Portfolio Optimization using Particle Swarm Optimization and Invasive Weed Optimization

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Abstract. The problem of efficient asset allocation is an important part of financial decision making and it can be done by the use of Portfolio Optimization. In this study we considered mean-variance and mean-semivariance portfolio models. Our aim is to find an efficient portfolio with minimum risk for a fixed lower bound of return. We solved those problems by Particle Swarm Optimization (PSO) and Invasive Weed Optimization (IWO). The results show that IWO produce lower risks and higher return values as compared to PSO.

Keywords: Portfolio Optimization, Particle Swarm Optimization, Invasive Weed Optimization

1 Introduction

Portfolio Optimization is based on the process of best allocation of wealth among several assets. This has been one of the burning research areas since when Harry Markowitz put a foundation stone on the Modern Portfolio Theory [1, 2]. Portfolio models are mainly concerned with investment with two criteria: expected return and risk. An investor wants the return to be high and the risk to be low. A portfolio is called efficient when it gives maximum return at a given level of risk and the curve showing all the efficient portfolios in a risk-return framework is called *efficient frontier*. Markowitz proposed the mean-variance model where the former is the mean of the portfolio return and the later is considered as the measure of risk. Over the years many heurestic methods such as Genetic Algorithm, Tabu Search, PSO etc. have been used to solve portfolio problems [3, 4] . In this paper we have discussed on portfolio optimization and two heurestic methods like PSO and IWO in section 2. Section 3 presents the results of the portfolio problems by these methods and we made a comparison among these methods. We have concluded this paper by some remarks in Section 4.

2 Risk-Return Portfolio Analysis

Consider n assets $S_j(j = 1, 2, \dots n)$ with returns $R_j(j = 1, 2, \dots n)$ respectively. Let μ_i and σ_i denote the expected return and the standard deviation of the i^{th} asset return. A portfolio is represented by the n dimensional vector $x =$

$[x_1, ..., x_n]^T$ where x_i denotes the proportion of the total funds invested in asset S_i. Let $\mu = [\mu_1, ..., \mu_n]^T$, and $\Sigma = (\sigma_{ij})$ be the $n \times n$ covariance matrix. The expected return and the variance of the resulting portfolio are respectively given by,

$$E[x] = \mu^T x = x_1\mu_1 + \cdots + x_n\mu_n \tag{1}$$

and

$$Var[x] = x^T \Sigma x = \sum_{i,j} x_i x_j \sigma_{ij} \tag{2}$$

The Markowitz Mean-Variance Model can be formulated as follows [5]:

$$\min \quad \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{3}$$

$$s.t: \quad \sum_{i=1}^{n} x_i\mu_i \geq t, \tag{4}$$

$$\sum_{i=1}^{n} x_i = 1, \qquad x_i \geq 0 \tag{5}$$

The above problem is a quadratic optimization problem. Here the constant $\frac{1}{2}$ is multiplied in the objective function for convenience in optimality conditions. The objective function minimizes the total risk associated with the portfolio subject to the expected return of the portfolio being above some target return value t. The non-negative constraint indicates that no short selling is allowed. However for non-normal return data some other risk measures such as, *Lower Semiabsolute Deviation*, *Lower Semivariance*, *Value at risk*, *Conditional Value at Risk* can be used instead of variance.[6]
Now let $R(x)$ be the rate of return of the portfolio given by $\sum_{j=1}^{n} R_j x_j$ and $\mu(x) = \sum_{j=1}^{n} \mu_j x_j$ be the expected value of $R(x)$. Then the Lower Semivariance of the portfolio is defined as:

$$V_-(x) = E(|R(x) - \mu(x)|_-^2) \tag{6}$$

where $|u|_- = \min\{0, u\}$. That means the lower semivariance considers only those return values which are below the expected return.

2.1 Particle Swarm Optimization

Particle Swarm Optimization (PSO)[7, 8] is based on the behavior of group of birds while searching for food. Here a randomly distributed set of particles (potential solution) is chosen and the PSO algorithm tries to improve the solutions based on the fitness function.
Suppose there are N number of particles $x_i, i = 1, 2, \ldots, N$ in the particle

swarm. Let the position vector and the velocity vector of the *ith* particle are $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})$ and $v_i = (v_{i1}, v_{i2}, \ldots, v_{in})$ respectively in n- dimensional search space. Let the personal best of the *ith* particle is given by the vector $p_{best_i} = (p_{i1}, p_{i2}, \ldots, p_{in})$ and the global best of the entire solution is given by the vector $g_{best} = (g_1, g_2, \ldots, g_n)$. Now we update the velocity of each particle as:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 r_1 [p_{ij}(t) - x_{ij}(t)] + c_2 r_2 [g_j(t) - x_{ij}(t)] \tag{7}$$

$$i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, n \tag{8}$$

where w is the weight vector for the velocity component, c_1 and c_2 are the accelerating factors, r_1 and r_2 are random numbers whose values lie in between 0 and 1, t is the iteration number.
Then we update the position of the particles by:

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \qquad i = 1, 2, \ldots, N, \quad j = 1, 2, \ldots, n \tag{9}$$

Then we compare the fitness values of the new position with the previous personal best (p_{best}) solution. If the new position is better than the previous p_{best} solution, then update $p_{best_i}(t+1) = x_i(t+1)$; otherwise $p_{best_i}(t+1) = p_{best_i}(t)$. Then update the g_{best} as the point at which we have the best fitness value among all the points in the population. The process is repeated until the termination condition is achieved.

