

DEPARTEMENT OF COMPUTER SCIENCE & ENGINEERING

Theory of Computation (CS1534)

CSE-C SEM-III

CNF

1. $S \rightarrow a|aA|B|C$, $A \rightarrow aB|\epsilon$, $B \rightarrow aA|\epsilon$, $C \rightarrow cCD$, $D \rightarrow ddd$

Sol: Eliminating ϵ -productions:

$V_n = \{A\}$

$P_1 \{S \rightarrow a|aA|B|C, A \rightarrow aB, B \rightarrow aA|a, C \rightarrow cCD, D \rightarrow ddd\}$

$G_1 = \{\{A, B, C, D\}, \{a, c, d\}, P_1, S\}$

Eliminating unit productions:

(S,S) $S \rightarrow a|aA|B|C$

(S,B) $S \rightarrow aA|a$

(S,C) $S \rightarrow cCD$

(A,A) $A \rightarrow aB$

(B,B) $B \rightarrow a|aA$

(C,C) $C \rightarrow cCD$

(D,D) $D \rightarrow ddd$

$P_2 \{S \rightarrow a|aA|cCD, A \rightarrow aB, B \rightarrow a|aA, C \rightarrow cCD, D \rightarrow ddd\}$

$G_2 = \{\{A, B, C, D\}, \{a, c, d\}, P_2, S\}$

Useful symbols:

C is not generating.

$P_3 \{S \rightarrow a|aA, A \rightarrow aB, B \rightarrow a|aA, D \rightarrow ddd\}$

D is not reachable

$P_4 \{S \rightarrow a|aA, A \rightarrow aB, B \rightarrow a|aA\}$

$G_3 = \{\{A, B\}, \{a\}, P_4, S\}$

CNF:

Let $C_1 \rightarrow a$

$P_5 \{S \rightarrow a|C_1A, A \rightarrow C_1B, B \rightarrow a|C_1A\}$

$G_4 = \{\{A, B, C_1\}, \{a\}, P_5, S\}$

2. $S \rightarrow ABC|BaB$, $A \rightarrow aA|BaC|aaa$, $B \rightarrow bBb|a|D$, $C \rightarrow CA|AC$, $D \rightarrow \epsilon$

Sol: Eliminating ϵ -productions:

$V_n = \{D, B\}$

$P_1 \{S \rightarrow ABC|AC|BaB|Ba|aB|a, A \rightarrow aA|BaC|aC|aaa, B \rightarrow bBb|bb|a|D, C \rightarrow CA|AC\}$

$G_1 = \{\{A, B, C\}, \{a, b\}, P_1, S\}$

Eliminating unit productions:

(S,S) $S \rightarrow ABC|BaB|AC|Ba|aB|a$

(A,A) $A \rightarrow aA|BaC|aaa|aC$

(B,B) $B \rightarrow bBb|bb|a|D$

(C,C) $C \rightarrow CA|AC$

(B,D)

$P_2 \{S \rightarrow ABC|BaB|AC|Ba|aB|a, A \rightarrow aA|BaC|aaa|aC, B \rightarrow bBb|bb|a, C \rightarrow CA|AC\}$

$G_2 = \{\{A, B, C\}, \{a, b\}, P_2, S\}$

Useful symbols:

C is not generating.

$P_3 \{S \rightarrow BaB|Ba|aB|a, A \rightarrow aA|aaa, B \rightarrow bBb|bb|a\}$

A is not reachable

$P_4 \{S \rightarrow BaB|Ba|aB|a, B \rightarrow bBb|bb|a|D\}$

$G_3 = \{\{B\}, \{a, b\}, P_4, S\}$

CNF:

Let $C_1 \rightarrow a$, $C_2 \rightarrow B$, $C_3 \rightarrow BC_1$, $C_4 \rightarrow BC_2$

$P_5 \{S \rightarrow C_3B|BC_1|C_1B|a, B \rightarrow C_2C_4|C_2C_2|a\}$

$G_4 = \{\{B, C_1, C_2, C_3\}, \{a\}, P_5, S\}$

3. $S \rightarrow 0A0|1B1|BB$, $A \rightarrow C$, $B \rightarrow S|A$, $C \rightarrow S|\epsilon$

