# EFFECT OF CHEMICAL REACTION AND RADIATION ON MHD CASSON FLOWALONG A MOVING VERTICAL POROUS PLATE

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**Abstract**: In this article, we consider effects of force buoyancy and magnetohydrodynamic on convectivemass and heat transfer flow past a touching vertical porous plate in the incidence ofthermal radiation and chemical reaction. By using similarity transformation the governing partial differential equations are rigorous to a system of self-similar equations. The subsequentcalculations are solved numerically by using the fourth order RK method along with the shooting technique. The results are initiate for the temperature, velocity, concentration. The possessions of several parameters on flowvariables are shown graphically and the physical aspects of the problem are conferred.

**Key words**:, MHD, porous medium, heat and mass transfer, thermal radiation, chemical reaction.

#### INTRODUCTION

In recent years, the work of free convective heat and mass transfer flows concluded a porous medium under the effect of the attention of many researchers was attracted by a magnetic field as many they have number of uses in engineering sciences, easy example transportation cooling, cross-hatching on ablative surfaces.Rajakumar et al.[1] studied chemical reaction and viscous indulgence possessions on MHD free convective flow past a semi-infinite moving vertical porous plate. Thammannaet al.[2] discussed three dimensional MHD flow of couple stress Casson fluid past an unsteady stretching surface with chemical reaction. Malleswari[3] analyzedheat and mass transfer analysis on mud radiating flow in presence of natural convection, porous medium and viscoelastic Rivlin-Ericksen fluid through numerical solutions. Srinivasa Rajuet al.[4] studied influence of angle of inclination on unsteady MHDCasson fluid flow past a vertical surface filled by porous medium in presence of constant heat flux, chemical reaction and viscous dissipation. Ramesh Babuet al.[5] expressed numerical investigation of heat transfer echanism in MHDC asson fluid flow past a vertically inclined plate in presence of hall current.G.V.R. Reddyet al.[6] studied Soret and Dufour effects on MHD micropolar fluid flow over a linearly stretching sheet through a non -Darcy porous medium.Ramana Reddyet al.[7]examined MHD free convective flow of Casson fluid past over an oscillating vertical porous plate. Vijayaet al.[8]presented Soret and radiation effects on an unsteady flow of a Casson fluid through porous vertical channel with expansion and contraction. GVRReddy[9]discussed numerical solutions of unsteady MHD flow heat transfer over a stretching surface with suction or injection. Hari Krishnaet al.[10] analyzedchemical reaction effect on MHD flow of Casson fluid with porous stretching sheet.Daset al.[11]expressed Newtonian heating effect on steady hydro magnetic Casson fluid flow a plate with heat and mass transfer. Dashet al.[12] discussed Casson fluid flow in a pipe

filled with a homogeneous porous medium. Gnaneswara Reddy[13] studiedunsteady Radiative-convective boundary layer flow of a Casson fluid with variable thermal conductivity. Hayaet al.[14] discussed Soret and Dufour effects on magnetohydrodynamic flow of Casson fluid. Hussananet al.[15] analyzed unsteady boundary layer flowand heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. Khalidet al.[16] discussed unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium.

Henceforward, the impartial of the research discussion is to analyse the radiation effect on MHD free convection flow past on a moving vertical porous plate in incidence of thermal radiation and chemical reaction. The central equations are converted to dimensionless equations by using shooting technique and similarity transformation. The effects of different governing parameters on the concentration, temperature, velocity, are obtained.

