Proceedings of the International Conference on Applied Mechanical Engineering Research (IC-AMER2019)
NIT Warangal, India – May 02-05, 2019
Paper No. XXX (The number assigned when the papaer is accepted)

Mathematical Modelling of Material Removal Rate using Buckingham Pi Theorem in Electrical Discharge Machining of Hastelloy C276

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In today's world, to meet the requirements of the extreme applications the need for precisely manufactured components become necessary. The new materials developed for the extreme applications are difficult to machine by conventional machining processes due to their high hardness. EDM is currently widely used in industry for high-precision machining of all types of conductive material such as metals, metallic alloys, graphite and even some ceramic materials, regardless of hardness. In this present work a prediction model for MRR in machining of Hastelloy C276 on EDM using Buckingham pi theorem is developed to study the influence of process parameters. Further, the linear programming using MS-solver was applied to perform the optimization and the sensitivity analysis for the model developed. The theoretical and experimental results are compared and found that the predicted model results are satisfactory.

Keywords**:** EDM, Hastelloy C276, Buckingham Pi theorem, MS-solver.

1. Introduction

Electrical Discharge Machining (EDM) is a non-traditional manufacturing process which removes material from a workpiece by a series of repeated generated electrical discharges between a tool electrode, and the workpiece electrode being machined which are separated by a small gap (5-100μm depending on electrical parameters) in the presence of a circulated dielectric fluid [1]. EDM has become extensively used since its inception in the mid 1940's for producing the components such as moulds/dies, aircraft engine parts, medical instruments, etc. Complicated shapes and sizes with high accuracy and precision can be obtained by this process [2].

A nickel alloy Hastelloy C276 having about 57% of nickel is widely used in chemical industries, nuclear plants, aerospace industries due to its high strength and corrosion resistance at elevated temperatures. Because of distinctive characteristics such as low thermal diffusivity, affinity to react with tool material, and work hardening makes it difficult to machine [3]. Hence a non-traditional method EDM is most advantageous to use for machining of these materials. The important responses in EDM are material removal rate(MRR) and surface roughness(Ra) of workpiece. Peak current(I), Pulse ON time(T_{on}), and Pulse OFF time(T_{off}) are the machining parameters which influence the performance measures. The influence of process parameters in machining of Inconel 718 on output responses material removal rate, electrode wear rate, half taper angle, and radial overcut has been reported by D.V.Ghewade [4]. E. Aliakbari[5] evaluated the effect of rotary tool in EDM on surface finish. An integrated approach of Taguchi method and Artificial Intelligence was applied to optimize response process parameter design and proved the improvement in setting the optimal parameters set[6].

Tsai and Wang [7] developed a semi-empirical model based on dimensional analysis between the process parameters and performance measures. Mangesh [8] employed Buckingham pi theorem to develop a model for the material removal rate and power consumption in turning of ferrous materials. R.S.Kadu et al. [9] formulated a mathematical model for the tool wear in boring operation on cast iron using carbide and CBN tools. A semi-empirical model for material removal rate in machining of metal matrix composites has been developed by employing dimensional analysis by P.B.Bains et al. [10]. B.Ravindranath et al. [11] developed a model for MRR and Ra employing dimensional analysis and considering thermo-mechanical properties.

Few researchers has been attempted to develop mathematical models for MRR in EDM based on thermo-mechanical approaches. In the present work, an attempt is made to establish the relation between process parameters, thermo-mechanical properties, and performance measures using Buckingham pi theorem. The linear programming using MS-solver was used to perform optimization and sensitivity analysis for the developed model. Based on the literature survey, several preliminary experiments were performed to select the influencing factors on performance measures. The selected machining variables are Peak current, Pulse ON time, Pulse OFF time, density, electrical resistivity, thermal conductivity, and specific heat.

1.1. Experimental Procedure

The experimental design has been selected in such a way that the objectives of the study can be fulfilled. The properties of Hastelloy C276 super alloy [12] are given in Tables 1.

<Insert Table.1 here>

1.2. Materials

The electrode material is pure copper (99.97%) of diameter 10 mm and density of 8.89g/cm^3. Commercial EDM oil (grade 30) is used as dielectric fluid and it is circulated by jet side flushing. The work pieces are cut to the size of 20 mm × 20 mm × 10mm using wire EDM machine. The work piece surfaces are further polished to a surface finish of $R_a \approx 1\mu m$ using emery papers before conducting the experiments. The electrode face is turned and polished using a very fine grade emery sheet before conducting each experiment. The detailed experimental conditions are given in Table.2 and process parameters and their levels [13] in Table.3.

