PUSHDOWN AUTOMATA

Definition of PDA

It has 7 components

P = (Q, Σ, Ґ, δ, q_0, Z_0, F)

Q -> Final set of states

Σ ->input symbols

Ґ ->finite state alphabet

q_0 ->start state

Z_0 ->accepting as final state

δ ->A transition function. It takes as arguments a δ(q, a, X)

(i) q is the state in Q
(ii) a is an input symbol in Σ or a = ε
(iii) X is a stack symbol

The o/p of δ is a finite set of pairs (P, V) where P is the new state and V is the string of the stack symbols that replaces X at the top of the stack.

Q1) Construct a PDA which accepts a language 0^n1^n where n≥1

L = {0^n1^n | n≥1}

L = {01, 0011, 000111......}

w = 000111

δ(q_0, 0, Z0) = (q_0, 0Z0)

δ(q_0, 0, 0) = (q_0, 00)

δ(q_0, 0, 0) = (q_0, 00)

δ(q_0, 1, 0) = (q_1, Eε)

δ(q_1, 1, 0) = (q_1, ε)

$\delta(q_1, 1, 0) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_2, Z_0)$

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q1, q2\}$

$\Sigma = \{0, 1\}$

$T = \{0\}$

q_0->start state

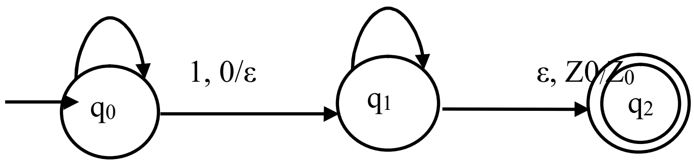
Z_0->stack start symbol

F-> $\{q_2\}$

Graphical notation of PDA

(1) Nodes correspond to the states of PDA

0, 0/00 0, Z0/0Z_0



2) Construct a PDA for $a^n b^m c^{n+m}$ n, m≥0

$L = \{\varepsilon, a^2, b^2, c^4, abc^2, a^3b^5c^8, b^2c^2a^2c^2\}$

w = aabbcccc

$\delta(q_0, a, Z0) = (q_0, aZ0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, ba)$

$\delta(q_1, b, b) = (q_1, bb)$

$\delta(q_0, c, b) = (q_2, \varepsilon)$

$\delta(q_2, c, a) = (q_4, \varepsilon)$

$\delta(q_4, \varepsilon, Z_0) = (q_3, Z_0)$

w = aacc

$\delta(q_0, a, Z0) = (q_0, aZ0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, c, a) = (q_4, \varepsilon)$

$\delta(q_0, \varepsilon, Z_0) = (q_3, Z_0)$

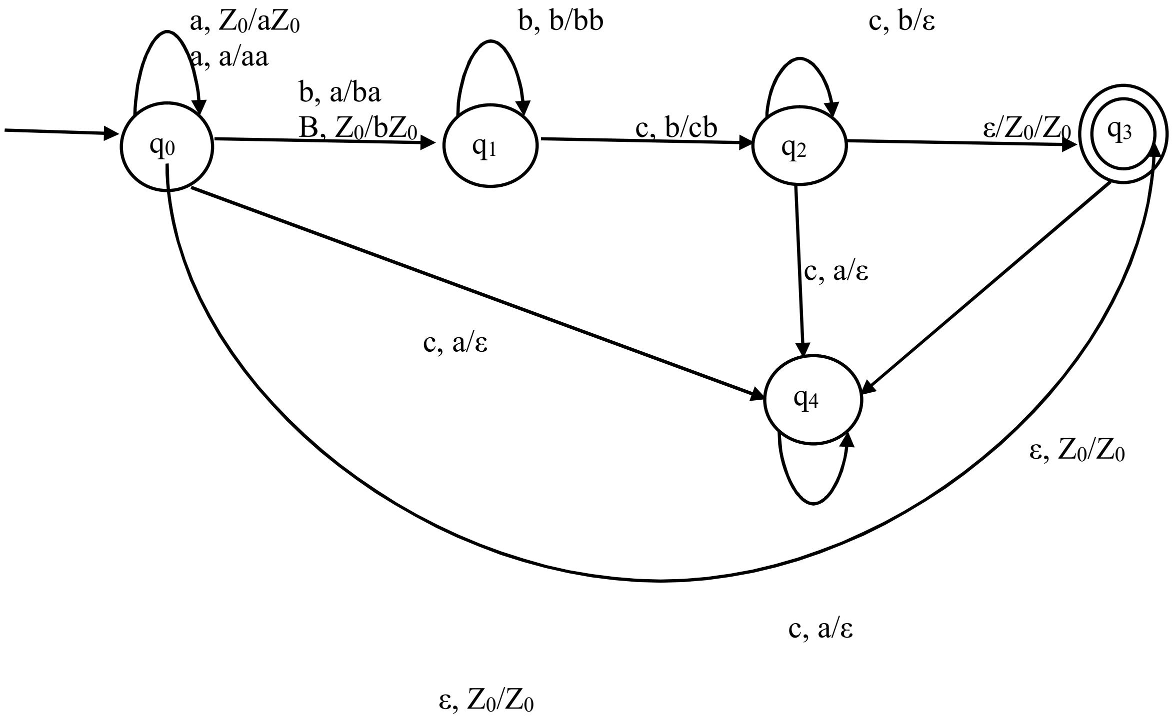
w = bbcc

$\delta(q_0, b, Z_0) = (q_1, bZ_0)$

$\delta(q_1, b, b) = (q_1, bb)$

$\delta(q_{01}c, b) = (q_2, \varepsilon)$

$\delta(q_2, \varepsilon, Z_0) = (q_3, Z_0)$



$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2, , q_3, q_4\}$

$\Sigma = \{a, b, c\}$

T = {a, b}

q_0->start state

Z_0->stack start symbol

F-> $\{q_3\}$

Instantaneous Description ID of PDA

We represent PDA by a triple (q, W, V) where

1) W is the remaining i/p

2) V is the stack content

We show the top of stack at the left end of V and the bottom at the right end, such a triple is called instantaneous description or ID of a PDA

Let P = $(Q, \Sigma, Ґ, \delta, q_0, Z_0, F)$ be a PDA define |- where P is understood as follows

Suppose $\delta(q, a, X)$ contains $(p, \alpha, w, X\beta)$ |- $(p, w, \alpha\beta)$

$(q_0, aabbcccc, Z_0)$ |- $(q_0, abbcccc, aZ_0)$|-$(q_0, bbcccc, Z_0)$ |- $(q_1, bcccc, ba)$|-$(q_1, cccc, bb)$|-(q_2, ccc, ba)|-(q_2, cc, aa)|-(q_4, c, aZ_0)|-(q_4, ε, Z_0)|-(q_3, ε, Z_0)

3) Construct a PDA for $a^n b^{2n}$ for $n \geq 1$

L = {abb, aabbbb …}

$\delta(q_0, a, Z_0) = (q_0, AAZ_0)$

W = aabbbb

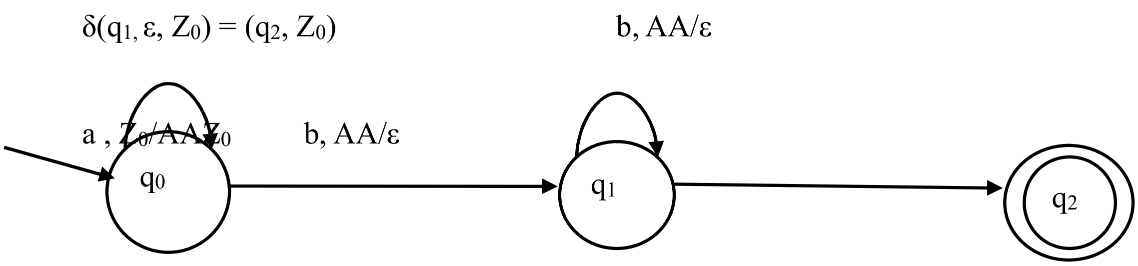
$\delta(q_0, a, Z_0) = (q_0, AAZ_0)$

$\delta(q_0, a, AA) = (q_0, AAAA)$

$\delta(q_0, b, AA) = (q_1, \varepsilon)$

$\delta(q_1, b, AA) = (q_0, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_2, Z_0)$



