

**M.S. Ramaiah Institute of Technology
(Autonomous Institute, Affiliated to VTU)**

Department of Computer Science and Engineering

Course Name: Artificial Intelligence

Course Code: CS52

Credits: 2:0:1

UNIT - 2

Term: 3rd Oct 2024 – 25th Jan 2025

UNIT -2 Outline

Knowledge-based agents(Logical Agents)

- Wumpus world
- Logic
- Propositional (Boolean) logic
- Reasoning patterns in Propositional logic
- Equivalence Proportional Model checking
- Agents based on Propositional logic

First Order Logic

- Representation revisited
- Syntax and semantics of first-order logic
- Using first-order logic
- Knowledge engineering in first-order logic

Inference in First Order Logic

- Propositional vs. First-order inference
- Unification and lifting
- Forward chaining
- Backward chaining
- Resolution

Introduction

Machine is like empty box, to make it a valuable machine, it is required to put the necessary knowledge in it. So that it could understand it and is able to take the right decisions.

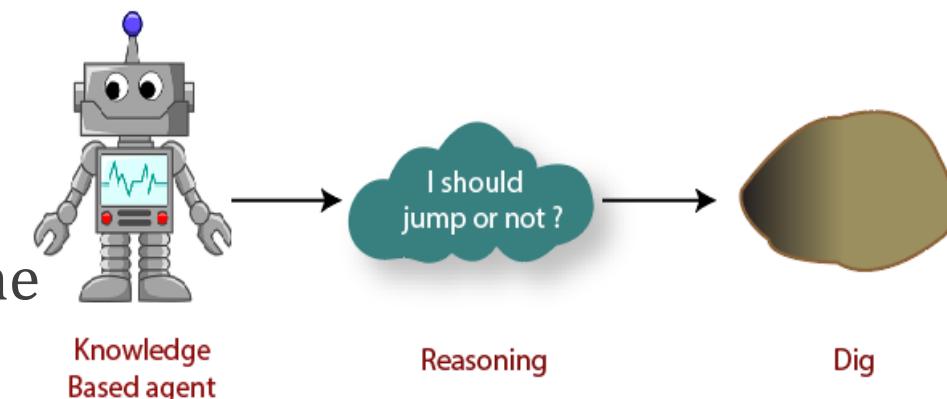
There are **three factors** which are put into the machine, which makes it valuable:

Knowledge: The information related to the environment is stored in the machine.

