THE EFFECT OF THE THICKNESS OF THE POROUS MATERIAL ON THE PARALLEL PLATE CHANNEL FLOW OF A JEFFREY FLUID WHEN THE WALLS ARE PROVIDED WITH POROUS LINING

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Abstract: Flow in a deformable porous channel bounded by moving parallel plates is investigated. The coupled governing equations are solved analytically and the expressions for the velocity field and solid displacement are obtained. The effects of the porous layer thickness and the drag on the flow velocity and displacement are analyzed graphically. It is observed that displacement increases with increasing in the drag, whereas the opposite behavior in the velocity.

Keywords: viscous flow; Porous layer; porous layer thickness.

Nomenclature

x, y	Cartesian coordinates
μ_a	The apparent viscosity of the fluid in the porous material
K	The drag coefficient
μ	The Lame constant
μ_f	The coefficient of viscosity
v	Velocity of the fluid through the rigid deformable layers
u	The displacement
h	width of the wall
ε	Porous layers thickness
ϕ	The volume fraction of the fluid
q	Fluid velocity in free flow
$\frac{\partial p}{\partial x} = G_0$	Typical pressure gradient
δ	Viscous drag
λ_1	Jeffrey parameter

1. Introduction

Fluid flow past soft, deformable solid surfaces is encountered in diverse settings: the flow of fluid through a tube with flexible walls occurs in biological systems, and some biotechnological applications involve flow past polymer matrices and membranes. The dynamics of fluid flow past deformable solids is qualitatively different from that of rigid surfaces because of the coupling between the fluid and solid dynamics and the elasticity of the solid could affect the fluid flow viscous flow through a porous media has several important applications in the field of Geology and Medicine. Most of the research works available in flow through porous media is mainly concentrated on the undeformable porous media. But when we consider the phenomena such as hemodynamic effects of the endothelial glycocalyx, the theory of undeformable porous media is not adequate to explain the biofluid behavior there. The coupled phenomenon of fluid flow and deformation of porous materials is a problem of prime importance in geomechanics biomechanics, biological soft tissue modeling including cartilage, skin myocardium and arterial walls. In new of these applications, the theory of mixtures is introduced by Biot [1] and others to describe the flow through deformable porous media. Mow et al. [2] also contributed important theory for the study of rectilinear cartilages and biological tissue mechanics. Channabasappa et al. [3] are discussed the thickness of the porous material on the parallel plate channel flow when the with porous lining .Oomens et al. [4] are discussed A mixture approach to the mechanics of skin. Huyghe et al. [5] are analyzed A two-phase finite element model of the diastolic left ventricle. Kenyon [6] has studied A mathematical model of water flux through aortic tissue. Rudraiah [7] have discussed impermeable parallel flows in a channel and a Bounding porous medium of finite thickness. Jayaraman [8] have discussed Water transport in the arterial wall-A theoretical study. Klanchar et al. [9] are studied Modelling water flow through arterial tissue.

In all these models flows in deformable porous layer. Barry et al. [10], Ranganatha and Siddagamma [11] and Sreenadh et al. [12-15] analyzed mathematical models involving deformable porous media.

In view of the above studies, the effect of the thickness of the porous material on the parallel plate channel flow of a Jeffrey fluid when the walls are provided with porous lining is investigated. The fluid velocity and displacement of the solid matrix are obtained. The effects of various physical parameters on the flow quantities are discussed through graphs and tables.

2. Mathematical Formulation

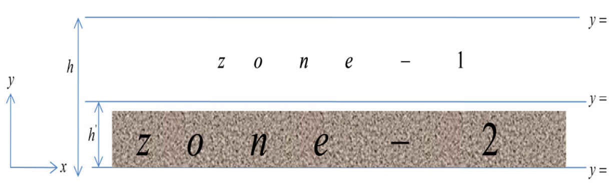

Fig 1: Schematic Diagram

Consider a steady, fully developed flow through deformable porous layers. These deformable layers are bounded by rigid walls at $y = h'$ and $y = h$ and deformable porous layer of thickness h' attached to the lower wall as shown in Fig.1. The flow region between the plates is divided into two regions. The flow region between the lower plate $y = 0$ and the interface $y = h'$ is termed as deformable porous layer whereas the flow region between the interface $y = h'$ and the upper plate $y = h$ is the free flow region. The fluid velocity in the free flow region and in the porous flow region are assumed respectively as $(q,0,0)$ and $(v,0,0)$. The displacement due to the deformation of the solid matrix is taken as $(u,0,0)$. A pressure gradient $\frac{\partial p}{\partial x} = G_0$ is applied, producing an axially directed flow in the channel.

