Modified model for LTNE effects on nanofluid convection

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Thermal instability of a nanofluid layer is studied under the effects of local thermal non-equilibrium (LTNE). The volume fraction of nanoparticles is taken to be invariable at the primary state to get the analytical expression of thermal Rayleigh number which is sensitive to both the properties; density as well as conductivity of nanoparticles. Three LTNE parameters; modified thermal diffusivity ratio, modified thermal capacity ratio and Nield number are introduced in the set of conservation equations which lead to significant impact on the onset of convection. The mode of instability is established as stationary. The thermal lagging within the fluid and particle results in quickening the convection in the layer. The critical wave number gets influenced appreciably with the presence of nanoparticles through LTNE parameters.

***Keywords*:** Thermal convection, Nanofluid, Local thermal non-equilibrium, Modified model

1. **Introduction**

Motivated by the extraordinary heat transfer enhancements in nanofluids, Buongiorno[1] developed equations based on conservation laws for nanofluids to identify the governing mechanisms. Tzou[2,3] initiated the mathematical investigations on the instability of nanofluids using the model given by Buongiorno[1] and found that addition of nanoparticles make the fluid more unstable. Thermal instability of a nanofluid layer was investigated analytically for different boundaries by Nield and Kuznetsov[4]. Further, Gupta et al.[5] and Agarwal et al.[6] carried forward their work by extending the nanofluid convection problems under magnetic field and rotation, respectively. To study the binary impact of heat and solute on convection in a nanofluid layer was conducted by Nield and Kuznetsov[7] and Yadav et al.[8]. The stabilizing impact of magnetic field and rotation on binary convection in nanofluids was established by Gupta et al.[9] and Sharma et al.[10], respectively. All these studies consider the thermal instability of nanofluids using local thermal equilibrium model (LTE) in which fluid and particle phase remain in thermal equilibrium. The possibility of thermal lagging between the fluid and particle phase results in reported breakthrough in heat transfer enhancement of nanofluids was proposed by Vadasz[11] and introduced a local thermal non-equilibrium (LTNE) model to study the convection problems. To explore his findings further, Kuznetsov and Nield [12,13] analyzed the local thermal non-equilibrium model for convective heat transfer of nanofluids in a porous and non-porous medium, respectively. Bhadauria and Agarwal [14,15] claimed that the LTNE effects on Rayleigh Bénard convection in a nanofluid layer are significant except for typical dilute nanofluids. The influence of double-diffusion and LTNE on the onset of convection in porous medium was considered by Nield and Kuznetsov [16]. They found that the system with LTNE exhibits less stability than LTE model.

All the above mentioned studies were conducted with the assumption that nanoparticle volume fraction on the boundaries can be controlled, (i.e. top heavy/bottom heavy) which is quite difficult to achieve physically. In order to meet realistic situations, Nield and Kuznetsov[17] revised the model with the assumption that nanoparticle’s flux vanishes across the boundaries. Using the revised boundary conditions on nanoparticle volume fraction, Chand and Rana[18] revisited the thermal convection in a nanofluid layer saturating porous medium using Darcy-Brinkman model. Agarwal[19] and Chand and Rana[20] considered the effects of Coriolis and Lorentz forces, respectively, on the onset of nanofluid convection in a porous medium under revised model. Realizing the fact that original and revised models mentioned so far are not sensitive to the conductivity of nanoparticles; Sharma et al.[21] modified the model by assuming initial constant nanoparticle volume fraction in the fluid layer. It was established that density of nanoparticles hastens the onset of convection in the fluid whereas increase in conductivity delays the same.

The present work is based on interesting effects of local thermal non-equilibrium (which are assumed to be responsible for significant heat transfer enhancement in nanofluids) on thermal convection in nanofluids using a model which is modified in the light of more realistic initial condition of constant nanoparticle volume fraction. The importance and novelty of the method lies in the fact that it uses both the physical properties of the nanoparticles i.e. density as well as conductivity for consideration of nanoparticle effects as compared to earlier models which were insensitive to the effects of conductivity of the nanoparticles. The analysis considers conservation equations for the system which are non-dimensionalized leading to additional LTNE parameters: modified thermal diffusivity ratio, modified thermal capacity ratio and Nield number. The expression for thermal Rayleigh number is obtained using normal mode technique for stationary convection. The possibility of thermal lagging between fluid phase and particle phase destabilizes the system significantly establishing itself to be the main feature in unexpected heat transfer enhancement of nanofluids.

1. **Problem formulation and conservation equations**

Let us consider a fluid layer of thickness d which is soluted and heated from below. Let  be the temperatures at bottom and top boundaries of the layer, respectively. The conservation equations (refer: Nield and Kuznetsov [13]) are

 (1)

 (2)

 (3)

 (4)

 (5)

where the fluid density ρ is given by

 (6)

Here,  respectively, are the fluid velocity, the particle volume fraction, the particle density, the Brownian diffusion coefficient, the thermophoretic diffusion coefficient, the viscosity of the fluid, the heat capacity of fluid, the heat capacity of nanoparticles and the thermal conductivity of medium. In addition to the above physical variables some other variables get introduced in our system of equations due to thermal non-equilibrium model. These additional variables denote, respectively, the effective thermal conductivity of the fluid and particle phase, denote the temperature of the fluid and particle phase andis the interphase heat transfer coefficient between the fluid/particle phases.

Let us write non-dimensional variables as

, (7)

where  and  is the initial volume fraction of nanoparticles which is so small that it is taken to be constant.

