# EFFECT OF CHEMICAL REACTION AND RADIATION ON MHD CASSON FLOWALONG A MOVING VERTICAL POROUS PLATE

## E. PRATAP SANKAR<sup>a,\*</sup>, G. V. RAMANA REDDY<sup>b</sup>, T.V. PRADEEP KUMAR<sup>c</sup> and Y. HARI KRISHNA<sup>d</sup>

<sup>a</sup> Research Scholar, Department of Basic Science, JNTUK Kakinada, AP.

\* Dept of Mathematics, Sri Mittapalli College of Engineering, Tummalapalem, Guntur (Dt.)-522019, Andhra Pradesh, India.
b.d Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur (Dt.)-522502, Andhra Pradesh, India.
College of Engineering and Technology, Acharya Nagarjuna University, Nagarjuna Nagar, Guntur (Dt.)-522510, Andhra Pradesh, India.

**Abstract**: In this article, we consider effects of force buoyancy and magnetohydrodynamic on convective mass and heat transfer flow past a touching vertical porous plate in the incidence of thermal radiation and chemical reaction. By using similarity transformation, the governing partial differential equations are rigorous to a system of self-similar equations. The subsequent calculations are solved numerically by using the fourth order RK method along with the shooting technique. The results are initiate for the temperature, velocity, concentration. The possessions of several parameters on flow variables are shown graphically and the physical aspects of the problem are conferred.

**Key words**: MHD, porous medium, heat and mass transfer, thermal radiation, chemical reaction.

#### INTRODUCTION

In recent years, the work of free convective heat and mass transfer flows concluded a porous medium under the effect of the attention of many researchers was attracted by a magnetic field as many they have Number of uses in engineering sciences, easy example transportation cooling, cross-hatching on ablative surfaces. Rajakumaret al. [1] studied chemical reaction and viscous indulgence possessions on MHD free convective flow past a semi-infinite moving vertical porous plate. Thammanna et al. [2] discussed three dimensional MHD flow of couple stress Casson fluid past an unsteady stretching surface with chemical reaction. Malleswari[3] analyzed heat and mass transfer analysis on mud radiating flow in presence of natural convection, porous medium and viscoelastic Rivlin-Ericksen fluid through numerical solutions. Srinivasa Rajuet al. [4] studied influence of angle of inclination on unsteady MHD Casson fluid flow past a vertical surface filled by porous medium in presence of constant heat flux, chemical reaction and viscous dissipation. Ramesh Babuet al. [5] expressed numerical investigation of heat transfer mechanism in MHD Casson fluid flow past a vertically inclined plate in presence of hall current. G.V.R. Reddy et al. [6] studied Soret and Dufour effects on MHD micropolar fluid flow over a linearly stretching sheet through a non -Darcy porous medium. Ramana Reddy et al. [7] examined MHD free convective flow of Casson fluid past over an oscillating vertical porous plate. Vijaya et al. [8] presented Soret and radiation effects on an unsteady flow of a Casson fluid through vertical channel with expansion and contraction. GVRReddy [9] discussed numerical solutions of unsteady MHD flow heat transfer over a stretching surface with suction or injection. Hari Krishna et al. [10] analyzed chemical reaction effect on MHD flow of Casson fluid with porous stretching sheet. Das et al. [11] expressed Newtonian heating effect on steady hydro magnetic Casson fluid flow a plate with heat and mass transfer. Dash et al. [12] discussed Casson fluid flow in a pipe filled with a homogeneous porous medium. Gnaneswara Reddy [13] studied unsteady Radiativeconvective boundary layer flow of a Casson fluid with variable thermal conductivity. Haya et al. [14] discussed Soret and Dufour effects on magnetohydrodynamic flow of Casson fluid. Hussananet al. [15] analyzed unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. Khalidet al. [16] discussed unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium.

Henceforward, the impartial of the research discussion is to analyse the radiation effect on MHD free convection flow past on a moving vertical porous plate in incidence of thermal radiation and chemical reaction. The central equations are converted to dimensionless equations by using shooting technique and similarity transformation. The effects of different governing parameters on the concentration, temperature, velocity, are obtained.

