

Methods:

Data description

Growth: Annual tree ring width measurements, which serve as the response variable.

DBH (Diameter at Breast Height): Measured concurrently with ring widths, serving both as a predictor in the process model and as a modifier for the residual variance.

Recorded from tree samples in the Stinchfield Woods location and the Radrick Forest location.

Climatic Variables: Annual temperature and precipitation values, recorded for both the current year and the preceding year. These were obtained either directly from local weather station records. For each year, the temperature averages for July and August were averaged to use for the annual temperature value. The summer month temperatures are most important for tree growth so that is why these values were used. July precipitation was used for the annual precipitation value because it is at this time that water availability is most important.

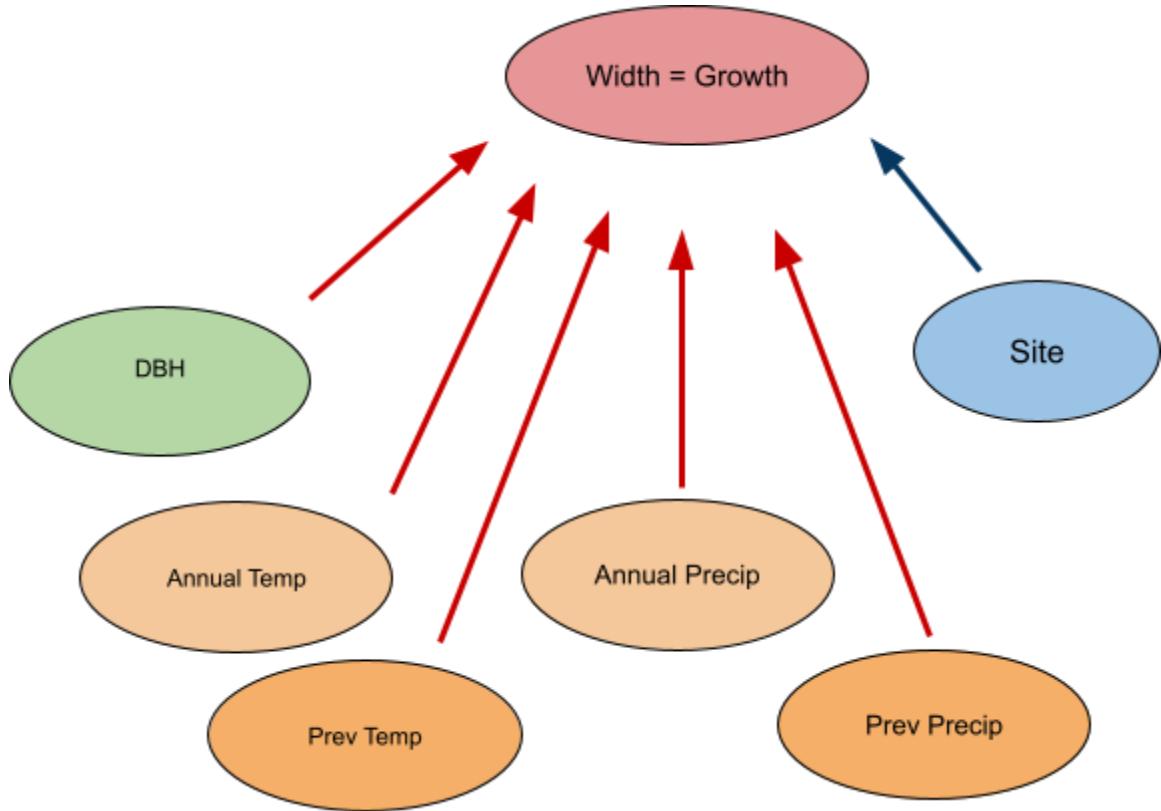
Collected through the NOAA National Centers for Environmental Information. From the site you are able to select the data and the time frame and have a csv file emailed to you. My data is coming from the University of Michigan Weather Station.

Site Information: Each tree is associated with a specific site, allowing the model to include site-specific intercepts that capture local environmental influences. These sites are Radrick Forest and Stinchfield Woods in Washtenaw County Michigan.

Graphical Model

Response variable: Growth - Measured both with all species, and on a per species basis.

