Unit 2- Counting inversions

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Topics
1. Counting inversions a. Problem. b. Designing algorithm based on divide and conquer approach. c. Analysis

Counting Inversions

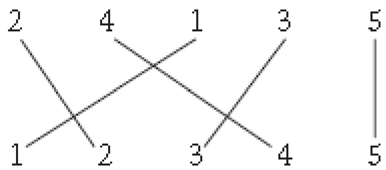
- We are given a sequence of n numbers $a_1, \ldots, a_n$; we will assume that all the numbers are distinct.
- We want to define a measure that tells us how far this list is from being in ascending order; the value of the measure should be 0 if $a_1 < a_2 < \ldots < a_n$, and should increase as the numbers become more scrambled.

Counting the number of *inversions*

The two indices $i < j$ form an inversion if $a_i > a_j$, that is, if the two elements *ai* and *aj* are "out of order." **Counting the number of *inversions*** is to determine the number of inversions in the sequence *a*1, . . . , *an*.

Example:

Sequence is 2, 4, 1, 3, 5.
There are three inversions in this sequence: (2, 1), (4, 1), and (4, 3).



- if the sequence is in descending order, then every pair forms an inversion, and so there are
- n(n-1)/2 pairs.

Example:

5	4	3	2	1

{5,4}, {5,3},{5,2},{5,1}
{4,3}, {4,2}, {4,1}
{3,2}, {3,1}
{2,1}

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Algorithm:

- Clearly, we could look at every pair of numbers (a_i , a_j) and determine whether they constitute an inversion; this would take $O(n^2)$ time.
- The basic idea is to follow the strategy of divide and conquer.
- We set m = _n/2_ and divide the list into the two pieces $a_1, \ldots, a_m$ and $a_{m+1}, \ldots, a_n$.
- We first count the number of inversions in each of these two halves separately.
- Then we count the number of inversions (a_i , a_j), where the two numbers belong to different halves.

- Note that these first-half/second-half inversions have a particularly nice form: they are precisely the pairs (a_i , a_j), where a_i is in the first half, a_j is in the second half.
- Suppose we have recursively sorted the first and second halves of the list and counted the inversions in each. We now have two sorted lists A and B, containing the first and second halves, respectively.
- We want to produce a single sorted list C from their union, while also counting the number of pairs (a, b) with $a \in A$, $b \in B$, and $a > b$.

Algorithm: Counting Inversions

//Purpose: To count the inversions for a given list L(a_1,a_2,……….a_n)
//Input: An unsorted list of distinct numbers L(a_1,a_2,……….a_n)
//Output: The number of inversions **r** for the list L(a_1,a_2,……….a_n)

```
Sort-and-Count(L)
If the list has one element
      then there are no inversions
Else
      Divide the list into two halves:
      A.contains the first _n/2_ elements
      B.contains the remaining _n/2 elements
      (rA, A) = Sort-and-Count(A)
      (rB, B) = Sort-and-Count(B)
      (r , L) = Merge-and-Count(A, B)
Endif
Return r = rA + rB + r, and the sorted list L
```

Algorithm: Merge two lists A and B from Sort-and-Count to count the number of inversions in the list L($a_1, a_2, \ldots\ldots\ldots a_n$)
//Purpose: To Merge two lists A and B from Sort-and-Count to count the number of inversions in the list L($a_1, a_2, \ldots\ldots\ldots a_n$)
//Input: An unsorted list of distinct numbers A($a_1, a_2, \ldots\ldots\ldots a_{n/2}$) and B($a_{n/2+1}, a_{n/2+2}, \ldots\ldots\ldots a_n$)
//Output: The number of inversions **r** for the sorted list L($a_1, a_2, \ldots\ldots\ldots a_n$)

```
Merge-and-Count(A,B)
     Maintain a Current pointer into each list, initialized to
     point to the front elements
     Maintain a variable Count for the number of inversions,
     initialized to 0
     While both lists are nonempty:
          Let ai and bj be the elements pointed to by the
          Current pointer
          Append the smaller of these two to the output list
               If bj is the smaller element then
                    Increment Count by the number of elements
               remaining in A
               Endif
          Advance the Current pointer in the list from which the
          smaller element was selected.
     EndWhile
     Once one list is empty, append the remainder of the other list
     to the output
     Return Count and the merged list
```

Analysis

- The running time of Merge-and-Count can be bounded by the analogue of the argument we used for the original merging algorithm at the heart of Mergesort: each iteration of the While loop takes constant time, and in each iteration we add some element to the output that will never be seen again.
- Thus the number of iterations can be at most the sum of the initial lengths of A and B, and so the total running time is O(n).
- The Sort-and-Count algorithm correctly sorts the input list and counts the number of inversions; it runs in O(n log n) time for a list with n elements.