$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$T = \{A\}$

q_0->start state

Z_0->stack start symbol

F-> $\{q_2\}$

4) Define a DPDA to accept strings with more a’s than b’s

L = {x belongs to {a, b} | $n_a(x)>n_b(x)$}

L = {aab, aaaaabbb, aaaa, abaa, ababa …}

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

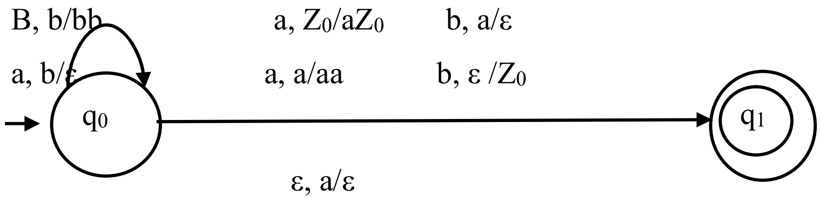
$\delta(q_0, b, a) = (q_0, \varepsilon)$

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, \varepsilon)$

$\delta(q_0, \varepsilon, a) = (q_1, \varepsilon)$



$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

$T = \{q_0, q_1\}$

q_0->start state

Z_0->stack start symbol

F-> $\{q_1\}$

ID for baaba

$(q_0, baaba, Z_0)$|-$(q_0, aaba, bZ_0)$|-(q_0, aba, Z_0)|-(q_0, ba, aZ_0)|-(q_0, a, Z_0)|-(q_0, ε, aZ_0)|-$(q_0, \varepsilon,$

Z_0)

5) Write PDPA for balanced parenthesis using {[( )]}

L = {[ ], [({ })], ((({ })))....}

$\delta(q_0, [, Z_0) = (q_0, [Z_0)$

$\delta(q_0, (, Z_0) = (q_0, (Z_0)$

$\delta(q_0, \{, Z_0) = (q_0, \{Z_0)$

$\delta(q_0, [, [) = (q_0, [[)$

$\delta(q_0, (, ( ) = (q_0, (( )$

$\delta(q_0, \{, \{) = (q_0, \{\{)$

$\delta(q_0,], [) = (q_0, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

$\delta(q_0,), ( ) = (q_0, \varepsilon)$

$\delta(q_0, \}, \{) = (q_0, \varepsilon)$

$\delta(q_0, \{, [ ) = (q_0, \{[ )$

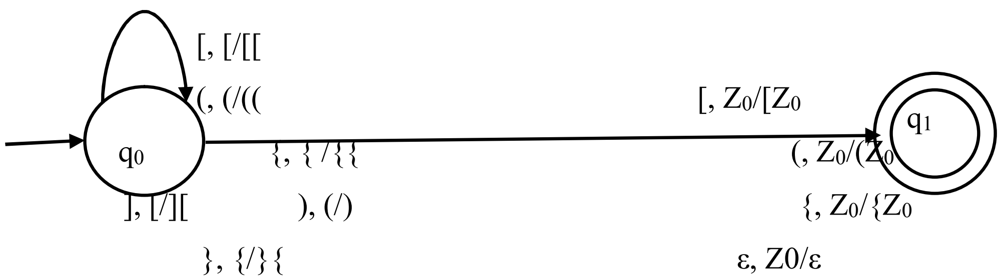
$\delta(q_0, \{, ( ) = (q_0, \{( )$

$\delta(q_0, (, [ ) = (q_0, ([ )$

$\delta(q_0, (, \{) = (q_0, (\{)$

$\delta(q_0, [, ( ) = (q_0, [, ( )$

$\delta(q_0, [, \{) = (q_0, [, \{)$



$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1\}$

$\Sigma = \{\{, [, (,),], \} \}$

$T = \{\{, [, (\}$

q_0->start state

Z_0->stack start symbol

F-> $\{q_1\}$

Languages of a PDA

Acceptance by final state

Let P = (Q, Σ, Ґ, δ, q_0, Z_0, F) be a PDA then L(P) the language accepted by P final state is w such that , $\{w|(q_0, w, Z_0)$ |----- (q, ε, ε)} for some state q in F and any stack string α i.e starting in the initial ID with w weighting on the i/p and enters an accepting state the contents of the stack at that time is irrelevant.

Acceptance by empty stack

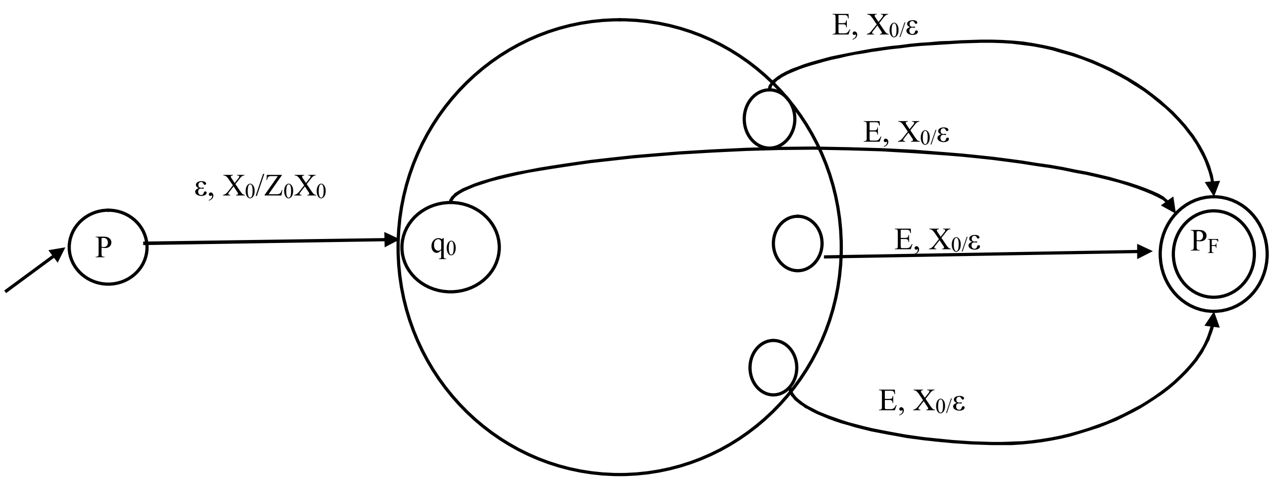
For each PDA P = (Q, Σ, Ґ, δ, q_0, Z_0, F)

N(P) = $\{w|(q_0, w, Z_0)$|-(q, ε, ε)} for any state q i.e N(P) is the set of i/p s w that P can consume and at the same time empty its stack

From empty stack to final state.

Theorem--

If L = N(P_N) for some PDA P_N = (Q, Σ, Ґ, δ_N, q_0, Z_0, F) then there is a PDA P_f such that L = L(P_F)



Proof-

We use a new symbol X_0 which is not a symbol of T, X_0 is the start symbol of P_F and also a marker on the button of the stack that tells us when P_N has reached an empty stack.

A new start p_0 is introduced whose function is to push Z_0 the start symbol of P_N

Then P_F stimulates P_N until the stack of P_N is empty which P_F detects since it sees X_0 on the top of the stack. We introduce another new state p_f which is accepting state of P_F wherever it discovers that P_N would have emptied its stack.

$P_F = (Q \cup \{p_0, p_f\}, \Sigma, T \cup \{X_0), \delta_N, X_0, \{p_f\})$

Where δ_F is defined by

1) $\delta_F (p_0, \varepsilon, X_0) = \{(q_0, Z_0, X_0)\}$

2) For all states q in Q inputs a in Σ are a = ε and stack symbol Y in T $\delta_F(q, a, Y)$ contains all the pairs in $\delta F(q, a, Y)$.