After applying Eqs. (7) to the system of Eqs. (1)-(6), we get equations in non-dimensional form (after dropping the asterisks) as

 (8)  (9)  (10) (11) (12)

where the non-dimensional numbers are redefined as

; the modified diffusivity ratio,

; the interphase heat transfer parameter or Nield number,

; the modified thermal capacity ratio,

; the modified thermal diffusivity ratio,

; the thermal Rayleigh number,

; the concentration Rayleigh number,

the Prandlt number,

the modified particle density increment,

the nanofluid Lewis number,

the basic density Rayleigh number. (13)

1. **Basic solution and perturbation equations**

Initially, the system is at rest and nanoparticle volume fraction is constant. Let us write

 (14)

Thus, Eqs. (8)-(12) take the form

 (15)

 (16)

 (17)  (18)

We get the initial solution of Eqs. (15)-(18) as

 (19)

Let us apply small perturbations on the initial solution and write

 (20)

On applying these perturbations and neglecting the product of perturbed quantities, Eqs. (8)-(12) become

 (21)

 (22)

 (23)

 (24)

 (25)

Using the identity  on Eq. (22) together (21), we get (26)

where is the Laplacian operator.

1. **Normal mode technique and stability analysis**

Let us write

= (27)

where *lx* and *my* represent the wave numbers of disturbance along x and y directions, respectively,

 is the resultant wave number and  is the growth rate parameter. Following the technique of normal modes (superposition of basic modes), the disturbance equations take the form

 (28)  (29)  (30)

 (31)

Trial functions in one term Galerkin weighted residual method which satisfy boundary conditions corresponding to both free surfaces; at z=0 and z=1 are taken as . By putting the solution in equations (28)-(31), the process of orthogonality is followed and the constants  are eliminated to get the eigenvalue equation .

1. **Analytical results and discussions**

**5.1 Stationary convection**

Let growth rate for stationary convection; the eigenvalue equation gives

 (32)

Note that coefficient of Nield numberin numerator is less than denominator as values of  meaning thereby that value of  is decreased by the usage of LTNE model. Also, critical Rayleigh number and critical wave number are strongly influenced by the LTNE parameter. Under the assumption of local thermal equilibrium, put  in Eq.(33) to get

 (33)

which coincides with the corresponding expression given by Sharma et.al.[21].

**5.2 Oscillatory Convection**

For oscillatory motions:  Complex expressions are obtained from Eq.(36) which cannot be studied analytically. Let us make the approximations  which give real and imaginary parts as:

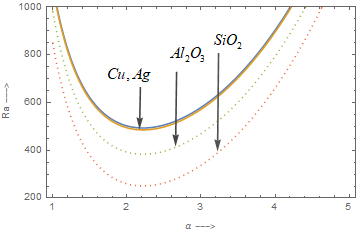
 (34) (35)

Substituting Nield number  in Eqs. (34) and (35), we get equations for oscillatory motions for local thermal equilibrium model which coincides with the corresponding equations given by Sharma et.al.[21] and establish the absence of oscillatory motions.

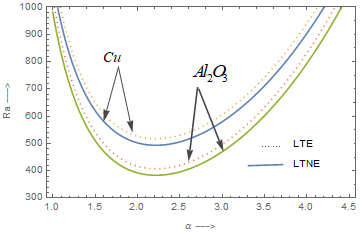
1. **Numerical Results and Discussions**

**6.1 Stationary Convection**

To show the efficacy of modified model and importance of LTNE effects, numerical calculations and graphical interpretations are accomplished for Eq. (32) with the help of Mathematica software. Figures 1 and 2 show the stability curves for fixed values of the nanofluid parameters as: copper ; silver;  alumina; ; silicon oxide:  and all other parameters as: . Figure 1 authenticates the need of modified model using different metallic and metallic oxide nanoparticles on the stability of nanofluid under LTNE model. The instability pattern followed by nanoparticles is: copper silver > alumina >> silica which could not be seen using previous (original and revised) models establishing the efficacy of the present model. The stability of copper nanoparticles is almost same as that of silver due to their comparable conductivities in spite of having different densities meaning thereby that density do not influence the stability to a greater extent. The important effect of local thermal non-equilibrium on different nanoparticles (alumina and copper) is depicted in Fig.2. The destabilizing influence of LTNE is observed which conclude that the thermal lagging effects hasten the onset of convection and results in unexpectedly large conductivity of nanofluids. This validates the result that addition of nanoparticles substantially increase the conductivity of fluids is due to LTNE, as anticipated by Vadasz[11]. Clearly, it can be observed that local thermal effects due to temperature difference between particle phase and fluid phase make the system more unstable at the same rate for metallic (copper) as well as non-metallic (alumina) nanoparticles.

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**Figure 1:** Comparison of Metallic and Non-Metallic nanoparticles

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**Figure 2**

**Figure 2:** Comparison of LTE and LTNE Models.

1. **Conclusions**

The local thermal non-equilibrium effects are studied on convection in a horizontal nanofluid layer. The model used to analyze the problem is modified effectively in the light of fact that density and conductivity of nanoparticles are essential properties to initiate convection in the layer. Under this model, volume fraction of nanoparticles in the fluid is assumed to be so small that it is treated as constant at the primary state. Due to possibility of thermal lagging within the fluid layer; three additional parameters: Nield number, modified thermal diffusivity ratio and modified thermal capacity ratio are introduced in the set of conservation equations. The impact of different nanoparticles on the stability is established as: copper-water  silver-water > alumina-water >> silica-water. The effect of nanoparticles and solute is to hasten the convection in fluid layer significantly. The destabilizing influence of LTNE is observed resulting in unexpected large conductivity of nanofluids. Absence of oscillatory motions is established numerically for alumina and copper nanoparticles.

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