Counting Inversions- Divide and Conquer

Counting Inversions

- Music site tries to match your song preferences with others.
  - You rank n songs.
  - Music site consults database to find people with similar tastes.

- Similarity metric: number of inversions between two rankings.
  - My rank: 1, 2, …, n.
  - Your rank: a_1, a_2, …, a_n.
  - Songs i and j inverted if i < j, but $a_i > a_j$.

Songs

	A	B	C	D	E
Me	1	2	3	4	5
You	1	3	4	2	5

Inversions

3-2, 4-2

Counting Inversions

Two elements a[i] and a[j] form an inversion if a[i]>a[j] and i<j

a

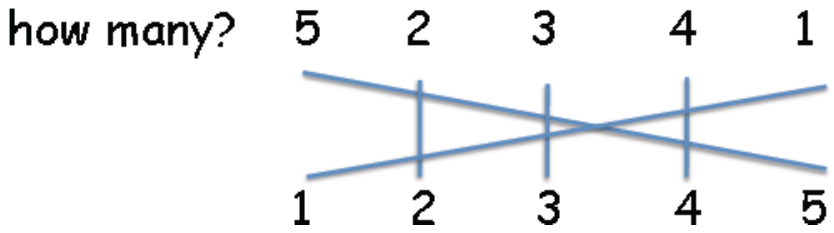
0	1	2	3	4
2	4	1	3	5

Inversions

(2,1), (4,1),(4,3)

Inversion Count for an array indicates:How far (close)the array is from being sorted
if array is already sorted inversion count is ??
if array is sorted in reverse order that inversion count is the ???

Visualizing inversions



Naive algorithm

```
# test all pairs
c=0
for i in 0 to n-1
  for j in i+1 to n-1
     compare r_i and r_j
     if inversion c++
```

Counting Inversions: Divide-and-Conquer

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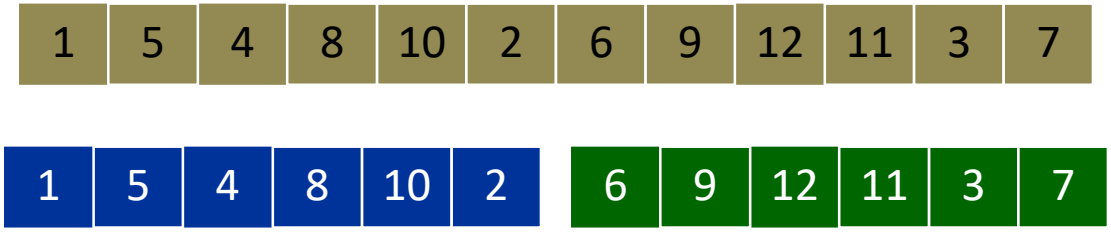
  - Divide: separate list into two pieces.



Divide: O(1).

Counting Inversions: Divide-and-Conquer

- Divide-and-conquer.
  - Divide: separate list into two pieces.
  - Conquer: recursively count inversions in each half.

Divide: O(1).

Conquer: 2T(n / 2)

5 blue-blue inversions

8 green-green inversions

5-4, 5-2, 4-2, 8-2, 10-2

6-3, 9-3, 9-7, 12-3, 12-7, 12-11, 11-3, 11-7

Counting Inversions: Divide-and-Conquer

- Divide-and-conquer.
  - Divide: separate list into two pieces.
  - Conquer: recursively count inversions in each half.
  - Combine: count inversions where a_i and a_j are in different halves, and return sum of three quantities.

Divide: O(1).

Conquer: 2T(n / 2)

5 blue-blue inversions

8 green-green inversions

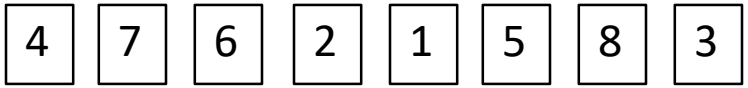
9 blue-green inversions

5-3, 4-3, 8-6, 8-3, 8-7, 10-6, 10-9, 10-3, 10-7

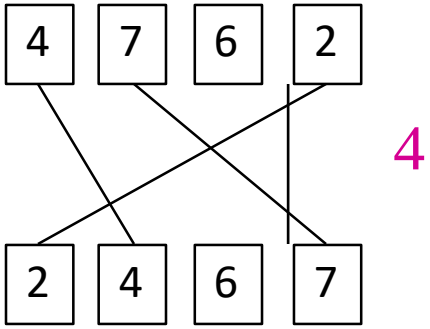
Combine: ???

Total = 5 + 8 + 9 = 22.

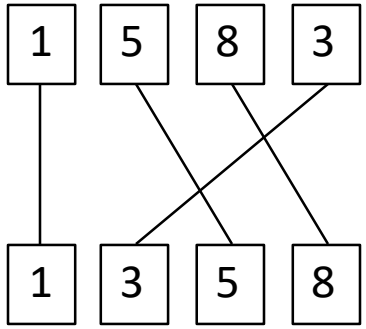
Counting Inversions



Divide into front and back halves



4



2

Count inversions within each half, and sort each half

Then, count inversions between front and back half

Counting Inversions: Implementation

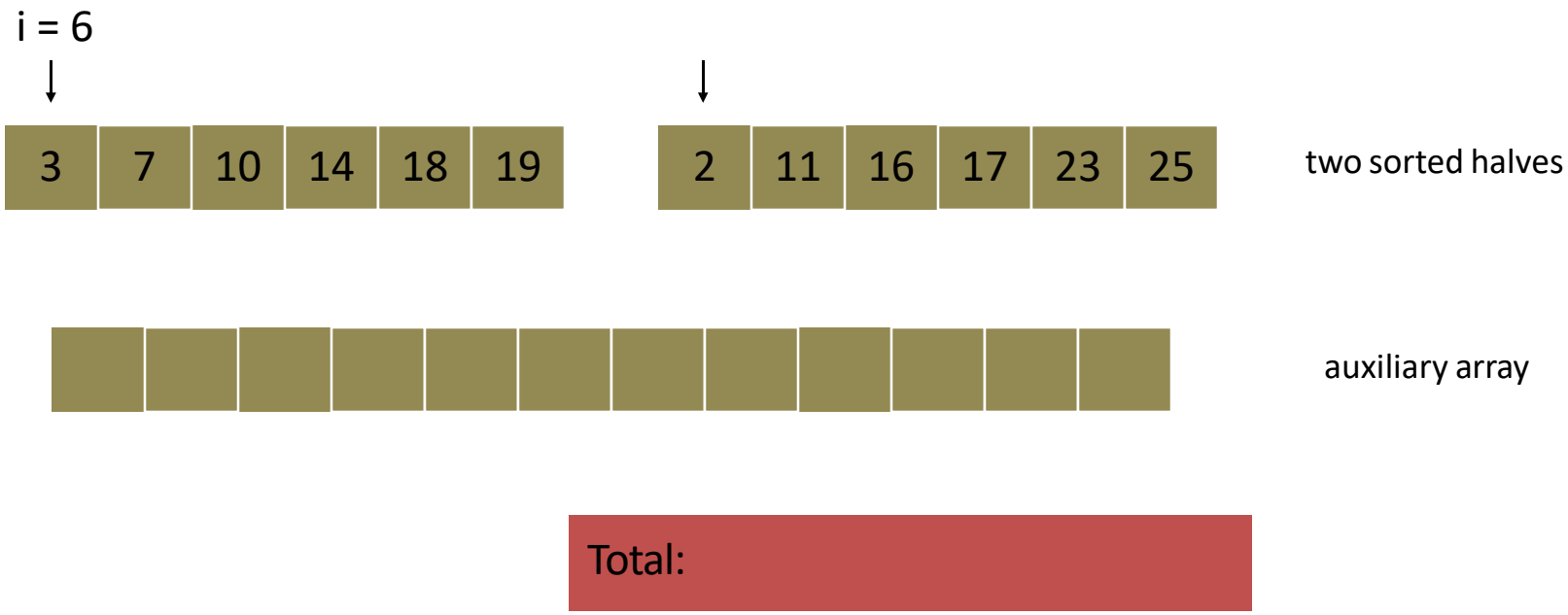
```
Sort-and-Count(L)
  If the list has one element then
      there are no inversions
  Else
      Divide the list into two halves:
         A contains the first ⌈n/2⌉ elements
         B contains the remaining ⌊n/2⌋ elements
      (r_A, A) = Sort-and-Count(A)
      (r_B, B) = Sort-and-Count(B)
      (r, L) = Merge-and-Count(A, B)
   Endif
   Return r = r_A + r_B + r, and the sorted list L
```

Counting Inversions: Implementation

```
Merge-and-Count(A,B)
  Maintain a Current pointer into each list, initialized to
    point to the front elements
  Maintain a variable Count for the number of inversions,
    initialized to 0
  While both lists are nonempty:
    Let a_i and b_j be the elements pointed to by the Current pointer
    Append the smaller of these two to the output list
    If b_j is the smaller element then
      Increment Count by the number of elements remaining in A
    Endif
    Advance the Current pointer in the list from which the
      smaller element was selected.
  EndWhile
  Once one list is empty, append the remainder of the other list
      to the output
  Return Count and the merged list
```

Counting Inversions

- Merge and count step.
  - Given two sorted halves, count number of inversions where a_i and a_j are in different halves.
  - Combine two sorted halves into sorted whole.



