

# Numerical Optimization of Pile foundation in Non-liquefiable and Liquefiable soils conditions

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## Abstract:

Numerical optimization techniques are used widely for different engineering fields. But there are limited applications of this method in geotechnical engineering. However in this study topology optimization of pile foundation for different site conditions and loading conditions is obtained through a finite element (FE) analysis study. The suitable topology of piles in foundation system offering minimum internal energy i.e. maximum stiffness for a given fraction of material is studied. The study is also enhanced to the piles which are located in the soils prone to liquefaction. In the present study the design methodology for cost optimization of construction of a pile group with a raft foundation is also presented through a case study. In the optimization algorithm the raft dimensions, no of piles, pile diameter, pile length are taken as the design variables.

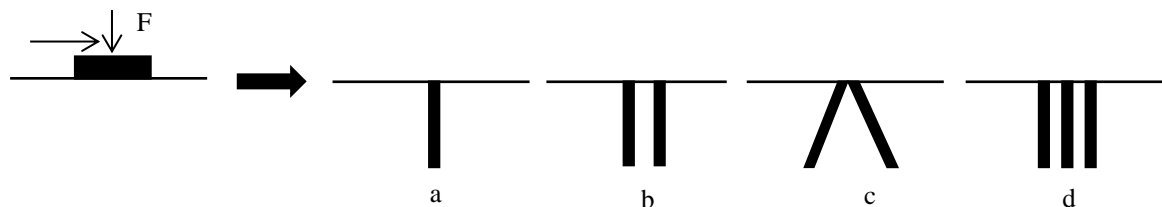
**Key words:** Pile foundation, Topology optimization, Liquefaction, Cost optimization

## 1. Introduction

Pile foundation are used for large number of purposes in geotechnical fields like to cater the heavy load of the structure to a harder strata, to support the structure where the uplift force is high, to resist lateral loads, supporting retaining walls, bridge piers, abutments etc. In general, there are two types of pile namely driven piles and bored piles. Once the decision for pile foundation has been taken, the engineer must choose the type, topology and size of the pile which is most suitable for a particular soil and loading condition. Using the numerical optimization technique it will be much easier to obtain the topology of pile foundation which is most suitable and will optimize the material use.

### 1.1. Topology optimization of pile foundation

When a structure is carrying heavy load and near ground soil strata is comparatively weak, it is often inevitable to go for pile foundation. At this stage topology optimization can be conducted which will improve deformational behaviour of structure along with cost saving, economy in material use. Possible topology of piles under a strip footing may be any one of the followings as depicted in Fig.1.a, b, c, d.



**Fig. 1.** Topology optimization of pile foundation: four different topologies

Depending on the magnitude and direction of the forces acting on the strip footing and the soil characteristics existing in site, the best suitable pile foundation topology can be established. Further using shape optimization more knowledge about the suitable shape of that chosen topology can be obtained (Fig.2.a, b, c, d).



**Fig. 2.** Shape optimization: four different shape for foundation

Further size optimization may be used to optimize size of pile (Fig. 2.a) or the dimensions of varied part in piles (Fig. 2. b, c, d).

Topology optimization was first used in geotechnical problem for the underground excavation in linear elastic rock material by Ren et al. [1]. Pucker & Garbe [2] presented topology optimization under a strip footing in granular hypoplastic material. In this paper the application of topology optimization is presented by FE analysis for both liquefiable and non-liquefiable soils under different loading conditions. The FE analysis of topology optimization is based on solid isotropic material with penalisation (SIMP) method which is illustrated briefly in section 2.

### 1.2. Cost optimization of pile foundation with a raft

In general the foundation cost of real-world structures can vary from 5% to 20% of the construction cost of the superstructure [3]. For a conservative design much attention is not paid to cost of the construction of such foundation. Hence, sometime it may happens that the number of pile used in foundation for design is much more than actual requirement and this unnecessary increases cost of the project. Hence an optimum design methodology fulfilling all structural criteria should be studied to minimize cost prior to the detail designing of the proposed structure. The main objective is to minimize the pile foundation cost with the consideration of all the constraints like bearing capacity of the soil beneath the raft, pile load bearing capacity, settlement criteria as in IS: 2911 Part IV [4]. The optimization formulation in the present study is carried out through a case study and is based on the evolutionary algorithm by Lagaros et al. [5].

