Numerical study of physiological hemodynamic Jeffery fluid model undertheimpact of rotation and hall current through an inclined vertical tapered channel with slip boundary conditions

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Abstract:

The key objective of this analysis is to present the novel aspects of the physiological hemodynamic Jeffery fluid model under the impact of rotation and hall current through an inclined vertical tapered channel with slip boundary conditions. The porous medium is also taken into the interpretation. Analytical results are found for the velocity, pressure gradient and pressure rise. An impact of different governing parameters were conferred and illustrated diagrammatically through a set of figures. It could be noted that the velocity of the fluid reduces by an increasing the values of porosity parameter and rotation parameter. The rate of pumping enhances in both retrograde and peristaltic pumping zones whereas the pumping rate reduces in the co-pumping zone.

Keywords: Rotation, slip boundary conditions, Jeffery fluid, hall current, inclined channel, porous medium, tapered channel.

1. Introduction

Past few decades, a good diversity of researchers consummates blood flow through various channels. The Literature of Biomathematics has provided with an enormous variety of applications in drugs and biology. The behaviour of the majority of the biological fluids, oil, polymer and hydrocarbons are acknowledged to be non-Newtonian. The study of the mechanisms of peristalsis, in every physiological and mechanical situation, has been the object of scientific investigation for quite it slow. The first attempt was made by Latham [1] and a few of the fascinating investigations within the same direction are given in references [2-5].

TasawarHayat et al. [6] examined the hall current and joule heating effects on peristaltic flow of viscous fluid in a rotating channel with convective boundary conditions. In this paper, we notice that Runge-Kutta scheme of order four is implemented for the results of axial and secondary velocities, temperature and heat transfer coefficient. Simultaneous effects of hall current and homogeneous/heterogeneous reaction on peristalsis pointed out by Hayat et al.

[7]. Seth et al. [8] investigated an effect of hall current, radiation and rotation on natural convection heat and mass transfer flow past a moving vertical plate. In this paper, exact solution of the governing equations is obtained in closed form by Laplace transform technique. The physical significance of magnetic field and rotation effects on peristaltic transport of a Jeffrey fluid in an asymmetric channel are studied by Abd-Alla and Abo-Dahab [9]. Combined effects of hall current and heat transfer on peristaltic transport of a nanofluid in a vertical tapered channel through a porous medium by Ahmed I. Abdellateef and Syed Z. UlHaque [10]. Few relevant studies may be seen via makes attempts [11-16].

2. Formulation for the problem

Consider the physiological hemodynamic Jeffery fluid model of an incompressible viscous fluid in a two dimensional uneven inclined vertical tapered channel under the influence of velocity slip boundary conditions, rotation and hall current. We tend to assume that the fluid is subject to a relentless transverse magnetic field B_0.The fluid is induced by sinusoidal wave trains propagating with constant speed *c* along the channel walls.

The geometry of the wall deformations are drawn by the subsequent expressions

$$Y = \overline{H_1} = -b - m'\overline{X} - d\sin\left[\frac{2\pi}{\lambda}\left(\overline{X} - c\overline{t}\right) + \phi\right] \tag{1}$$

$$Y = \overline{H_2} = b + m'\overline{X} + d\sin\left[\frac{2\pi}{\lambda}\left(\overline{X} - c\overline{t}\right)\right] \tag{2}$$

In the above equations, d is the wave amplitude of the peristaltic wave, *c* is the wave velocity, b is the mean half-width of the channel, m' is dimensional the non-uniform parameter, $\lambda$is the wavelength, t is the time, X is the direction of wave propagation and ϕ is the phase variance.

The equations governing the flow in the wave frame of reference are given by

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{3}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - \rho\left(\Omega^2 u + 2\Omega\frac{\partial v}{\partial t}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(S_{xx}\right) + \frac{\partial}{\partial y}\left(S_{xy}\right) + \frac{\sigma B_0^{\,2}}{1+m^2}\left(mv - \left(u+c\right)\right)$$
$$-\frac{\mu}{k_1}\left(u+c\right) + \rho g\sin[\alpha] \tag{4}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) + \rho\left(-\Omega^2 v + 2\Omega\frac{\partial u}{\partial t}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(S_{xy}\right) + \frac{\partial}{\partial y}\left(S_{yy}\right) - \frac{\sigma B_0^{\,2}}{1+m^2}\left(m\left(u+c\right) + v\right)$$
$$-\rho g\cos[\alpha] \tag{5}$$