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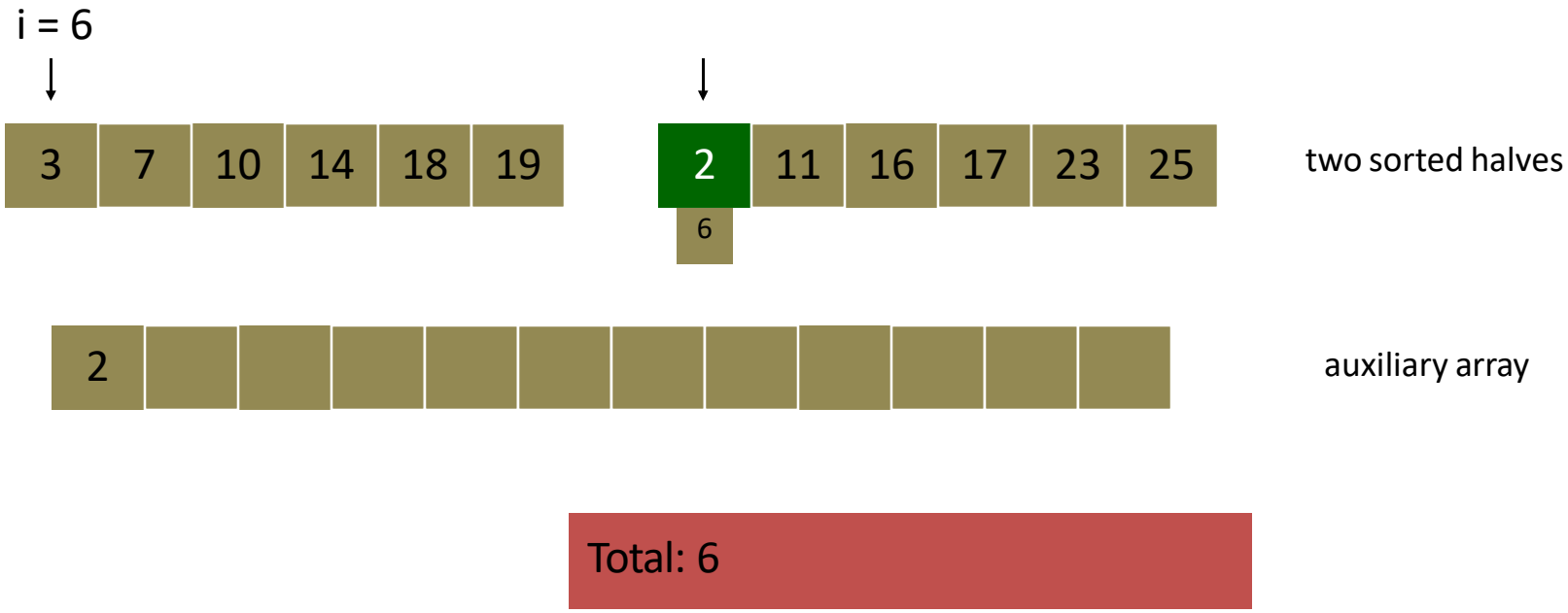

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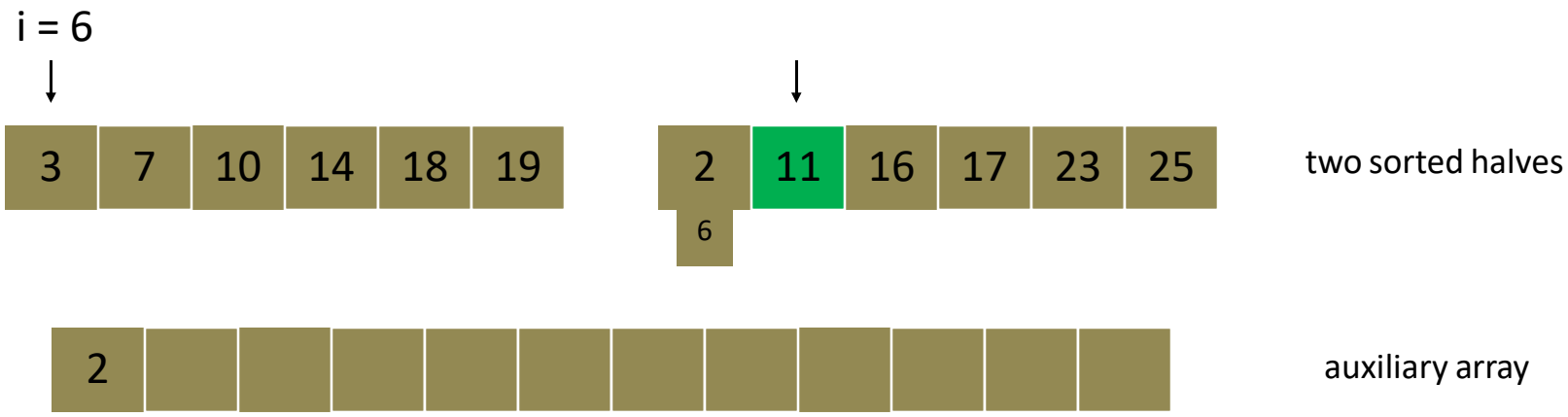

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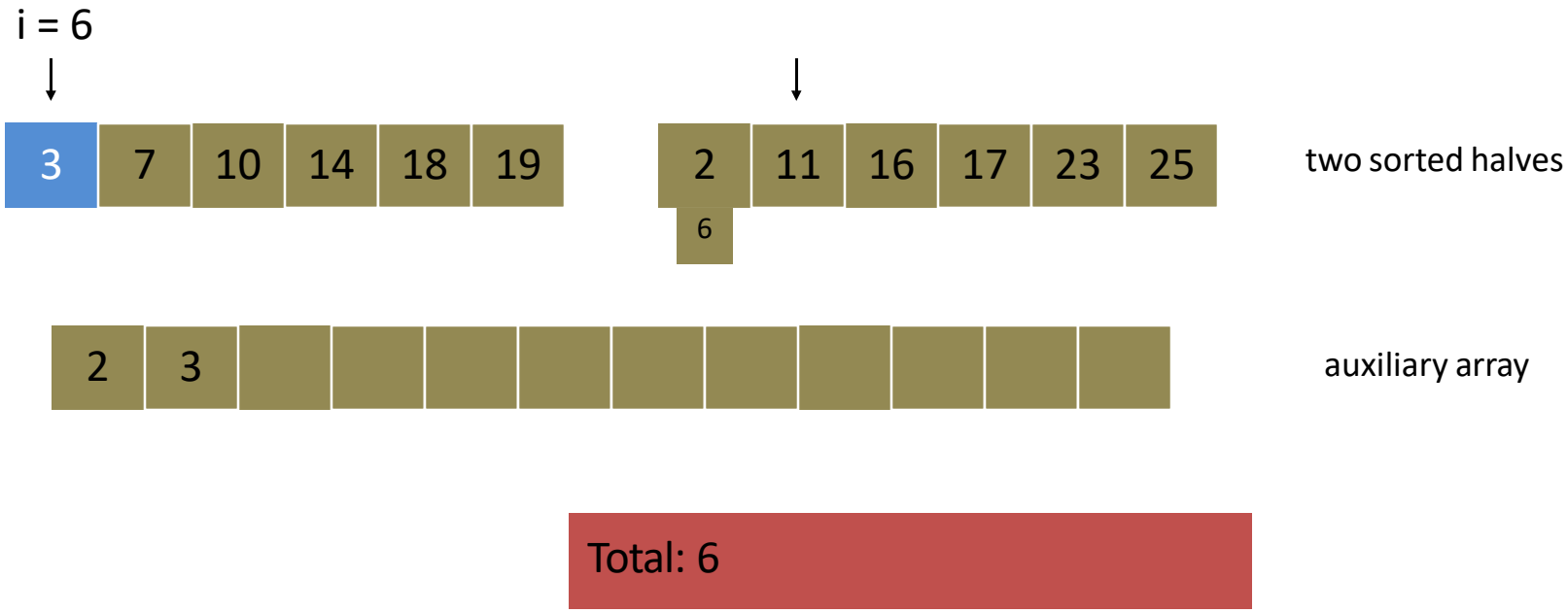

