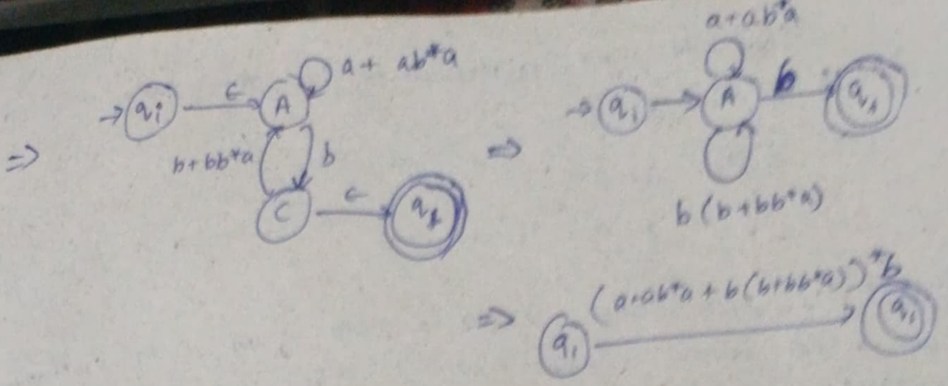


Obtain regex using Kleen's theorem

Let $M = (Q, \Sigma, \delta, q_0, F)$ for the language L then there exists equivalent regular language $L = L(R)$

Let $Q = \{q_1, q_2, \ldots, q_n\}$ ~~the~~ represent the different states where 'n' indicates no. of states. The path from 'i' to 'j' through an intermediate state whose number is not greater than k is given by the regular expression R_{ij}^{k} where ~~i, j is~~ i and j ~~be~~ are greater than k. The string 'w' can be written as $w = xy$ where x is greater than zero & y is greater than zero.

$\delta(i, x) = k$ and $\delta(x, y) = j$

Basis: $k = 0$:- This indicates that there is no intermediate stage & path from state 'i' to state 'j' is given by 2 condition

condition 1 :- There is a direct edge ~~&~~ between i and j. This ~~is~~ is possible when $i \neq j$. The DFA with all input symbols 'a' such that there is a transition from i to j is considered by following cases :-

i) No input symbol ~~&~~ then the corresponding regular expression is given by $R_{ij}^{(0)} = \phi$

ii) There is exactly one input ~~symbol~~ symbol 'a' between i & j then corresponding regex is given by $R_{ij}^{(0)} = a$

iii) There are multiple inputs $a, a_1 \ldots a_k$ where there is transition from each symbol from state i to state j then the corresponding regex is given by $R_{ij}^{(0)} = a_1 + a_2 + \cdots + a_k$

condition 2 :- There is only one state such that $i = j$ and there exists a path from i to itself on input symbol 'a' forming a self loop or path of length zero and is denoted by ϵ

Thus the regex corresponding to these cases will be

$R_{ij}^{(0)} = \phi + \epsilon$

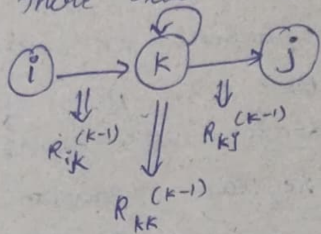
$R_{ij}^{(0)} = a + \epsilon$

$R_{ij}^{(0)} = a_1 + a_2 + \cdots + a_k + \epsilon$

Induction: Suppose there exists a path from i to j through a state which is not higher than 'k'. This situation leads to two cases.

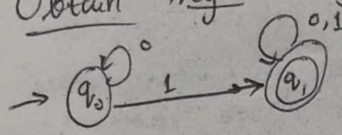
case 1 There exists a path from i to j which doesn't go through k so the language accepted is $R_{ij}^{(k-1)}$

case 2 There exist a path from i to j through k

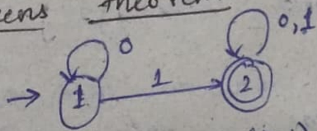


$$\boxed{\underbrace{R_{ij}^{(k-1)}}_{\text{no passing through } k} + \underbrace{R_{ik}^{(k-1)} \left(R_{kk}^{(k-1)}\right)^* R_{kj}^{(k-1)}}_{\text{passing through } k}}$$

Obtain regex using Kleens theorem



$\Rightarrow$



k=0

$R_{11}^{(0)} = 0 + \epsilon$

$R_{12}^{(0)} = 1$

$R_{21}^{(0)} = \phi$

$R_{22}^{(0)} = 0 + 1 + \epsilon$

k = 1 $\quad R_{ij}^{k} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} \left(R_{kk}^{(k-1)}\right)^* R_{kj}^{(k-1)}$

$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)}$

$= 0 + \epsilon + (0+\epsilon)(0+\epsilon)^*(0+\epsilon)$

$= (0+\epsilon) + 0^* = 0^*$ //

$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} = 1 + (0+\epsilon)(0+\epsilon)^* 1 = 1 + 0^*1$

$= 0^*1$ //

$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)} = \phi + \phi(0+\epsilon)^*(0+\epsilon)$

$= \phi$ //

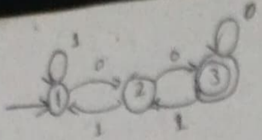
$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} = 0 + 1 + \epsilon + \phi(0+\epsilon)^* 1$

$= 0 + 1 + \epsilon$ //

k=2: $\quad R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} \left(R_{22}^{(1)}\right)^* R_{22}^{(1)}$

$= 0^*1 + 0^*1\,(0+1+\epsilon)^*(0+1+\epsilon)$

$= 0^*1 + 0^*1\,(0+1)^* = 0^*1(0+1)^*$ //



K=0:

$R_{11}^{(0)} = 1+\epsilon$

$R_{12}^{(0)} = 0$

$R_{13}^{(0)} = \phi$

$R_{21}^{(0)} = 1$

$R_{22}^{(0)} = \phi + \epsilon = \epsilon$

$R_{23}^{(0)} = 0$

$R_{31} = \phi$

$R_{32} = 1$

$R_{33} = 0+\epsilon$

K=1:

$R_{ij}^{(1)} = R_{ij}^{(0)} + R_{i1}^{(0)} (R_{11}^{(0)})^* R_{1j}^{(0)}$

$R_{11}^{(1)} = 1+\epsilon + (1+\epsilon)(1+\epsilon)^*(1+\epsilon)$

$R_{11}^{(1)} = 1+\epsilon+1^* = 1^*$ //

$R_{12}^{(1)} = 0 + (1+\epsilon)(1+\epsilon)^*(0)$

$0 + 1^*0 = 1^*0$ //

$R_{13}^{(1)} = \phi + (1+\epsilon)(1+\epsilon)^*\phi = \phi$ //

$R_{21}^{(1)} = 1 + (1)(1+\epsilon)^*(1+\epsilon) = 1+1^+ = 1^+$ //

$R_{22}^{(1)} = \epsilon + (1)(1+\epsilon)^*(0) = \epsilon + 1^+0$ //

$R_{23}^{(1)} = 0 + (1)(1+\epsilon)^*(\phi) = 0+\phi = 0$ //

$R_{31}^{(1)} = \phi + \epsilon\phi = \phi$ //

$R_{32}^{(1)} = 1 + \phi = 1$ /

$R_{33}^{(1)} = 0+\epsilon+\phi = 0+\epsilon$ //

K=2:-

$R_{11}^{(2)} = 1^* + (1^*0)(\epsilon+1^+0)^*(1^+) = 1^* + (1^*0)(1^+0)^*(1^+) =$

$R_{12}^{(2)} = 1^*0 + (1^*0)(\epsilon+1^+0)^*(\epsilon+1^+0) = \cancel{1^*0 + 1^*0(\epsilon+1^+0)^+ = 1^*0(\epsilon+1^+0)^*}$

$1^*0 + 1^*0(1^+0)^* = 1^*0(1^+0)^*$ //

$R_{13}^{(2)} = \phi + (1^*0)(\epsilon+1^+0)^*(0) = 1^*0(1^+0)^*0$ //

$R_{21}^{(2)} = 1^+ + (\epsilon+1^+0)(\epsilon+1^+0)^*(1^+) = (\epsilon+1^+0)^* 1^+ \Rightarrow (1^+0)^* 1^+$ //

$R_{22}^{(2)} = \epsilon+1^+0 + (\epsilon+1^+0)(\epsilon+1^+0)^*(\epsilon+1^+0) = \epsilon+1^+0 + (\epsilon+1^+0)^*$

$= (1^+0)^*$ //

$R_{23}^{(2)} = 0 + (\epsilon+1^+0)^*(\epsilon+1^+0)^*(0) = 0 + (1^+0)^*0$

$= (1^+0)^*0$ //

$R_{31}^{(2)} = \phi + (1)(\epsilon+1^+0)^* 1^+ = 1(1^+0)^* 1^+$ //

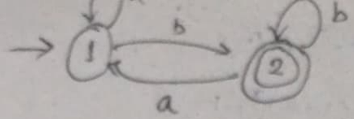
$R_{32}^{(2)} = 1 + (1)(\epsilon+1^+0)^*(\epsilon+1^+0) = 1 + 1(1^+0)^* = 1(1^+0)^*$ //

$R_{33}^{(2)} = 0+\epsilon + (1)(\epsilon+1^+0)^*(0) = 0+\epsilon+1(1^+0)^*0$ //

$R_{13}^{(3)} = 1^*0(11^*0)^*0 + (1^*0(11^*0)^*0)(0+\epsilon+1(11^*0)^*0)^*(0+\epsilon+1(110)^*0)$