#### MATHEMATICAL ANALYSIS

Let us choose a two-dimensional unsteady free convection flow of chemical reacting fluid flow, thermal radiating and a viscous incompressible electrical conducting on a moving vertical porous plate immersed in a porous medium. The y-axis is chosen along normal to the plate. x-axis is chosen along the plate in the upward direction. The fluid is assumed to be a gray, absorbing and emitting radiation but non-scattering medium. Rosseland approximation is used to depict the radiation heat flux in the energy equation. A uniform magnetic field is applied in the direction vertical to the plate. The fluid is taken to be slightly conducting, and so the magnetic Reynolds Number is much fewer than unity and the induced magnetic field is not that much important when compared to the applied magnetic field. It is taken that the external electrical field is '0' and the electric field owing to the polarization of charges is insignificant. Primarily, the plate and the fluid have the same temperature  $(T_n)$  and the concentration  $(C_{\infty})$ . The plate temperature and concentration are increased to  $(T_{\omega})$  and  $(C_{\omega})$  at time t > 0 respectively and put these fixed. Besides one can consider that all fluid properties have no variations; except that the effect of the density change with temperature and concentration in the body force. In addition to these, chemical reaction exits between the fluid and the diffusing species. The foreign mass presence in the flow is taken to be at low level and so Soret and Dufour effects are insignificant. Owing to these assumptions, the governing boundary layer equations of the flow field are:

Conservation of mass:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Conservation of momentum:

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = v \left( 1 + \frac{1}{\beta_0} \right) \frac{\partial^2 u}{\partial y^2} + g \beta \left( T - T_{\infty} \right) + g \beta^* \left( C - C_{\infty} \right) - \left( \frac{\sigma B_0^2}{\rho} + \frac{v}{K^*} \right) u$$
 (2)

Conservation of energy (Heat):

$$\rho c_{p} \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^{2} T}{\partial y^{2}} - \frac{\partial q_{r}}{\partial y} + Q \left( T - T_{\infty} \right)$$
(3)

Conservation of species (Concentration):

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - Kr^* \left( C - C_{\infty} \right) \tag{4}$$

where u is velocity in x axis direction and v is the velocity in y axis direction,  $\sigma$ -the electrical conductivity of the fluid,  $B_0$ -the magnetic induction, k-the thermal conductivity,  $Kr^*$ -the chemical reaction parameter.  $q_r$ -the local radiative heat flux, C-the species concentration in the boundary layer,  $C_\infty$ -the species concentration in fluid far away from the plate, D-the mass diffusivity  $T_\infty$ -the temperature of the fluid far away from the plate, T-the temperature of the fluid in the boundary layer, v-the kinematic viscosity,  $\tau$ -Thermo porosity parameter, Q- heat source parameter, g- Casson parameter, g-the acceleration due to gravity, g-the fluid density, g-the thermal and concentration expansion coefficients respectively, and g-the thermal diffusivity.

The 2<sup>nd</sup> term on the RHS of the momentum equation (2) is the thermal buoyancy effects and the 3<sup>rd</sup> terms on the RHS of the momentum equation (2) denote the concentration buoyancy effect.

The boundary conditions for the velocity, temperature and concentration fields are:

$$t \le 0$$
:  $u = 0, v = 0, T = T_{\infty}, C = C_{\infty}$  for all  $y$ 

$$t > 0: \begin{cases} u = U, v = v(t), T = T_{w}, C = C_{w} & \text{at } y = 0\\ u \to 0, v \to 0, T = T_{\infty}, C = C_{\infty} & \text{as } y \to \infty \end{cases}$$
 (5)

Here U is said to be the characteristic velocity of plate. Thermal radiation is expected in a unidirectional flux in y - direction i.e.,  $q_r$  by using Rosseland approximation value the radiative heat flux  $q_r$  is given by

$$q_r = -\frac{4\sigma_s}{3k_a} \frac{\partial T^4}{\partial y} \tag{6}$$

It must be identified that by applying the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature deviations within the flow are small enough, then equation (6) can become linear model by applying Taylor series expansion for  $T^4$  in the nbd of  $T_{\infty}$  and neglect the higher order terms one can get:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