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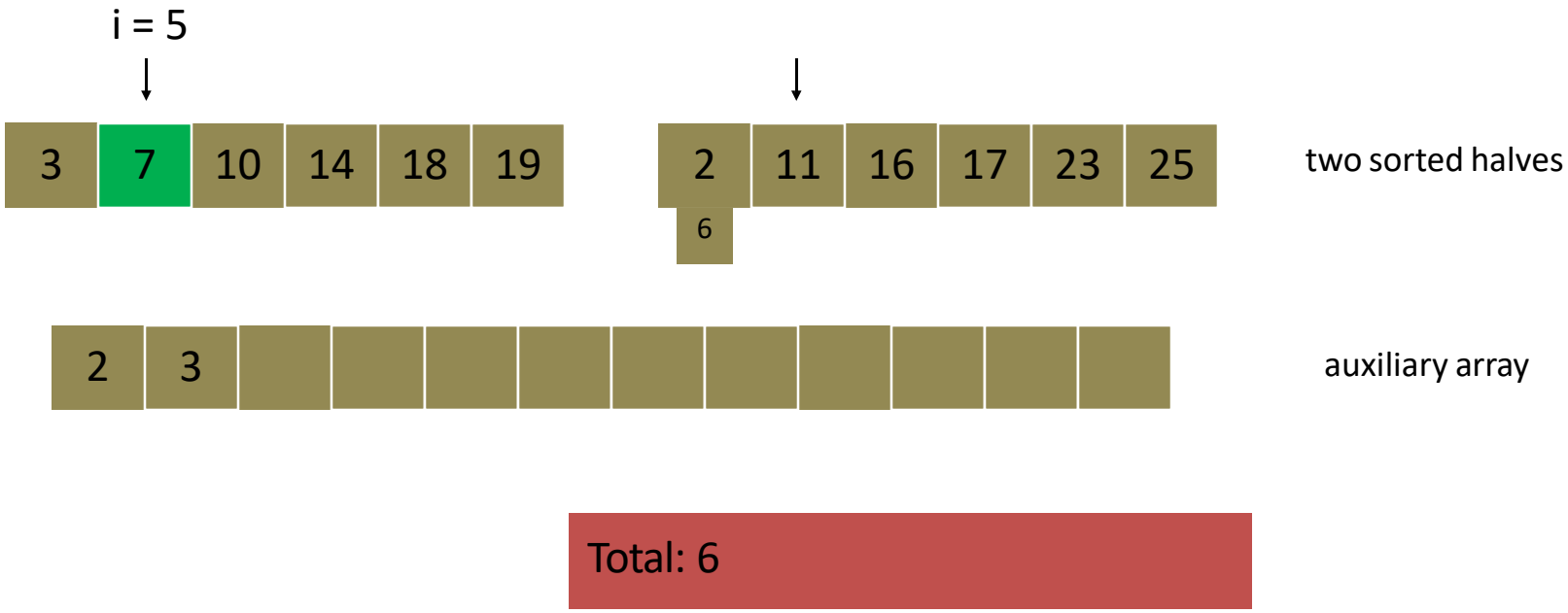

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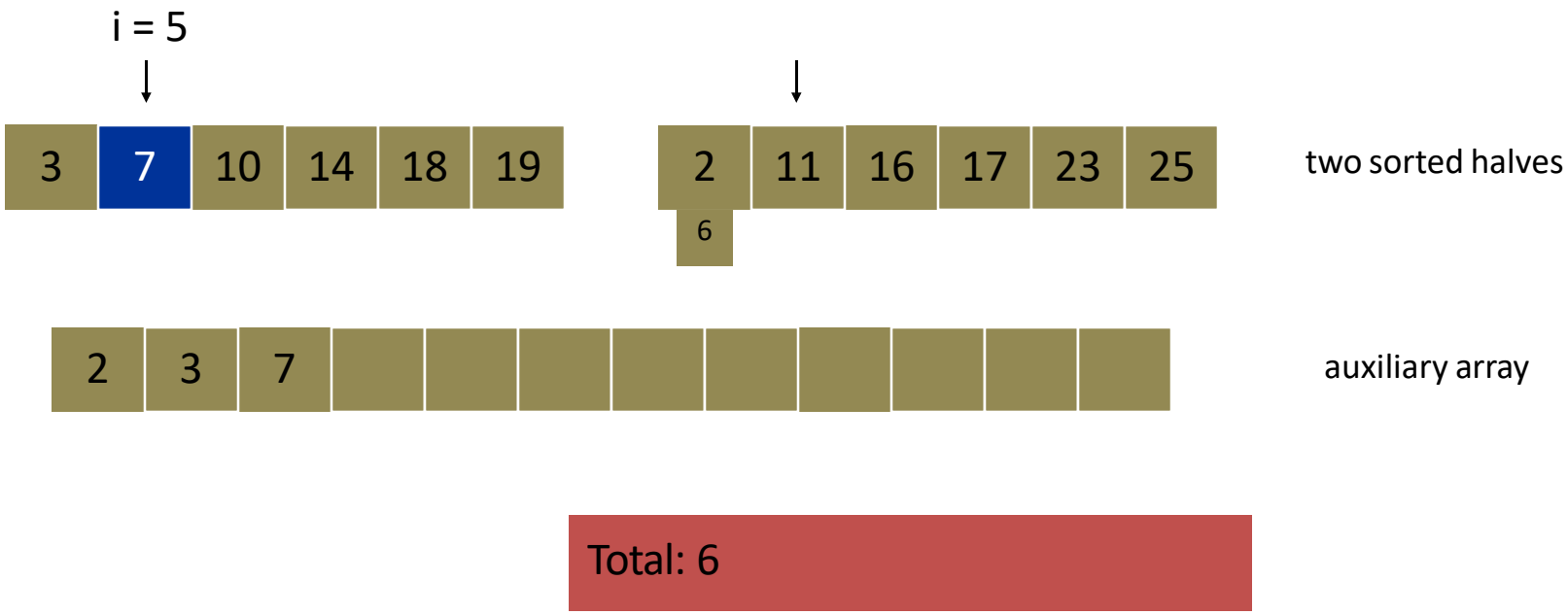

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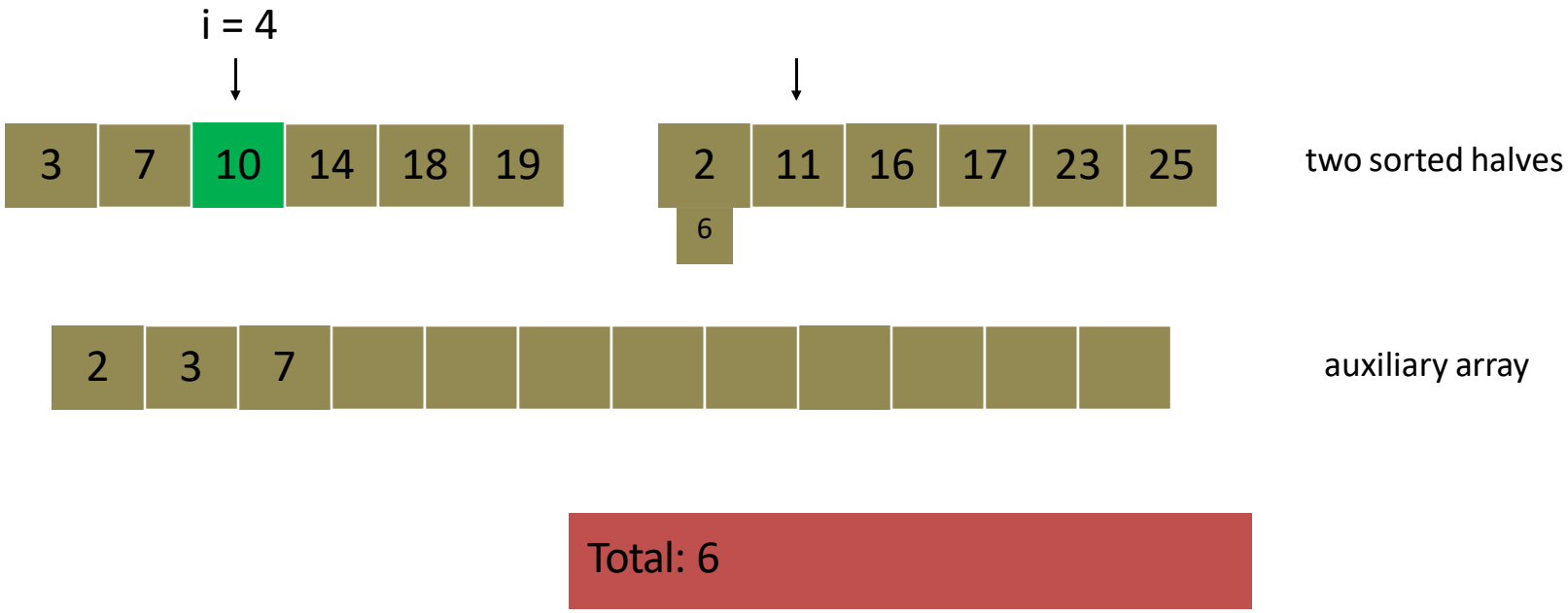

