**THEORETICAL ANALYSIS OF VISCOELASTIC NON-NEWTONIAN FLUID IN TURBULENT FLOW THROUGH A PIPE**

**USING EDDY DIFFUSIVITY MODEL**

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The problem of momentum and heat transfer in turbulent pipe flow of a viscoelastic non-Newtonian liquid is investigated theoretically by solving the momentum and energy balance equations. The expressions for momentum and thermal eddy diffusivities available in literature are taken and are used after small modifications in the balance equations for fully developed flow neglecting entrance effects and Weissenberg (rod climbing) effects. From the numerical results for velocity and temperature profiles, the friction and heat transfer coefficients are calculated theoretically for apparent Reynolds numbers from 10000 to 50000, and a wide range of concentrations of viscoeleastic liquids. The theoretical results are validated by comparing with empirical equations available in literature. An outcome of the present theoretical study is that the expressions available in literature for the momentum and thermal diffusivities are modified so as to fit with the existing experimental data and empirical equations existing in literature for wide range of concentrations of viscoelastic liquids. Further equations obtained for friction and heat transfer coefficients in the explicit form.

***Keywords*:** Viscoelastic fluids, Turbulent flow, Eddy diffusivity, heat transfer coefficients.

1. **Introduction**

The viscoelatic liquids became prominent due to the reason that they possess better drag reduction properties than purely viscous non-Newtonian liquids and Newtonian liquids. Some specific properties of these fluids placed them in lime light of research area of non-Newtonian fluids, such as rod climbing, die swell, recoil and tubeless siphon. The applications of these fluids are found in biological, pharmalogical and food industries. They are also used in the reduction of friction drag during flow. Poly acryl amide (PAM) and polyethylene oxide (PEO) are non-Newtonian liquids possessing elastic property and hence are called viscoelastic liquids.

Tom (1948) eventually identified the drag reducing phenomenon of these polymeric fluids and known to be Tom’s phenomenon. The good number experimental works were carried out by various researchers on viscoelastic fluids both on momentum and heat transfer of turbulent pipe flows.

Ng et al. (1979) developed an empirical equation using their experimental data in terms of Re׳, neglecting entrance effects and time. The experimental data of Kwack et al. (1981) are correlated as a function of apparent Reynolds number by Cho and Hartnett (1981). Virk et al. (1970) proposed an empirical equation from their experimental data with polyethylene oxide solutions in which entrance effects (x/d>100) are negligible and beyond the critical Weinsenberg number for turbulent flows. Several researchers (Pruitt et al. (1966), Wells (1968), Poreh and Paz (1968)) tackled the problem of viscoelastic liquids using methods such as Reynolds analogy and Reichardt method and Karman-Martinelli analysis. Further experimental works on heat transfer also found in literature. Kwack et al. (1982) conducted experimental studies to calculate heat transfer coefficients of viscoelastic liquids in turbulent pipe flows on polyacrylamide solutions in once through pipe test section. They measured the dimensionless heat transfer factor, jH factor for different concentrations ranging from 10 to 1000 ppm and at thermally fully developed region (x/d>430). A good amount of research is made in literature on the eddy momentum and thermal diffusivities appear in the momentum and energy balance equations. Khabakhpasheva and Perepelitsa (1973) and Mizushina and Usui (1977) were the first to establish the equation for friction and heat transfer coefficients using the eddy diffusivity method. Khabakhpasheva and Perepelitsa (1973) developed equations for momentum and thermal eddy diffusivities from their experimental data. Mizushina and Usui (1977) established correlations in terms of eddy diffusivity using van Driest damping factor model for momentum and heat transfer separately. Though different models are in existence the continuous eddy diffusivity model of Deissler is widely used.

Kale (1977), and Cho and Hartnett (1980) neglected entrance effects and Weissenberg’s rod climbing effect, and developed eddy diffusivity equations for viscoelastic liquids based on the eddy diffusivity equations proposed by Diessler and van Driest for Newtonian liquids.

1. **Physical Model and Formulation of the Problem**

The physical model considers a viscoelastic non-Newtonian liquid in turbulent flow through a pipe. The entrance effects and rod climbing (or Weissenberg) effect are neglected and the viscoelastic properties are assumed to be independent of time. The problem is governed by the momentum and energy balance equations.

1. **1. Expressions for Eddy Diffusivities**

The following eddy diffusion equations are chosen in the present analysis by making slight modifications in the equations existing in literature.

**Momentum eddy diffusivity** (εm):

(4.1)

where

**Thermal eddy diffusivity** ():

(4.2)

y+ and u+ are the normalized distance from the wall and velocity defined as y+ = y u\*/νa and u+ = u/u\*. The shear velocity, u\* is defined as .

1. **2. Governing Equations**

### The governing equations for momentum and energy are solved for the present problem using boundary conditions and yields

### Momentum balance equation

(4.3)

### Energy balance equation

(4.4)

K and n are flow consistency and flow behaviour indexes respectively. The liquid is heated while flowing in the pipe and pipe surface temperature is maintained constant. Eqs. (4.3) and (4.4) are integrated from r=r to r=R.