#### **MATHEMATICAL ANALYSIS**

Let us choose a two-dimensional unsteady free convection flow of chemical reacting fluid flow, thermal radiating and a viscous incompressible electrical conducting on a moving vertical porous plate immersed in a porous medium. The y-axis is chosenalong normal to the plate. x-axis is chosen along the plate in the upward direction. The fluid is assumed to be a gray, absorbing and emitting radiation but non-scattering medium. Rosseland approximation is used to depict the radiation heat flux in the energy equation. A uniform magnetic field is applied in the direction vertical to the plate. The fluid is taken to be slightly conducting, and so the magnetic Reynolds number is much fewer than unity and the induced magnetic field is not that much importantwhen compared to the applied magnetic field. It is taken that the external electrical field is '0' and the electric field owing to the polarization of charges is insignificant. Primarily, the plate and the fluid have the same temperature  $(T_m)$  and the concentration ( $C_{\infty}$ ). The plate temperature and concentration are increased to ( $T_{\omega}$ ) and ( $T_{\omega}$ ) at time t > 0 respectively and put these fixed. Besides one can consider that all fluid properties have no variations; except that the effect of the density change with temperature and concentration in the body force. In addition to these, chemical reaction exits between the fluid and the diffusing species. The foreign mass presence in the flow is taken to be at low level and so Soret and Dufour effects are insignificant. Owing to these assumptions, the governing boundary layer equations of the flow field are:

Conservation of mass:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Conservation of momentum:

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = v \left( 1 + \frac{1}{\beta_0} \right) \frac{\partial^2 u}{\partial y^2} + g \beta \left( T - T_{\infty} \right) + g \beta^* \left( C - C_{\infty} \right) - \left( \frac{\sigma B_0^2}{\rho} + \frac{v}{K^*} \right) u \tag{2}$$

Conservation of energy (Heat):

$$\rho c_{p} \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^{2} T}{\partial y^{2}} - \frac{\partial q_{r}}{\partial y} + Q \left( T - T_{\infty} \right)$$
(3)

Conservation of species (Concentration):

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial v} = D \frac{\partial^2 C}{\partial v^2} - Kr^* \left( C - C_{\infty} \right) \tag{4}$$

where u is velocity in x axis direction and v is the velocity in y axis direction,  $\sigma$  -the electrical conductivity of the fluid,  $B_0$  -the magnetic induction, k-the thermal conductivity,  $Kr^*$ -the chemical reaction parameter.  $q_r$ -the local radiative heat flux ,C-the species concentration in the boundary layer,  $C_\infty$  -the species concentration in fluid far away from the plate, D-the mass diffusivity  $T_\infty$  -the temperature of the fluid far away from the plate, T-the temperature of the fluid in the boundary layer, V-the kinematic viscosity,  $\tau$ -Thermo porosity parameter,Q- heat source parameter, g- Casson parameter, g-the acceleration due to gravity, g-the fluid density, g-the thermal and concentration expansion coefficients respectively, and g-the thermal diffusivity.

The 2<sup>nd</sup> term on the RHS of the momentum equation(2) is the thermal buoyancy effects and the 3<sup>rd</sup> terms on the RHS of the momentum equation (2) denote the concentration buoyancy effect.

The boundary conditions for the velocity, temperature and concentration fields are:

$$t \le 0: u = 0, v = 0, T = T_{\infty}, C = C_{\infty} \quad \text{for all} \quad y$$

$$t > 0: \begin{cases} u = U, v = v(t), T = T_{w}, C = C_{w} & \text{at} \quad y = 0\\ u \to 0, v \to 0, T = T_{\infty}, C = C_{\infty} & \text{as} \quad y \to \infty \end{cases}$$
(5)

Here U is said to be the characteristic velocity of plate. Thermal radiation is expected in a unidirectional flux in y - direction i.e.,  $q_r$  by using Rosseland approximation value the radiative heat flux  $q_r$  is given by

$$q_r = -\frac{4\sigma_s}{3k_s} \frac{\partial T^4}{\partial y} \tag{6}$$

It must be identified that by applying the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature deviations within the flow are small enough, then equation (6) can become linear model by applying Taylor series expansion for  $T^4$  in the nbd of  $T_{\infty}$  and neglect the higher order terms one can get:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

In the light of equations (6) & (7), equation (3) is transformed to:

$$\rho c_{p} \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^{2} T}{\partial y^{2}} + \frac{16\sigma_{s}}{3k_{s}} T_{\infty}^{3} \frac{\partial^{2} T}{\partial y^{2}} + Q_{0} \left( T - T_{\infty} \right)$$

