

# EFFECT OF CHEMICAL REACTION AND RADIATION ON MHD CASSON FLOW ALONG A MOVING VERTICAL POROUS PLATE

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**Abstract:** In this article, we consider effects of force buoyancy and magneto hydrodynamic on convective mass and heat transfer flow past a touching vertical porous plate in the incidence of thermal radiation and chemical reaction. The governing partial differential equations are concentrated to a system of self-similar equations using the similarity transformations. The resulting equations are solved numerically using the 4<sup>th</sup> order Runge-Kutta method along with the shooting technique. The outcomes are found for the velocity, temperature, concentration, Nusselt number, Sherwood number and skin-friction. The effects of various parameters on flow variables are illustrated graphically, and the physical aspects of the problem are discussed.

**Key words:**, MHD, porous medium, heat and mass transfer, thermal radiation, chemical reaction.

## INTRODUCTION

In recent years, the problems of free convective heat and mass transfer flows through a porous medium under the effect of a magnetic field have attracted the attention of many researchers because of their possible applications in many branches of technology and science, such as its applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. Rajakumar et al.[1] studied chemical reaction and viscous dissipation effects on MHD free convective flow past a semi-infinite moving vertical porous plate with radiation absorption. Thammanna et al.[2] discussed three dimensional MHD flow of couple stress Casson fluid past an unsteady stretching surface with chemical reaction. Malleswari[3] analyzed heat and mass transfer analysis on mud radiating flow in presence of natural convection, porous medium and viscoelastic Rivlin-Ericksen fluid through numerical solutions. Srinivasa Raju et al.[4] studied influence of angle of inclination on unsteady MHD Casson fluid flow past a vertical surface filled by porous medium in presence of constant heat flux, chemical reaction and viscous dissipation. Ramesh Babu et al.[5] expressed numerical investigation of heat transfer mechanism in MHD Casson fluid flow past a vertically inclined plate in presence of hall current. G.V.R. Reddy et al.[6] studied Soret and Dufour effects on MHD micropolar fluid flow over a linearly stretching sheet through a non -Darcy porous medium. Ramana Reddy et al.[7] examined MHD free convective flow of Casson fluid past over an oscillating vertical porous plate. Vijaya et al.[8] presented Soret and radiation effects on an unsteady flow of a Casson fluid through porous vertical channel with expansion and contraction. GVRReddy [9] discussed numerical solutions of unsteady MHD flow heat transfer over a stretching surface with suction or injection. Hari Krishna et al.[10] analyzed chemical reaction effect on MHD flow of Casson fluid with porous stretching sheet. Das et al.[11] expressed Newtonian heating effect on steady hydro magnetic Casson fluid flow a plate with heat and mass transfer. Dash et al.[12] discussed Casson fluid flow in a pipe filled with a

homogeneous porous medium. Gnanaswara Reddy[13] studied unsteady Radiative-convective boundary layer flow of a Casson fluid with variable thermal conductivity. Haya et al.[14] discussed Soret and Dufour effects on magnetohydrodynamic flow of Casson fluid. Hussanan et al.[15] analyzed unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. Khalid et al.[16] discussed unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium.

Henceforward, the impartial of this research paper is to analyses the effect of radiation on MHD free convection flow past along a moving vertical porous plate in presence of thermal radiation and chemical reaction. The governing equations are changed by the resultant dimensionless equations are resolved numerically by using shooting technique and using unsteady similarity transformation. The effects of different governing parameters on the concentration, temperature, velocity, are obtained.

## MATHEMATICAL ANALYSIS

Consider an unsteady two-dimensional free convection flow of a viscous incompressible electrical conducting, thermal radiating and chemical reacting fluid flow along a moving vertical porous plate immersed in a porous medium. The x-axis is taken along the plate in the upward direction and y-axis is taken normal to the plate. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiation heat flux in the energy equation. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field. It is assumed that the external electrical field is zero and the electric field due to the polarization of charges is negligible. Initially, the plate and the fluid are at the same temperature  $T_\infty$  and the concentration  $C_\infty$ . At time  $t > 0$ , the plate temperature and concentration are raised to  $T_w$  and  $C_w$  respectively and are maintained constantly thereafter. It is also assumed that all fluid properties are constant except that the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation). Also, there is chemical reaction between the diffusing species and the fluid. The foreign mass present in the flow is assumed to be at low level and hence Soret and Dufour effects are negligible. Under these assumptions, the governing boundary layer equations of the flow field are:

Conservation of mass:

$$\frac{\partial v}{\partial y} = 0 \quad (1)$$

Conservation of momentum:

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \left( 1 + \frac{1}{\beta_0} \right) \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_\infty) + g \beta^* (C - C_\infty) - \left( \frac{\sigma B_0^2}{\rho} + \frac{\nu}{K^*} \right) u \quad (2)$$

Conservation of energy (Heat):

$$\rho c_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q(T - T_\infty) \quad (3)$$

Conservation of species (Concentration):

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - Kr^* (C - C_\infty) \quad (4)$$

where  $u$  and  $v$  are the velocity components in  $x$  and  $y$  directions respectively,  $\sigma$  -the electrical conductivity of the fluid,  $B_0$  -the magnetic induction,  $k$  -the thermal conductivity,  $Kr^*$  -the chemical reaction parameter,  $q_r$  -the local radiative heat flux,  $C$  -the species concentration in the boundary layer,  $C_\infty$  -the species concentration in fluid far away from the plate,  $D$  -the mass diffusivity,  $T_\infty$  -the temperature of the fluid far away from the plate,  $T$  -the temperature of the fluid in the boundary layer,  $\nu$  -the kinematic viscosity,  $\tau$  -Thermo porosity parameter,  $Q$  - heat source parameter,  $\beta_0$  - Casson parameter,  $g$  -the acceleration due to gravity,  $\rho$  -the fluid density,  $\beta, \beta^*$  -the thermal and concentration expansion coefficients respectively, and  $\alpha$  -the thermal diffusivity.

The second and third terms on the right hand side of the momentum equation (2) denote the thermal and concentration buoyancy effects respectively.

The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} t \leq 0: & u = 0, v = 0, T = T_\infty, C = C_\infty \text{ for all } y \\ t > 0: & \begin{cases} u = U, v = v(t), T = T_w, C = C_w \text{ at } y = 0 \\ u \rightarrow 0, v \rightarrow 0, T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty \end{cases} \end{aligned} \quad (5)$$

where  $U$  is said to be the plate characteristic velocity. Thermal radiation is expected in a unidirectional flux in  $y$  - direction i.e.,  $q_r$  by using Rosseland approximation value the radiative heat flux  $q_r$  is given by

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y} \quad (6)$$

It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then equation (6) can be linearized by expanding  $T^4$  in Taylor series about  $T_\infty$  which after neglecting higher order terms takes the form:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

In view of equations (6) and (7), equation (3) reduces to:

$$\rho c_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_s}{3k_e} T_\infty^3 \frac{\partial^2 T}{\partial y^2} + Q_0 (T - T_\infty) \quad (8)$$

We introduce similarity variables and the dimensionless quantities i.e.,

$$\begin{aligned} \eta &= \frac{y}{2\sqrt{\nu t}}, u = Uf(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}, \\ Gc &= \frac{4g\beta^*(C_w - C_\infty)t}{U}, Gr = \frac{4g\beta(T_w - T_\infty)t}{U}, \\ M &= \frac{4\sigma B_0^2 t}{\rho}, K^* = \frac{K\nu}{tc}, R = \frac{16\sigma_s(T_w - T_\infty)^3}{3k_e k}, \\ N &= \frac{T_\infty}{T_w - T_\infty}, Pr = \frac{\mu c_p}{k}, Sc = \frac{\nu}{D}, Kr^* = \frac{Kr}{4t} \end{aligned} \quad (9)$$

From equation (1),  $\nu$  is either a constant or a function of time. Following (Singh and Soundalgekar [21], we choose

$$\nu = -c \left( \frac{\nu}{t} \right)^{\frac{1}{2}} \quad (10)$$

where  $c > 0$  is the suction parameter.

In view of equations (10) and (9), the equations (8), (2) and (4) reduce to

$$f'' \left( 1 + \frac{1}{\beta_0} \right) + 2(\eta + c) f' + Gr\theta + Gc\phi - \left( M + \frac{1}{K} \right) f = 0 \quad (11)$$

$$\theta'' + 2(\eta + c) Pr \theta' + R(3(N + \theta)^2 \theta'^2 + (N + \theta)^3 \theta'') + \frac{Q}{Pr} \theta = 0 \quad (12)$$

$$\phi'' + 2(\eta + c) Sc \phi' - Kr Sc \phi = 0 \quad (13)$$

where the primes denote the differentiation with respect to  $\eta$ ,  $Pr$  is the Prandtl number,  $M$  is the magnetic field parameter,  $Gr$  is the thermal Grashof number,  $Sc$  is the Schmidt number,  $R$  is radiation parameter,  $Gc$  is the modified Grashof number,  $Kr$  is the chemical reaction parameter,  $N$  is the temperature difference parameter.  $\tau$ -Thermo porosity parameter,  $Q$ - heat source parameter and  $\beta_0$  - Casson parameter.

