A study of MHD Boundary Layer Flow of Casson fluid over a Stretching Sheet Through Porous Medium

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The present paper deals with the study of MHD Casson fluid flow over a stretching sheet in porous media. The liquid surrounding stretching sheet serves the purpose of cooling and the level of stretching& cooling enhances the quality of the final product.It has tremendous applications in processing industries such as polymer processing, glass fiber production ,etc. In view of this an attempt is made to study the concept of Casson fluid flow over stretching sheet under uniform Magnetic field.The governing partial differential equations describing the phenomena are converted into ordinary differential equations, and then solved by using analytical technique with numerical computations. The findings of this study are explained graphically by using MATLAB to identify the influence of different critical parameters.

***Keywords*:MHD,** Casson fluid,stretching sheet

1. Introduction

The flow over a strenching sheet has many applications in manufacturing processes.One of the important aspects of stretching sheet is the flow behaviour which determines the rate of cooling. Casson fluid is one such type of such non-Newtonian fluid, which behaves like an elastic solid The study of non Newtonian fluids has many industrial applications in the process of glass fibre plastic sheets etc.Because of diversity in physical structure several models have been developed in non Newtonian fluids. Among them one of the most popular models is casson fluid model.It behaves as solid when the stress magnitude applied on the fluid is less than the yield stress and it begins to move when stress magnitude exceeds yield stress.Materials such as syrups,nail polish,chocolates,cosmetics etc are examples of casson fluids .Its origin in modelling of biological fluid is blood .

In fluid dynamics boundary layer flow of Non newtonian fluid over a stretching is classical.The solution has tremendous applications in many Engineering processes. V.Kumaran studied on the flow of a fluid over a stretching sheet(1966)Then the Problem was extended by Crane, L.J (1970) then Ioan Pop explained the concept of Unsteady flow past a stretching sheet (1996) Nadeema stressedleMHD flow of a Casson fluid over an exponentially shrinking sheet,(2012) Swati Mukhopadhyay (2013) showed a vivid picture on Casson fluid flow over an unsteady stretching surface. Krishnendu Bhattacharyya and Tasawar Hayat probed in Exact solution for boundary layer flow of Casson fluid over a permeable stretching and shrinking sheet. rAdamu Gizachew Chanie1, Bandari Shankar throw some light on the concept of “Mass Transfer on MHD Flow of a Casson Fluid Through a Porous Media. Abid hussain. analysed unsteady heat transfer flow of casson fluid with Newtonian heating T.Hymavathi & W.Sridhar explored the topic of Numerical solution to mass transfer on MHD flow of casson fluid Animasaun Isaac Lare (2015) explained the topic casson fluid flow with variable viscosity embedded in thermally stratified medium.

M. Eswara Rao and S. Sreenadh (2017)gave meaningful contributions towards the MHD Boundary Layer Flow of Casson Fluid Over a Stretchingand Shrinking Sheet Through Porous Mediume Further K.Bhattacharya, brought valuable data on the concept. flow of casson fluid over a stretching surface with heat transfer effects.Krishnamurthy (2014) revealed the solution of casson fluid flow over porous stretching surface with wall mass transfer. Imran Ullah (2015) explored the topic of Sorret and Dufour effects on unsteady mixed convection slip flow of casson fluid over non linearly stretching sheet. M.Krishnamurthy(2016) brought valuable data on Analytical solution for MHD Casson FluidFlow past Porous surface with wall masstransfer.

2. Formulation of the problem

Consider the steady flow of Casson fluid over a stretching sheet through porous medium.The sheet is situated at y=0 and the flow is combined in y>0. A Uniform magnetic field of strength B_0 is applied perpendicular to the stretching sheet.The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is given by

$$\tau_{ij} = \left(\mu_B + \frac{p_{y/}}{\sqrt{2\pi}}\right)2e_{ij}, \pi > \pi_c$$

$$\left(\mu_B + \frac{p_{y/}}{\sqrt{2\pi}}\right)2e_{ij}, \pi < \pi_c$$

Where μ_B is plastic dynamic viscosity of the non-Newtonian fluid, $p_{y/}$ is the yield stress of fluid, π is the product of the component of deformation rate with itself.