$= 1^*0(11^*0)^*0 + (11^*0(11^*0)^*0)(0+1(11^*0)^*0)^*$

$= 1^*0(11^*0)^*0\ (0+1(11^*0)^*0)^*$



ϵ

$\underline{k=0:}$

$R_{11}^{(0)} = \epsilon + a$

$R_{12}^{(0)} = b$

$R_{21}^{(0)} = a$

$R_{22}^{(0)} = \epsilon + b$

$\underline{k=1:}$

$R_{11}^{(1)} = \epsilon + a + (\epsilon+a)(\epsilon+a)^*(\epsilon+a) = a^*$

$R_{12}^{(1)} = b + (\epsilon+a)(\epsilon+a)^*(b) = b + a^*b = a^*b$

$R_{21}^{(1)} = a + a(\epsilon+a)^*(\epsilon+a) = a + aa^* = aa^*$

$R_{22}^{(1)} = \epsilon + b + a(\epsilon+a)^*(b) = \epsilon + b + aa^*b$

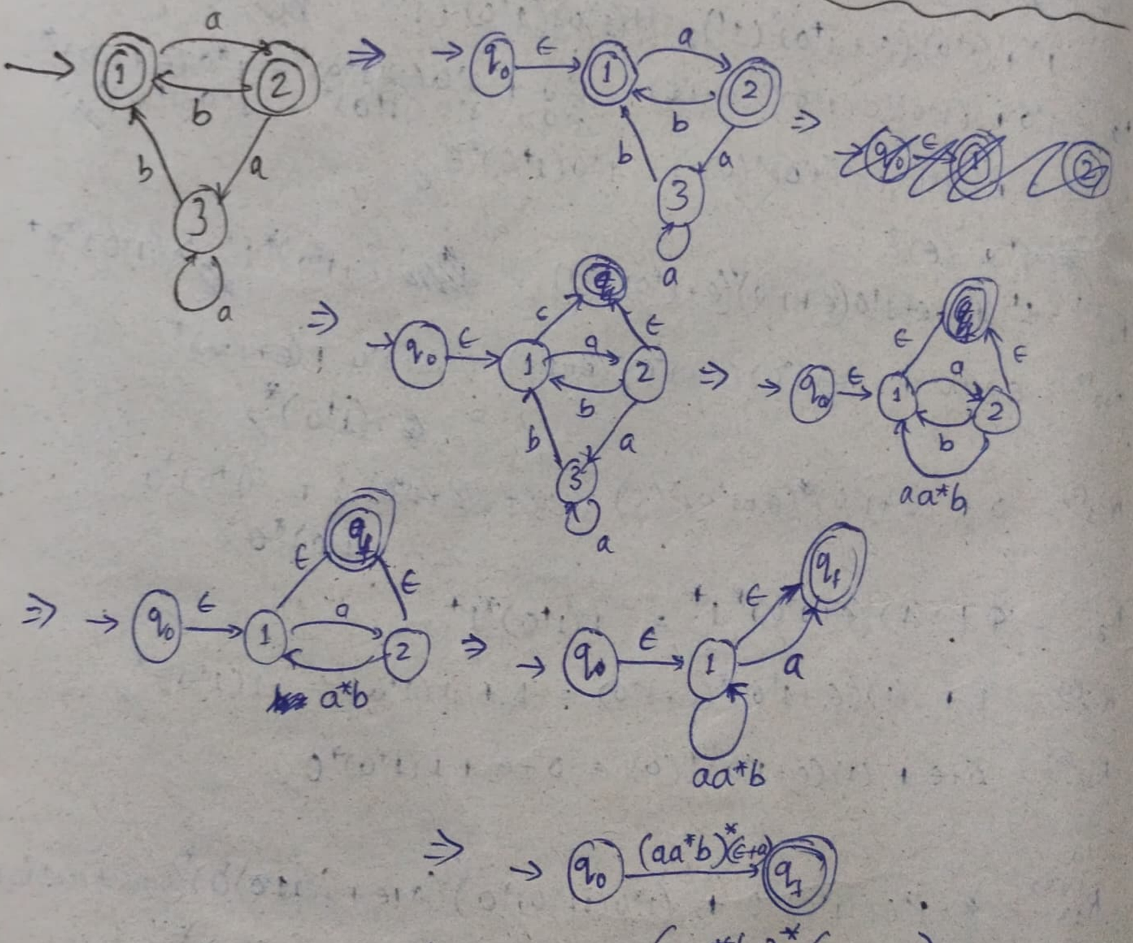
$\cancel{R_{12}^{(2)} = a^*b + a^*(a^*b)^*}$

$$\begin{aligned} R_{12}^{(2)} &= a^*b + a^*b(\epsilon + b + aa^*b)^*(\epsilon + b + aa^*b) \\ &= a^*b + a^*b(b + aa^*b)^* \\ &= a^*b(b + aa^*b)^* \\ &= a^*b(a^*b)^* \\ &= (a^*b)^+ \end{aligned}$$

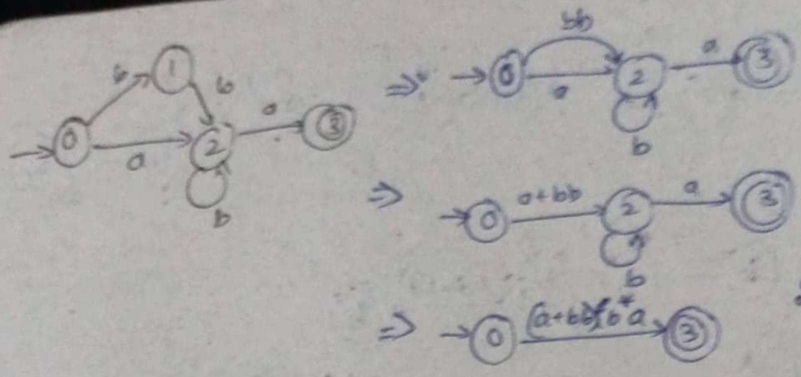
b ab aab

$a^+ b$

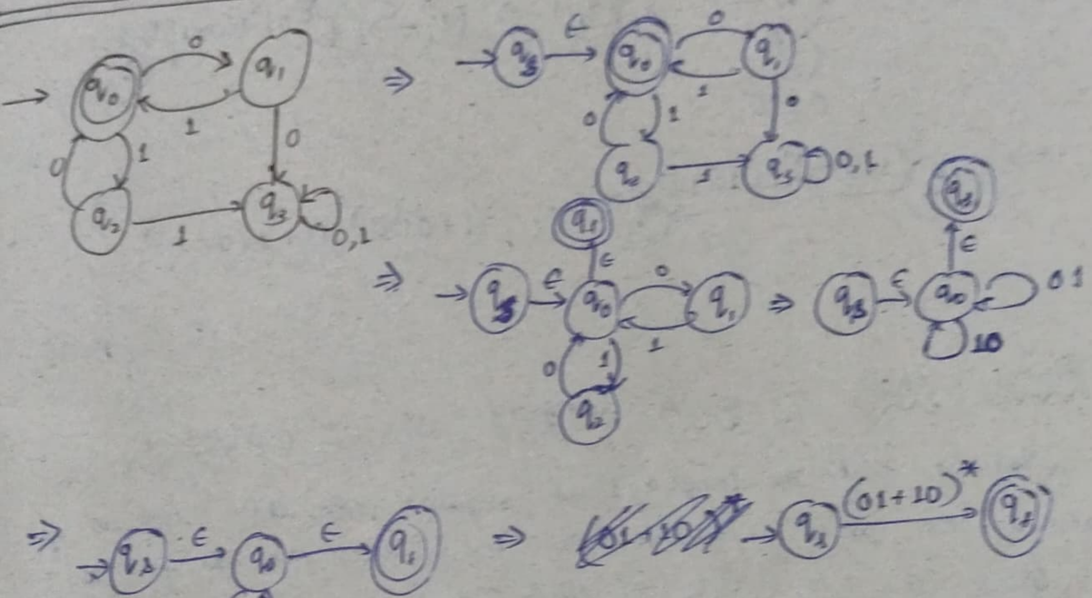
2



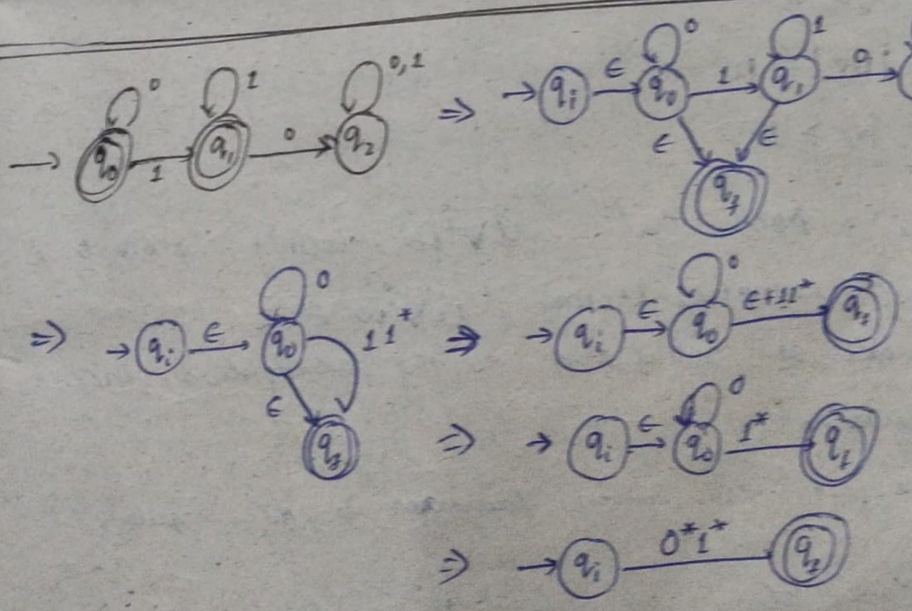
$\therefore$ regex $= (aa^*b)^*(\epsilon + a)$



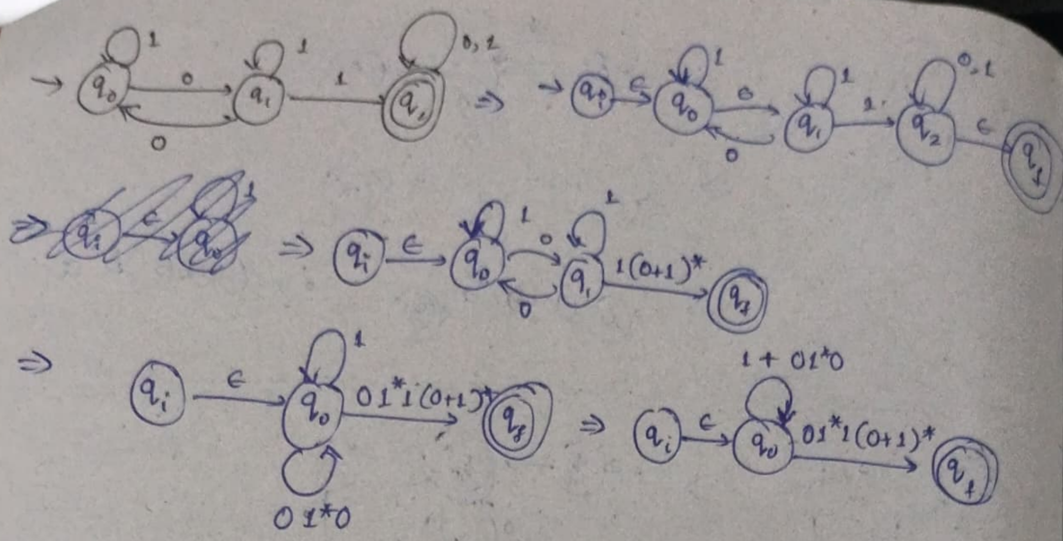
$\therefore (a+bb)b^*a$

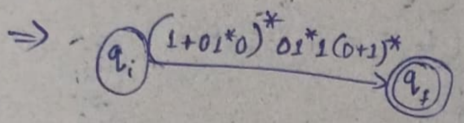


$\therefore (01+10)^*$



$\therefore 0^*1^*$





$\therefore\ (1+01^*0)^*01^*1(0+1)^*$ //

Prove that a given language is not regular

Assume that the language is regular ~~the~~ and the no. of states in the finite automata be 'n'. Select the strings 'x' and break it into u, v, w such that $x = uvw$ where the constraits are:-

i) $|x| \geq n$

ii) $|uv| \leq n$

iii) $|v| \geq 1$

Find any i such that uv^iw doesn't belong to L

According to pumping Lemma ; $uv^iw \in L$

The result is contradiction to the assumption that L is regular.

Therefore the given language is not regular

Example:- Show that $L = \{ww^R \mid w \in (0,1)^*\}$ is not regular.

Let L be regular and 'n' be no. of states in the ~~the~~ finite automata.

consider the string

$$x = \overbrace{\underbrace{1\ldots1}_{n}\ \underbrace{0\ldots0}_{n}}^{w}\ \overbrace{\underbrace{0\ldots0}_{n}\ \underbrace{1\ldots1}_{n}}^{w^R}$$

Where; n is no. of state in the finite automata.

$w = 1\ldots10\ldots0$

and $w^R = 0\ldots01\ldots1$

Since $x \geq n$, x can be split into u, v, w such that

$|uv| \leq n$ and $|v| \geq 1$

$$x = \underbrace{1\ldots\ldots1}_{u}\underbrace{1}_{v}\underbrace{0\ldots\ldots00\ldots\ldots01\ldots\ldots1}_{w}$$

where $|u| = n-1$, $|v| = 1$, $|uv| = |u| + |v| = n-1+1$

$|uv| = n$

According to pumping lemma uv^iw belongs to L

for $i = 0, 1, 2, \ldots\ldots$

If i is 0, 'v' doesn't appear so the no. of 1's on left hand side of x will be less than the no. of 1's on right hand side of x. So the string isn't in the form of ww^R so

$uv^iw \notin L$ when $i = 0$

This is condradictory to the assumption that L is regular.

