

27/6/24

Pumping Lemma for Context Free Language

A method used to prove that the language is not context free.

- 1] Let L be a CFL.
- 2] Let n be an integer constant
- 3] Select a string z from L such that $|z| \geq n$
- 4] Divide the string z into 5 parts u, v, w, x, y such that $|vwx| \leq n$ and $|vx| \geq 1$
- 5] For $i \geq 0$, $u v^i w x^i y$ is in L .

① Show that $L = \{a^n b^n c^n \mid n \geq 1\}$ is not a CFL.

Step 1: $L = \{abc, aabbcc, aaabbbccc, \dots\}$

Step 2: Let $n = 3$.

Step 3: Let $z = aaabbbccc$

~~so~~ $(|z| = 9) \geq (n = 3)$.

Step 4: Divide into 5 parts $z = aaabbbccc$

$u = aaa$

$v = b$

$w = b$

$x = b$

$y = ccc$

$$|vwx| \leq n \quad |vx| \geq 1$$

$$3 \leq 3 \quad \checkmark \quad 2 \geq 1 \quad \checkmark$$

Step 5: For $i=0$, $uwy = aaabccc \notin L$.

Hence the given language is not a CFL.

② Prove that $L = \{a^p \mid p \text{ is prime}\}$ is not CFL.

Step 1: $L = \{aa, aaa, aaaaa, aaaaaaa, \dots\}$

Step 2: Let $n = 4$.

Step 3: Let $z = aaaaaaa$

$$|z| \leq n \Rightarrow 7 \leq 4 \quad \checkmark$$

Step 4: Divide into 5 parts $z = aaaaaaa$

$$u = aa \quad v = a \quad w = a \quad x = aa \quad y = a$$

$$|vwx| \leq n \Rightarrow 4 \leq 4 \quad \checkmark \quad |vx| \geq 1 \Rightarrow 3 \geq 1$$

UNIT-V

classmate

Date

Page

Step 5:

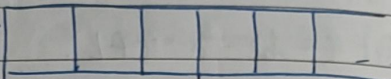
$$\text{For } i=0, uv^0w^0y = uwy \\ = aadaa \notin L.$$

From above, we proved that given language is not CFL.

Self Study

Turing machine

infinite tape



- It is a mathematical model which consists of infinite length tape divided into cells on which input is given.

- A state register stores the state of Turing machine.

- After reading input, it is replaced by another symbol, its internal state changes and it moves from one cell to the right or left.

- A TM is described by 7 tuples $(Q, \Sigma, \Gamma, \delta, q_0, b, F)$

i) Q - finite set of states

ii) Γ - tape alphabet/symbols

iii) Σ - input alphabets/symbols

iv) δ - transition function

$\delta: Q \times \Sigma \rightarrow Q \times \{L, R\}$
where L is left shift, R is right shift.

v) q_0 - initial state

vi) b - blank symbol

vii) F - set of final states (ACCEPT & REJECT) state

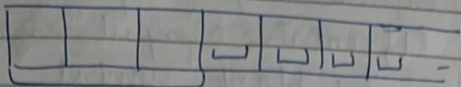
- It is deterministic with infinite tape structure.

Rules of operation

- 1] At each step of computation,
 - Read the current symbol
 - Update (i.e. write) the same cell
 - Move exactly one cell LEFT or RIGHT

head ← can only read symbol pointed by head

* \downarrow



Input
symbol

□ - blank
symbol

4 operations:

- * read (head is pointing) * move one step Left
- * write * move one step Right

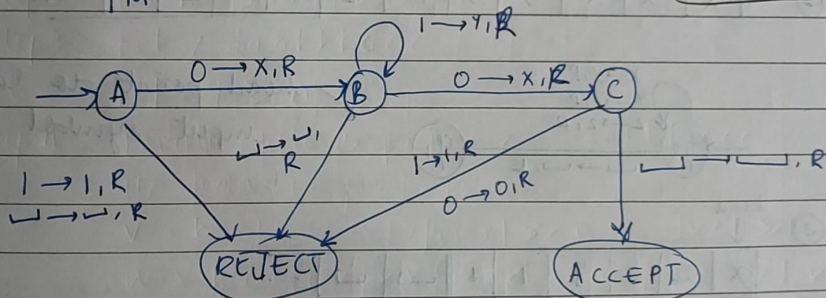
~~not~~

→ We have two final states: (only two)
ACCEPT and REJECT

eg: NFA:

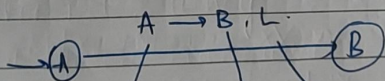
$\{01^*0\}$

TM:



~~A, B, C~~

note:

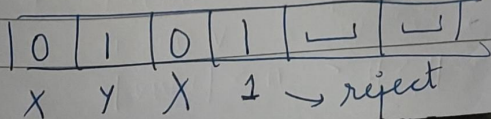
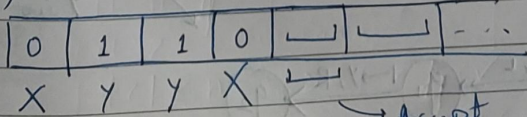


Read

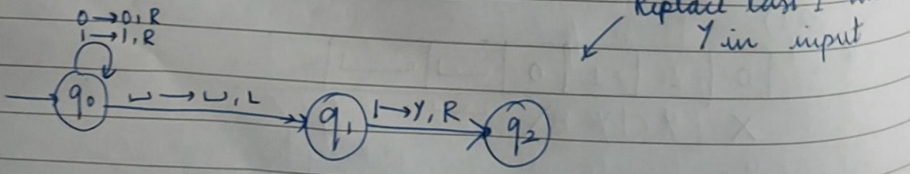
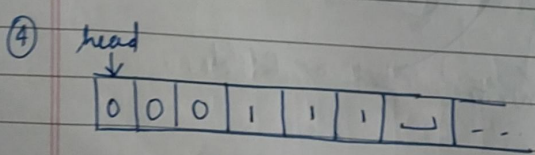
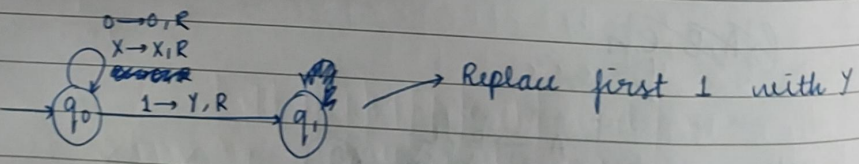
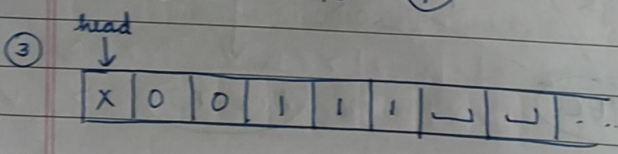
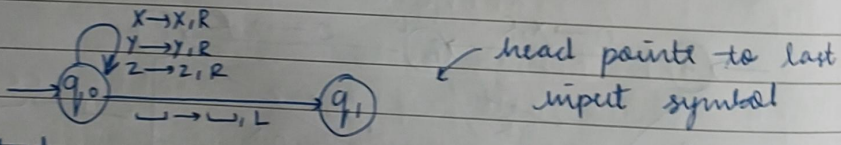
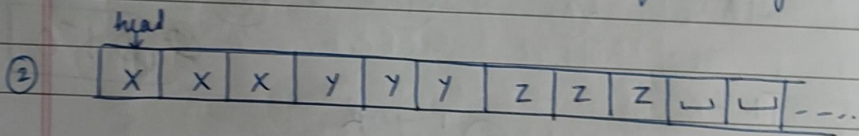
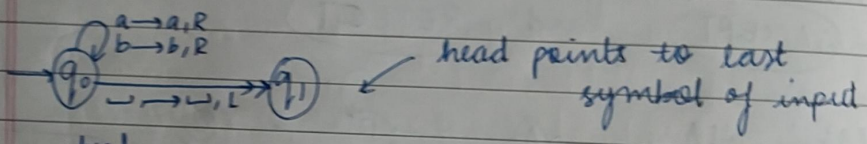
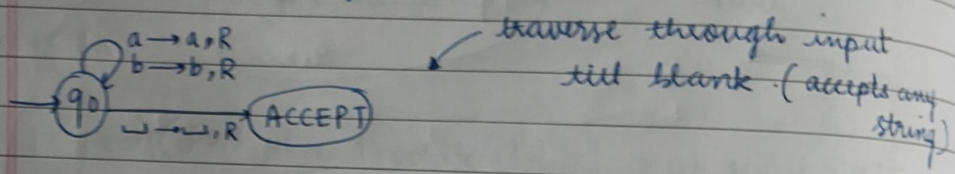
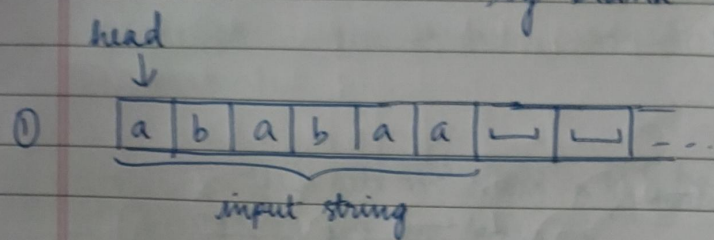
Write