Where u,v are the velocity components along x and y directions, t is the time, p is the fluid pressure, ρ is the density of the fluid, μ is the coefficient of the viscosity of the fluid, m is the hall current parameter and Ω is the rotation parameter.

$$S_{xx}=\frac{2\mu}{1+\lambda_1}\left(1+\lambda_2\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\frac{\partial u}{\partial x}\; S_{xy}=\frac{\mu}{1+\lambda_1}\left(1+\lambda_2\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)$$

$$S_{yy}=\frac{2\mu}{1+\lambda_1}\left(1+\lambda_2\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\frac{\partial u}{\partial y}$$

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x=X-ct\,,\; y=Y\,, u=U-c\,,\; v=V\,,\; p(x)=P(x,t) \tag{6}$$

Introducing the following non-dimensional quantities:

$$\left.\begin{array}{l}\bar{x}=\dfrac{x}{\lambda},\bar{y}=\dfrac{y}{b},\bar{t}=\dfrac{ct}{\lambda},\bar{u}=\dfrac{u}{c},\varepsilon=\dfrac{d}{b},\bar{v}=\dfrac{v}{c\delta},h_1=\dfrac{H_1}{b},\\ h_2=\dfrac{H_2}{b},\delta=\dfrac{b}{\lambda},\mathrm{Re}=\dfrac{\rho cb}{\mu},M=B_0 d\sqrt{\dfrac{\sigma}{\mu}},\bar{p}=\dfrac{b^2\rho}{c\lambda\mu}\end{array}\right\} \tag{7}$$

Where $\epsilon=\frac{a}{d}$ is the non-dimensional amplitude of channel, $\delta=\dfrac{b}{\lambda}$ is the wave number, Re is the Reynolds number and M is the Hartmann number.

3. Solution of the problem

In view of the above transformations (6) and non-dimensional variables (7), equations (3-5) are reduced to the following non-dimensional form after dropping the bars

$$\delta\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{8}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)-\frac{\rho b^2\Omega^2}{\mu}u-2\mathrm{Re}\,\delta^2\Omega\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial x}+\delta\frac{\partial}{\partial x}\left(S_{xx}\right)+\frac{\partial}{\partial y}\left(S_{xy}\right)+\frac{M^2}{1+m^2}\left(m\delta v-(u+1)\right)$$
$$-\frac{1}{Da}u-\frac{1}{Da}+\eta\sin[\alpha] \tag{9}$$

$$Re\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)-\frac{\rho\Omega^2 b^2\delta^2}{\mu}v+2\mathrm{Re}\,\Omega\delta^2\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y}$$
$$-\frac{\delta M^2}{1+m^2}\left(m(u+1)+\delta v\right)-\eta_1\cos[\alpha] \tag{10}$$

Where

$$S_{xx}=\frac{2\delta}{1+\lambda_1}\left(1+\frac{\lambda_2\delta c}{d}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\frac{\partial u}{\partial x}\; S_{yy}=\frac{2}{1+\lambda_1}\left(1+\frac{\lambda_2\delta c}{d}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\frac{\partial v}{\partial y}$$

$$S_{xy}=\frac{1}{1+\lambda_1}\left(1+\frac{\lambda_2\delta c}{d}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right)\left(\frac{\partial u}{\partial x}+\delta^2\frac{\partial v}{\partial x}\right)$$

Applying long wavelength approximation and neglecting the wave number along with low-Reynolds numbers. Equations (8-10) become

$$\frac{\partial^2 u}{\partial y^2}-E\left(1+\lambda_1\right)u=\frac{dp}{dx}+F\left(1+\lambda_1\right) \quad (11)$$

$$\frac{\partial p}{\partial y}=0 \quad (12)$$

Where

$$E=\left(\frac{M^2}{1+m^2}+\frac{1}{Da}-\frac{\rho\Omega^2 b^2}{\mu}\right) \quad F=\left(\frac{M^2}{1+m^2}+\frac{1}{Da}-\eta\sin[\alpha]\right)$$

The dimensionless boundary conditions

$$u=-\beta\frac{\partial u}{\partial y}\text{ at } y=h_1=-1-k_1x-\varepsilon\sin\left[2\pi\left(x-t\right)+\phi\right] \quad (13)$$

$$u=\beta\frac{\partial u}{\partial y}\text{ at } y=h_2=1+k_1x+\varepsilon\sin\left[2\pi\left(x-t\right)\right] \quad (14)$$