* $L = \{a^ib^j \mid i > j\}$

Let L be regular & n be no. of state in the finite automata

consider the string

$$x = \underbrace{a\ldots\ldots a}_{n}\underbrace{a}_{1}\underbrace{b\ldots\ldots b}_{n} \quad \text{i.e } a^{n+1}b^n$$

Sin $|x| \geq n$, x can be split into u, v, w such that

$|uv| \leq n$ & $|v| \geq 1$

$$x = \underbrace{a\ldots\ldots a}_{u}\underbrace{a}_{v}\underbrace{ab\ldots\ldots b}_{w}$$

where $|u| = n-1$ $|v| = 1$, $|uv| = |u| + |v| = n-1+1 = n$

According to pumping lemma uv^iw belongs to L for $i = 0, 1, 2, \ldots$

If i is '0', then no. of a's decreases by one i.e we get $a^n b^n$ ~~which doesn't belong to L~~ . But no. of a's is not strictly greater that no. of b's

Thus $a^n b^n \notin L$ i.e

$$uv^iw \notin L \text{ when } i=0$$

This is contradictory & hence L is not regular

* $$L = \{a^{n!} \mid n \geq 0\}$$

Let L be regular and n be no. of states in the finite automata

consider the string

$$x = a^j a^k a^{n!-j-k}, \text{ thus } |x| = n!$$

Since $|x| \geq n$, x can be split into u, v, w such that $|uv| \leq n$ and $|v| \geq 1$

$$x = \underbrace{a^j}_{u} \underbrace{a^k}_{v} \underbrace{a^{n!-j-k}}_{w}$$

where $|u| = j$, $|v| = k$, $|uv| = |u| + |v| = j + k$

consider $j + k \leq n$

According to pumping lemma uv^iw belongs to L for $i = 0, 1, \ldots$

If i is 0 then a^k disappears i.e the string becomes

$$x = a^j a^{n!-j-k}$$

$$x = a^{n!-k}$$

If $k=1$ then $a^{n!-k} = a^{n!-1} \neq a^{n!}$

$$\therefore x = a^j a^{n!-j-k} \notin L$$

$$uv^iw \notin L \text{ for } i=0$$

This is contradictory and hence L is not regular

* $L = \{ n_a(w) < n_b(w) \mid n \geq 0 \}$

Let L be regular and n be no. of states in the finite automata

consider the string

$x = a^{n-1} b^{n}$, here $n_a(w) = n-1$ and $n_b(w) = n$

i.e $n_a(w) < n_b(w)$

since $|x| \geq n$, x can be split into u, v, w such that $|uv| \leq n$ and $|v| \geq 1$

$$x = \underbrace{a^{n-1}}_{u} \underbrace{b^{1}}_{v} \underbrace{b^{n-1}}_{w}$$

where $|u| = n-1$, $|v| = 1$, $|uv| = n-1+1 = n$

According to pumping lemma uv^iw belongs to L for $i = 0, 1, 2, \ldots$

If i is 0, v disappears and no. of b decreases

i.e x becomes

$$x = a^{n-1} b^{n-1}$$

where $n_a(w) = n-1 = n_b(w)$

thus $x = a^{n-1} b^{n-1} \notin L$

i.e $uv^iw \notin L$ when $i = 0$

This is contradictory and hence L is not regular.

* $L = \{ ww \mid w \in (a+b)^* \}$

Let L be regular and n be no of states in the finite automata.

Consider the string

$$x = \overbrace{\underbrace{a \cdots a}_{n}\underbrace{b \cdots b}_{n}}^{w}\ \overbrace{\underbrace{a \cdots a}_{n}\underbrace{b \cdots b}_{n}}^{w}$$

$|x| = 4n \geq n$

since $|x| \geq n$, x can be split into u, v, w such that $|uv| \leq n$ and $|v| \geq 1$

$$x = \underset{u}{a^j}\ \underset{v}{a^{n-j}}\ \underbrace{b^n a^n b^n}_{w}$$

where $|u| = j$ and $|v| = n-j$ and

$|uv| = |u| + |v| = j + n - j = n$

i.e $|uv| \leq n$ is true

According to pumping Lemma uv^iw belongs to L for $i = 0, 1, 2, \ldots$

If i is 0 then v disappears & no. of a's on lefthand side decreases from no. of a's on right hand side. i.e

$$x = a^j b^n a^n b^n$$

where $a^j b^n \neq a^n b^n$

Thus $x \notin L$

i.e $uv^iw \notin L$ when $i = 0$

This is contradictory and ~~then~~ hence L is not regular.

* $L = \{a^n \mid n = k^2, k \geq 0\}$

Let L be regular and n be no. of states in the finite automata.

consider the string

$x = (a^n)^n$ where $|x| = n * |a^n| = n * n = n^2$

since $|x| \geq n$ and $|x| = k^2$ where $k = n$ & x can be split into u, v, w such that $|uv| \leq n$ and $|v| \geq 1$

$$x = \underset{u}{a^j}\ \underset{v}{a^{n-j}}\ \underset{w}{(a^n)^{n-1}}$$

where $|u| = j$ and $|v| = n-j$ and $|uv| = |u| + |v|$

$= j + n - j$

$|uv| = n$

According to pumping lemma uv^iw belongs to L for $i = 0, 1, 2, \ldots$

If i is 0 then v disappears and x becomes

$$x = a^j (a^n)^{n-1}$$

$$|x| = |a^j| + |(a^n)^{n-1}| = j + (n-1)(n)$$
$$= j + n^2 - n$$
$$= n^2 - n + j$$

when $j = 1$ $|x| = n^2 - n + 1 \neq n^2$

thus $x = a^j (a^n)^{n-1} \notin L$

i.e $uv^iw \notin L$ when $i = 0$

This is contradictory and hence L is not regular.

Regular Grammar And Regular Languages

A Grammar 'G' is 4 tuple machine where

$G = (V, T, P, S)$ where;

- V :- Set of variables or non-terminals
- T :- Set of terminals
- P :- Set of production where each production is in the form $\alpha \to \beta$ where ~~α belongs~~
$$\alpha \in (V \cup T)^+$$
$$\beta \in (V \cup T)^*$$
- S :- Start symbol

Types of Grammars

1) Type 0 grammar or ~~past~~ phrase structured grammar.
The grammar is said to be in type 0 or also called as unrestricted grammar if all the productions are in the form of $\alpha \to \beta$ where $\alpha \in (V \cup T)^+$
$$\beta \in (V \cup T)^*$$

Example: $S \to aAb$
$aA \to ab \mid \epsilon$

2) Type 1 grammar or context sensitive grammar.
A grammar ~~G~~ $G = (V, T, P, S)$ is said to be Type 1 grammar or context sensitive grammar if all the productions are in the form of

$\alpha \to \beta$ but there is a restriction on the length of 'β'.
The length of $|\beta| \geq |\alpha|$ and $(\alpha, \beta) \in (V \cup T)^+$
i.e., $\alpha \in (V \cup T)^+$
$\beta \in (V \cup T)^+$

Example:

$S \to aAb$
$aA \to aBb$
$Bb \to dBc \mid dc$

3) Type 2 grammar or context free grammar

A grammar $G = (V, T, P, S)$ is said to be type 2 grammar or context free grammar if all the productions are of the form $A \to \alpha$ where $\alpha \in (V \cup T)^*$

Example:

$S \to aA \mid bB$
$A \to a \mid \epsilon$
$B \to b \mid \epsilon$

4) Type 3 grammar or regular grammar

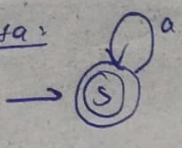
The grammar $G = (V, T, P, S)$ is said to be type 3 grammar or regular grammar if all the productions are of the form $A \to wB$ or $A \to w$ where $A, B \in V$ and $w \in T^*$

Example:

$S \to aA \mid bB$
$A \to a \mid \epsilon$
$B \to b \mid \epsilon$

1) Obtain a grammar to generate strings consisting of any no. of a's.

Sol^n:- dfa:



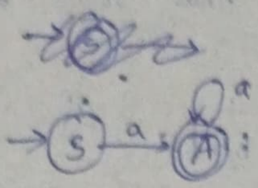
grammar

$S \to aS \mid \epsilon$

2) Obtain a grammar to generate strings consisting of atleast 1 a.

Sol^n:

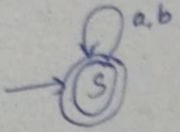
sol:



grammar:

$S \to aA$
$A \to aA \mid \epsilon$ } OR $S \to aS \mid a$

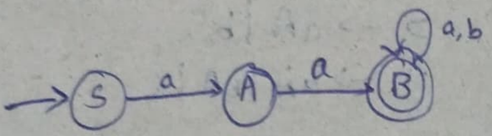
3) consisting of any no. of a's and b's



grammar:

$S \to aS$
$S \to bS$
$S \to \epsilon$ } $S \to aS \mid bS \mid \epsilon$

4) consisting of atleast 2 a's



grammar:

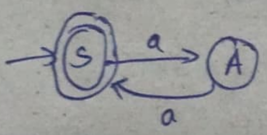
$S \to aA$
$A \to aB$
$B \to aB \mid \epsilon$

OR

$S \to aS \mid aa$

5) consisting of even no. of a's

$S \to aaS \mid \epsilon$

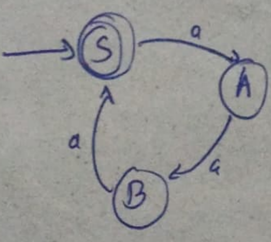


grammar

$S \to aA$
$S \to \epsilon$
$A \to aS$

6) consisting of multiples of 3 a's

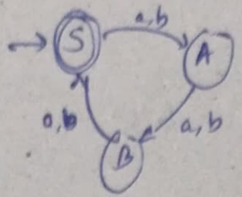
$S \to aaaS \mid \epsilon$



grammar:

$S \to aA \mid \epsilon$
$A \to aB$
$B \to aS$

7) Strings of a's and b's such that string length is multiple of 3



$S \to aA \mid bA \mid \epsilon$

$A \to aB \mid bB$

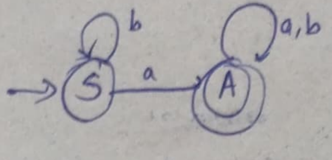
$B \to aS \mid bS$

OR

$S \to AAAS \mid \epsilon$

$A \to a \mid b$

8) Strings consisting of any no. of a's and b's with atleast 1 'a'!

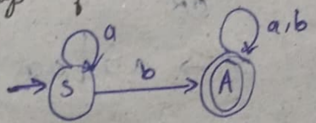


~~S → aA|bS~~

$S \to aA \mid bS$

$A \to aA \mid bA \mid \epsilon$

9) Strings of a's and b's with atleast 1 b:



$S \to aS \mid bA$

$A \to aA \mid bA \mid \epsilon$

Regex to grammar

1) Strings of a's & b's having substring 'ab'!

→ regex: $(a+b)^* ab (a+b)^*$

$S \to ABA$

$A \to aA \mid bA \mid \epsilon$

$B \to ab$

2) Strings of a's and b's ending with 'ab'

→ regex $(a+b)^* ab$

$S \to AB$

$A \to aA \mid bA \mid \epsilon$

$B \to ab$

3) Strings of a's and b's starting with 'ab'

→ regex : $ab(a+b)^*$

$S \to AB$

$A \to ab$

$B \to aB \mid bB \mid \epsilon$

4) Grammar for $L = \{ w \mid n_a(w) \bmod (2) = 0 \}$ $w \in \{a,b\}$

→ regex = $(b^*ab^*ab^*)^*$

$S \to AS \mid B$

$A \to BaBaB$

$B \to bB \mid \epsilon$

aaabbaabbaaa

5) $L = \{ ww^R \mid w \in (a+b)^* \}$

→ $S \to aSa \mid bSb \mid \epsilon$

6) $L = \{ a^m b^n \mid m \neq n \}$

→ $S \to aSb \mid A \mid B$

$A \to aA \mid a$

$B \to bB \mid b$

7) $L = \{ w \mid n_a(w) \geq n_b(w) \}$

~~S → aSbS | bSaS | A~~

~~A → aA | a~~

→ $S \to aSb \mid bSa \mid A \mid \epsilon$

$A \to aA \mid \epsilon$

bsa asb

bbbaaaaab

babaa

8) $L = \{ a^n b^n \mid n \geq 0 \}$

→ $S \to aSb \mid \epsilon$

9) $L = \{ a^n b^m \mid m > n \text{ and } n \geq 0 \}$

→ $S \to aSb \mid B$

$B \to bB \mid b$

10) $L = \{ w \mid n_a(w) \neq n_b(w) \}$

→ $S \to aSb \mid bSa \mid A \mid B$

$A \to aA \mid a$

$B \to bB \mid b$

11) Regex: $(011+1)^*(01)^*$

$S \rightarrow AB$

$A \rightarrow 011A \mid 1A \mid \epsilon$

$B \rightarrow 01B \mid \epsilon$

12) $L = \{a^n b^{2n}\}$

$S \rightarrow aSbb \mid \epsilon$

13)

~~S → AcA~~

~~A → aa | ba | ab | bb~~

$S \rightarrow aSa \mid bSb \mid C$

$C \rightarrow c$

14) $(0+1)^*$

$S \rightarrow 0S \mid 1S \mid \epsilon$

15) $a^n b^n c^n \mid n \geq 0$

~~S → ABC~~

S

~~S → ABC~~

~~A →~~

16) $\Sigma = \{(,), \{,\}, [,]\}$ Obtain grammar to balance the parenthesis

$S \rightarrow S(S)S \mid S\{S\}S \mid S[S]S \mid \epsilon$

Derivation: The process of obtaining a terminal / non-terminal from the start symbol by applying some set of productions is called derivation.

