sigma^* T_\infty'^3}{3kk^*}, Q=\dfrac{\nu Q_0}{\rho c_p U_0^2}, Ec=\dfrac{U_0^2}{c_p\ (T'_w-T'_\infty)}, Kr=\dfrac{\nu Kr'}{U_0^2},\ Sc=\dfrac{\nu}{D} \end{array}\right\} \quad (2.8)$$

In view of equations (2.8), equations (2.7) and (2.3) reduce to the following dimensionless form:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+GrT+GcC-\left(M+\frac{1}{K}\right)u \quad (2.9)$$

$$\frac{\partial T}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}-QT-RT+Ec\left(\frac{\partial u}{\partial y}\right)^2 \quad (2.10)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KrC \quad (2.11)$$

Using (2.8) the initial boundary conditions (2.4), in non-dimensional form reduces to

$$\left.\begin{array}{l} u=0,\ \ T=0, C=0 \ \ \ for \ \ y\ge 0 \ \ and \ t\le 0 \\ u=1 \ \ at \ \ y=0 \ \ \ for \ \ \ t>0, \\ T=t, C=t \ at \ y=0 \ \ for \ \ 0<t\le 1, \\ T=1, C=1 \ at \ y=0 \ \ for \ t>1, \\ u\to 0,\ T\to 0, C\to 0 \ \ as \ y\to\infty \ \ \ for \ t>0 \end{array}\right\} \quad (2.12)$$

3. Method of solution

3.1. Numerical solution by FEM

The dimensionless non-linear partial differential equations (2.9) - (2.11) along with associated boundary conditions (2.12) are solved by finite element method. This method comprises of following five fundamental steps: discretization of the domain, derivation of element equations, assembly of element equations, imposition of boundary conditions and solution of the system of equations. An amazing depiction of these steps exhibited in in the text books by Reddy (1996), and Bathe (2006). The expressions for Skin-friction, Nusselt number and Sherwood number are

$$\tau = \left[\frac{\partial u}{\partial y}\right]_{y=0} \quad (3.1); \quad Nu = -\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad (3.2); \quad Sh = -\left[\frac{\partial C}{\partial y}\right]_{y=0} \quad (3.3)$$

4. Validation of numerical solution

To authenticate the numerical strategy utilized for the solution of the present investigation under a few assumptions (without Eckert number) was contrasted with Barik (2014) and the corresponding comparisons are shown in tables 1-3. These correlations indicate great understanding between the outcomes upto four decimal places. Accordingly, the created code can be utilized with an incredible trust in the numerical outcomes exhibited therefore to the problem considered in this paper.

Table 1: Comparison of Skin friction τ when $Ec = 0$

										Barik (2014)		Present Results	
Kr	Q	t	Pr	Sc	Gc	K	Gr	M	R	Ramped	Isothermal	Ramped	Isothermal
0.2	1	0.2	0.71	2.7	0.4	0.5	2	2	4	2.0855	1.609	2.085501	1.60901
0.4	1	0.2	0.71	2.7	0.4	0.5	2	2	4	2.0859	1.6094	2.085901	1.609401
0.2	3	0.2	0.71	2.7	0.4	0.5	2	2	4	2.0926	1.6317	2.092601	1.631701
0.2	1	0.5	0.71	2.7	0.4	0.5	2	2	4	1.753	1.3535	1.75301	1.353501
0.2	1	0.2	7	2.7	0.4	0.5	2	2	4	2.1111	1.8985	2.111101	1.898501
0.2	1	0.2	0.71	0.6	0.4	0.5	2	2	4	2.083	1.6065	2.08301	1.606501
0.2	1	0.2	0.71	2.7	3	0.5	2	2	4	2.0262	1.5497	2.026201	1.549701
0.2	1	0.2	0.71	2.7	0.4	4	2	2	4	1.7391	1.147	1.739101	1.14701
0.2	1	0.2	0.71	2.7	0.4	0.5	4	2	4	2.0259	1.0729	2.025901	1.072901
0.2	1	0.2	0.71	2.7	0.4	0.5	2	4	4	1.8454	0.536	1.845401	0.53601
0.2	1	0.2	0.71	2.7	0.4	0.5	2	2	10	2.0916	1.6315	2.091601	1.631501

Table 2: Comparison of Nusselt Number (Nu) when $Ec = 0$

				Barik (2014)		Present Results	
Pr	Q	t	R	Ramped	Isothermal	Ramped	Isothermal
2	1	0.2	4	1.3859	2.1297	1.385901	2.129701
0.7	1	0.2	4	0.8257	1.2689	0.825701	1.268901
2	3	0.2	4	1.2678	2.7592	1.267801	2.759201
2	1	0.5	4	2.0404	1.6499	2.040401	1.649901
2	1	0.2	10	2.1516	2.2785	2.151601	2.278501

