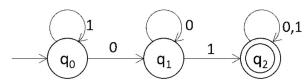
# Techniques for DFA

- 1. If the L is equal to a finite value, if possible, draw that base condition state flow, figure out all the possible transitions for each state.
- 2. If L is not equal to a finite value, draw that base condition flow, and mark the initial ones as final but not the last state. Figure out all the possible transitions for each state.
  - Design a DFA which accepts all strings with a substring 01.

 $L = \{01, 001, 101, 010, 011, 0001, 0010, 0100 \dots\}$ 



 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, q_1, q_2\}$ 

 $\Sigma = \{0, 1\}$ 

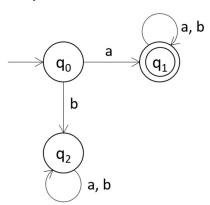
 $q_0 = q_0$  (start state)

 $F = \{q_2\}$ 

δ	0	1
<b>→</b> q₀	$q_1$	$q_0$
$q_1$	$q_1$	$q_2$
*q <sub>2</sub>	$q_2$	$q_2$

2. Construct a DFA over {a, b} which accepts language for all strings starting with symbol 'a'.

L = {a, aa, ab, aaa, aab, aba, abb ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

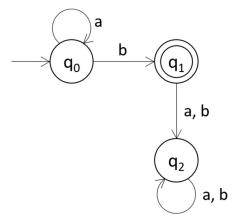
$$q_0 = q_0$$
 (start state)

$$F = \{q_1\}$$

δ	а	b
<b>→</b> q₀	$q_1$	$q_2$
*q <sub>1</sub>	$q_1$	$q_1$
q <sub>2</sub>	$q_2$	$q_2$

**NOTE:** q<sub>2</sub> is trap state or sink state.

3. Construct DFA to accept all strings with arbitrary no. of a's followed by a single b.



 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, q_1, q_2\}$ 

 $\Sigma = \{a, b\}$ 

 $q_0 = q_0$  (start state)

 $F = \{q_1\}$ 

δ	а	b
<b>→</b> q₀	$q_0$	$q_1$
*q <sub>1</sub>	$q_2$	$q_2$
q <sub>2</sub>	$q_2$	$q_2$

Construct DFA for  $\Sigma = \{0, 1\}$  which accepts strings starting with 2 0's & ending with 2 1's.

0

1

0

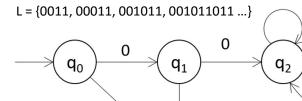
 $q_3$ 

0

1

 $q_4$ 

1



1  $q_5$ 0,1

 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$ 

1

 $\Sigma = \{0, 1\}$ 

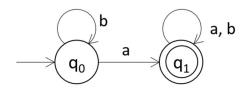
 $q_0 = q_0$  (start state)

 $F = \{q_4\}$ 

δ	0	1
→q₀	$q_1$	<b>q</b> <sub>5</sub>
$q_1$	$q_2$	<b>q</b> <sub>5</sub>
q <sub>2</sub>	$q_2$	q <sub>3</sub>
q <sub>3</sub>	$q_2$	$q_4$
*q <sub>4</sub>	$q_2$	$q_4$
q <sub>5</sub>	<b>q</b> <sub>5</sub>	<b>q</b> <sub>5</sub>

### **S**. Design automata for $\Sigma = \{a, b\}$ strings with at least one 'a'.

L = {a, aa, ab, ba, aaa, aab, baa, abb ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1\}$$

