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Investigating the Influence of Higher-order NURBS Discretization on Contact Force Oscillation for Large Deformation Contact Using Isogeometric Analysis

Vishal Agrawal, Sachin S. Gautam*
Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati 781039, Assam, India
*Corresponding author Email: ssg@iitg.ac.in

The present work studies the influence of higher-order Non-Uniform Rational B-Splines (NURBS) based discretization on contact reaction force oscillation for large deformation contact problem using the isogeometric analysis. The segment-to-segment based Gauss-point-to-surface contact algorithm is used to model the contact. The penalty method is adopted for the regularization of impenetrability contact constraint. The frictionless sliding contact problem involving two deformable bodies is considered to examine the variation in the distribution of contact force oscillations with increasing the order of the NURBS discretization. The study shows that the accuracy of the contact solution improves on increasing the order of NURBS discretization.

***Keywords*:** Isogeometric analysis, computational contact mechanics, large deformation sliding, NURBS.

1. Introduction and Motivation

Isogeometric analysis uses the computer-aided design (CAD) polynomials as a basis for the modelling of complex geometry exactly and approximation of unknown solution fields (Hughes et al. [1]). The key purpose of isogeometric analysis technique is to circumvent the computationally expensive mesh generation process by merging the CAD modelling and finite element analysis (FEA) into a unified framework. Apart from fulfilling its original purpose, it is shown that isogeometric analysis delivers superior results and is robust per degree of freedom in comparison to its counterpart FEA due to its inherent features, i.e., capability to represent the complex shape geometry exactly even with a coarse discretization, tailorable inter-element continuity, and non-negativeness of the underlying basis functions. For the detailed description, the reader is referred to [1] and a monograph by Cottrell et al. [2]. Due to its unique intrinsic properties, isogeometric analysis has emerged as an advantageous computational technology for the treatment of contact problems, especially for large deformation. The NURBS discretized geometry directly provides a unique normal vector field across the contact element boundary, and thus, eliminates the need for additional surface smoothening strategies that were required in the context of traditional finite element (FE) based approaches. In the past few years, a considerable amount of research efforts have been devoted for the treatment of large deformation contact problems with or without considering friction within the framework of isogeometric analysis [3-5]. A comprehensive overview on the recent growth of isogeometric analysis and its advantages with respect to traditional finite element based approach in the field of contact mechanics is presented by De Lorenzis et al. [6].

Based on the available literature on the treatment of contact using the isogeometric analysis, it has been observed that a very fine mesh is predominantly utilized to get accurate results [3-6]. However, from the analysis point of view, a considerable amount of computational efforts are associated with this approach, which is undesirable. Therefore, an alternative solution approach, which sidesteps the necessity of very fine mesh and accurately resolves the contact interface is needed. To address this issue, the present work utilizes the higher-order NURBS polynomials for the isogeometric contact analysis. To the best of authors' knowledge, to this date, no study make use of more than quartic-order of NURBS for the treatment of contact. In this paper, the Gauss-point-to-surface contact algorithm of Dimitri et al. [7] is used to model the contact between two deformable bodies. In this, the impenetrability contact constraint is directly enforced at the Gauss points of the slave bodies. For the regularization of the impenetrability constraint, the penalty method is adopted. For the determination of active Gauss-points, an active set strategy is used. A MATLAB code incorporating the NURBS toolbox [8] is developed for the simulation of considered large deformation contact problem. For the implementation of isogeometric analysis technology, a tutorial paper by Agrawal and Gautam [9] can also be referred.