Sol: Eliminating ϵ -productions:

$V_n = \{C, A, B, S\}$

$P_1 \{ S \rightarrow 0A0|00|1B1|11|BB|B, A \rightarrow C, B \rightarrow S|A, C \rightarrow S \}$

$G_1 = \{\{A, B, C\}, \{0, 1\}, P_1, S\}$

Eliminating unit productions:

(S,S) $S \rightarrow 0A0|00|1B1|11|BB|B$

(S,B) $S \rightarrow S|A$

(S,A) $S \rightarrow C$

(S,C) $S \rightarrow S$

(A,A) $A \rightarrow C$

(A,C) $A \rightarrow S$

(A,S) $A \rightarrow 0A0|00|1B1|11|BB|B$

(A,B) $A \rightarrow S|A$

(B,B) $B \rightarrow S|A$

(B,A) $B \rightarrow C$

(B,C) $B \rightarrow S$

(B,S) $B \rightarrow 0A0|00|1B1|11|BB|B$

(C,C) $C \rightarrow S$

(C,S) $C \rightarrow 0A0|00|1B1|11|BB|B$

(C,B) $C \rightarrow S|A$

(C,A) $C \rightarrow C$

$P_2 \{ S \rightarrow 0A0|00|1B1|11|BB, A \rightarrow 0A0|00|1B1|11|BB, B \rightarrow 0A0|00|1B1|11|BB,$

$C \rightarrow 0A0|00|1B1|11|BB \}$

$G_2 = \{\{A, B, C\}, \{0, 1\}, P_2, S\}$

Useful symbols:

All are generating.

C is not reachable

$P_3 \{ S \rightarrow 0A0|00|1B1|11|BB, A \rightarrow 0A0|00|1B1|11|BB, B \rightarrow 0A0|00|1B1|11|BB \}$

$G_3 = \{\{A, B\}, \{0, 1\}, P_3, S\}$

CNF:

Let $C_1 \rightarrow 0, C_2 \rightarrow 1, C_3 \rightarrow C_1A, C_4 \rightarrow C_2B$

$P_4 \{ S \rightarrow C_3C_1|C_1C_1|C_4C_2|C_2C_2|BB, A \rightarrow C_3C_1|C_1C_1|C_4C_2|C_2C_2|BB, B \rightarrow C_3C_1|C_1C_1|C_4C_2|C_2C_2|BB \}$

$G_4 = \{A, B, C_1, C_2, C_3, C_4\}, \{0, 1\}, P_4, S\}$

4. $S \rightarrow AAA|B, A \rightarrow aA|B, B \rightarrow \epsilon$

Sol: Eliminating ϵ -productions:

$V_n = \{A, B, S\}$

$P_1 \{S \rightarrow AAA|AA|A|B, A \rightarrow aA|B|a\}$

$G_1 = \{A, B\}, \{a\}, P_1, S\}$

Eliminating unit productions:

$(S, S) S \rightarrow AAA|AA|A|B$

$(S, A) S \rightarrow aA|B|a$

(S, B)

$(A, A) S \rightarrow aA|B|a$

(A, B)

$P_2 \{S \rightarrow AAA|AA|aA|a, A \rightarrow aA|a\}$

$G_2 = \{A\}, \{a\}, P_2, S\}$

Useful symbols:

All are generating and reachable

$P_3 \{S \rightarrow AAA|AA|aA|a, A \rightarrow aA|a\}$

$G_3 = \{A\}, \{a\}, P_2, S\}$

CNF:

Let $C_1 \rightarrow a, C_2 \rightarrow AA$

$P_4 \{S \rightarrow C_2A|AA|C_1A|a, A \rightarrow C_1A|a\}$

$G_4 = \{A, C_1, C_2\}, \{a\}, P_4, S\}$

5. $S \rightarrow aAa|bBb| \epsilon, A \rightarrow C|a, B \rightarrow C|b, C \rightarrow CDE| \epsilon, D \rightarrow A|B|ab$

