Logical Agents

Chapter 7

Why Do We Need Logic?

- Problem-solving agents were very inflexible: hard code every possible state.
- Search is almost always exponential in the number of states.
- Problem solving agents cannot infer unobserved information.

Knowledge & Reasoning

To address these issues we will introduce

- A knowledge base (KB): a list of facts that are known to the agent.

- Rules to infer new facts from old facts using rules of inference.

- Logic provides the natural language for this.

Knowledge Bases

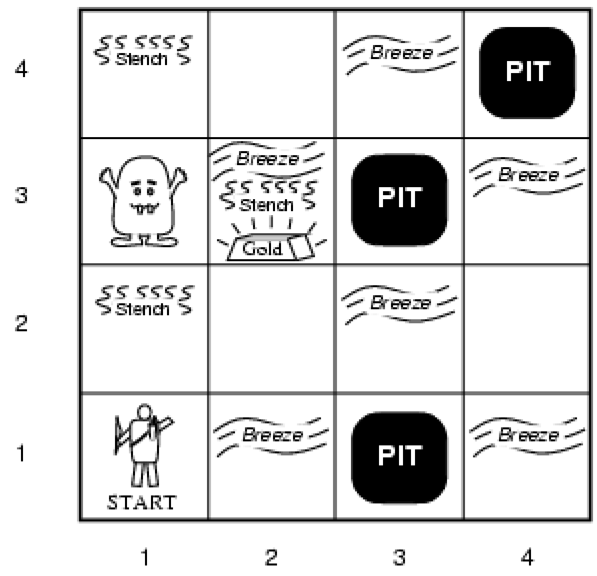
- Knowledge base:
  - set of sentences in a formal language.

- Declarative approach to building an agent:
  - `Tell` it what it needs to know.
  - `Ask` it what to do → answers should follow from the KB.

Wumpus World PEAS description

- Performance measure
  - gold: +1000, death: -1000
  - -1 per step, -10 for using the arrow

- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square

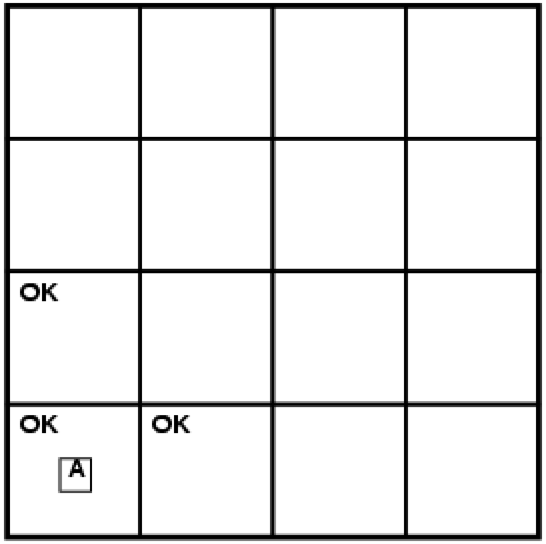


- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – things we do have an impact.
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent? Yes – Wumpus is essentially a natural feature

Exploring a wumpus world



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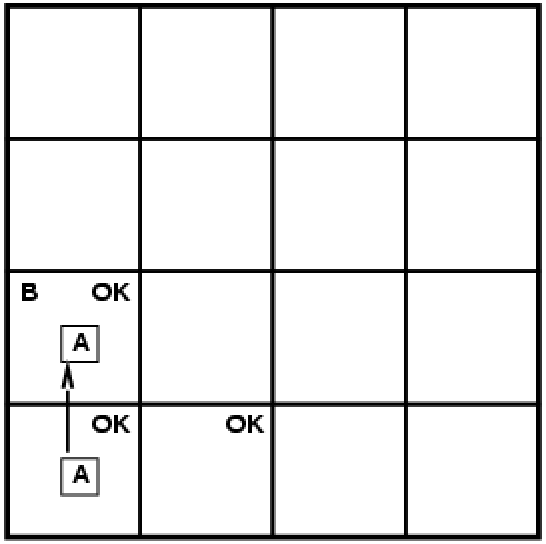