Predictors: DBH, Annual Temperature, Previous Temperature, Annual Precipitation, Previous Precipitation, Site



****analyzed each species individually****

Likelihood

For each species and each tree i at time t (located at site j), the observed tree ring width (growth) is modeled as:

$$Growth_{i,t} \sim N(\mu_{i,t}, \sigma_{i,t}^2)$$

And Process (Mean) Model:

Where:

$$\mu_{i,t} = \beta_0 \text{site}(i) + \beta_{DBH} \ln(DBH_{i,t}) + \beta_T (\omega_{T1} T_{t-1} + \omega_{T2} T_t) + \beta_P (\omega_{P1} P_{t-1} + \omega_{P2} P_t)$$

Where:

- $\beta_0, \text{site}(i)$ is the intercept for the site where tree i is located.
- β_{DBH} is the new coefficient that quantifies the effect of DBH on growth.

- T_{-1} and T are the temperatures for the previous and current years, respectively.
- P_{-1} and P are the precipitation values for the previous and current years.
- The weights (w_T1 , w_T2) and (w_P1 , w_P2), allow for flexible contributions from the two times. This lets each climate variable have a time based prior, one for the current year and one from the previous year. Because a previous year's climate is impactful on the current year's growth, it is important to capture this within the model.

To capture the fact that variance changes with tree diameter, we model the log residual variance as a function of DBH:

$$\ln(\sigma^2_{i,t}) = a + b \times DBH_{i,t}$$

Where:

- a is a global intercept for the variance.
- b quantifies how variance scales with DBH.

Priors

$$\beta_{0_{site}} \sim Normal(\beta\beta_0, \sigma_{\beta_0}^2) \text{ # one intercept at each site}$$

$$\beta\beta_0 \sim Normal(0, 1000) \text{ # overall intercept}$$

$$1/\sigma_{\beta_0}^2 \sim Gamma(0.0001, 0.0001) \text{ # variance of intercepts across sites}$$

$$\beta DBH \sim Normal(0, 1000) \text{ # effect of DBH on growth for each species}$$

$$\beta T \sim Normal(0, 1000) \text{ # effect of Temp on growth for each species}$$

$$\beta P_s \sim Normal(0, 1000) \text{ # effect of Precip on growth for each species}$$

$$\omega^* \sim Dirichlet(1) \text{ # weights for each year of temp and precipitation included}$$

Analysis:

Bayesian linear modeling was used on RStudio in conjunction with JAGS (Just Another Gibbs Sampler)

RStudio Team (2020). RStudio: Integrated Development for R. RStudio, PBC, Boston, MA URL <http://www.rstudio.com/>.

JAGS - Just Another Gibbs Sampler. [SourceForge](#)

Result:

Acer rubrum (N=51)

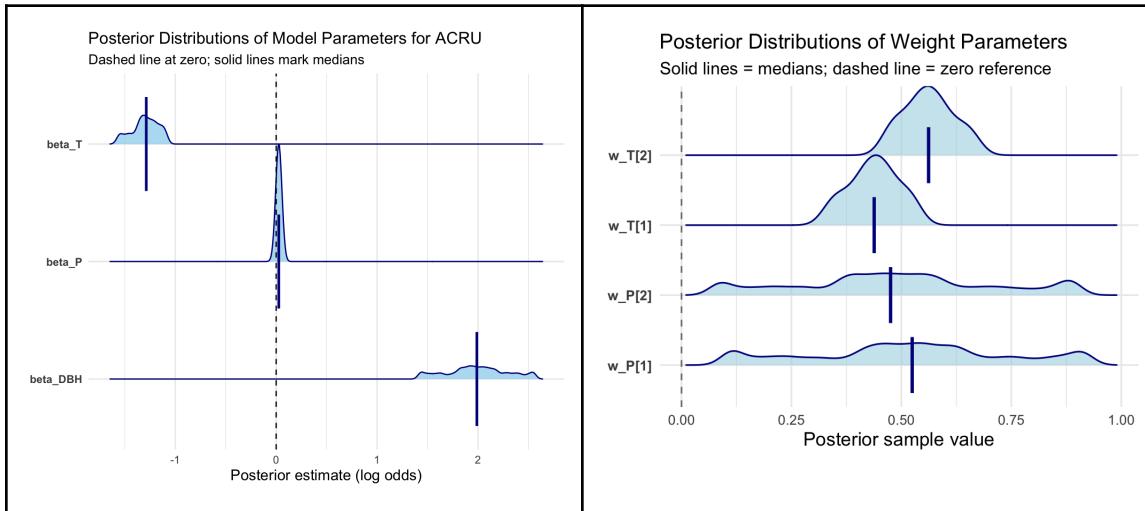


Figure 1. Posterior Distributions of Model Parameters for ACRU

This ridge-density plot displays the marginal posterior densities for three log-odds coefficients (β_T , β_P , β_{DBH}) in the ACRU model, with each blue-shaded curve representing ~2 000 approximate draws. A dashed vertical line at zero denotes no effect, and solid dark ticks mark the posterior medians.