2.2 Invasive Weed Optimization

Invasive Weed Optimization (IWO)[8, 9] is based on the behavior of weeds . The IWO method is described as follows:

1. *Initialization:* An initial population is randomly generated with n number of weeds in search space.
2. *Reproduction:* Every weed produces a number of seeds depending on its fitness value. Let S_{min} and S_{max} be the number of seeds produced by the weakest and strongest weed respectively.
3. *Dispersion of Seeds:* The produced seeds are randomly distributed over the search area with zero mean and standard deviation given by,

$$\sigma_{iter} = \left(\frac{iter_{max} - iter}{iter_{max}}\right)^n (\sigma_{initial} - \sigma_{final}) + \sigma_{final} \tag{10}$$

4. *Elimination:* When the number of plants exceeds a maximum limit (P_{max}), the weed with worst fitness value is removed from the colony.
5. The process continues till the maximum number of iterations is reached.

3 Results and Discussions

Here we have considered three portfolio problems. In *Problem-1*, we have taken the closing price data of three stocks IBM (IBM), WMT (Walmart), SEHI

(Southern Electric) in the last trading day of November 2000 to November 2001. Our aim is to make such an allocation that minimize the risk with the lower bound of the expected portfolio return being 0.05. Let the weights associated with these three stocks be w_1, w_2 and w_3 respectively. After finding the expected return vector and the covariance matrix of these assets, we formulated the mean-variance portfolio model as:

$$\begin{aligned}
\min \quad & \frac{1}{2}\big\{0.017092333w_1^2 + 0.005878743w_2^2 + 0.062948688w_3^2 + \\
& 2(0.0033025w_1w_2) + 2(0.004493854w_1w_3) + 2(0.004493854w_2w_3)\big\} \\
s.t: \quad & 0.026w_1 + 0.008083333w_2 + 0.07375w_3 \geq 0.05, \\
& \sum_{i=1}^{3} w_i = 1, \qquad w_i \geq 0
\end{aligned}$$

In *Probelm-2*, we have taken the daily return data of three stocks AAPL (Apple Inc.), BAC (Bank of America Corp), TEVA (Teva Pharmaceutical Industries) for the year 2018 from www.kaggle.com. Let their weights in the portfolio be x_1, x_2 and x_3 respectively. Here the lower bound of the expected portfolio return is 0.001. So the mean-variance model for this problem is given by:

$$\begin{aligned}
\min \quad & \frac{1}{2}\big\{0.000209147x_1^2 + 2.14E-04x_2^2 + 6.57E-04x_3^2 + \\
& 2(9.73E-05x_1x_2) + 2(8.63E-05x_1x_3) + 2(1.54E-04x_2x_3)\big\} \\
s.t: \quad & 0.001456478x_1 + 0.000183735x_2 + 0.000685123x_3 \geq 0.001, \\
& \sum_{i=1}^{3} x_i = 1, \qquad x_i \geq 0
\end{aligned}$$

In *Probelm-3*, we formulated the mean-semivariance model by using the same return data which was used in *Probelm 2*. The problem is given by:

$$\begin{aligned}
\min \quad & \frac{1}{2}\big\{6.97E-05x_1^2 + 9.37E-05x_2^2 + 0.000255371x_3^2 + \\
& 2(3.43E-05x_1x_2) + 2(3.22E-05x_1x_3) + 2(6.21E-05x_2x_3)\big\} \\
s.t: \quad & 0.001456478x_1 + 0.000183735x_2 + 0.000685123x_3 \geq 0.001, \\
& \sum_{i=1}^{3} x_i = 1, \qquad x_i \geq 0
\end{aligned}$$

We have solved these three portfolio problems using PSO and IWO (taking the initial solution as $w_1 = 0, w_2 = 0, w_3 = 1$ for *Probelm 1* and $x_1 = 1, x_2 = 0, x_3 = 0$ for *Probelms 2 and 3*) and made a comparison of the results in Table 1, 2 and 3. The Convergence Characteristic Curves of the two approaches for the above three problems are shown in Fig.s 1, 2 and 3.