The governing equations that describe the flow are as follows under the usual boundary layer approximations

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0{}^2 u}{\rho} - \frac{\upsilon u}{k} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = K\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_1 \tag{4}$$

Here, u and v are the components of velocity in the x and y directions,ϑ is the kinematic fluid viscosity, ρ is the fluid density β is parameter of the Casson fluid, K_1 is the permeability parameter and B_0 is the strength of magnetic field applied in the y direction T is the temperature, and K is the thermal diffusivity of the fluid D the mass diffusion, C the concentration field and K_1 the reaction rate

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The boundary conditions are

$u=U_w$ $v=-v_w$ at y=0, $u\to 0$ as $y\to\infty$ (5)

U_w is the stretching velocity of the sheet and v_w is the wall mass transfer U_w=cx where c is stretching constant. Introducing the stream function , the velocity components u and v can be written as

$$u=\frac{\partial\psi}{\partial x} \text{ and } v=-\frac{\partial\psi}{\partial y} \tag{6}$$

and also the boundary conditions in (4) reduce to

$\frac{\partial \psi}{\partial x}$ =- v_w and $\frac{\partial \psi}{\partial y}$ = U_w (7)

Introducing similarity transformations

$\psi = \sqrt{c\upsilon}\, f(\eta)$ and $\eta = y\sqrt{\frac{c}{\upsilon}}$

u=cxf'(η), v=- $\sqrt{c\upsilon f(\eta)}$ (8)

The equations takes the following forms

$\left(1+\frac{1}{\beta}\right) f^{i11}(\eta)+ f(\eta)f^{11}(\eta)-(f^{1}(\eta))^{2}-(M+K_1)(f^{1}(\eta))=0$ (9)

$\theta'' + Prf\theta' = 0$ (10)

$\varphi'' + S_C f\varphi' - S_C\, \gamma\, \varphi = 0$ (11)

$\Phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$, $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$

$f(\eta)=m$, $f^1(\eta)= 1$ at $\eta=0$, $f^1(\eta) \to 0$, $\eta \to \infty$,

$\theta=1$ at $\eta=0$, $\theta \to 0$ as $\eta \to \infty$, $C=C_w$ at $\eta=0$, $C \to 0$ as $\eta \to \infty$ (12)

3. Solution of the problem

Let $f(\eta)=A+B\, e^{-\lambda\, \eta}$

In porous medium the momentum equation is

$\left(1+\frac{1}{\beta}\right) f^{i11}(\eta)+ f(\eta)f^{11}(\eta)-(f^{1}(\eta))^{2}-(M+K_1)(f^{1}(\eta))=0$ where K_1 is the permeability Parameter and M is the magnetic parameter (13)

Substituting the values of $f(\eta)$, $f^1(\eta)$, $f^{11}(\eta)$ along with boundary conditions

A= m+$\frac{1}{\lambda}$, B= -$\frac{1}{\lambda}$

the Solution $f(\eta)= m+\frac{1}{\lambda}-\frac{1}{\lambda} e^{-\lambda\eta}$

Where $\lambda = \frac{m \pm \sqrt{m^2+4\left(1+\frac{1}{\beta}\right)(1+M+K_1)}}{2\left(1+\frac{1}{\beta}\right)}$ (14)

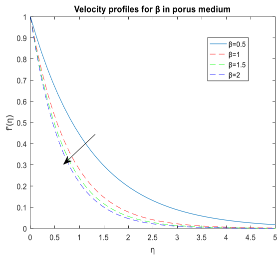

Fig-1

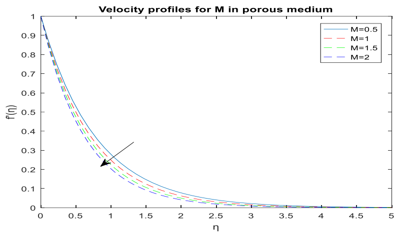

Fig-2

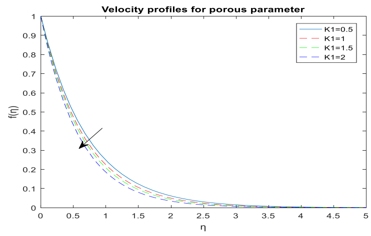

Fig-3

In the absence of porous parameter the equation is

$$\left(1+\frac{1}{\beta}\right) f^{i11}(\eta)+ f(\eta)f^{11}(\eta)-(f^{1}(\eta))^{2}-M(f^{1}(\eta))=0 \qquad (15)$$

