# Identification of parameters in moving load dynamics problem using statistical process recognition approach

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**Abstract.** The present work is focused to develop an indirect approach for structural health monitoring analogy for moving load dynamics problem using the concepts of statistical process recognition (SPR) approach. The objectives of the proposed analogy are to identify and locate the subsistence of cracks on the structure in supervised manner. The statistical process reorganization approach is based on the concepts of the time domain Auto-Regressive (AR) method. Numerical studies are carried out for a damaged simply supported beam under a moving mass for the exactness of the proposed method. The numerical study evidenced that the proposed method is perceptive to structural damage identification parameters for moving load dynamics problem.

Keywords: AR., moving load, crack,

## 1 Introduction

Structural health monitoring problem has received major attention in the present scenario of civil and mechanical industries. The early detection of fault in structures becomes an interesting topic for engineers and researchers for structural integrity and healthy environment. Jena and Parhi [1-4] have carried out several studies like numerical, finite element analysis (FEA) and experimental to determine the moving load induced dynamic responses on the structures. Apart from various studies, many researchers also focused their work on statics based approach for structural health monitoring problem. Xue et al. [5] have applied the Monte Carlo and likelihood estimation methods to identify the damage parameters on structures. Mao et al. [6] have developed a statistical crack detection approach based on the concept of sensitivity of dynamic response. Farahani and Penumadu [7] investigated a noble damage detection procedure for girder bridge structure using vibration data and timeseries analysis. Mechbal et al. [8] have developed an algorithm based upon multiclass pattern reorganization approach for structural health monitoring problem. Yang et al. [9] explored a static algorithm for localization of damage in structure by traversing mass. Ma et al. [10] have considered the measurement of the consequence of noise and performance of sensors for detection of damage in structures.

So far the literatures are considered the moving load induced dynamic response in the domain of statistical process reorganization approach for structural health monitoring

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era is a little. The novelty of the present approach is to develop a structural health monitoring analogy in the domain of AR method by using the measured responses of the structure. The present analogy has done in supervised manner.

### 2 The Problem Definition.

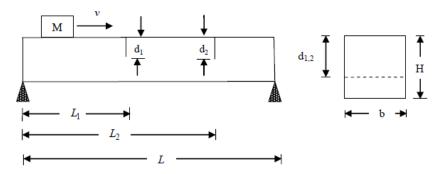


Fig 1. Cracked simply supported structure under moving mass

For the inverse approach, a problem has been formulated. As per the Fig 1, a simply supported with two cracks subjected to a transit mass has been analyzed in this work. The governing equation of motion of the system has been articulated as per Euler-Bernoulli's equation i.e.

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = F(x,t), \text{ where } F(x,t) = P(t)\delta(x-\beta) + r(x,t)$$
 (1)

The solution of the governing equation (1) has been already obtained by Jena and Parhi [1] earlier which is represented in equation (2) i.e.

$$EI\lambda_n^{\ 4}T_n(t) + mT_{n,tt}(t) - \left(\frac{M}{V_n}\right) \left[g - \sum_{q=1}^{\infty} \left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial \beta}\right)^2 w_q(\beta)T_q(t)\right] w_n(\beta) = 0 \quad (2)$$

Where F(x,t) is the applied force, P(t) is the driving force,  $\delta$  is the Dirac delta function,  $\beta = vt$  is the position of the transit mass,  $w_q$  is the shape function of the beam,  $T_n(t)$  is the amplitude function, v is the speed, M is the transit mass. The detailed procedures are already discussed [1]. For the said analysis a numerical problem has been formulated.

Beam dimensions=  $(150\times5\times0.5)$  cm,  $\alpha_{1,2}=\frac{d_{1,2}}{H}=$  Relative crack depth, M=2.5kg, v=6.3~m/s,  $\eta_{1,2}=\frac{L_{1,2}}{H}=$  Relative crack position.  $L_{1,2}=$  Position of cracks. L= Length of the structure.  $\alpha_{1,2}=0.35,0.32$  and 0.42,0.25,  $\eta_{1,2}=0.3,0.5$  and 0.4,0.67.

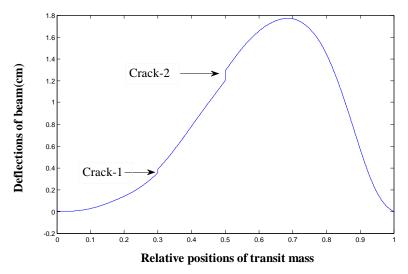


Fig 2. For Deflections vs. Relative positions  $\beta_{1,2} = 0.3, 0.5, \alpha_{1,2} = 0.35, 32,$ M = 2.5kg, v = 6.3m/s

For the response analysis of the system, one numerical example has been described in Fig 2. From the observed responses, it has been proven that there are sudden rise in amplitudes at certain positions of the structure. The sudden rise in amplitude and the corresponding positions indicate the existence and positions of crack in that particular structure respectively. The said problem has been considered in statistical process control approach in a supervised manner.

# 3 Statistical Process Recognition (SPR) approach:

Statistics based methodology is the compilation of methods for taking the pronouncement about an approach that depend on the exploration of data controlled in a specified sample. The SPR approach presents the primary features by means a product can be sampled, trained and monitored. The proposed fault detection approach is developed with the implementation of time series analysis in the SPR domain. The mechanism of control chart analysis has been applied to detect the existence of cracks on the structure.

Lower Control Limit (LCL) = 
$$\mu_s - z \frac{\sigma_s}{\sqrt{n}}$$
 (3-a)

Upper Control Limit (UCL) = 
$$\mu_s + z \frac{\sigma_s}{\sqrt{n}}$$
 (3-b)

Where  $\mu_s$  and  $\sigma_s$  and are the mean, standard deviation of the sample size (n) respectively. z is the sample statistics. The value of z is normally obtained from different sample sizes [11]. The mean and standard deviations of the samples are determined from beam responses at various damage parametric conditions.