In view of the assumptions mentioned above, the equations of motion in the deformable porous layer and free flow region are (See for details Barry et al.[10]).

$$\mu \frac{\partial^2 u}{\partial y^2} - (1-\phi) G_0 + K v = 0 \tag{1}$$

$$\frac{2\mu_a}{1+\lambda_1} \frac{\partial^2 v}{\partial y^2} - \phi G_0 - \mathrm{K} v = 0 \tag{2}$$

$$\frac{\mu_f}{1+\lambda_1} \frac{\partial^2 q}{\partial y^2} = G_0 \tag{3}$$

NON DIMENSIONALIZATION OF THE FLOW QUANTITIES

It is convenient to introduce the following non-dimensional quantities.

$$\frac{y}{h} = y^*, u^* = \frac{-\mu u}{h^2 G_0} u,\ v^* = \frac{-\mu_f v}{h^2 G_0},\ \ q^* = \frac{-\mu_f q}{h^2 G_0} q^*, \varepsilon = \frac{h^1}{h}$$

In view of the above dimensionless quantities, after neglecting the stars $(*)$, the equations (1) – (3) take the following form

$$\frac{d^2 u}{dy^2} = -(1-\phi) - \delta v \tag{4}$$

$$\frac{d^2 v}{dy^2} - (1+\lambda_1)\delta\eta v = -\phi(1+\lambda_1)\eta \tag{5}$$

$$\frac{d^2 q}{dy^2} = -(1+\lambda_1) \tag{6}$$

The boundary conditions are

$$\left.\begin{array}{l} y = 0 : v = 0,\ u = 0 \\ y = \varepsilon : q = \phi v \\ \qquad\qquad \frac{dq}{dy} = \frac{1}{\eta\phi}\frac{dv}{dy} \\ \qquad\qquad \frac{dq}{dy} = \frac{1}{(1-\phi)}\frac{du}{dy} \\ y = 1 : \quad q = 0 \end{array}\right\} \tag{7}$$

3 SOLUTION OF THE PROBLEM

Equations (4) - (6) are coupled with differential equations that can be solved by using the boundary conditions (7). The solid displacement and fluid velocities in the free flow region and deformable porous layer are obtained as below,

$$u(y) = -\frac{y^2}{2} - \frac{c_1\delta}{a^2}e^{ay} - \frac{c_2\delta}{a^2}e^{-ay} + c_3 y + c_4 \quad (8)$$

$$v(y) = c_1 e^{ay} + c_2 e^{-ay} + \left(\frac{\phi^f}{\delta}\right) \quad (9)$$

$$q(y) = \frac{-(1+\lambda_1)y^2}{2} + c_5 y + c_6 \quad (10)$$

Where $a = \sqrt{\delta\eta(1+\lambda_1)}$, $b = -\phi(1+\lambda_1)\eta$

$$c_6 = (1+\lambda_1)(0.5-\varepsilon) - \frac{a}{\eta\phi}(c_1 e^{a\varepsilon} - c_2 e^{-a\varepsilon}) ,$$

$$c_5 = (1+\lambda_1)\varepsilon + \frac{a}{\eta\phi}\left(c_1 e^{a\varepsilon} - c_2 e^{-a\varepsilon}\right),$$

$$c_3 = \left[c_1 e^{a\varepsilon} - c_2 e^{-a\varepsilon}\right]\left(\frac{a(1-\phi)}{\eta\phi}\right) + \frac{\delta}{a} + \varepsilon ,$$

$$c_1 = \frac{\left\{\begin{array}{l}\left(\frac{1+\lambda_1}{2}\right) + (1+\lambda_1)\left(\frac{\varepsilon^2}{2} - \varepsilon\right) \\ + \frac{\phi e^{-a\varepsilon}}{\delta}\left[\phi + \frac{a(\varepsilon-1)}{\eta\phi}\right] - \frac{\phi^2}{\delta}\end{array}\right\}}{\left\{\begin{array}{l} e^{a\varepsilon}\left[\phi - \frac{a(\varepsilon-1)}{\eta\phi}\right] \\ -e^{-a\varepsilon}\left[\phi + \frac{a(\varepsilon-1)}{\eta\phi}\right]\end{array}\right\}}$$