3) $\delta_F(q, \varepsilon, X_0)$ contains (p_f, ε) for every state q in Q

We must show that w is in $L(P_F)$ if and only if w is in $N(P_N)$ $(q_0, w, Z_0)|\text{-}(q, \varepsilon, \varepsilon)$ for some state q as we insert X_0 at the bottom of the stack and conclude $(q_0, w, Z_0, X_0)|\text{-}(q, \varepsilon, X_0)$

P_F has all the moves of P_N so we can conclude that $(q_0, w, Z_0, X_0)|\text{-}(q, \varepsilon, X_0)|\text{-}(p_f, \varepsilon, \varepsilon)$

-------.1

Thus P_F accepts w by final state [only if]

If the stack of P_F contains only X_0 we can use rule 3

Any computations of P_F that accepts w must look like equ 1 also the first and last step must also be a computation of P_N must give $(q, \varepsilon, \varepsilon)$ i.e. w is in $N(P_N)$

6. Design a PDA to accept $a^i b^j c^k$ such i-j such that i = j or j = k.

$L = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$

L = {ab, bc, abc}

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \varepsilon)$

$\delta(q_1, b, a) = (q_1, \varepsilon)$

$\delta(q_1, c, Z_0) = (q_f, cZ_0)$

$\delta(q_1, \varepsilon, Z_0) = (q_f, Z_0)$

$\delta(q_f, c, c) = (q_f, cc)$

$\delta(q_0, b, Z_0) = (q_2, bZ_0)$

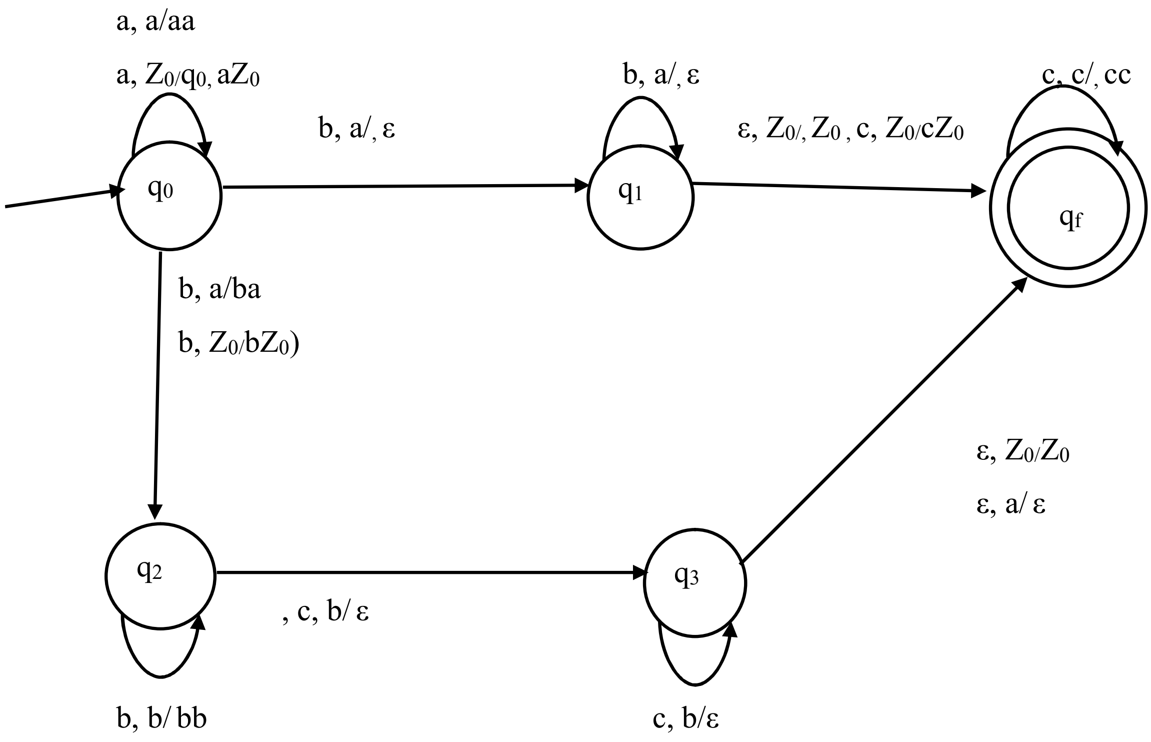
$\delta(q_2, b, b) = (q_0, bb)$

$\delta(q_0, b, a) = (q_2, ba)$

$\delta(q_2, c, b) = (q_3, \varepsilon)$

$\delta(q_3, \varepsilon, a) = (qf, \varepsilon)$

$\delta(q_0, \varepsilon, Z_0) = (q_f, \varepsilon)$



$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2, q_3, q_f\}$

$\Sigma = \{a, b, c\}$

$T = \{a, b, c\}$

q_0->start state

Z_0->stack start symbol

F-> $\{q_f\}$

7. Construct a PDA $L = \{wcw^R/w$ belongd to $\{0, 1\}^*\}$ by empty stack

$L = \{c, 0c0, 0101c0101, ...\}$

$\delta(q_0, c, Z_0) = (q_1, Z_0)$

$\delta(q_0, 0, Z_0) = (q_0, 0Z_0)$

$\delta(q_0, 0, 0,) = (q_0, 00)$

$\delta(q_0, 1, Z_0) = (q_0, 1Z_0)$

$\delta(q_0, 1, 1) = (q_0, 11)$

$\delta(q_0, 0, 1) = (q_0, 01)$

$\delta(q_0, 1, 0) = (q_0, 10)$

$\delta(q_0, c, 0) = (q_1, 0)$

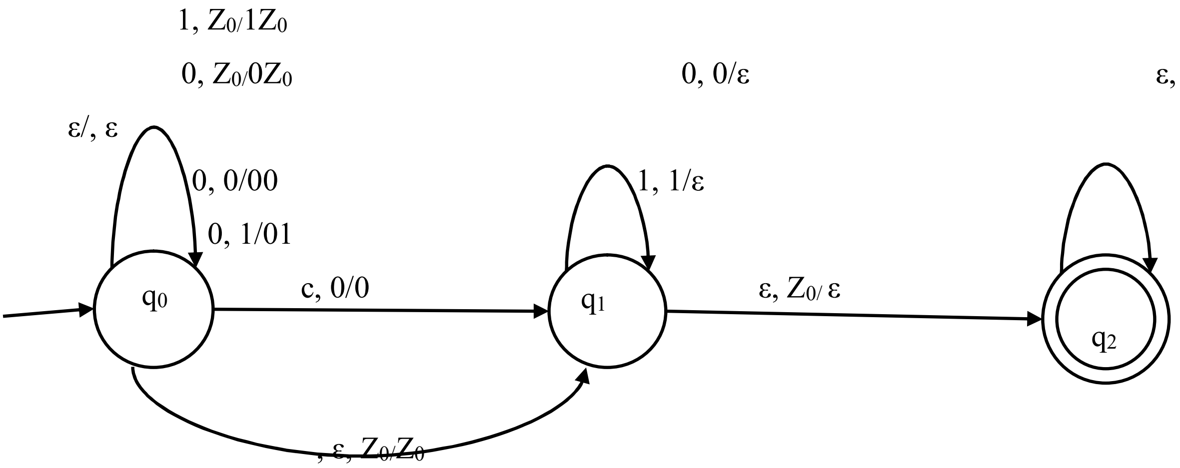
$\delta(q_0, c, 1) = (q_1, 1)$

$\delta(q_{1,} 0, 0) = (q_1, \varepsilon)$

$\delta(q_{1,} 1, 1) = (q_1, \varepsilon)$

$\delta(q_0, \varepsilon, Z_0) = (q_{2,} \varepsilon)$

$\delta(q_0, \varepsilon, \varepsilon) = (q_2, \varepsilon)$



$P = (Q, \Sigma, Γ, \delta, q_{0,} Z_0, F)$

$Q = \{q_0, q_1, q_{2\}}$

$\Sigma = \{0, c, 1\}$

$T = \{0, 1\}$

q_0->start state

Z_0->stack start symbol

F-> $\{q_2\}$

w = 010c010

$(q_0, 010c010, Z_0)|\!-(q_0, 10c010, 0Z_0)|\!-(q_0, 0c010, 10Z_0)|\!-(q_0, c010, 010Z_0)|\!-(q_1, 010, 010Z_0)|\!-(q_1, 10, 10Z_0)|\!-(q_1, 0, 0Z_0)|\!-(q_1, \varepsilon, Z_0)|\!-(q_0, \varepsilon, \varepsilon)$