Table 3: Comparison of Sherwood Number (Sh) when $Ec = 0$

			Barik (2014)		Present Results	
Sc	Kr	t	Ramped	Isothermal	Ramped	Isothermal
2	2	0.5	1.4716	2.1005	1.471601	2.100501
2	2	0.7	1.8855	2.0467	1.885501	2.046701
4	2	0.7	2.6665	2.8944	2.666501	2.894401
4	4	0.7	2.3321	2.8356	2.332101	2.835601

5. Results and discussion

To analyze the impacts of different parameters on stream field in the boundary layer region, numerical values of fluid velocity, temperature and concentration are computed from the numerical solutions, are depicted graphically versus boundary layer co-ordinate y in Figs. 2–12(c) for different estimations of thermal Grashof number, solutal Grashof number, magnetic parameter, permeability parameter, Prandtl number, thermal heat absorption coefficient, radiation parameter, Eckert number, chemical reaction parameter, Schmidt number and time. In the present investigation we embraced the accompanying default parameter estimations of finite element computations.

$$Gr = 1.0, Gc = 1.0, M = 2.0, K = 0.2, \Pr = 0.71, R = 1.0, Q = 1.0, Ec = 0.01, Sc = 2.0, \quad Kr = 2.0, t = 0.5.$$

Fig. 2 shows the behavior of speed profiles for different estimations of Grashof number. Since thermal Grashof number Gr is the proportion of the thermal buoyancy force and viscous hydrodynamic force. From this Fig it is seen that speed profile increments with the expanding of Gr. This is expected the reason that the thermal buoyancy force quicken liquid stream.

Fig. 3 delineates velocity profiles for different values of solutal Grashoff number which is the ratio of the species buoyancy force and viscous hydrodynamic force. From Fig.3 it is seen that velocity profile increases with the expanding of Solutal Grashof number Gc. This because of the reason that concentration buoyancy force has a tendency to accelerate fluid velocity.

Fig. 4 displays velocity profiles for dissimilar values of magnetic field. From Fig. 4 it is identified that the speed profile starts to diminish with expanding of M . The presence of magnetic field normal to the flow in an electrically conducting fluid introduces a Lorentz force which acts against the stream. This resistive force will in general back off the stream and subsequently the liquid speed diminishes with expanding magnetic field parameter.

From fig. 5 it is conformed that velocity profile get increased against permeability parameter K . Since when the holes of porous medium become larger, the resistive of the medium may be neglected. This implies that the resistance in porous medium which tends to accelerate flow of the fluid.