$$\Sigma = \{a, b\}$$

$$q_0 = q_0 \text{ (start state)}$$

 $F = \{q_1\}$ 

δ	а	b
→q₀	$q_1$	$q_0$
*q <sub>1</sub>	$q_1$	$q_1$

#### Extending the transition function to strings ( $\delta^*$ ):-

Example: 1) baab

$$\delta^*(q_0, \epsilon) = q_0$$

$$\delta^*(q_0, b) = \delta(\delta^*(q_0, \epsilon), b)$$

$$= \delta(q_0, b)$$

$$= q_0$$

$$\delta^*(q_0, ba) = \delta(\delta^*(q_0, b), a)$$

$$= \delta(q_0, a)$$

$$= q_1$$

$$\delta^*(q_0, baa) = \delta(\delta^*(q_0, ba), a)$$

$$= \delta(q_1, a)$$

$$= q_1$$

$$\delta^*(q_0, baab) = \delta(\delta^*(q_0, baa), b)$$

$$= q_1$$

$$\delta^*(q_0, baab) = \delta(\delta^*(q_0, baa), b)$$

$$= q_1$$

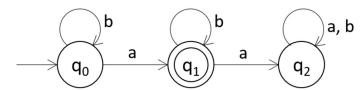
Since  $q_1 \in F$ , baab is valid.

Example: 2) bbb

$$\begin{split} \delta^*(q_0,\,\epsilon) &= q_0 \\ \delta^*(q_0,\,\epsilon) &= \delta(\delta^*(q_0,\,\epsilon),\,b) \\ &= \delta(q_0,\,b) \\ &= q_0 \\ \delta^*(q_0,\,b) &= \delta(\delta^*(q_0,\,b),\,b) \\ &= \delta(q_0,\,b) \\ &= q_0 \\ \delta^*(q_0,\,bbb) &= \delta(\delta^*(q_0,\,bb),\,b) \\ &= \delta(q_0,\,b) \\ &= q_0 \\ \text{Since } q_0 \not\in F,\,bbb \text{ is invalid.} \end{split}$$

β. Design an automata with  $Σ = {a, b}$  that accepts string with exactly one 'a'.

L = {a, ab, ba, abb, bab, bba ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

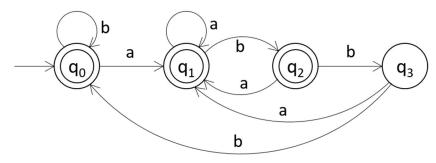
$$q_0 = q_0$$
 (start state)

$$F = \{q_1\}$$

δ	а	b
→q <sub>0</sub>	$q_1$	$q_0$
*q <sub>1</sub>	$q_2$	$q_1$
q <sub>2</sub>	$q_2$	$q_2$

7. Design an automata with  $\Sigma = \{a, b\}$  such that it accepts all strings except those which end with abb.

 $L = \{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abba ...\}$ 



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

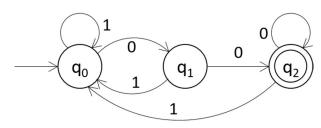
$$\Sigma = \{a, b\}$$

$$q_0 = q_0$$
 (start state)

$$F = \{q_0, q_1, q_2\}$$

δ	а	b
→*q <sub>0</sub>	$q_1$	$q_0$
*q <sub>1</sub>	$q_1$	$q_2$
*q <sub>2</sub>	$q_1$	$q_3$
q <sub>3</sub>	$q_1$	$q_0$

8. Design an automata with  $\Sigma = \{0, 1\}$  that accepts set of all strings ending with 00. L =  $\{00, 000, 100, 1000, 0100, 0000, 1100 ...\}$ 



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2\}$$

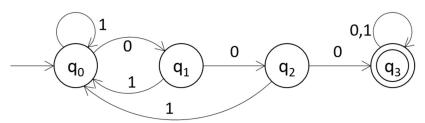
$$\Sigma = \{0, 1\}$$

 $q_0 = q_0$  (start state)

$$F=\{q_2\}$$

δ	0	1
→q₀	$q_1$	$q_0$
$q_1$	$q_2$	$q_0$
*q <sub>2</sub>	$q_2$	$q_0$

9 Draw an automata with  $\Sigma = \{0, 1\}$  that accepts set of all strings with 3 consecutive 0's.  $L = \{000, 1000, 0000, 0001, 101000, 00010001 ...\}$ 



$$A = {Q, Σ, δ, q_0, F}$$

Q = 
$$\{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{0, 1\}$$

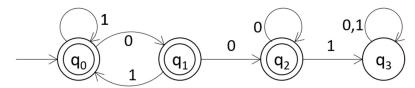
$$q_0 = q_0$$
 (start state)

$$F = \{q_3\}$$

δ	0	1
<b>→</b> q₀	$q_1$	$q_0$
$q_1$	$q_2$	$q_0$
q <sub>2</sub>	$q_3$	$q_0$
*q <sub>3</sub>	$q_3$	$q_3$