The corresponding dimensionless boundary conditions are

$$\begin{cases} f = 1, \theta = 1, \phi = 1, & \text{at } \eta = 0 \\ f \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 & \text{as } \eta \rightarrow \infty \end{cases} \quad (14)$$

## SOLUTION OF THE PROBLEM

The set of coupled non-linear governing boundary layer equations (11)-(13) together with the boundary conditions (14) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (11)-(13) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size  $\Delta\eta = 0.005 = 0.05$  is used to obtain the numerical solution with decimal place accuracy as the criterion of convergence.

## RESULTS AND DISCUSSION

The problem considering for unsteady MHD free convection fluid flow past a moving vertical porous plate embedded in porous medium with thermal radiation and chemical reaction in presence of suction. The numerical values of velocity ( $f$ ), temperature ( $\theta$ ) and concentration ( $\phi$ ) with the boundary layer have been computed for different parameters as the thermal Grashof number  $Gr$ , solutal Grashof number  $Gc$ , magnetic field parameter  $M$ , Permeability parameter  $K$ , Prandtl number  $Pr$ , thermal radiation parameter  $R$ , Schmidt number  $Sc$ ,  $\tau$ -Thermo porosity parameter,  $Q$ - heat source parameter,  $\beta_0$  - Casson parameter and suction parameter  $c$ . In the present study, we adopted the following default parametric values:  $Gr = 5$ ,  $Gc = 10$ ,  $M = 1.0$ ,  $K = 0.5$ ,  $Pr = 0.71$ ,  $R = 0.5$ ,  $N = 0.1$ ,  $Sc = 0.6$ ,  $Kr = 0.5$ ,  $c = 0.5$ ,  $\tau = 0.01$ ,  $B = 10$ ,  $c = 0.5$  and  $Q = 0.05$ . All the graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

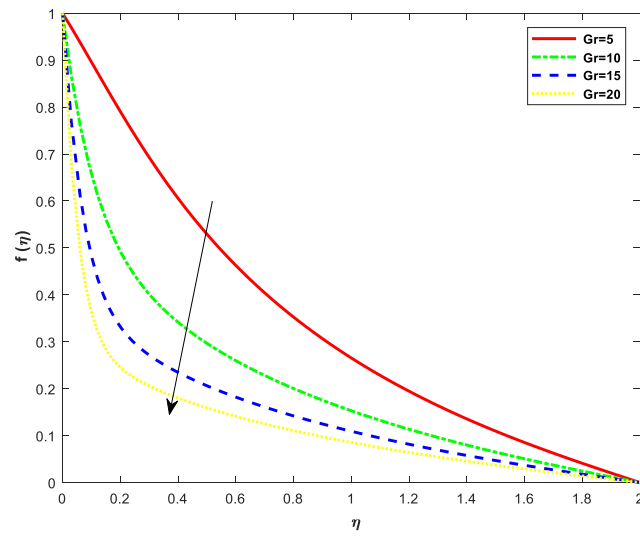


Fig. 1 – Velocity profiles for dissimilar values of Grashoff Number( $Gr$ ).

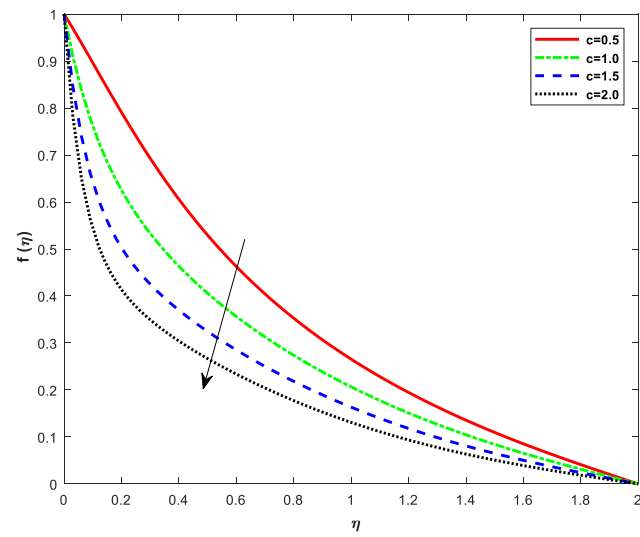


Fig. 2 – Velocity profiles for dissimilar values of suction parameter ( $c$ )

