

EFFECTS OF RADIATION AND RADIATION ABSORPTION ON UNSTEADY MHD FLOW PAST A VERTICAL POROUS FLAT PLATE IN A ROTATING SYSTEM WITH CHEMICAL REACTION IN A NANOFLUID

J.L. Rama Prasad^{1*}, G. Dharmaiah² and K. S. Balamurugan³

^{1*}Department of Mathematics, P B Siddhartha College of Arts and Science, Vijayawada
Andhra Pradesh, India, Email: jlrprasad@gmail.com

²Department of Mathematics, Narasaraopeta Engineering College, Narasaraopet,
Andhra Pradesh, India, Email: dharma.g2007@gmail.com

³Department of Mathematics, R V R & J C College of Engineering, Guntur,
Andhra Pradesh, India, Email: muruganbalaks@gmail.com

ABSTRACT

In the presence of radiation, heat absorption and chemical reaction, an analysis of the MHD Ag-nanofluid boundary flow in a rotating system was presented, where the flow of a nanofluid is enclosed by a semi-infinite flat plate under a rotating frame of reference with an assumption that the plate oscillates with steady frequency so that the solutions of the boundary layer are the similar oscillatory kind. The partial differential equations of system were solved using perturbation technique. The changes in velocity, temperature and concentration of the boundary layer Ag-water based nanofluid flow with respect to various parameters are discussed in detail.

KEYWORDS

Radiation, Ag-water Nanofluid, sphere shape particles, heat transfer, porous.

INTRODUCTION

Choi [1] was the first one who discussed about the fluids, called nanofluids in which particles of nanoscale are suspended in the base fluid. Aluminum, copper, iron and titanium or their oxides are some of the common nanoparticles which are used widely. Due to high thermal conductivity, nano fluids have several useful applications and they are used on a large scale in manufacturing industries, fuel cells, solid-state lighting, and as coolants in auto mobiles etc. Numerous methods have been developed to improve the thermal conductivity of these fluids by suspending nano particles in liquids.

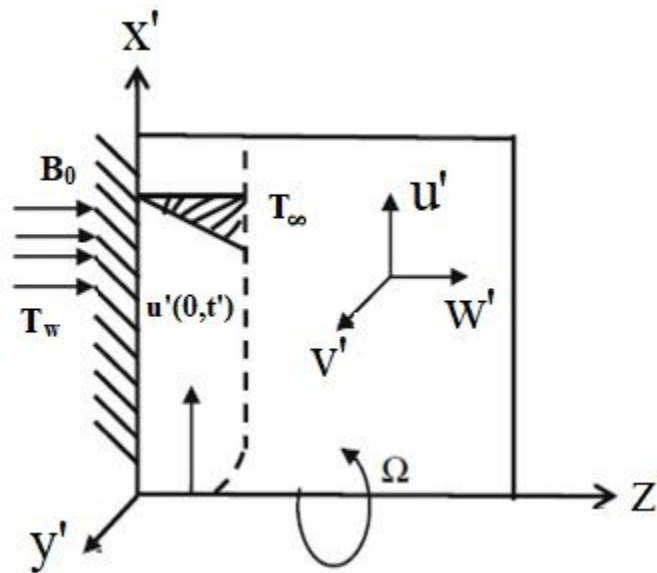
Khanafer et al. [2] analyzed 'buoyancy driven heat transfer enhancement in a two dimensional enclosure utilizing nanofluids'. Ishak [4] studied 'similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition'. Yacob[5] discussed 'boundary layer flow past a stretching/shrinking surface beneath an external uniform shear flow with a convective surface boundary condition in a nanofluid'. KhairyZaimi et al. [12] inspected the flow and heat transfer over a shrinking sheet in a nanofluid with suction at the boundary. Kim et.al. [14] investigated convective instability and heat transfer characteristics of the nanofluids.

Hamad and Pop [3] studied ‘unsteady MHD free convection flow past a vertical permeable flat plate in a rotating frame of reference with constant heat source in a nanofluid’. The fluid flow over a rotating and stretching disk and the flow between two stretching disks was studied by Fang [6,7]. Prasad et al. [13], Turkyilmazoglu [8] worked on MHD fluid flows over a stretching sheet in different environments. Fang and Hua [9], Rashidi et al. [10] investigated the fluid flows over stretchable rotating disks. Bachok et al. [11] used Keller-Box technique for steady Nano fluid flow over a porous rotating disk.