10. Design an automata with  $\Sigma = \{0, 1\}$  that accepts set of all strings except those containing substring 001.

 $L = \{\epsilon, \, 0, \, 1, \, 00, \, 11, \, 01, \, 10, \, 101, \, 011, \, 010, \, 010100 \, \ldots \}$ 



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

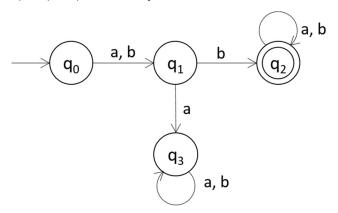
$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$
 (start state)

$$F = \{q_0, q_1, q_2\}$$

δ	0	1
<b>→</b> *q <sub>0</sub>	$q_1$	$q_0$
*q <sub>1</sub>	$q_2$	$q_0$
*q <sub>2</sub>	$q_2$	q <sub>3</sub>
q <sub>3</sub>	$q_3$	q <sub>3</sub>

11. Design an automata with  $\Sigma = \{a, b\}$  that accepts set of all strings with b as second letter. L = { ab, bb, aba, abb, bba, bbb, abbabab ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

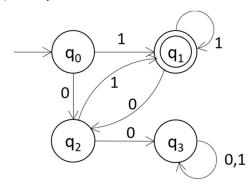
$$\Sigma = \{a, b\}$$

$$q_0 = q_0$$
 (start state)

$$F = \{q_2\}$$

δ	а	b
<b>→</b> q₀	$q_1$	$q_1$
$q_1$	$q_3$	$q_2$
*q <sub>2</sub>	$q_2$	$q_2$
q <sub>3</sub>	$q_3$	q <sub>3</sub>

12 Obtain DFA that accepts all strings on  $\Sigma = \{0, 1\}$  that ends with 1 and do not contain 00.



 $A = {Q, Σ, δ, q_0, F}$ 

 $Q = \{q_0, q_1, q_2, q_3\}$ 

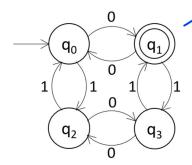
 $\Sigma = \{0, 1\}$ 

 $q_0 = q_0$  (start state)

 $F = \{q_1\}$ 

δ	0	1
<b>→</b> q₀	$q_2$	$q_1$
*q <sub>1</sub>	$q_2$	$q_1$
q <sub>2</sub>	$q_3$	$q_1$
q <sub>3</sub>	$q_3$	$q_3$

13. Obtain DFA to accept the language  $L = \{w \mid n_0(w) \text{ is odd and } n_1(w) \text{ is even}\}$ .



 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, q_1, q_2, q_3\}$ 

 $\Sigma = \{0, 1\}$ 

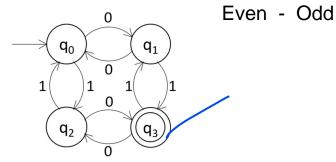
 $q_0 = q_0$  (start state)

 $F = \{q_1\}$ 

δ	0	1
→q <sub>0</sub>	$q_1$	$q_2$
*q <sub>1</sub>	$q_0$	<b>q</b> <sub>3</sub>
q <sub>2</sub>	$q_3$	$q_0$
q <sub>3</sub>	$q_2$	$q_1$

Ü	U	Position
Even -	 · Even	1
Odd -	Even	2
Odd -	Odd	3

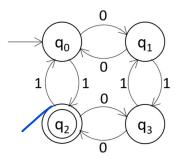
14.	$L = \{w \mid n_0(w) \text{ is } \underline{\text{odd}} \text{ and } n_1(w) \text{ is } \underline{\text{odd}} \}$
,	L = {01, 10, 0010, 1000, 010101}



A = {Q, 
$$\Sigma$$
,  $\delta$ ,  $q_0$ , F}  
Q = {q\_0, q\_1, q\_2, q\_3}  
 $\Sigma$  = {0, 1}  
 $q_0$  =  $q_0$  (start state)  
F = {q\_3}