In the light of equations (6) & (7), equation (3) is transformed to:

$$\rho c_{p} \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^{2} T}{\partial y^{2}} + \frac{16\sigma_{s}}{3k_{e}} T_{\infty}^{3} \frac{\partial^{2} T}{\partial y^{2}} + Q_{0} \left( T - T_{\infty} \right)$$

$$(8)$$

Similarity variables and the dimensionless quantities are now introduced,

$$\eta = \frac{y}{2\sqrt{vt}}, u = Uf(\eta), \theta = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \phi = \frac{C - C_{\infty}}{C_{w} - C_{\infty}}, Gc = \frac{4g\beta^{*}(C_{w} - C_{\infty})t}{U}, Gr = \frac{4g\beta(T_{w} - T_{\infty})t}{U},$$

$$M = \frac{4\sigma B_{0}^{2}t}{\rho}, K^{*} = \frac{Kv}{tc}, R = \frac{16\sigma_{s}(T_{w} - T_{\infty})^{3}}{3k_{s}k}, N = \frac{T_{\infty}}{T_{w} - T_{\infty}}, \Pr = \frac{\mu c_{p}}{k}, Sc = \frac{v}{D}, Kr^{*} = \frac{Kr}{4t}$$
(9)

v is a constant or a function of time but not both from equation (1).

one can choose

$$v = -c\left(\frac{v}{t}\right)^{\frac{1}{2}} \tag{10}$$

where c > 0 is the suction parameter.

In light of equations (10)& (9), the equations (8), (2) and (4) are changed to

$$f''\left(1+\frac{1}{\beta_0}\right)+2\left(\eta+c\right)f'+Gr\theta+Gc\phi-\left(M+\frac{1}{K}\right)f=0\tag{11}$$

$$\theta'' + 2(\eta + c) \Pr \theta' + R(3(N + \theta)^{2} \theta'^{2} + (N + \theta)^{3} \theta'') + \frac{Q}{\Pr} \theta = 0$$
(12)

$$\phi'' + 2(\eta + c)Sc\phi' - KrSc\phi = 0 \tag{13}$$

where the primes denote the differentiation with respect to  $\eta$ , Pr is the Prandtl Number, M is the magnetic field parameter, Gr is the thermal Grashof Number, Sc is the Schmidt Number, R is radiation parameter, Gc is the modified Grashof Number, Kr is the chemical reaction parameter, R is the temperature difference parameter. R -Thermo porosity parameter, R - heat source parameter and R - Casson parameter.

The corresponding dimensionless boundary conditions are

$$\begin{cases} f = 1, \theta = 1, \phi = 1, & \text{at } \eta = 0 \\ f \to 0, \ \theta \to 0, \ \phi \to 0 & \text{as } \eta \to \infty \end{cases}$$
 (14)

#### **SOLUTION OF THE PROBLEM**

The boundary layer equations (11),(12), (13) with the boundary conditions (14) can be solved by applying Runge-Kutta 4<sup>th</sup> order methods along with shooting method in Applied Numerical Analysis. Primary non-linear higher order differential equations (11),(12),(13) can be transformed to 1<sup>st</sup>order SLDE(simultaneous linear differential equations) and they are further changed into IVP(initial value problem) by using the Shooting Technique. The resulting IVP can be solved by applying fourth order Runge-Kutta method. To fulfill the convergence of methods one can, choose the step size as  $\Delta \eta = 0.005$  and calculations are carried up to desired place of accuracy.

### RESULTS AND DISCUSSION

The problem dealing with unsteady MHD free convection fluid flow past a moving vertical porous plate embedded in porous medium with chemical reaction and thermal radiation in occurrence of suction. The numerical values of temperature, velocity, and concentration with the boundary layer can be calculated for different parameters as the thermal Grashof Number Gr, magnetic field parameter M, Q- heat source parameter, Permeability parameter K, Prandtl Number Pr, solutal Grashof Number Gc, Schmidt Number Sc,  $\tau$ -Thermo porosity parameter,  $\theta_0$ - Casson parameter, thermal radiation parameter Rand suction parameter c. In the present study, we adopted the following default parametric values: Gr = 5, Gc = 10, M = 1.0, K = 0.5, Pr = 0.71, R = 0.5, N = 0.1, Sc = 0.6, Kr = 0.5, c = 0.5,  $\tau$  =0.01,B=10,c=0.5 and Q=0.05. All the graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

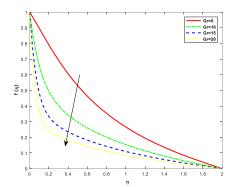


Fig. 1 – Velocity profiles for dissimilar values of Grashof Number (Gr).