The following equations in normalised form are obtained from the momentum and energy balance equations written above

(4.5)

(4.6)

where and , ;

The position coordinate y is introduced in above equations y is measured from the wall, and hence y=R–r. Further the same y presented in dimensionless form as η = y/R.

The apparent Reynolds number Rea is computed using the equation shown below.

(4.7)

The normalized mean velocity U+ is calculated by the following equation.

(4.8)

where (4.9)

Eqs. (4.5) and (4.6) are solved numerically to give the distribution of the velocity and temperature from the wall to the centre line of the pipe.

From the numerical results, the mean velocity of the liquid, U and normalised bulk temperature, are calculated from the equations given below.

(4.10)

(4.11)

here

(4.12)

The friction and heat transfer coefficients are calculated from the following equations.

**Friction coefficient, f:**

(4.13)

## Nusselt number, Nu:

(4.14)

1. **Results and Discussions**

Results are obtained for viscoelastic fluids PAM and PEO. The rheological properties of these fluids are taken from literature (Rupesh Bhatia (2011)). However results of PAM solutions only considered for presentation in this paper. The theoretical results are compared with the experimental correlations of Ng et al. (1979). The velocity and temperature profiles are presented in Figs. (4.1) and (4.2) for Rea=10000 and Rea=50000 which represent the lowest and the highest values of apparent Reynolds numbers considered in the analysis.



Fig.1. Normalized velocity profiles for PAM



Fig.2. Normalized wall temperature profiles for PAM

The results shows that normalized velocity and temperature profiles are flat for the higher apparent Reynolds numbers than the lower values and this trend begins at normalized distance (y+/R+)=0.1, measured from the pipe wall.

The friction coefficients and Nusselt numbers calculated from theoretical results are shown in Figs. 3 and 4. The theoretical results of the present analysis agree satisfactorily with the experimental data of Ng et al. [5]. The Blasius equation for friction coefficient for Newtonian liquids is also shown in Fig.3. An important outcome of the present investigation is the drag reduction property of the viscoelastic liquids, which is evident through their low value of friction coefficient compared to that of water as shown in Fig. 3.

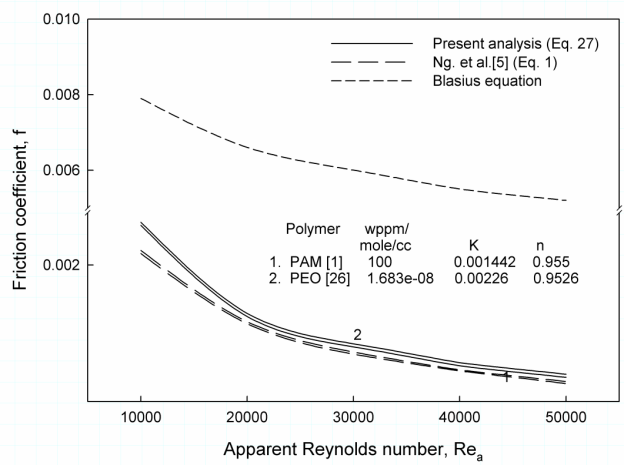


Fig.3. Friction coefficients of PAM, PEO and Newtonian fluid

The Nusselt numbers obtained are compared with Kwack et al., [] shown in Fig. 4. An increment in Nusselt number observed with increase in concentration i.e., n. At higher apparent Reynolds number theoretical equation could predict Nusselt numbers reasonably good.

Upto 65% drag reduction is obtained with viscoelastic liquids (PAM), where as a drag reduction of 35% is obtained with pseudoplastic liquids (CMC).

## Regression equations

For the purpose of using in design, the theoretical results are translated into explicit equations for friction coefficient and Nusselt number making use of non-linear regression analysis. These equations are given below.

Std. Devn = ±2.5%

Std. Devn = ±3.0%

The above equations are valid for the following range of system parameters:

10000< Rea <50000; 0.49 < n <0.96; 7 <Pra<553



Fig.4. Comparison of Nusselt numbers for PAM



Fig.5. Comparison of % drag reduction of viscous and viscoelastic fluids with Newtonian fluid

## Conclusions

1. A theoretical analysis is presented for obtaining the velocity and temperature profiles in turbulent flow of a viscoelastic non-Newtonian fluid in a circular pipe.

2. The theoretical results, after validation with the experimental data, provide eddy momentum and thermal diffusivities for viscoelastic liquids.

3. The results indicate that the friction coefficients are by far low (about 70%) in viscoelastic non-Newtonian liquids compared to those in Newtonian liquids that is agree with literature..

4. The analytical method can be used to predict the friction and heat transfer coefficients.

5. Equations in explicit form are provided for friction coefficients and Nusselt numbers using non-linear regression analysis.

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