## 2. Topology optimization of pile foundation

In FE analysis the topology optimization is carried out by using numerical optimization algorithm i.e. SIMP by Sigmund [6] through an iterative procedure. SIMP method assumes that material is uniformly placed in the design domain. The material property depends on the relative density  $\rho$ . The relative density varies over the design domain. It is considered that material is concentrated at highly loaded regions. The relative density should be either zero or one in the design domain after optimization completed. Zero relative density signifies no material and unit value signifies material.

The aim of the optimization procedure is to minimise the compliance i.e. the internal energy of the structure in the design domain so that stiffness of the structure can be maximised for a given fraction of material in the design domain.

Optimization task is basically,

$$\text{Minimise: internal energy } c(x) = U^T KU \text{ i.e also the objective function} \quad (1)$$

$$\text{Subject to: } KU = F \quad (2)$$

$$V_\delta = V_0 \cdot \delta \text{ These are the constraints.} \quad (3)$$

where  $U$  is the global deformation tensor,  $K$  the global stiffness matrix and  $x$  is tensor of design parameters. Where  $F$  is the external forces and  $V_0$  is the initial volume in the design domain,  $V_\delta$  is the volume after optimization and  $\delta$  is fraction of initial volume. The relative density  $\rho$  of the element changes as iteration progresses. The material change over of material with a relative density  $\rho$  and a chosen penalty term  $p$  from elastic modulus  $E_1$  to  $E_2$  is given by

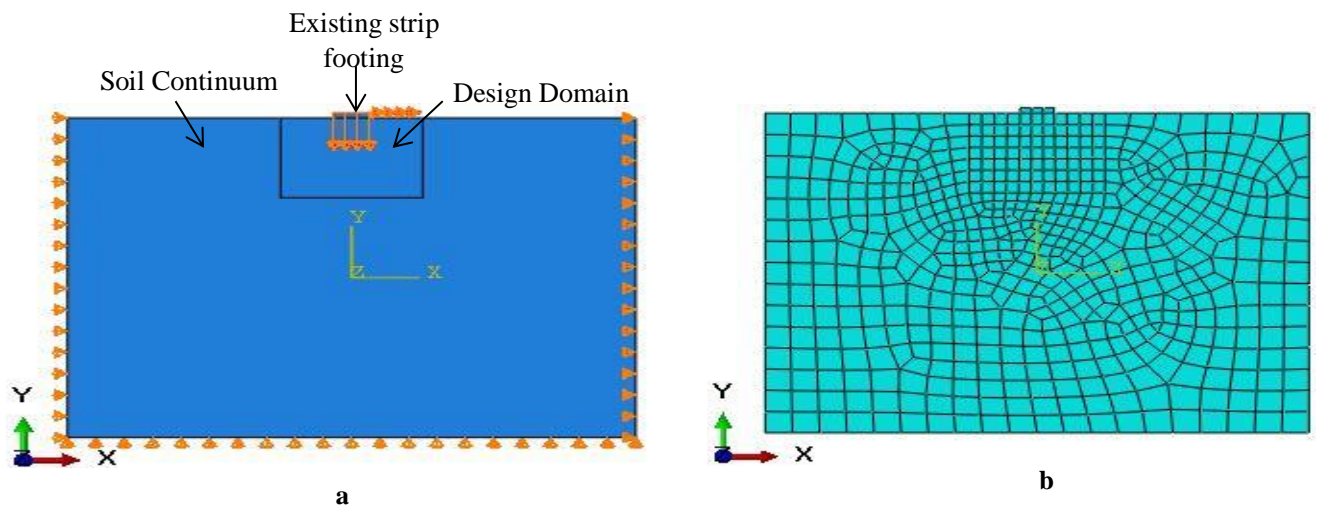
$$E_2 = E_1 \cdot \rho^p \quad (4)$$

## 2.1. FE Modelling

Optimized topology of piles is obtained using this method in a 2D-FE analysis with a strip footing for both non-liquefiable and liquefiable condition of soil under different type of loadings.

### 2.1.1. In non-liquefiable soil

The existing strip footing of 5m wide and 1m depth carries both vertical and horizontal loads is resting on a soil of elastic modulus of 70 MPa and poison ratio 0.3. The elastic modulus of concrete is 25000 MPa. The soil mass continuum of 80m width and 60m deep under the strip footing is considered for developing the numerical model. The design domain is of 20m wide and 15m deep as shown in Fig. 3.a. The x and y displacement of the soil continuum is restricted in both the y and x plane respectively. FE analysis has been carried out for both horizontal and vertical loading.