Table 1. Comparison of Results for Problem-1

	PSO	**IWO**
Best Portfolio Weights	[0 0.35816 0.63696]	[0.50057 0.0054544 0.49869]
Portfolio Risk	0.014183	0.0103018
Portfolio Return	0.0500000	0.0499615
No. of Iterations for Convergence	34	245

Table 2. Comparison of Results for Problem-2

	PSO	**IWO**
Best Portfolio Weights	[0.642273 0.352727 0]	[0.613040 0.305270 0.078025]
Portfolio Risk	7.851610E-05	7.728261E-05
Portfolio Return	0.001000265	0.001002425
No. of Iterations for Convergence	112	30

Table 3. Comparison of Results for Problem-3

	PSO	**IWO**
Best Portfolio Weights	[0.621166 0.3064053 0.067430]	[0.62671 0.30367 0.061954]
Portfolio Risk	2.75858393E-05	2.74445599E-05
Portfolio Return	0.00100721	0.00101103
No. of Iterations for Convergence	21	174

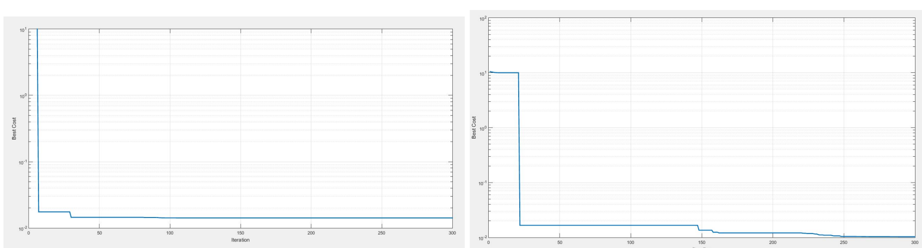

Fig. 1. Convergence Characteristic Curves of PSO (left) and IWO (right) for Problem-1

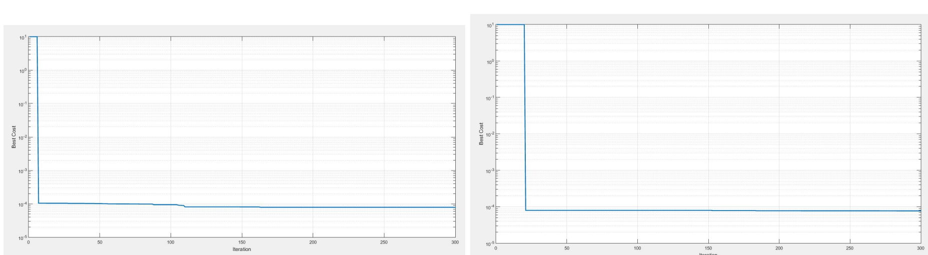

Fig. 2. Convergence Characteristic Curves of PSO (left) and IWO (right) for Problem-2

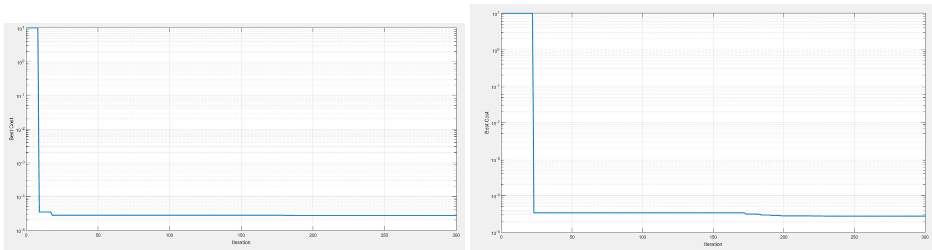

Fig. 3. Convergence Characteristic Curves of PSO (left) and IWO (right) for Problem-3

4 Concluding Remarks

The results show that IWO gives the better portfolio returns and risk values in all the three problems. But in *Probelms 1 and 3* we got faster convergence in PSO method. This tells that none of the discussed methods is more efficient than the other in every aspect. However since in portfolio problems our main aim is to get the best portfolio and not to get the faster convergence, so IWO may be considered as a better method.

References

1. Markowitz, H., Portfolio Selection, Journal of Finance, Vol. 7, 77-91 (1952)
2. Markowitz, H., Portfolio Selection: efficient diversification of investments, John Wiley and Sons, Inc., New York (1959)
3. Chang, T. J., Yang, S. C., and Chang, K. J. , Portfolio optimization problems in different risk measures using genetic algorithm, Expert Systems with Applications, Vol. 36(7), pp. 10529-10537 (2009)
4. Golmakani, H. R., and Fazel, M., Constrained portfolio selection using particle swarm optimization, Expert Systems with Applications, Vol. 38(7), pp. 8327-8335 (2011)
5. Cornuejols, G., and Tutuncu, R., Optimization methods in finance(Vol.5),pp. 141-169. Cambridge University Press (2006)
6. Konno, H., Waki, H., and Yuuki, A., Portfolio optimization under lower partial risk measures, Asia-Pacific Financial Markets, Vol. 9(2), pp. 127-140 (2002)
7. Parsopoulos, K. E., and Vrahatis, M. N., Particle swarm optimization method for constrained optimization problems, Intelligent TechnologiesTheory and Application: New Trends in Intelligent Technologies, Vol. 76(1), 214-220 (2002)
8. Shahriar, M. S., et. al., Comparison of Invasive Weed Optimization (IWO) and Particle Swarm Optimization (PSO) in improving power system stability by UPFC controller employing a Multi-objective approach, In 1st International Conference on Advanced Information and Communication Technology 2016 (ICAICT 2016), pp. 1-7 (2016)
9. Naidu, Y. R., and Ojha, A. K., Solving nonlinear constrained optimization problems using invasive weed optimization.In Intelligent Computing, Communication and Devices.. Springer, New Delhi, pp. 127-133 (2015)