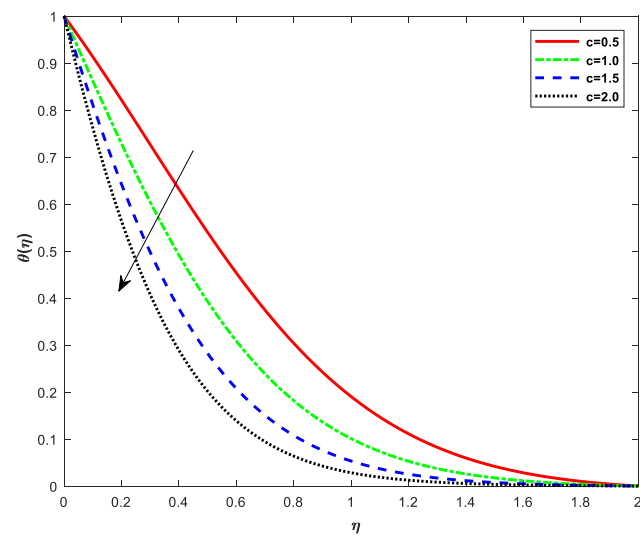


Fig. 3 – Temperature profiles for dissimilar values of suction parameter ( $c$ )

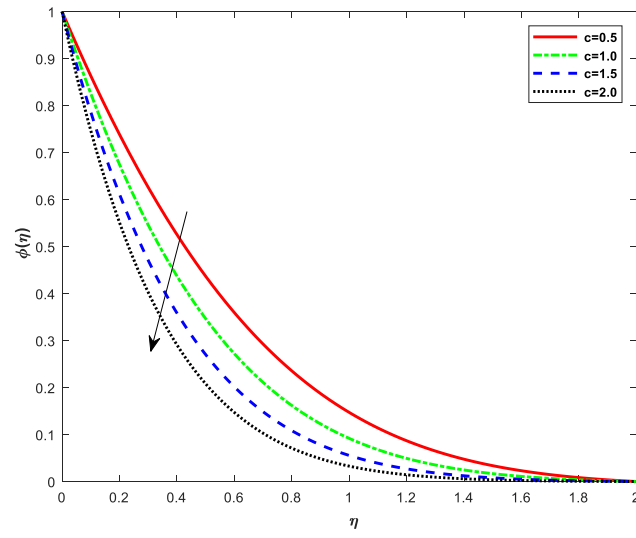


Fig. 4 – Concentration profiles for dissimilar values of suction parameter ( $c$ )

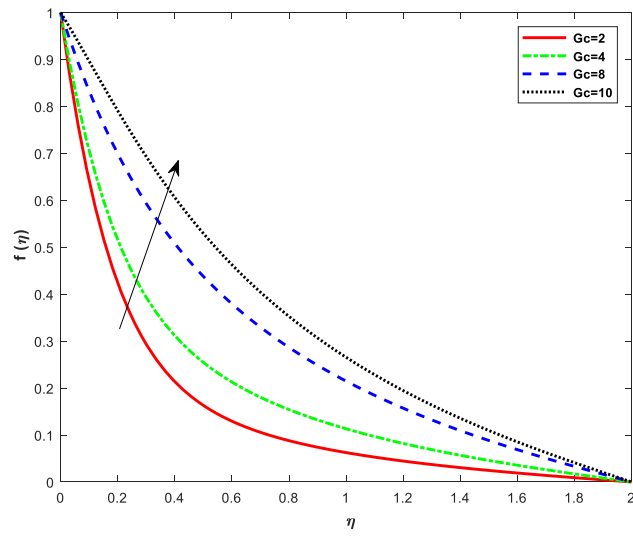


Fig. 5 – Velocity profiles for dissimilar values of solutal Grashof number ( $G_c$ )