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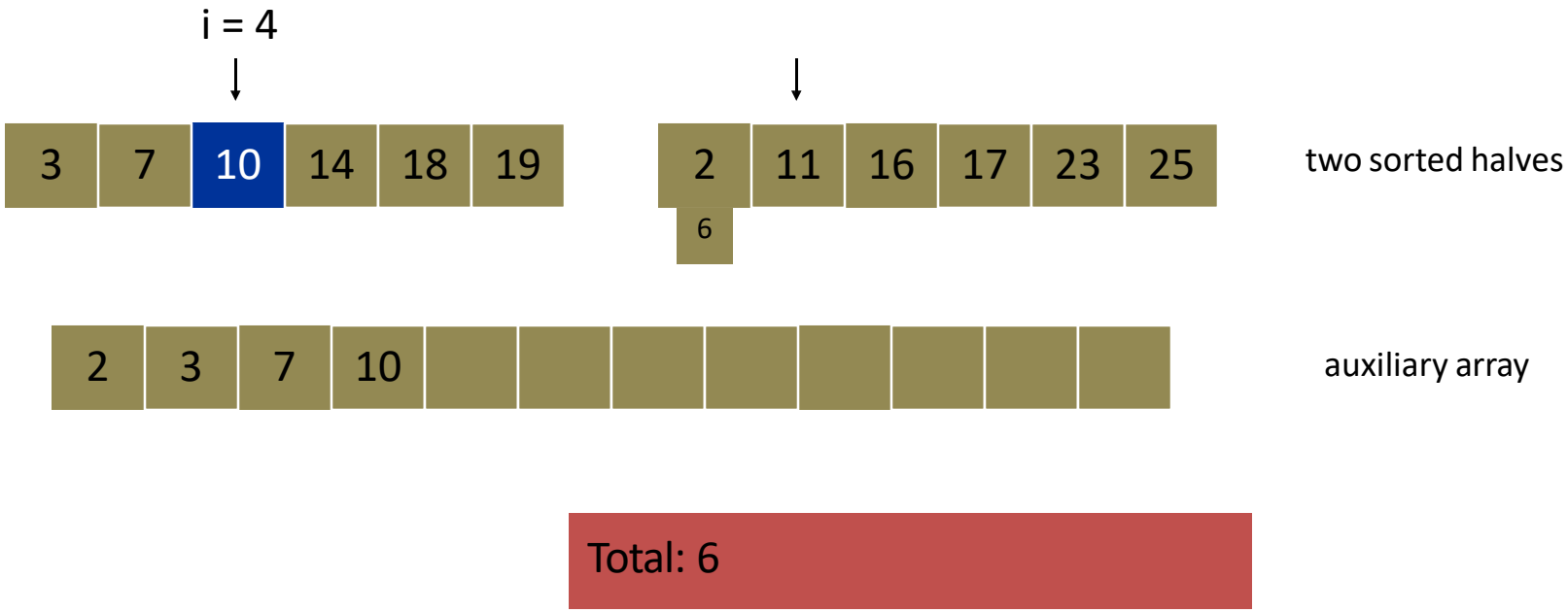

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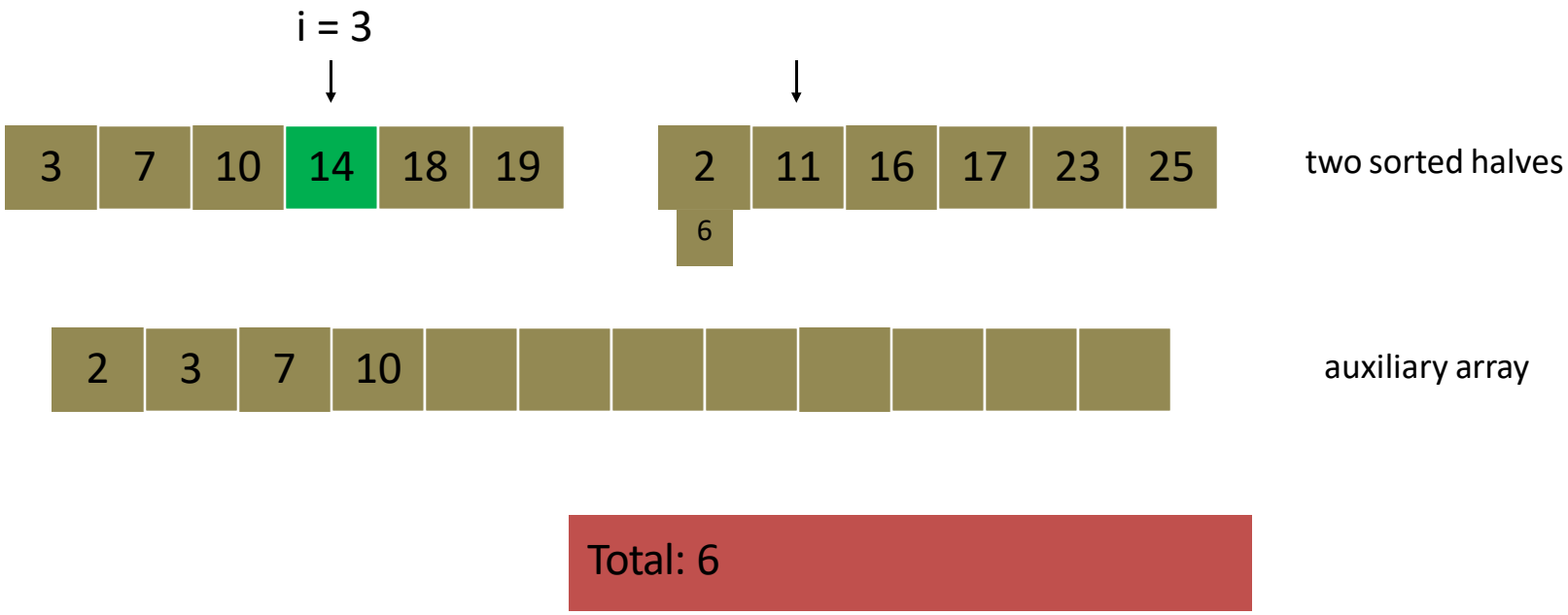

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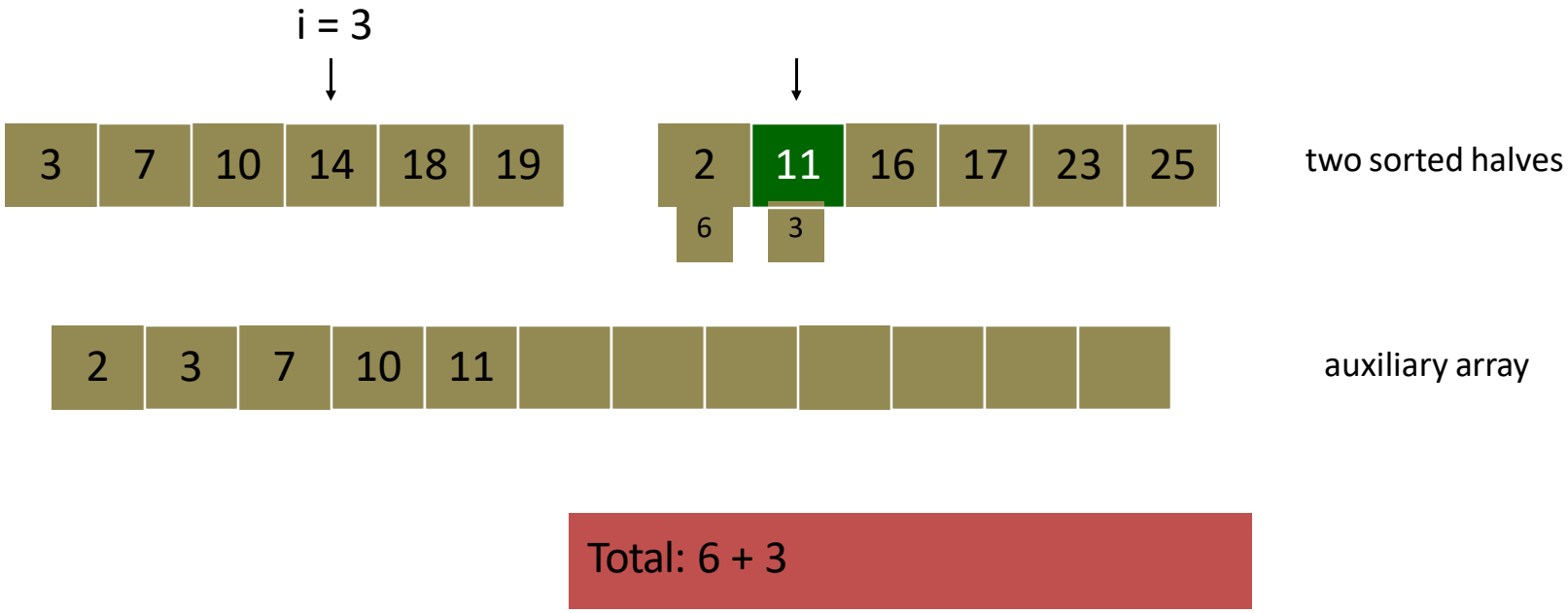