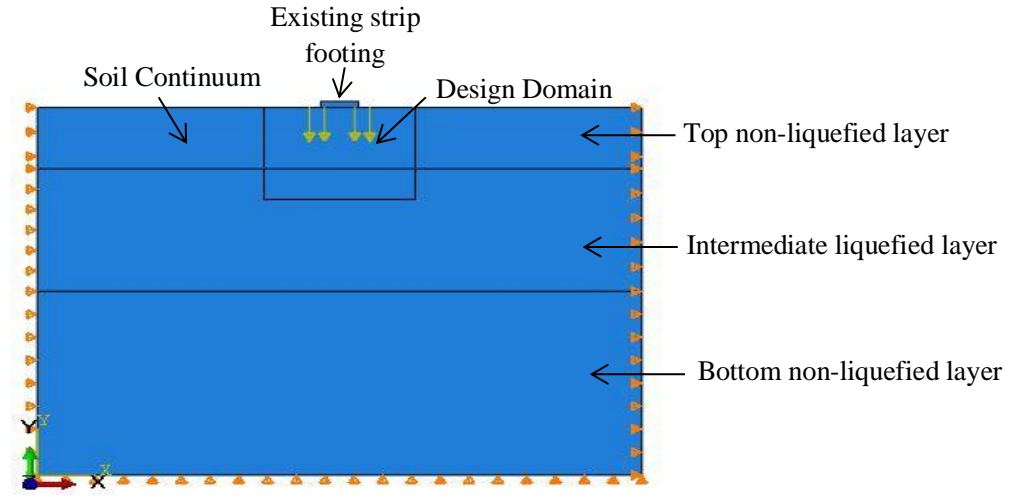


**Fig. 3.** a) Boundary and loading condition of the soil mass under the strip footing tube optimized b) Meshing of the soil continuum, design domain and the strip footing

In the present study, the soil and concrete has been analysed using linear elastic material property. For meshing free mesh control for both soil and strip footing with plane stress linear element of reduced integration and hourglass control has been chosen. As the design domain is our area of concern, the size of meshing is reduced to capture the results efficiently and is shown in Fig. 3.b. For modelling the strip footing and soil, it is assumed that no relative movements occurred between them. Hence they are modelled integrally. The main objective of this modelling is to minimise the strain energy of the whole model by using 10% of the material that is in the design domain. Thus the optimum topology for pile foundation can be obtained.

### 2.1.2. Liquefiable soil

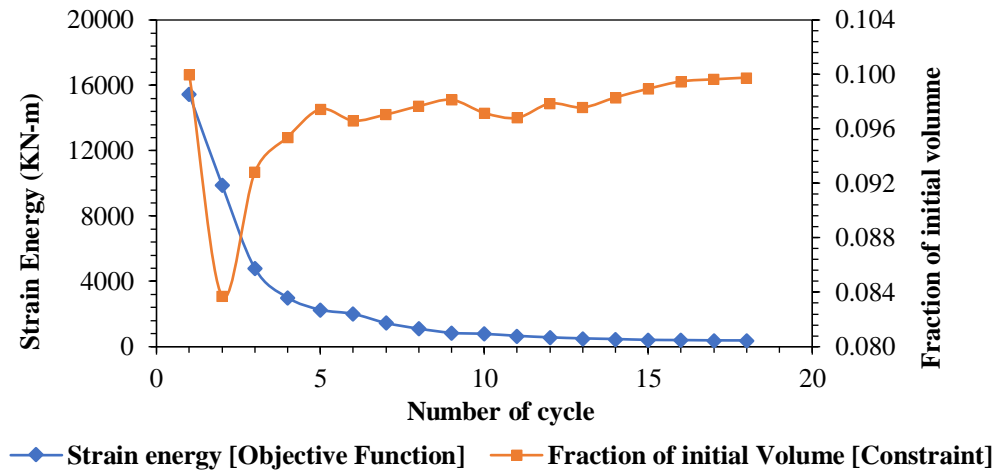
In this analysis it is considered that the soil is susceptible to liquefaction under seismic condition. Hence to optimise the topology for the pile foundation for such soils the post liquefaction modulus of elasticity has been considered i.e. generally one tenth of initial elastic modulus. As it can be observed in Fig. 4 the soil has been divided into three layers of 10m, 20m and 30m from top to bottom. Considering post liquefied state in the middle layer elastic modulus is taken as 7MPa. Concrete property and other soil layers property has been considered same as described in the earlier section for non-liquefiable soil condition. The geometric condition, boundary condition, element type are identical to the previous section. The analysis carried out for a vertical loading of 500KN.