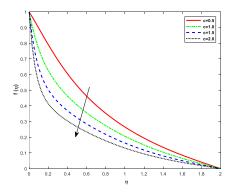


Fig. 2 – Velocity profiles for dissimilar values of suction parameter (c)

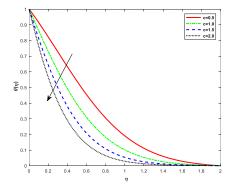


Fig. 3 – Temperature profiles for dissimilar values of suction parameter (c)

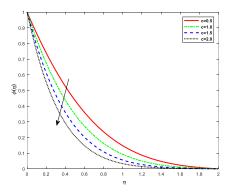


Fig. 4 – Concentration profiles for dissimilar values of suction parameter (c)

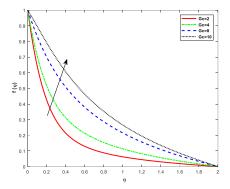


Fig. 5 – Velocity profiles for dissimilar values of solutal Grashof Number (Gc)

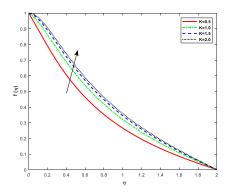


Fig. 6 – Velocity profiles for dissimilar values of Permeability parameter (K)

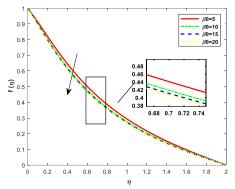
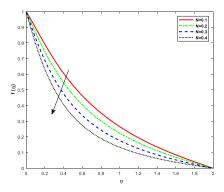
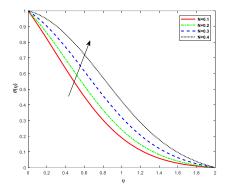


Fig. 7 – Velocity profiles for dissimilar values of Casson Parameter(B0)



 $Fig. \ 8-Velocity\ profiles\ for\ dissimilar\ values\ of$   $temperature\ difference\ parameter\ N$ 



 $Fig. \ 9-Velocity \ profiles \ for \ dissimilar \ values \ of \\ temperature \ difference \ parameter \ N$ 

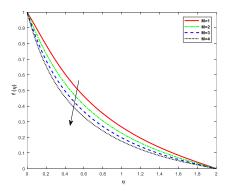


Fig. 10 - Velocity profiles for dissimilar values ofmagnetic field parameter(M)

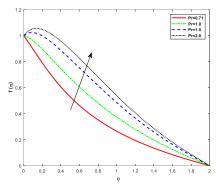


Fig. 11 – Velocity profiles for dissimilar values of Prandtl Number (Pr)

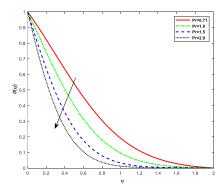


Fig. 12 – Temperature profiles for dissimilar values of Prandtl Number (Pr)

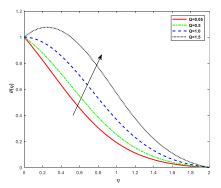


Fig. 13 – Temperature profiles for dissimilar values of Q

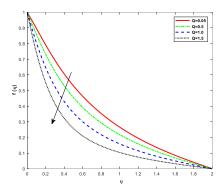


Fig. 14 - Velocity profiles for dissimilar values of  $\ensuremath{Q}$ 

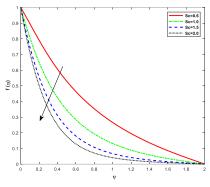


Fig. 15 – Velocity profiles for dissimilar values of Schmidt Number (Sc)

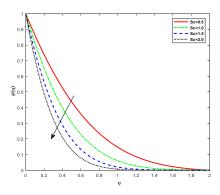


Fig. 16 – Concentration profiles for dissimilar values of Schmidt Number (Sc)