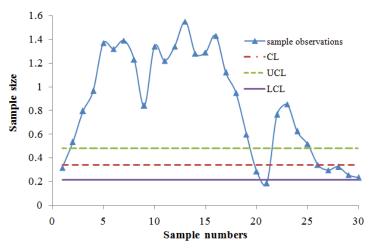


Fig 3. Control chart analysis of the beam under transit mass

The values of the centre line (CL), LCL and UCL in the control chart analysis (Fig 2) are evaluated by considering the response analysis of undamaged beam under transit mass only. The points below LCL and above UPL exhibit the feasible presence of cracks on the structure. The implementation of AR process is here to locate the cracks positions. The general equation of AR approach at a given time by considering the time history data (Beam deflection vs. time) of the given sample size can be articulated as:

$$\psi_i(t_i) = [\psi_1(t_i) \quad \psi_2(t_i) \quad ... y_n(t_i)]^T$$
 (4)

Where,  $\psi_i(t_j)$  is the data from the time history at 'n' number of observations and sampled at 'r' number of intervals. The time history data are correlated with previous data along with each other's data also. The above equation can be rearticulated as:

$$\psi_t = \left[\psi_{1t} \quad \psi_{2t} \quad \dots y_{nt}\right]^T \tag{5}$$

 $\psi_t$  is the response data from the time history at time 't'.

Now, the covariance  $(\phi)$  which is  $n \times n$  matrix among all the observed points over

the whole time interval can be represented as: 
$$\phi = \sum_{j=1}^{n} \psi(t_j) \psi(t_j)^T$$
 (6)

Using the concepts of feature extraction approach with the covariance, the damage sensitive parameters can be obtained from the measured time history data which can differentiate between damaged and undamaged states. In the AR (q) approach, the current data in the time series study is characterized as the linear combination of the precedent 'q' data. The AR model with order 'q', AR (q), can be expressed as:

$$\psi_t = \sum_{i=1}^p \varphi_j \psi_{t-q} + a_t \tag{7}$$

Where,  $a_t$  is shock value with zero mean & constant variance.

 $\psi_t$  is the coefficient of AR approach which acts like damage sensitive parameters.

$$\psi_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \dots \varphi_q y_{t-q} + a_t$$
 (8)

The values of ' $\varphi_j$ ' are obtained by feeding the AR (q) form to the time vs. deflection data by using the Yule-Walker approach [11], Where the coefficients of the AR process have been calculated with the liner linear least squares regression analysis by corroborating the Gaussianity and randomness of the predicted errors by trial and error methods.

The moving load induced displacement-time data of the undamaged and damaged states are contrasted to extort the damage sensitive parameters each of the response data. The entire undamaged data set are divided into two groups; reference data sample (DS<sup>R</sup>) and healthy data sample (DS<sup>H</sup>), while those of damaged set are named as DS<sup>D</sup>. The DS<sup>R</sup> has been applied to get the damage sensitive parameters for all sets of data for further comparison. The damage sensitive parameters for DS<sup>H</sup> and DS<sup>D</sup> are also found out. The damage sensitive parameters or coefficients of AR (q) model from both the DS<sup>H</sup> and DS<sup>D</sup> states are corroborated with DS<sup>R</sup>. By observing deviation of damage sensitive parameters that take place in the anticipated coefficients of AR model indicate the continuation of cracks in that system. The Fisher Criterion approach has been implemented to obtain the authentic disparity of coefficients of AR model for both the the DS<sup>H</sup> and DS<sup>D</sup>. The Fisher criterion approach is as follows:

$$F_{criterion} = \frac{\left(\mu_D - \mu_H\right)^2}{\phi_D + \phi_H} \tag{8}$$

Where ' $\mu_D$  and  $\mu_H$ ' are the values of mean, ' $\phi_D$  and  $\phi_H$ ' are the values of variances of damaged sensitive parameters of damaged and undamaged states respectively. The Fisher criterion values are greater at the potential crack locations where the sudden rise in the structure's response occur which indicates the probable locations of cracks.

#### 4. Results and Discussions

The present work discusses a noble damage detection procedure for in the concepts SPR approach for moving load dynamics problems. The proposed approach includes two parts that is training and monitoring. The control chart approach is established to recognize the probable existence of cracks. Later, the AR process and Fisher criterion methodologies are applied to indentify the potentiality of cracks at any locations. For the present analogy, 300 numbers of patterns are generated, out of which 250 patterns are from healthy and 50 patterns are damaged states respectively. From the control chart analysis, it can be predicted that system is stable or not i.e. presence of cracks. The AR approach along with the implementation of Fisher criterion has predicted the probable locations of cracks. To corroborate the authentication of the said approach, a numerical example has been formulated. The same example which has been taken in the problem definition part is also considered for authentication of SPR approach. The predicted results from SPR approach has been found out and compared with those of the numerical example. It has been traced that the variation of results on crack

location from SPR approach converge (Table 1) with those of the numerical with an average discrepancy about 4.64% which seemed to be well.

4.64

Table 1. Comparison of SPR Results with Numerical Data

# **5 Conclusions**

Total percentage of error

In the present investigation, a structural health monitoring problem in the domain of SPR approach has been developed for damage detection in moving load induced structural dynamics problem. The entire study has been carried out in a supervised manner. The control chart mechanism along with AR process with the implementation of Fisher criterion has been applied for the proposed problem to spot the potential existence of damage in structure. Numerical examples has been devised and compared with the predicted results from SPR approach. It has been proven that the SPR approach converge well with the supervised problem.

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