Anti Identity Matrix- J and its Properties

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***Abstract:*** In this article we have a new known matrix named Anti Identity matrix denoted by J , derive its properties and the role of Anti Identity Matrix

*Keywords:* Identity matrix, Characteristic Equation, Gauss Jordan Method, Normal Form, Echolon Form

**I. Introduction**

**Definition 1.1**: A square matrix J of order nXn is said to be Anti Identity matrix if J represented as ** =**

The Anti-Identity matrix of the different order for n=2, 3, 4, 5,. , .are given below.

** =**,** =, =..**

**II.Methodology**

In this section we are going to derive the following

1. Properties of Anti Identity matrix

**2.1. Properties of Anti Identity matrix**

In this section we have derive properties of anti Identity matrix

**Property 2.1.1:** If  =is an Anti -Identity matrix,  and an equation of the form

 =

If we replace λ with matrix A is satisfied then

=

**Proof:** Given that

 and an equation of the form

 =

If we replace λ with matrix A is satisfied then

= ---- (1)

For example (1)



 ==

If we replace λ with matrix A is satisfied then



=2

For example (2)



 =

=

If we replace λ with matrix A is satisfied then



=2

Therefore

 and an equation of the form

 =

If the matrix A is satisfied by A then

=

**Property 2.1.2**: If is an Anti -Identity matrix of order nXn , and let  be any square matrix of order n then

=Where is an Identity matrix of order nXn

**Proof:** By using mathematical induction

**Case (i)**For n=2 and m=1,2,…k,k+1

Let  is an Anti -Identity matrix of order 2X2 **,**

**I,e,. =**

For m=1



For m=2

=

For m=k ,Assume it is true true



For m=k+1

 =

For n=2 the theorem is true for every m

=Where is an Identity matrix of order nXn

**Case (ii)**For n=3 and m=1,2,…k,k+1

Let  is an Anti -Identity matrix of order 3X3**,**

**I,e,. =**

For m=1

**=**

For m=2

****=****

For m=k ,Assume it is true true

 

For m=k+1

  = 

For n=3 the theorem is true for every m

=Where is an Identity matrix of order nXn

**Case (iii)**For n=4 and m=1,2,…k,k+1

Let  is an Anti -Identity matrix of order 4X4**,**

Let ** =**

For m=1

**=**

For m=2

****=****

For m=k ,Assume it is true true

 ****

For m=k+1

 **** ****= ****

For n=4 the theorem is true for every m

=Where is an Identity matrix of order nXn

By Mathematical induction

For each n belongs to N the theorem is true for every m belongs to N

=Where is an Identity matrix of order nXn

**Property 2.1.3**: If is an Anti -Identity matrix of order nXn, and is an Identity matrix of order nXn then

**i)**

 ,[for n=2,3,6,7,10,11,…]

n= (2 + (-1)^n + (-1)^(n+1))\*n - (1 + (-1)^n)/2, n >= 1

[ [Paolo P. Lava](https://oeis.org/wiki/User:Paolo_P._Lava)[1] ]

 , [for n=4, 5, 8, 9, 12, 13,..]

n=[(2 + (-1)^n + (-1)^(n+1))\*n - (1 + (-1)^n)/2]+2,n >= 1

**ii)** The Roots of the equations and are same in any order 

are λ=1 corresponding to the order of the matrix.

i.e, if order of matrix n=2 means λ=1,1

i.e, if order of matrix n=3 means λ=1,1,1

and respectively.

**Proof:**

For n=2

Let  is an Anti -Identity matrix of order 2X2 **,**

**I,e,. =**

And let ** =**

 ==

 ==



The roots of the equations



are λ=1,1

For n=3

Let ** =**

 ==

And let ** =**

 ==

==



The roots of the equations



are λ=1,1,1



The roots of the equations



are λ=1,1,1 ,1and 1

Therefore we can prove that

 ,[for n=2,3,6,7,10,11,…]

n= (2 + (-1)^n + (-1)^(n+1))\*n - (1 + (-1)^n)/2, n >= 1

[Paolo P. Lava](https://oeis.org/wiki/User:Paolo_P._Lava), Feb 15 2008

 , [for n=4, 5, 8, 9, 12, 13…]

n=[(2 + (-1)^n + (-1)^(n+1))\*n - (1 + (-1)^n)/2]+2,n >= 1

**III.Conclusion:**

We have obtained properties of J matrix and derived its properties and found J matrix .Further theorems and its applications may be obtained

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