**Radiation and Chemical reaction effects on MHD Casson Fluid Flow Past a Semi-infinite Vertical Moving Porous Plate**

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**Abstract:**

A speculative investigation has been presented to explore the significant features of MHD convective flow of a Casson fluid past a semi-infinite moving vertical porous plate with heat source/sink are included in the flow configuration. The governing partial differential equations are remodeled into ordinary differential equations using appropriate non-dimensional variables. The ensuing differential equations are solved analytically using two term perturbation technique method. The result of flow heat and mass transfer analysis on the velocity, temperature and concentration profiles are given graphically. The numerical values of the physical parameters like Skin friction, Nusselt number, and Sherwood number are shown in tabular form, results shows that Casson parameter enhances the velocity, temperature and concentration fields are decreases for increasing the values radiation and chemical reaction.

**Keywords:** Casson parameter, MHD, Heat source/sink, Heat and mass transfer.

1. **INTRODUCTION**

The study of non-Newtonian Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rates of shear, a yield stress below which no flows occurs and a zero viscosity at an infinite rate of shear. If a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, where as if a shear stress greater than yield stress is applied and it starts to move. Few examples of Casson fluids are jelly, tomato sauce, honey, concentrated fruit juice, blood etc. Casson model is sometimes stated to fit rheological data better than general viscoelastic model for many materials.

Shehzad et al.[1] analyzed effects of mass transfer on MHD flow of Casson fluid with chemical reaction. Vajravelu et al. [2] discussed diffusion of chemically reactive species in Casson fluid flow over an unsteady permeable stretching surface. [Abid et al.[3]](https://www.ncbi.nlm.nih.gov/pubmed/?term=Hussanan%20A%5BAuthor%5D&cauthor=true&cauthor_uid=25302782) presented unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. Sekhar et al.[4] studied unsteady MHD convective heat and mass transfer of a Casson fluid past a semi-infinite vertical permeable moving plate with heat source/sink. Animasaun [5] studied effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of Non-Darcian MHD dissipative Casson fluid flow with suction and nth order of chemical reaction. Suresh et al.[6] discussed free convective heat transfer flow of a Casson fluid with radiative and dissipative effect due to variable thermal conductivity and internal heat generation past a stretching sheet. [Falodun](http://psjd.icm.edu.pl/psjd/contributor/17cd6e13eec330ad873bd97b02e95ab1) et al. [7] computed numerically heat transfer on unsteady magnetohydrodynamics (MHD) boundary layer flow of an incompressible fluid a moving vertical plate. Falodun[8] analyzed MHD heat and mass transfer of Casson fluid flow past a semi-infinite vertical plate with thermophoresis effect. Makinde et al.[9] considered chemical reaction effect on MHD flow of Casson fluid with porous stretching sheet. Rama Krishna Reddy et al. [10] presented MHD Free convective flow past a porous plate. Nagasantoshi [11] described heat and mass transfer of non-Newtonian nanofluid flow over a stretching sheet with non-uniform heat source and variable viscosity. Gvrreddy [12] discussed Soret and Dufour effects on MHD micropolar fluid flow over a linearly stretching sheet, through a non-Darcy porous medium. Suneetha et al. [13] presented radiation and heat source effects on MHD flow over a permeable stretching sheet through porous stratum with chemical reaction. Vijaya et al. [14] developed Soret and radiation effects on an unsteady flow of a Casson fluid through porous vertical channel with expansion and contraction. Ramana Reddy et al.[15] presented numerical solutions of unsteady MHD flow heat transfer over a stretching surface with suction or injection.

In view of these an investigation, the major concerns of present pattern are to consider the magnetohydrodynamic convective flow of a Casson fluid past a semi-infinite moving vertical porous plate with heat source/sink are included in the flow. The mathematical modelling of flow arrangement yields simultaneous non-linear partial differential equations. The appropriate two term perturbation technique method employed to governing equations to deduce two non-dimensional ordinary differential equations. The numerical values of the physical parameters like Skin friction, Nusselt number, and Sherwood number are shown in tabular form.

