

# Context Free Grammar

I. Write a CFG for each of the following languages

1)  $L = \{a^n b^m a^n \mid m, n > 0\}$

$$S \rightarrow aSa \mid aAa$$

$$A \rightarrow bA \mid b$$

2)  $L = \{a^n b^m c^m d^{2n} \mid m, n \geq 0\}$

$$S \rightarrow aSdd \mid A$$

$$A \rightarrow bAc \mid \varepsilon$$

3) Odd length palindromes over  $\{a, b\}$

$$S \rightarrow a \mid b \mid aSa \mid bSb$$

4)  $L = \{a^n b^m \mid 0 \leq n \leq m \leq 2n\}$

$$S \rightarrow aSb \mid aSbb \mid \varepsilon$$

5)  $L = \{a^+ b^*\}$

$$S \rightarrow aS \mid aA$$

$$A \rightarrow bA \mid \varepsilon$$

6) Strings over  $\{a, b\}$  that contain exactly 2 b's

$$S \rightarrow AbAbA$$

$$A \rightarrow aA \mid \varepsilon$$

7) Strings over  $\{a, b\}$  that contain at least 2 b's

$$S \rightarrow AbAbA$$

$$A \rightarrow aA \mid bA \mid \varepsilon$$

8) Even length strings over  $\{a, b\}$

$$S \rightarrow aA \mid bA \mid \varepsilon$$

$$A \rightarrow aS \mid bS \mid \varepsilon$$

9) Strings over  $\{a, b\}$  with even number of b's

$$S \rightarrow aS \mid bA \mid \varepsilon$$

$$A \rightarrow aA \mid bS$$

10) Strings with even number of a's and b's

$$S \rightarrow aA \mid bB \mid \varepsilon$$

$$A \rightarrow aS \mid bC$$

$$B \rightarrow aC \mid bS$$

$$C \rightarrow aB \mid bA$$

11) Strings with even number of a's and odd number of b's

$$S \rightarrow aA \mid bB$$

$$A \rightarrow aS \mid bC$$

$$B \rightarrow aC \mid bS \mid \varepsilon$$

$$C \rightarrow aB \mid bA$$

12) Strings with odd number of a's and even number of b's

$$S \rightarrow aA \mid bB$$

$$A \rightarrow aS \mid bC \mid \varepsilon$$

$$B \rightarrow aC \mid bS$$

$$C \rightarrow aB \mid bA$$

13) Strings with odd number of a's and b's

$$S \rightarrow aA \mid bB$$

$$A \rightarrow aS \mid bC$$

$$B \rightarrow aC \mid bS$$

$$C \rightarrow aB \mid bA \mid \varepsilon$$

14) Strings over  $\{a, b, c\}$  that do not contain abc as substring

$$S \rightarrow aA \mid bS \mid cS \mid \varepsilon$$

$$A \rightarrow aA \mid bB \mid cS \mid \varepsilon$$

$$B \rightarrow aA \mid bS \mid \varepsilon$$

15) Strings with  $n_a(w) = n_b(w)$

$$S \rightarrow aSb \mid bSa \mid SS \mid \varepsilon \quad \text{OR}$$

$$S \rightarrow aSbS \mid bSaS \mid \varepsilon \quad \text{OR}$$

$$S \rightarrow aA \mid bB \mid \varepsilon$$

$$A \rightarrow aAA \mid bS$$

$$B \rightarrow aS \mid bBB$$

16) Strings without ba as substring

$$S \rightarrow aS \mid bA \mid \varepsilon$$

$$A \rightarrow bA \mid \varepsilon \quad \text{OR}$$

$$S \rightarrow aS \mid Sb \mid \varepsilon$$

17)  $L = \{0^*1(0+1)^*\}$

$$S \rightarrow 0S \mid 1A$$

$$A \rightarrow 0A \mid 1A \mid \varepsilon$$

18)  $L = \{(011+1)^*(01)^*\}$

$$S \rightarrow 011S \mid 1S \mid S01 \mid \varepsilon$$

19)  $L = \{a^n b^n \mid n > 0\}$

$$S \rightarrow aSb \mid ab$$

20)  $L = \{a^n b^n c^m \mid m, n > 0\}$

$$S \rightarrow AB$$

$$A \rightarrow aAb \mid ab$$

$$B \rightarrow cB \mid c$$

21)  $L = \{a^n b^n c^m d^m \mid m, n > 0\}$

$$S \rightarrow AB$$

$$A \rightarrow aAb \mid ab$$

$$B \rightarrow cBd \mid cd$$

22)  $L = \{a^n b^m \mid n > m > 0\}$

$$S \rightarrow aSb \mid aAb$$

$$A \rightarrow aA \mid a$$

23)  $L = \{wcw^r \mid w \in \{a, b\}^*\}$

$$S \rightarrow aSa \mid bSb \mid c$$

24)  $L = \{a^n \mid n > 0\}$

$$S \rightarrow aS \mid a$$

25)  $L = \{a^n b^m \mid m, n > 0\}$

$$S \rightarrow aS \mid aA$$

$$A \rightarrow bA \mid b$$

26)  $L = \{a^{2n} \mid n > 0\}$

$$S \rightarrow aaS \mid aa$$

27)  $L = \{(ab)^n \mid n > 0\}$

$$S \rightarrow abS \mid ab$$

28)  $L = \{0^i 1^j 0^k \mid j > i + k\}$

$$S \rightarrow ABC$$

$$A \rightarrow 0A1 \mid \varepsilon$$

$$B \rightarrow 1B \mid 1$$

$$C \rightarrow 1C0 \mid \varepsilon$$

29)  $L = \{a^i b^j \mid i \leq 2j\}$

$$S \rightarrow aSb \mid aaSb \mid Sb \mid \varepsilon$$

30)  $L = \{w \mid n_a(w) = n_b(w) + 1\}$

$$S \rightarrow AaA$$

$$A \rightarrow aAb \mid bAa \mid AA \mid \varepsilon$$

31) Strings of even length such that the two symbols in the middle are same

$$S \rightarrow aSa \mid aSb \mid bSa \mid bSb \mid aa \mid bb$$

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