2. Variational Formulation

In this section, computational formulation for 2D contact between hyperelastic bodies without friction is briefly described. To model the contact between two bodies, the full pass contact algorithm is considered. In this, one body is taken as the slave, $\mathcal{B}^s$, and the other as the master, $\mathcal{B}^m$. The current configuration of a generic point $\mathbf{x}^r$ of body $\mathcal{B}^r$, where $r = \{s, m\}$, is given by: $\mathbf{x}^r = \mathbf{X}^r + \mathbf{u}^r$, where $\mathbf{X}^r$and $\mathbf{u}^r$ denote the reference configuration and displacement field, respectively. In the current configuration, the contact interface for slave and master bodies are described as : $\Gamma_c \coloneqq \Gamma_c^s = \Gamma_c^m$. In case of active contact, the contribution of the contact traction to virtual work is non-zero and is given by [11]

$$\delta W_c = \int_{\Gamma_c^s} t_N \, \delta g_\mathrm{N} \, \mathrm{d}\Gamma \tag{1}$$

In order to solve the above equation using the Newton-Raphson iterative solution method, the linearization of the contact virtual work δW_c is carried out that leads to [11]

$$\Delta\delta W_c = \int_{\Gamma_c^s} (\Delta t_N \delta g_N + t_N \Delta\delta g_N) \, \mathrm{d}\Gamma \tag{2}$$

Here, $g_N = (\mathbf{x}^s - \overline{\mathbf{x}}^m) \cdot \mathbf{n}$ denotes the normal gap between the given slave point $\mathbf{x}^s \in \Gamma_c^s$ on the contact surface of the slave body and $\overline{\mathbf{x}}^m \in \Gamma_c^m$ is a corresponding contact point on the contact surface of $\mathcal{B}^m$. The corresponding contact point $\overline{\mathbf{x}}^m$ on master surface Γ_c^m is determined by drawing a line that passes through the given slave point $\mathbf{x}^s$ in the direction of normal $\mathbf{n}$ to Γ_c^m. The penalty based regularization of normal traction yields $t_N = \epsilon_N < g_N >$. Further details on NURBS can be followed from [12]. Within the context of isogeometric analysis, the parametrization for the contact boundary layer is inherently imported from the description of bulk domain. Using the isoparametric concept, the displacement field $\mathbf{u}^e$, its variation $\delta\mathbf{u}^\mathrm{e}$, and the coordinates of the discretized form of the contact surface within its current configuration $\mathbf{x}^e$are given by [11, 13]

$$\mathbf{u}^e = \sum_{k=1}^{n_p^e} R_k \, \mathbf{u}_k, \quad \delta\mathbf{u}^e = \sum_{k=1}^{n_p^e} R_k \, \delta\mathbf{u}_k, \quad \text{and} \quad \mathbf{x}^e = \sum_{k=1}^{n_p^e} R_k \, \mathbf{x}_k \tag{3}$$

where $n_p^e = 1 + p$ represents the total number of control points within an element 'e'. In the Gauss-point-to-surface based contact algorithm, the impenetrability constraint is directly enforced at the Gauss-quadrature points on the slave body contact surface Γ_c^s. The virtual work is numerically computed on Γ_c^s as [7]

$$\delta W_c = \delta u^T \int_{\Gamma_c^s} \epsilon_N g_N \, \mathbf{N} \; d\Gamma = \delta u^T \, \mathbf{R_c} \tag{4}$$

where $\mathbf{R_c}$ denotes the contact contribution to the residual vector. It is computed over the active points using the Gauss-Legendre quadrature rule as [7]

$$\mathbf{R_c} = \epsilon_N \sum_{gp,\ act}^{n_{gp}^e} g_{N_{gp}} \mathbf{N}_{gp} J_{gp} w_{gp} \tag{5}$$

where n_{gp}^e denots the total numer of Gauss-points, w_{gp} and J_{gp} are the weight and jacobian associated to the active Gauss-quadrature point indiacted using 'gp'. The Gauss-point is considered active if $\mathrm{g_N} < 0$. The consistent contact stiffness matrix and other contact matrices can be diretly followed from reference [7].