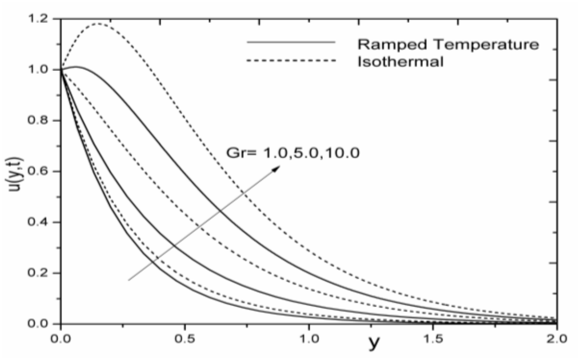

Fig. 2. Effect of Gr on velocity profile

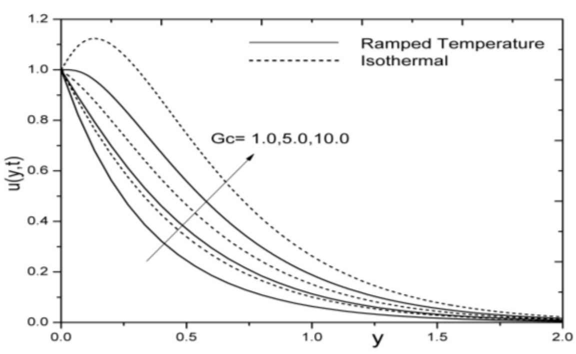

Fig. 3. Effect of Gc on velocity profile

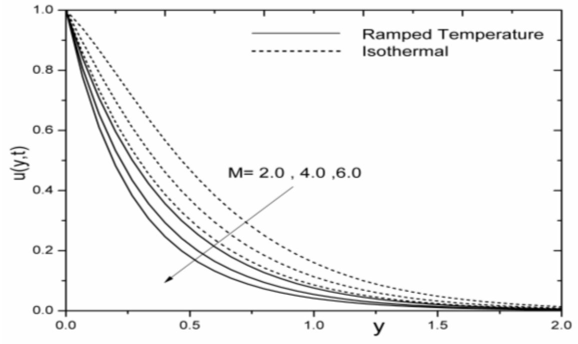

Fig. 4. Effect of M on velocity profile

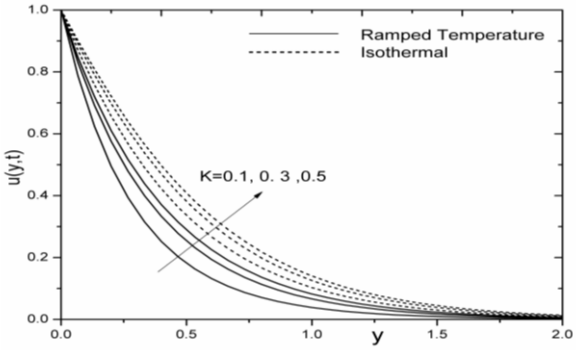

Fig. 5. Effect of K on velocity profile

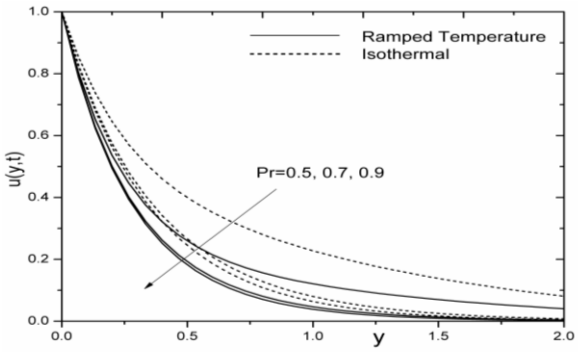

Fig. 6(a). Effect of Pr on velocity profile

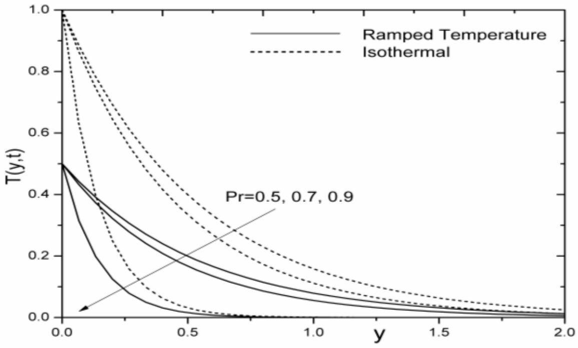

Fig. 6(b). Effect of Pr on temperature profile

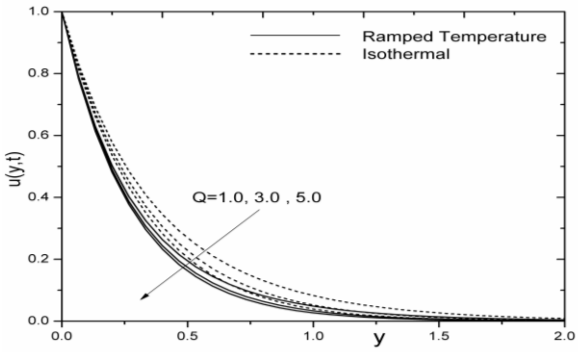

Fig. 7(a). Effect of Q on velocity profile

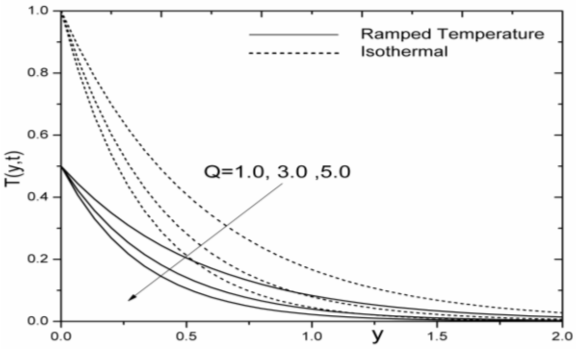

Fig. 7(b). Effect of Q on temperature profile

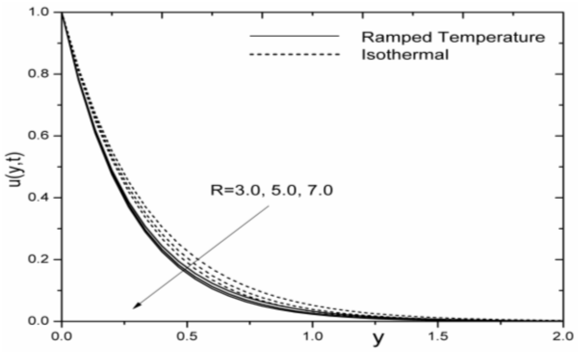

Fig. 8(a). Effect of R on velocity profile

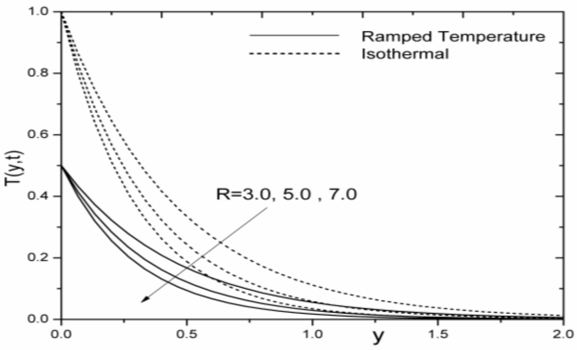

Fig. 8(b). Effect of R on temperature profile

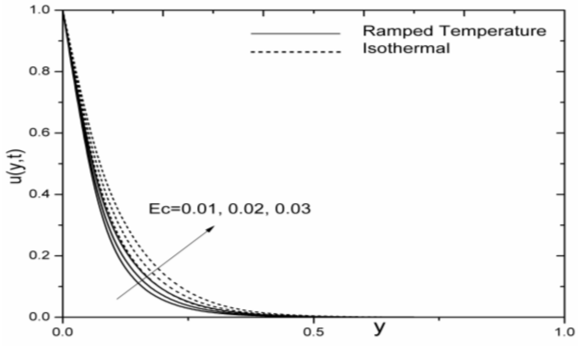

Fig. 9(a). Effect of Ec on velocity profile

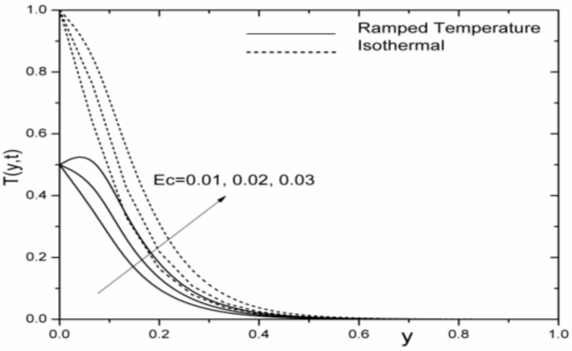

Fig. 9(b). Effect of Ec on temperature profile

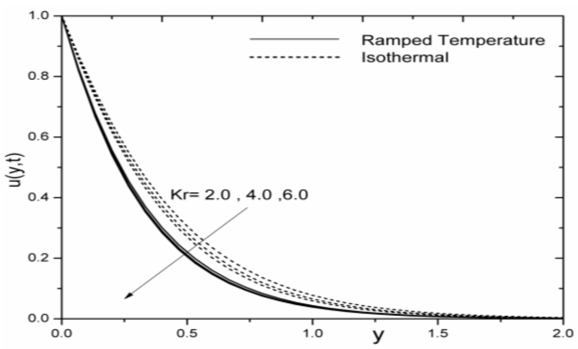

Fig. 10(a). Effect of Kr on velocity profile

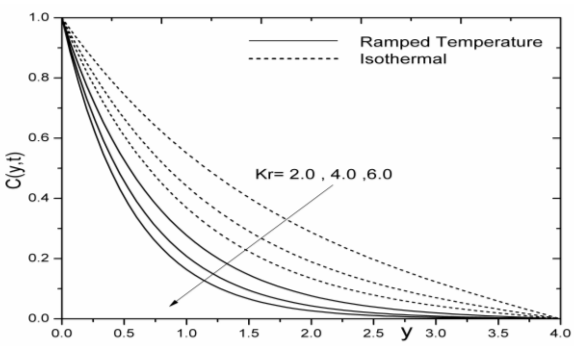

Fig. 10(b). Effect of Kr on concentration profile

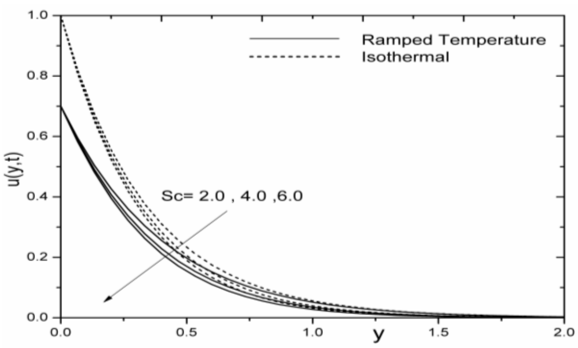

Fig. 11(a). Effect of Sc on velocity profile

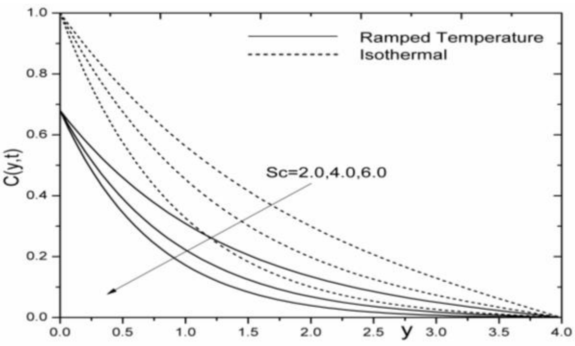

Fig. 11(b). Effect of Sc on concentration profile

Figs. 6(a) and 6(b) show the velocity and temperature profiles for various values of Prandtl number. Prandtl number is the proportion of momentum diffusivity (kinematic thickness) to thermal diffusivity. Fig 6(a) uncovers that speed profile diminishes with the expanding of Pr. This infers temperature dispersion will in general quicken liquid throughout boundary layer region. This occurs because of the way that thermal dispersion gives a stimulus to thermal buoyancy force. Fig 6(b) clarifies that temperature profile diminishes with the expanding of Pr. Since Pr means the relative effects of viscosity to thermal conductivity. This suggests, thermal diffusion will in general improve liquid temperature.