Left or Right

Q1)



2/7/24 Main component : infinite length tape
1, contains input string followed by blank symbol

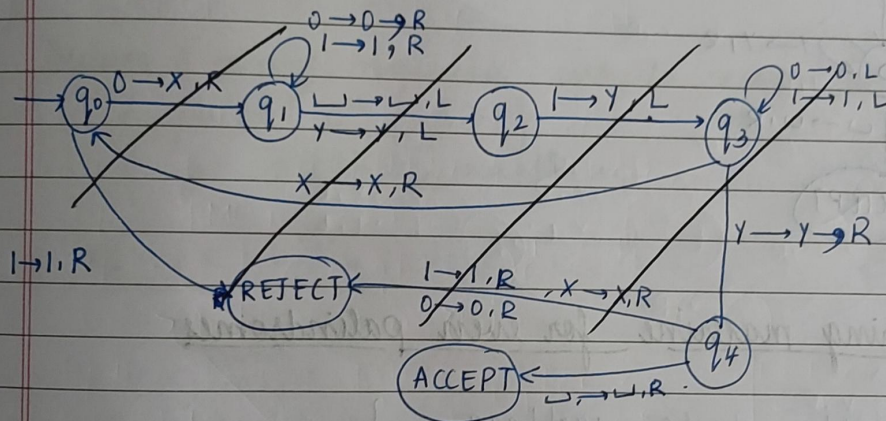


Design a turing machine which recognizes the language $L = \{0^n 1^n \mid n \geq 0\}$

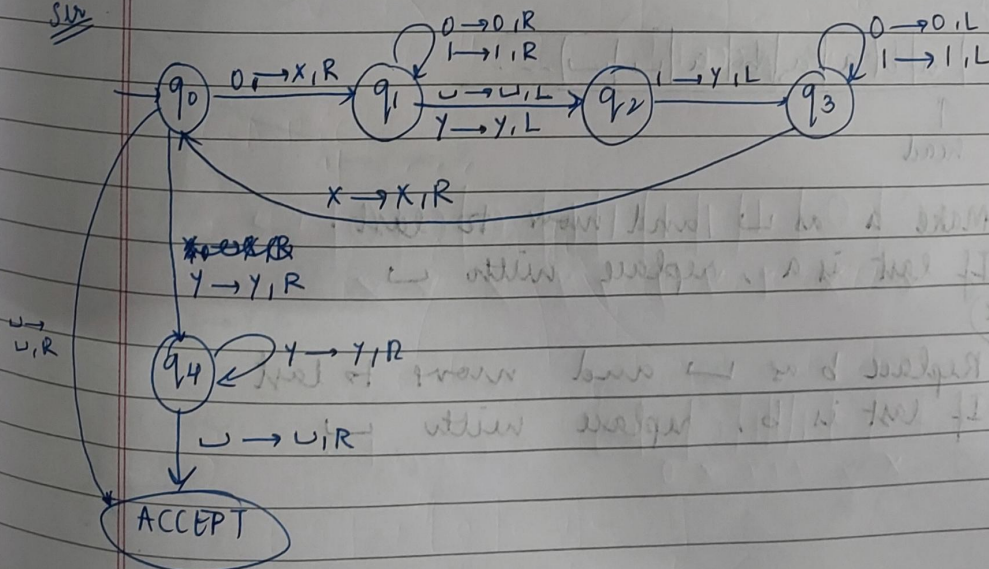
eg:

0	0	0	1	1	1	␣	␣	...
---	---	---	---	---	---	---	---	-----

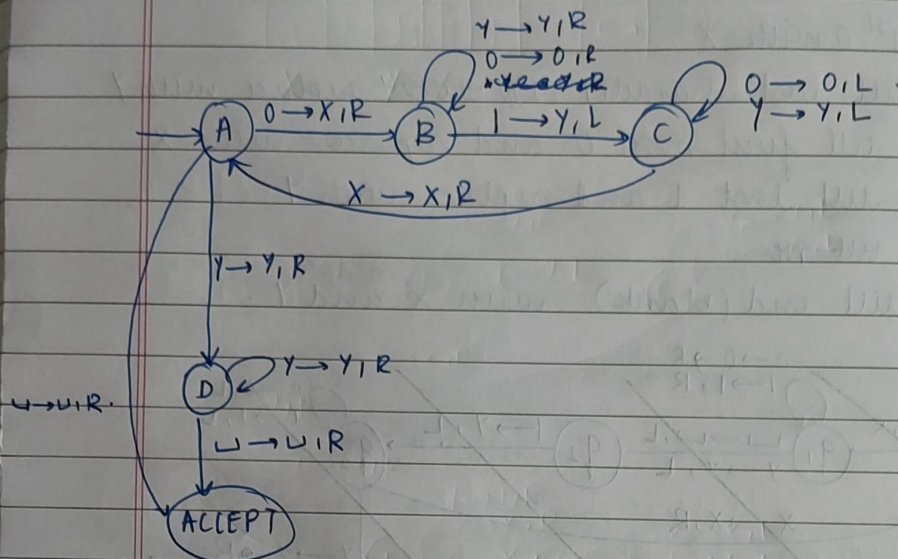
- Replace 1st 0 with X
- Move R till last character (1) ~~and~~ replace with Y
- Move L till first most 0 and replace with X
- Move R till last 1 and replace with Y
- Repeat till no
- Move till end (blank) with X and Y.



Sol

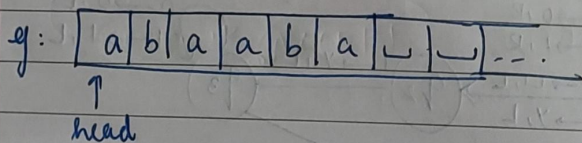


- Replace 0 with X
- Move R till 1st '1' and replace with Y
- Move L till recent 0 and replace with X
- Move R till recent 1 and replace with Y.
- Repeat till blank.