Equivalence of PDA and context free languages grammars

Let G = (V T Q S) be a CFG construct PDA P that accepts L(G) y the empty stack as follows

P = ({q}, T, VUT, δ, q, S) where transition function δ is defined by

1) For each variable 'A' δ(q, ε, A) = {(q, β) where A-> β is a production of P}

2) For each terminal 'a' δ(q, a, a) = {(q, ε)}

1. Convert the expression grammer to PDA

I->a|b|Ia|I0|I1|Ib

E->I|E*E|E+E|(E)

The set of terminals for PDA is {a, b, 0, 1, (,), +, *} . These 8 symbols and the symbols I and E form the stack alphabet. The transition function for PDA is

δ(q, ε, I) = {(q, Ia), (q, I0), (q, Ib), (q, I1)}

δ(q, ε, E) = (q, I), (q, E*E), (q, E+E), (q, (E))

δ(q, a, a) = (q, ε)

δ(q, b, b) = (q, ε)

δ(q, 0, 0) = (q, ε)

δ(q, 1, 1) = (q, ε)

δ(q,),)) = (q, ε)

δ(q, (, ( ) = (q, ε)

δ(q, +, +) = (q, ε)

δ(q, *, *) = (q, ε)

2. Convert an equivalent PDA for the CFG

S-> 0BB

B->0S/1S/0

The set of terminals for PDA is {0, 1}

These 2 symbols and symbols S&B form stack alphabet. The transition function for PDA is

$\delta(q, \varepsilon, S) = (q, 0BB)$

$\delta(q, \varepsilon, B) = \{(q, 0S), (q, 1S), (q, 0)\}$

$\delta(q, 0, 0) = (q, \varepsilon)$

$\delta(q, 1, 1) = (q, \varepsilon)$

$P = (\{q\}, T, VUT, \delta, q, S)$

$T = \{0, 1\}$

q = q

S = start symbol

$V = \{S, B\}$

$VUT = \{S, B, 0, 1\}$

3.Construct equivalent PDA

S-> aABB/aAA

A->aBB/a

B->bBB/A

The set of terminals are a, b

The symbols a, b and S, A, B form stack alphabet

For each variable 'A' $(q, \varepsilon, A) = \{(q, \beta)$ where A-> β is a production of A

$\delta(q, \varepsilon, S) = \{(q, aABB), (q, aAA)\}$

$\delta(q, \varepsilon, A) = \{(q, aBB), (q, a)\}$

$\delta(q, \varepsilon, B) = \{(q, bBB), (q, a)\}$

For each terminal 'a' $\delta(q, a, a) = \{(q, \varepsilon)\}$

$\delta(q, a, a) = (q, \varepsilon)$

$\delta(q, b, b) = (q, \varepsilon)$

$P = (\{q\}, T, VUT, \delta, q, S)$

$T = \{a, b\}$

$q = q$

S = start symbol

$V = \{S, B\}$

$VUT = \{S, B, A, a, b\}$

4. Construct equivalent PDA for

S->aA

A->aABC|bB|a

B->b

C->c

The set of terminals are a, b, c

The symbols a, b and S, A, B, C form stack alphabet

For each variable 'A' $(q, \varepsilon, A) = \{(q, \beta)$ where $A \to \beta$ is a production of A

$\delta(q, \varepsilon, S) = \{(q, aA)\}$

$\delta(q, \varepsilon, A) = \{(q, aABC), (q, bB), (q, a)\}$

$\delta(q, \varepsilon, B) = \{(q, b)\}$

$\delta(q, \varepsilon, C) = \{(q, c)\}$

For each terminal 'a' $\delta(q, a, a) = \{(q, \varepsilon)\}$

$\delta(q, a, a) = (q, \varepsilon)$

$\delta(q, b, b) = (q, \varepsilon)$

$\delta(q, c, c) = (q, \varepsilon)$

$P = (\{q\}, T, VUT, \delta, q, S)$

$T = \{a, b\}$

$q = q$

S = start symbol

V = {S, B}

VUT = {S, B, A, a, b}

Normal forms for CGF's

Safe sequence

Eliminate

(i) E production

(ii) Unit productions

(iii) useless symbols

The grammar G obtained into CNF

Eliminating useless symbols

There are 2 things a symbol has to be able to do to be useful

(i) We say X is <u>generating</u> if

$X \Rightarrow^* w$ for some terminal string w

(ii) We say X is <u>reachable</u> if there is a derivation

$S \Rightarrow^* \alpha \times \beta$ for some $\alpha \& \beta$

Ex 1

$S \rightarrow asb/A/E$

$A \rightarrow aA$

G = (V T P S)

V = {S, A}

T = {a, b}

V′ = {s}

P′ = {$S \rightarrow asb$. $S \rightarrow E$}

$A \rightarrow aA$

$A \rightarrow aa\underline{A}$ it is not generating a terminating string hence eliminated

Ex 2

$S \to AB/a$

$A \to a$

$G = (V\ T\ P\ S)$ $\quad$ $T' = \{a\}$

$T = \{a\}$ $\quad$ $V' = \{S, A\}$

$V = \{A, B\}$ $\quad$ $P' = \{S \to a, A \to a\}$

$V'' = \{S\}$

$P'' = \{S \to a\}$

Ex 3

$S \to A$

$A \to aA/E$

$B \to bA$

1) $V' = \{S, A, B\}$

$P' = \{S \to A$

$A \to aA \notin \varsigma A \to E$

$B \to bA\}$

2) $V'' = \{S, A\}$

$P'' = \{S \to A$

$A \to aA/E\}$

Define $G = (VTPS)$

$V = \{S, A\}$

$T = \{a\}$

$P = \{S \to A$

$A \to aA/E\}$

a) $S \to AB/CA$

$B \to B\cancel{C}C/AB$

$A \to a$

$C \to aB/b$

V′ = {S, A, ~~B~~, C}

P′ = {S→~~AB~~×CA

A→a

C→b

}

V′′ = {S, A, C}

{S→CA}

A→a

C→b

}

G = (VTPS)

V = {SAC}

T = {a, b}

P = {S→CA

A→a

C→b

}

5) S→AB

A→a

B→b/Ɠ

E→Ɠ

V′ = {S, A.B}

P′ = {S→AB

A→a

B→b

}

V′ = V′′

P′ = P′′

G = (VTPS)

$V = \{S, A, B\}$

$T = \{a, b\}$

$P = \{S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b\}$

Eliminating $\in$ production

$A \rightarrow \in$

$\in$production is of this form

A is called nullable

6) $S \rightarrow aS/AB$

$B \rightarrow \in$

$A \rightarrow \in$

$D \rightarrow b$

$V_n = \{A, B, S\}$

$S \rightarrow as/a$

$S \rightarrow AB/A/B$

$D \rightarrow b$

$G = (VTPS)$

$V = \{S, A, B, D\}$

$T = \{a, b\}$

$P = \{S \rightarrow as/a$

$S \rightarrow AB/A/B$

$D \rightarrow b\}$

2) $S \rightarrow a/Xb/aYa$

$X \rightarrow Y/E$

$Y \rightarrow b/X$

$V_n = \{X, Y\}$

$S \rightarrow a/b/aa/Xb/aYa$

$X \rightarrow X \rightarrow Y$

$Y \rightarrow b/X$

3) $S \to XY$

$X \to Zb$

$Y \to bW$

$W \to Z$

$A \to aA/bA/\in$

$B \to Ba/Bb, \in$

$Z \to AB$

$V_n = \{A, B, Z, W\}$

$S \to XY/\cancel{Y/X}$

$X \to Zb/b$

$Y \to b/W/b$

$W \to Z$

II Eliminating unit productions

1) $I \to a/b/I_a/I_b/I_o/Ii$

$F \to I/€$

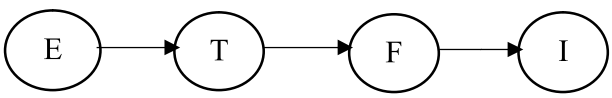
$T \to F/T*F$

$E \to T/E+T$

<u>BASICS:-</u>(A, A) is a unit pair for any variable A i.e; $\cancel{a}A \Rightarrow^* A$ by 0 steps

<u>INDUCTION:-</u>Suppose we hace determined that (A, B) is a unit pair and $B \Rightarrow C$ is a production where C is a variable then (A, C) is a unit pair.