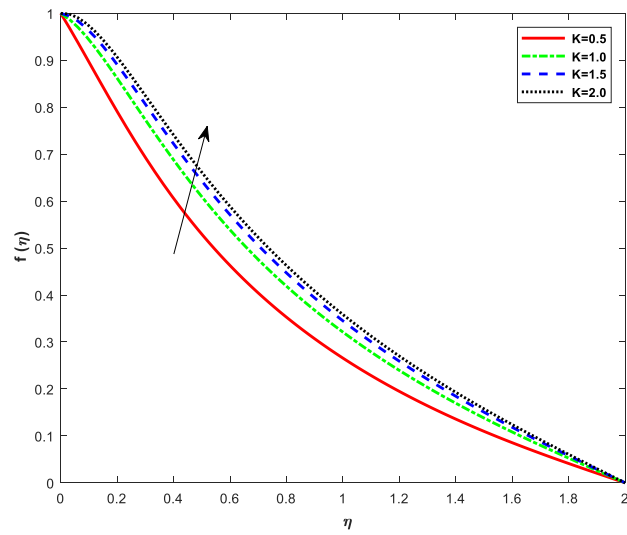


Fig. 6 – Velocity profiles for dissimilar values of Permeability parameter(K)

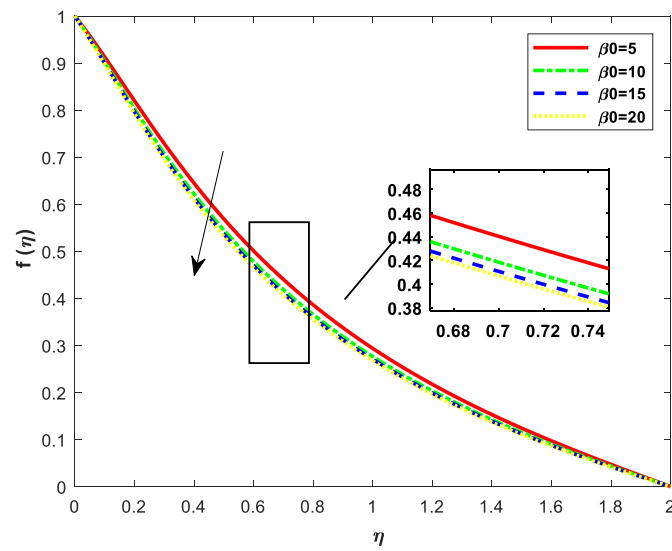


Fig. 7 – Velocity profiles for dissimilar values of Casson Parameter( $\beta_0$ )

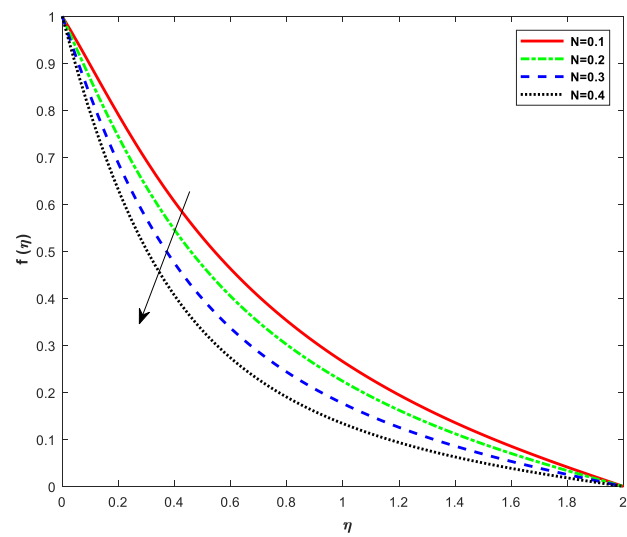


Fig. 8 – Velocity profiles for dissimilar values of temperature difference parameter N

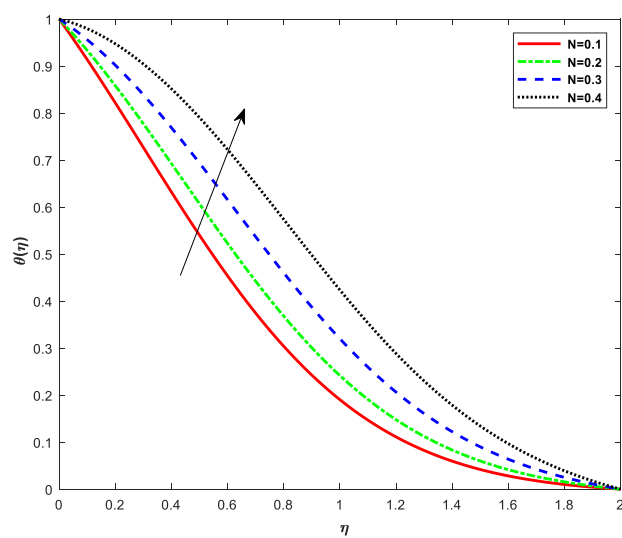


Fig. 9 – Velocity profiles for dissimilar values of temperature difference parameter N