δ	0	1
<b>→</b> q₀	$q_1$	$q_2$
$q_1$	$q_0$	q <sub>3</sub>
q <sub>2</sub>	$q_3$	$q_0$
*q <sub>3</sub>	$q_2$	$q_1$

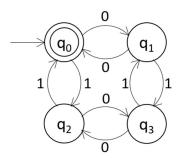
15. L = {w |  $n_0(w)$  is even and  $n_1(w)$  is odd} L = {1, 100, 010, 001, 111, 00111 ...}



$A = \{Q, \Sigma, \delta, q_0, F\}$
$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1\}$
$q_0 = q_0$ (start state)
$F = \{a_2\}$

δ	0	1
<b>→</b> q₀	$q_1$	$q_2$
$q_1$	$q_0$	$q_3$
*q <sub>2</sub>	$q_3$	$q_0$
q <sub>3</sub>	$q_2$	$q_1$

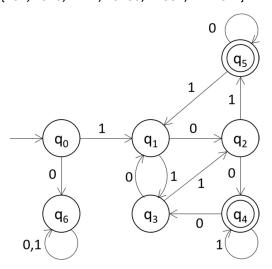
16. L = {w |  $n_0(w)$  is even and  $n_1(w)$  is even} L = { $\epsilon$ , 11, 00, 0101, 01011010 ...}



A = {Q, 
$$\Sigma$$
,  $\delta$ ,  $q_0$ , F}  
Q = {q<sub>0</sub>, q<sub>1</sub>, q<sub>2</sub>, q<sub>3</sub>}  
 $\Sigma$  = {0, 1}  
q<sub>0</sub> = q<sub>0</sub> (start state)  
F = {q<sub>0</sub>}

δ	0	1
→*q <sub>0</sub>	$q_1$	$q_2$
$q_1$	$q_0$	$q_3$
$q_2$	$q_3$	$q_0$
q <sub>3</sub>	$q_2$	$q_1$

17. DFA to accept binary numbers that are divisible by 5 and start with 1.



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$

$$\Sigma = \{0, 1\}$$

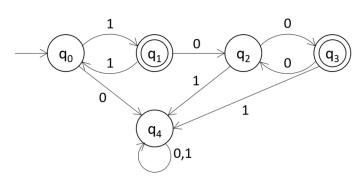
$$q_0 = q_0 \text{ (start state)}$$

$$F = \{q_4, q_5\}$$

δ	0	1
⇒q <sub>0</sub>	$q_6$	$q_1$
$q_1$	$q_2$	$q_3$
q <sub>2</sub>	$q_4$	<b>q</b> <sub>5</sub>
q <sub>3</sub>	$q_1$	$q_2$
*q <sub>4</sub>	$q_3$	$q_4$
*q <sub>5</sub>	<b>q</b> <sub>5</sub>	$q_1$
q <sub>6</sub>	$q_6$	$q_6$

**18.** DFA for L =  $\{w \mid w \text{ has odd no of 1's followed by even no of 0's}$ 

L = {1, 100, 111, 1110000 ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

 $Q = \{q_0, q_1, q_2, q_3, q_4\}$ 

 $\Sigma = \{0, 1\}$ 

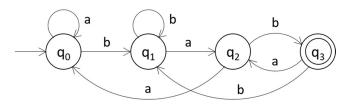
 $q_0 = q_0$  (start state)

 $F = \{q_1, q_3\}$ 

δ	0	1
<b>→</b> q₀	$q_4$	$q_1$
*q <sub>1</sub>	$q_2$	$q_0$
q <sub>2</sub>	$q_3$	$q_4$
*q <sub>3</sub>	$q_2$	$q_4$
q <sub>4</sub>	$q_4$	$q_4$

19. DFA for L = {wbab |  $w \in \{a, b\}^*$ }

L = {bab, abab, bbab, aabab, abbab, abbbababbab ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

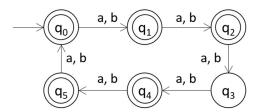
$$q_0 = q_0$$
 (start state)

$$F=\{q_3\}$$

δ	а	b
<b>→</b> q₀	$q_0$	$q_1$
$q_1$	$q_2$	$q_1$
q <sub>2</sub>	$q_0$	$q_3$
*q₃	$q_2$	$q_1$

