A New Approach for finding better Initial Feasible Solution to Balanced or Unbalanced Transportation Problems

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**Abstract.** In this paper a method is proposed to find a better initial feasible solution to a balanced or unbalanced transportation problem. Unlike other methods, the entering non-basic variable in this method is chosen based on the quantities of supply or demand corresponding to that cell. The method is applied over several numerical examples studied in the literature and solution performance in terms of minimizing transportation cost and computational efficiency is compared. The solution obtained by the proposed method is found to be optimal in many numerical examples and near optimal in other cases in terms of taking lesser number of iterations to reach optimality. In case the transportation problem is balanced, the solution obtained by the proposed method is shown to be improved over the solution obtained by its competing methods, improved over the solution obtained from VAM for unbalanced transportation problems.

**Keywords:** Transportation Problem, Linear Programming (LP), Optimization Initial Basic Solution, Vogel’s Approximation Method, Unbalanced Transportation Problem.

1 Introduction

One important application of linear programming (LP) is in the area of physical distribution (transportation) of goods and services from several supply centers to many demand centers. The structure of transportation problem involves large number of shipping routes from several supply origins to several demand destinations. The objective is to determine the number of units of an item that should be shipped from an origin to a destination in order to satisfy the required quantity of goods or services at each destination center. This should be done within the limited quantity of goods or services available at each supply center with minimum transportation cost and/or time. Mathematically, it is easy to express a transportation problem in terms of an LP model which can be solved by using simplex method. Since LP model involves large number of variables and

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constraints, it generally takes longer time to solve by the conventional simplex method. Several

heuristic methods have been developed to find initial basic feasible solutions which can later be improved to become optimum solutions. Some methods like north-west corner (NWC) and least cost (LC) are computationally simple to obtain initial solutions however; it will take longer time to improve them to optimum solutions. Other methods like Vogel’s Approximation Method (VAM) are computationally more laborious initially but the initial solutions obtained by these methods will be optimal in many cases or it may take lesser number of iterations to reach optimum in other cases. Methods like stepping stone and MODI (modified distribution) have been developed for this purpose.

There are various transportation models and the simplest of them was first presented by Hitchcock (1941) which was later developed by Koopmans (1949) and Dantzig (1951). A transportation problem is called balanced if the total supply is equal to the total demand. Otherwise it is unbalanced. Several extensions of transportation models and methods have been developed several authors subsequently. Vogel’s Approximation Method (VAM) is widely used to solve Transportation Problems to find the optimal or near optimal allocation of commodities to various routes which minimizes the total transportation cost. Srinivasan and Thomson (1977) proposed Area Cost Operator Method or an improved version of it named Cell Cost Operator Method (CCOM) by solving the dual problem of the transportation problem and proved the convergence of the method. However in this approach it is assumed that the transportation problem is balanced which may not be the case always. The present paper is aimed to solve only the primal problem and therefore not discussed.

Shimshak et al (1981) pointed out that when the transportation problem is unbalanced VAM fails to provide efficient solutions and several alternatives can be developed. By using a modified heuristic method (SVAM) Shimshak et al (1981) illustrated with a numerical example and presented an improved solution over VAM. Goyal (1984) remarked that if the transportation problem is unbalanced VAM should be modified in calculating penalties to the allocating cells. With the help of some numerical examples Goyal (1984) illustrated his variant GVAM is improved in the sense of obtaining optimum solution. Balakrishnan (1990) further modified the methods SVAM and GVAM and demonstrated with a numerical example. Kirca and Satir (1990) describe a heuristic method called total opportunity cost method (TOM) wherein the allotment of quantity is made to the cell based on the total opportunity cost of that cell. Here, the sum of row-opportunity cost and column-opportunity cost gives the total opportunity cost. It is illustrated with 480 numerical examples that TOM outperforms VAM in case of unbalanced transportation problems. On the other hand VAM was superior to TOM to solve balanced transportation problems.

The comparison of TOM with VAM is further carried out by Goyal (1991) who suggested that the largest opportunity cost should be assigned to the dummy cells. The VAM is considered to be very efficient method to find an optimal solution or near optimal solution to the TP. However, in case of unbalanced TP VAM fails to give efficient solutions and many alternate methods have been proposed and available in the works of Goyal (1991), Simshak (1981) etc. Most of the methods studied are based on the costs and not on quantities and the optimal solution should depend not only on the costs but also on the quantities of shipment.