the Solution $f(\eta)= A+B\, e^{-\lambda\,\eta}$

$$\text{Where } \lambda = \frac{m\pm\sqrt{m^2+4\left(1+\frac{1}{\beta}\right)(1+M)}}{2\left(1+\frac{1}{\beta}\right)}, \quad A= m+\frac{1}{\lambda}, \quad B= -\frac{1}{\lambda} \qquad (16)$$

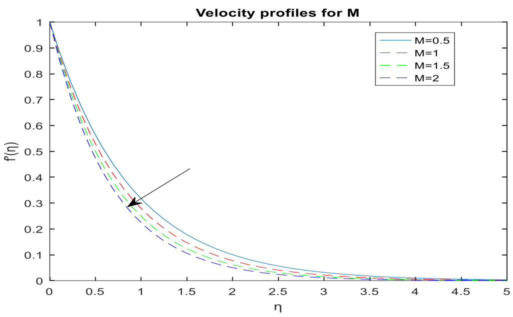

Fig-4

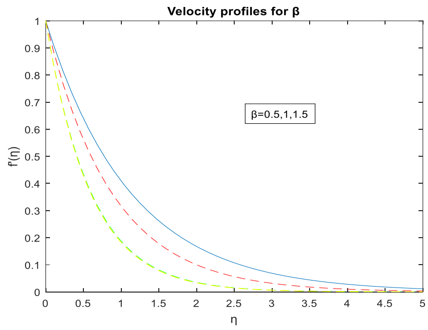

Fig-5

<u>The solution for steady case , M=0</u>

If M=0,

$$\left(1+\frac{1}{\beta}\right) f^{i11}(\eta)+ f(\eta)f^{11}(\eta)-(f^{1}(\eta))^{2}=0 \qquad (17)$$

The solution for steady case $f(\eta)= \frac{1}{\lambda}-\frac{1}{\lambda}\, e^{-\lambda\eta}$

$$\text{Where } \lambda = \frac{1}{\sqrt{1+\frac{1}{\beta}}} \qquad (18)$$

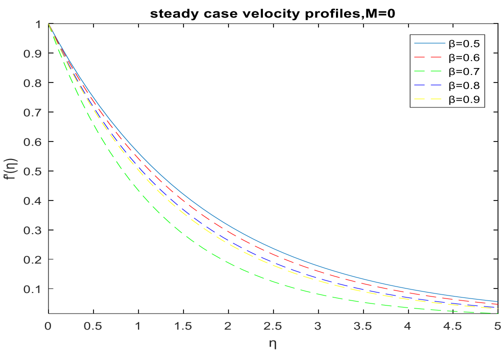

Fig-6

The energy equation is $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=K\frac{\partial^2 T}{\partial y^2}$ (19)

The transformed equation $\theta^{11}+Prf\theta^{1}=0$ (20)

Let $\theta^{1}=A$

The equation becomes $A^{'}+PrfA=0$ (21)

$\Rightarrow A\,e^{\int \Pr f d\eta}=C_1$

$\Rightarrow A=e^{-\int \Pr f d\eta}$

And $c_1=\frac{d\theta_0}{d\eta}$

To find c_1 put f(η)=0 at η=0

θ=1 at η=0, θ→0 as $\eta \to \infty$,

Hence $c_1=\frac{d\theta_0}{d\eta}=\frac{1}{\int_0^{\infty} e^{-\frac{\Pr}{2}\int_0^{\infty} f d\eta}\, d\eta}$

$\frac{d\theta_0}{d\eta}=0.332(\Pr)^{\frac{1}{3}}$ (22)