$$c_2 = \frac{\left\{\begin{array}{l}\left(\frac{1+\lambda_1}{2}\right) + (1+\lambda_1)\left(\frac{\varepsilon^2}{2} - \varepsilon\right) \\ + \frac{\phi e^{a\varepsilon}}{\delta}\left(\phi - \frac{a(\varepsilon-1)}{\eta\phi}\right) - \frac{\phi^2}{\delta}\end{array}\right\}}{\left\{\begin{array}{l} e^{a\varepsilon}\left[\phi + \frac{a(\varepsilon-1)}{\eta\phi}\right] \\ -e^{-a\varepsilon}\left[\phi - \frac{a(\varepsilon-1)}{\eta\phi}\right]\end{array}\right\}}$$

$$c_4 = \frac{-\phi}{a^2}$$

4. RESULTS AND DISCUSSION

In this paper, steady flow through an inclined channel with deformable porous media is investigated and the results are discussed for various physical parameters such as the volume fraction of the fluid ϕ, thickness ε, viscosity parameter η, drag δ and Jeffrey parameter λ_1. In this study for numerical computation we used $\phi = 0.6, \delta = 1$ $\eta = 0.5, \varepsilon = 0.2$ and $\lambda_1 = 0.5$. These values are kept as common in the entire study except for varied values as displayed in Figures 2 to 15.

The variation of free flow velocity q with y is calculated from equation (10) for different values of η, λ_1, ϕ and ε and is shown in Figures 2, 3, 4 and 5. The effect of η on free flow velocity is depicted in Figure 2, which shows that free flow velocity enhances as η increases. From Figure 3 it is observed that the free flow velocity increases with increase the Jeffrey parameter λ_1. It is seen from Figure 4 that the free flow velocity decreases with increasing volume fraction ϕ. From Figure 5 it is observed that the free flow velocity decreases with the increase porous layer thickness ε.

The variation of flow velocity v in the deformable porous layer with y is calculated from equation (9) for different values of ϕ, η, λ_1, ε and δ and is shown in Figures 6, 7, 8, 9 and10. It is observed that the velocity increases with the increasing volume fraction ϕ, viscous parameter η. From figure 8, it is seen that the flow velocity increases with increasing Jeffrey parameter λ_1.The effect of ε on flow velocity is depicted Figure 9, which is shows that flow velocity enhances as ε increases. Figure 10 it is observed that the flow velocity decreases with increasing drag δ.

The variation of solid displacement u with y is calculated from equation (8) for different values of ε, δ, λ_1, ϕ and η and is shown in Figures 11, 12, 13, 14 and 15. From figure 11 effect of the thickness parameter is increases with increasing solid displacement. Figures 12 and 13 show that the solid displacement increases with increase drag δ

and Jeffrey parameter λ_1. It is observed that the velocity decreases with the increasing volume fraction ϕ and viscosity parameter η figures 14 and 15.

Table I. illustrates the free flow and rigid deformable flow variation in volume flow rate $Q_D\left(0 \le v \le 1\right)$ with different values of drag δ , viscosity parameter η , thickness ε and volume fraction of the fluid ϕ. The table of comparison is presented for the fixed values of $\lambda_1 = 0.2$. It is noticed that the values of volume flow rate $Q_D\left(0 \le v \le 1\right)$ with deformable porous medium decreases with increasing values of drag parameter δ and thickness parameter ε for fixed value of Jeffrey parameter $\lambda_1 = 0.2$. It is also observed that that the values of volume flow rate $Q_D\left(0 \le v \le 1\right)$ with deformable porous medium increases with increasing values of volume fraction ϕ and viscosity parameter η for fixed value of Jeffrey parameter $\lambda_1 = 0.2$.