20. DFA for L = {w| |w| mod 3 >= |w| mod 2} where  $w \in \Sigma^*$  and  $\Sigma$  = {a, b}.

$$L = \{|w| = 1, |w| = 2, |w| = 4, |w| = 5, |w| = 6, |w| = 7 ...\}$$



 $A = {Q, Σ, δ, q_0, F}$ 

 $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$ 

 $\Sigma = \{a, b\}$ 

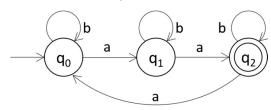
 $q_0 = q_0$  (start state)

 $F = \{q_0, q_1, q_2, q_4, q_5\}$ 

δ	a	b
→*q <sub>0</sub>	$q_1$	$q_1$
*q <sub>1</sub>	$q_2$	$q_2$
*q <sub>2</sub>	$q_3$	$q_3$
q <sub>3</sub>	$q_4$	$q_4$
*q <sub>4</sub>	<b>q</b> <sub>5</sub>	<b>q</b> <sub>5</sub>
*q <sub>5</sub>	$q_0$	$q_0$

21.  $L = \{w \mid n(a) \mod 3 > 1\} \text{ for } \Sigma = \{a, b\}.$ 

L = {aa, aab, baa, babab, abba, ababaabba ...}



 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, q_1, q_2\}$ 

 $\Sigma = \{a, b\}$ 

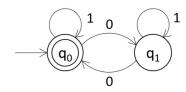
 $q_0 = q_0$  (start state)

 $F=\{q_2\}$ 

δ	а	b
->q₀	$q_1$	$q_0$
$q_1$	$q_2$	$q_1$
*q <sub>2</sub>	$q_0$	$q_2$

22. Detect even number of 0's for  $\Sigma = \{0, 1\}$ .

 $L = \{\epsilon, 1, 00, 0000, 010, 100, 1001, 110010101 ...\}$ 



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

 $Q = \{q_0, q_1\}$ 

 $\Sigma = \{0, 1\}$ 

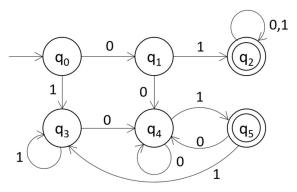
 $q_0 = q_0$  (start state)

 $F=\{q_0\}$ 

δ	0	1
→*q <sub>0</sub>	$q_1$	$q_0$
q <sub>1</sub>	$q_0$	$q_1$

23. Accept all strings that either begin or end or both with 01.

L = {01, 010, 011, 001, 101, 1001, 0111 ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

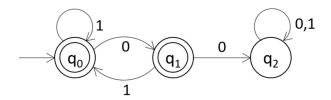
$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$
 (start state)

$$F = \{q_2, q_5\}$$

δ	0	1
⇒q <sub>0</sub>	$q_1$	$q_3$
$q_1$	$q_4$	$q_2$
*q <sub>2</sub>	$q_2$	$q_2$
q <sub>3</sub>	$q_4$	$q_3$
q <sub>4</sub>	$q_4$	<b>q</b> <sub>5</sub>
*q <sub>5</sub>	$q_4$	$q_3$

24. Accept strings that doesn't contain two consecutive 0's.



$$A = {Q, Σ, δ, q_0, F}$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

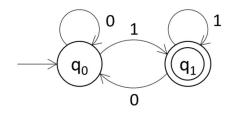
$$q_0 = q_0$$
 (start state)

$$F = \{q_0, q_1\}$$

δ	0	1
→*q <sub>0</sub>	$q_1$	$q_0$
*q <sub>1</sub>	$q_2$	$q_0$
q <sub>2</sub>	$q_2$	$q_2$

25 Detect odd binary numbers.

L = {1, 01, 11, 101, 001, 011, 111, 00101011 ...}



$$A = {Q, Σ, δ, q_0, F}$$

$$Q = \{q_0, q_1\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$
 (start state)