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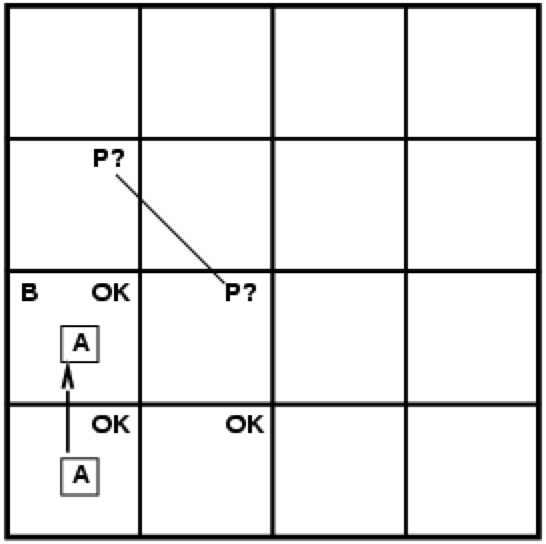

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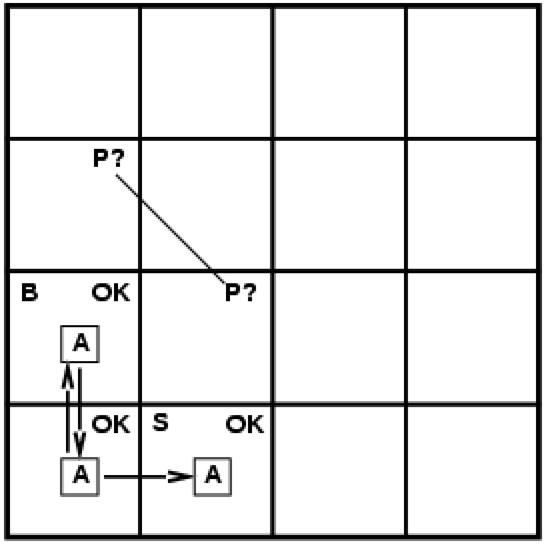
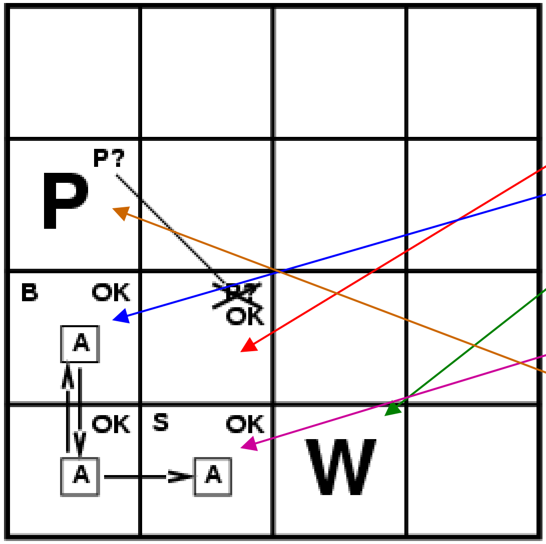


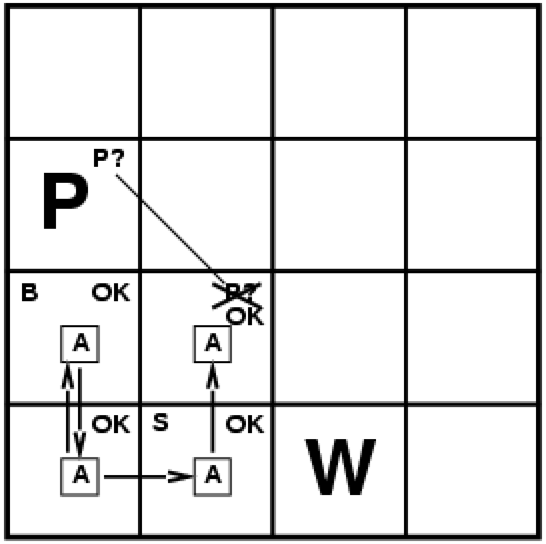
Exploring a Wumpus world



If the Wumpus were here, stench should be here. Therefore it is here.
Since, there is no breeze here, the pit must be there

We need rather sophisticated reasoning here!