$\Rightarrow \theta^{'}=A=\frac{d\theta}{d\eta}=0.332(\Pr)^{\frac{1}{3}}\,e^{-\Pr/2\int f d\eta}$

Integrating $\theta=-0.664(\Pr)^{\frac{-2}{3}}\,e^{-\Pr/2\int f d\eta}$ (23)

The concentration equation is

$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_1$

The transformed equation is $\varphi^{''}+S_C f\phi^{'}-S_C\gamma\phi=0$ (24)

Where Schmidt number $s_c=\frac{\upsilon}{D}$, If γ =0 the Mathematical laws for governing equations of heat and mass transfer are equal

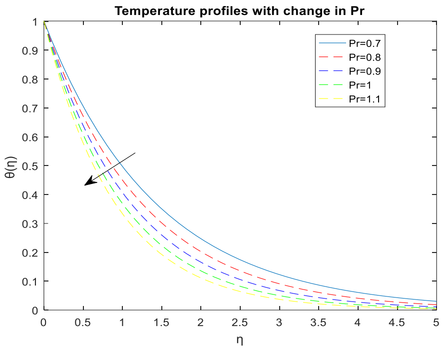

Fig-7

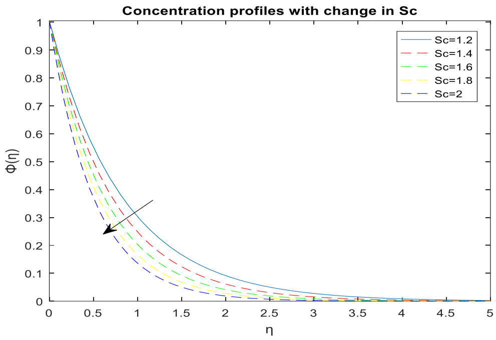

Fig-8

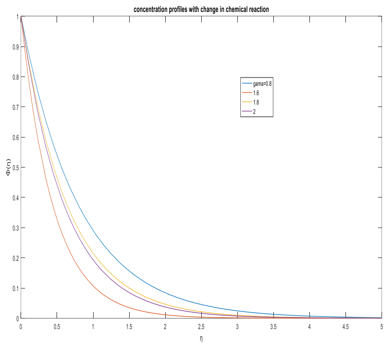

Fig-9

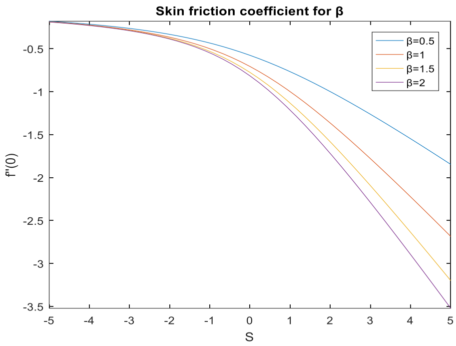

Fig-10

4 .Results&Discussions

In order to analyse the results, numerical computations are carried out for various values of casson fluid parameter β , Magnetic parameter M and permeability parameterK_1 .The observations are

1. Fig-1 Exhibit the variation of velocity profile for various values of β, M, K_1. It is clear that in both cases of unsteady and steady, velocity is found to be a decreasing function of β.

   Increase of casson fluid parameter causes decrease in velocity which was shown in figure -1. This is because of dynamic viscosity.The plastic, dynamic viscosity of Casson fluid increases with increase of β and fluid becomes more viscous.Fig-2 It portrays the influence of M on velocity profile. The increasing values of M lead to decrease in the fluid velocity because the resistive force known as Lorentz force produces electrically conducting fluid and the force has capability to slow down the fluid flow Figure – 3 depicts that raise in porous parameter causes fall in velocity. This is due to the fact increase of porous parameter causes fall in boundarylayer thickness, which in results fall in velocity profiles. Figures –4&5 depict variation in β, M in the absence of porous medium. Figures – 6 depicts steady case velocity profiles if M=0
2. Figures –7 depicts variation in temperature profiles with respect to Prandtl number. Increase of prandtl number decreases the thermal boundary layer thickness of the fluid and hence decreases in temperature Moreover,the thermal boundary layer thickness decreases by increasing Prandtl numbers.
3. Figure –8 shows influence of chemical reaction parameter on concentration profiles. When chemical reaction increases distribution of concentration decreases, which in turn results in increase in transport phenomena and hence results in reduction in concentration distribution.

