Discrete Mathematical Structures Tutorial

Portions: POSETS, Lattices

1. Consider the set U={1,2,3} and A=P{U}, R is the subset relation on A.
Determine if (A,R) is a POSET and draw the Hasse diagram for the same.

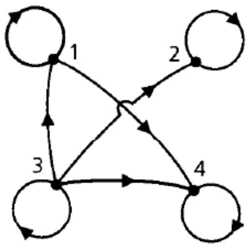
2. Solve the question 1 using the set U={1,2,3,4}.

3.

Let $A = \{1, 2, 3, 6, 9, 18\}$, and define $\mathfrak{R}$ on A by $x\,\mathfrak{R}\,y$ if $x \mid y$. Draw the Hasse diagram for the poset $(A, \mathfrak{R})$.

4. The directed graph G for a relation R on the set A={1,2,3,4} is given.

Verify that (A,R) is a POSET and find its Hasse diagram.



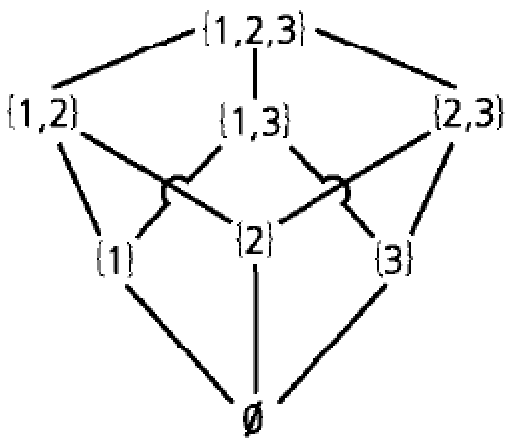
LB, UB, LUB, GLB, Lattice.

Let $(A, \mathfrak{R})$ be a poset with $B \subseteq A$. An element $x \in A$ is called a *lower bound* of B if $x \mathfrak{R} b$ for all $b \in B$. Likewise, an element $y \in A$ is called an *upper bound* of B if $b \mathfrak{R} y$ for all $b \in B$.

An element $x' \in A$ is called a *greatest lower bound* (glb) of B if it is a lower bound of B and if for all other lower bounds x'' of B we have $x'' \mathfrak{R} x'$. Similarly $y' \in A$ is a *least upper bound* (lub) of B if it is an upper bound of B and if $y' \mathfrak{R} y''$ for all other upper bounds y'' of B.

The poset $(A, \mathfrak{R})$ is called a *lattice* if for any $x, y \in A$ the elements $\text{lub}\{x, y\}$ and $\text{glb}\{x, y\}$ both exist in A.

Example:



If B={{1},{2},{1,2}} then

$\{1,2\}$, $\{1,2,3\}$, $\{1,2,4\}$, and $\{1,2,3,4\}$ are all upper bounds

$\{1,2\}$ is a least upper bound

Can you determine the lower bounds and the greatest lower bound of B?

5.

For $A = \{a, b, c, d, e, v, w, x, y, z\}$, consider the poset $(A, \mathcal{R})$ whose Hasse diagram is shown in Fig. 7.23. Find

a) $\text{glb}\{b, c\}$ **b)** $\text{glb}\{b, w\}$ **c)** $\text{glb}\{e, x\}$ **d)** $\text{lub}\{c, b\}$

e) $\text{lub}\{d, x\}$ **f)** $\text{lub}\{c, e\}$ **g)** $\text{lub}\{a, v\}$

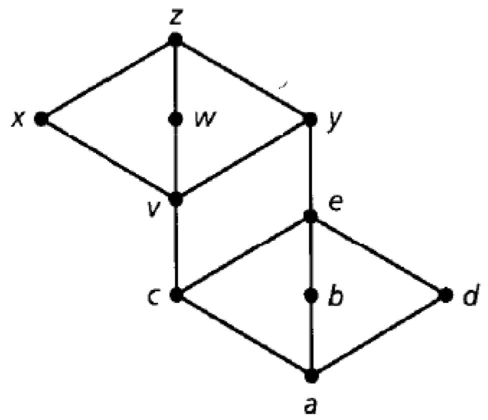


Figure 7.23

6. Draw the Hasse diagrams and determine which of the following POSETs (A, |) are lattices and why?

i) A={1,2,3,5,30}

ii) A= {1,2,3,4,6,12}

iii) A={2,3,4,12}