Table II. Illustrates the rigid flow variation in volume flow rate $Q_{D1}\left(0 \le v \le 1\right)$ with different values of drag δ , viscosity parameter η and volume fraction of the fluid ϕ. The table of comparison is presented for the fixed values of $\lambda_1 = 0.2$. .It is noticed that the values of volume flow rate $Q_{D1}\left(0 \le v \le 1\right)$ with deformable porous medium decreases with increasing values of drag parameter δ and viscosity parameter η for fixed value of Jeffrey parameter $\lambda_1 = 0.2$. It is also observed that that the values of volume flow rate $Q_{D1}\left(0 \le v \le 1\right)$ with deformable porous medium increases with increasing values of volume fraction ϕ for fixed value of Jeffrey parameter $\lambda_1 = 0.2$.

Table I: For fixed value $\lambda_1 = 0.2$

δ	ϕ	η	ε	Q_D
1	0.6	0.5	0.2	0.0631
2				0.0630
3				0.0629
4				0.0628
1	0.2			0.0534
	0.4			0.0574
	0.6			0.0631
	0.8			0.0700
	0.6	0.333		0.0592
		0.666		0.0667
		1		0.0738
		0.5	0.1	0.0786
			0.2	0.0631
			0.3	0.0528

Table II: For fixed value $\lambda_1 = 0.2$

δ	ϕ	η	Q_{D1}
1	0.6	0.5	1.6481
2			1.0286
3			0.8110
1	0.2		0.8670
	0.4		1.2575
	0.6		1.6481
	0.8		2.0387
	0.6	0.333	1.6648
		0.666	1.6322
		1	1.6018