1. **MATHEMATICAL ANALYSIS**

We consider an unsteady two-dimensional MHD free convective flow of a viscous, an incompressible, heat absorbing and electrically conducting fluid past a semi-infinite vertical permeable plate embedded in a uniform porous medium which is subject to boundary condition at the interface of porous medium and fluid layers. A uniform transversal magnetic field of strength B0 is applied in the presence of concentration and radiation buoyancy effects in the direction of -axis. The transversely applied magnetic field and magnetic Reynolds number are considered to be very small so that Hall Effect and induced magnetic field are negligible. It is assumed that there is no applied voltage which implies the deficiency of electric field. The length of the plate is large enough and the motion is two-dimensional so all the physical variables are independent of. The wall is maintained at constant concentration Cw and temperature Tw, higher than the surrounding concentration and temperature , respectively. Also, it is assumed that there exists a first-order homogeneous Casson fluid and the heat source. It is considered to be that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. Remaining properties of the porous medium and the fluid are assumed to be constant. Under these assumptions, the governing equations can be expressed as:

 (1)

 (2)

 (3)

 (4)

where  and are the dimensional distances along to the plate.  and  are the of dimensional velocity components along  and  directions. g is the gravitational acceleration,  is the fluid dimensional temperature near the plate,  is the stream dimensional temperature far away from the plate ,  is the dimensional concentration of the fluid,  is the stream dimensional concentration far away from the plate. βT and βC - expansion coefficients of the thermal and concentration respectively.  is the pressure, Cp is the specific heat, B0 is the coefficient of magnetic field, µ is fluid viscosity, ρ is the density, K is the thermal conductivity, σ is the density of the fluid magnetic permeability,  is the kinematic viscosity, D is the diffusivity of the molecular, Q0 is the dimensional coefficient of the heat absorption and β is the Casson parameter. The 3rd and 4th terms of RHS of Eq. (2) denote the thermal and concentration buoyancy effects, respectively.

 (5)

 (6)

where  are the wall dimensional velocity, temperature and concentration, respectively. are the free stream dimensional velocity, temperature and concentration, respectively are constants.

By using the Rosseland approximation, the radiative flux vector can be written as:

 (7)

Where,  are the mean absorption coefficient and Stefan-Boltzmann constant respectively. We assume that within the flow the temperature difference is sufficiently small such that  can be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about the free stream temperature  and neglecting higher order terms, thus

 (8)

It is clear from Eq.(1) that the suction velocity at the plate surface is a function of time only. Considering that it takes the following exponential form:

 (9)

Where A is a real positive constant,  is a scale of suction velocity which has non-zero positive constant and  and are small less than unity . Outside the boundary layer, Eq. (2) gives

 (10)

Introducing the non-dimensional quantities

 (11)

In the view of the above dimensionaless variables, the basic field of Eqs. (2)-(4) can be expressed in non-dimensional form as

 (12)

 (13)

 (14)

Where, 

The corresponding boundary conditions (5) and (6) in non-dimensionl form are

 (15)

 (16)

1. **SOLUTION OF A PROBLEM**

Eqs. (12)-(14) represent a set of partial differential equations that cannot be solved in closed-form. However, it can be reduced to a set of ordinary differential equations in dimensionless form that can be solved analytically. This can be done by representing the velocity, temperature and concentration as

 (17)

 (18)

 (19)

Substituting (17)-(19) into Eqs.(12)-(14) and equating the harmonic and non-harmonic terms, and neglecting the higher order , and simplifying to get the following pairs of equations for 

 (20)

 (21)

 (22)

 (23)

 (24)

 (25)

Where the prime denotes ordinary differentiation with respect to y,

The corresponding boundary conditions are

 (26)

