Unit Productions

Example 1:

S → 0A | 1B | C
A → 0S | 00
B → 1 | A
C → 01

S → C is a unit production. But while removing S → C we have to consider what C gives. So, we can add a rule to S.
S → 0A | 1B | 01
Similarly, B → A is also a unit production so we can modify it as
B → 1 | 0S | 00
Thus finally we can write CFG without unit production as
S → 0A | 1B | 01
A → 0S | 00
B → 1 | 0S | 00
C → 01

Example 2:

S -> AB
A -> a
B -> C / b
C -> D
D -> E
E -> a
Solution:
There are 3 unit production in the grammar
B -> C
C -> D
D -> E
For production D -> E there is E -> a so we add D -> a to the grammar and add D -> E from the grammar. Now we have C -> D so we add a production C -> a tp the grammar and delete C -> D from the grammar. Similarly we have B -> C by adding B -> a and removing B -> C we get the final grammar free of unit production as:
S -> AB
A -> a
B -> a / b
C -> a
D -> a
E -> a
We can see that C, D and E are unreachable symbols so to get a completely reduced grammar we remove them from the CFG. The final CFG is :
S -> AB
A -> a
B -> a / b

Example 3:

S -> S + T/ T
T -> T * F/ F
F -> (S)/a

S -> T and T -> F are the two unit productions in the CFG.
For productions T -> F we have F -> (S)/a so we add T -> (S)/a to the grammar and remove T-> F from the grammar. Now for production S -> T we have production T -> T * F/(S)/a so we add S -> T * F/(S)/a to the grammar. So the grammar after removal of unit production is:
S->S + T/ T * F/ (S)/ a
T -> T * F/ F
F -> (S)/ a

Example 4:

Remove unit productions from a grammar (G1) whose production rule is given by
P: S→XY, X→a, Y →Z | b, Z→M, M→N, N→a // Grammar (G1)

In above grammar (G1) Unit Productions are
Y → | Z
Z→ M
M → N

The production unit which is removed easily is considered first. Let see,
For the Removal of Third Unit Production (M → N)
As N→a So, Unit Production M → N is updated to M→a.
For the Removal of Second Unit Production (Z → M)
As we derived M→a in above case, So, Unit Production Z → M is updated to Z→a
For the Removal of First Unit Production (Y → Z)
As we derived Z→a, So, Unit Production Y→Z is updated to Y→a
After Removal Unit Productions the Updated Grammar (G2) is given below
P: S→XY, X→a, Y →a| b, Z→a, M→a, N→a // Grammar (G2)
We can remove the unreachable states from above grammar (G2). So Finally, Grammar (G2) is given below
P: S→XY, X→a, Y →a| b. // Grammar (G2)

Example 5:

Remove unit productions from a grammar (G1) whose production rule is given by
P: S → aA | B, A → ba | bb, B → A | bba // Grammar (G1)
In above grammar Unit Production is
S → B
B→ A
The production unit which is removed easily is considered first. Let see,
For the Removal of 2nd Unit Production (B→ A)
As A→ba | bb. So, Unit Production B → A | bba is updated to B→ba | bb.

For the Removal of first Unit Production (S → B)
As B→A | ba | bb and A →ba|bb Therefore B→ ba | bb | bba. So, Unit Production S → B is updated to S→ ba | bb | bba.

After Removal Unit Productions the Updated Grammar (G2) is given below
P: S → aA | ba | bb | bba, A → ba | bb, B → A | bba // Grammar (G1)

We can remove the unreachable states from above grammar (G2). So Finally, Grammar (G2) is given below
P: S → aA | ba | bb | bba, A → ba | bb // Grammar (G2)

Example 6:

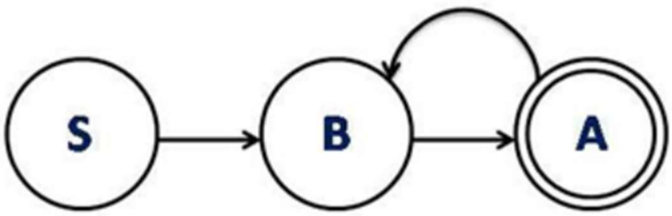
consider a grammar as an example.

G_1: S→ Aa|B, B→ A|bb, A → a|bc|B

Step 1:

First, we create a dependency graph of all unit production.

S→B, BA and A→B



So, SB,SA,B⇒ A and A ⇒ B
Step 2:
Now we write grammar without unit production:
S→ Aa S→ bb + S→ a|bc [Reason S⇒ B, S⇒ A]
B→ bb + B → a|bc
[Reason B⇒ A]
A→ abc + A → bb
[Reason A⇒ B]
Whatever we derive from B, we same way derive from A because A ⇒ B, and same things happen for all production.
New grammar:
G2: S→ Aa|bb|a|bc
A→ a|bc|bb
B→ bb|a|bc
So, $G_1 = G_2$ and $L(G_1) = L(G2)$
Example 7:
S -> Aa | B

A -> b | B
B -> A | a
Lets add all the non-unit productions of ‘G’ in ‘Guf’. ‘Guf’ now becomes –

S -> Aa
A -> b
B -> a
Now we find all the variables that satisfy ‘X *=> Z’. These are ‘S*=>B’, ‘A *=> B’ and ‘B *=> A’. For ‘A *=> B’ , we add ‘A -> a’ because ‘B ->a’ exists in ‘Guf’. ‘Guf’ now becomes

S -> Aa
A -> b | a
B -> a
For ‘B *=> A’ , we add ‘B -> b’ because ‘A -> b’ exists in ‘Guf’. The new grammar now becomes

S -> Aa
A -> b | a
B -> a | b
We follow the same step for ‘S*=>B’ and finally get the following grammar –

S -> Aa | b | a
A -> b | a
B -> a | b
Now remove B -> a|b , since it doesnt occur in the production ‘S’, then the following grammar becomes,
S->Aa|b|a
A->b|a