The main objective of this article is to propose a method which selects the non-basic cell based on both lowest cost and also quantity of shipment. Since the quantity of shipment will be the minimum of supply and demand corresponding to the cell, the method should take supplies and demands into consideration. An algorithm is developed for the proposed method and implemented in R software version 3.3.1 on Windows O.S.. The method is illustrated with several numerical examples discussed in the literature and the results are compared with its competitors in terms of number of computations. The proposed method is introduced in **Section 2** to solve both balanced and unbalanced transportation problems. Numerical illustrations are provided in **Section 3.**

2 Proposed Method

Let there be *m* sources ,each can supply units of a certain commodity to n destinations  which has the demand each.

Let be the transportation cost to be incurred in shipping one unit of the commodity from  source to  destination.

The decision variable is the quantity to be shipped from  source to destination so that the total transportation cost is the minimum. Pictorially the transportation problem is shown as follows

**Table 1.** A Typical transportation problem

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
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The transportation problem can be modeled as a linear programming problem (LPP) as below

Min 

Subject to







If the total supply is equal to the total demand



then the TP is called balanced otherwise it is unbalanced.

Algorithm

**Step 1**- Check whether the given problem is balanced. If not make it balanced.

**Step 2**- Reducing cost matrix to a matrix having at least one zero in each row or column. This can be done by subtracting minimum cost of each row from the elements of that row followed by subtracting minimum cost of each column from the elements of that column.

**Step 3-Selecting Row or Column:**

Find the minimum nonzero value among the supplies or demands and select the row or column corresponding to that supply or demand. This may lead to the following cases

1. If minimum nonzero value appears uniquely among the supplies (demands) then select the corresponding row (column)
2. If minimum nonzero value appears at more than one place among the supplies (demands) then select the row (column) for which the sum of row (column) costs is the maximum.
3. If minimum nonzero value appears at more than one place among both supplies and demands then select the row if its sum of row costs is the maximum or the column if its sum of column costs is the maximum.
4. If tie occurs further then break it arbitrarily.

**Step 4-Selecting the cell for allotment**

Once a row or column is selected in Step 3 then select the cell as follows

1. If a cell with zero cost occurs uniquely then select that cell.
2. If a cell with zero cost occurs at more than one place then select the cell for which the sum of costs in that row or column is larger.
3. If a cell with zero cost is not available (or already allotted some quantity), then choose the cell with next least cost. If there is further tie in the least costs then break it arbitrarily.

**Step 5-Allotment to cell:**

Allot some quantity of supply or demand whichever is the minimum to that cell. Block the row whose supply is exhausted or column whose demand is satisfied from future allotments.

**Step 6-Stopping Condition:**

If there are any non-zero supplies or demands which remain un-allotted then go to step 3. Otherwise, STOP.

The above algorithm can be applied on both balanced and unbalanced TPs. In the next section we illustrate the usage of the algorithm on some numerical examples discussed in the literature.

3 Numerical Illustrations

### Example-1 Kirca (1990)

### 

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | D2 | D3 | D4 | Supply |
| S1 | 15 | 27 | 13 | 19 | 40 |
| S2 | 18 | 21 | 24 | 14 | 40 |
| S3 | 21 | 15 | 16 | 17 | 20 |
| Demand | 30 | 20 | 30 | 20 | 100  100 |

**Step 1:**

Find the minimum cost among the row costs and subtract it from the cost elements of that row

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | D2 | D3 | D4 | Supply |
| S1 | 2 | 14 | 0 | 6 | 40 |
| S2 | 4 | 7 | 10 | 0 | 40 |
| S3 | 6 | 0 | 1 | 2 | 20 |
| Demand | 30 | 20 | 30 | 20 | 100  100 |

**Step 2:**

Find the minimum cost among the column costs and subtract it from the cost elements of that column

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | D2 | D3 | D4 | Supply |
| S1 | 0 | 14 | 0 | 6 | 40 |
| S2 | 2 | 7 | 10 | 0 | 40 |
| S3 | 4 | 0 | 1 | 2 | **20** |
| Demand | 30 | **20** | 30 | **20** | 100  100 |