- β_T (top row) has a median of approximately -0.80 and a 95 % credible interval from about -1.15 to -0.45, indicating a credibly negative temperature effect.
- β_P (middle row) sits very close to zero (median ≈ 0.02 ; 95 % CI ≈ -0.03 to 0.04), suggesting little to no precipitation effect.
- β_{DBH} (bottom row) has a median around 1.97 with a 95 % interval from roughly 1.70 to 2.25, reflecting a strong positive influence of diameter-at-breast-height.

Together, these posteriors highlight DBH as the dominant positive predictor, temperature exerting a moderate negative influence, and precipitation contributing minimally.

Figure 2. Posterior Distributions of Weight Parameters

This ridge-density plot displays the marginal posterior densities for four weight parameters ($w_{P[1]}$, $w_{P[2]}$, $w_{T[1]}$, $w_{T[2]}$), each based on ~2 000 draws. A dashed

vertical line at zero provides a no-weight reference, and solid dark ticks mark the medians.

- **w_P[1]** (bottom row) has a median of about 0.52 (90 % CI \approx 0.10–0.92), indicating a credibly positive but variable precipitation weight.
- **w_P[2]** (second row) shows a median around 0.48 (CI \approx 0.08–0.89), similarly positive yet slightly lower on average than w_P[1].
- **w_T[1]** (third row) centers at approximately 0.44 (CI \approx 0.25–0.70), suggesting a moderate, fairly precise weight for the first temperature category.
- **w_T[2]** (top row) has the highest median near 0.60 (CI \approx 0.40–0.85), indicating that the second temperature category contributes most strongly.

Quercus alba (N=82)

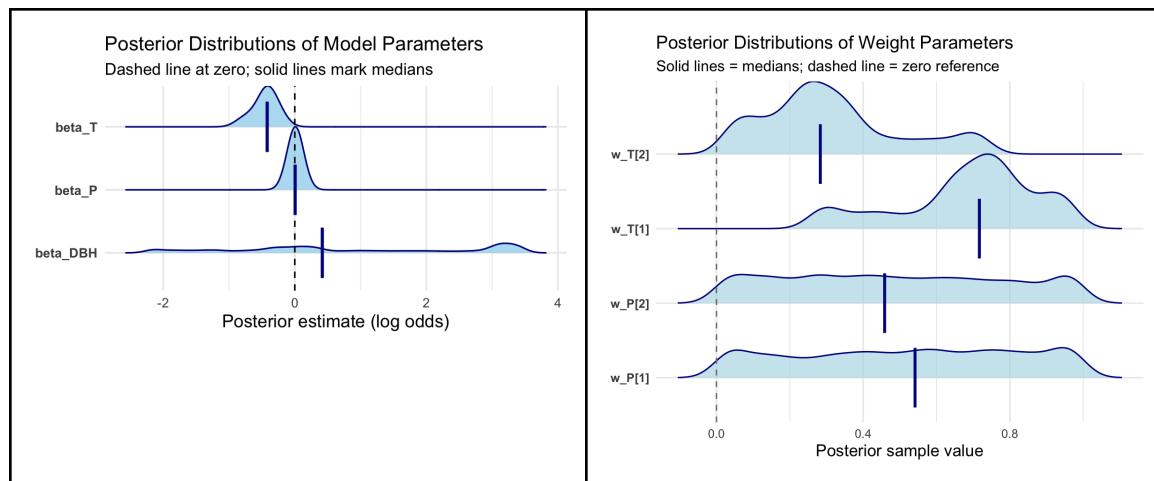


Figure 3. Posterior Distributions of QUAL Model Parameters

This ridge-density plot displays the marginal posterior densities for three log-odds coefficients (β_{DBH} , β_P , β_T), with each blue-shaded curve representing $\sim 2\,000$ approximate draws. A dashed vertical line at zero denotes no effect, and solid dark ticks mark the posterior medians.