Types :- 1) Leftmost 2) Rightmost

Leftmost derivation

The process of obtaining a string of terminals from a sequence of replacements such that only left most non-terminal is replaced at each and every step

Rightmost derivation

The process of obtaining a string of terminals from a sequence of replacements such that only right most non-terminal is replaced at each and every step

Example:

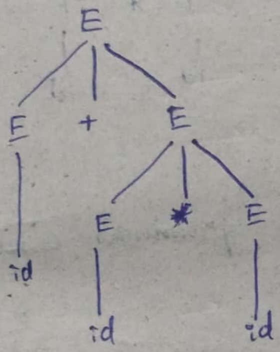
$E \rightarrow E+E$

$E \rightarrow E*E$

$E \rightarrow (E)$

$E \rightarrow id$

Leftmost	Rightmost
E	E
$E+E$	$E+E$
$id+E$	$E+E*E$
$id+E*E$	$E+E*id$
$id+id*E$	$E+id*id$
$id+id*id$	$id+id*id$



If tree is same for leftmost & rightmost then the grammar is unambiguous.
Else the grammar is ambiguous.

Parse tree / Derivation tree

Let $G = (V, T, P, S)$ be a context free grammar. the tree is a derivation tree if the following properties are satisfied:

i) The root has the label 'S'.
ii) Every vertex has the label: $V \cup T \cup \epsilon$
iii) Every leaf node has label from T and an interior vertex has a label V
iv) If the vertex is labelled A and if $X_1, X_2, X_3, \ldots, X_n$ are its children then there exists a production

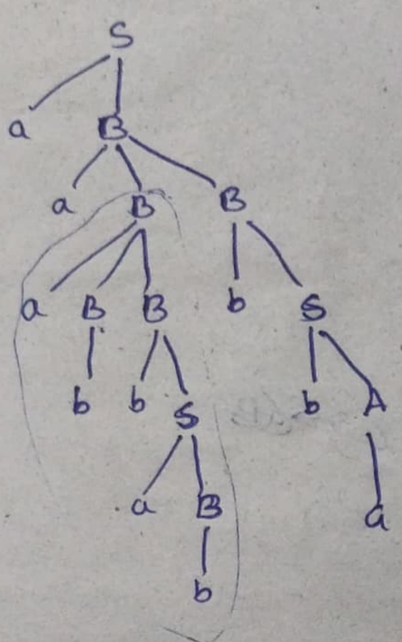
$$A \longrightarrow X_1, X_2, X_3, \ldots, X_n$$

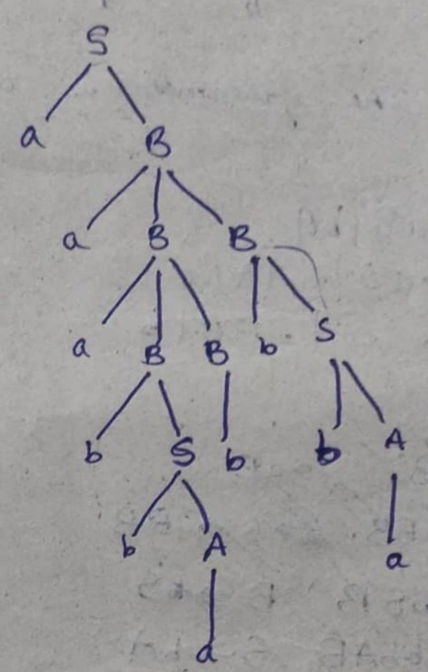
Ambiguous Grammar:

Let $G = (V, T, P, S)$ be a context free grammar the grammar G is ambiguous iff there exists a consider string aaabbabbba

S
aB   S → aB
aaBB   B → aBB
aaaBBB   B → aBB
aaabBB   B → b
aaabbSB   B → bS
aaabbaBB   S → aB
aaabbabB   B → b
aaabbabbS   B → bS
aaabbabbbA   S → bA
aaabbabbba   A → a

S
aB   S → aB
aaBB   B → aBB
aaaBBB   B → aBB
aaabSBB   B → bS
aaabbABB   S → bA
aaabbaBB   A → a
aaabbabB   B → b
aaabbabbS   B → bS
aaabbabbbA   S → bA
aaabbabbba   A → a





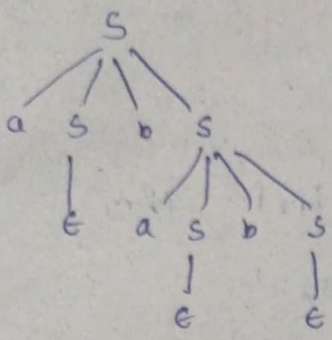
∴ The grammar is ambiguous

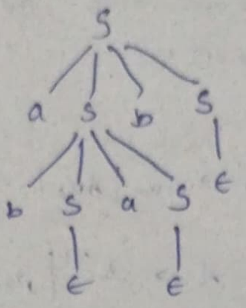
4) S → aSbS
S → bSaS
S → ϵ

abab

i) S
aSbS   S → aSbS
abS   S → ϵ
abaSbS   S → aSbS
ababS   S → ϵ
abab   S → ϵ

S
aSbS   S → aSbS
abSaSbS   S → bSaS
abaSbS   S → ϵ
ababS   S → ϵ
abab   S → ϵ





⇒ aababb

i) S

aSbS S → aSbS

aaSbSbS S → aSbS

aabSbS S → ∈

aabaSbSbS S → aSbS

aababSbS S → ∈

aababbS S → ∈

aababb S → ∈

ii) ~~S~~

~~aSbS S → aSbS~~

~~aaSbSbS S → aSbS~~

S

aSbS S → aSbS

aaSbSbS S → aSbS

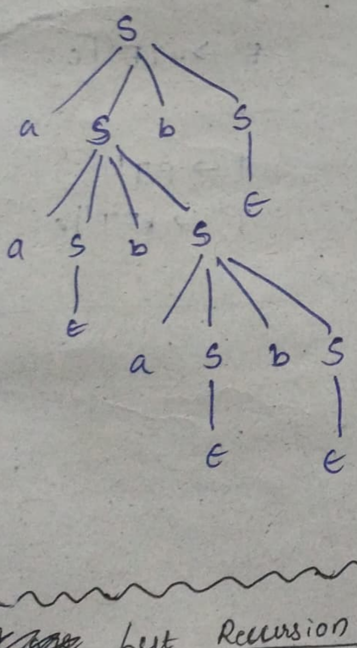
aabSaSbSbS S → bSaS

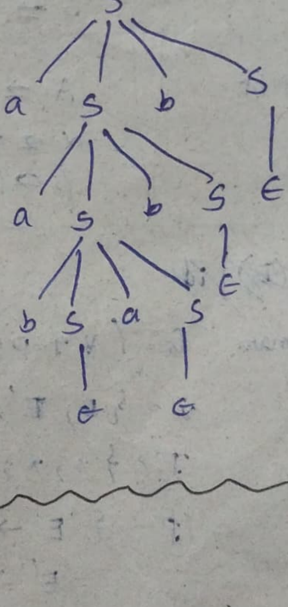
aabaSbSbS S → ∈

aababSbS S → ∈

aababbS S → ∈

aababb S → ∈ .





Left Recursion

A grammar is said to be left recursive if there is some non-terminal A such that $A \rightarrow A\alpha$

If the first symbol on the right is a variable and if the derivation is obtained from the same non - terminal then the grammar is said to have left recursion.

Consider $A \to A\alpha_1 \mid A\alpha_2 \mid \cdots \mid A\alpha_n \mid \beta_1 \mid \beta_2 \mid \cdots \mid \beta_m$.

without left recursion $A \to \beta_1 A' \mid \beta_2 A' \mid \cdots \mid \beta_m A'$

$A' \to \alpha_1 A' \mid \alpha_2 A' \mid \cdots \mid \alpha_n A' \mid \epsilon$

Example:

$E \to E + T \mid T$

$T \to T * F \mid F$

$F \to (E) \mid id$

Given	Substitution	Without left recursion
$A \to A\alpha \mid \beta$		$A \to \beta A'$ $A' \to \alpha A' \mid \epsilon$
$E \to E+T \mid T$	$A \Rightarrow E$ $\alpha \Rightarrow +T$ $\beta \Rightarrow T$	$E \to TE'$ $E' \to +TE' \mid \epsilon$
$T \to T*F \mid F$	$A \Rightarrow T$ $\alpha \Rightarrow *F$ $\beta \Rightarrow F$	$T \to FT'$ $T' \to *FT' \mid \epsilon$

Grammar $G = (V, T, P, S)$

$V = \{E, E', T, T', F\}$

$T = \{+, *, id, (,)\}$

$P = \{ E \to TE'$

$E' \to +TE' \mid \epsilon$

$T \to FT'$

$T' \to *FT' \mid \epsilon$

$F \to (E) \mid id$

$\}$

$S = E$

Example:

$S \rightarrow Ab \mid a$

~~[illegible]~~

$A \rightarrow Ab \mid Sa$

$\Rightarrow$

$S \rightarrow Ab \mid a$

$A \rightarrow Ab \mid Aba \mid aa$

$\Rightarrow$

$S \rightarrow Ab \mid a$

$A \rightarrow aaA'$

$A' \rightarrow bA' \mid baA' \mid \epsilon$

Eliminating ϵ productions

$S \rightarrow AB$ } Indirectly produces ϵ
$A \rightarrow \epsilon$
$B \rightarrow \epsilon$ } Produce ϵ directly
} These are Nullable productions

also

$S \rightarrow AB \mid Ca$
$A \rightarrow \epsilon \mid aA \mid a$
$B \rightarrow \epsilon \mid b$
$C \rightarrow c$

There is a chance that S can become nullable and A and B are nullable.