 (27)

without going into the details, the solutions of Eqs . (20)- (25) With the help of boundary conditions (26) and (27), we get

 (28)

 (29)

 (30)

 (31)

 (32)

 (33)

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

 (34)

 (35)

 (36)

The Skin-friction coefficient, the Nusselt number and the Sherwood number are significant physical parameters for this type of boundary-layer flow. These parameters can be defined and determined as follows:

1. **SKIN FRICTION**

Knowing the velocity field, the skin friction on the plate y = 0 in non-dimensional form is given by

 (37)

1. **NUSSELT NUMBER**

Knowing the temperature field, the rate of heat transfer coefficients can be obtained which is in non-dimensional form in terms of Nusselt number is given by

 (38)

1. **SHERWOOD NUMBER**

Knowing the temperature field, the rate of heat transfer coefficients can be obtained which is in the non-dimensional form in terms of Sherwood number is given by

 (39)

Here the constants are not given due sake of brevity.

1. **RESULTS AND DISCUSSION**

The system of coupled extremely non-linear ordinary differential equations (12)-(14) subject to the boundary conditions is resolved analytically, using two term perturbation techniques. This method has been successfully used by the present authors to solve numerous problems associated with boundary layer flow, heat and mass transfer fields. Numerical values obtained for the problem are expressed in terms of graphs for various flow parameters. Impact of magnetic parameter (M), Casson parameter (β), permeability parameter (K), Grashof number (Gr), modified Grashof number (Gc), Prandtl number (Pr), heat source (Q), radiation parameter (R), chemical reaction (Kr) and Schmidt number (Sc) on the velocity, temperature and concentration profiles are discussed.

Figs.2&3 were represents the velocity profiles for different values of magnetic parameter (M) and permeability parameter (K) respectively. In Fig.2 the velocity profile decreases with the raising of magnetic parameter values, as a result of the presence of a magnetic field in an electrically conducting fluid introduced a force referred to the Lorentz force that acts against the flow, if the magnetic field applied in the normal direction as within the study. But the reveres trend is observed, the velocity profiles for increasing values of permeability parameter. Velocity distribution for various values of Grashof number (Gr) and modified Grashof number (Gc) are represented in Figs. 4&5 respectively as seen in these figures the maximum peak value is observed in the absence of buoyancy force, this is due to that buoyancy forces enhances the fluid velocity and the boundary layer thickness increases in the increasing the values of the Grashof number and modified Grashof number. The impact of Prandtl number (Pr) for the velocity and temperature profiles are as shown in the Figs. 6&7 respectively at the velocity and temperature both are decreases for increasing values of Prandtl number, the reason is that smaller values of Prandtl number are similar to increasing the thermal conductivities and so heat is ready to decreases far away from the heated surface for higher values of Prandtl number.

The impact of the casson parameter (β) and the velocity profiles as shown in the Fig. 8 it is noticed that the velocity profiles increases for increasing values of casson fluid parameter. Figs. 9&10 represent the temperature profiles for different values of heat source/sink parameter (Q), radiation parameter (R) respectively. It is observed that as the value of heat source and radiation parameters increases, the temperature profiles decreases. Figs. 11&12 represent the velocity and concentration profiles for different values of chemical reaction parameter (Kr). It is noticed that both the velocity and concentration profiles are decreases for increasing value of chemical reaction parameter. Figs. 13&14 exhibit the effect of the velocity and concentration profiles for different values of Schmidt number (Sc). It is noticed that both the velocity and concentration profiles are decreases for increasing value of Schmidt number.



Fig.2. Velocity profiles for different values of magnetic parameter.



Fig.3. Velocity profiles for different values of permeability parameter.



Fig.4. Velocity profiles for different values of Grashof number.



Fig.5. Velocity profiles for different values of modified Grashof number.



Fig.6. Velocity profiles for different values of Prandtl number.



Fig.7. Temperature profiles for different values of Prandtl number.