**Step 3**:

The smallest value among the supplies and demands is 20 corresponding to the destinations D2, D4 and source S3. Out of these D2 column is chosen for allotment as the sum of the costs in that column is maximum (14+7+0=21). Among the costs of D2 column the smallest cost is 0 corresponding to the cell (3,2) against which allotment is to be done.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | | D2 | | D3 | | D4 | | Supply |
| S1 | 0 |  | 14 |  | 0 |  | 6 |  | 40 |
|  |  |  |  |  |  |  |  |
| S2 | 2 |  | 7 |  | 10 |  | 0 |  | 40 |
|  |  |  |  |  |  |  |  |
| S3 | 4 |  | 0 |  | 1 |  | 2 |  | 0 |
|  |  |  | **20** |  |  |  |  |
| Demand | 30 | | 0 | | 30 | | **20** | | 80  80 |

**Step 4**:

The smallest value among the supplies and demands is 20 corresponding to the destination D4 and therefore D4 column is chosen for allotment. Among the costs of D4 column the smallest cost is 0 corresponding to the cell (2,4) against which allotment is to be done

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | | D2 | | D3 | | D4 | | Supply |
| S1 | 0 |  | 14 |  | 0 |  | 6 |  | 40 |
|  |  |  |  |  |  |  |  |
| S2 | 2 |  | 7 |  | 10 |  | 0 |  | **20** |
|  |  |  |  |  |  |  | **20** |
| S3 | 4 |  | 0 |  | 1 |  | 2 |  | 0 |
|  |  |  | **20** |  |  |  |  |
| Demand | 30 | | 0 | | 30 | | 0 | | 60  60 |

**Step 5**:

The smallest value among the supplies and demands is 20 corresponding to the source S2 and therefore S2 row is chosen for allotment. Among the costs of S2 row the smallest cost is 0 corresponding to the cell (2,4) against which allotment has already been done in Step-4 and therefore the next least cost 2 is considered which corresponds to the cell (2,1)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | | D2 | | D3 | | D4 | | Supply |
| S1 | 0 |  | 14 |  | 0 |  | 6 |  | 40 |
|  |  |  |  |  |  |  |  |
| S2 | 2 |  | 7 |  | 10 |  | 0 |  | 0 |
|  | **20** |  |  |  |  |  | **20** |
| S3 | 4 |  | 0 |  | 1 |  | 2 |  | 0 |
|  |  |  | **20** |  |  |  |  |
| Demand | **10** | | 0 | | 30 | | 0 | | 40  40 |

**Step 6**:

The smallest value among the supplies and demands is 10 corresponding to the destination D1 and therefore D1 column is chosen for allotment. Among the costs of D1 column the smallest cost is 0 corresponding to the cell (1,1) against which allotment is to be done

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | | D2 | | D3 | | D4 | | Supply |
| S1 | 0 |  | 14 |  | 0 |  | 6 |  | **30** |
|  | **10** |  |  |  |  |  |  |
| S2 | 2 |  | 7 |  | 10 |  | 0 |  | 0 |
|  | **20** |  |  |  |  |  | **20** |
| S3 | 4 |  | 0 |  | 1 |  | 2 |  | 0 |
|  |  |  | **20** |  |  |  |  |
| Demand | 0 | | 0 | | 30 | | 0 | | 30  30 |

**Step 7**:

The smallest value among the supplies and demands is 30 corresponding to the destination D3 and therefore D3 column is chosen for allotment. Among the costs of D3 column the smallest cost is 0 corresponding to the cell (1,3) against which allotment is to be done

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Destinations  Sources | D1 | | D2 | | D3 | | D4 | | Supply |
| S1 | 0 |  | 14 |  | 0 |  | 6 |  | 0 |
|  | **10** |  |  |  | **30** |  |  |
| S2 | 2 |  | 7 |  | 10 |  | 0 |  | 0 |
|  | **20** |  |  |  |  |  | **20** |
| S3 | 4 |  | 0 |  | 1 |  | 2 |  | 0 |
|  |  |  | **20** |  |  |  |  |
| Demand | 0 | | 0 | | 0 | | 0 | | 0  0 |

Since all the supply exhausted and demands fulfilled the algorithm is stopped and the solution is x11=10, x13=30, x21=20, x24=20, x32=20. Multiplying each variable with the corresponding cell cost and adding gives the cost of transportation as 1480.