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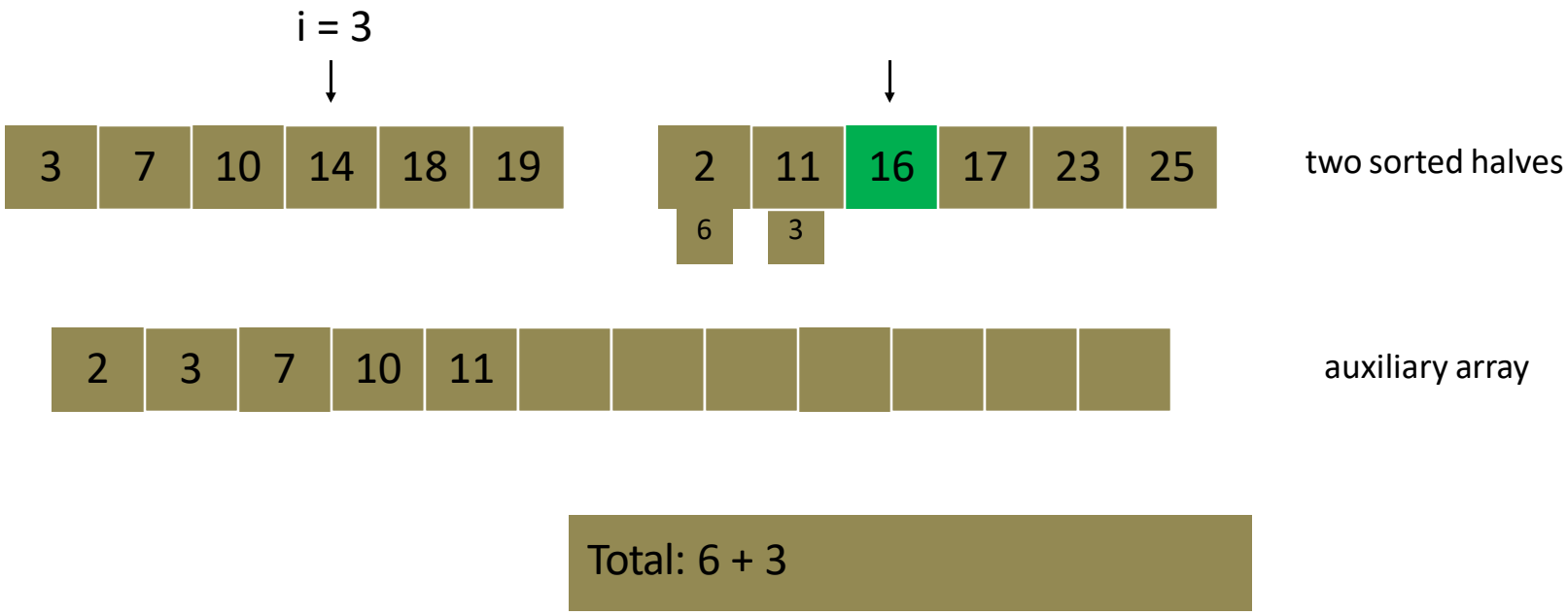

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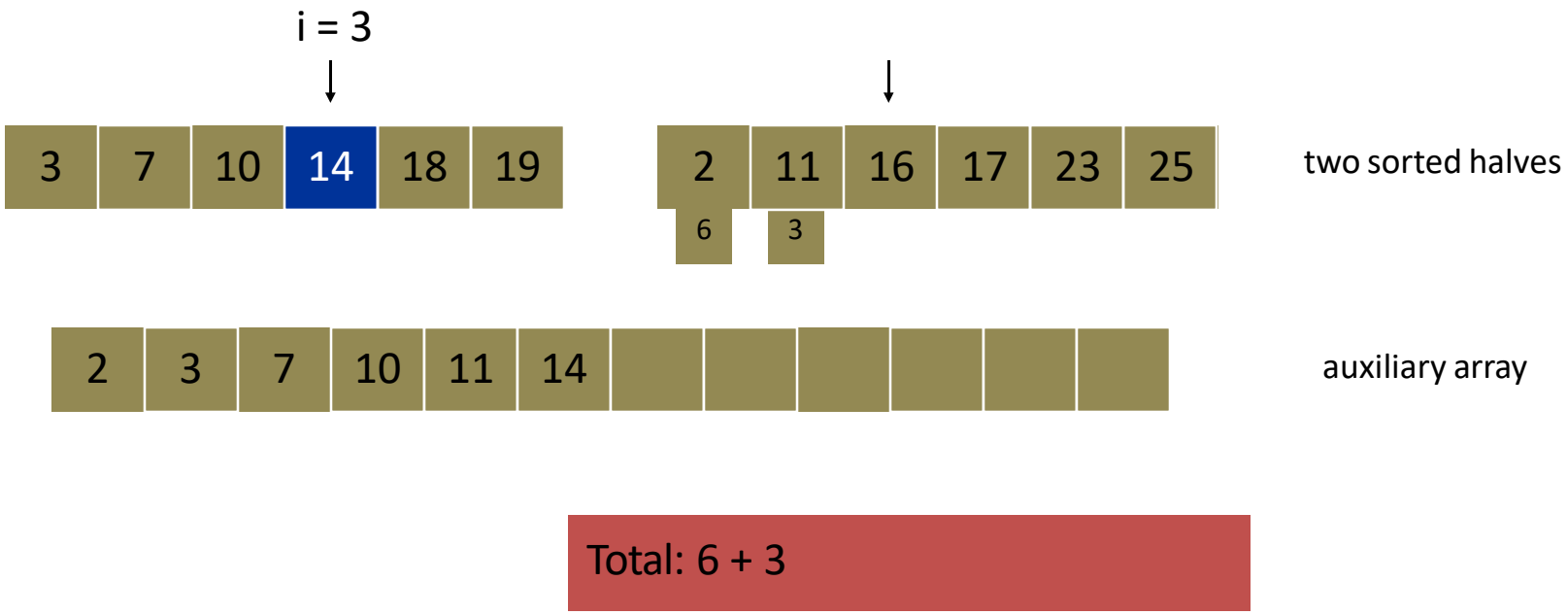

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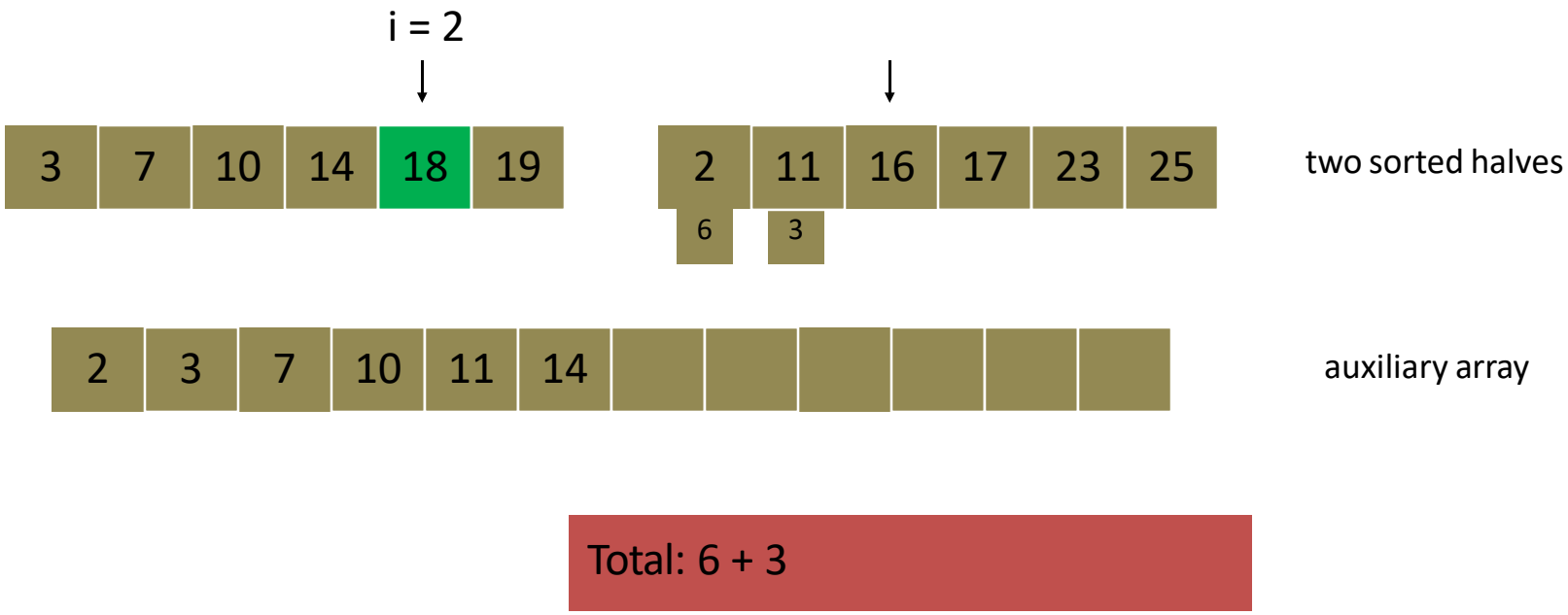