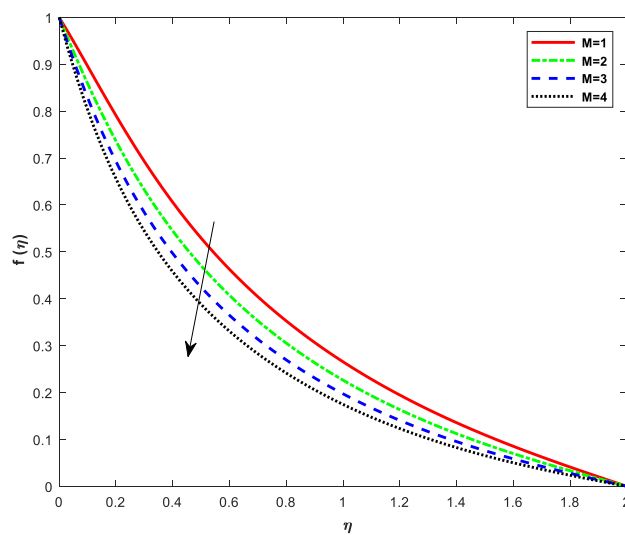


Fig. 10 – Velocity profiles for dissimilar values of magnetic field parameter(M)



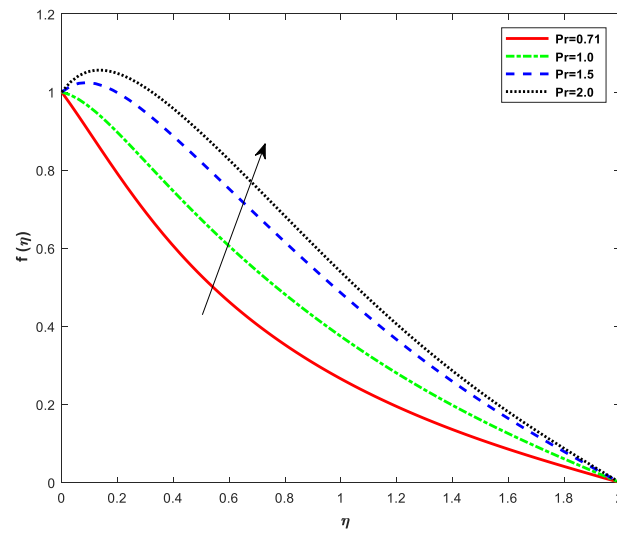


Fig. 11 – Velocity profiles for dissimilar values of Prandtl Number(Pr)

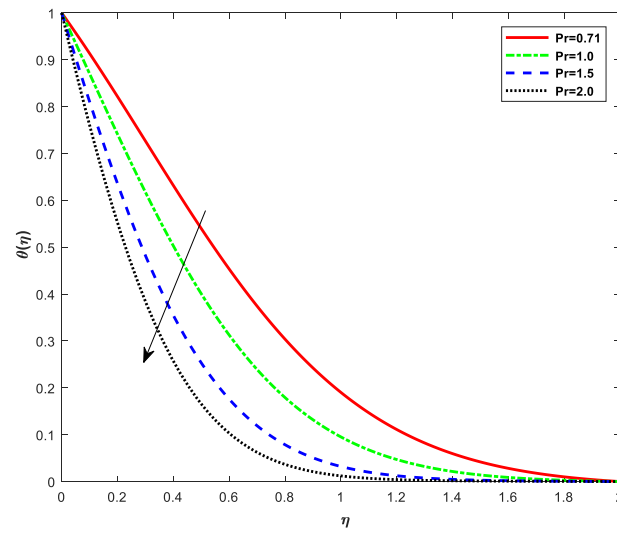


Fig. 12 – Temperature profiles for dissimilar values of Prandtl Number(Pr)