Exploring a wumpus world



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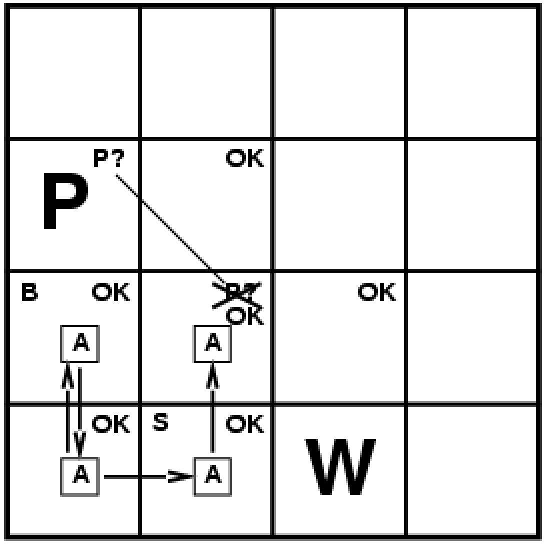

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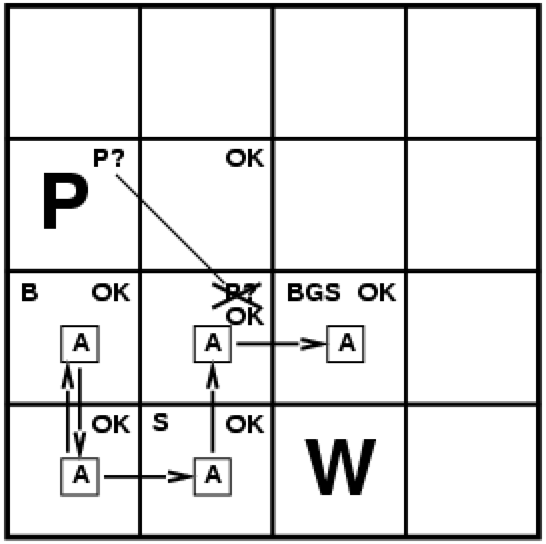


Logic

- We used logical reasoning to find the gold.
- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences;
  - i.e., define truth of a sentence in a world

- E.g., the language of arithmetic
  - $x+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence → syntax
  -
  - $x+2 \geq y$ is true in a world where $x = 7$, $y = 1$
  - $x+2 \geq y$ is false in a world where $x = 0$, $y = 6$

semantics

Entailment

- Entailment means that one thing follows from another:

$$KB \models \alpha$$

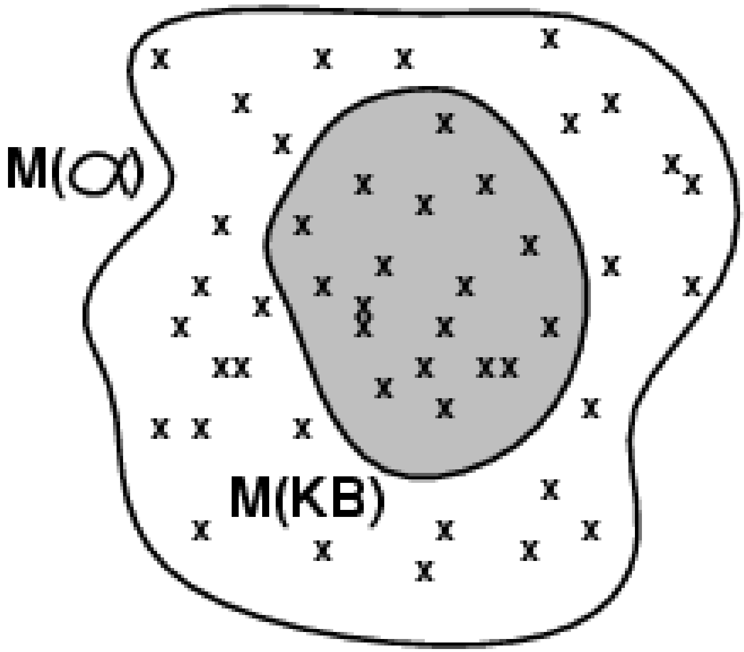
- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true

  – E.g., the KB containing “the Giants won and the Reds won” entails “The Giants won”.