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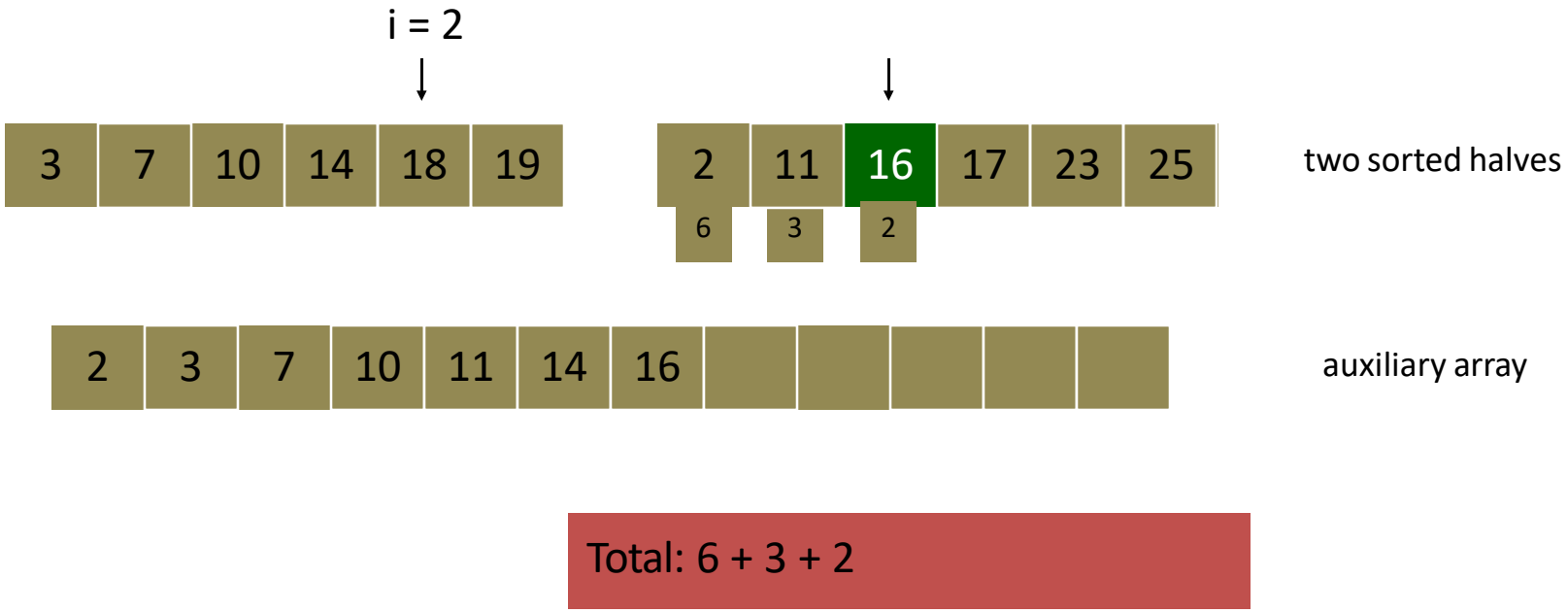

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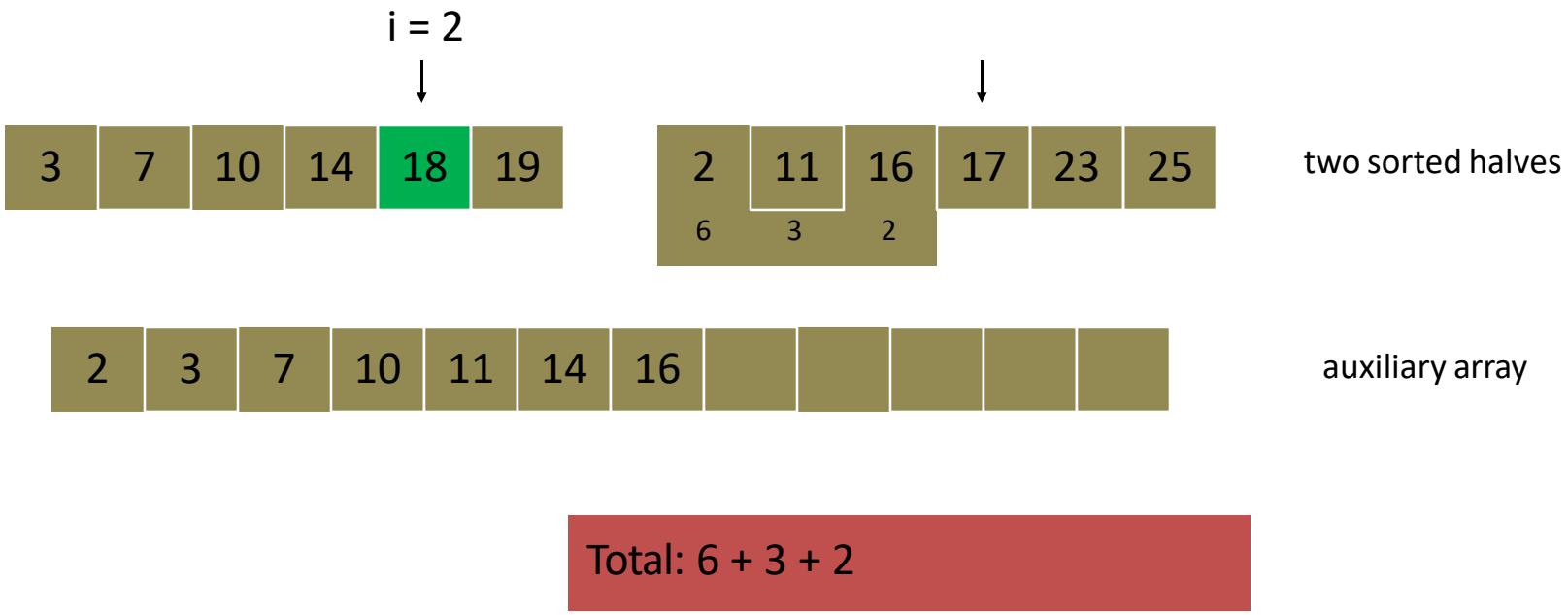

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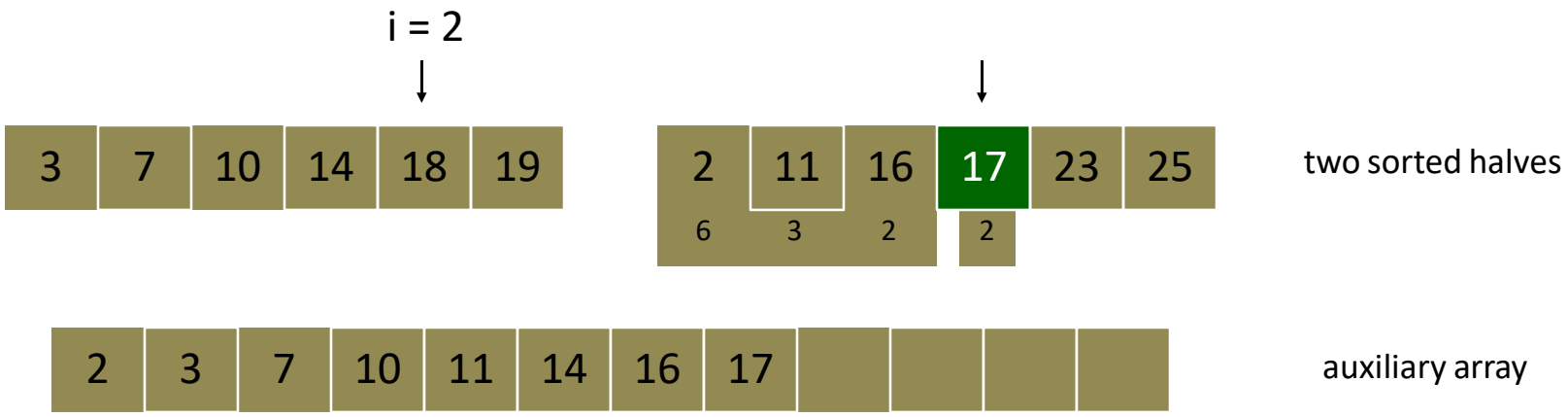

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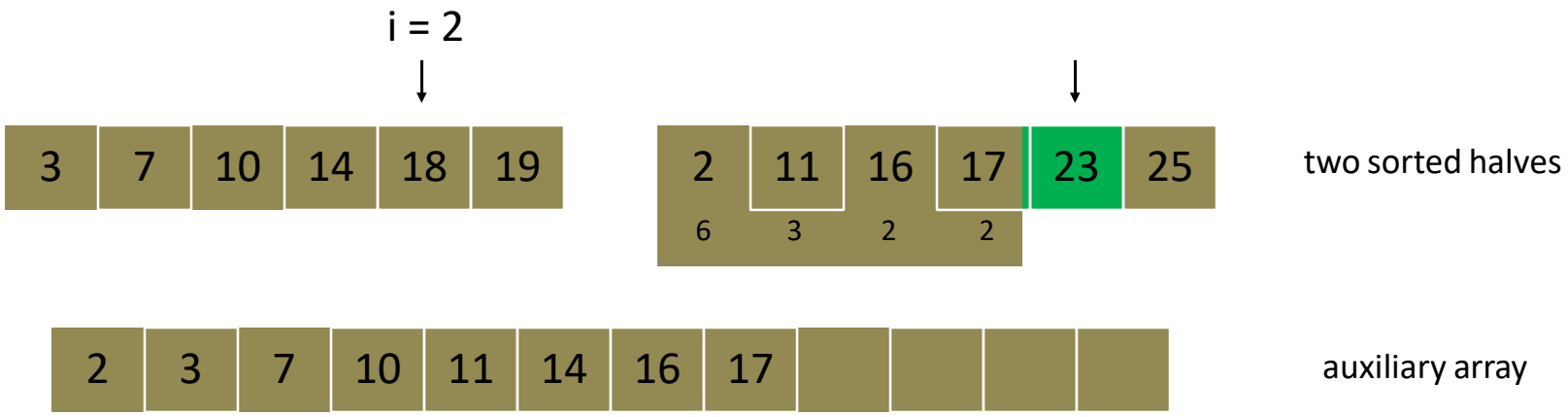