(i) (E, E) and the production $E \to T$ gives us unit pair(E, T)
(ii) (E, T and the production $T \to F$ gives us unit pait(E, F)
(iii) (E, F) and the production $F \to I$ gives us unit pair (E, T)
(iv) (T, T) and the production $T \to F$ gives us unit pair (T, F)
(v) (T, F) and the production $F \to I$ gives us unit pair (t, i)
(vi) (F, F) and the production $F \to I$ gives us the unit pair (F, I)



To eliminate the unit production we proceed as follows given a CFG G = (VTPS) construct CFG G' = (V, T, P_1, S)

(i) Find all the unit of pairs of G

(ii) For each unit pair (A, B) add tp P_1 all the production s $A \rightarrow \alpha$ where $B \rightarrow \alpha$ is a non unit production in P. Note that A = B is possible, P_1 contains all the non unit productions in P

Pairs	Productions
(E, E)	$E \rightarrow E+T$
(E, T)	$E \rightarrow T*F$
(E, F)	$E \rightarrow (E)$
(E, I)	$E \rightarrow a/b/I_a/I_b/I_O/I_i$
(T, T)	$T \rightarrow T*F$
(T, F)	T→€
(T, I)	$T \rightarrow a/b/I_a/I_b/I_o/I_i$
(F, F)	$F \rightarrow (E)$
(F, I)	$F \rightarrow a/b/I_a/I_b/I_o/I_i$
(I, I)	$I \rightarrow a/b/I_a/I_b/I_o/I_i$

The resulting grammar

$E \rightarrow E+T/T*F/(E)/\ a/b/I_a/I_b/I_O/I_i$

$T \rightarrow T*F/(E)/\ a/b/I_a/I_b/I_O/I_i$

$F \to (E) / a/b/I_a/I_b/I_O/I_i$

$I \to a/b/I_a/I_b/I_O/I_i$

2) $S \to A/bb$

$A \to B/b$

$B \to S/a$

(i) (B, B) and the production $B \to S$ gives us a pair (B, S)

(ii) (B, S) and the production $S \to A$ gives us a pair (B, A)

(iii) (B, A) and the production $A \to B$ gives us a pair (B, B)

(iv) (S, A) and the production $A \to B$ gives us a pair (S, B)

(v) (S, B) and the production $B \to S$ gives us a pair (S, S)

(vi) (S, S) and the production $S \to A$ gives us a pair (S, A)

(vii) (A, A) and the production $A \to B$ gives us a pair (A, B)

(viii)(A, B) and the production $B \to S$ gives us a pair (A, S)

(ix) (A, S) and the production $S \to A$ gives us a pair (A, A)

Pair	Production
(B, B)	$B \to a$
(B, S)	$B \to bb$
(B, A)	$B \to b$
(S, S)	$S \to bb$
(S, A)	$S \to b$
(S, B)	$S \to a$
(A, A)	$A \to b$
(A, B)	$A \to a$
(A, S)	$A \to bb$

The resultant grammar is

B→a/bb/b

S→b/bb/a

A→b/bb/a

3) S→AB

B→C/b

D→E

A→a

C→D

E→a

(i) (B, B) and the production B→C gives (B, C)

(ii) (B, C) and the production C→Dgives (B, D)

(iii) (B, D) and the production D→Egives (B, E)

(iv) (C, C) and the production C→Dgives (C, D)

(v) (C, D) and the production D→Egives (C, E)

(vi) (D, D) and the production D→Egives (D, E)

Pairs	Production
(B, B)	B→b
(B, C)	
(B, D)	
(B, E)	B→a
(C, D)	
(C, E)	C→a
(C, C)	
(D, D)	
(D, E)	D→a
(E, E)	E→a
(S, S)	S→AB

The resulting grammar is

S→AB

A→A

B→b/a

C→a

D→a

E→a

Reduce to CNF

1.S->aAD

A->aB/bAB

B->b

D->d

P = { S->aAD

c_1->a

S->C_1AD

C_2->AD

S->C_1C_2

A->aB

A->C_1B

A->bAB

C_3->b

A->C_3AB

C_4->AB

A->C_3C_4

B->b

D->d}

2.S->aSa/bSB/a/b/aa/bb

P = { S->aSa

C_1->a

S->C_1SC_1

C_2->SC_1

S->C_1C_2

S->bSb

C_3->b

S->C_3SC_3

C_4->SC_3

S->C_3C_4

S->a

S->b

S->C_1C_1

$S \to C_3C_3\}$

$P1 = \{S \to C1C2/C3C4/a/b/C1C1/C3C3$

$C_1 \to a$

$C_3 \to b$

$C_2 \to SC_1$

$C_4 \to SC_3\}$

3. $S \to ABa$

$A \to aab$

$B \to Ac$

$S \to ABa$

$C_1 \to a$

$S \to ABC_1$

$C_2 \to BC_1$

$S \to AC_2$

$A \to aab$

$A \to C_1C_1b$

$C_3 \to b$

$A \to C_1C_1C_3$

$C_4 \to C_1C_3$

$A \to C_1C_4$

$B \to Ac$

$C_5 \to c$

$B \to AC_5$

$P^{1=} = \{ S \to AC_2$

$A \to C_1C_4$

$B \to AC_5$

$C_1 \to a$

$C_3 \to b$

$C_2 \to BC_1$

$C_4 \to C_1C_3 \quad C_5 \to c\}$

1. Eliminate ε, unit and uselss nad convert to CNF form

S->a/aA/B/C

A->aB/ε

B->aA

C->cCD

D->ddd

$V_n = \{A\}$

S->a/aA/B/C

A->aB

B->aA/a

C->cCD

D->ddd

$G_1 = (V\ T\ P\ S)$

$V_1 = \{S, A, B, C, D\}$

$T_1 = \{a, c, d\}$

$P_1 = \{$S->a/aA/B/C

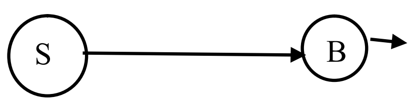
A->aB

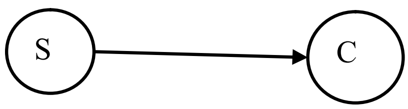
B->aA/a

C->cCD

D->ddd

}





(i) (S, S) and the production S->B gives the pair (S, B)

(ii) (S, S) and the production S->C gives the pair (S, C)

Pairs	Productions
1. (S, S)	S->a/aA
2. (S, B)	S->aA/a
3. (B, B)	B->aA/a
4. (S, C)	S->cCD
5. (A, A)	A->aB
6. (D, D)	D->ddd
7. (C, C)	C->cCD

G_2 = (VTPS)

V_2 = {S, A, B, C, D}

T_2 = {a, c, d}

P_2 = {S->a/aA/cCD

B->aA/a

A->aB

D->ddd

C->cCD

}

V_2^1 = {S, A, B, D}

$P^1{}_2$ = {S->a,

S->aA,

A->aB,

B->aA/a}

G_3 = (VTPS)

V_3 = {S A B}

T_3 = {a}

P_3 = {S->a/aA

A->aB

B->aA/a}

S->aA	A->aB	B->aA
X->a	A->XB	B->XA
S->XA		

G_4 = (VTPS)

