Discrete Mathematical Structures Tutorial
Week 2

Portions

- Laws of logic
- Logical equivalence

1.

Verify the first Absorption Law by means of a truth table.

Ans:

p	q	$p \wedge q$	$p \vee (p \wedge q)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

2.

Let p, q, r denote primitive statements.

a) Use truth tables to verify the following logical equivalences.

i) $p \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$

ii) $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$

iii) $[p \rightarrow (q \vee r)] \Leftrightarrow [\neg r \rightarrow (p \rightarrow q)]$

b) Use the substitution rules to show that $[p \rightarrow (q \vee r)] \Leftrightarrow [(p \wedge \neg q) \rightarrow r]$.

Ans:

(a)

(i)

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

(ii)

p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

(iii)

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$p \rightarrow q$	$\neg r \rightarrow (p \rightarrow q)$
0	0	0	0	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	1	1	0	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

b)

$[p \rightarrow (q \vee r)] \iff [\neg r \rightarrow (p \rightarrow q)]$ From part (iii) of part (a)

$\iff [\neg r \rightarrow (\neg p \vee q)]$ By the 2nd Substitution Rule, and $(p \rightarrow q) \iff (\neg p \vee q)$

$\iff [\neg(\neg p \vee q) \rightarrow \neg\neg r]$ By the lst Substitution Rule, and $(s \rightarrow t) \iff (\neg t \rightarrow \neg s)$, for primitive statements s, t

$\iff [(\neg\neg p \wedge \neg q) \rightarrow r]$ By DeMorgan's Law, Double Negation and the 2nd Substitution Rule

$\iff [(p \wedge \neg q) \rightarrow r]$ By Double Negation and the 2nd Substitution Rule

3.

Negate and express each of the following statements in smooth English.

a) Kelsey will get a good education if she puts her studies before her interest in cheerleading.

b) Norma is doing her mathematics homework, and Karen is practicing her piano lessons.

c) If Logan goes on vacation, then he will enjoy himself if he doesn't worry about traveling by airplane.

d) If Harold passes his Pascal course and finishes his data structures project, then he will graduate at the end of the semester.

Ans:

a) Kelsey placed her studies before her interest in cheerleading, but she (still) did not get a good education.

b) Norma is not doing her mathematics homework or Karen is not practicing her piano lesson.

c) Harold did pass his C++ course and he did finish his data structures project, but he did not graduate at the end of the semester.

4. Prove the logical equivalence using laws of logic

$$(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$$

Ans:

	Reasons
$(p \vee q) \wedge \neg(\neg p \wedge q)$	
$\Leftrightarrow (p \vee q) \wedge (\neg\neg p \vee \neg q)$	DeMorgan's Law
$\Leftrightarrow (p \vee q) \wedge (p \vee \neg q)$	Law of Double Negation
$\Leftrightarrow p \vee (q \wedge \neg q)$	Distributive Law of $\vee$ over $\wedge$
$\Leftrightarrow p \vee F_0$	Inverse Law
$\Leftrightarrow p$	Identity Law

5. Prove the logical equivalence using laws of logic

$$\neg[\neg[(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow q \wedge r$$

Ans:

	Reasons
$\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$	
$\Leftrightarrow \neg\neg[(p \vee q) \wedge r] \wedge \neg\neg q$	DeMorgan's Law
$\Leftrightarrow [(p \vee q) \wedge r] \wedge q$	Law of Double Negation
$\Leftrightarrow (p \vee q) \wedge (r \wedge q)$	Associative Law of $\wedge$
$\Leftrightarrow (p \vee q) \wedge (q \wedge r)$	Commutative Law of $\wedge$
$\Leftrightarrow [(p \vee q) \wedge q] \wedge r$	Associative Law of $\wedge$
$\Leftrightarrow q \wedge r$	Absorption Law (as well as the Commutative Laws for $\wedge$ and $\vee$)

6. Write the dual of

(a) $q \rightarrow p$, (b) $p \rightarrow (q \wedge r)$, (c) $p \leftrightarrow q$,

Ans:

7.

Write the converse, inverse, contrapositive, and negation of the following statement: "If Sandra finishes her work, she will go to the basketball game unless it snows."

Ans:

8. Verifiy the following logical equivalences using laws of logic

a)

$$[(p \vee q) \wedge (p \vee \neg q)] \vee q \quad \Leftrightarrow p \vee q$$

Ans:

b)

$$\neg(p \vee q) \vee [(\neg p \wedge q) \vee \neg q] \quad \Leftrightarrow \neg(q \wedge p)$$

Ans:

c)

$$(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \quad \Leftrightarrow \neg(q \vee p)$$

Ans:

9.

Write the converse, inverse, and contrapositive of each of the following implications. For each implication, determine its truth value as well as the truth values of its corresponding converse, inverse, and contrapositive.

a) If today is Labor Day, then tomorrow is Tuesday.

b) If $-1 < 3$ and $3 + 7 = 10$, then $\sin\left(\frac{3\pi}{2}\right) = -1$.

c) If Wesley lives in New England, then Wesley lives in Vermont.

Ans: