IIITB COMET FOUNDATION

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POLYNOMIALS

2.1 Introduction

In Class IX, you have studied polynomials in one variable and their degrees. Recall that if $p(x)$ is a polynomial in x, the highest power of x in $p(x)$ is called the **degree of the polynomial** $p(x)$. For example, $4x + 2$ is a polynomial in the variable x of degree 1, $2y^2 - 3y + 4$ is a polynomial in the variable y of degree 2, $5x^3 - 4x^2 + x - \sqrt{2}$

is a polynomial in the variable x of degree 3 and $7u^6 - \frac{3}{2}u^4 + 4u^2 + u - 8$ is a polynomial

in the variable u of degree 6. Expressions like $\frac{1}{x-1}$, $\sqrt{x+2}$, $\frac{1}{x^2+2x+3}$, etc., are not polynomials.

A polynomial of degree 1 is called a **linear polynomial**. For example, $2x - 3$, $\sqrt{3}x + 5$,

$y + \sqrt{2}$, $x - \frac{2}{11}$, $z + 4\frac{2}{3} + 1$, etc., are all linear polynomials. Polynomials

such as $2x + 5 - x^2$, $x^3 + 1$, etc., are not linear polynomials.

A polynomial of degree 2 is called a **quadratic polynomial**. The name 'quadratic'

has been derived from the word 'quadrate', which means 'square'. $2x^2 + 3x - \frac{2}{5}$

$y^2 - 2$, $2 - x^2 + \sqrt{3}x$, $\frac{u}{3} - 2u^2 + 5$, $\sqrt{5}v^2 - \frac{2}{3}v$, $4z^2 + \frac{1}{7}$ are some examples of

quadratic polynomials (whose coefficients are real numbers).More generally, any quadratic

polynomial in x is of the form $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.A

polynomial of degree 3 is called a **cubic polynomial**. Some examples of a cubic polynomial are $2 - x^3$, x^3, $\sqrt{2}x^3$, $3 - x^2 + x^3$, $3x^3 - 2x^2 + x - 1$. In fact,the most general form of a cubic polynomial is

$$ax^3 + bx^2 + cx + d,$$

where a, b, c, d are real numbers and $a \neq 0$.

Now consider the polynomial $p(x) = x^2 - 3x - 4$. Then, putting $x = 2$ in the polynomial, we get $p(2) = 2^2 - 3 \cdot 2 - 4 = -6$. The value '–6', obtained by replacing x by 2 in $x^2 - 3x - 4$, is the value of $x^2 - 3x - 4$ at $x = 2$. Similarly, $p(0)$ is the value of $p(x)$ at $x = 0$, which is –4. If $p(x)$ is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in $p(x)$, is called **the value of** $p(x)$ **at** $x = k$, and is denoted by $p(k)$.

What is the value of $p(x) = x^2 - 3x - 4$ at $x = -1$? We have :

$$p(-1) = (-1)^2 - \{3 \times (-1)\} - 4 = 0$$

Also, note that $p(4) = 4^2 - (3x4) - 4 = 0$.

As $p(-1) = 0$ and $p(4) = 0$, –1 and 4 are called the **zeroes** of the quadratic polynomial $x^2 - 3x - 4$. More generally, a real number k is said to be a **zero of a polynomial** $p(x)$, if $p(k) = 0$.

You have already studied in Class IX how to find the zero of a linear polynomial. For example, if k is a zero of $p(x) = 2x + 3$, then p(k) = 0 gives us
2k + 3 = 0, i.e., $k = \frac{3}{2}$

In general, if k is a zero of $p(x) = ax + b$, then

$$p(k) = ak + b = 0 \Rightarrow ak = -b \Rightarrow k = -\frac{b}{a}.$$

So, the zero of the linear polynomial $ax + b$ is $-\frac{b}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x}$

Thus, the zero of a linear polynomial is related to its coefficients. Does this happen in the case of other polynomials too? For example, are the zeroes of a *quadratic* polynomial also related to its coefficients?

In this chapter, we will try to answer these questions. We will also study the division algoirithm for polynomials

2.2 Geometrical Meaning of the Zeroes of a Polynomial

You know that a real number k is a *zero* (or *root*) of a polynomial $p(x)$ if $p(k) = 0$. But why are these zeroes of a polynomial so important?To answer this, first we see the **geometrical** representations of linear and quadratic polynomials and the geometrical meaning of their zeroes.