From figs 7(a) and 7(b) obviously liquid speed and temperature profiles decline with expanding of Q. This is because of the reason that, heat absorption will in general block liquid speed and. This might be perceived to the way that the tendency of heat absorption (thermal sink) is to diminish the liquid temperature which causes the strength of thermal buoyancy force to diminish bringing about a net decrease in the liquid speed.

Figs. 8(a) and 8(b) depict velocity and temperature profiles for several values of radiation parameter. From these two figs it is observed that fluid velocity and temperature increase with the increasing of radiation parameter in the boundary layer region which implies that thermal radiation tends to enhance fluid velocity and temperature.

Figs. 9(a) and 9(b) portray speed and temperature profiles for several values of Eckert number on. The Eckert number is the connection between kinetic energy in the flow and the enthalpy. It speaks to the change of kinetic energy into internal energy by work done against the viscous fluid stresses. From Figs 9(a) and 9(b) obviously there is an expansion in liquid speed and temperature because of the way that more prominent thick viscous dissipative heat causes an ascent in the speed and temperature.

Figs. 10(a) and 10(b) portray velocity and concentration profiles for different values of chemical reaction. From 10(a) and 10(b) it is observed that an increase in chemical reaction parameter leads to a decrease in both the values of velocity and concentration. This is due to fact that distinct velocity acceleration occurs near the wall after which profiles decay smoothly to the stationary value in free stream. Chemical reaction therefore boosts momentum transfer, i.e., accelerates the flow.