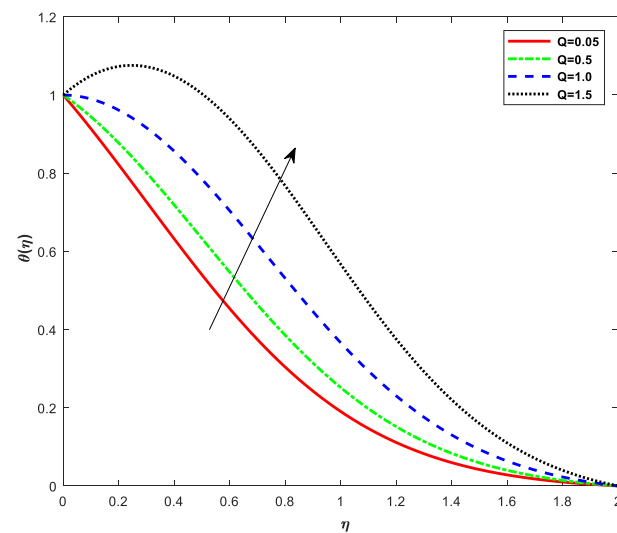


Fig. 13 – Temperature profiles for dissimilar values of  $Q$

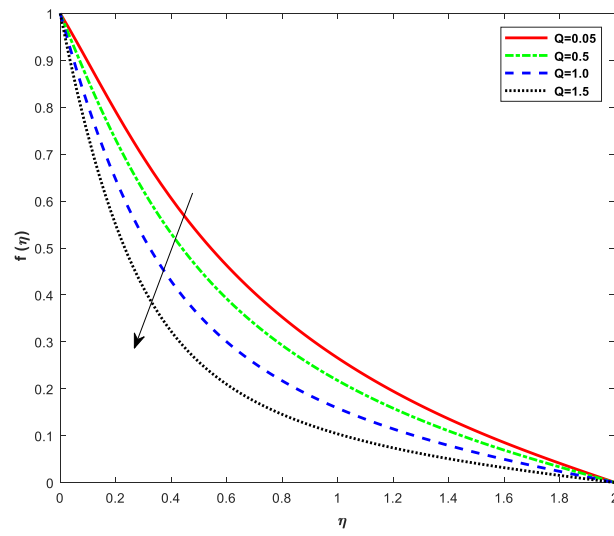


Fig. 14 – Velocity profiles for dissimilar values of  $Q$

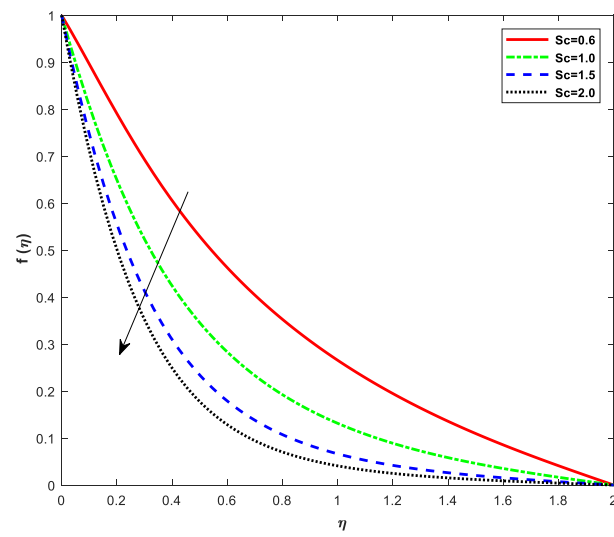


Fig. 15 – Velocity profiles for dissimilar values of Schmidt Number( $Sc$ )

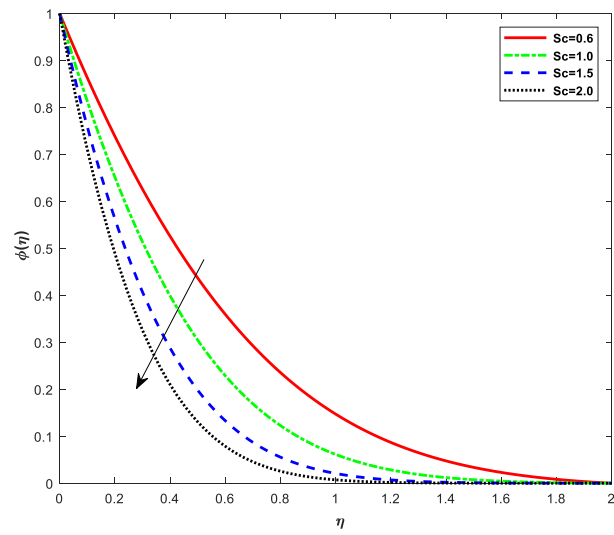


Fig. 16 – Concentration profiles for dissimilar values of Schmidt Number(Sc)

