

Minimization of DFA

1)

δ	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
$*q_4$	q_4	q_4

Solution:

q_1	×			
q_2	×			
q_3	×			
q_4	×	×	×	×
	q_0	q_1	q_2	q_3

$$(q_0, q_1) \xrightarrow{0} (q_1, q_2)$$

$$\simeq^1 (q_3, q_4)$$

$$(q_0, q_2) \xrightarrow{0} (q_1, q_1)$$

$$\simeq^1 (q_3, q_4)$$

$$(q_0, q_3) \xrightarrow{0} (q_1, q_2)$$

$$\simeq^1 (q_3, q_4)$$

$$(q_1, q_2) \xrightarrow{0} (q_2, q_1)$$

$$\simeq^1 (q_4, q_4)$$

$$(q_1, q_3) \xrightarrow{0} (q_2, q_2)$$

$$\simeq^1 (q_4, q_4)$$

$$(q_2, q_3) \xrightarrow{0} (q_1, q_2)$$

$$\simeq^1 (q_4, q_4)$$

$(q_1, q_2), (q_1, q_3), (q_2, q_3)$ are indistinguishable.

$$Q = \{\{q_0\}, \{q_1, q_2, q_3\}, \{q_4\}\}$$

δ	0	1
$\rightarrow q_0$	q_{123}	q_{123}
q_{123}	q_{123}	q_4
$*q_4$	q_4	q_4

2)

δ	0	1
$\rightarrow q_0$	q_1	q_2
$*q_1$	q_3	q_4
$*q_2$	q_5	q_5
$*q_3$	q_3	q_4
$*q_4$	q_5	q_5
q_5	q_6	q_5
q_6	q_6	q_6

Solution:

q_1	\times					
q_2	\times	\times				
q_3	\times		\times			
q_4	\times	\times		\times		
q_5	\times	\times	\times	\times	\times	
q_6	\times	\times	\times	\times	\times	
	q_0	q_1	q_2	q_3	q_4	q_5

$Q = \{\{q_0\}, \{q_1, q_3\}, \{q_2, q_4\}, \{q_5, q_6\}\}$

δ	0	1
$\rightarrow q_0$	q_{13}	q_{24}
$*q_{13}$	q_{13}	q_{24}
$*q_{24}$	q_{56}	q_{56}
q_{56}	q_{56}	q_{56}

3)

δ	0	1
$\rightarrow *q_0$	q_1	q_2
q_1	q_1	q_1
q_2	q_3	q_4
$*q_3$	q_1	q_2
q_4	q_1	q_1

Solution:

q ₁	×			
q ₂	×	×		
q ₃		×	×	
q ₄	×		×	×
	q ₀	q ₁	q ₂	q ₃

$Q = \{\{q_0, q_3\}, \{q_1, q_4\}, \{q_2\}\}$

δ	0	1
\rightarrow^*q_{03}	q ₁₄	q ₂
q ₁₄	q ₁₄	q ₁₄
q ₂	q ₀₃	q ₁₄

4)

δ	0	1
$\rightarrow a$	b	f
b	g	c
$*c$	a	c
d	c	g
e	h	f
f	c	g
g	g	e
h	g	c

Solution:

b	×						
c	×	×					
d	×	×	×				
e		×	×	×			
f	×	×	×		×		
g	×	×	×	×	×	×	
h	×		×	×	×	×	×
	a	b	c	d	e	f	g

$$Q = \{ae, bh, c, df, g\}$$

δ	0	1
$\rightarrow ae$	bh	df
bh	g	c
*c	ae	c
df	c	g
g	g	ae

5)

δ	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_4
* q_3	q_5	q_5
q_4	q_3	q_3
* q_5	q_5	q_5

Solution:

q_1					
q_2	×	×			
q_3	×	×	×		
q_4	×	×	×	×	
q_5	×	×	×		×
	q_0	q_1	q_2	q_3	q_4

$$Q = \{\{q_0, q_1\}, \{q_2\}, \{q_3, q_5\}, \{q_4\}\}$$

δ	0	1
$\rightarrow q_{01}$	q_{01}	q_{35}
q_2	q_{01}	q_4
* q_{35}	q_{35}	q_{35}
q_4	q_{35}	q_{35}

6)

δ	0	1
$\rightarrow q_0$	q_1	q_5
q_1	q_6	q_2
$*q_2$	q_2	q_0
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

Solution:

$$Q = \{\{q_0, q_4\}, \{q_1, q_7\}, \{q_2\}, \{q_3, q_5\}, \{q_6\}\}$$

δ	0	1
$\rightarrow q_{04}$	q_{17}	q_{35}
q_{17}	q_6	q_2
$*q_2$	q_2	q_{04}
q_{35}	q_2	q_6
q_6	q_6	q_{04}

7)

δ	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
$*q_3$	q_3	q_0
q_4	q_3	q_5
q_5	q_2	q_4
q_6	q_5	q_6
q_7	q_6	q_3

Solution:

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$

δ	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
$*q_3$	q_3	q_0
q_4	q_3	q_5
q_5	q_2	q_4
q_6	q_5	q_6
q_7	q_6	q_3

8)

δ	0	1
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
$*q_6$	q_1	q_3

Solution:

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$

δ	0	1
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
$*q_6$	q_1	q_3

9)

δ	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
$*q_3$	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

Solution:

$$Q = \{\{q_0, q_6\}, \{q_1, q_5\}, \{q_2, q_4\}, \{q_3\}\}$$

δ	0	1
$\rightarrow q_{06}$	q_{15}	q_{06}
q_{15}	q_{06}	q_{24}
q_{24}	q_3	q_{15}
$*q_3$	q_3	q_{06}

10)

δ	a	b
$\rightarrow 1$	2	3
2	3	5
3	4	3
4	3	5
$*5$	2	5

Solution:

$$Q = \{13, 24, 5\}$$

δ	a	b
$\rightarrow 13$	24	13
24	13	5
$*5$	24	5

11)

δ	a	b
$\rightarrow 1$	2	1
*2	4	1
3	2	5
4	3	6
*5	4	5
6	5	2

12)

δ	a	b
$\rightarrow 1$	2	6
2	1	3
* 3	2	4
4	4	2
5	4	5
*6	5	4

13)

δ	a	b
$\rightarrow 1$	4	6
2	1	7
3	2	4
4	6	5
5	7	5
*6	3	6
*7	3	7

14)

δ	a	b
$\rightarrow 1$	2	3
2	4	5
3	6	7
*4	4	5
5	6	7
*6	4	5
*7	6	ϕ

15)

δ	a	b
$\rightarrow *1$	2	3
*2	4	5
3	6	7
4	5	4
5	7	5
6	ϕ	7
*7	7	4

16)

δ	a	b
$\rightarrow 1$	2	3
*2	4	5
3	6	7
4	4	5
5	6	7
*6	4	5
7	6	7