V_4 = {S, X, A, B}

T_4 = {a}

P_4 = {

S->XA/a

X->a

A->XB

B->XA/a}

2. S->ABC/BaB

A->aA/BaC/aaa

B->bBb/a/D

C->CA/AC

D->ε

Nullable variable

V_n = {D, B}

P_1 = { S->ABC/AC/Bab/a/aB/Ba

A->aA/BaC/aC/aaa

B->bBb/bb/a/D

C->CA/AC

G_1 = (VTPS)

V_1 = {S, A, B, C}

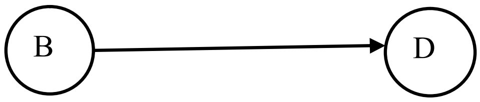
T_1 = {a, b}

S->start symbol

Elimination of unit production

Pairs	Productions
1.(S, S)	
2.(A, A)	
3.(B, B)	P_2 = {P_1->B->D}
4.(C, C)	

$G_1 = G_2$



Elimination of usless symbols

V^1_3 = {S, A, B}

P^1_3 = {A->aA A->aaa

B->bBb B->bb B->a

S->BaB S->a}

V_3^{11} = {S, B}

P_3^{11} = {S->BaB/a/aB/Ba

B->bBb/bb/a}

G_3(VTPS)

Reduce to CNF form

S->Bab	B->bBb	B-bb
X->a	Z->b	B->ZZ
S->BXB	B->ZBZ	S->aB
Y->XB	H->BZ	S->XB
S->BY	B->ZH	S->BX

G_4(VTPS)

V_4 = {S, B, X, Y, Z, H}

T_4 = {b, a}

P_4 = { S->BY/a/BX/XB B->ZH/ZZ/a

X->a Y->XB Z->b H->BZ}

3. S->0A0|1B1|BB

A->C

B->S|A

C->S|ε

Elimination of ε

V_n = {A, C}

P_1 = {S->0A0|00|1B1|BB

B->S/A

C->S

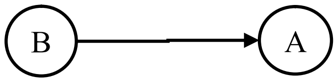
A->C}

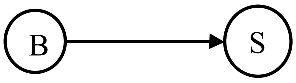
G_1->(VTPS)

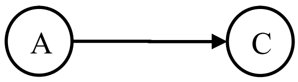
V_1->{S, B, C, A}

T_1->{0, 1)

Elimination of unit productions







1.(B, B) and the production B->S gives a pairs (B, S)

2.(C, C) and the production C->S gives a pair (C, S)

	Pairs	Productions
1.	(B, B)	
2.	(B, S)	B->0A0/00/1B1/BB
3.	(S, S)	S->0A0/00/1B1/BB
4.	(C, C)	
5.	(C, S)	C->0A0/00/1B1/BB

G_2 = (VTPS)

V_2 = {S, B, A, S}

T_2 = {0, 1}

P_2 = { S->0A0/00/1B1/BB

B->0A0/00/1B1/BB

C->0A0/00/1B1/BB

Eliminating usless symbols

V_3^1 = {S, B, C}

P_3^1 = { S->00 S->1B1 S->BB

B->00 B->1B1 B->BB

C->00 C->1B1 C->BB}

$V_3^{11 =}$ {S, B}

P_3^{11} = {S->00|1B1|BB

B->X|00|1B1|BB}

G_3 = (VTPS)

V_3 = {S, B}

T_3 = {0, 1}

Reduce to CNF form

S->1B1	B->1B1
X->1	B->XBX
S->XBX	B->XY
S->XY	
S->00	B->ZZ
Z->0	
S->ZZ	

G_4 = (VTPS)

V_4 = {S, Z, X, Y, B}

T_4 = {0, 1}

P_4 = {S-> XY/ZZ/BB

B->XY/ZZ/BB

}

4.S->AAA/B

A->aA/B

B->ε

Eliminating ε productions

V_n = {B}

P_1 = {S->AAA

A->aA

}

G_1 = (VTPS)

V_1 = {S, A}

T = {a}

Eliminating unit productions

$G_2 = G_1$

Eliminating usless symbols

$G_3 = G_2$

Reduce to CNF

S->AAA	A->aA
X->AA	Y->a
S->AX	A->YA

G_4 = (VTPS)

V_4 = {A, X, Y, S}

T_4 = {a}

P_4 = { S->AX

A->YA

X->AX

Y->a

}

5.BS->aAa/bBb/ε

A->C/a

B->C/b

C->CDE/ε

D->A/B/ab

Eliminating ε

V_n = {S, C}

P_1 = {S->aAa/bBb

A->a

B->b

C->CDE/DE

D->A/B/ab

}

G_1 = (VTPS)

V_1 = {S, A, B, C, D}

T_1 = {a, b}

Eliminating unit productions

$D \longrightarrow A$ $\qquad$ $D \longrightarrow B$

1.(D, D) and the production D->A forms a pairs D, A

2.(D, D) and the production D->B gives a pair D, B

Pairs	Productions
(D, D)	D->ab
(D, A)	D->a
(A, A)	A->a
(D, B)	A->a
(B, B)	B->b
(S, S)	S->aAa/bBb
(C, C)	C->CDE/DE

G_2 = (VTPS)

V_2 = {S, A, B, C, D}

$T_2 = \{a, b\}$

$P_2 = \{$ S->aAa/bBb

A->a

B->b

C->CDE/DE

D->ab/a/b }

Eliminiating useless symbols

$V^1{}_3 = \{S, A, B, D\}$

$P_3{}^1 = \{$ S->aAa/bBb

A->a

B->b

D->ab/a/b }

$V^{11}{}_3 = \{S, A, B\}$

$P_3{}^{11} = \{$ S->aAa/bBb

A->a

B->b }

$G_3 = (VTPS)$

$V_3 = \{S, A, B\}$

$T_3 = \{a, b\}$

Reduce to CNF form

S->aAa	S->bBb
X->a	Z->b
S->XAX	S->ZBZ
Y->AX	U->BZ
S->XY	S->ZU

$G_{4\,=}$ (VTPS)

$V_{4\,=}$ {X, Y, Z, U, S, A, B}

$T_4 = \{a, b\}$

$P_4 = \{$S->XY/ZU

A->a B->B

X->a Y->AX

Z->b   U->BZ}

HW

1. S->aA/a/B/C

A->aB/ε

B->aA

C->cC

D->abd

2. S->BAAB

A->0A2/2A0/ε

B->AB/1B/ε

3. (i) S->ABA

A->aA/ε

B->bB/ε

(ii) S->aSa/bSb/ε

A->aBb/bBa

B->aB/bB/ε

(iii) S->A/B/C

A->aAa/B

B->bB/bb

C->aCaa/D

D->baD/abD/aa

4. S->AaA/CA/BaB

A->aaBA/CDa/aa/DC

B->bB/bAB/bb/aS

C->Ca/bC/D

D->bD/ε

5. S->aSaSbS/aSbSaS/bSaSaS/ε

6. S->AaB/aaB

A->ε

B->bbA/ε

1. S->aA/a/B/C

A->aB/ε

B->aA

C->cC

D->abd

(i) Eliminating ε productions

V_n = {A}

P_1 = {S->aA/a/B/C

B->aA/a

C->cC

D->a

A->aB}

(ii) Eliminating unit productions

(S, S) and production (S->B) gives (S, B)

(S, S) and productions (S->C) gives (S, C)

Unit pair	productions
(S, S)	S->aA/a
(S, B)	S->aA
(A, A)	A->aB
(B, B)	B->Aa/a
(C, C)	C->cC
(D, D)	D->abcd
(S, C)	S->cC