The main aim is to study the effects of different parameters on velocity, temperature and concentration distributions of magneto hydro dynamics Ag-nanofluid boundary flow in a rotating system in the presence of heat absorption, radiation and chemical reaction.

MATHEMATICAL MODEL OF THE FLOW

In presence of radiation, heat absorption and absence of an electric field, consider the flow of a three-dimensional nanofluid, which is grey absorbing/emitting but not scattering medium as shown the figure. Here the flow in the x' – direction is unsteady, electrically conducting, incompressible and passes through a semi-infinite vertical permeable plate, B_0 is external magnetic field, z' axis is normal to the plate, whole system is rotates about the z' -axis with a constant vector Ω , The radiation heat flux in x' -direction is negligible in comparison to that in the z' - direction and the flow variables are functions of z and time t only.



Physical Model and Co-ordinate system

Assumed that the regular fluid and the suspended nano particles are in thermal equilibrium and no slip occurs between them. Boundary layer approximations, the boundary layer equations governing the flow, temperature and concentration along with the Boussinesq are:

$$\frac{\partial w'}{\partial z'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + w' \frac{\partial u'}{\partial z'} - 2\Omega v' = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u'}{\partial z'^2} + \frac{[\rho\beta]_{nf}}{\rho_{nf}} g(T - T_\infty) - \frac{1}{\rho_{nf}} \sigma B_0^2 u' - \frac{1}{\rho_{nf}} \frac{v_f u'}{K'} \quad (2)$$

$$\frac{\partial v'}{\partial t'} + w' \frac{\partial v'}{\partial z'} + 2\Omega u' = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v'}{\partial z'^2} - \frac{1}{\rho_{nf}} \sigma B_0^2 v' - \frac{1}{\rho_{nf}} \frac{v_f v'}{K'} \quad (3)$$

$$\frac{\partial T}{\partial t'} + w' \frac{\partial T}{\partial z'} = \alpha_{nf} \frac{\partial^2 T}{\partial z'^2} - \frac{1}{[\rho Cp]_{nf}} \left(Q'(T - T_\infty) + \frac{\partial q_r}{\partial z'} \right) + Q_1^*(C - C_\infty) \quad (4)$$

$$\frac{\partial C}{\partial t'} + w' \frac{\partial C}{\partial z'} = D^* \frac{\partial^2 C}{\partial z'^2} - K_1^*(C - C_\infty) \quad (5)$$

The boundary conditions are

$$u'(z', t') = 0, v'(z', t') = 0, T = T_\infty, C = C_\infty \quad \text{for } t' \leq 0 \text{ and any } z' \quad (6)$$

$$u'(\infty, t') = U_0 \left[1 + \frac{\varepsilon}{2} (e^{\text{int}'} + e^{-\text{int}'}) \right], T(\infty, t') \rightarrow T_\infty, C(\infty, t') \rightarrow C_\infty \quad \text{for } t' \geq 0 \quad (7)$$

Here u' , v' and w' are the velocity components along the x' , y' and z' axis respectively.

The properties of nanofluids are defined as [cf. 16]

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s,$$

$$(\rho Cp)_{nf} = (1 - \phi)(\rho Cp)_f + \phi(\rho Cp)_s,$$

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s,$$

$$\mu_{nf} = \frac{\mu_f}{[1 - \phi]^{2.5}}, \quad (8)$$

$$\alpha_{nf} = \frac{K_{nf}}{(\rho Cp)_{nf}},$$

$$\frac{K_{nf}}{K_f} = \left[\frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)} \right]$$

The thermo physical properties of the base fluid(water) and silver are given below.

| Nanofluid | ρ | Cp | K | $\beta \times 10^5$ |
|------------|--------|------|-------|---------------------|
| Silver(Ag) | 10,500 | 235 | 429 | 1.89 |
| Pure water | 997.1 | 4179 | 0.613 | 21 |

By using Rosseland approximation, the radiative flux vector q_r can be written as:

$$q'_r = -\frac{4\sigma^*}{3k'_1} \frac{\partial T'^4}{\partial z} \quad (9)$$

where, σ^* and k'_1 are respectively the Stefan-Boltzmann constant and the mean absorption coefficient.