- facts, skills, education, experience

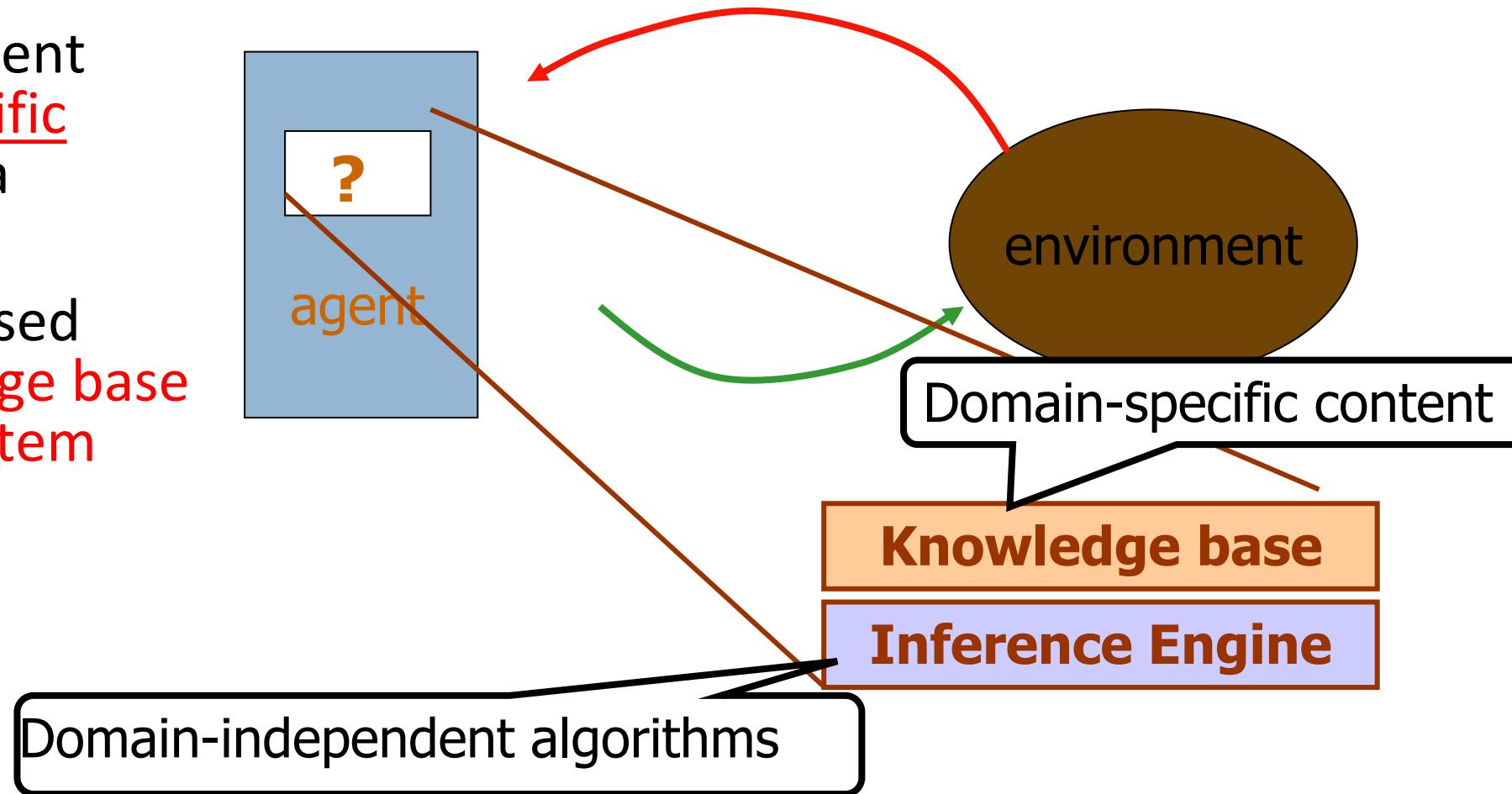
Reasoning: The ability of the machine to understand the stored knowledge.

Intelligence: The ability of the machine to make decisions on the basis of the stored information.