Solving equations (11&12) using the boundary conditions (13&14), we get

$$u=H\sinh\left[ay\right]+G\cosh\left[ay\right]-pa_1-a_2 \quad (15)$$

Where

$$H=\left(\frac{pa_1+a_2-Ga_4}{a_3}\right) G=\left(\frac{-pa_8-a_9}{a_7}\right) a_1=\left(\frac{1}{E(1+\lambda_1)}\right) a_2=\left(\frac{F}{E}\right)$$

$$a_3=\left(\sinh[ah_1]+\beta a\cosh[ah_1]\right) \; a_4=\left(\cosh[ah_1]+\beta a\sinh[ah_1]\right)$$

$$a_5=\left(\sinh[ah_2]-\beta a\cosh[ah_2]\right) \; a_6=\left(\cosh[ah_2]-\beta a\sinh[ah_2]\right)$$

$$a_7=\left(a_4a_5-a_3a_6\right) \; a_8=\left(a_1a_3-a_1a_5\right) \; a_9=a_2\left(a_3-a_5\right) \; a=\sqrt{E(1+\lambda_1)}$$

The volumetric flow rate in the wave frame is defined by

$$q=\int_{h_2}^{h_1}u\,dy$$

$$=\int_{h_2}^{h_1}\left(H\sinh\left[ay\right]+G\cosh\left[ay\right]-pa_1-a_2\right)dy=pa_{14}+a_{15} \quad (16)$$

Where $a_{10}=\left[\frac{\cosh[ah_1]-\cosh[ah_2]}{a}\right]$ $a_{11}=\left[\frac{\sinh[ah_1]-\sinh[ah_2]}{a}\right]$ $a_{12}=a_1\left(h_1-h_2\right)$

$$a_{13}=a_2\left(h_1-h_2\right) \; a_{14}=\left[\frac{a_1a_{10}}{a_3}+\frac{a_4a_8a_{10}}{a_3a_7}-\frac{a_8a_{11}}{a_7}-a_{12}\right] \; a_{15}=\left[\frac{a_2a_{10}}{a_3}+\frac{a_4a_9a_{10}}{a_3a_7}-\frac{a_9a_{11}}{a_7}-a_{13}\right]$$

The pressure gradient obtained from equation (16) can be expressed as

$$\frac{dp}{dx}=\frac{q-a_{15}}{a_{14}} \tag{17}$$

The instantaneous flux Q (x, t) in the laboratory frame is

$$Q=\int_{h_2}^{h_1}\left(u+1\right)dy=q-h \tag{18}$$

The average volume flow rate over one wave period (T = λ/c) of the peristaltic wave is defined as

$$\bar{Q}=\frac{1}{T}\int_{0}^{T}Qdt=q+1+d \tag{19}$$

From the equations (17) and (19), the pressure gradient can be expressed as

$$\frac{dp}{dx}=\left(\frac{(\bar{Q}-1-d)-a_{14}}{a_{15}}\right) \tag{20}$$

4. Discussion of the problem

The motivation behind the study has been to analyze the numerical study of physiological hemodynamic Jeffery fluid model under the impact of rotation and hall current through an inclined vertical tapered channel with slip boundary conditions. **Mathematica** software is used to find out numerical results.

4.1 Velocity Distribution

Impact of porosity parameter on axial velocity is depicted in figure (1). It can be seen that the fluid flow of the axial velocity reduces by the increase in porosity parameter. Figure (2) reveals the impact of rotation parameter on axial velocity. We observe from this figure that the results in velocity distribution reduces when the increase in rotation parameter. The aim of the figure (3) was to study the effect of slip parameter on axil velocity. Graphical results indicate that the axial velocity enhances near the channel walls by an increase in slip parameter and we notice that the fluid of the velocity is not noteworthy, this due to the impact of slip parameter on axil velocity. Influence of hall current parameter on axial velocity is exhibited in figure (4). It is interesting to note that the velocity of the fluid enhances in $x\in(-0.75, 0.75)$ by an increase in hall current parameter.