Suppose A and B are nullable variables in a production

Step-1 :- First add the production to P'

Step-2 :- Replace A with ϵ in the given production and add the resulting production to P'

Step-3 :- Replace B with ϵ in the given production and add the resulting production to P'

Step-4 :- Replace A and B with ϵ and all the production to P'

Step-5 :- If all symbols on the right-hand-side of production are nullable variables, the resulting production is a ϵ production & do not add this to P'

Example:-

$S \rightarrow ABCa \mid bD$
$A \rightarrow BC \mid b$
$B \rightarrow b \mid \epsilon$
$C \rightarrow c \mid \epsilon$
$D \rightarrow d$

old variables	new variables	Productions
ϕ	B, C	$B \rightarrow \epsilon$, $C \rightarrow \epsilon$
B, C	B, C, A	$A \rightarrow BC$
B, C, A	B, C, A	—

The nullable variables are A, B, C

Given	P'
$S \rightarrow ABCa$	$S \rightarrow BCa \mid ACa \mid ABa \mid Ca \mid Ba \mid Aa \mid a \mid ABCa$
$S \rightarrow bD$	$S \rightarrow bD$
$A \rightarrow BC \mid b$	$A \rightarrow BC \mid \epsilon \mid B \mid b$

Given	P'
$B \rightarrow b \mid \epsilon$	$B \rightarrow b$
$C \rightarrow c \mid \epsilon$	$C \rightarrow c$
$D \rightarrow d$	$D \rightarrow d$

$\therefore G' = (V', T', P', S')$

$V' = (S, A, B, C, D)$

$T' = (a, b, c, d)$

$P' = \{$ $S \to ABCa \mid ABa \mid ACa \mid BCa \mid Aa \mid Ba \mid Ca \mid a \mid bD$

$A \to BC \mid C \mid B \mid b$

$B \to b$

$C \to c$

$D \to d$

$\}$

2) $S \to BAAB$

$A \to 0A2 \mid 2A0 \mid \epsilon$

$B \to AB \mid 1B \mid \epsilon$

old variables	new variables	Production
ϕ	A, B	$A \to \epsilon$ $B \to \epsilon$
A, B	S, A, B	$S \to BAAB$
S, A, B	S, A, B	–

The nullable variables are S, A, B

Given	P'
$S \to BAAB$	$S \to AAB \mid BAB \mid BAA \mid AB \mid AA \mid BB \mid BA \mid A \mid B \mid BAAB$
$A \to 0A2 \mid 2A0 \mid \epsilon$	$A \to 0A2 \mid 02 \mid 2A0 \mid 20$
$B \to AB \mid 1B \mid \epsilon$	$B \to AB \mid A \mid B \mid 1B \mid 1$

$\boxed{\text{Write } G'}$

3) $S \to XYX$

$X \to 0X \mid \epsilon$

$Y \to 1Y \mid \epsilon$

old variables	new variables	production
ϕ	X, Y	$X \to \epsilon$ $Y \to \epsilon$
X, Y	X, Y, S	$S \to XYX$
X, Y, S	X, Y, S	–

The nullable variables are S, X, Y

Given	
S → XYX	S → XYX | YX | XX | XY | X | Y
X → 0X | ε	X → 0X | 0
Y → 1Y | ε	Y → 1Y | 1

2) S → 0X0 | 1Y1 | ε
X → Z | 0
Y → ZY0 | ε
Z → 0Z1 | XZWZ | ε
W → X | X | 01

4) S → 0E0 | 1FF | ε
E → G
F → S | E
G → g | E

3) S → Sa | aSSSb | ε
A → ε | aAy | AA

5) S → Aa | B
A → Bb | Sc | ε
B → d

6) S → Aa | Bb | c
A → Bd | ε
B → Ae | ε

7) S → 0AA | 1BB | ε
A → AC | 0
B → CB | 1
C → CDE | ε
D → A | B | 01

2)

old variables	new variables	Production
φ	S, Y, Z	S → ε Y → ε Z → ε
S, Y, Z	S, X, Y, Z, W	X → Z W → Y
S, X, Y, Z, W.	S, X, Y, Z, W	

The nullable variables are
S, X, Y, Z, W

P'

Given	
S → 0X0 | 1Y1 | ε	S → 0X0 | 00 | 1Y1 | 11
X → Z | 0	X → Z | 0
Y → ZY0 | ε	Y → ZY0 | Z0 | Y0 | 0
Z → 0Z1 | XZWZ | ε	Z → 0Z1 | 01 | XZWZ | XZW | XZZ | XWZ | ZWZ | XZ | XW | ZW | ZZ | WZ | Z | W | X

$W \to X \mid Y \mid 0L$	$W \to X \mid Y \mid 01$

3)

old variables	new variables	Production
ϕ	S, A	$S \to \epsilon$ $A \to \epsilon$
S,A	S,A	–

Nullable variables are S and A

Given	Production
$S \to Sa \mid asssb \mid \epsilon$	$S \to Sa \mid a \mid asssb \mid assb \mid asb \mid ab$
$A \to \epsilon \mid aAy \mid AA$	$A \to aAy \mid ay \mid AA \mid A$

4)

~~S~~ old variables	New variables	Production
ϕ	S	$S \to \epsilon$
S	S, F	$F \to S$
S, F	S, F	–

Nullable variables are S and F

Given	Production
$S \to 0E0 \mid 1FF \mid \epsilon$	$S \to 0E0 \mid 1FF \mid 1F \mid 1$
$E \to G$	$E \to G$
$F \to S \mid E$	$F \to S \mid E$
$G \to g \mid E$	$G \to g \mid E$

5)

old vars	New vars	Production
ϕ	A	$A \to \epsilon$
A	A	–

Nullable variables are A

Given	Production
$S \to Aa \mid B$	$S \to Aa \mid a \mid B$
$A \to Bb \mid Sc \mid \epsilon$	$A \to Bb \mid Sc$
$B \to d$	$B \to d$

6)

Old var	New var	Production
ϕ	A, B	$A \to \epsilon$ $B \to \epsilon$
A, B	A, B	–

Nullable variables are A and B

Given	Production
$S \to Aa \mid Bb \mid c$	$S \to Aa \mid a \mid Bb \mid b \mid c$
$A \to Bd \mid \epsilon$	$A \to Bd \mid d$
$B \to Ae \mid \epsilon$	$B \to Ae \mid e$

7)

Old var	New var	Production
ϕ	S, C	$S \to \epsilon$ $C \to \epsilon$
S, C	S, C	–

Nullable variables are S and C

Given	Production
$S \to 0AA \mid 1BB \mid \epsilon$	$S \to 0AA \mid 1BB$
$A \to AC \mid 0$	$A \to AC \mid A \mid 0$
$B \to CB \mid 11$	$B \to CB \mid B \mid 11$
$C \to CDE \mid \epsilon$	$C \to CDE \mid DE$
$D \to A \mid B \mid 01$	$D \to A \mid B \mid 01$

Eliminating Unit Production

Unit Production:- Let grammar $G = (V, T, P, S)$ be a context free grammar and if there exists a production of the form $A \rightarrow B$ where A and B both belong to V i.e $A, B \in V$ is a unit production

Example:

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow C \mid b$
$C \rightarrow D$
$D \rightarrow E \mid bC$
$E \rightarrow d \mid Ab$

Non unit Productions

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow b$
$D \rightarrow bC$
$E \rightarrow d \mid Ab$

Unit Productions

$B \rightarrow C$
$C \rightarrow D$
$D \rightarrow E$

Dependency graph

$B \rightarrow C \rightarrow D \rightarrow E$

Now:

$\Rightarrow E \rightarrow d \mid Ab$
$\Rightarrow D \rightarrow E \mid bC$
$\Rightarrow D \rightarrow d \mid Ab \mid bC$
$\Rightarrow C \rightarrow D$
$C \rightarrow d \mid Ab \mid bC$
$\Rightarrow B \rightarrow C \mid b$
$B \rightarrow d \mid Ab \mid bC \mid b$

Grammar

$G' = (V', T', P', S')$
$V' = \{S, A, B, C, D, E\}$
$T' = \{a, b, d\}$
$P' = \{$
$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow d \mid Ab \mid bC \mid b$
$C \rightarrow d \mid Ab \mid bC$
$D \rightarrow d \mid Ab \mid bC$
$E \rightarrow d \mid Ab$
$\}$
$S' = S$

$S \rightarrow A0 \mid B$
$B \rightarrow A \mid 11$
$A \rightarrow 0 \mid 12 \mid B$

Non unit

$S \rightarrow A0$
$B \rightarrow 11$
$A \rightarrow 0 \mid 12$

Unit

$S \rightarrow B$
$B \rightarrow A$
$A \rightarrow B$

$S \rightarrow B \rightleftarrows A$

$\Rightarrow A \rightarrow 0 \mid 12$
$\Rightarrow B \rightarrow A \mid 11$
$B \rightarrow 0 \mid 12 \mid 11$
$\Rightarrow A \rightarrow 0 \mid 12 \mid B$
$A \rightarrow 0 \mid 12 \mid 11$
$\Rightarrow S \rightarrow A0 \mid B$
$S \rightarrow A0 \mid 0 \mid 12 \mid 11$

$G' = (V', T', P', S')$
$V' = (S, A, B)$
$T' = \{0, 1, 2\}$
$P' = \{ S \rightarrow A0 \mid 0 \mid 12 \mid 11$
$A \rightarrow 0 \mid 12 \mid 11$
$B \rightarrow 0 \mid 12 \mid 11$
$S' = S \}$

3) $S \to Aa \mid B \mid Ca$
$B \to aB \mid b$
$C \to Db \mid D$
$D \to E \mid d$
$E \to ab$

~~Non~~ unit
$S \to B$
~~$B \to$~~
$C \to D$
$D \to E$

Non Unit
$S \to Aa \mid Ca$
$B \to aB \mid b$
$C \to Db$
$D \to d$
$E \to ab$

$S \to B$

$C \to D \to E$

$\Rightarrow B \to aB \mid b$
$\Rightarrow S \to B \mid Aa \mid Ca$
$S \to aB \mid b \mid Aa \mid Ca$

$\Rightarrow E \to ab$
$\Rightarrow D \to E \mid d$
$D \to ab \mid d$
$\Rightarrow C \to Db \mid D$
$C \to Db \mid ab \mid d$

$G' = (V', T', P', S')$
$V' = (S, A, B, C, D, E)$
$T' = (a, b, d)$
$P' = \{$
$S \to aB \mid b \mid Aa \mid Ca$
$B \to aB \mid b$
$C \to Db \mid ab \mid d$
$D \to ab \mid d$
$E \to ab$
$\}$
$S' = S$

$S \to AB$
$A \to B$
$B \to aB \mid Bb \mid A$

Non unit
$S \to AB$
$B \to aB \mid Bb$

Unit
$A \to B$
$B \to A$

$\Rightarrow B \to aB \mid Bb$
$\Rightarrow A \to B$
$A \to aB \mid Bb$
$\Rightarrow B \to aB \mid Bb \mid A$
$B \to aB \mid Bb$

$G' = (V', T', P', S')$
$V' = (S, A, B)$
$T' = (a, b)$
$P' = \{$
$S \to AB$
$A \to aB \mid Bb$
$B \to aB \mid Bb$
$\}$
$S' = S$

$S \to XY$
$X \to A$
$A \to B \mid a$
$B \to b$
$Y \to T$
$T \to Y \mid c$

Non unit
$S \to XY$
$A \to a$
$B \to b$
$T \to c$

Unit
$X \to A$
$A \to B$
$Y \to T$
$T \to Y$

$\Rightarrow B \to b$
$\Rightarrow A \to B \mid a$
$A \to b \mid a$
$\Rightarrow X \to A$
$\Rightarrow X \to b \mid a$

$\Rightarrow T \to c$
$\Rightarrow Y \to T \Rightarrow Y \to c$
$\Rightarrow T \to Y \mid c \Rightarrow T \to c$

$G' = (V', T', P', S')$ $V' = (S, A, B, X, Y, T)$

$T' = (a, b, c)$. $P' = \{ S \to XY, A \to b|a, B \to b, X \to b|a, Y \to a, T \to c \}$

$S' = S$

$S \to aB|bA$
$A \to B|C|id$
$B \to ba|Sc$
$C \to A|int|rb$

Non unit
$S \to aB|bA$
$A \to id$
$B \to ba|Sc$
$C \to int|rb$

Unit
$A \to B|C$
$C \to A$

$\Rightarrow B \to ba|Sc$
$\Rightarrow C \to int|rb$
$\Rightarrow A \to B|C|id$
$A \to ba|Sc|int|rb|id$
$\Rightarrow C \to A|int|rb$
$\Rightarrow C \to ba|Sc|int|rb|id$