$$(8)$$

Similarity variables and the dimensionless quantities are now introduced,

$$\eta = \frac{y}{2\sqrt{vt}}, u = Uf(\eta), \theta = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \phi = \frac{C - C_{\infty}}{C_{w} - C_{\infty}}, 
Gc = \frac{4g\beta^{*}(C_{w} - C_{\infty})t}{U}, Gr = \frac{4g\beta(T_{w} - T_{\infty})t}{U}, 
M = \frac{4\sigma B_{0}^{2}t}{\rho}, K^{*} = \frac{Kv}{tc}, R = \frac{16\sigma_{s}(T_{w} - T_{\infty})^{3}}{3k_{e}k}, 
N = \frac{T_{\infty}}{T_{w} - T_{w}}, \Pr = \frac{\mu c_{p}}{k}, Sc = \frac{v}{D}, Kr^{*} = \frac{Kr}{4t}$$
(9)

v is a constant or a function of time but not both from equation (1). one can choose

$$v = -c\left(\frac{v}{t}\right)^{\frac{1}{2}} \tag{10}$$

where c > 0 is the suction parameter.

In light of equations (10)& (9), the equations (8), (2) and (4) are changed to

$$f''\left(1 + \frac{1}{\beta_0}\right) + 2(\eta + c)f' + Gr\theta + Gc\phi - \left(M + \frac{1}{K}\right)f = 0$$
 (11)

$$\theta'' + 2(\eta + c) \Pr \theta' + R(3(N + \theta)^{2} \theta'^{2} + (N + \theta)^{3} \theta'') + \frac{Q}{\Pr} \theta = 0$$
(12)

$$\phi'' + 2(\eta + c)Sc\phi' - KrSc\phi = 0 \tag{13}$$

where the primes denote the differentiation with respect to  $\eta$ , Pr is the Prandtl number, M is the magnetic field parameter, Gr is the thermal Grashof number, Sc is the Schmidt number, R is radiation parameter, R is the modified Grashof number, R is the chemical reaction parameter R is the temperature difference parameter. R -Thermo porosity parameter, R heat source parameter and R - Casson parameter.

The corresponding dimensionless boundary conditions are

$$\begin{cases} f = 1, \theta = 1, \phi = 1, & \text{at } \eta = 0 \\ f \to 0, \ \theta \to 0, \ \phi \to 0 & \text{as } \eta \to \infty \end{cases}$$
 (14)

## **SOLUTION OF THE PROBLEM**

The boundary layer equations (11),(12), (13) with the boundary conditions (14) can be solved by applying Runge-Kutta 4<sup>th</sup> order methods along with shooting method in Applied Numerical Analysis. Primarly,non-linear higher order differential equations (11),(12),(13) can be transformed to 1<sup>st</sup>order SLDE(simultaneous linear differential equations) and they are further changed into IVP(initial value problem) by using the Shooting Technique. The resultingIVPcan be solved by applyingfourth order Runge-Kutta method. To fulfill the convergence of methods one can choose the step size as  $\Delta \eta = 0.005$  and calculations are carried up to desired place of accuracy.

## **RESULTS AND DISCUSSION**

The problem dealing with unsteady MHD free convection fluid flow past a moving vertical porous plate embedded in porous medium with chemical reaction andthermal radiation in occurrence of suction. The numerical values of temperature ( $\theta$ ), velocity (f), and concentration ( $\phi$ ) with the boundary layer can be calculated for different parameters as the thermal Grashof number Gr, magnetic field parameter M, Q- heat source parameter, Permeability parameter K, Prandtl number Pr, solutal Grashof number Gc, Schmidt number Sc,  $\tau$ -Thermo porosity parameter,  $\theta$ -Casson parameter, thermal radiation parameter Rand suction parameter c. In the present study, we adopted the following default parametric values: Gr = 5, Gc = 10, M = 1.0, K = 0.5, Pr = 0.71, R = 0.5, N = 0.1, Sc = 0.6, Kr = 0.5, c = 0.5,  $\tau$  = 0.01,B=10,c=0.5 and Q=0.05. All the graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

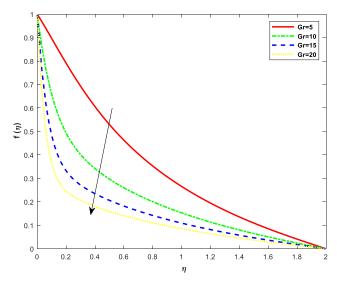


Fig. 1 – Velocity profiles for dissimilar values of Grashoff Number(Gr).