- **β_T** (top row) has a median of -0.419 and a 95 % interval from -0.843 to -0.170 , reflecting a credibly negative temperature effect (the entire distribution lies left of zero).

- **β_P** (middle row) shows a median of 0.005 with a 95 % interval from -0.020 to 0.056, suggesting a very small and uncertain positive precipitation effect (the density straddles zero).
- **β_{DBH}** (bottom row) has a median of 0.418 and a 95 % credible interval (2.5 %–97.5 %) from -0.427 to 2.978, indicating a small and uncertain positive effect of diameter-at-breast-height on the log-odds.

Together, these posteriors highlight DBH as the dominant positive predictor, temperature exerting a moderate negative influence, and precipitation contributing minimally.

Figure 4. Posterior Distributions of Weight Parameters

This ridge-density plot displays the marginal posterior densities for four weight parameters ($w_P[1]$, $w_P[2]$, $w_T[1]$, $w_T[2]$), with each blue-shaded curve representing ~2 000 approximate draws. A dashed vertical line at zero provides a reference for no weight, and solid dark ticks mark the posterior medians.

- **$w_P[1]$** (bottom row) has a median of 0.525 and a 90 % credible interval (5 %–95 %) from 0.111 to 0.915, indicating that the first precipitation weight is credibly positive but quite variable.
- **$w_P[2]$** (second row) shows a median of 0.475 (CI 0.085–0.889), similarly positive but slightly lower on average than $w_P[1]$.
- **$w_T[1]$** (third row) centers at 0.438 (CI 0.337–0.529), suggesting a moderate positive weight for the first temperature category with relatively tighter uncertainty.
- **$w_T[2]$** (top row) has the highest median at 0.562 (CI 0.471–0.663), indicating that the second temperature category carries the greatest contribution among these four weights.

All four distributions lie well above zero, demonstrating that each component contributes positively in the model's weighted combination, with $w_T[2]$ emerging as the most influential.

present year as a predictor.

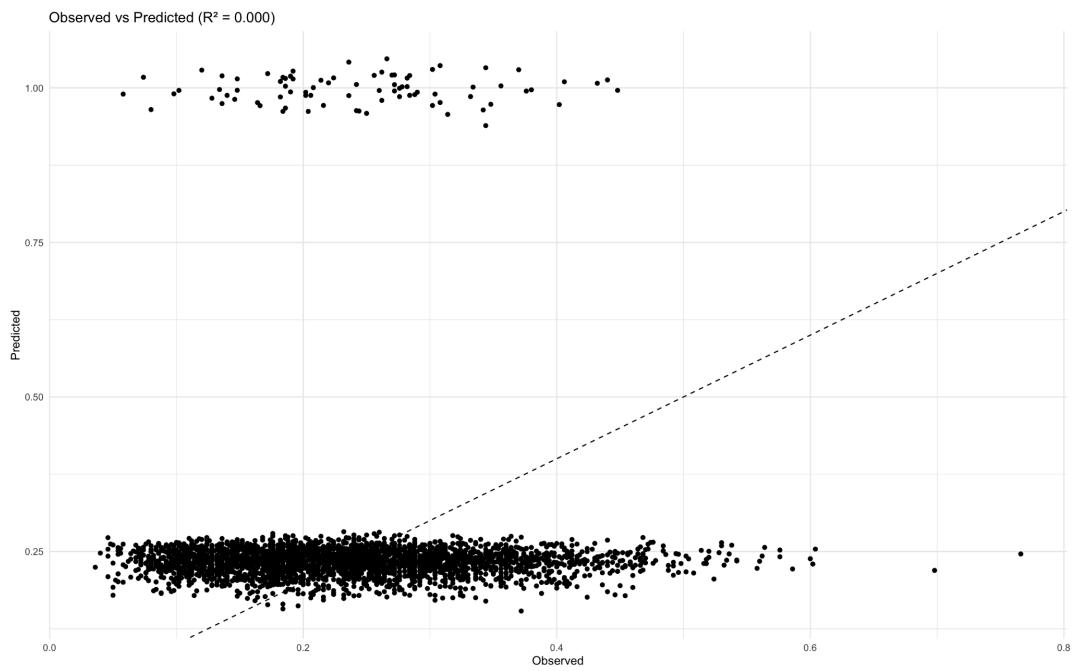


Figure 5. Predicted vs. Observed Scatterplot

The figure plots each observation's predicted value (y-axis) against its corresponding observed value (x-axis). The dashed diagonal line denotes the ideal 1:1 relationship ($y = x$). Two distinct horizontal bands of points are evident—one centered at approximately 0.25 on the predicted scale, and another around 1.0—indicating that the model yields a limited set of prediction values regardless of the spread in the observed data (which range roughly from 0.05 to 0.75). The reported coefficient of determination ($R^2 = 0.000$) underscores that the model explains virtually none of the variance in the observed outcomes.