$S \to aB|bA$
$A \to ba|Sc|int|rb|id$
$B \to ba|Sc$
$C \to ba|Sc|int|rb|id$

$S \to B|SB|ab$
$A \to a|SA$
$B \to A|SB|b$

Non unit
$S \to SB|ab$
$A \to a|SA$
$B \to SB|b$

Unit
$S \to B$
$B \to A$

$A \to a|SA$
$\Rightarrow B \to A|SB|b$
$B \to a|SA|SB|b$
$\Rightarrow S \to B|SB|ab$
$S \to a|SA|SB|ab|b$

$S \to a|SA|SB|ab|b$
$A \to a|SA$
$B \to a|SA|SB|b$

$S \to ABa|B$
$B \to b|bB|A$
$A \to aA|a|B$

Non unit
$S \to ABa$
$B \to bB$
$A \to aA|a$

Unit
$S \to B$
$B \to A$
$A \to B$

$B \to bB$
$\Rightarrow A \to aA|a|B$
$A \to aA|a|bB$
$\Rightarrow B \to b|bB|A$
$B \to \cancel{a}\, b|bB|a|aA$
$\Rightarrow A \to aA|a|B$
$A \to aA|a|b|bB$
$\Rightarrow S \to ABa|B$
$S \to ABa|b|bB|a|aA$

$S \to ABa|b|bB|a|aA$
$A \to aA|a|b|bB$
$B \to aA|a|b|bB$

Chomsky Normal Form

The grammar $G = (V, T, P, S)$ be a context free grammar if all the productions are of the form $A \rightarrow BC$ or $A \rightarrow a$

Steps 1: Eliminate start symbol from RHS: If ~~the~~ start symbol 'S' is on the R.H.S of any production in the grammar, Create a new production $S_0 \rightarrow S$ where S_0 is the new start symbol

Step 2: Eliminate NULL, UNIT and ~~USELESS~~ USELESS productions.
If the grammar contains ϵ - productions, unit productions and useless productions then there must be eliminating

Step 3: Eliminate terminals in RHS if they exist with other terminal / non-terminals.

Example: $X \rightarrow xy$, $X \rightarrow xY$

Decompose the production into:-

consider $X \rightarrow xY$

$$\boxed{\begin{array}{l} X \rightarrow ZY \\ Z \rightarrow x \end{array}}$$

Step 4: Eliminate RHS with more than 2 non - terminals.

$X \rightarrow XYZ$ could be replaced by

$$\boxed{\begin{array}{l} X \rightarrow PZ \\ P \rightarrow XY \end{array}}$$

Question:

$S \rightarrow 0A \mid 1B$
$A \rightarrow 0AA \mid 1S \mid 1$
$B \rightarrow 1BB \mid 0S \mid 0$

A
1) $S \rightarrow S_0$
$S \rightarrow 0A \mid 1B$
$A \rightarrow 0AA \mid 1S \mid 1$
$B \rightarrow 1BB \mid 0S \mid 0$

2) No null, no useless. There are unit

$S_0 \rightarrow 0A \mid 1B$
$S \rightarrow 0A \mid 1B$
$A \rightarrow 0AA \mid 1S \mid 1$
$B \rightarrow 1BB \mid 0S \mid 0$

Given	Action	Production
$S_0 \to 0A \mid 1B$	$D_0 \to 0$ $D_1 \to 1$	$S \to D_0A \mid D_1B$ $D_0 \to 0$ $D_1 \to 1$
~~$A \to 0AA \mid 1S \mid 1$~~ $S \to 0A \mid 1B$	$D_0 \to 0$ $D_1 \to 1$	$S \to D_0A \mid D_1B$ $D_0 \to 0$ $D_1 \to 1$
$A \to 0AA \mid 1S \mid 1$	~~$A \to D_0AA \mid D_1S \mid 1$~~ $D_0 \to 0$ $D_1 \to 1$	$A \to D_0AA \mid D_1S \mid 1$ $D_0 \to 0$ $D_1 \to 1$
$B \to 1BB \mid 0S \mid 0$	$D_0 \to 0$ $D_1 \to 1$	$B \to D_1BB \mid D_0S \mid 0$ $D_0 \to 0$ $D_1 \to 1$

$S_0 \to D_0A \mid D_1B$
$S \to D_0A \mid D_1B$
$A \to D_0AA \mid D_1S \mid 1$
$B \to D_1BB \mid D_0S \mid 0$
$D_0 \to 0$
$B_1 \to 1$

$\boxed{\text{Step 4}}$

Given	Action	Production
$A \to D_0AA$	$D_2 \to D_0A$	$A \to D_2A$ $D_2 \to D_0A$
$B \to D_1BB$	$D_3 \to D_1B$	$B \to D_3B$ $D_3 \to D_1B$

G:-

$S_0 \to D_0A \mid D_1B$
$S \to D_0A \mid D_1B$
$A \to D_2A \mid D_1S \mid 1$
$B \to D_3B \mid D_0S \mid 0$
$D_0 \to 0$
$D_1 \to 1$
$D_2 \to D_0A$
$D_3 \to D_1B$

Question

$S \to ASB$
$A \to aAS \mid a \mid \epsilon$
$B \to SbS \mid A \mid bb$

i) $S_0 \to S$
$S \to ASB$
$A \to aAS \mid a \mid \epsilon$
$B \to SbS \mid A \mid bb$

ii) $S_0 \to S$ removed null
$S \to ASB \mid SB \mid AS \mid S$
$A \to aAS \mid a \mid aS$
$B \to SbS \mid A \mid bb$

~~S → ASB~~

removed unit :-

$S_0 \rightarrow ASB \mid AS \mid SB$

$S \rightarrow ASB \mid AS \mid SB$

$A \rightarrow aAS \mid a \mid aS$

$B \rightarrow SbS \mid aAS \mid a \mid aS \mid bb$

no useless ∴

Step 3 :-

Given

Action

Production

Given	Action	Production
$S_0 \rightarrow ASB \mid AS \mid SB$	–	$S_0 \rightarrow ASB \mid AS \mid SB$
$S \rightarrow ASB \mid AS \mid SB$	–	$S \rightarrow ASB \mid AS \mid SB$
$A \rightarrow aAS \mid a \mid aS$	$D_0 \rightarrow a$	$A \rightarrow D_0AS \mid a \mid D_0S$, $D_0 \rightarrow a$
$B \rightarrow SbS \mid aAS \mid a \mid aS \mid bb$	$D_1 \rightarrow b$, $D_0 \rightarrow a$	$B \rightarrow SD_1S \mid D_0AS \mid a \mid D_0S \mid D_1D_1$

Thus

$S_0 \rightarrow ASB \mid AS \mid SB$

$S \rightarrow ASB \mid AS \mid SB$

$A \rightarrow D_0AS \mid a \mid D_0S$

$B \rightarrow SD_1S \mid D_0AS \mid a \mid D_0S \mid D_1D_1$

$D_0 \rightarrow a$

$D_1 \rightarrow b$

Step 4 :-

Given	Action	Production
$S_0 \rightarrow ASB \mid AS \mid SB$	$D_2 \rightarrow AS$	$S_0 \rightarrow D_2B \mid AS \mid SB$, $D_2 \rightarrow AS$
$S \rightarrow ASB \mid AS \mid SB$	$D_2 \rightarrow AS$	$S \rightarrow D_2B \mid AS \mid SB$, $D_2 \rightarrow AS$
$A \rightarrow D_0AS \mid a \mid D_0S$	$D_3 \rightarrow D_0A$	$A \rightarrow D_3S \mid a \mid D_0S$, $D_3 \rightarrow D_0A$
$B \rightarrow SDS \mid D_0AS \mid a \mid D_0S \mid D_1D_1$	$D_3 \rightarrow D_0A$, $D_4 \rightarrow SD_1$	$B \rightarrow D_4S \mid D_3S \mid a \mid D_0S \mid D_1D_1$, $D_3 \rightarrow D_0A$, $D_4 \rightarrow SD_1$

∴

$S_0 \rightarrow D_2B \mid AS \mid SB$

$S \rightarrow D_2B \mid AS \mid SB$

$A \rightarrow D_3S \mid a \mid D_0S$

$B \rightarrow D_4S \mid D_3S \mid a \mid D_0S \mid D_1D_1$

$D_0 \rightarrow a$

$D_1 \rightarrow b$

$D_2 \rightarrow AS$

$D_3 \rightarrow D_0A$

$D_4 \rightarrow SD_1$

1) $S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \epsilon$

$\Rightarrow S_0 \to S$

$S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \epsilon$

$\Rightarrow$ removing null (A, B)

$S_0 \to S$

$S \to ASA \mid SA \mid AS \mid S \mid aB \mid a$

$A \to B \mid S$

$B \to b$

$\Rightarrow$ removing unit

$S_0 \to ASA \mid SA \mid AS \mid aB \mid a$

$S \to ASA \mid SA \mid AS \mid aB \mid a$

$A \to b \mid ASA \mid SA \mid AS \mid aB \mid a$

$B \to b$

No useless

$\Rightarrow S_0 \to ASA \mid SA \mid AS \mid D_0B \mid a$

$S \to ASA \mid SA \mid AS \mid D_0B \mid a$

$A \to b \mid ASA \mid SA \mid AS \mid D_0B \mid a$

$B \to b$

$D_0 \to a$

$\Rightarrow S_0 \to D_1A \mid SA \mid AS \mid D_0B \mid a$

$S \to D_1A \mid SA \mid AS \mid D_0B \mid a$

$A \to b \mid D_1A \mid SA \mid AS \mid D_0B \mid a$

$B \to b$

$D_0 \to a$

$D_1 \to AS$

---

2) $S \to AS \mid BABC$

$A \to A \mid 0A1 \mid 01$

$B \to 0B \mid 0$

$C \to 1C \mid 1$

$\Rightarrow S_0 \to S$

$S \to AS \mid BABC$

$A \to A \mid 0A1 \mid 01$

$B \to 0B \mid 0$

$C \to 1C \mid 1$

No null, no useless

$\Rightarrow$ removing unit

$S_0 \to AS \mid BABC$

$S \to AS \mid BABC$

$A \to 0A1 \mid 01$

$B \to 0B \mid 0$

$C \to 1C \mid 1$

$\Rightarrow S_0 \to AS \mid BABC$

$S \to AS \mid BABC$

$A \to D_0AD_1 \mid D_0D_1$

$B \to D_0B \mid 0$

$C \to D_1C \mid 1$

$D_0 \to 0$

$D_1 \to 1$

$\Rightarrow S_0 \to AS \mid D_2D_3$

$S \to AS \mid D_2D_3$

$A \to D_4D_1 \mid D_0D_1$

$B \to D_0B \mid 0$

$C \to D_1C \mid 1$

$D_0 \to 0$

$D_1 \to 1$

$D_2 \to BA$

$D_3 \to BC$

$D_4 \to D_0A$

3) $S \rightarrow aXbX$

$X \rightarrow aY \mid bY \mid \epsilon$

$Y \rightarrow X \mid \epsilon$

No 'S' in RHS

$\Rightarrow$ removing null (X, Y)

$S \rightarrow aXbX \mid aXb \mid abX \mid ab$

$X \rightarrow aY \mid a \mid bY \mid b$

$Y \rightarrow X$

$\Rightarrow$ removing unit

$S \rightarrow aXbX \mid aXb \mid abX \mid ab$

$X \rightarrow aY \mid a \mid bY \mid b$

$Y \rightarrow aY \mid a \mid bY \mid b$

No useless

$\Rightarrow$ $S \rightarrow D_0XD_1X \mid D_0XD_1 \mid D_0D_1X \mid D_0D_1$

$X \rightarrow D_0Y \mid a \mid D_1Y \mid b$

$Y \rightarrow D_0Y \mid a \mid D_1Y \mid b$

$D_0 \rightarrow a$

$D_1 \rightarrow b$

$\Rightarrow$ $S \rightarrow D_2D_3 \mid D_2D_1 \mid D_0D_3 \mid D_0D_1$

$X \rightarrow D_0Y \mid a \mid D_1Y \mid b$

$Y \rightarrow D_0Y \mid a \mid D_1Y \mid b$

$D_0 \rightarrow a$

$D_1 \rightarrow b$

$D_2 \rightarrow D_0X$

$D_3 \rightarrow D_1X$

4) $S \rightarrow ABS$

$S \rightarrow \epsilon$

$A \rightarrow \epsilon$

$A \rightarrow xyz$

$B \rightarrow wB$

$B \rightarrow v$

$\Rightarrow$ $S_0 \rightarrow S$

$S \rightarrow ABS \mid \epsilon$

$A \rightarrow xyz \mid \epsilon$

$B \rightarrow wB \mid v$

$\Rightarrow$ removing null (S, A)