<Insert Table.2 and Table.3 here respectively>

1.3. Experimental Design

Using Taguchi technique [7], L9 orthogonal array is selected and performed 9 experiments. All experiments are conducted randomly according to L9 orthogonal array. The experiments are replicated three times at each parametric combination to reduce experimental error. MRR is calculated and then the average of three trials is used for final analysis. The MRR (m^3/s) is calculated using formula,

$$MRR = (m_1-m_2) / \rho.T$$

Where, m_1 = mass of the work piece before machining in kg

m_2 = mass of the work piece after machining in kg

ρ = density of the workpiece in kg/m^3 T = machining time in seconds

2. Dimensional Analysis

The dimensional analysis is a systematic approach to identify variables in a machining process and correlating them to form a set of dimensionless group. The following are the two methods for dimensional analysis:

a). Buckingham Pi theorem b). Rayleigh's method

Buckingham Pi theorem:

The dimensional analysis for the experimental data leads to some non-dimensional parameters which are referred as *pi* terms. Based on the concept of dimensional homogeneity, these dimensionless parameters may be grouped and expressed in functional forms. In this work Buckingham Pi theorem is used for dimensional analysis to elicit the functional relationship between the inputs and responses in the form of an exponential equation.

The independent variables and their dimensional formulae used for predicting the response MRR are shown in Table.4.

<Insert Table.4 here>

Number of *pi* terms = Number of variables – Number of primary dimensions
I, T_{on}, k, ρ, C_v are the selected repeating variables.
Calculation of pi terms:
$\pi_1 = I^{A1}\, Ton^{B1}\, \rho^{C1}\, k^{D1}\, C_v{}^{E1}$ (MRR)
$[M^0\, L^0\, T^0\, I^0\, \theta^0] = [\,I\,]^{A1}\, [T]^{B1}\, [ML^{-3}]^{C1}\, [MLT^{-3}\theta^{-1}]^{D1}\, [L^2T^{-2}\theta^{-1}]^{E1}\, [L^3T^{-1}]$
$\pi_2 = I^{A2}\, Ton^{B2}\, \rho^{C2}\, k^{D2}\, p^{E2}$ (Toff)
$[M^0\, L^0\, T^0\, I^0\, \theta^0] = [\,I\,]^{A2}\, [T]^{B2}\, [ML^{-3}]^{C2}\, [MLT^{-3}\theta^{-1}]^{D2}\, [L^2T^{-2}\theta^{-1}]^{E2}\, [T]$
$\pi_3 = I^{A3}\, Ton^{B3}\, \rho^{C3}\, k^{D3}\, p^{E3}$ (R)
$[M^0\, L^0\, T^0\, I^0\, \theta^0] = [\,I\,]^{A3}\, [T]^{B3}\, [ML^{-3}]^{C3}\, [MLT^{-3}\theta^{-1}]^{D3}\, [L^2T^{-2}\theta^{-1}]^{E3}\, [ML^3T^{-3}I^{-2}]$
Equating the powers of the fundamental units on both sides, the set of simultaneous linear equations are obtained and solved for the constants. The resulted equations are

$$\pi_1 = I^0\, T_{on}{}^{-0.5}\, \rho^{1.5}\, k^{-1.5}\, C_v{}^{1.5}\ (MRR) \tag{1}$$
$$\pi_2 = I^0\, T_{on}{}^{-1}\, \rho^{0}\, k^0\, C_v{}^0\ (T_{off}) \tag{2}$$
$$\pi_3 = I^2\, T_{on}{}^{0}\, \rho^{2}\, k^{-3}\, C_v{}^{3}\ (R) \tag{3}$$

$\pi_1 = f(\pi_2, \pi_3)$
A probable exact mathematical form for the dimensional equations of the phenomenon could be assumed to be of exponential form.
$\pi_1 = M\, (\pi_2)^a\, (\pi_3)^b$
$$\left[\frac{MRR\ \rho^{1.5} C_v{}^{1.5}}{T_{on}{}^{0.5} k^{1.5}}\right] = M \left(\frac{I^2\ \rho^2 C_v{}^3 R}{k^3}\right)^a \left(\frac{T_{off}}{T_{on}}\right)^b$$
Taking logarithm on both sides

$$\log\left[\frac{MRR\ \rho^{1.5} Cv^{1.5}}{T_{on}{}^{0.5} k^{1.5}}\right] = \log M + a \log\left(\frac{I^2\ \rho^2 C_v{}^3 R}{k^3}\right) + b \log\left(\frac{T_{off}}{T_{on}}\right) \tag{4}$$