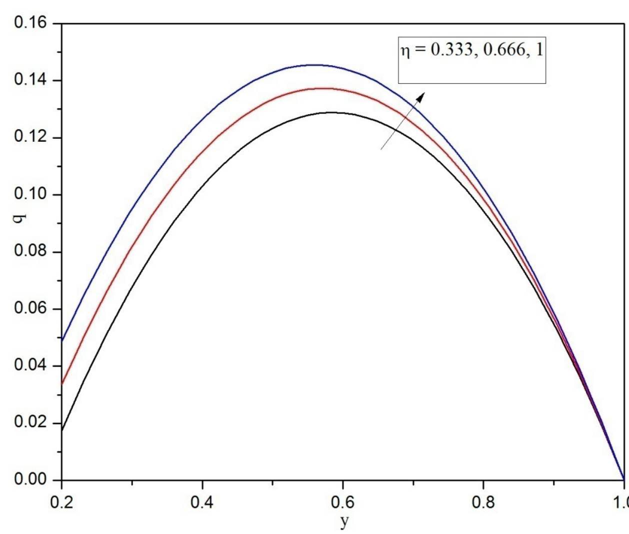

Figure 2: free flow velocity profiles for different values of η

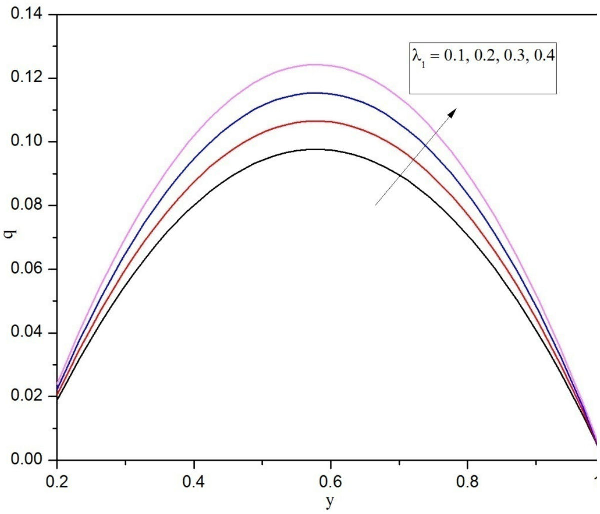

Figure 3: free flow velocity profiles for different values of λ_1

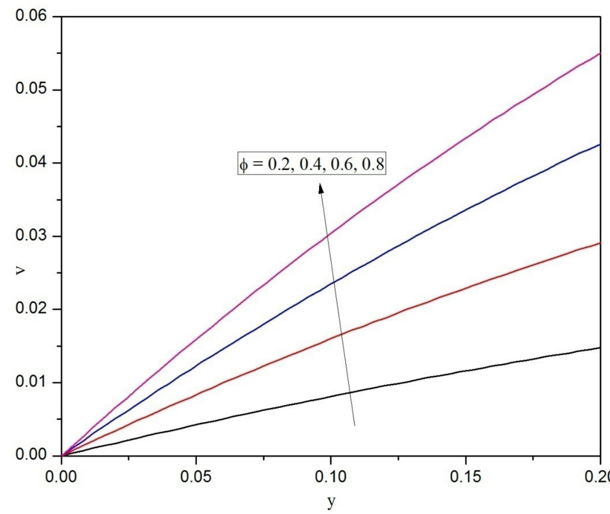

Figure 6: velocity profiles for different values of ϕ

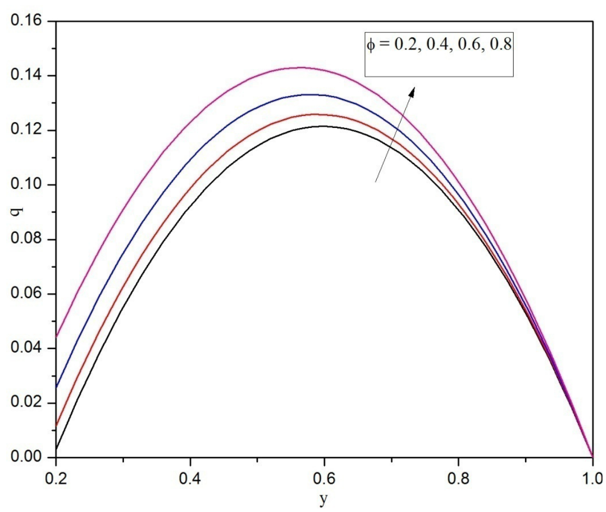

Figure 4: free flow velocity profiles for different values of ϕ

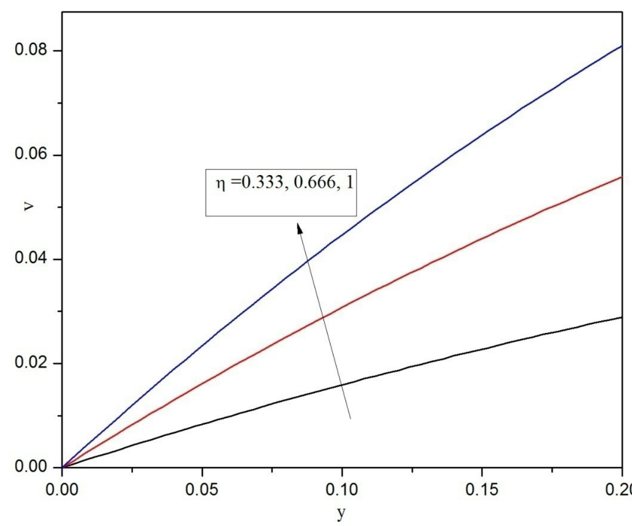

Figure 7: velocity profiles for different values of η