$S_0 \rightarrow S$

$S \rightarrow ABS \mid BS \mid AB \mid B$

$A \rightarrow xyz$

$B \rightarrow wB \mid v$

$\Rightarrow$ removing unit

$S_0 \rightarrow ABS \mid BS \mid AB \mid wB \mid v$

$S \rightarrow ABS \mid BS \mid AB \mid wB \mid v$

$A \rightarrow xyz$

$B \rightarrow wB \mid v$

No useless

$\Rightarrow$ $S_0 \rightarrow ABS \mid BS \mid AB \mid D_0B \mid v$

$S \rightarrow ABS \mid BS \mid AB \mid D_0B \mid v$

$A \rightarrow D_1D_2D_3$

$B \rightarrow D_0B \mid v$

$D_0 \rightarrow w$

$D_1 \rightarrow x$

$D_2 \rightarrow y$

$D_3 \rightarrow z$

$\Rightarrow$ $S_0 \rightarrow D_4S \mid BS \mid AB \mid D_0B \mid v$

$S \rightarrow D_4S \mid BS \mid AB \mid D_0B \mid v$

$A \rightarrow D_5D_3$

$B \rightarrow D_0B \mid v$

$D_0 \rightarrow w$

$D_1 \rightarrow x$

$D_2 \rightarrow y$

$D_3 \rightarrow z$

$D_4 \rightarrow AB$

$D_5 \rightarrow D_1D_2$

5) $S \to ABC$

$A \to aC \mid D$

$B \to bB \mid \epsilon \mid A$

$C \to Ac \mid \epsilon \mid Cc$

$D \to aa$

No 'S' in RHS

$\Rightarrow$ removing null (B, C)

$S \to ABC \mid AB \mid AC \mid A$

$A \to aC \mid a \mid D$

$B \to bB \mid b \mid A$

$C \to Ac \mid Cc \mid c$

$D \to aa$

$\Rightarrow$ removing unit

$S \to ABC \mid AB \mid AC \mid aC \mid a \mid aa$

$A \to aC \mid a \mid aa$

$B \to bB \mid b \mid aC \mid a \mid aa$

$C \to Ac \mid Cc \mid c$

$D \to aa$ $\to$ useless

$\Rightarrow$ $S \to ABC \mid AB \mid AC \mid D_0C \mid a \mid D_0D_0$

$A \to D_0C \mid a \mid D_0D_0$

$B \to D_1B \mid b \mid D_0C \mid a \mid D_0D_0$

$C \to AD_2 \mid CD_2 \mid c$

$D_0 \to a$

$D_1 \to b$

$D_2 \to c$

$\Rightarrow$ $S \to D_3C \mid AB \mid AC \mid D_0C \mid a \mid D_0D_0$

$A \to D_0C \mid a \mid D_0D_0$

$B \to D_1B \mid b \mid D_0C \mid a \mid D_0D_0$

$C \to AD_2 \mid CD_2 \mid c$

$D_0 \to a$

$D_1 \to b$

$D_2 \to c$

$D_3 \to AB$

PDA = ~~Based~~ Push Down Automata

$$M = (Q, \Sigma, \Gamma, \delta, q_0, F, Z_0)$$

set of stack symbols (Γ)

initial symbol in the stack (Z_0)

$$\delta = Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma$$

Transitions

i) $\delta(q, a, z) = (P, az)$

$\Rightarrow$ there is a transition from current state 'q' to 'p' when input symbol is a and top of the stack is 'z' and push op. occurs i.e. 'a' is pushed onto the stack

ii) $\delta(q, a, z) = (P, \epsilon)$

$\Rightarrow$ z is popped from the stack

iii) $\delta(q, a, z) = (P, \gamma)$

$\Rightarrow$ z is replaced by γ

iv) $\delta(q, \epsilon, z) = (p, \gamma)$

$\Rightarrow$ z is replaced by γ when no more input string is available.

v) $\delta(q, \epsilon, \epsilon) = (p, z)$

$\Rightarrow$ z is pushed onto the stack

$$\Rightarrow \quad q \xrightarrow{\epsilon, \epsilon \mid z} p$$

Examples $L = \{ wcw^R \mid w \in (a+b)^* \}$ Non deterministic

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, c, a) = (q_0, a)$

$\delta(q_0, c, b) = (q_0, b)$

$\delta(q_0, c, z_0) = (q_1, z_0)$

$\delta(q_0, a, a) = \delta(q_1, \epsilon)$

$\delta(q_0, b, b) = \delta(q_1, \epsilon)$

$\delta(q_1, a, a) = \delta(q_1, \epsilon)$

$\delta(q_1, b, b) = \delta(q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_f, z_0)$

$L = \{ww^R \mid w \in (a+b)^*\}$ Non deterministic

$\delta(q_0, a, Z_0) \Rightarrow (q_0, aZ_0)$
$\delta(q_0, b, Z_0) \Rightarrow (q_0, bZ_0)$
$\delta(q_0, a, a) \rightarrow (q_0, aa)$
$\delta(q_0, b, a) \rightarrow (q_0, ba)$
$\delta(q_0, a, b) \rightarrow (q_0, ab)$
$\delta(q_0, b, b) \rightarrow (q_0, bb)$

$\delta(q_0, \epsilon, Z_0) \Rightarrow (q_f, Z_0)$
$\delta(q_0, a, a) \rightarrow (q_1, \epsilon)$
$\delta(q_0, b, b) \rightarrow (q_1, \epsilon)$

$$\boxed{\begin{array}{l} \delta(q_1, a, a) \rightarrow (q_1, \epsilon) \\ \delta(q_1, b, b) \rightarrow (q_1, \epsilon) \\ \delta(q_1, \epsilon, Z_0) \Rightarrow (q_f, Z_0) \end{array}}$$ Not needed

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It is clear from the language $L(M) = ww^R$ and if $w = abb$ then $w^R = bba$ So the language

$L = \underbrace{abb}_{w}\underbrace{bba}_{w^R}$ ~~than~~

To check for a palindrome lets push all the scanned symbols onto the stack until string w completes, Once we start with the string w^R, there should be a corresponding symbol on the stack. If there is not input and stack is empty then we say the given string is a palindrome

step 1 :- * Input symbol can be 'a' or 'b'

* Let q_0 be the initial state and Z_0 be the initial symbol on the stack. In state q_0, when top of the stack is Z_0, the input symbol whether a or b can be pushed onto the stack & remain in the state q_0, The transition for this can be written as

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$
$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

* Once the 1st input symbol is pushed onto the stack, the stack may contain either an 'a' or 'b' on top of the stack. Irrespective of whatever is the input, until 'w' completes we have to push all elements onto the stack which can be indicated by the transitions

$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_0, ba)$
$\delta(q_0, a, b) = (q_0, ab)$
$\delta(q_0, b, b) = (q_0, bb)$

String 'w' completes with these 4 transitions.

step 2:- The string w^R starts, input symbol scanned can be an 'a' or 'b'

To be a palindrom for each input symbol, there should be corresponding symbol in the stack. So whenever the input symbol is same as symbol on the stack make a transition to state q_1 and delete the symbol from the stack.

∴ Which can be represented by the 2 transitions

$$\delta(q_0, a, a) = (q_1, \epsilon)$$
$$\delta(q_0, b, b) = (q_1, \epsilon)$$

Further if it is a palindrome the scanned symbols & the corresponding stack symbols will match and all the symbols that match can be popped which can be represented by the transition

$$\delta(q_1, a, a) = (q_1, \epsilon)$$
$$\delta(q_1, b, b) = (q_1, \epsilon)$$

Finally if the current state is q_1 and no more input symbols to be scanned and top of the stack is 'Z_0' then we can change the state to final state q_f and let Z_0 remain on top of the stack which can be indicated by the transition

$$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$$

Lastly since $w \in (a+b)^*$, w can be empty which means without any input symbols we can reach the final state which can be indicated by

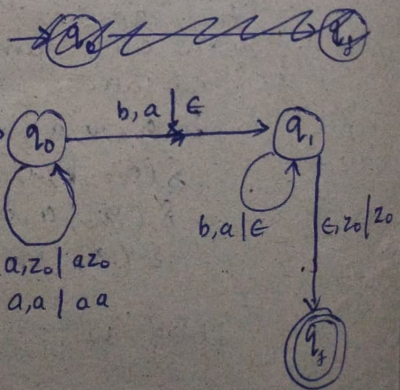
$$\delta(q_0, \epsilon, Z_0) = (q_f, Z_0)$$

---

2) Obtain PDA for a language $L = \{a^n b^n \mid n \geq 1\}$

sol:-

$$\delta(q_0, a, Z_0) = (q_0, aZ_0)$$
$$\delta(q_0, a, a) = (q_0, aa)$$
$$\delta(q_0, b, a) = (q_1, \epsilon)$$
$$\delta(q_1, b, a) = (q_1, \epsilon)$$
$$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$$



Deterministic

3) ~~[illegible]~~

$L = \{ n_a(w) = n_b(w) \mid w \in (a+b)^* \}$

Sol^n:

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

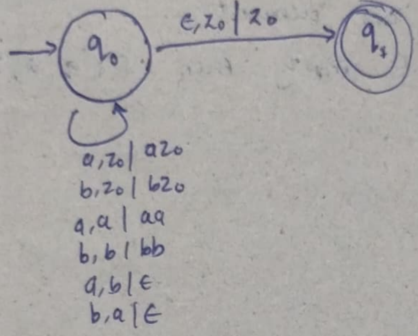
$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, \epsilon)$

$\delta(q_0, b, a) = (q_0, \epsilon)$

$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$

~~[illegible]~~ Non deterministic



4) $L = \{ n_a(w) > n_b(w) \mid w \in (a,b)^* \}$ Deterministic

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

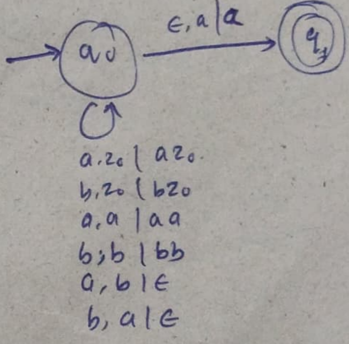
$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, \epsilon)$

$\delta(q_0, b, a) = (q_0, \epsilon)$

$\delta(q_0, \epsilon, a) =$ ~~[illegible]~~ (q_f, a)



5) ~~[illegible]~~ Obtain a pda ~~[illegible]~~ to obtain strings of balanced parenthesis. The parethesis can be ( ) or [ ].

$\delta(q_0, (, z_0) = (q_0, (z_0)$

$\delta(q_0,), z_0) = (q_0,)z_0)$

$\delta(q_0, [, z_0) = (q_0, [z_0)$

$\delta(q_0,], z_0) = (q_0,]z_0)$

$\delta(q_0, (, () = (q_0, (()$

$\delta(q_0,), () = (q_0, \epsilon)$

$\delta(q_0, (,)) = (q_0, ())$

$\delta(q_0,),)) = (q_0,)))$

$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$

6) $L = \{a^n b^{2n} \mid n \geq 1\}$

Deterministic

$\delta(q_0, a, Z_0) = \delta(q_0, aaZ_0)$
$\delta(q_0, a, a) = \delta(q_0, aaa)$
$\delta(q_0, b, a) = \delta(q_1, \epsilon)$
$\delta(q_1, b, a) = \delta(q_1, \epsilon)$
$\delta(q_1, \epsilon, Z_0) = \delta(q_f, Z_0)$

Deterministic and non-deterministic push down automata

Deterministic PDA:- Let $M = (Q, \Sigma, T, \delta, q_0, Z_0, F)$ be a pushdown automata. The PDA is said to be deterministic if i) $\delta(q, a, z)$ has only one element.

ii) if $\delta(q, \epsilon, z)$ is not empty then $\delta(q, a, z)$ should be empty.