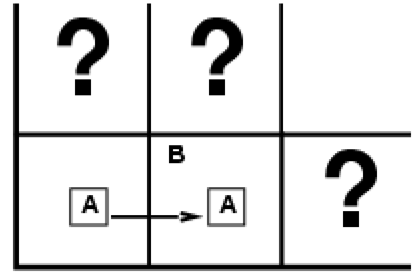
  – E.g., x+y = 4 entails 4 = x+y

Models

- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
- *M(α)* is the set of all models of α
- Then KB ⊨ α iff *M(KB)* ⊆ *M*(α)
  - E.g. *KB* = Giants won and Reds won α = Giants won



Entailment in the wumpus world

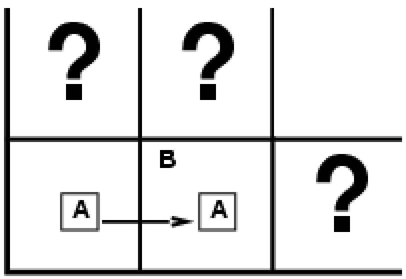


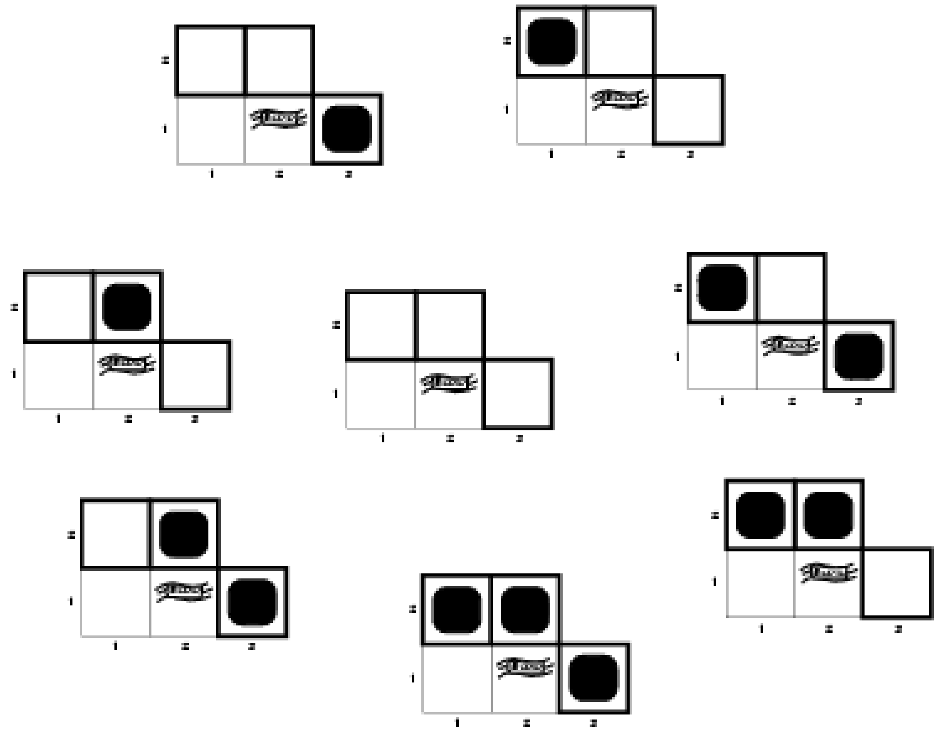
- Consider possible models for *KB* assuming only pits and a reduced Wumpus world

- Situation after detecting

nothing in [1,1], moving right, breeze in [2,1]