**Fig. 4.** Boundary and loading condition of the soil mass prone to liquefaction under the strip footing

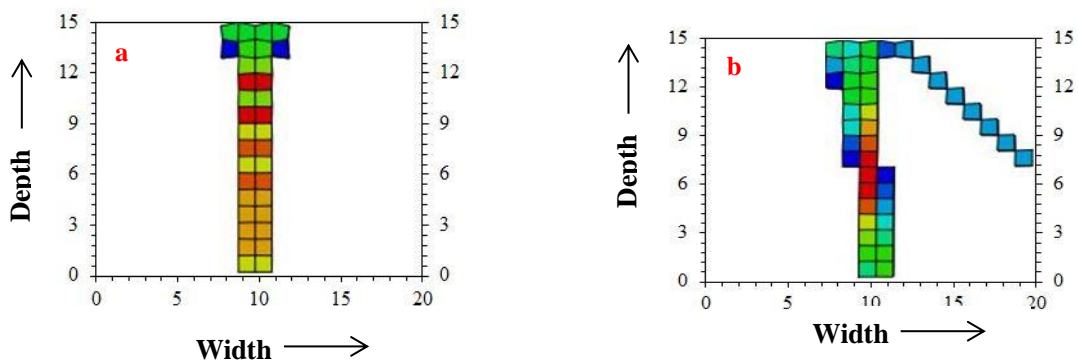
## 2.2. Results and discussion

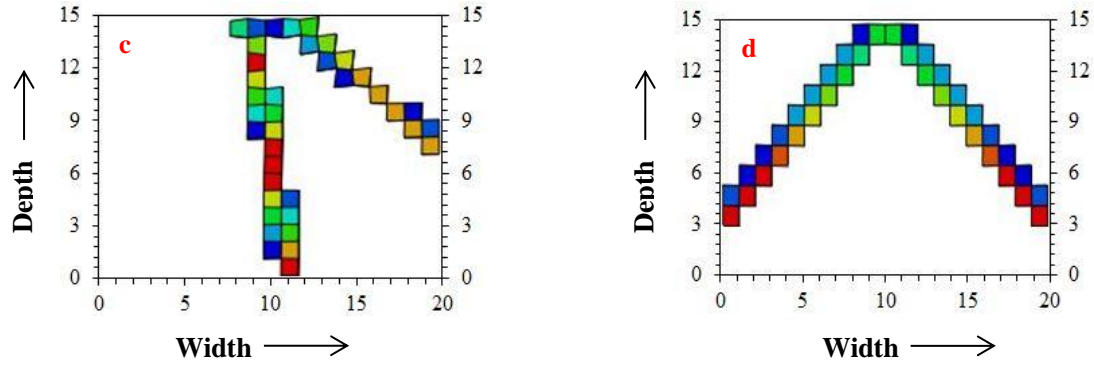
Fig.5. depicts the progress of FE analysis in successive iterations by minimising the strain energy and keeping material fraction 10% of original volume.



**Fig. 5.** Objective function minimisation and material fraction vs. Number of cycle

It can be observed from Fig. 6.a that when there is no eccentricity in loading a simply vertical profile of pile is obtained. From Fig. 6.b it can be seen that a vertical pile along with an inclined pile be the efficient topology for pile for a vertical and horizontal loading condition. Fig. 6.c shows the same topology but with a thicker inclined pile is required than the previous one as the horizontal load increases in this case. Now a day any profile of piles can be constructed efficiently by concrete jet grouting method. Hence, this method realizes pile construction with any optimized topology. In the case of the liquefiable soil it remarked from Fig. 6.d that the batter pile or inclined pile will be most suitable to provide a very stiff foundation within minimum concrete use. But due to being stiff the design for induced seismic forces and ductility check under seismic condition is very important to consider.