Fig.8. velocity profiles for different values of cason parameter.



Fig.9. Temperature profiles for different values of heat source parameter.



Fig.10. Temperature profiles for different values of radiation parameter.



Fig.11. Concentration profiles for different values of Chemical reaction parameter.



Fig.12. Velocity profiles for different values of Chemical reaction parameter.

Fig.13. Concentration profiles for different values of Schmidt number.



Fig.14. Velocity profiles for different values of Chemical Schmidt number.

**Table –I Numerical Results**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Sf | Nu | Sh |
| M | 0.5 | -4.2031 | 1.434 | 0.9378 |
|  | 1.0 | -3.7164 | 1.434 | 0.9378 |
|  | 1.5 | -3.4932 | 1.434 | 0.9378 |
| Gc | 0.5 | -3.6269 | 1.434 | 0.9378 |
|  | 1.0 | -3.8190 | 1.434 | 0.9378 |
|  | 1.5 | -4.0111 | 1.434 | 0.9378 |
| Gr | 0.5 | -1.9205 | 1.434 | 0.9378 |
|  | 1.0 | -2.1741 | 1.434 | 0.9378 |
|  | 1.5 | -2.4277 | 1.434 | 0.9378 |
| K | 0.5 | -4.2031 | 1.434 | 0.9378 |
|  | 1.0 | 0.0131 | 1.434 | 0.9378 |
|  | 1.5 | -1.9649 | 1.434 | 0.9378 |
| Sc | 0.5 | -4.7099 | 1.434 | 0.821 |
|  | 1.0 | -3.1279 | 1.434 | 1.3841 |
|  | 1.5 | -2.5261 | 1.434 | 1.9193 |
| Q | 0.5 | -4.2031 | 1.434 | 0.9378 |
|  | 1.0 | 5.256 | 1.65 | 0.9378 |
|  | 1.5 | -1.7741 | 1.8346 | 0.9378 |
| Pr | 0.5 | -3.7586 | 1.2971 | 0.9378 |
|  | 1.0 | 11.6712 | 1.6382 | 0.9378 |
|  | 1.5 | -2.2153 | 2.0243 | 0.9378 |
| R | 0.5 | -4.2031 | 1.434 | 0.9378 |
|  | 1.0 | 5.256 | 1.65 | 0.9378 |
|  | 1.5 | -1.7741 | 1.8346 | 0.9378 |

1. **The numerical results of Skin friction, Nusselt number, Sherwood number at the boundary surface are expressed in Table**

At the plate (y=0) when the Grashof number (Gr) and modified Grashof number (Gc) increases, the Skin friction (Cf) is decreasing, but the Nusselt number (Nu) and Sherwood number (Sh) constants is observed. But the reverse trend is observed for increasing the value of magnetic field (M) and the Permeability parameter(K). The rate of Skin friction (Cf) and Sherwood number (Sh) increases for the increasing values of Schmidt number (Sc) and Nusselt number (Nu) is constant. For the increasing values of Heat source parameter(Q), thermal radiation(R), and Prandtl number (Pr), the changes observed at Skin friction (τ) and Nusselt number (Nu) but there is no change in Sherwood number (Sh).

1. **CONCLUSION**

Numerical results for velocity, temperature and concentration profiles are obtained for constant variation of different ranges and for the different values of flow significant parameters. The outcomes of the problem are summarized as follows;

* For the increased values of Permeability parameter (K), Grashof number (Gr), modified Grashof number (Gc) and casson parameter (β) increase the velocity profiles, but the reverse trend is observed in magnetic parameter(M).
* The fluid velocity and temperature decreases when Prandtl number (Pr) increases.
* The Temperature level of the fluid decreases when the Heat source parameter (Q) and Radiation parameter (R) increases.
* Higher chemical reaction parameter (Kr) and Schmidt number (Sc) causes the numerous reductions in velocity and Concentration profiles.

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