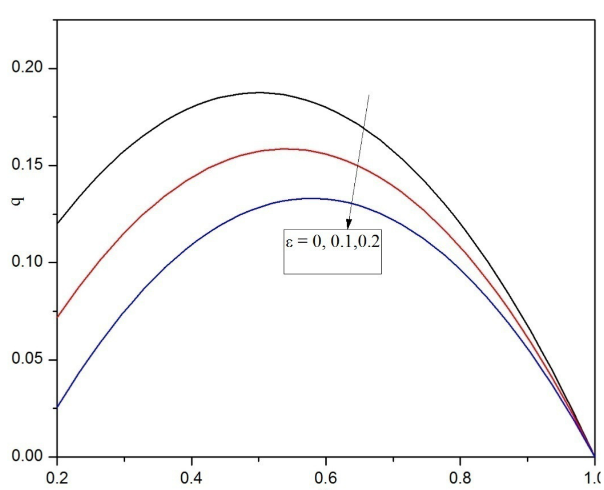

Figure 5: free flow velocity profiles for different values of ε

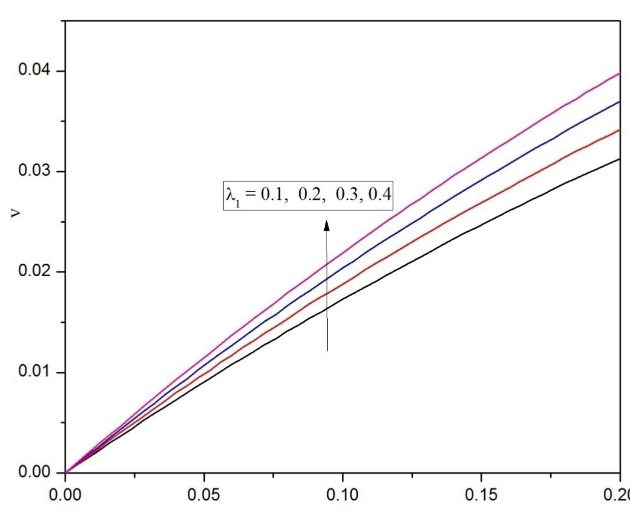

Figure 8: velocity profiles for different values of λ_1

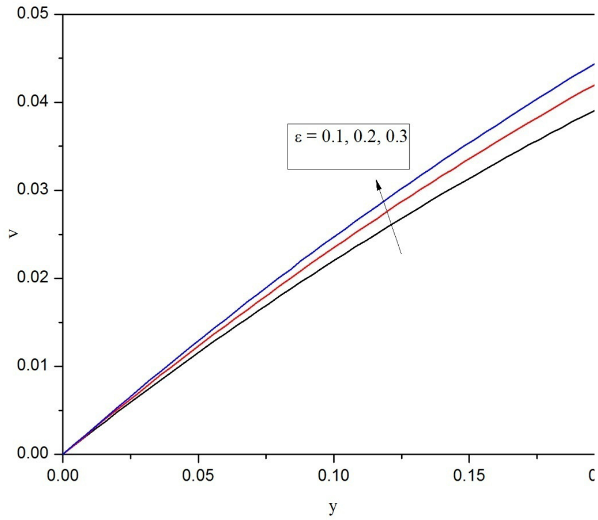

Figure 9: velocity profiles for different values of ε

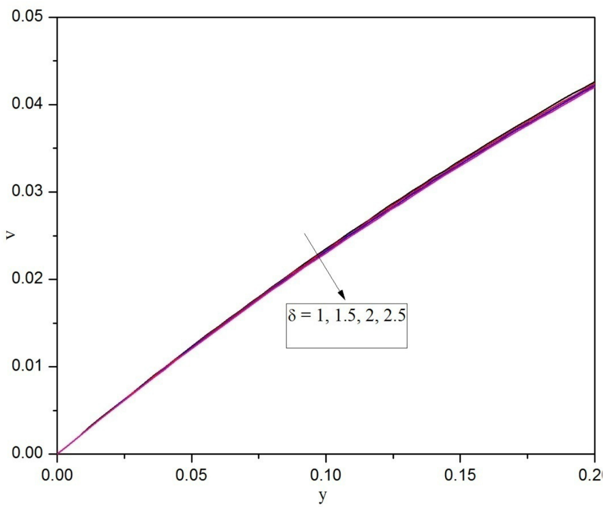

Figure 10: velocity profiles for different values of δ

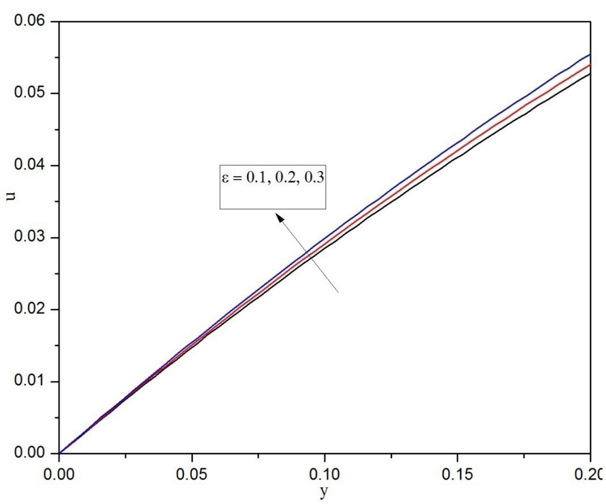