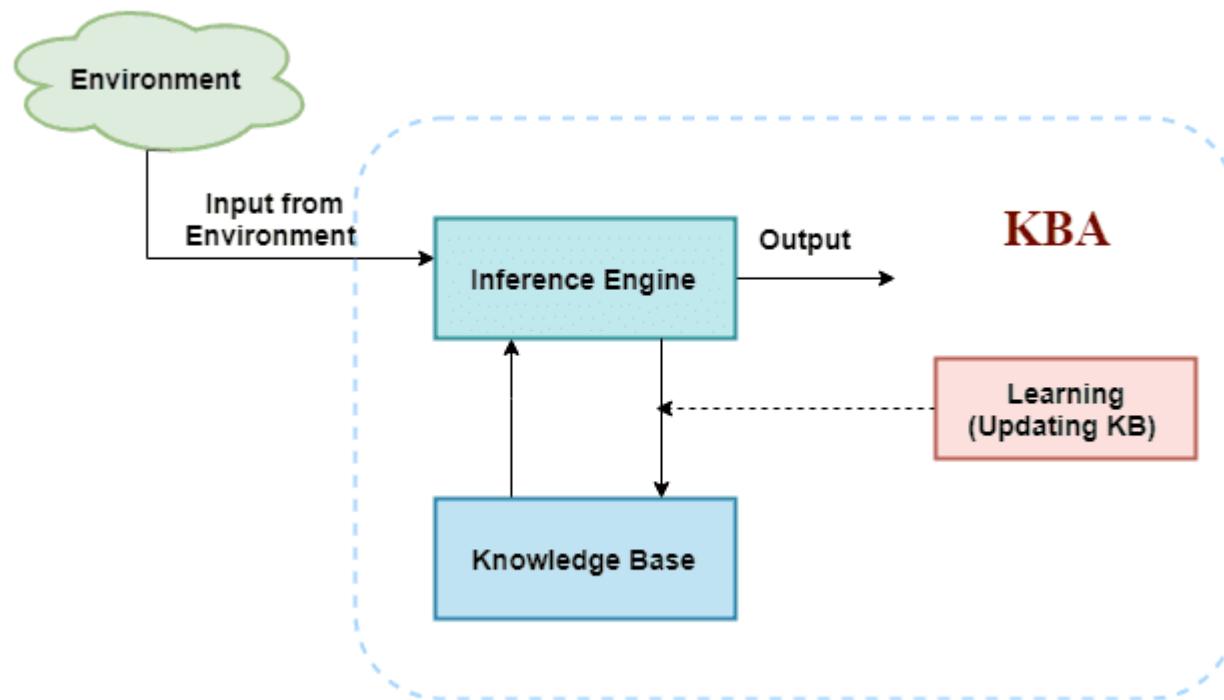


Knowledge based Agent

- Knowledge-based agent uses some task-specific knowledge to solve a problem efficiently.
- Every Knowledge-based agent has a **knowledge base** and an **inference system**



Knowledge based Agent



Knowledge based Agent

Central component of Knowledge based Agent is a Knowledge-base(KB)

KB = set of **sentences** in a **formal** language

Two generic functions are

1. TELL – add new sentence to KB (TELL IT WHAT IT NEEDS TO KNOW)
2. ASK – Query what is known from KB (ASK WHAT TO DO NEXT)

For example:

- ▶ TELL: Father of John is Bob
- ▶ TELL: Jane is John's sister
- ▶ TELL: John's Father is the same as John's sister's father
- ▶ ASK: Who's Jane's Father?

Tell facts
Ask for inference

Knowledge based Agent

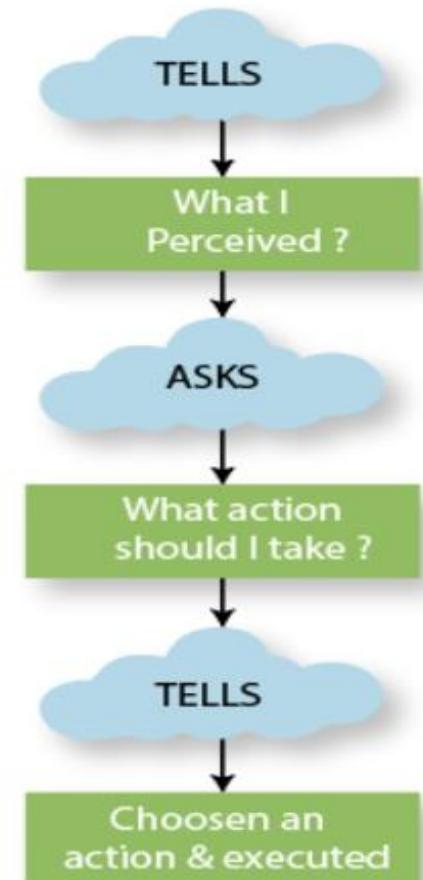
A knowledge-based agent must able to do the following:

- An agent should be able to represent states, actions, etc.
- An agent Should be able to incorporate new percepts
- An agent can update the internal representation of the world
- An agent can deduce the internal representation of the world
- An agent can deduce appropriate actions.

Generic KB-Based Agent

Agent maintains a knowledge base KB in 3 steps:

- First it TELLS the KB what it perceives
- It asks the KB what action it should perform
- Process of answering the query requires lots of reasoning