Let, $Z = \log\left[\frac{MRR\ \rho^{1.5} C_v{}^{1.5}}{T_{on}{}^{0.5} k^{1.5}}\right]$ $\quad Y = \log\left(\frac{T_{off}}{T_{on}}\right)$ $\quad X = \log\left(\frac{I^2\ \rho^2 R}{T_{on}{}^3\ p^{-3}}\right)$ $\quad c = \log M$
Then equation (4) is written as,
$Z = aX + bY + c$
The above equation is a regression equation of Z on X and Y. The equation consists of 3 unknowns and it requires minimum three experimental data to determine the unknown quantities. To accomplish all the experimental data in determination of unknown parameter values, it has been applied Least Square method.
Writing normal equations
$$\Sigma Z = a\Sigma X + b\Sigma Z + Nc \tag{5}$$
$$\Sigma XZ = a\Sigma X^2 + b\Sigma XY + c\Sigma X \tag{6}$$
$$\Sigma YZ = a\Sigma XY + b\Sigma Y^2 + c\Sigma Y \tag{7}$$

On solving the normal equations (5), (6), and (7) the constants are as follows:
a = -0.0773713, b = 0.3033778, c = 1.548602, M = 35.36731
Finally the mathematical equation for MRR is written as

$$MRR = 35.36731\left(\frac{I^2\ \rho^2 C_v{}^3 R}{k^3}\right)^{-0.0773713} \left(\frac{T_{off}}{T_{on}}\right)^{0.3033778} \left(\frac{T_{on}{}^{0.5} k^{1.5}}{\rho^{1.5} Cv^{1.5}}\right) \tag{8}$$

MRR Mean:

Experimental – 10.407 mm^3/min
Model – 10.551 mm^3/min

3. RESULTS AND DISCUSSION

3.1. Model formulation

It is required to correlate quantitatively the various dependent and independent terms involved in this very complex phenomenon. The correlation is nothing but a mathematical model as a design tool for such situation. The mathematical model for material removal rate is shown in below:

$$\text{MRR} = 35.36731\left(\frac{I^2\,\rho^2 C_v{}^3\text{R}}{k^3}\right)^{-0.0773713}\left(\frac{T_{off}}{T_{on}}\right)^{0.3033778}\left(\frac{T_{on}{}^{0.5}k^{1.5}}{\rho^{1.5}\text{Cv}^{1.5}}\right) \quad (9)$$

The above formulated model is of interest to look at further comparisons between the experimental results and predictions based on the model for material removal rate. In order to validate it the average error is calculated as follows:

$$\text{Error (\%)} = \left|\frac{Experimental\ results - Predicted\ value}{Experimental\ results}\right| \times 100 \quad (10)$$

The average prediction error based on model is 1.38%. It is obvious that the predicted values by the dimensional analysis are very close to the experimental results. Therefore the model is well fitted between machining parameters and response.

3.2. Model optimization

The ultimate goal of this work is not only developing model but also to find out the best set of independent variables, which will result in maximization of the objective function material removal rate. The model has non-linear form, hence it is to be converted into a linear form for optimization purpose. This can be done by taking log on both sides of the model and is explained as follows:
Taking log on both sides of the equation (9), we get

$$\log(\text{MRR}) = \log(35.36731) - 0.0773713\log\left(\frac{I^2\,\rho^2 C_v{}^3\text{R}}{k^3}\right) + 0.3033778\log\left(\frac{T_{off}}{T_{on}}\right) + \log\left(\frac{T_{on}{}^{0.5}k^{1.5}}{\rho^{1.5}C_v{}^{1.5}}\right)$$

$Z = 1.548602 - 0.0773713\log\pi_4 + 0.3033778\log\pi_5 + \log\pi_6$

$Z = 1.548602 - 0.0773713X_1 + 0.3033778X_2 + X_3$ (11)

Where $Z = \log(\text{MRR})$, $X_1 = \log\pi_4$, $X_2 = \log\pi_5$, $X_3 = \log\pi_6$

The objective function is written as

Max $Z = 1.548602 - 0.0773713X_1 + 0.3033778X_2 + X_3$

Subject to the following constraints

$1 \times X_1 + 0 \times X_2 + 0 \times X_3 \leq 9.38777$ (12)