Sol: Eliminating ϵ -productions:

$V_n = \{C, D, B, A, S\}$

$P_1 \{S \rightarrow aAa|aa|bBb|bb, A \rightarrow C|a, B \rightarrow C|b, C \rightarrow CDE|CE|DE, D \rightarrow A|B|ab\}$

$G_1 = \{A, B, C, D\}, \{a, b\}, P_1, S\}$

Eliminating unit productions:

(S,S) $S \rightarrow aAa \mid aa \mid bBb \mid bb$

(A,A) $A \rightarrow C \mid a$

(A,C) $A \rightarrow CDE \mid CE \mid DE$

(B,B) $B \rightarrow C \mid b$

(B,C) $B \rightarrow CDE \mid CE \mid DE$

(C,C) $C \rightarrow CDE \mid CE \mid DE$

(D,D) $D \rightarrow A \mid B \mid ab$

(D,A) $D \rightarrow A \mid a$

(D,C) $D \rightarrow CDE \mid CE \mid DE$

(D,B) $D \rightarrow C \mid b$

$P_2 \{ S \rightarrow aAa \mid aa \mid bBb \mid bb, A \rightarrow CDE \mid CE \mid DE \mid a, B \rightarrow CDE \mid CE \mid DE \mid b, C \rightarrow CDE \mid CE \mid DE, D \rightarrow CDE \mid CE \mid DE \mid b \mid ab \}$

$G_2 = \{ \{A, B, C, D\}, \{a, b\}, P_2, S \}$

Useful symbols:

C is not generating.

$P_3 \{ S \rightarrow aAa \mid aa \mid bBb \mid bb, A \rightarrow DE \mid a, B \rightarrow DE \mid b, D \rightarrow DE \mid b \mid ab \}$

D is not reachable

$P_4 \{ S \rightarrow aAa \mid aa \mid bBb \mid bb, A \rightarrow a, B \rightarrow b \}$

$G_3 = \{ \{A, B\}, \{a, b\}, P_4, S \}$

CNF:

Let $C_1 \rightarrow a, C_2 \rightarrow b, C_3 \rightarrow C_1A, C_4 \rightarrow C_2B$

$P_4 \{ S \rightarrow C_3C_1 \mid C_1C_1 \mid C_4C_2 \mid C_2C_2, A \rightarrow a, B \rightarrow b \}$

$G_4 = \{ \{A, B, C_1, C_2, C_3, C_4\}, \{a, b\}, P_4, S \}$

6. $S \rightarrow aA \mid a \mid B \mid C, A \rightarrow aB \mid \epsilon, B \rightarrow aA, C \rightarrow cC, D \rightarrow abd$

Sol: Eliminating ϵ -productions:

$V_n = \{A\}$

$P_1 \{ S \rightarrow aA \mid a \mid B \mid C, A \rightarrow ab, B \rightarrow aA \mid a, C \rightarrow cC, D \rightarrow abd \}$

$G_1 = \{\{A, B, C, D\}, \{a, b, c\}, P_1, S\}$

Eliminating unit productions:

$(S, S) \quad S \rightarrow aA \mid a \mid B \mid C$

$(S, B) \quad S \rightarrow aA \mid a$

$(S, C) \quad S \rightarrow cC$

$(A, A) \quad A \rightarrow aB$

$(B, B) \quad B \rightarrow aA \mid a$

$(C, C) \quad C \rightarrow cC$

$(D, D) \quad D \rightarrow abd$

$P_2 \{S \rightarrow aA \mid a \mid cC, A \rightarrow aB, B \rightarrow aA \mid a, C \rightarrow cC, D \rightarrow abd\}$

$G_2 = \{\{A, B, C, D\}, \{a, b, c\}, P_2, S\}$

Useful symbols:

C is not generating.