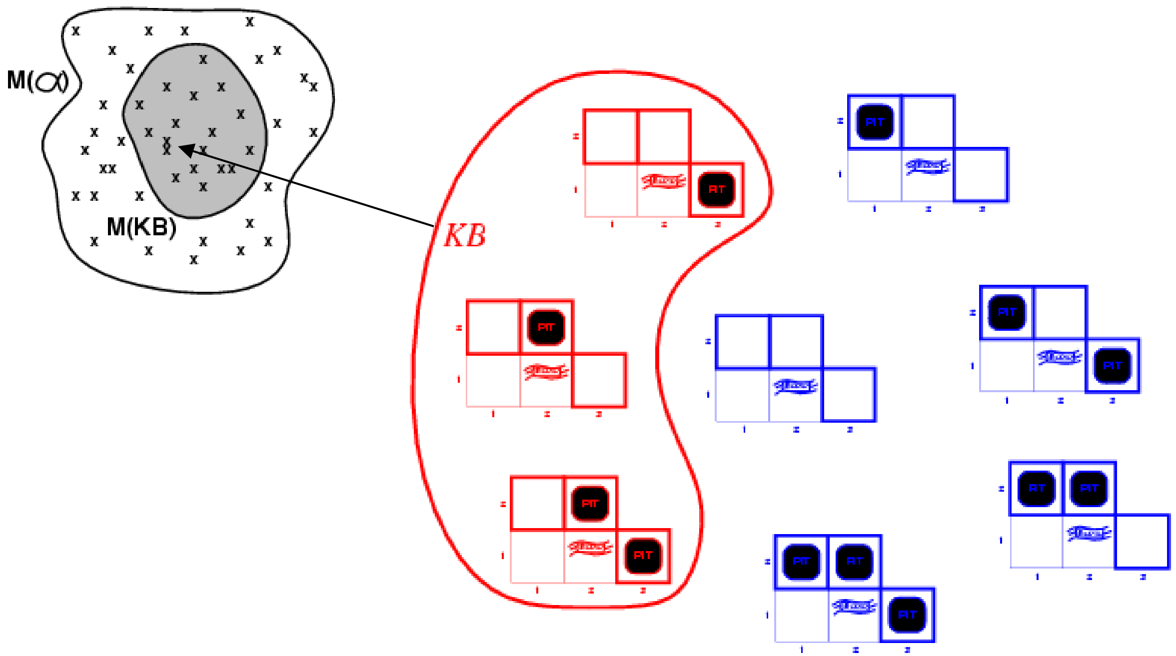
Wumpus models





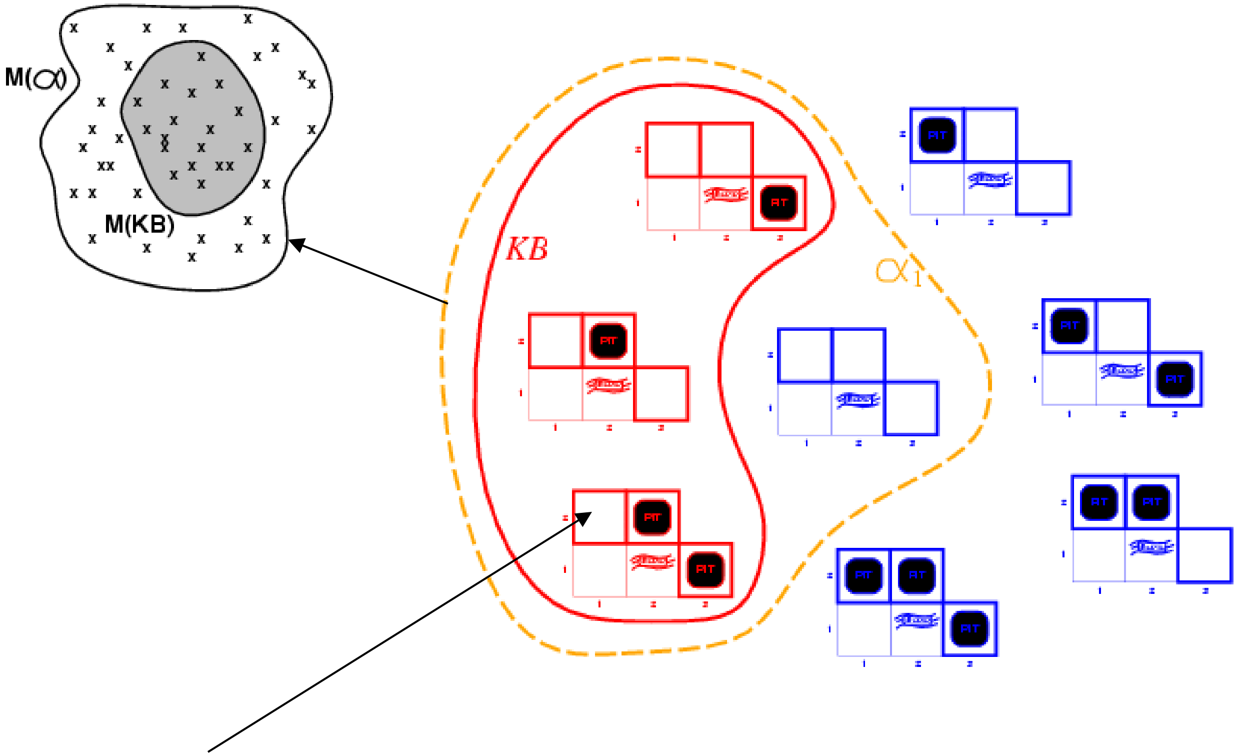
All possible models in this reduced Wumpus world.