A simple knowledge-based agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time
    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action  $\leftarrow$  ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t  $\leftarrow$  t + 1
    return action
```

- **MAKE-PERCEPT-SENTENCE** generates a sentence, what the agent perceived at the given time.
- **MAKE-ACTION-QUERY** generates a sentence to ask which action should be done at the current time.
- **MAKE-ACTION-SENTENCE** generates a sentence which asserts that the chosen action was executed.

Approaches to designing a knowledge-based agent

There are mainly two approaches to build a knowledge-based agent:

Declarative approach: We can create a knowledge-based agent by initializing with an empty knowledge base and telling the agent all the sentences with which we want to start with.

Example : Constraints and rules

Procedural approach: We directly encode desired behavior as a program code. Which means we just need to write a program that already encodes the desired behavior or agent.

Example: Functions

Wumpus World

Wumpus world – example to illustrate the worth of a knowledge based agent and to represent knowledge

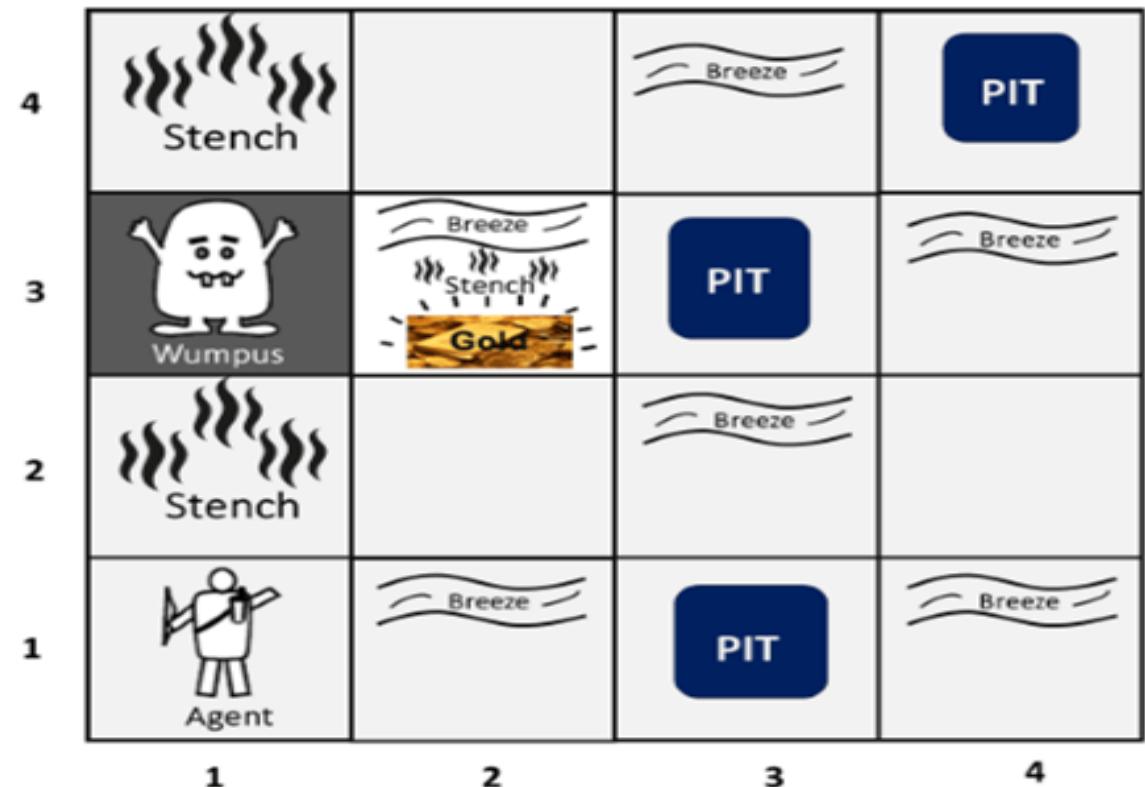