P_2 = {S-> a/aA/cC

A->aB

B->aA/a

C->cC

D->abd}

(iii) eliminating usless symbols

$V_3^1 = \{S, A, B, D\}$

$P_3^1 = \{$S-> a/aA

B->aA/a

A->aB

D->abd

}

$V_3^{11} = \{S, A, B\}$

$P_3^{11} = \{$ S->a/aA

A->aB

B->aA/a}

(iv) CNF form

S->aA

X->a

B->XB

S->XA

A->XB

$P_4 = \{$S->a/XB

A->XB

B->XB/a}

2. S->BAAB

A->0A2/2A0/ε

B->AB/1B/ε

(i) Eliminating ε productions

$V_n = \{B, A\}$

$P_1 = \{$S->BAAB/AAB/BAA/BAB/BA/AB/AA/BB/B/A

A->0A2/2A0/02/20

B->AB/1B/A/1/B

(ii) Eliminating unit productions

(S, S) and S->B gives unit pair (S, B)

(S, S) and S->A gives unit pair (S, A)

(B, B) and B->A gives unit pair (B, A)

(B, B) and B->B gives unit pair (B, B)

Pair	Production
(S, S)	S->BAAB/AAB/BAA/BAB/BA/AB/AA/BB/B/A
(S, B)	S->AB/1B/A/1/B
(S, A)	S->0A2/2A0/02/20
(B, B)	B->AB/1B/A/1/B
(B, A)	B->0A2/2A0/02/20
(A, A)	A->0A2/2A0/02/20

P_2 = {S->BAAB/AAB/BAA/BAB/BA/AB/AA/BB/B/A/AB/1B/A/1/B/0A2/2A0/02/20

A->0A2/2A0/02/20

B->AB/1B/A/1/B

}

7. 3.S->ABA

A->aA/ε

B->bB/ε

(i) Eliminating ε transitions

V_n = {A, B}

P_1 = { S = ABA/AB/BA/B/AA/A

A->aA/a

B->bB/B}

(ii) Eliminating unit productions

(S, S) and S->B gives (S, B)

(S, S) and S->A gives (S, A)

Pairs	Productions
(S, S)	S->ABA/AB/BA/AA
(S, B)	S->bB/b
(S, A)	S->aA/a
(A, A)	A->aA/a
(B, B)	B->bB/b

P_2 = {S->ABA/AB/BA/AA/bB/b/aA/a

A->aA/a

B->bB/b}

(iii) Eliminiting usless forms

V_3^1 = {S, A, B}

P_3^1 = {S->ABA/AB/BA/AA/bB/b/aA/a

A->aA/a

B->bB/b}

$V_3^{11} = V_3^1$

$P_3^{11} = P_3^{11}$

(iv) CNF form

S->ABA	S->bB	S->aA
S->AC_1	Y->b	X->a
S->AC_1	S->YB	S->XA

A->aA	B->bB
A->XA	B->YB

P_4 = { S->a/b/AA/AB/BA/AC_1/YB/XA

A->XA/a

B->YB/b

}

TURING MACHINES

We describe TM by 7 tuples

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Q->finite set of states of finite control

Σ->finite set of input symbols

Γ->complete set of 8 symbols

(Σ subset of Γ)

δ-> transition function the arguments of $\delta(q, X)$ are a state q and take symbol X.The value of $\delta(q, X)$ is a triple (p, Y, D) where

(i) p is the next srate in Q

(ii)Y is the symbol in Γ written in the cell being scanned replacing whatever symbol was there

(iii) D is a direction Lor R

q_{0}-> start state , number of Q

B-> The blank symbol , this symbol is in Γ but not in Σ

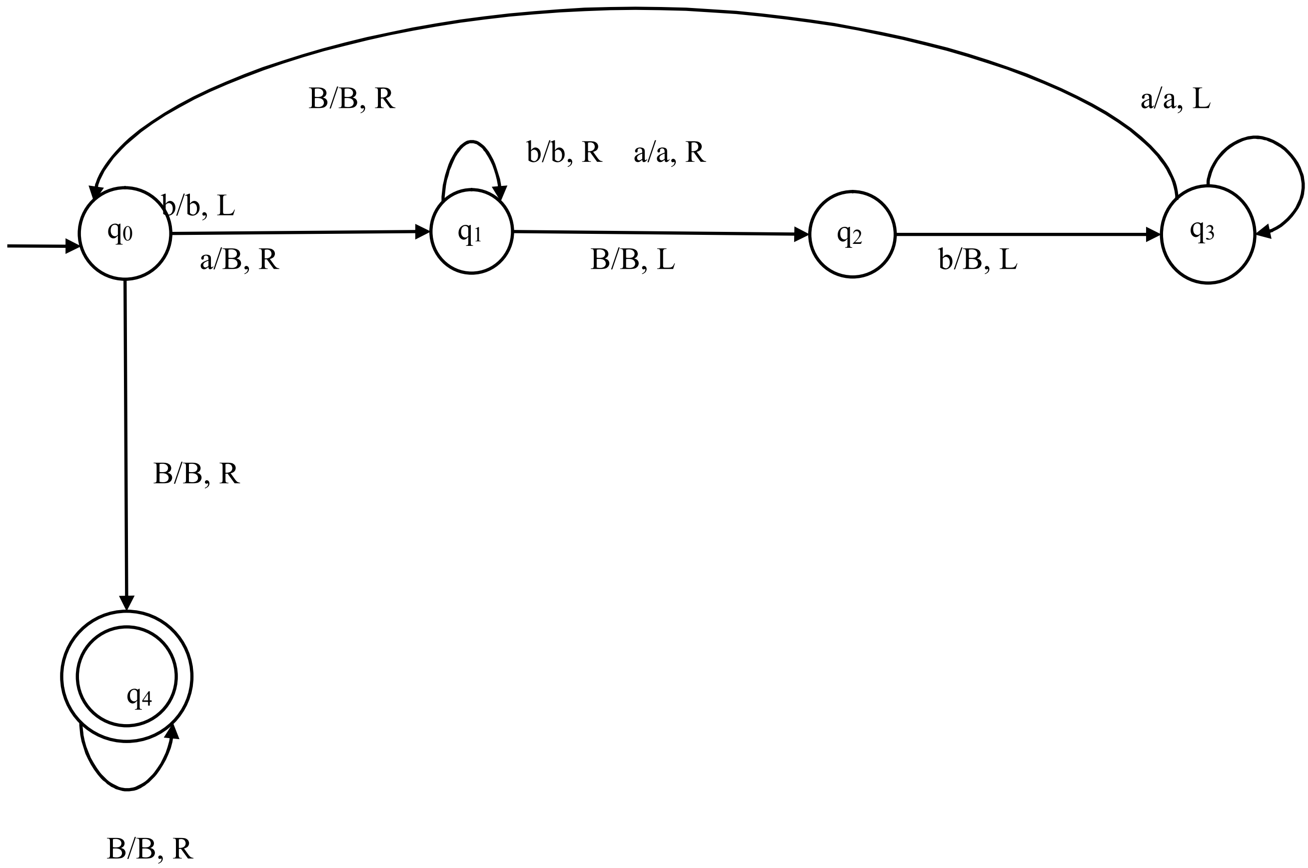
F-> set of accepting states a is subset of Q

1. Construct a turning machine for the language

$L = \{a^n b^n \mid n \geq 1\}$

w = aaabbb

B	a	a	a	b	b	b	B
B	B	a	a	b	b	b	B
B	B	a	a	b	b	B	B
B	B	B	a	b	b	B	B
B	B	B	a	b	B	B	B
B	B	B	B	b	B	B	B
B	B	B	B	B	B	B	B