We can assume that

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (10)$$

because temperature difference within the flow is sufficiently small.

$$\text{The solution of Eq. (1) is considered as } w' = -w_0 \quad (11)$$

Introducing dimensionless variables

$$\begin{aligned} u &= \frac{u'}{U_0}, v = \frac{v'}{U_0}, z = \frac{z'U_0}{v_f}, t = \frac{t'U_0^2}{v_f}, n = \frac{v_f n'}{U_0^2}, \\ K &= \frac{K' \rho_f U_0^2}{v_f^2}, S = \frac{w_0}{U_0}, R = \frac{2\Omega v_f}{U_0^2}, Q_H = \frac{Q' v_f^2}{K_f U_0^2}, \\ Q_1 &= \frac{Q_1^* (C_w - C_\infty)}{(T_w - T_\infty) U_0^2}, F = \frac{4\sigma^* T_\infty^3}{k k'_1}, \text{Pr} = \frac{v_f (\rho C p)_f}{K_f} \end{aligned} \quad (12)$$

Substituting Eq. (12) into Eqs. (2) – (5) gives the following dimensionless equations:

$$\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial z} - Rv = B_1 \frac{\partial^2 u}{\partial z^2} + B_2 \theta - B_3 u \left[M + \frac{1}{K} \right] \quad (13)$$

$$\frac{\partial v}{\partial t} - S \frac{\partial v}{\partial z} + Ru = B_1 \frac{\partial^2 v}{\partial z^2} - B_3 v \left[M + \frac{1}{K} \right] \quad (14)$$

$$\frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial z} = \frac{B_4}{\text{Pr}} \frac{\partial^2 \theta}{\partial z^2} - \frac{B_5 Q_H}{\text{Pr}} \theta + Q_1 \psi \quad (15)$$

$$\frac{\partial \psi}{\partial t} - S \frac{\partial \psi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \psi}{\partial z^2} - Kr \psi \quad (16)$$

The corresponding boundary conditions (5) in dimensionless form are:

$$\begin{aligned} u(z,t) = 0, v(z,t) = 0, \theta(z,t) = 0, \psi(z,t) = 0 \quad \text{for } t \leq 0 \text{ and any } z \\ u(0,t) = 1 + \frac{\varepsilon}{2}[e^{\text{int}} + e^{-\text{int}}], v(0,t) = 0, \theta(z,t) = 1, \psi(z,t) = 1 \left\{ \text{for } t \geq 0 \right. \\ \left. u(\infty,t) \rightarrow 0, v(\infty,t) \rightarrow 0, \theta(\infty,t) \rightarrow 0, \psi(\infty,t) \rightarrow 0 \right\} \end{aligned} \quad (17)$$

Using Eq. (13) the velocity characteristic U_0 is defined as

$$U_0 = \sqrt[3]{g\beta_f\nu_f(T_w - T_\infty)}$$

we assume that the fluid velocity in the complex form as

$$\chi(z,t) = u(z,t) + iv(z,t) \quad (18)$$

By using Eq. (18) we can simplify Eqs. (13) and (14) to the following equation

$$\frac{\partial \chi}{\partial t} - S \frac{\partial \chi}{\partial z} - Rv = B_1 \frac{\partial^2 \chi}{\partial z^2} + B_2 \theta - B_3 \chi \left[M + \frac{1}{K} \right] \quad (19)$$

The boundary conditions (17) become

$$\begin{aligned} \chi(z,t) = 0, \theta(z,t) = 0, \psi(z,t) = 0 \quad \text{for } t \leq 0 \text{ and any } z \\ \chi(0,t) = 1 + \frac{\varepsilon}{2}[e^{\text{int}} + e^{-\text{int}}], \theta(z,t) = 1, \psi(z,t) = 1 \left\{ \text{for } t \geq 0 \right. \\ \left. \chi(\infty,t) \rightarrow 0, \theta(\infty,t) \rightarrow 0, \psi(\infty,t) \rightarrow 0 \right\} \end{aligned} \quad (20)$$

To solve Eqs. (13)- (16) under the boundary conditions (17) in the neighborhood of the plate, we assume that (see Ganapathy [15])

$$\chi(z,t) = \chi_0(z,t) + \frac{\varepsilon}{2} \{ e^{\text{int}} \chi_1 + e^{-\text{int}} \chi_2 \} \quad (21)$$

$$\theta(z,t) = \theta_0(z,t) + \frac{\varepsilon}{2} \{ e^{\text{int}} \theta_1 + e^{-\text{int}} \theta_2 \} \quad (22)$$

$$\psi(z,t) = \psi_0(z,t) + \frac{\varepsilon}{2} \{ e^{\text{int}} \psi_1 + e^{-\text{int}} \psi_2 \} \quad (23)$$