It was inspired by a video game Hunt the Wumpus by Gregory Yob in 1973

A Wumpus World

The wumpus world is a cave consisting of rooms connected by passageways.

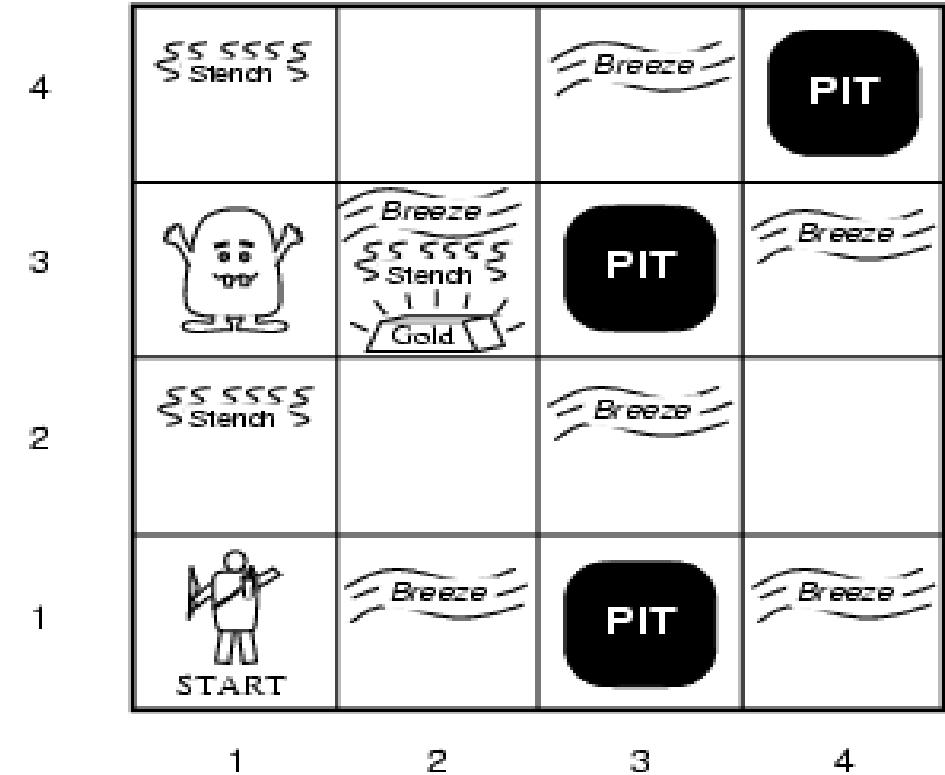
Goal – Agent should pick the Gold present in a Particular room

Agent has only one arrow



Wumpus World PEAS description

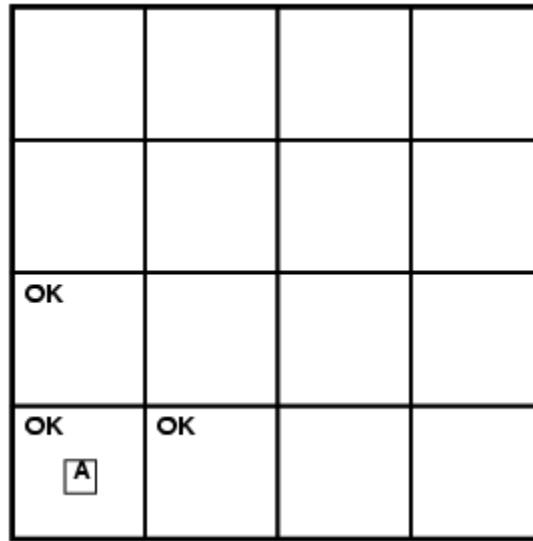
- Performance measure
 - gold +1000, death -1000
 - -1 per step, -10 for using the arrow
- Environment
 - Squares adjacent to wumpus are smelly
 - Squares adjacent to pit are breezy
 - Glitter if gold is in the same square
 - Shooting kills wumpus if you are facing it
 - Shooting uses up the only arrow
 - Grabbing picks up gold if in same square
 - Releasing drops the gold in same square
- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot



Wumpus world characterization

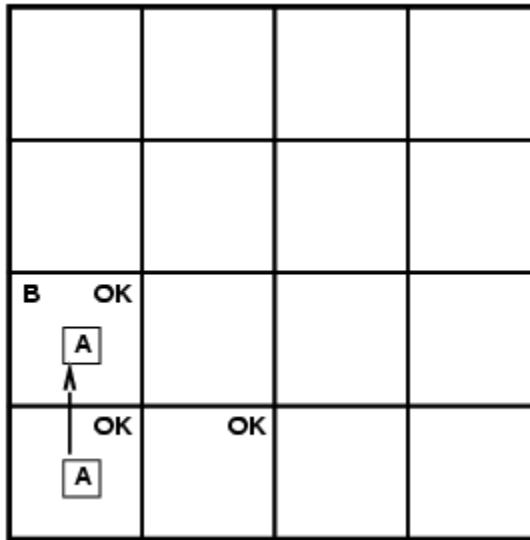
- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent? Yes – Wumpus is essentially a natural feature

Exploring a wumpus world

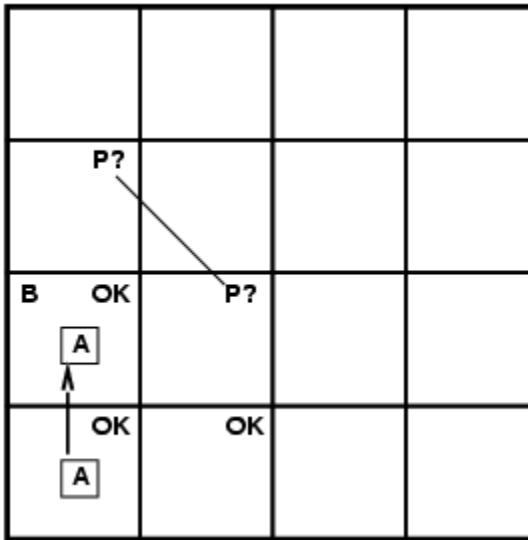


[None, None, None, None, None] - neighboring squares, [1,2] and [2,1], are free of dangers—they are OK

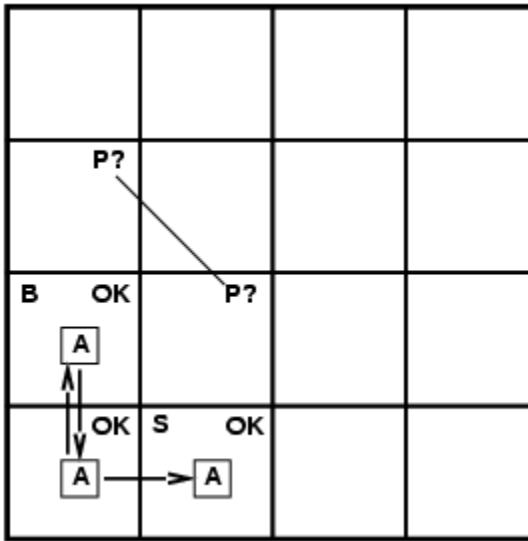
Exploring a wumpus world