B a a a b b b B

X a a b b b B

a a b b Y

X a b b

a b Y

X b

X Y

2. L = $\{a^n b^n c^n \,/\, n \geq 1\}$

w = aaabbbccc

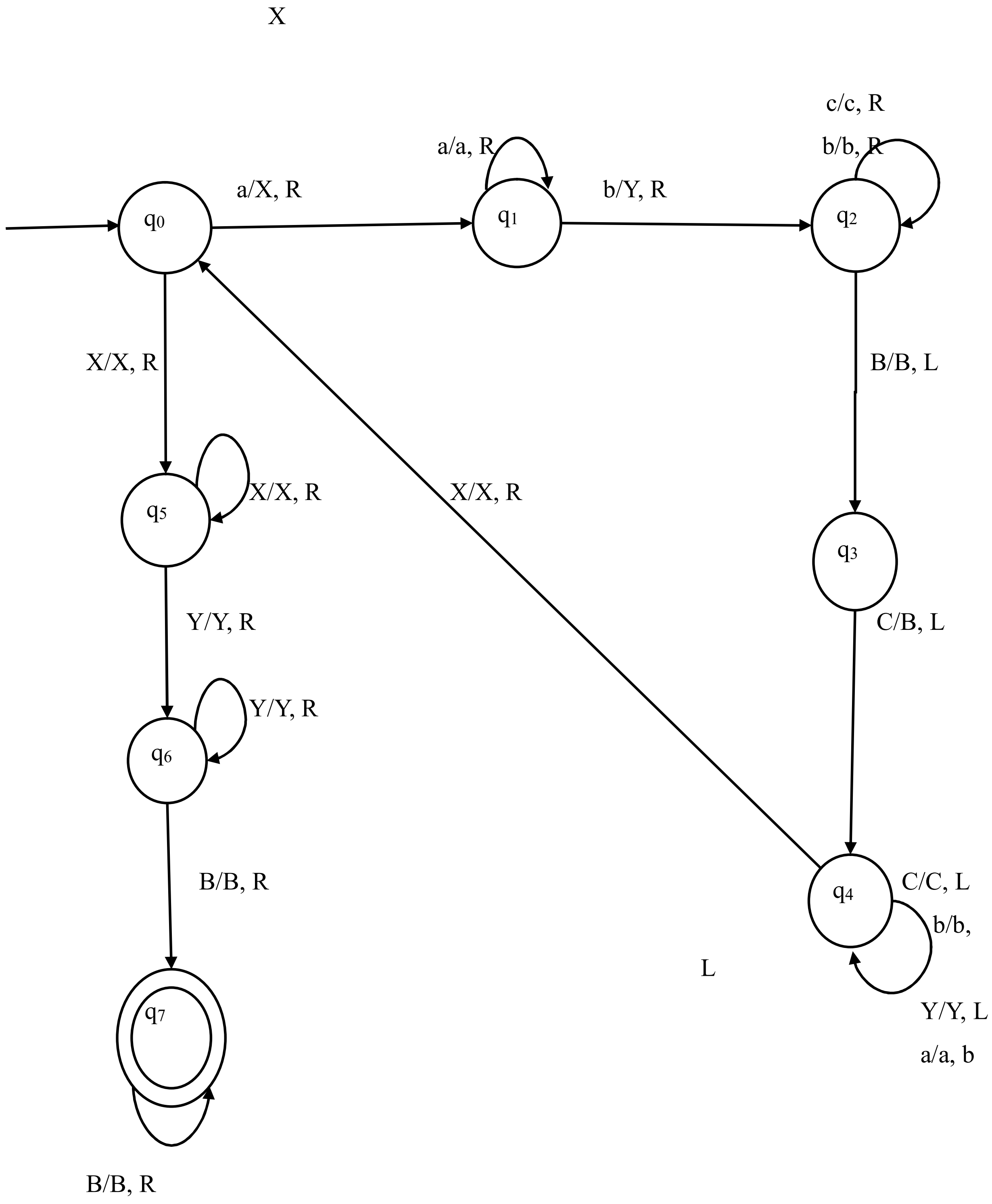
B a a a b b b c c c B

X a a Y b b c c c

a a Y b b c c B

X a Y Y b c c

a Y Y b c B



X
c/c, R
b/b, R
a/a, R
a/X, R
b/Y, R
q0
q1
q2
X/X, R
B/B, L
q5
X/X, R
X/X, R
q3
Y/Y, R
C/B, L
q6
Y/Y, R
B/B, R
q4
C/C, L
b/b,
L
q7
Y/Y, L
a/a, b
B/B, R

3.L = {w w^R}

B a b a a a b a B

B b a a b a

b a a b B

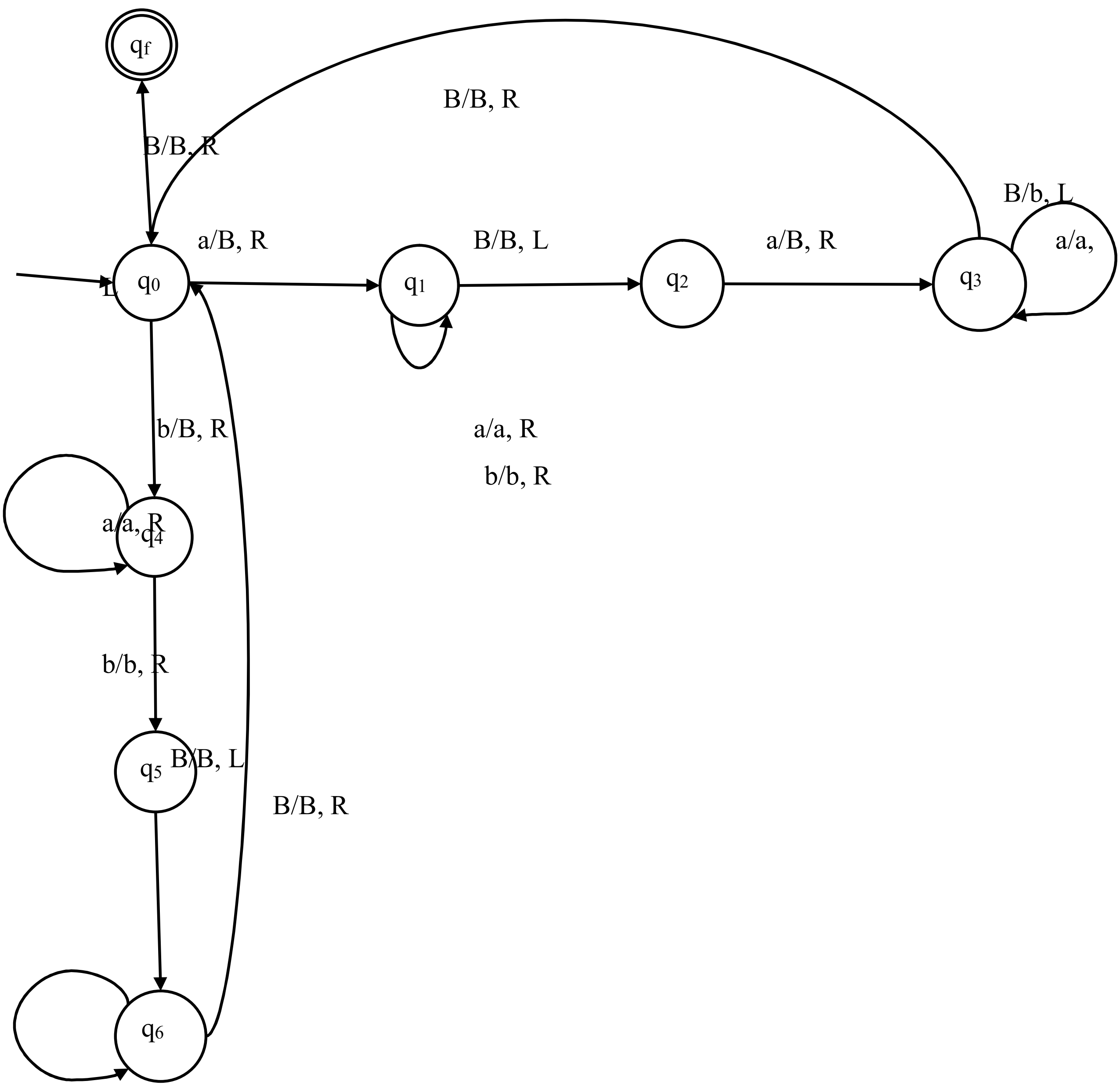
B a a b

a a B

B a

a

B



b/B, L

b/b, L

a/a, L

4. L = {a^n, b^m| n>m}

w = aaaabbb

B a a a a b b b B