From 11(a) and 11(b) it is noticed that both velocity and concentration distribution diminishes at all points of the flow field with the increase of the Schmidt number. This confirms that the heavier diffusing species have a greater retarding influence on velocity and concentration distribution of the flow field in case of both ramped temperature and isothermal plates.

6. Conclusion

The present numerical analysis deals with the effect of viscous dissipation on natural convection flow past an impulsively moving vertical plate with ramped temperature. The numerical solutions of momentum, energy and concentration equations are obtained by Finite element method. Graphical results of velocity, temperature and concentration against different flow parameters are obtained by MATLAB. Effect of Skin friction, Nusselt number and Sherwood number on flow field is presented in a tabular form and at the same time they are compared with the earlier results presented by Barik (2014).The overall conclusions for both ramped and isothermal plates are: the velocity increases with the increase of thermal buoyancy force concentration buoyancy force permiability parameter and Ecert number and it has reverse tendency with the increase of magnetic parameter Prandtl number heat absorption radiation parameter Chemical reaction parameter and schmidt number. Temperature increases with the increase of Ecert number and it has reverse trend with the increase of Prandtl number heat absorption and radiation parameter Concentration decreases with the increase of Chemical reaction parameter and Schmidt number.

Acknowledgements

We are grateful to the esteemed reviewers for their comments and suggestions which led to the improvement of the paper.

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