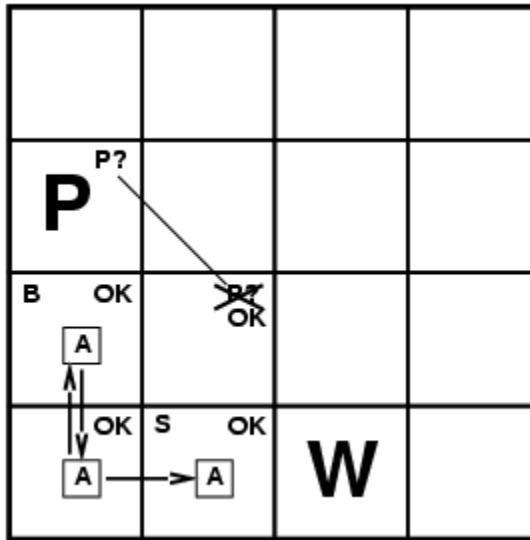
Exploring a wumpus world



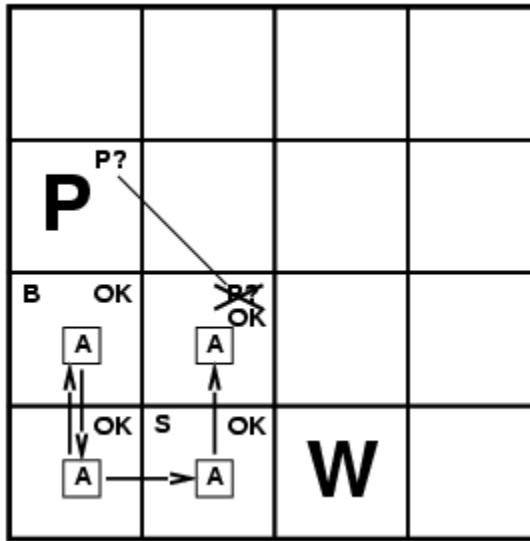
Exploring a wumpus world



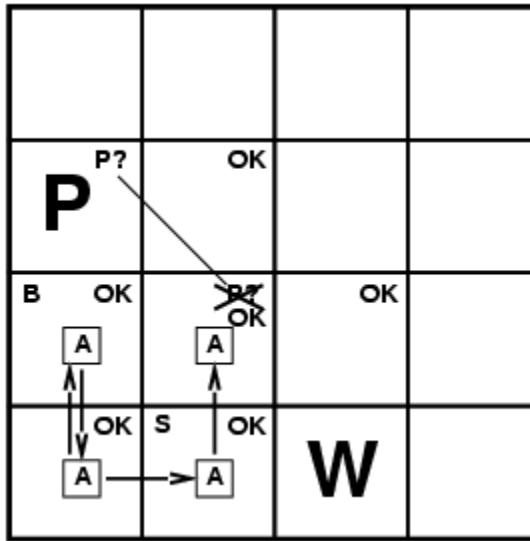
Exploring a wumpus world



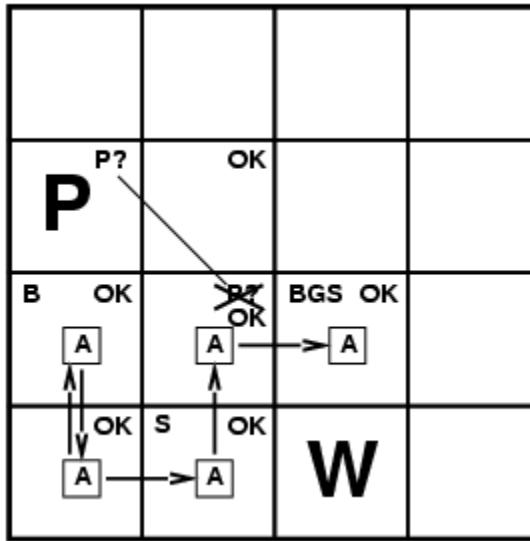
Exploring a wumpus world



Exploring a wumpus world



Exploring a wumpus world



Wumpus World PEAS description

Performance measure

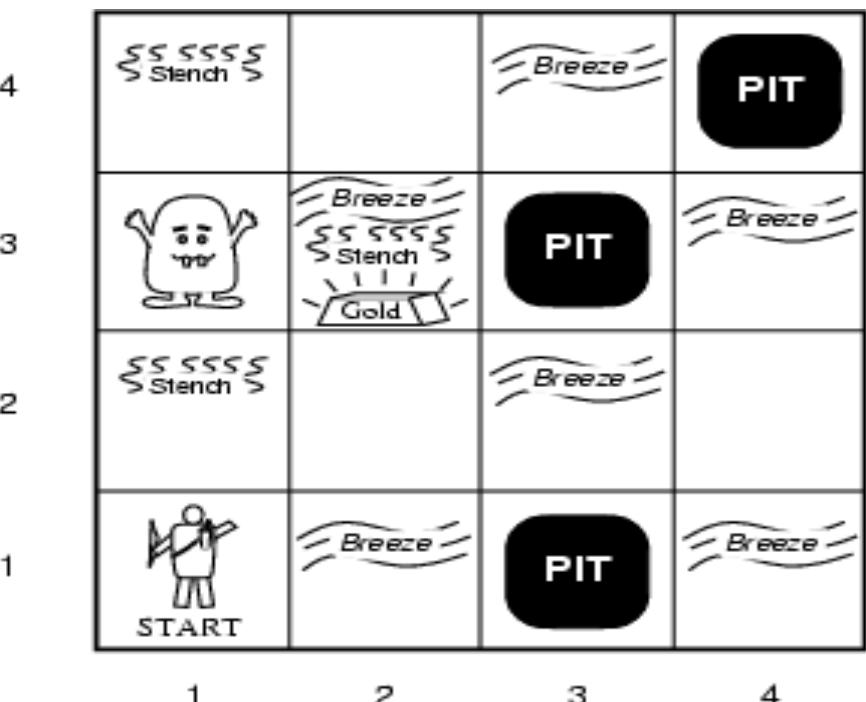
- gold +1000, death -1000
- -1 per step, -10 for using the arrow

Environment: 4 x 4 grid of rooms

- [1,1] start of an Agent

Sensors: Stench, Breeze, Glitter, Bump(Hit wall), Scream (shot Wumpu)

Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot



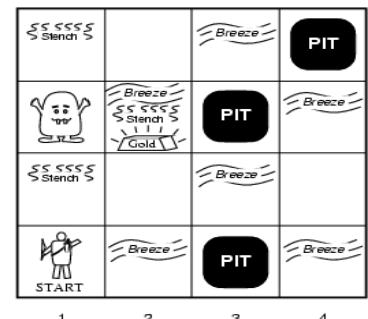
Exploring the Wumpus World

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
1,1	2,1	3,1	4,1

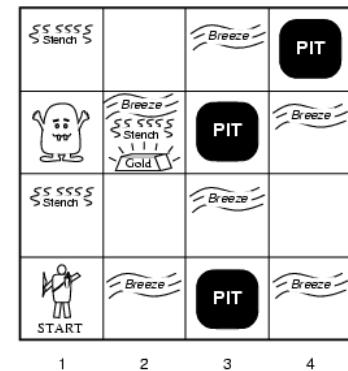
- A** = Agent
- B** = Breeze
- G** = Glitter, Gold
- OK** = Safe square
- P** = Pit
- S** = Stench
- V** = Visited
- W** = Wumpus

Percepts given to the agent

1. Stench
2. Breeze
3. Glitter
4. Bumb (ran into a wall)
5. Scream (wumpus has been hit by arrow)



Exploring the Wumpus World



Initial situation:

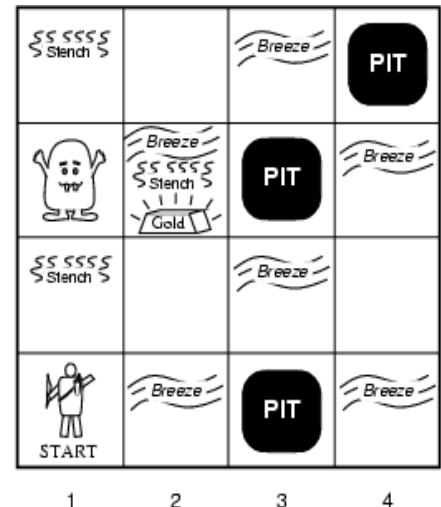