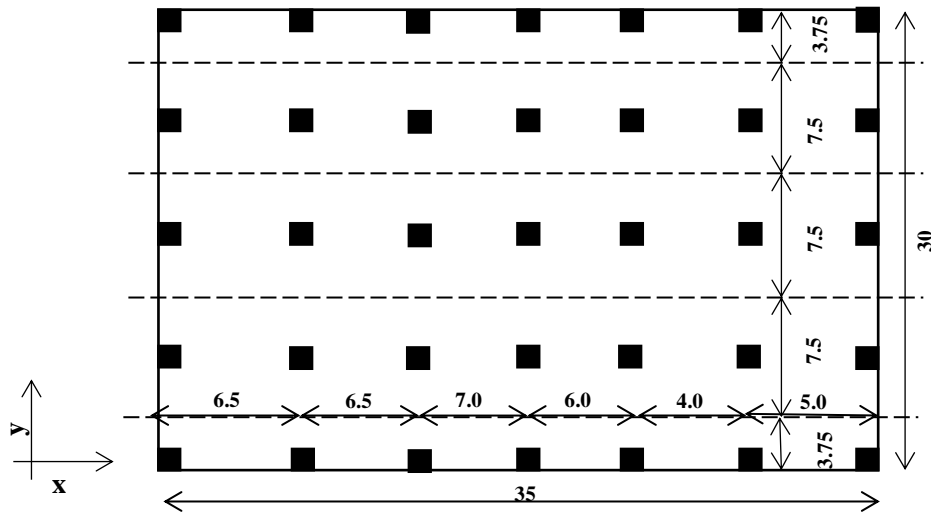


**Fig. 6.** Topology optimization under strip footing for a) Vertical loading,  $P=500$  KN, Horizontal loading,  $H=0$  KN in non-liquefiable soil; b)  $P=500$  KN,  $H=100$  KN in non-liquefiable soil; c)  $P=500$  KN,  $H=250$  KN in non-liquefiable soil; d)  $P=500$  KN,  $H=0$  KN in liquefiable soil.

It is also observed in the present study that as material volume percentage increases the settlement of the foundation gets reduced.

### 3. Cost optimization of a pile foundation with raft

Cost optimization design methodology is described for the structure with the column arrangements as shown in Fig. 7. It is assumed that foundation rests on a soil strata of medium stiff clayey soil upto 20m below foundation level. The undrained compressive strength of soil is 100 kPa. The factored loads in the columns in x direction are 2200KN, 2200KN, 2200KN, 2400KN, 2200KN, 2000KN and 2200KN respectively on the edge strips and in the middle strip columns loads are twice that of the edge strip columns.



[All the dimensions are in meter]

**Fig. 7.** Column arrangement for the case study [7]

#### 3.1. Objective function and constraints of the optimization algorithm

The main objective of this study is to observe how the design methodology is to be applied so that after satisfying all the design criteria the foundation can be built with optimum cost. The overall cost of raft including excavation, reinforcement, casting of concrete is 0.5 lakh per cum and for pile is 2.6 lakh per cum in Mumbai region. The constraints are – bearing pressure under raft, the pressure induced in each pile and pile group settlement. Pile group settlement is calculated by using formula given by Randolph et al. [8]. The length of the raft has been assumed as  $1.16B$ . The solution of the optimization problem is performed with the evolutionary algorithms [5].

### 3.2. Results and discussion

The variables and the result obtained are discussed in Table. 1.

**Table 1.** Variables range and the optimum values of variable in the algorithm

Sl. No.	Variable	Range	Design Values
1	Width of raft (m)	35-45	38
2	Thickness of raft (m)	0.35-0.5	0.43
3	Pile dia (m)	0.6-2.5	2.1
4	Pile length (m)	10-40	15
5	No of pile	50-350	72

Total optimized cost of the pile foundation along with raft is 46.5 Crore considering the structure to be in Mumbai region. The soil pressure as obtained from the numerical formulation under raft is 82 kPa. As the soil is medium stiff clayey soil the observed pressure is less than allowable bearing capacity of soil. It is considered in this analysis that 60% load is shared by pile and 40% load is shared by raft. Each pile has to carry a load of 810KN which is within safe load carrying capacity of each pile. The group settlement of the pile is 10mm which is lower than the value mentioned in IS: 2911 Part (IV) (1979).

### 4. Conclusion

From the present Numerical Optimisation study it can be concluded that vertical piles are best suited for loading without eccentricity. If there is eccentric loading, vertical pile along with inclined pile is most suitable. In case of liquefiable soil batter pile provides the stiffer solution than vertical pile. But the associated cost for realization of these topologies of piles in site must be investigated. Nevertheless, optimization of topology of piles provides significant opportunities in the design process of pile foundation. In the present cost optimization study approximate costs are considered for pile and raft construction in clayey soil. For other type of soil as the cost of pile and raft construction changes the objective function also changes. So a new function should be developed with the presented constraints to minimize the cost. Hence both topology and cost optimization of pile foundation is very much required to be considered in design from both structural and economic perspective.

### 5. Acknowledgement

The academic and research support provided by institution HBNI, BARC and ECIL is gratefully acknowledged by the authors.

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