

Turing Machines

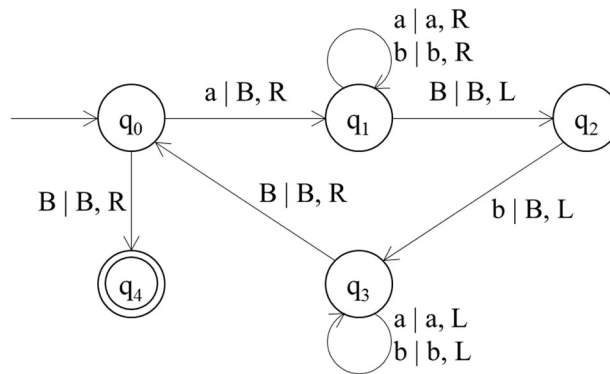
Design a TM for each of the following languages

1. $L = \{a^n b^n \mid n \geq 0\}$

Let $w = aaabbb$

We define a TM which converts the given string to a smaller one such that it accepts the smaller string only if the initial string is valid.

It can be done by converting $w = aaabbb$ to $w = aabb$ by removing the first 'a' and the last 'b'. We continue this process and the string is accepted if $w = \epsilon$ at the end.



$$M = (Q, \Sigma, \tau, \delta, q_0, B, F)$$

$$Q = \{q_0, q_1, q_2, q_3, q_4\}, \Sigma = \{a, b\}, \tau = \{a, b, B\}, F = \{q_4\}$$

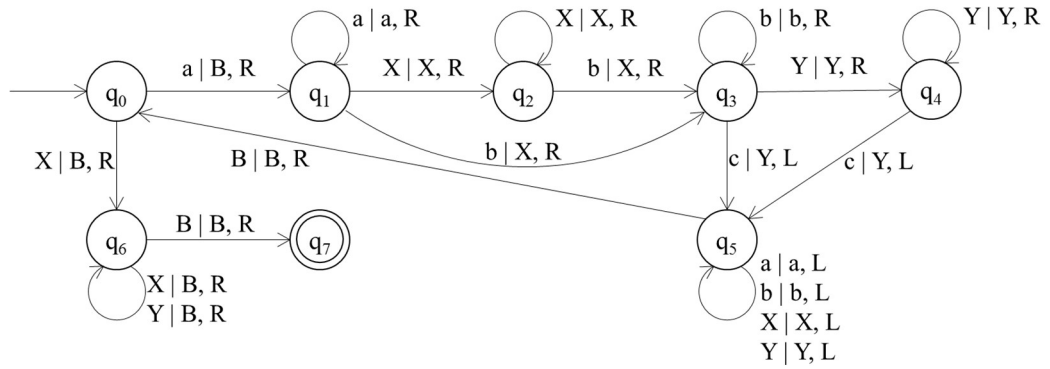
q_0 is start state, B is Blank tape symbol

δ	a	b	B
$\rightarrow q_0$	(q1, B, R)	-	(q4, B, R)
q1	(q1, a, R)	(q1, b, R)	(q2, B, L)
q2	-	(q3, B, L)	-
q3	(q3, a, L)	(q3, b, L)	(q0, B, R)
*q4	-	-	-

ID for $w = aaabbb$

$Bq_0aaabbb \vdash Bq_1aabbb \vdash aq_1abbb \vdash aaq_1bbb \vdash aabq_1bb \vdash aabbq_1b \vdash$
 $aabbbq_1B \vdash aabbq_2b \vdash aabbq_3B \vdash aabq_3b \vdash aaq_3bb \vdash aq_3abb \vdash Bq_3aabb \vdash$
 $Bq_3Baabb \vdash Bq_0aabb \vdash Bq_1abb \vdash aq_1bb \vdash abq_1b \vdash abbq_1B \vdash abq_2b \vdash abq_3B \vdash$
 $aq_3b \vdash Bq_3ab \vdash Bq_3Bab \vdash Bq_0ab \vdash Bq_1b \vdash bq_1B \vdash Bq_2b \vdash Bq_3B \vdash Bq_3B \vdash Bq_4B$

$$2. L = \{a^n b^n c^n \mid n \geq 1\}$$



$$M = (Q, \Sigma, \tau, \delta, q_0, B, F)$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \Sigma = \{a, b\}, \tau = \{a, b, B, X, Y\}, F = \{q_7\}$$