Substituting the above equations (21) - (23) into the Eqs. (13) - (16) with boundary conditions (17), we obtain

$$\chi(z,t) = P_3 e^{-\xi_1 z} + P_4 e^{-\xi_2 z} + P_5 e^{-\xi_3 z} + \frac{\varepsilon}{2} \{ e^{-\xi_4 z} e^{\text{int}} + e^{-\xi_5 z} e^{-\text{int}} \} \quad (24)$$

$$\theta(z,t) = P_1 e^{-\xi_1 z} + P_2 e^{-\xi_2 z} \quad (25)$$

$$\psi(z,t) = e^{-\xi_1 z} \quad (26)$$

The skin-friction coefficient, Nusselt number and Sherwood number are given by

$$C_f = \left(\frac{\partial \chi}{\partial z} \right)_{z=0} = (-1) \left\{ (P_3 \xi_1 + P_4 \xi_2 + P_5 \xi_3) + \frac{\varepsilon}{2} (\xi_4 \exp(int) + \xi_5 \exp(-int)) \right\} \quad (27)$$

$$Nu = - \left(\frac{\partial \theta}{\partial z} \right)_{z=0} = P_1 \xi_1 + P_2 \xi_2 \quad (28)$$

$$Sh = - \left(\frac{\partial C}{\partial z} \right)_{z=0} = \xi_1 \quad (29)$$

RESULTS AND DISCUSSION

The effects on the velocity, the temperature and the concentration profiles are discussed through graphs. In the present study the following default parameter values are adopted. $n=10$, $nt = \pi/2$, $Pr = 6.72$, $\varepsilon = 0.001$, $F = 1$, $K = 0.5$, $Kr = 0.5$, $M = 1$, $Q_H = 10$, $Q_1 = 1$, $R = 0.02$, $S = 1$, $Sc = 0.16$, $\phi = 0.10$.