Figure 11: Displacement profiles for different values of ε

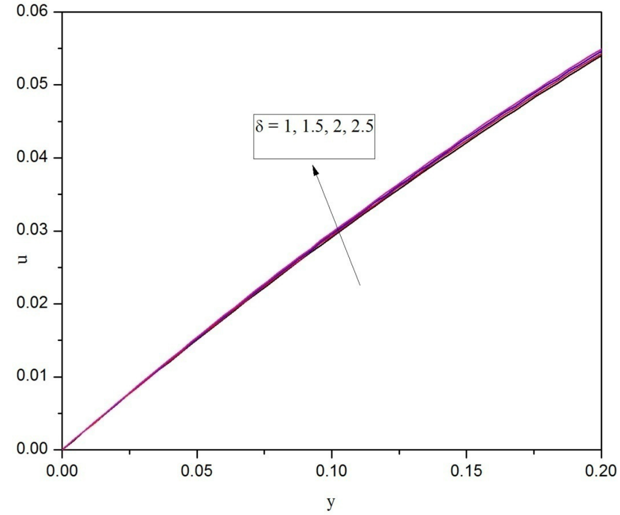

Figure 12: Displacement profiles for different values of δ

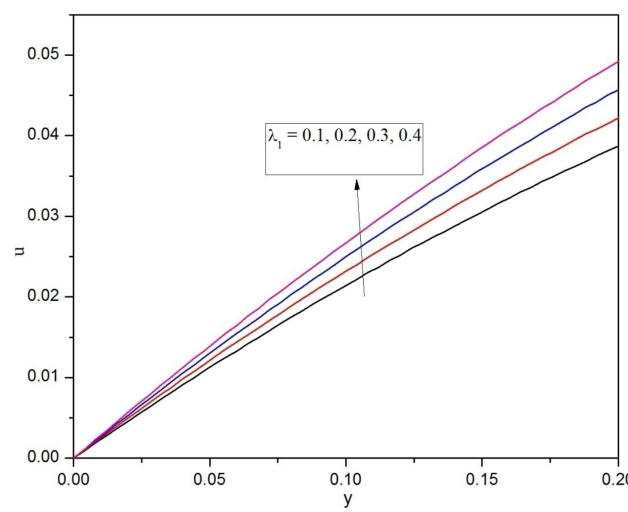

Figure 13: Displacement profiles for different values of λ_1

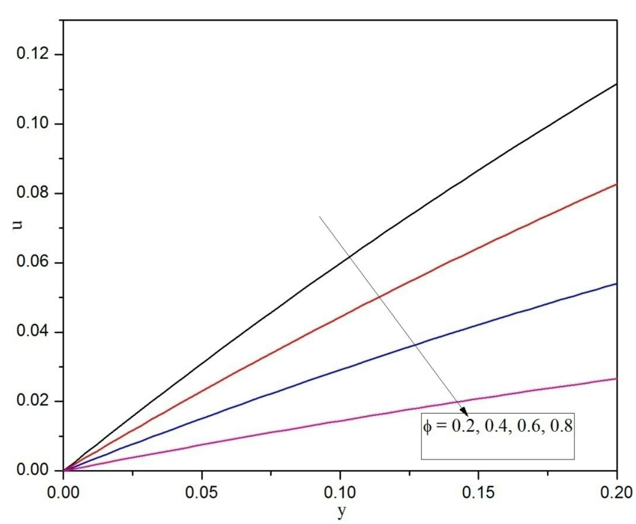

Figure 14: Displacement profiles for different values of ϕ

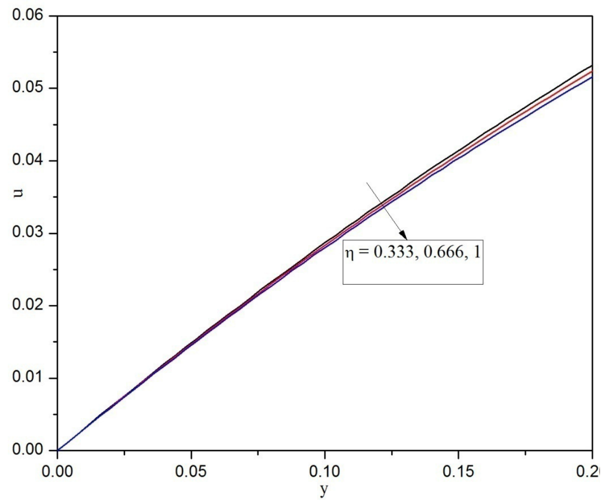

Figure 15: Displacement profiles for different values of η

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