Consider first a linear polynomial $ax + b$, where $a \neq 0$.You have studied in Class IX that the graph of $y = ax + b$ is a straight line. For example, the graph of $y = 2x + 3$ is a straight line passing through the points $(-2, -1)$ and $(2, 7)$.

x	-2	2
$y = 2x + 3$	-1	7

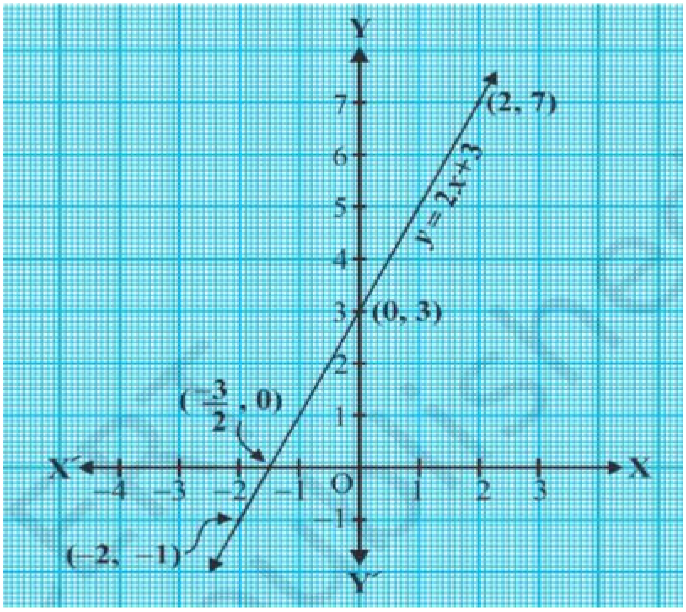

Fig. 2.10

From Fig. 2.1, you can see that the graph of $y = 2x + 3$ intersects the x-axis mid-way between $x = -1$ and $x = -2$, that is, at the point $\left(-\frac{3}{2},\ 0\right)$.

You also know that the zero of $2x + 3$ is $-\frac{3}{2}$. Thus, the zero of the polynomial $2x + 3$ is the x-coordinate of the point where the graph of $y = 2x + 3$ intersects the x-axis.

In general, for a linear polynomial $ax + b$, $a \neq 0$, the graph of $y = ax + b$ is a straight line which intersects the x-axis at exactly one point, namely, $\left(\frac{-b}{a},\ 0\right)$. Therefore, the linear polynomial $ax + b$, $a \neq 0$, has exactly one zero, namely, the x-coordinate of the point where the graph of $y = ax + b$ intersects the x-axis.

Now, let us look for the geometrical meaning of a zero of a quadratic polynomial. Consider the quadratic polynomial $x^2 - 3x - 4$. Let us see what the graph* of $y = x^2 - 3x - 4$ looks like.Let us list a few values of $y = x^2 - 3x - 4$ corresponding to a few values for x as given in Table 2.1.

* Plotting of graphs of quadratic or cubic polynomials is not meant to be done by the students,nor is to be evaluated.

4. The number of zeroes is 1. (Why?)
5. The number of zeroes is 1. (Why?)
6. The number of zeroes is 4. (Why?)

EXERCISE 2.1

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$.Find the number of zeroes of $p(x)$, in each case

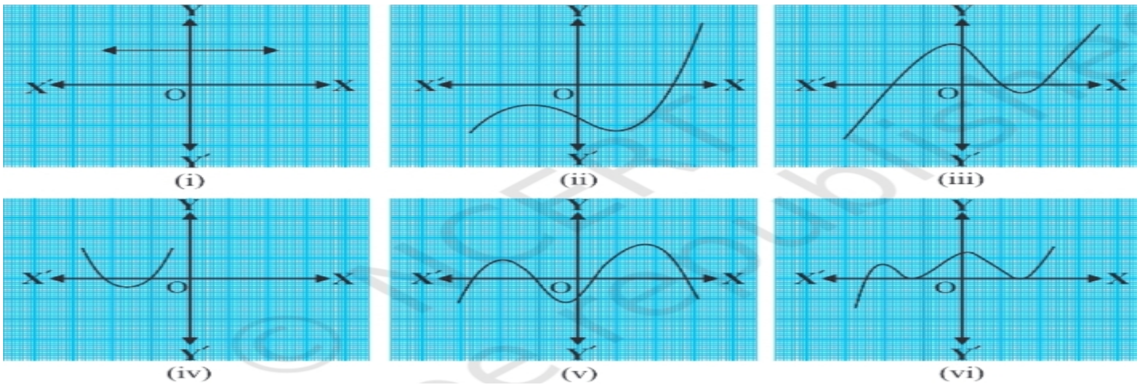

Fig. 2.10

2.3 Relationship between Zeroes and Coefficients of a Polynomial

You have already seen that zero of a linear polynomial $ax + b$ is $-\frac{b}{a}$.We will now try to answer the question raised in Section 2.1 regarding the relationship between zeroes and coefficients of a quadratic polynomial.For this, let us take a quadratic polynomial, say $p(x) = 2x^2 - 8x + 6$.In Class IX, you have learnt how to factorise quadratic polynomials by splitting the middle term.So, here we need to split the middle term $-8x$' as a sum of two terms,whose product is $6 \times 2x^2 = 12x^2$.So, we write:

$$2x^2 - 8x + 6 = 2x^2 - 6x - 2x + 6 \qquad = 2x(x-3) - 2(x-3)$$

$$= (2x-2)(x-3) \qquad = 2(x-1)(x-3)$$