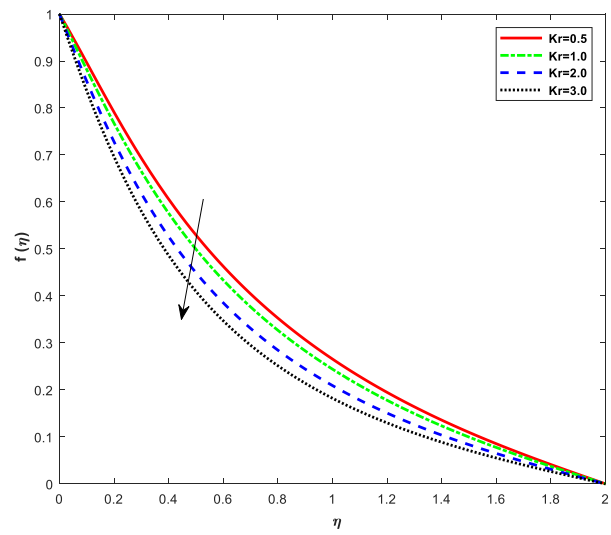


Fig. 17 – Velocity profiles for dissimilar values of Chemical reaction parameter(Kr)

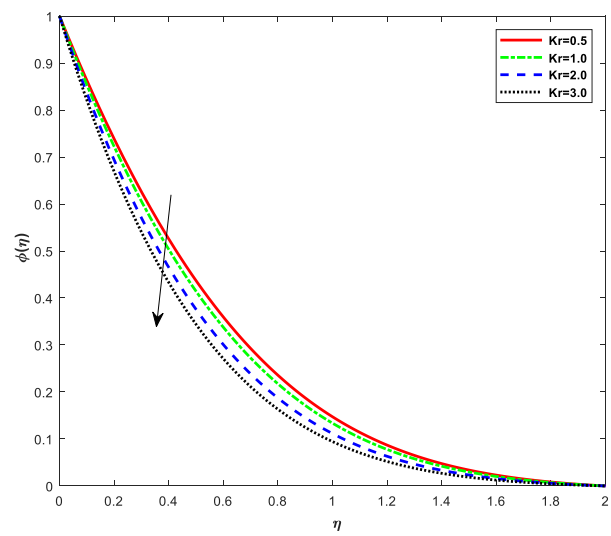


Fig. 18 – Concentration profiles for dissimilar values of Chemical reaction parameter( $K_r$ )

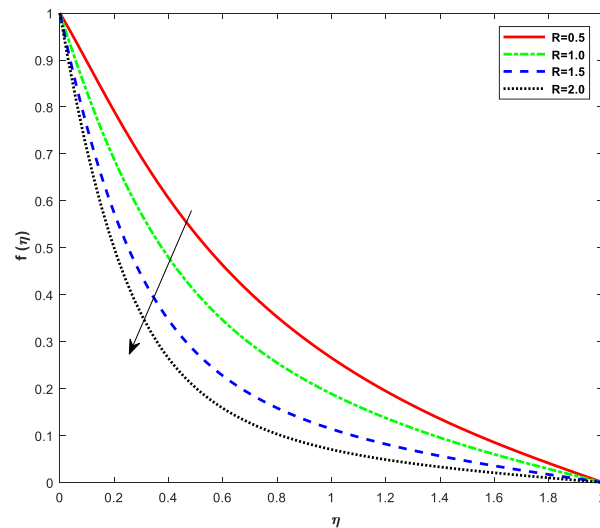


Fig. 19 – Velocity profiles for dissimilar values of thermal radiation parameter( $R$ )

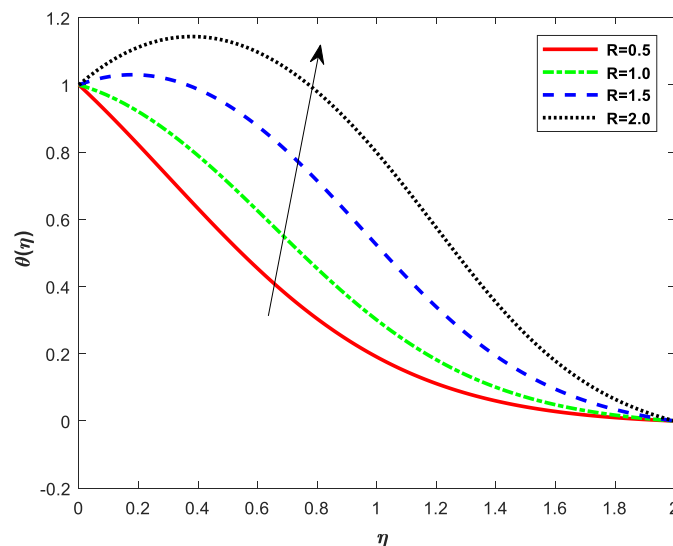


Fig. 20 – Temperature profiles for dissimilar values of thermal radiation parameter( $R$ )

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