Wumpus models



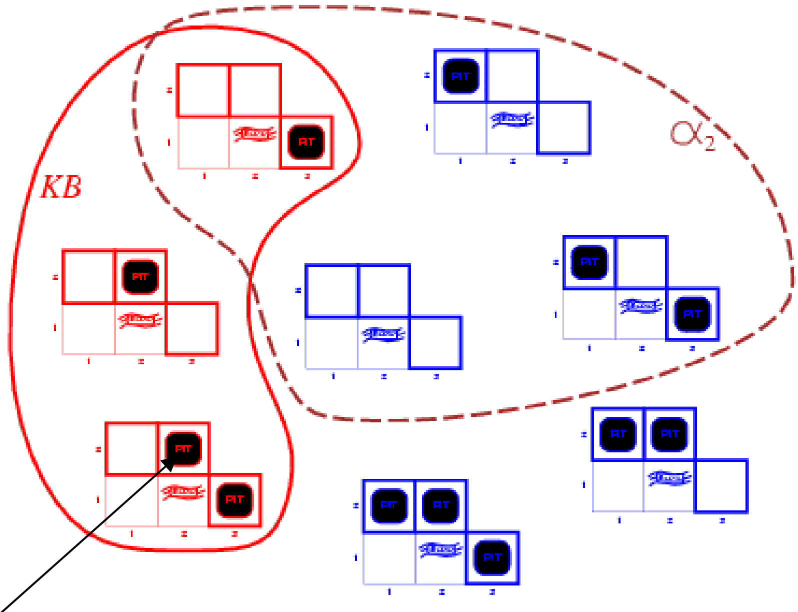
- *KB* = all possible wumpus-worlds consistent with the observations and the "physics" of the Wumpus world.

Wumpus models



α_1 = "[1,2] is safe", *KB* $\models \alpha_1$, proved by model checking

Wumpus models



α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Inference Procedures

- *KB* $\vdash_i$ α = sentence α can be derived from *KB* by procedure *i*
- Soundness: *i* is sound if whenever *KB* $\vdash_i$ α, it is also true that *KB* $\models$ α *(no wrong inferences, but maybe not all inferences)*
- Completeness: *i* is complete if whenever *KB* $\models$ α, it is also true that *KB* $\vdash_i$ α *(all inferences can be made, but maybe some wrong extra ones as well)*

Propositional logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas
- The proposition symbols P_1, P_2 etc are sentences
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
  - If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
  - If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model/world specifies true or false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{true} \vee \mathit{false}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$$

Truth tables for connectives

P	Q	$\neg P$	$P \land Q$	$P \lor Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

OR: P or Q is true or both are true.
XOR: P or Q is true but not both.

Implication is always true when the premises are False!

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j].

Let $B_{i,j}$ be true if there is a breeze in [i, j].

start: $\neg P_{1,1}$

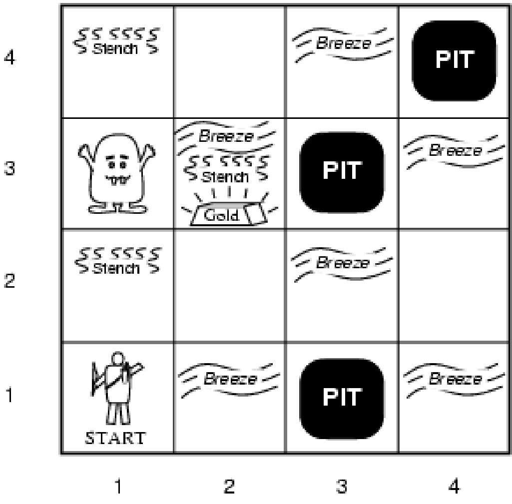
$\neg B_{1,1}$

$B_{2,1}$

- "Pits cause breezes in adjacent squares"

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$



Inference by enumeration

- Enumeration of all models is sound and complete.
- For n symbols, time complexity is $O(2^n)$...
- We need a smarter way to do inference!
- In particular, we are going to infer new logical sentences from the data-base and see if they match a query.

Logical equivalence

- To manipulate logical sentences we need some rewrite rules.
- Two sentences are logically equivalent iff they are true in same models: α ≡ ß iff α ╞ β and β ╞ α

You need to know these !

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Validity and satisfiability

A sentence is valid if it is true in all models,

e.g., *True*, $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \lor B$, C

A sentence is unsatisfiable if it is false in all models

e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \land \neg \alpha)$ is unsatisfiable

(there is no model for which KB=true and α is false)