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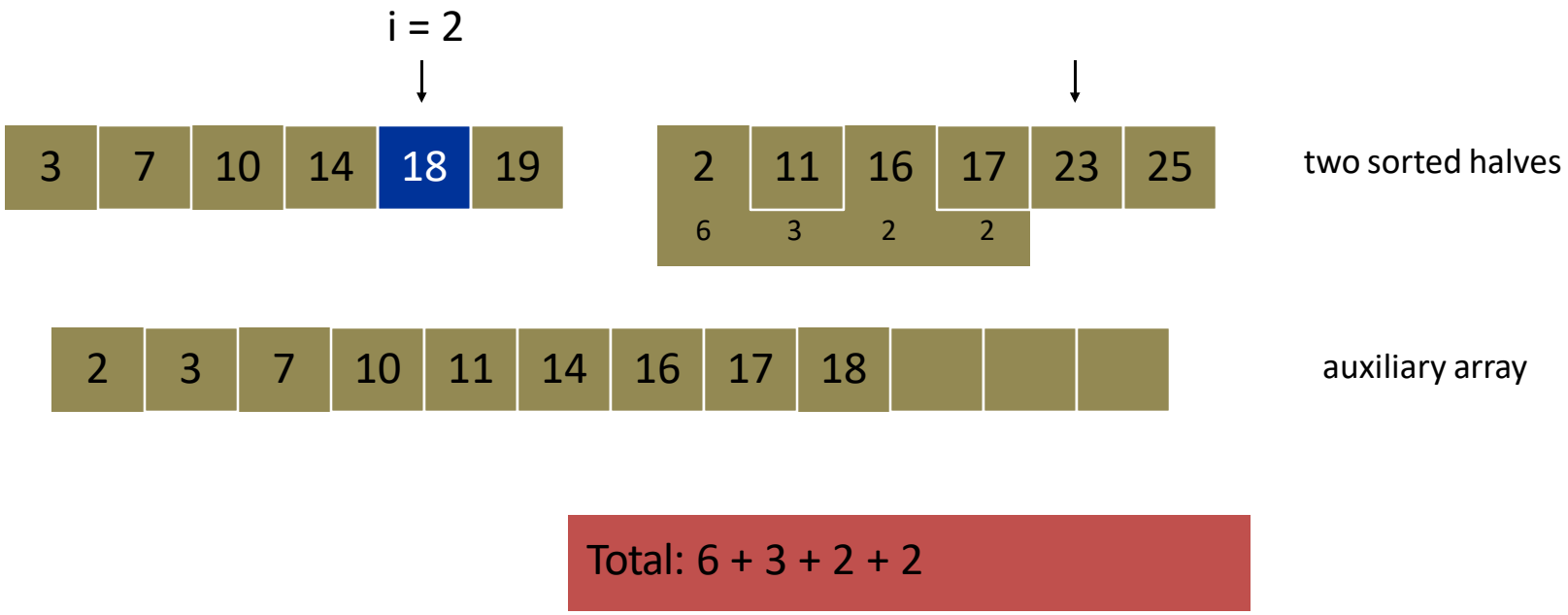

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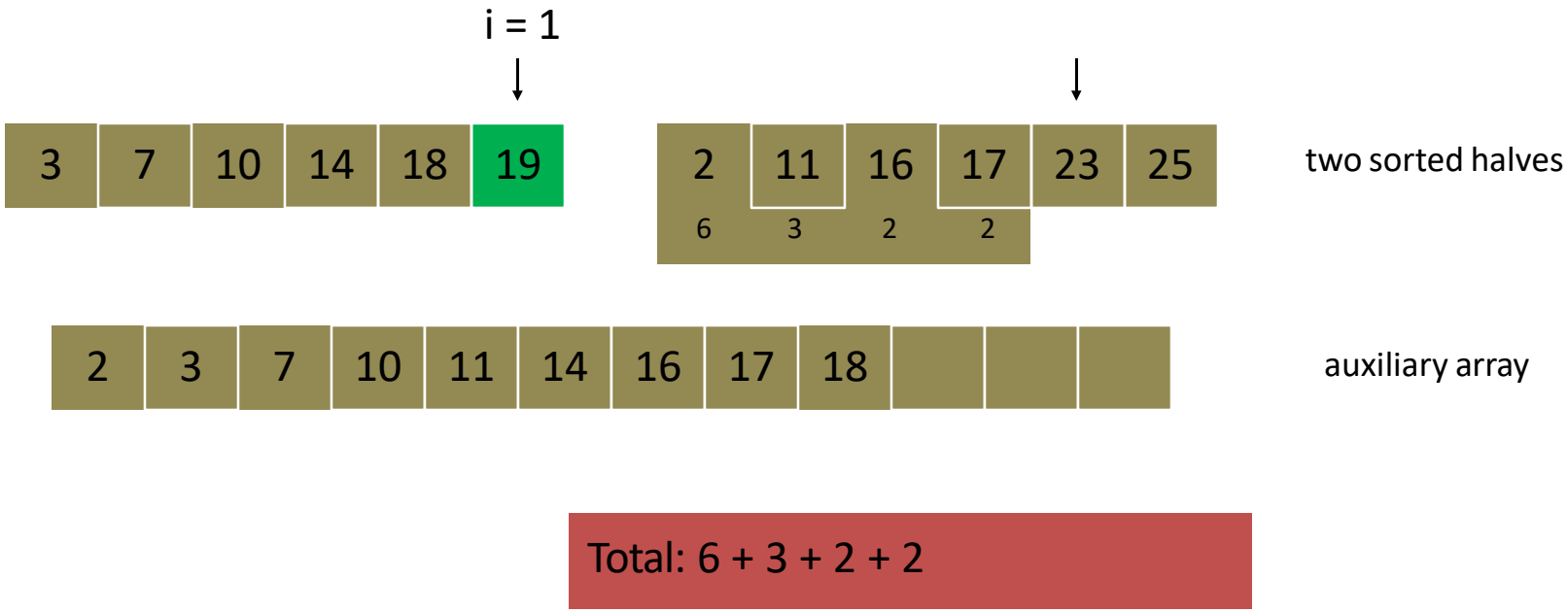

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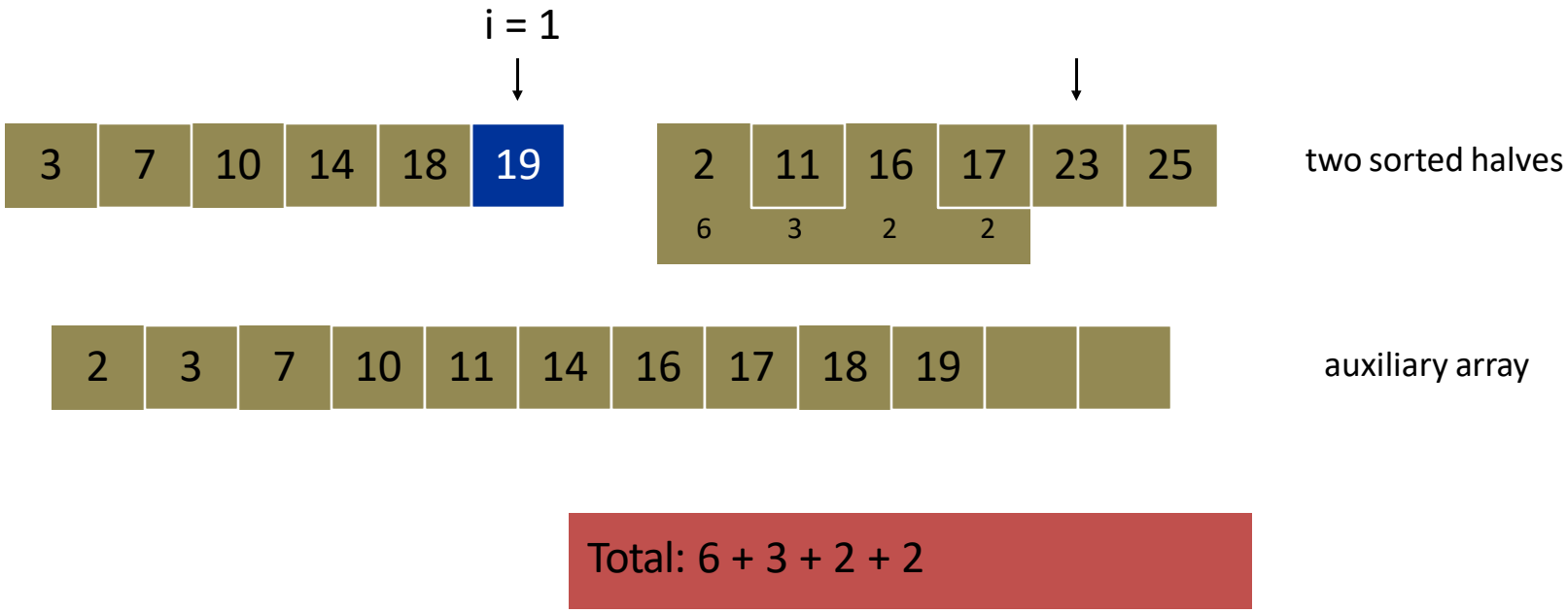