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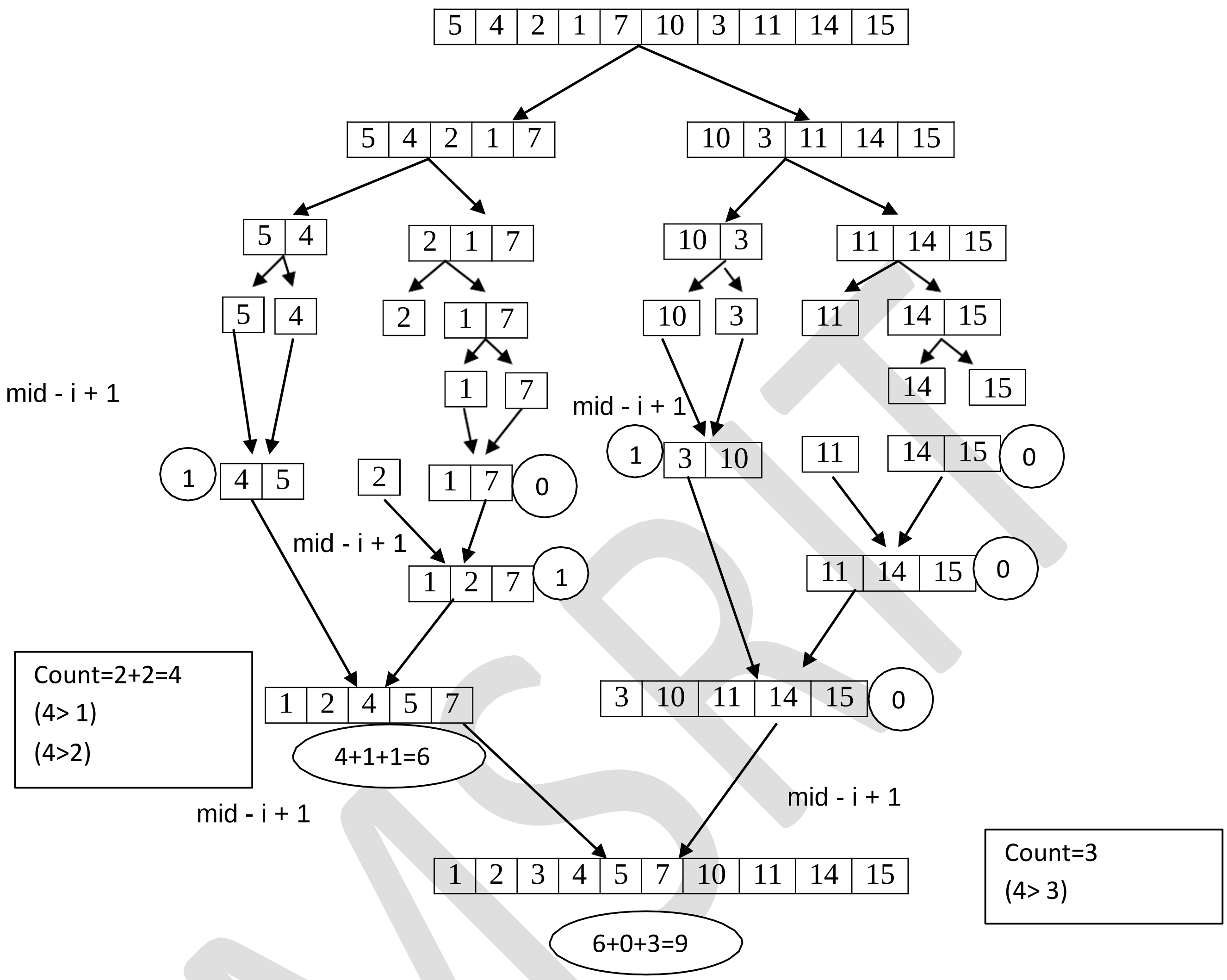
Examples using divide and conquer approach

- Count the number of inversions for the following array.



The total no of inversions=9

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