$1 \times X_1 + 0 \times X_2 + 0 \times X_3 \geq 9.03559$ (13)

$0 \times X_1 + 1 \times X_2 + 0 \times X_3 \leq 1.30103$ (14)

$0 \times X_1 + 1 \times X_2 + 0 \times X_3 \geq -0.39794$ (15)

$0 \times X_1 + 0 \times X_2 + 1 \times X_3 \leq -10.50661$ (16)

$0 \times X_1 + 0 \times X_2 + 1 \times X_3 \geq -11.00661$ (17)

On solving the above optimization problem using MS solver, we get the X_1, X_2, X_3 and are listed in the Table.5. The optimum value of MRR is 32.79 mm^3/min.

<Insert Table.5 here>

3.3. Sensitivity Analysis

The influence of various independent π terms have been studied by analyzing the indices of the various model π terms. By the sensitivity analysis technique, the change in value of dependent π term due to introduced change in the value of independent π term is evaluated. Here, the change of $\pm 10\%$ is introduced in the independent π terms (one at a time). This defines sensitivity. The total percentage in output for $\pm 10\%$ change in input is shown in Table. 6.

<Insert Table.6 here>

4. Conclusion

Dimensional analysis modelling has been found to be the easiest technique to perform the analysis of MRR with respect to various independent parameters including thermo-mechanical properties of workpiece. A designer can subsequently select the best combination of design variables with the mathematical model obtained for achieving optimum MRR. This will eventually reduce the machining time. The error calculated is 1.38% only which means the model is very closely approached the experimental results. Hence the dimensional analysis(DA) technique can be employed to obtain a mathematical model for performance measures involving many process parameters in electrical discharge machining process.

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List of Tables

Table.1: Properties of Hastelloy C276 super alloy

Melting range (^{0}C)	1,325–1,370
Density (g/cm^3)	8.89
Modulus of Elasticity(GPa)	205
Specific Heat(J/kg ^{0}K)	427
Electrical Resistivity(μΩ-m)	1.3
Thermal conductivity (W/m °K)	10.2

Table.2: Experimental conditions in EDM

Work piece	20 mm × 20 mm × 10 mm plate
Electrode	Copper tool (diameter = 10 mm)
Dielectric	commercial EDM oil (grade 30)
Dielectric flushing	Side flushing with pressure
Polarity	Positive (work piece '+ve' and tool '–ve')
Work time	20 minutes
Gap voltage	30 V (constant)

Table.3: Process parameters and their levels

Process Parameter	Symbol	Selected Levels		
		1	2	3
Peak current (Amps.)	A	12	15	18
Pulse on time (μs)	B	5	20	50
Pulse off time (μs)	C	20	50	100

Table.4. Process parameters of EDM and their dimensional formulae

Process parameters	Units	Dimensional formula
Current (I)	Amp	A (or) I
Pulse-on Time (T_{on})	μS	T
Pulse-off Time (T_{off})	μS	T
Density (ρ)	Kg/m^3	ML^{-3}
Electrical Resistivity (R)	μΩ-m	$ML^3T^{-3}I^{-2}$
Thermal Conductivity (k)	W/m °K	$MLT^{-3}\theta^{-1}$
Specific heat (C_v)	J/kg °K	$L^2T^{-2}\theta^{-1}$
Metal removal rate (MRR)	m^3/S	L^3T^{-1}
SR (Ra)	μm	L

Table.5: Optimized values of response variables

	Log values of π terms	Antilog values of π terms
Z	-9.2624	5.465×10^{-10} m^3/s (or) 32.79mm^3/min
X_1	9.03559	1.0854×10^{9}
X_2	1.30103	20
X_3	-10.5066	3.11458×10^{-11}

Table.6: Sensitivity analysis of MRR

π_4	π_5	π_6	MRR (mm^3/min)
$1.0854x10^{9}$	2	$3.11452x10^{-11}$	16.30704
$1.19394x10^{9}$	2	$3.11452x10^{-11}$	16.18722
$0.97686x10^{9}$	2	$3.11452x10^{-11}$	16.44051
		Change(%)	1.55%
$1.0854x10^{9}$	2.2	$3.11452x10^{-11}$	16.78543
$1.0854x10^{9}$	1.8	$3.11452x10^{-11}$	15.79403
		Change(%)	6.08%
$1.0854x10^{9}$	2	$3.425972x10^{-11}$	17.93773
$1.0854x10^{9}$	2	$2.491616x10^{-11}$	13.04562
		Change(%)	30%