$P_3 \{S \rightarrow aA \mid a, A \rightarrow aB, B \rightarrow aA \mid a, D \rightarrow abd\}$

D is not reachable

$P_4 \{S \rightarrow aA \mid a, A \rightarrow aB, B \rightarrow aA \mid a\}$

$G_3 = \{\{A, B\}, \{a, b\}, P_4, S\}$

CNF:

Let $C_1 \rightarrow a$

$P_4 \{S \rightarrow C_1A \mid a, A \rightarrow C_1B, B \rightarrow C_1A \mid a\}$

$G_4 = \{\{A, B, C_1\}, \{a\}, P_4, S\}$

7. $S \rightarrow BAAB, A \rightarrow 0A2 \mid 2A0 \mid \epsilon, B \rightarrow AB \mid 1B \mid \epsilon$

Sol: Eliminating ϵ -productions:

$V_n = \{A, B, S\}$

$P_1 \{S \rightarrow BAAB \mid B \mid A \mid BA \mid AB \mid AA \mid BB \mid BAA \mid BAB \mid AAB, A \rightarrow 0A2 \mid 02 \mid 2A0 \mid 20, B \rightarrow AB \mid A \mid B \mid 1B \mid 1\}$

$G_1 = \{\{A, B\}, \{0, 1, 2\}, P_1, S\}$

Eliminating unit productions:

$(S, S) \quad S \rightarrow BAAB \mid B \mid A \mid BA \mid AB \mid AA \mid BB \mid BAA \mid BAB \mid AAB$

(S,A) $S \rightarrow 0A2 \mid 02 \mid 2A0 \mid 20$

(S,B) $S \rightarrow AB \mid A \mid B \mid 1B \mid 1$

(A,A) $A \rightarrow 0A2 \mid 02 \mid 2A0 \mid 20$

(B,B) $B \rightarrow AB \mid A \mid B \mid 1B \mid 1$

(B,A) $B \rightarrow 0A2 \mid 02 \mid 2A0 \mid 20$

$P_2 \{ S \rightarrow BAAB \mid BA \mid AB \mid AA \mid BB \mid BAA \mid BAB \mid AAB \mid 0A2 \mid 02 \mid 2A0 \mid 20 \mid AB \mid 1B \mid 1, A \rightarrow 0A2 \mid 02 \mid 2A0 \mid 20,$

$B \rightarrow AB \mid 0A2 \mid 02 \mid 2A0 \mid 20 \mid 1B \mid 1 \}$

$G_2 = \{ \{A,B\}, \{0,1,2\}, P_2, S \}$

Useful symbols:

All are generating and reachable.

$P_3 \{ S \rightarrow BAAB \mid BA \mid AB \mid AA \mid BB \mid BAA \mid BAB \mid AAB \mid 0A2 \mid 02 \mid 2A0 \mid 20 \mid AB \mid 1B \mid 1, A \rightarrow 0A2 \mid 02 \mid 2A0 \mid 20,$

$B \rightarrow AB \mid 0A2 \mid 02 \mid 2A0 \mid 20 \mid 1B \mid 1 \}$

$G_3 = \{ \{A,B\}, \{0,1,2\}, P_3, S \}$

CNF:

Let $C_1 \rightarrow 1, C_2 \rightarrow 0, C_3 \rightarrow 2, C_4 \rightarrow C_2A, C_5 \rightarrow C_3A, C_6 \rightarrow BA, C_7 \rightarrow AB$

$P_4 \{ S \rightarrow C_4A \mid C_2C_3 \mid C_5C_2 \mid C_3C_2 \mid AB \mid C_1B \mid 1 \mid BA \mid AB \mid AA \mid BB \mid C_6A \mid C_6B \mid AC_7 \mid C_6C_7,$

$A \rightarrow C_4C_3 \mid C_2C_3 \mid C_5C_2 \mid C_3C_2, B \rightarrow C_4C_3 \mid C_2C_3 \mid C_5C_2 \mid C_3C_2 \mid C_1B \mid AB \mid 1 \}$

$G_4 = \{ \{A,B,C_1,C_2,C_3,C_4,C_5,C_6,C_7\}, \{0,1,2\}, P_4, S \}$