3. Numerical Example, Results and Discussion

In this section, a 2D large deformation frictionless ironing problem is considered to study the variation in the non-physical oscillation of contact reaction forces for the different order of NURBS discretizations. The geometrical setup along with the considered mesh for the die and slab is shown in Fig. 1, where $R_o = 0.5$ mm, $R_i = 0.3$ mm, $L = 3.0$ mm and $h = 1$ mm. The bottom surface of the slab is constrained along the both vertical and horizontal directions. This problem is simulated in two steps. In the first, the top surface of the die is subjected to downward vertical displacement $U_y = -0.2$ mm in 10 time steps, and then, moved horizontally by $U_x = 1.5$ mm along the slab in next 140 time steps. A Neo-Hookean hyperelastic material model with $E_{die} = 1000$ GPa and $E_{slab} = 1$ GPa, and $\nu_{die} = \nu_{slab} = 0.3$ under the plane strain conditions are considered for the modelling. The penalty parameter is $\epsilon_N = 100$. The deformed configurations of the setup for the quadratic order of NURBS discretization at different time steps are shown in Figs. 1(b) - (d). The resultant vertical contact reaction force P_y, evaluated at the top line of the die, is accounted at the end of each time step. Fig. 2(a) illustrates the variation of vertical contact force P_y versus load step t for quadratic and quartic order of NURBS discretizations. It can be observed that during the first step of simulation, when the die is pressed against the slab, the magnitude of the vertical contact force increases linearly. After that, the vertical contact reaction force continuously oscillates around a mean line during sliding. In theory, the magnitude of vertical contact reaction force remains constant during sliding and no such oscillations are present. The commonly employed solution approach to alleviate such non-physical oscillation of contact reaction force is to use a very fine mesh that can accurately capture the variation of contact forces across the contact interface [10]. However, this requires undesirably large amount of computational efforts. The present work makes use of higher-order NURBS as an alternative to very fine meshes. For this, the fifth- and sixth order NURBS are utilized for discretizing the contact surface along with the bulk domain and the corresponding plots of the vertical reaction forces are shown in Fig. 2(a). The enlargement of P_y for the different order of NURBS discretizations is shown in Fig. 2(b). It can be observed that on increasing the order of NURBS discretization from $p = 4$ to $p = 6$ the amplitude of the oscillation error reduces. This is due to utilizing higher-order NURBS that improves the approximation of contact quantities at coarse mesh compared to quartic-order of NURBS discretization.

4. Conclusion

This paper studies the higher-order NURBS based isogeometric analysis for large deformation contact problem involving two deformable bodies. It is observed that the accuracy of the vertical contact reaction force improves on increasing the order of NURBS discretization. The sixth-order NURBS delivers the most accurate results among the all tested discretizations. The future objective is to modify the existing NURBS discretization procedure in a way that it becomes oriented to contact interface rather than the bulk domain.