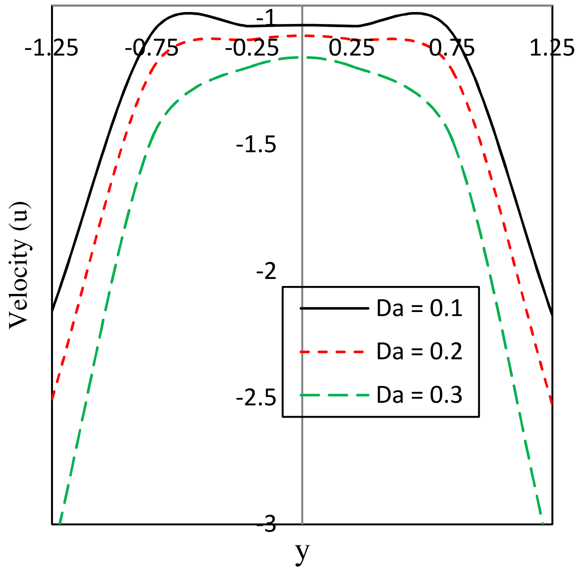

Fig. 1 Effect of Da on axial velocity (u)

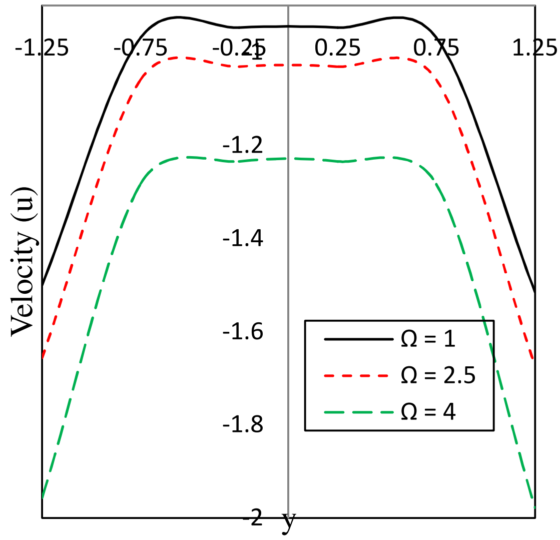

Fig. 2 Effect of Ω on axial velocity (u)

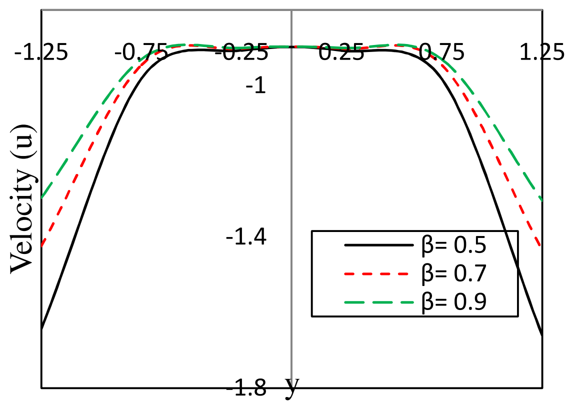

Fig. 3 Effect of β on axial velocity (u)

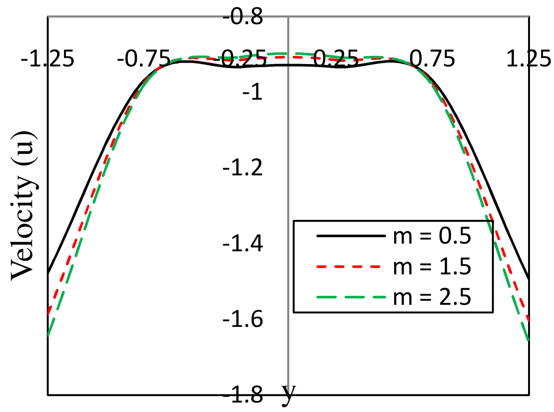

Fig. 4 Effect of m on axial velocity (u)

4.2 Pressure Rise

Influence of various parameters on pressure rise is exhibited in figures (5-8). Moreover, the pumping region is divided into the four pumping regions, which are defined as retrograde pumping zone ($\Delta p > 0, \overline{Q} < 0$), peristaltic pumping region ($\Delta p > 0, \overline{Q} > 0$), free pumping region ($\Delta p = 0$) and co-pumping region ($\Delta p < 0$). Influence of hall current parameter and porosity parameter on pressure rise is depicted in figures (5-6). We notice from these figures that the pressure rate diminishes in the retrograde pumping region and also in peristaltic pumping region whereas the pumping rate enhances in co-pumping region and also we notice that the pumping curves coincide at free pumping zone by an increase in hall current parameter and porosity parameter. The aim of figure the (7) was to study the impact of rotation parameter on pressure rise. We notice from this graph that pumping rate diminishes in both retrograde pumping region and also in the peristaltic pumping region whereas the pumping rate enhances in free pumping and co-pumping zones when we increase in rotation parameter. Effect of hartmann number on pressure rise is depicted in figure (8). It is interesting to note that the pumping rate enhances in both retrograde and peristaltic pumping zones whereas the pumping rate reduces in co-pumping zone by an increase in hartmann number.