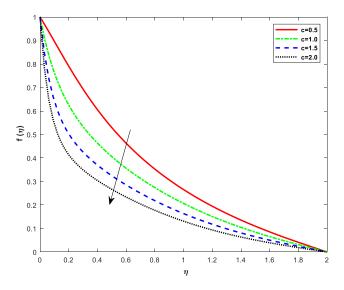


Fig. 2 – Velocity profiles for dissimilar values of suction parameter (c)

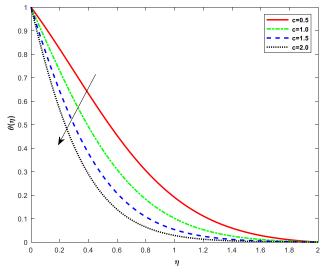


Fig. 3 – Temperature profiles for dissimilar values of suction parameter (c)

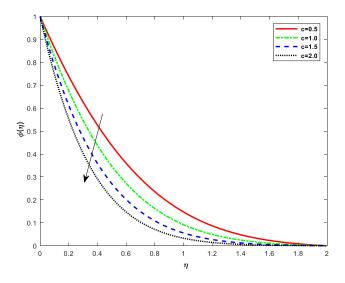


Fig. 4 – Concentration profiles for dissimilar values of suction parameter (c)

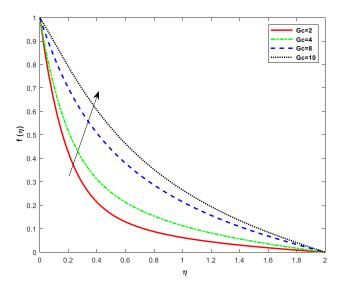
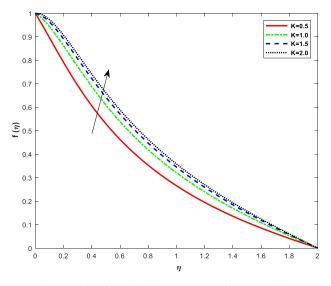


Fig. 5 – Velocity profiles for dissimilar values of solutal Grashof number (Gc)



 $Fig.\ 6-Velocity\ profiles\ for\ dissimilar\ values\ of\ Permeability\ parameter(K)$ 

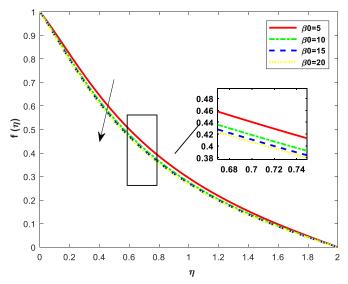


Fig. 7 – Velocity profiles for dissimilar values of Casson Parameter(B0)

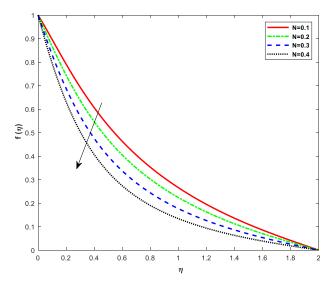


Fig. 8 - Velocity profiles for dissimilar values of temperature difference parameterN

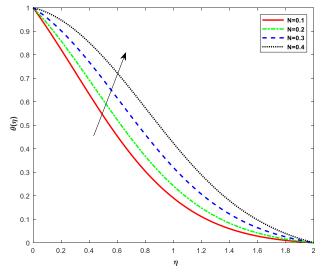


Fig. 9 – Velocity profiles for dissimilar values of temperature difference parameterN

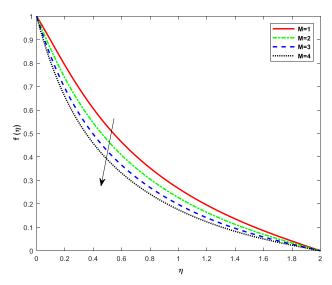
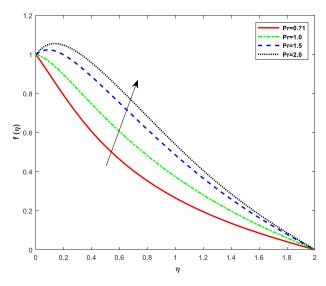