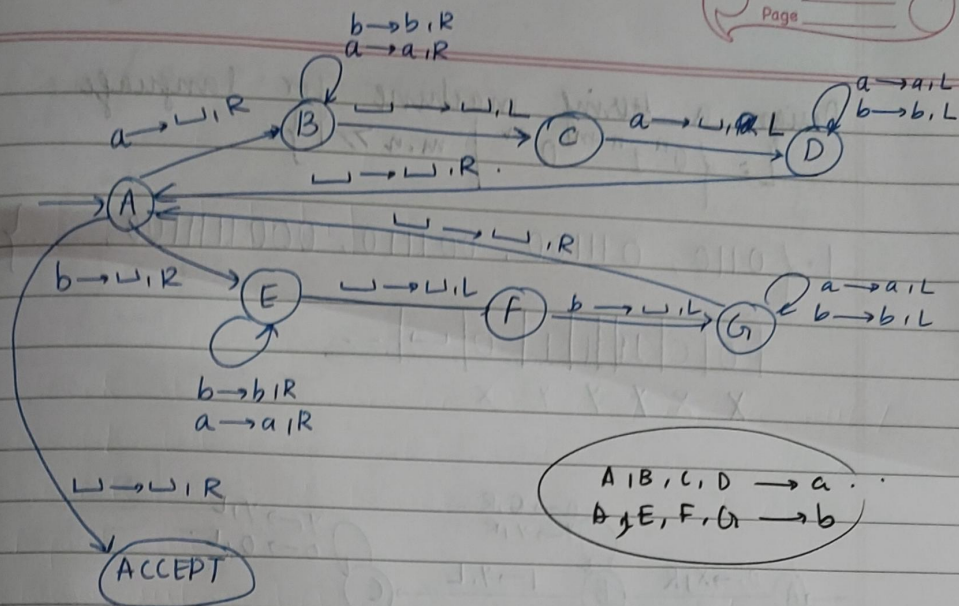


Turing machine for even palindromes

$L = \{abba, abaaba, aaabaaa, \dots\}$

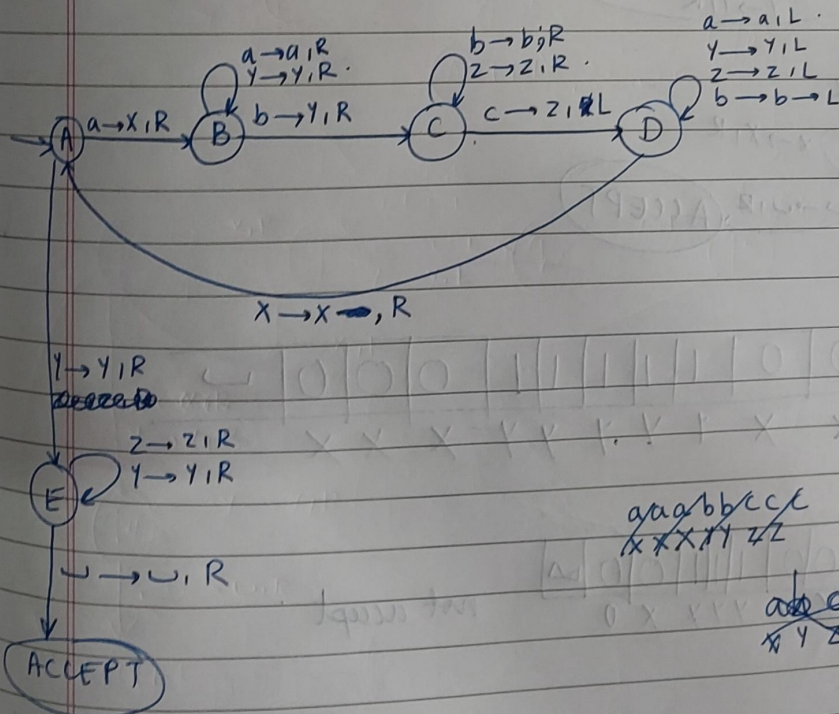
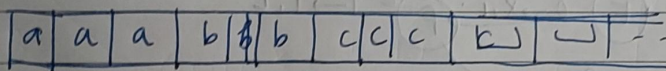


- Make a as \sqcup and move to last.
- If last is a, replace with \sqcup
- ok
- Replace b as \sqcup and move to last
- If last is b, replace with \sqcup



Design a Turing machine which recognizes language
 $L = \{a^n b^n c^n \mid n \geq 1\}$

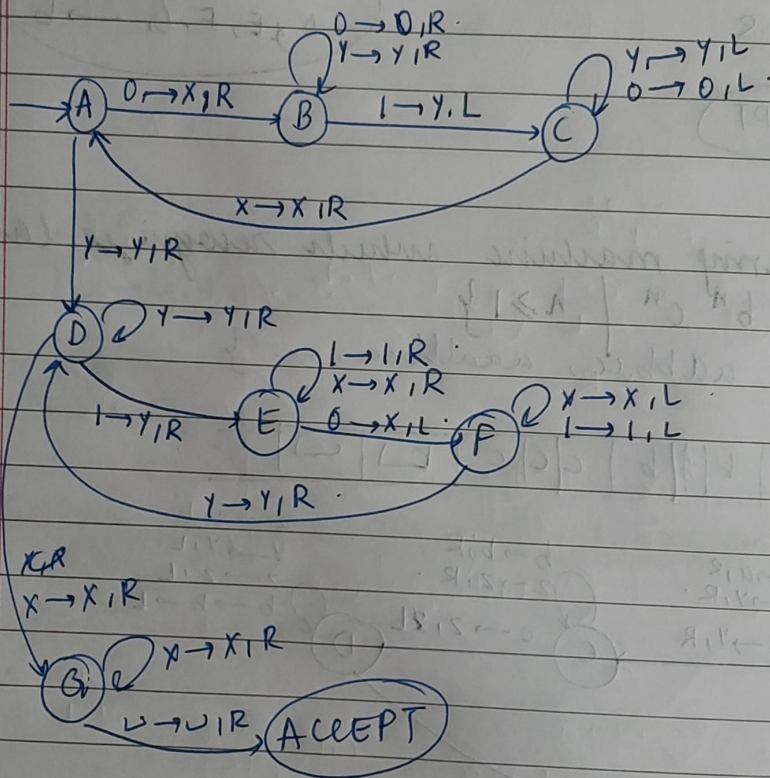
$L = \{abc, aabbcc, aaabbbccc, \dots\}$



Design a Turing machine for language
 $L = \{0^n 1^{n+m} 0^m \mid m, n \geq 1\}$

$L = \{0110, 011100, 001110, 000111100, \dots\}$

0	0	1	1	1	0	␣	...
X	X	Y	Y	Y	X		



0	0	1	1	1	1	0	0	0	␣
X	X	Y	Y	Y	Y	X	X	X	

0	0	1	1	1	0	0	␣
X	X	Y	Y	Y	X	0	

not accept