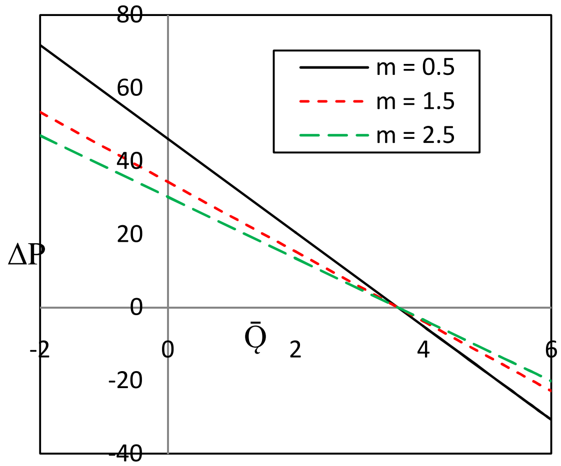

Fig. 5 Effect of m on Pressure rise (Δp)

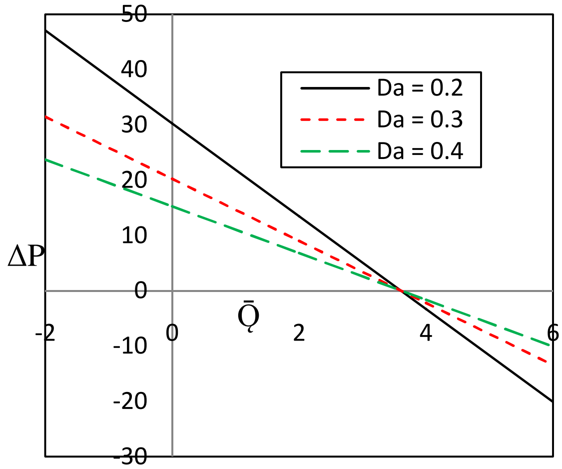

Fig. 6 Effect of Da on Pressure rise (Δp)

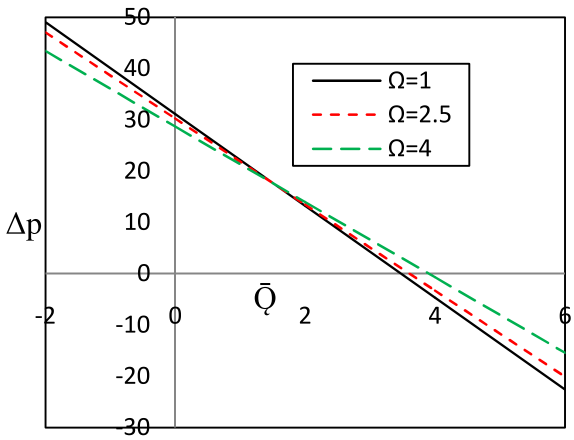

Fig. 7 Effect of Ω on Pressure rise (Δp)

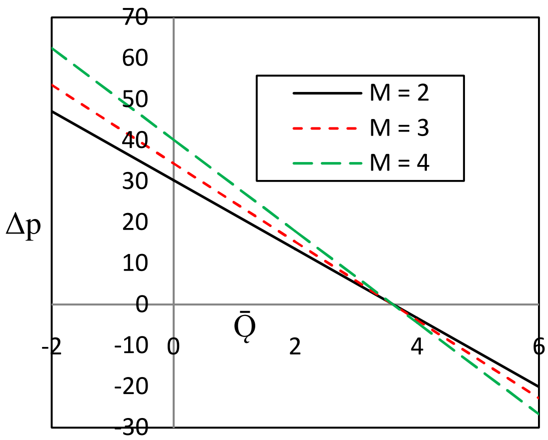

Fig. 8 Effect of M on Pressure rise (Δp)

5. Conclusions

Numerical study of physiological hemodynamic Jeffery fluid model under the impact of rotation and hall current through an inclined vertical tapered channel with slip boundary conditions have been investigated. The main findings are cited below.

1. The velocity of the fluid reduces by increasing the values of the porosity parameter and the rotation parameter.
2. Velocity enhances near the channel walls by an increase in slip parameter and we notice that the fluid of the velocity is not noteworthy.
3. Pumping rate diminishes in retrograde and peristaltic pumping zones whereas the pumping rate enhances in co-pumping zone by an increase in hall current parameter, rotation parameter and porosity parameter.
4. The rate of pumping enhances in both retrograde and peristaltic pumping zones whereas the pumping rate reduces in co-pumping zone.

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