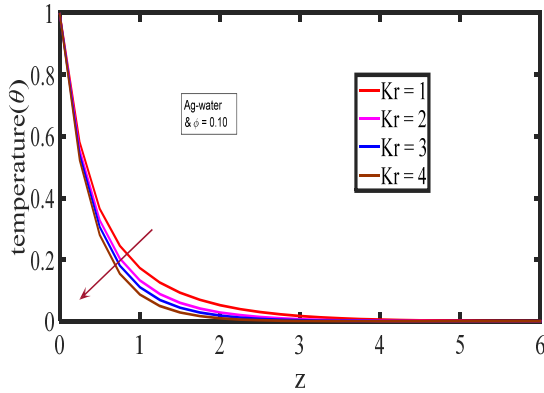


Fig. 1

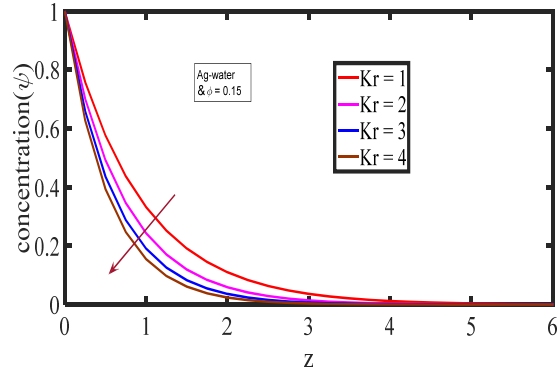


Fig. 2

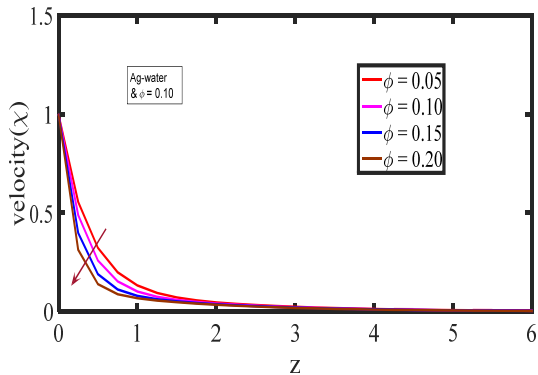


Fig. 3

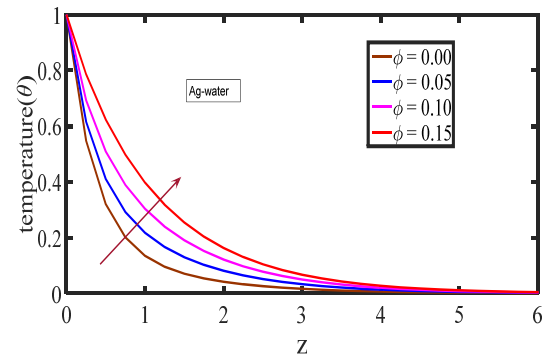


Fig. 4

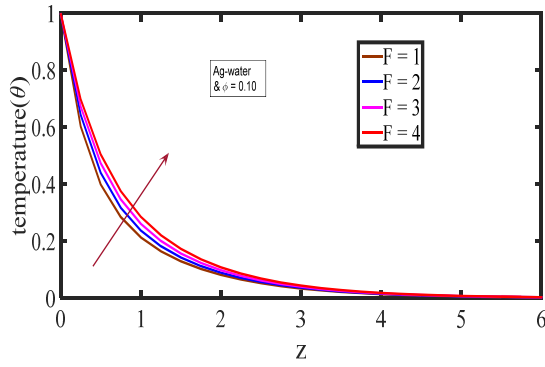


Fig. 5

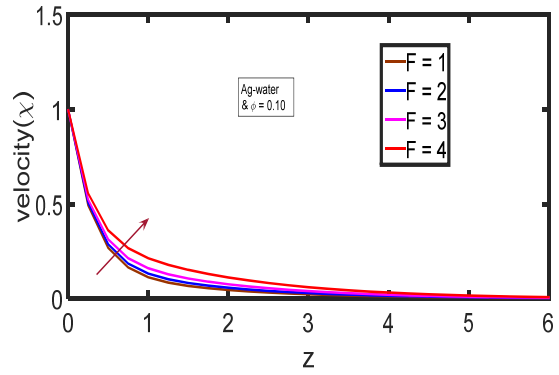


Fig. 6

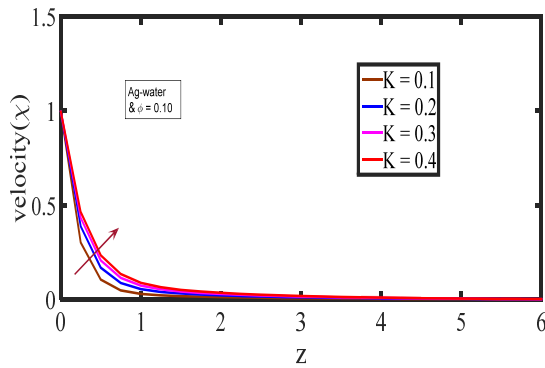


Fig. 7

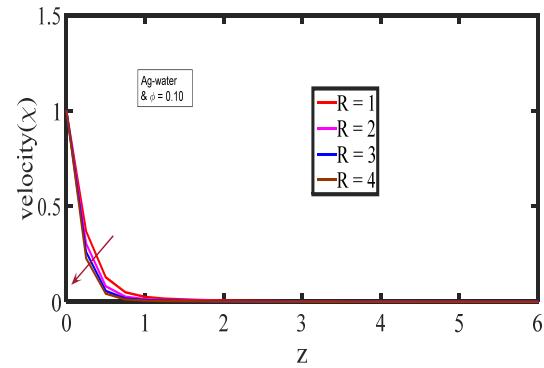


Fig. 8

The effect of chemical reaction parameter K_r on the nanofluid temperature and concentration profiles for the Ag-water nano fluid particles are illustrated in Figs. 1 and 2 and an increase in K_r leads to the decrease in the nanofluid temperature and concentration distributions. The effect of the nanoparticle volume fraction ϕ on the velocity and temperature profiles is given Figures 3 and 4. It is clear that as ϕ increases, the nanofluid velocity decreases and temperature distributions increases.

The effect of thermal radiation parameter F on the nanofluid temperature and velocity profiles is given in Figures 5 and 6 respectively. It is clear that, as F increases, the nano fluid velocity and temperature distributions across the boundary layer increases. The increase in radiation parameter results to the release of heat energy from the flow region and hence temperature x increases as the thermal boundary layer thickness become thin.