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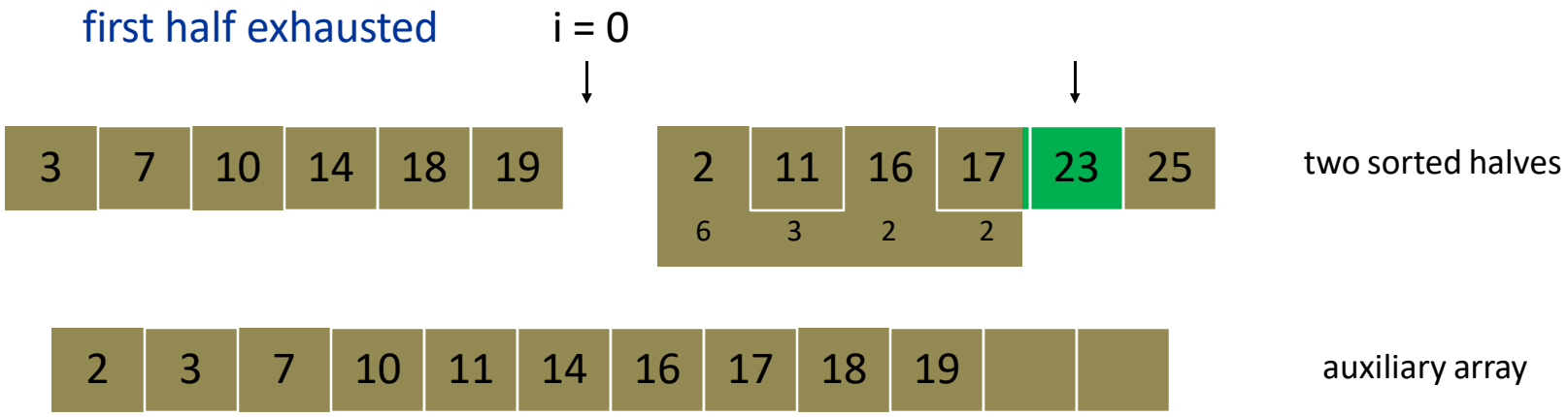

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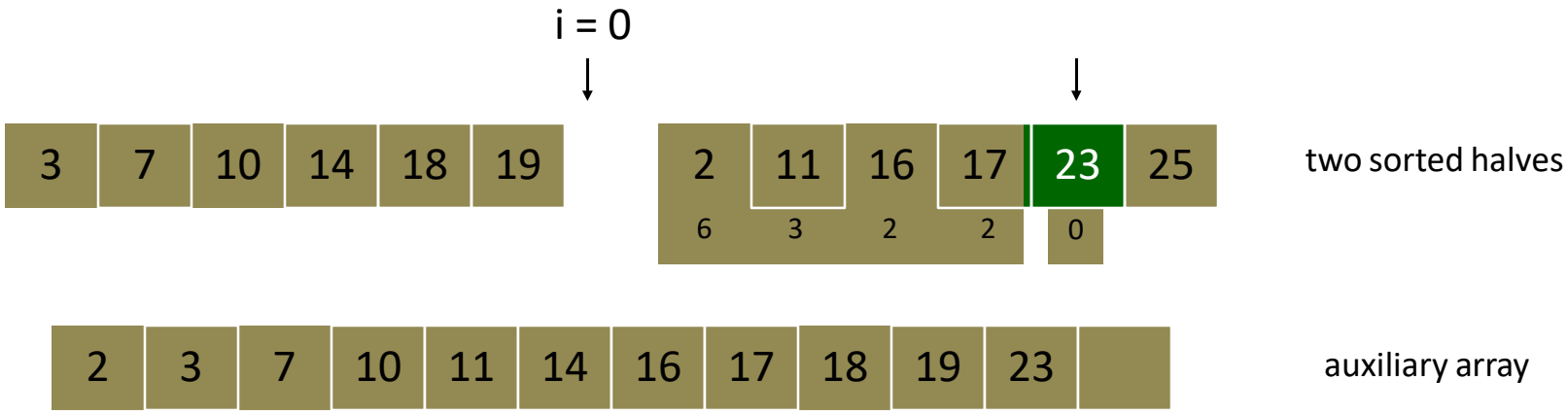

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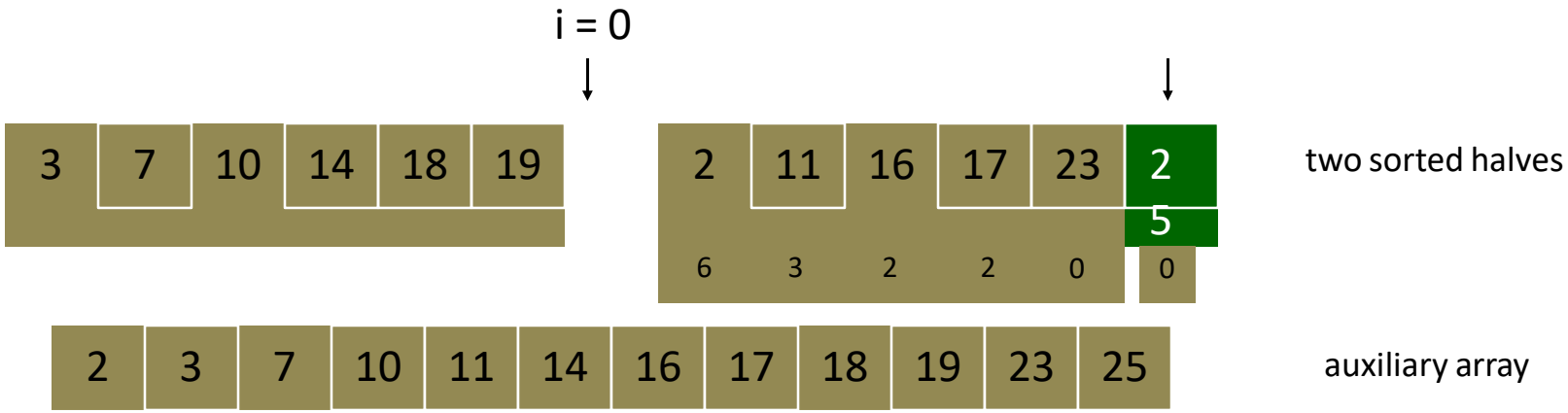

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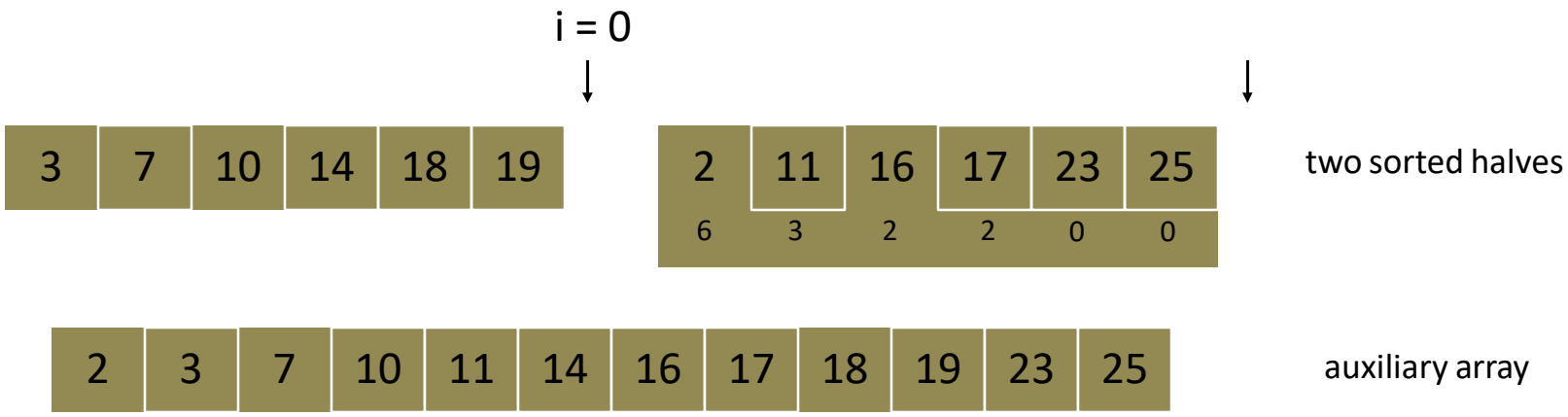

- **The e-commerce application**
  Have you ever noticed on any e-commerce website, they have this section of "You might like", they have maintained an array for all the user accounts and then whichever has the least number of inversion with your array of choices, they start recommending what they have bought or they like