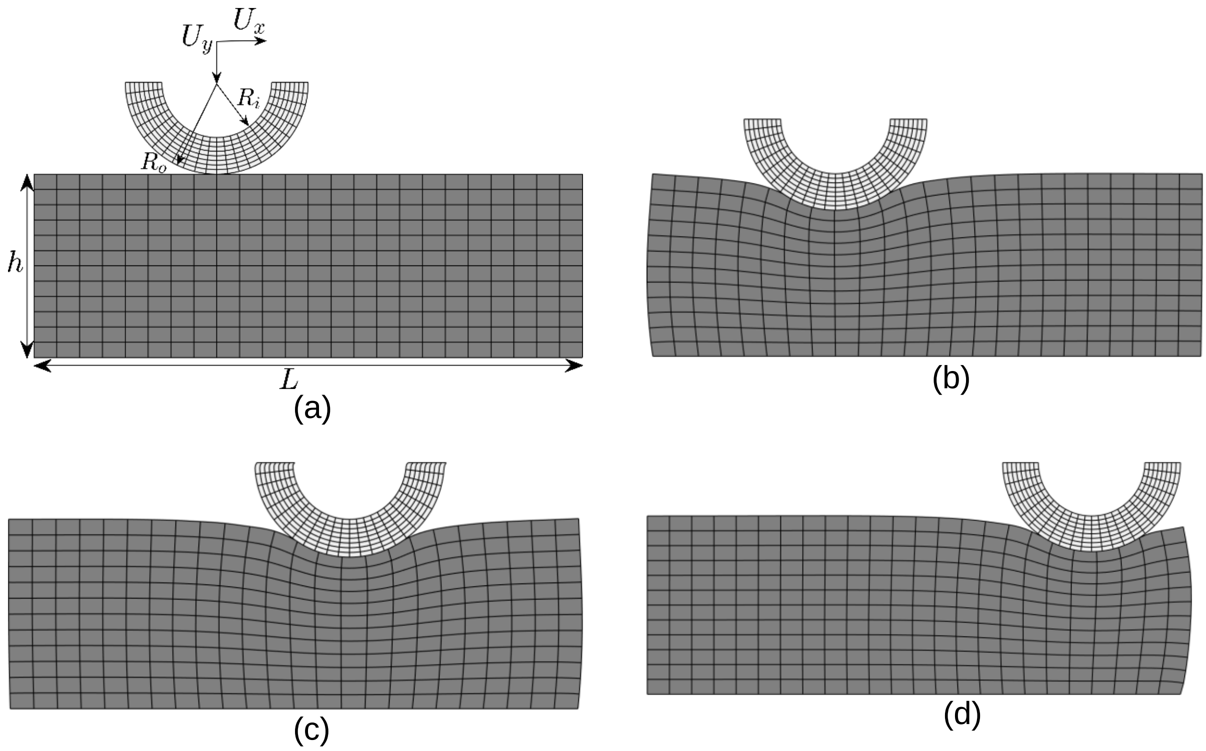

Fig. 1. Ironing problem: (a) geometrical setup of the problem along with the considered mesh arrangement for the die and the slab. Deformed configurations: (b) at time step $t = 11$, (c) at $t = 46$, and (d) at $t = 141$.

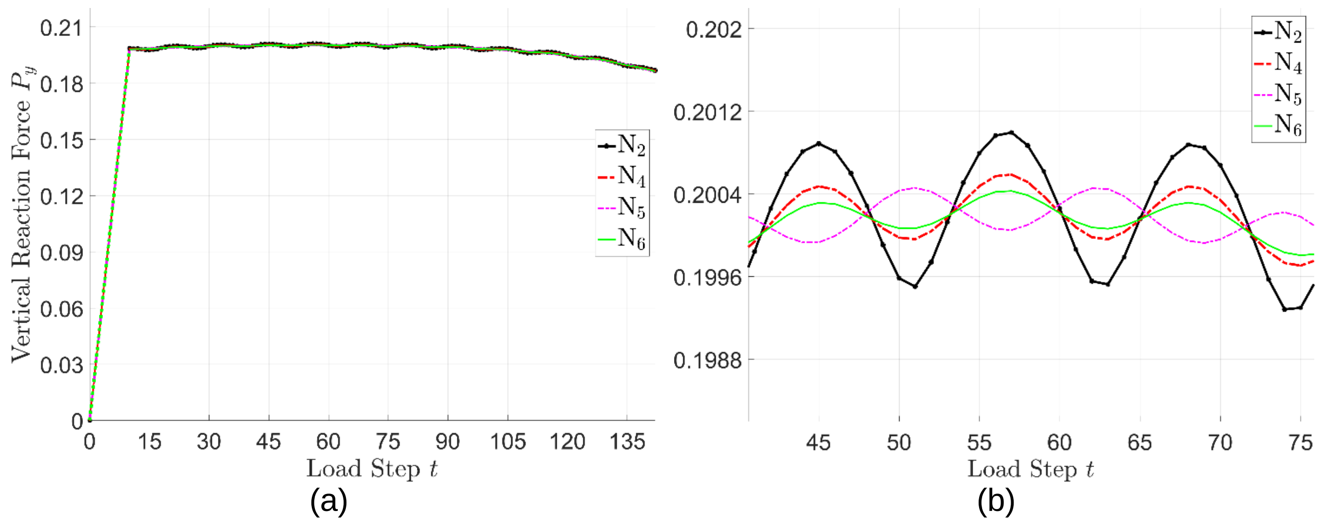

Fig. 2. (a) The resultant vertical contact reaction force P_y for the overall simulation process, (b) Zoomed view of P_y for different order of NURBS discretizations.

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