q_0 is start state, B is Blank tape symbol

δ	a	b	c	X	Y	B
$\rightarrow q_0$	(q ₁ , B, R)	-	-	(q ₆ , B, R)	-	-
q ₁	(q ₁ , a, R)	(q ₃ , X, R)	-	(q ₂ , X, R)	-	-
q ₂	-	(q ₃ , X, R)	-	(q ₂ , X, R)	-	-
q ₃	-	(q ₃ , b, R)	(q ₅ , Y, L)	-	(q ₄ , Y, R)	-
q ₄	-	-	(q ₅ , Y, L)	-	(q ₄ , Y, R)	-
q ₅	(q ₅ , a, L)	(q ₅ , b, L)	-	(q ₅ , X, L)	(q ₅ , Y, L)	(q ₀ , B, R)
q ₆	-	-	-	(q ₆ , B, R)	(q ₆ , B, R)	(q ₇ , B, R)
*q ₇	-	-	-	-	-	-

ID for $w = aabbcc$

$$Bq_0aabbcc \vdash Bq_1abbcc \vdash aq_1bbcc \vdash aXq_3bcc \vdash aXbq_3cc \vdash aXq_5bYc \vdash$$

$$aq_5XbYc \vdash Bq_5aXbYc \vdash Bq_5BaXbYc \vdash Bq_0aXbYc \vdash Bq_1XbYc \vdash Xq_2bYc \vdash$$

$$XXq_3Yc \vdash XXYq_4c \vdash XXq_5YY \vdash Xq_5XYY \vdash Bq_5XXYY \vdash Bq_5BXXY \vdash$$

$$Bq_0XXYY \vdash Bq_6XYY \vdash Bq_6YY \vdash Bq_6Y \vdash Bq_6B \vdash Bq_7B$$

$$3. L = \{ww^r \mid w \in \{a, b\}^*\}$$

$$M = (Q, \Sigma, \tau, \delta, q_0, B, F)$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \Sigma = \{a, b\}, \tau = \{a, b, B\}, F = \{q_6\}$$

q_0 is start state, B is Blank tape symbol

δ	a	b	B
$\rightarrow q_0$	(q ₁ , B, R)	(q ₃ , B, R)	(q ₆ , B, R)
q ₁	(q ₁ , a, R)	(q ₁ , b, R)	(q ₂ , B, L)
q ₂	(q ₅ , B, L)	-	-
q ₃	(q ₃ , a, R)	(q ₃ , b, R)	(q ₄ , B, L)
q ₄	-	(q ₅ , B, L)	-
q ₅	(q ₅ , a, L)	(q ₅ , b, L)	(q ₀ , B, R)
*q ₆	-	-	-

Let $w = abaaba$

$$Bq_0abaaba \vdash Bq_1baaba \vdash bq_1aaba \vdash baq_1aba \vdash baaq_1ba \vdash baabq_1a \vdash baabaq_1B$$

$$\vdash baabq_2a \vdash baaq_5bB \vdash baq_5ab \vdash bq_5aab \vdash Bq_5baab \vdash Bq_5baab \vdash Bq_5Bbaab \vdash$$

$$Bq_0baab \vdash Bq_3aab \vdash aq_3ab \vdash aaq_3b \vdash aabq_3B \vdash aaq_4b \vdash aq_5aB \vdash Bq_5aa \vdash$$

$$Bq_5Baa \vdash Bq_0aa \vdash Bq_1a \vdash Baq_1B \vdash Bq_2a \vdash Bq_5B \vdash Bq_0B \vdash Bq_6B$$

4. $L = \{0^n 1^{n+m} 0^m \mid m, n \geq 1\}$

Let $w = 001110$

We will now look at 2 ways of solving this problem.