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_0, a, b) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_2, \epsilon)$ $\qquad Q = \{q_0, q_1, q_2\}$

Formula: $\delta(q_i, a, Z) = (q_j, AB)$

$q_i Z q_k \rightarrow a(q_j A q_l)(q_l B q_k)$

i) $\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$q_0 Z_0 q_0 \rightarrow a(q_0 a q_0)(q_0 Z_0 q_0) \mid a(q_0 a q_1)(q_1 Z_0 q_0) \mid a(q_0 a q_2)(q_2 Z_0 q_0)$

$q_0 Z_0 q_1 \rightarrow a(q_0 a q_0)(q_0 Z_0 q_1) \mid a(q_0 a q_1)(q_1 Z_0 q_1) \mid a(q_0 a q_2)(q_2 Z_0 q_1)$

$q_0 Z_0 q_2 \rightarrow a(q_0 a q_0)(q_0 Z_0 q_2) \mid a(q_0 a q_1)(q_1 Z_0 q_2) \mid a(q_0 a q_2)(q_2 Z_0 q_2)$

ii) $\delta(q_0, a, a) = (q_0, aa)$

$q_0 a q_0 \rightarrow a(q_0 a q_0)(q_0 a q_0) \mid a(q_0 a q_1)(q_1 a q_0) \mid a(q_0 a q_2)(q_2 a q_0)$

$q_0 a q_1 \rightarrow a(q_0 a q_0)(q_0 a q_1) \mid a(q_0 a q_1)(q_1 a q_1) \mid a(q_0 a q_2)(q_2 a q_1)$

$q_0 a q_2 \rightarrow a(q_0 a q_0)(q_0 a q_2) \mid a(q_0 a q_1)(q_1 a q_2) \mid a(q_0 a q_2)(q_2 a q_2)$

iii) $\delta(q_0, b, a) = (q_1, \epsilon)$ $\qquad$ iv) $\delta(q_0, a, b) = (q_1, \epsilon)$ $\qquad$ v) $\delta(q_1, \epsilon, Z_0) = (q_2, \epsilon)$

$q_0 a q_1 \rightarrow b$ $\qquad$ $q_0 b q_1 \rightarrow a$ $\qquad$ $q_1 Z_0 q_2 \rightarrow \epsilon$

Note: $\delta(q_0, a, A) \rightarrow (q_1, A) \Rightarrow \delta(q_0, a, A) \rightarrow (q_3, \epsilon)$

$\delta(q_3, \epsilon, Z_0) \rightarrow (q_1, AZ_0)$

Pumping Lemma for context-free language ~~grammar~~

Let L be a context free language and is infinite. Let z be a sufficiently long string where $z \in L$ so that $|z| \geq n$ where n is a positive integer. The string z can be decomposed into combination of strings ~~such that~~ i.e

$$z = uvwxy$$

such that $|vwx| \leq n$, $|vx| \geq 1$ then

$z = uv^i w x^i y \in L$ for $i = 0, 1, 2, \ldots\ldots$

Question 1 $L = \{a^n b^n c^n \mid n \geq 0\}$

~~z~~ $z = a^n b^n c^n$ ie $|z| = 3n$ ie $|z| > n$

$z = \underbrace{a^{n+2}}_{uv^iwx^i} \underbrace{b^n c^2}_{y}$

$z = \underbrace{a \cdots\cdots a}_{uv^iwx^i} \underbrace{b \cdots\cdots b c \cdots\cdots c}_{y}$

~~when~~

1) Let L be a context free & infinite language. Let $z = a^n b^n c^n \in L$.

2) Note that $|z| \geq n$ so we can split 'z' into 'uvwxy' such that $|vwx| \leq n$ and $|vx| \geq 1$

3) $z = \underbrace{\overbrace{a \cdots\cdots a}^{n}}_{uv^iwx^i} \underbrace{\overbrace{b \cdots\cdots b}^{n} \overbrace{c \cdots\cdots c}^{n}}_{y}$

When $i = 2$ we have

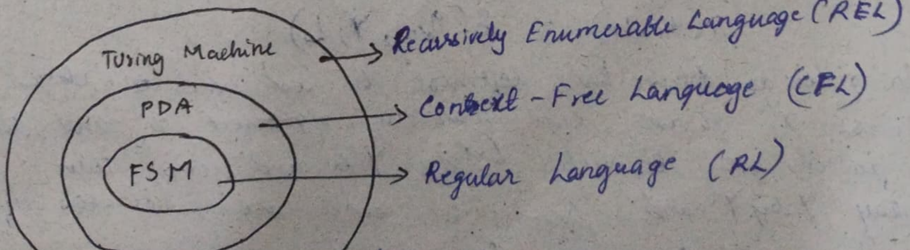
$z = \underbrace{\overbrace{a \cdots\cdots a}^{n+j+k}}_{uv^2wx^2} \underbrace{\overbrace{b \cdots\cdots b}^{n} \overbrace{c \cdots\cdots c}^{n}}_{y}$

Thus $uv^2wx^2y = a^{n+j+k} b^n c^n \notin L$

when $j+k \geq 1$, the string given should have some no of a's followed by equal no. of b's followed by equal no. of c's but when $i = 2$, the no. of a's becomes more than no. of b's and c's which is a contradiction according to pumping Lemma.

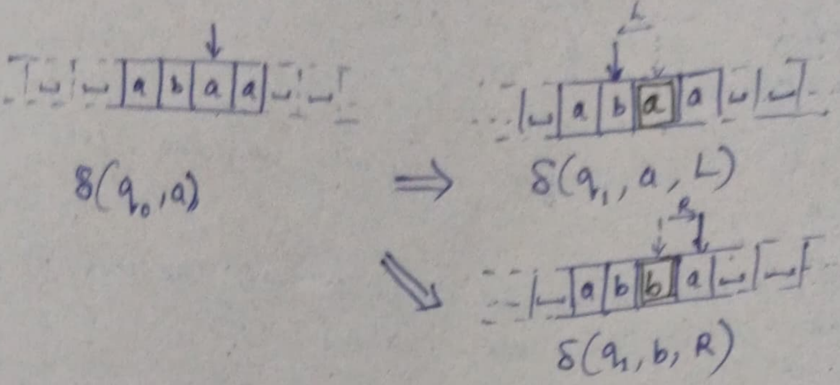
Hence the given language is not a context free language.

Turing Machine



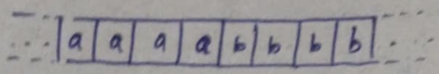
$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$



Construct a TM for the language $L = \{a^n b^n \mid n \geq 1\}$

Ex:



For the given language, Consider the string $W = aaaabbbb$

Let q_0 be the start state and ~~[illegible]~~ read-write head ~~[illegible]~~ be pointing to the first symbol of the string. replace the leftmost a by 'x' and change the state to q_1

$$\delta(q_0, a) = (q_1, X, R)$$

Search for the left most B ~~[illegible]~~ to replace it by Y and repeat the procedure. In state q_1, we have to obtain the leftmost b and replace it 'Y'. During this process we come across many input symbols. If the input symbol is a then replace 'a' by 'a' itself and move towards right. This can be represented by the transition

$$\delta(q_1, a) = \delta(q_1, a, R)$$
$$\delta(q_1, Y) = \delta(q_1, Y, R)$$

In state q_1, if the input symbol scanned is a b then replace 'b' by 'y' and move towards left

$$\delta(q_1, b) = \delta(q_2, Y, L)$$

In order to obtain the leftmost a we need to obtain the rightmost 'x' so we can scan the rightmost 'x' and during the process we encounter ~~[illegible]~~ Y's and a's ; then replay Y by Y and 'a' by 'a' and move towards left

$$\delta(q_2, Y) = (q_2, Y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, X) = (q_0, X, R)$$

Now we have obtained rightmost X, to get leftmost 'a', replace X by X move right & transition to state q_0

$$\therefore \delta(q_2, X) = (q_0, X, R)$$

In state q_0 if input symbol scanned is Y then there are no a's. If there are no a's then we should make sure that there are no b's.

$$\delta(q_0, Y) = (q_3, Y, R)$$
$$\delta(q_3, Y) = (q_3, Y, R) \; * \leftarrow$$

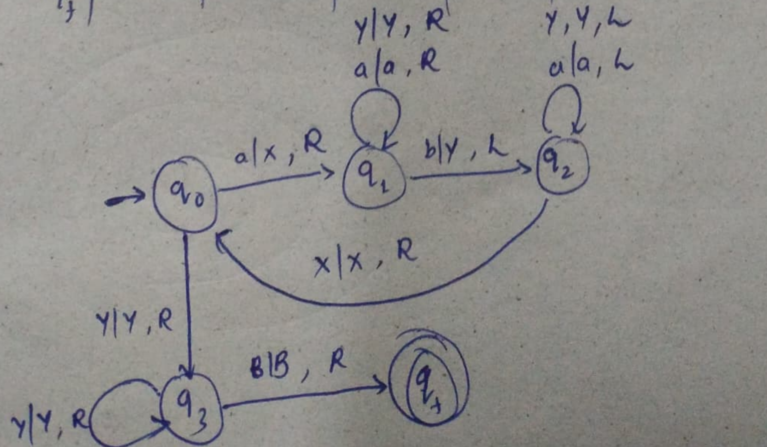
Replace Y by Y and move right

$$\delta(q_3, B) = (q_f, B, R)$$

In state q_3 we should see that there are only Y's and no more b's so we scan and replace Y by Y and remain in state q_3. The transition is (*)

Note that the string ends with a infinite no. of ␣ (blanks) so in state q_3, if we encounter a the string is encountered blank symbol (B), it means end of the string is followed by 'n' no. of B's which means 'n' no. of a's so change to state q_f. Replace B by B and move the pointer towards the right & the string is accepted.

	a	b	X	Y	B
q_0	(q_1, X, R)	-	-	(q_3, Y, R)	-
q_1	(q_1, a, R)	(q_2, Y, L)	-	(q_1, Y, R)	-
q_2	(q_2, a, L)	-	(q_0, X, R)	(q_2, Y, L)	-
q_3	-	~~(q_3, Y, R)~~	-	(q_3, Y, R)	(q_f, B, R)
q_f					



$L = \{ 0^n 1^n 2^n \mid n \geq 1 \}$

soln:

000Y11222

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, 1) = (q_2, Y, R)$

$\delta(q_2, 1) = (q_2, 1, R)$

$\delta(q_2, 2) = (q_3, Z, L)$

$\delta(q_3, Z) = (q_3, Z, L)$

$\delta(q_3, 1) = (q_3, 1, L)$

$\delta(q_3, Y) = (q_3, Y, L)$

$\delta(q_3, 0) = (q_3, 0, L)$

$\delta(q_3, X) = (q_0, X, R)$

$\delta(q_0, Y) = (q_4, Y, R)$

$\delta(q_4, Y) = (q_4, Y, R)$

$\delta(q_4, Z) = (q_5, Z, R)$

$\delta(q_4, B) = (q_f, B, R)$

$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_2, Z) = (q_2, Z, R)$

	0	1	2	X	Y	Z	B
q_0	q_1, X, R	-	-	-	q_4, Y, R	-	-
q_1	$q_1, 0, R$	q_2, Y, R	-	-	q_1, Y, R	-	-
q_2	-	$q_2, 1, R$	q_3, Z, L	-	-	q_2, Z, R	-
q_3	$q_3, 0, L$	$q_3, 1, L$	-	q_0, X, R	q_3, Y, L	q_3, Z, L	-
q_4	-	-	-	-	q_4, Y, R	q_5, Z, R	$q_f, B,$
q_5							
q_f	-	-	-	-	-	-	-