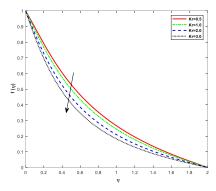


Fig. 17 – Velocity profiles for dissimilar values of Chemical reaction parameter (Kr)

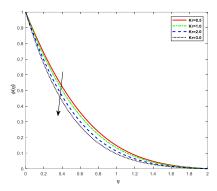


Fig. 18 – Concentration profiles for dissimilar values of Chemical reaction parameter (Kr)

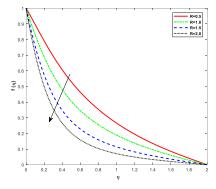
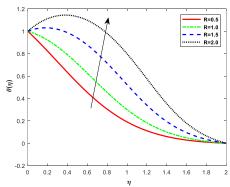


Fig. 19 – Velocity profiles for dissimilar values of thermal radiation parameter(R)



 $Fig.\ 20-Temperature\ profiles\ for\ dissimilar\ values\\ of\ thermal\ radiation\ parameter(R)$ 

Figure 1 and 2 represents the attributes of Grashoff number and suction parameter on the velocity profiles. It can be observed that the velocity profiles are decreases for the increasing the Grashoff number and Suction parameter. Figure 3 presents the impact of suction on temperature profile. It is clear to observe that temperature profile decreases up to the thermal boundary region of  $\varepsilon = 2$  and after that the trend is constant to meet the boundary condition. Further, it is noticed that as suction increases the temperature reduces at all points in the thermal boundary layer. Similar effect is encountered for the concentration profile as shown in the Fig. 4 which represents the effect of suction on concentration distribution. It is noteworthy that concentration decreases up to  $\varepsilon = 2$  and then it tends to be constant and profile is parabolic in nature. So far as effect of suction is concerned with increase in suction parameter the concentration decreases. Figure 5 present the effect of solutal Grashof number Gc on the velocity profile of the flow field. It is observed by increasing the solutal Grashof number Gc which also helps in accelerating the velocity of the flow field with a considerable reduction in the solutal boundary layer thickness. Figure 6 shows the effects of the permeability of the porous medium parameter K on the velocity distribution. As shown, the velocity is increasing with the increasing dimensionless porous medium parameter. The effect of the dimensionless porous medium parameter K becomes smaller as K increases. Physically, this result can be achieved when the holes of the porous medium are very large so that the resistance of the medium may be neglected. It is observed in figure 7 that the velocity decreases when  $\beta 0$  increases. In practice, increasing  $\beta$  results in an increase in the plastic dynamic viscosity that produces a resistance in the flow and a decrease in fluid velocity. Behavior of velocity and temperature profiles is demonstrated via figure 8 and 9 verses increasing the values of temperature difference parameter (N) respectively. It is noticed that as increasing values of N the velocity profiles decrease but reverse trend is observed in temperature profiles due to thermal diffusivity. Figure 10 portraits variation of magnetic parameter on the velocity profiles. As increment in magnetic parameter (M) brings enhancement in Lorenz force. Since it is resists in nature, so the fluid particles encountered strong resistance in the context of the movement. Therefore, the average velocity of the fluid decreases. The physical outcomes of Prandtl number on the velocity and temperature profiles are highlighted with the help of figure 11 and 12 respectively. It is observed that the velocity profiles increase, and temperature profiles decreases for increasing values of Prandtl number. One can observed that the fluid with low Prandtl number possesses high thermal energy and vice versa. It embraces the physical because Prandtl number correspondence inversely with competency of conduction. Behavior of the temperature and velocity profiles for different values of heat source parameter (Q) as shown in figure 13 and 14 respectively. The influence of heat source parameter increases, the temperature profiles increases but the reverse trend is observed in velocity profiles. The modification on the velocity and concentration profiles verses Schmidt number as shown in figure 15 and 16 respectively. It is observed that both the velocity and concentration profiles decrease for decreasing values of Schmidt number. Figure 17 and 18 are demonstrated the effect of chemical reaction on the velocity and concentration profiles respectively. It is noticed that for increasing the values chemical reaction parameter the velocity and concentration profiles decreases. Figure 19 and 20 addresses the effect of radiation parameter on the velocity and temperature profiles. It can be observed that the radiation parameter increases the velocity profiles decreases but the reverse trend is observed in temperature profiles.

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