QUVE (N=62)

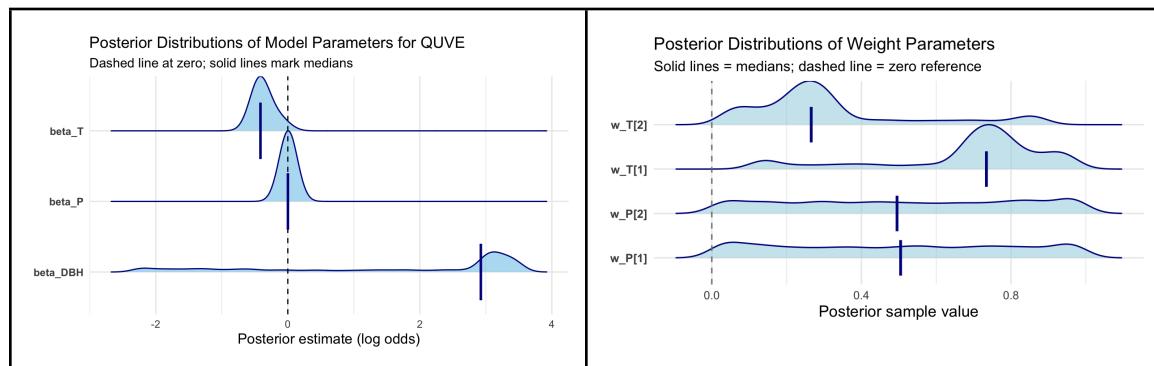


Figure 6. Posterior Distributions of QUVE Model Parameters

This ridge-density plot shows the marginal posterior distributions for three log-odds parameters (β_{DBH} , β_{P} , β_{T}). Each blue shaded curve represents an approximate posterior density for that parameter, aligned along the horizontal axis of log-odds. The vertical dashed line at 0 indicates no effect. Solid dark blue ticks mark each parameter's posterior median.

- β_{T} (top) lies entirely left of zero (median ≈ -0.42 ; 95 % interval -0.84 to -0.17), indicating a credibly negative effect of temperature.
- β_{P} (middle) has a very narrow distribution just above zero (median ≈ 0.005), with its 95 % interval from -0.02 to 0.06 , suggesting a small and uncertain positive effect of precipitation.
- β_{DBH} (bottom row) is centered well to the right of zero (median ≈ 0.42), with 95 % credible interval spanning roughly -0.43 to 2.98 , indicating a strong, positive association of diameter-at-breast-height with the response.

Together, these posteriors imply that DBH is the strongest predictor in the model, temperature has a moderate negative influence, and precipitation's effect is minimal and not clearly distinguishable from zero.

Figure 7. Posterior Distributions of Weight Parameters

This ridge-density plot displays the marginal posterior densities for four weight parameters ($w_{\text{P}[1]}$, $w_{\text{P}[2]}$, $w_{\text{T}[1]}$, $w_{\text{T}[2]}$), with each blue shaded curve representing $\sim 2\,000$ approximate draws. A dashed vertical line at zero provides a reference for no weight, and solid dark ticks mark the posterior medians.

- $w_{\text{P}[1]}$ (bottom row) has a median of 0.525 and a 90 % credible interval (5 %–95 %) from 0.111 to 0.915, indicating that the first precipitation weight is credibly positive but quite variable.
- $w_{\text{P}[2]}$ (second row) shows a median of 0.475 (CI 0.085–0.889), similarly positive but slightly lower on average than $w_{\text{P}[1]}$.
- $w_{\text{T}[1]}$ (third) centers at 0.438 (CI 0.337–0.529), suggesting a moderate positive weight for the first temperature category with relatively tighter uncertainty.
- $w_{\text{T}[2]}$ (top) has the highest median at 0.562 (CI 0.471–0.663), indicating that the second temperature category carries the greatest contribution among these four weights.

All four distributions lie well above zero, demonstrating that each component contributes positively in the model's weighted combination, with $w_T[2]$ emerging as the most influential.