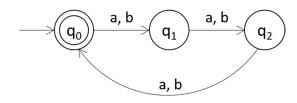
$$F=\{q_1\}$$

δ	0	1
->q₀	$q_0$	$q_1$
*q <sub>1</sub>	$q_0$	$q_1$

## n Mod length is basically a circle with n states

26. L =  $\{w \mid |w| \mod 3 = 0\}$  for  $\Sigma = \{a, b\}$ 

L = {aaa, aab, aba, abb, baa, bab, bba, bbb, abaaba ...}



 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, q_1, q_2\}$ 

 $\Sigma = \{a, b\}$ 

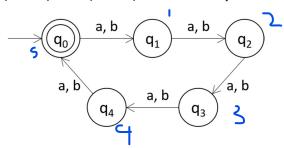
 $q_0 = q_0$  (start state)

 $F = \{q_0\}$ 

δ	а	b
→*q <sub>0</sub>	$q_1$	$q_1$
$q_1$	$q_2$	$q_2$
$q_2$	$q_0$	$q_0$

27.  $L = \{w \mid |w| \mod 5 = 0\} \text{ for } \Sigma = \{a, b\}$ 

L = {aaaaa, bbbbb, ababa, abbaa, baabb, babba, aababbaabb ...}



 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, q_1, q_2, q_3, q_4\}$ 

 $\Sigma = \{a, b\}$ 

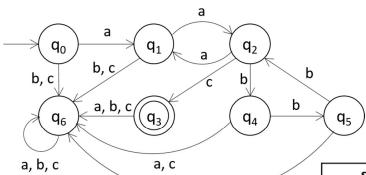
 $q_0 = q_0$  (start state)

 $F = \{q_0\}$ 

δ	а	b
→*q <sub>0</sub>	$q_1$	$q_1$
$q_1$	$q_2$	$q_2$
q <sub>2</sub>	$q_3$	q <sub>3</sub>
q <sub>3</sub>	$q_4$	$q_4$
q <sub>4</sub>	$q_0$	$q_0$

### 28. $L = \{a^{2n}b^{3m}c \mid n \ge 1 \text{ and } m \ge 0\}$

L = {aac, aabbbc, aaaac, aaaabbbc, aabbbbbbc ...}



a, c

 $A = \{Q, \Sigma, \delta, q_0, F\}$ 

 $Q = \{q_0, \, q_1, \, q_2, \, q_3, \, q_4, \, q_5, \, q_6\}$ 

 $\Sigma = \{a, b\}$ 

 $q_0 = q_0$  (start state)

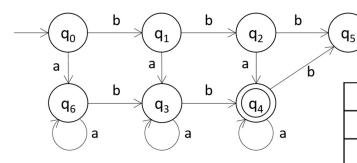
 $F=\{q_3\}$ 

δ	а	b	С
→q₀	$q_1$	$q_6$	$q_6$
q <sub>1</sub>	$q_2$	$q_6$	$q_6$
q <sub>2</sub>	$q_1$	$q_4$	$q_3$
*q <sub>3</sub>	$q_6$	$q_6$	$q_6$
q <sub>4</sub>	$q_6$	<b>q</b> <sub>5</sub>	$q_6$
q <sub>5</sub>	$q_6$	$q_2$	$q_6$
q <sub>6</sub>	$q_6$	$q_6$	$q_6$

a, b

### 29. At least one 'a' and exactly two b's.

L = {abb, bab, bba, abab, aaba, aabaabaa ...}



$$A = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$

$$\Sigma = \{a, b\}$$

 $q_0 = q_0$  (start state)

$$F = \{q_4\}$$

δ	а	b
<b>→</b> q₀	$q_6$	$q_1$
q <sub>1</sub>	<b>q</b> <sub>3</sub>	$q_2$
$q_2$	$q_4$	<b>q</b> <sub>5</sub>
q <sub>3</sub>	$q_3$	$q_4$
*q <sub>4</sub>	$q_4$	<b>q</b> <sub>5</sub>
q <sub>5</sub>	<b>q</b> <sub>5</sub>	<b>q</b> <sub>5</sub>
q <sub>6</sub>	$q_6$	q <sub>3</sub>