Schmidt number gives the relative effectiveness of momentum to mass diffusion of spieces concentration, which gives decrease in concentration. This was shown in figure – 9

Nusselt number gives the rate of heat transfer at the plate and is given as

$$Nu = \left.\frac{\partial\theta}{\partial y}\right|_{y=0}$$

Sherwood number measures the rate of mass transfer at the plate given by

$$sh = \left.\frac{\partial\varphi}{\partial y}\right|_{y=0}$$

Influence of different physical parameters on skin friction cf=- $(1+1/\beta) f''(0)$

The local skill friction coefficient , nusselt number and sharewood number are described through numerical values for various parameters of interest. The velocity, temperature and concentration fields are depicted graphycally for different flow parameters

β	M	m	K_1	cf
0.5	0.5	1	0.5	2.43648
1	0.5	1	0.5	2.56154
1.5	0.5	1	0.5	1.9148
2	0.5	1	0.5	2.30274
0.5	0.5	1	0	2.67942
1	1	1	0	2.30277
1.5	1	1	0	2.23350
2	1	1	0	2.55036

Nusselt number	
Pr	Nu
0.7	0.207911
0.8	0.206740
0.9	0.204450
1	0.201368

Sherwood number		
Sc	γ	sh
0.8	0.1	0.800001
1	0.1	0.999998
1.2	0.1	0.364617
1.4	0.1	1.399005
0.8	0.2	0.800000
0.8	0.3	0.800004
0.8	0.4	0.800008

5. Conclusion

Effects of casson fluid parameter, magnetic parameter, permeability parameter on MHD flow of casson fluid model are presented. The observations are

1. The casson parameter, magnetic and permeability parameters have similar effects on velocity
2. Increase of prandtl number causes decrease in temperature profile, this is due to increase in kinematic viscocity.
3. Increase of Schmidt number, chemical reaction parameter causes decrease in concentration

References

[1].V.Kumaran ,Acta mathematica 116,229-233(1966)

[2]. Crane, L.J., ZAMP, 21, pp. 645–655 (1970).

[3]. Ioan Pop ,Mechanical research communications , Vol-23 no.4 p413-432(1996)

[4] . S. Nadeema,,,Scientia Iranica B ,19 (6), 1550–1553, (2012)

[5] .Swati Mukhopadhyay, Ain Shams Engineering Journal, 4, 933–938 (2013)

[6] **. Bhattacharyya K, Hayat T, Alsaedi A. Chin Phys B);22(2),(2013)**

[7] .Krishnendu Bhattacharyya, Tasawar Hayat, **and Ahmed Alsaedi**,ZAMM,Z. Angew. Math. Mech., 1 – 7 (2013)

[8]. Adamu Gizachew Chanie1, Bandari Shankar, **Imperial Journal of Interdisciplinary Research (IJIR),Vol-3, Issue-10, 2017**

[9]M. Eswara Rao S. Sreenadh, Chemical and Process Engineering Research Vol.47, 2017

[10].S. A. Shehzad, T. Hayat, M. Qasim and S. Asghar, Brazilian Journal of Chemical Engineering Vol. 30, No. 01, pp. 187 - 195, January – March,(2013)

[11].Abid hussain "Eur.Phys.J (2014)129,46

[12].N Saidulu and A Venkata Lakshmi,European Journal of Advances in Engineering and **Technology, 3(1): 47-55, (2016)**

[13].M. Eswara Rao and S. Sreenadh, Global Journal of Pure and Applied Mathematics. 10 (2017)

[14] . Abid Hussain Jounal Teknology(2016)78,1-7

[15]. T.Hymavathi & W.Sridhar IJCS Vol-14 (issue 4,2016)

[16]. Imran Ullah 7.,1113(2017)

[17]Domukundwar & Arora "A course in Heat & Mass Transfer" Dhanapati rai&co,seventh Edition(2013)