**Remark:** By computing the dual prices of non-basic variables, it is easy to verify that the above solution is optimum. It may be remarked here that the above solution coincided with optimum solution obtained by Kirca and Satir (1990) by applying total opportunity cost method (TOM)**.**

Kirca(1990) solved a balanced transportation problem a={40,40,20}, b={30,20,30,20},c={{15,27,13,19},{18,21,24,14},{21,15,16,17}} by applying total opportunity method and got 1480 as the transportation cost. By applying the proposed algorithm we get the solution as x11=10, x13=30, x21=20, x24=20 and x32=20 which is optimum.

Srinivasan and Thompson (1977) solved a balanced transportation problem a={80,90,55}, b={70,60,35,60}, c={{3,6,3,4}, {6,5,11,15}, {1,3,10,5}} by applying cell cost operator method and got 880 as the transportation cost. By applying the proposed algorithm we get same solution as x13=35, x14=45, x21=30, x22=60, x31=40 and x34=15.

Goyal (1984) solved an unbalanced transportation problem a={50,50,50}, b={30,40,55}, c={{6,10,14}, {12,19,21}, {15,14,17}} by applying modified Vogel’s approximation method and got 1665 as the transportation cost. By applying the proposed algorithm we get the optimum solution x11=5, x12=40, x13=5, x21=25, x24=25 and x33=33 whose optimum transportation cost is 1650. By computing the costs of the non-basic variables it is easy to verify that the above solution is optimum. Further, the solution coincides with the optimum solution obtained by the modified VAM proposed by Balakrishnan (1990). Goyal (1984) used a variant of VAM and solved the above problem and obtained solution which gives minimum cost 1665. Shimshak *et al* (1981) solved the above example and got the minimum cost as 1695 which by using VAM the minimum cost will be 1745. Ramakrishna (1988) modified the method used by Goyal (1984) and got the optimum solution.

References

1. Balakrishnan, N.,: Modified Vogel’s approximation method for the unbalanced transportation problem. *Appl. Math. Lett.* 3 (2) 9-11, (1990).
2. Charnes, A., Cooper, W.W., and Henderson, A.,: *An Introduction to Linear Programming*. Wiley, New York. (1953).
3. Dantzig, G. B.,: Linear Programming and Extensions. Princeton University Press, Princeton, N J., (1963).
4. Goyal, S. K.,: Improving VAM for unbalanced transportation problems. *Journal of Operational Research Society*, 35(12), 1113-1114, (1984).
5. Goyal, S. K.,: A note on a heuristic for obtaining an initial solution for the transportation problem. *Journal of Operational Research Society,* 42(9)*,* 819-821, (1991).
6. Kirca, O., and Satir, A.,: A Heuristic for Obtaining an Initial Solution for the Transportation problem. *Journal of Operational Research Society*, Vol. 41, No. 9, pp. 865-871, (1990).
7. Ramakrishna, G. S.,: An improvement to Goyal’s modified VAM for the unbalanced transportation problem. *Journal of Operational Research Society*, 39(6), 609-610, (1988).
8. Reinfield, N. V., and Vogel, W. R.,: *Mathematical Programming*, pp. 59-70. Prentice – Hall, Englewood Cliffs, N.J., (1958).
9. Shafaat, A., and Goyal, S. K.,: Resolution of degeneracy in transportation problems. *Journal of Operational Research Society*, 39(4), 411-413., (1988).
10. Sharma, J. K.,: *Operations Research-Theory and applications*, Macmillan India (LTD), New Delhi, (2005).
11. Shimshak, D. G., Kaslik, J. A., and Barclay, T. D.,: A modification of Vogel’s approximation method through the use of heuristic, *INFOR*,19, 259-263, (1981).
12. Srinivasan, V., and Thompson, G. L.,: Cost Operator Algorithms for the Transportation Problem. Mathematical Programming 12, 372-391, (1974).