The usual approach:

$001110 \rightarrow X0Y110 \rightarrow XXYY10 \rightarrow XXYYYYX$

$M = (Q, \Sigma, \tau, \delta, q_0, B, F)$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \Sigma = \{0, 1\}, \tau = \{0, 1, X, Y, B\}, F = \{q_7\}$

q_0 is start state, B is Blank tape symbol

δ	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)	-	-	(q_3, Y, R)	-
q₁	$(q_1, 0, R)$	(q_2, Y, L)	-	(q_1, Y, R)	-
q₂	$(q_2, 0, L)$	-	(q_0, X, R)	(q_2, Y, L)	-
q₃	-	(q_4, Y, R)	(q_6, X, R)	(q_3, Y, R)	-
q₄	(q_5, X, L)	$(q_4, 1, R)$	(q_4, X, R)	-	-
q₅	-	$(q_5, 1, L)$	(q_5, X, L)	(q_3, Y, R)	-
q₆	-	-	(q_6, X, R)	-	(q_7, B, R)
*q₇	-	-	-	-	-

Let $w = 001110$

$Bq_0001110 \vdash Xq_101110 \vdash X0q_11110 \vdash Xq_20Y110 \vdash Bq_2X0Y110 \vdash Xq_00Y110$

$\vdash XXq_1Y110 \vdash XXq_1Y110 \vdash XXYq_1110 \vdash XXq_2YY10 \vdash Xq_2XYY10 \vdash$

$XXq_0YY10 \vdash XXYq_3Y10 \vdash XXYq_310 \vdash XXYq_40 \vdash XXYq_5YX \vdash$

$XXYq_3X \vdash XXYq_6B \vdash XXYq_7B$

Optimized approach:

001110 \rightarrow 0110 \rightarrow 10 $\rightarrow \epsilon$

If the string begins with 01, replace both of them by blank and move to a different state which accepts strings of the form $1^m 0^m$.

If it begins with 00, replace both of them by blank and change the first 1 to 0. It is equivalent to removing a 0 and a 1. i. e. **00011110** = **BB001110**.

Repeat the same for the other half.

This method can be tweaked easily to work for cases where $m, n = 0$.

$M = (Q, \Sigma, \tau, \delta, q_0, B, F)$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}, \Sigma = \{0, 1\}, \tau = \{0, 1, B\}, F = \{q_9\}$

δ	0	1	B
$\rightarrow q_0$	(q ₁ , B, R)	-	-
q ₁	(q ₂ , B, R)	(q ₄ , B, R)	-
q ₂	(q ₂ , 0, R)	(q ₃ , 0, L)	-
q ₃	(q ₃ , 0, L)	-	(q ₀ , B, R)
q ₄	-	(q ₅ , B, R)	-
q ₅	(q ₈ , B, R)	(q ₆ , B, R)	-
q ₆	(q ₇ , 1, L)	(q ₆ , 1, R)	-
q ₇	-	(q ₇ , 1, L)	(q ₄ , B, R)
q ₈	-	-	(q ₉ , B, R)
*q ₉	-	-	-

Let $w = 0001111100$

$Bq_00001111100 \vdash BBq_1001111100 \vdash BBBq_201111100 \vdash BBB0q_21111100 \vdash$

$BBBq_300111100 \vdash BBq_3B00111100 \vdash Bq_000111100 \vdash Bq_10111100 \vdash$

$Bq_2111100 \vdash Bq_3011100 \vdash Bq_3B011100 \vdash Bq_0011100 \vdash Bq_111100 \vdash Bq_41100$

$\vdash BBq_5100 \vdash BBBq_600 \vdash BBBq_7B10 \vdash Bq_410 \vdash Bq_50 \vdash Bq_8B \vdash Bq_9B$

It is clear that this approach is faster since the ID is smaller for a longer string.

$$5. L = \{w \mid n_a(w) = n_b(w)\}$$

$$M = (Q, \Sigma, \tau, \delta, q_0, B, F)$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}, \Sigma = \{a, b\}, \tau = \{a, b, X, Y, B\}, F = \{q_5\}$$

q_0 is start state, B is Blank tape symbol

δ	a	b	X	Y	B
$\rightarrow q_0$	(q ₁ , X, R)	(q ₀ , b, R)	(q ₀ , X, R)	(q ₀ , Y, R)	(q ₄ , B, L)
q₁	(q ₁ , a, R)	(q ₁ , b, R)	(q ₁ , X, R)	(q ₁ , Y, R)	(q ₂ , B, L)
q₂	(q ₂ , a, L)	(q ₃ , Y, L)	(q ₂ , X, L)	(q ₂ , Y, L)	-
q₃	(q ₃ , a, L)	(q ₃ , b, L)	(q ₃ , X, L)	(q ₃ , Y, L)	(q ₀ , B, R)
q₄	-	-	(q ₄ , B, L)	(q ₄ , B, L)	(q ₅ , B, R)
*q₅	-	-	-	-	-