Fig. 10 - Velocity profiles for dissimilar values of magnetic field parameter(M)



 $Fig.\ 11-Velocity\ profiles\ for\ dissimilar\ values\ of\ Prandtl\ Number(Pr)$ 

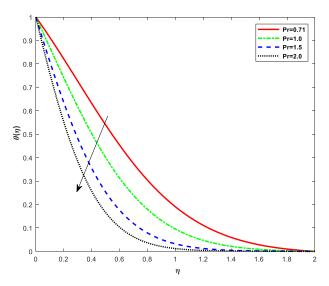


Fig. 12 – Temperature profiles for dissimilar values of Prandtl Number(Pr)

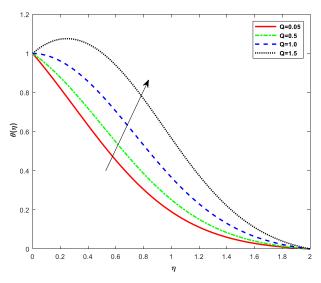


Fig. 13 – Temperature profiles for dissimilar values of Q

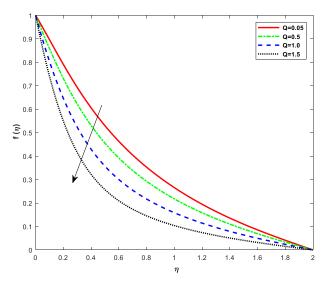


Fig. 14 – Velocity profiles for dissimilar values of Q

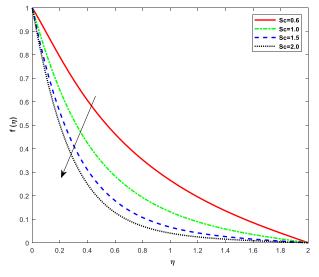
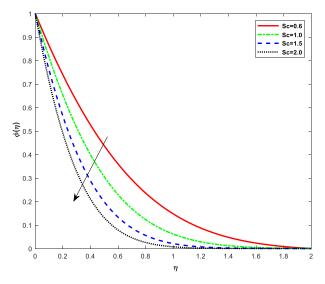


Fig. 15 – Velocity profiles for dissimilar values of Schmidt Number(Sc)



 $Fig.\ 16-Concentration\ profiles\ for\ dissimilar\ values\ of\ Schmidt\ Number(Sc)$ 

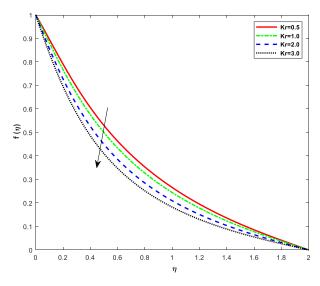


Fig. 17 – Velocity profiles for dissimilar values of Chemical reaction parameter(Kr)

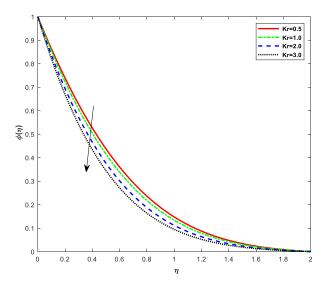


Fig. 18 – Concentration profiles for dissimilar values of Chemical reaction parameter(Kr)

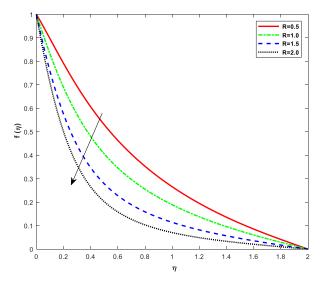


Fig. 19 – Velocity profiles for dissimilar values of thermal radiation parameter(R)

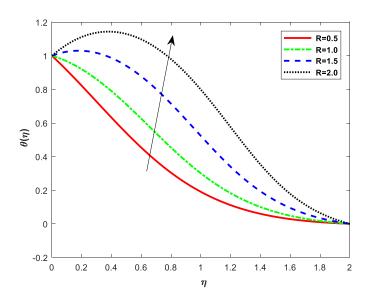


Fig. 20 – Temperature profiles for dissimilar values of thermal radiation parameter(R)

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