The effect of permeability parameter K on the nano fluid velocity distribution can be seen from Figure 7. It is clear that as K increases, the nano fluid velocity increases. The effect of rotation parameter R on χ for different values of Ag nano particles is given in Fig. 8 and observed that with an increase of R , the nano fluid velocity distribution across the boundary layer decreases.

| Table 1 | | | | | |
|----------------|-------------|----------|------------|------------|----------------|
| S | ϕ | Q_H | F | Q_1 | Nu |
| 2 | 0.10 | 0.1 | 1.0 | 0.1 | 4.2597 |
| 3 | 0.10 | 0.1 | 1.0 | 0.1 | 6.6137 |
| 4 | 0.10 | 0.1 | 1.0 | 0.1 | 9.0643 |
| 5 | 0.10 | 0.1 | 1.0 | 0.1 | 11.5333 |
| 1 | 0.05 | 0.1 | 1.0 | 0.1 | 2.0994 |
| 1 | 0.10 | 0.1 | 1.0 | 0.1 | 2.2352 |
| 1 | 0.15 | 0.1 | 1.0 | 0.1 | 2.4240 |
| 1 | 0.20 | 0.1 | 1.0 | 0.1 | 2.6763 |
| 1 | 0.10 | 2 | 1.0 | 0.1 | 0.5746 |
| 1 | 0.10 | 4 | 1.0 | 0.1 | 1.1705 |
| 1 | 0.10 | 6 | 1.0 | 0.1 | 1.6034 |
| 1 | 0.10 | 8 | 1.0 | 0.1 | 1.9472 |
| 1 | 0.10 | 0.1 | 0.1 | 0.1 | 2.8532 |
| 1 | 0.10 | 0.1 | 0.2 | 0.1 | 2.7496 |
| 1 | 0.10 | 0.1 | 0.3 | 0.1 | 2.6589 |
| 1 | 0.10 | 0.1 | 0.4 | 0.1 | 2.5786 |
| 1 | 0.10 | 0.1 | 1.0 | 0.1 | 3.3450 |
| 1 | 0.10 | 0.1 | 1.0 | 0.2 | 3.2217 |
| 1 | 0.10 | 0.1 | 1.0 | 0.3 | 3.0984 |
| 1 | 0.10 | 0.1 | 1.0 | 0.4 | 2.9751 |

| Table 2 | | | |
|----------------|-------------|------------|---------------|
| S | Sc | Kr | Sh |
| 2 | 0.30 | 0.5 | 1.2928 |
| 3 | 0.30 | 0.5 | 1.7396 |
| 4 | 0.30 | 0.5 | 2.2100 |
| 5 | 0.30 | 0.5 | 2.6937 |
| 1 | 0.45 | 0.5 | 1.1925 |
| 1 | 0.60 | 0.5 | 1.4832 |
| 1 | 0.78 | 0.5 | 1.8212 |
| 1 | 0.96 | 0.5 | 2.1520 |
| 1 | 0.30 | 0.1 | 0.6240 |
| 1 | 0.30 | 0.2 | 0.7062 |
| 1 | 0.30 | 0.3 | 0.7743 |
| 1 | 0.30 | 0.4 | 0.8339 |

From Table 1, it is clear that with the increasing values of S, ϕ and Q_H , Nusselt number increases and with the increasing values of F and Q_1 , Nusselt number decreases. From Table 2, it is observed that Sherwood number increases with the increasing values of S, Sc and Kr.

CONCLUSIONS

In this work, the authors theoretically studied the effects of various parameters on velocity, temperature and concentration profiles are discussed through graphs, of the unsteady MHD free convection heat and mass transfer flow of an incompressible, Ag-water based nano fluid along a semi-infinite vertical flat plate in a rotating frame of reference and arrived at the following conclusions.

1. The nano fluid velocity increases with an increase of K.
2. The nanofluid velocity and temperature distributions across the boundary layer increases with an increase of F.
3. An increase of Kr results to the decrease in the nanofluid velocity, temperature and concentration distributions.
4. The nanofluid velocity and temperature profiles increase with an increase of Q_1 .
5. It is clear that as the nanoparticle volume fraction increases, the nanofluid velocity decreases and temperature distributions increases.

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