1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
OK			
1,1	2,1	3,1	4,1
A	OK	OK	

**Agent in 1,1 and percept is [None, None, None, None, None]
[Stench,Breeze,Glitter,Bumb,Scream]**

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
OK			
1,1	2,1	3,1	4,1
A	B	P?	OK

[None, Breeze, None, None, None]

Exploring the Wumpus World



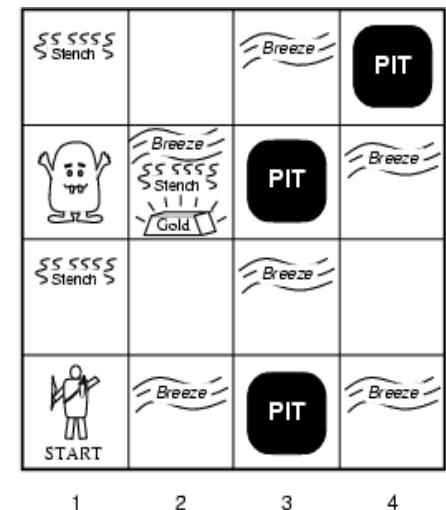
1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 A S OK	2,2 OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

[Stench,None,None,None,None]

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 V OK	2,2 A V OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

[None, None, None, None, None]

Exploring the Wumpus World



1,4	2,4 P?	3,4	4,4
1,3 W!	2,3 A S G B	3,3 P?	4,3
1,2 S V OK	2,2 V OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

[Stench,Breeze,Glitter,None,None]

Logic in general

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences;
 - i.e., define truth of a sentence in a possible world
- E.g., the language of arithmetic
 - $x+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence
 - $x+2 \geq y$ is true iff the number $x+2$ is no less than the number y
 - $x+2 \geq y$ is true in a world where $x = 7, y = 1$
 - $x+2 \geq y$ is false in a world where $x = 0, y = 6$

Logic - Logical Reasoning

In each case where the agent draws a conclusion from the available information, that conclusion is guaranteed to be correct if the available information is correct.

The above is the fundamental of logical reasoning

Syntax

- Syntax is the way sentences are expressed.
- The **syntax** are the rules that determine whether or not a sentence is well formed.
- Example : name John my is
my name is John
- $1+2$ 3
- $1+2=3$

Semantics

- Semantics is the meaning behind the sentences.
- In logic, semantics talk about whether a sentence is true or false.
- Example : my name is Paul - False
my name is John - True

Models

- A model is a version of the environment where variable states take on a specific value, and this helps us evaluate the truthfulness of sentences.
- Example: $x+2 \geq y$ is true in a world where $x = 7, y = 1$
- When a sentence is true under a particular model and we use the word “satisfies”

Models

- Knowledge Base (KB): a collection of sentences.

Example KB				
A	B	C	KB	
F	F	F	F	
F	F	T	F	
F	T	F	T	
F	T	T	F	
T	F	F	T	
T	F	T	T	
T	T	F	T	
T	T	T	T	

KB:

$A \vee B$

$\neg C \vee A$

Type	Connective	Symbol	Notation	Read as
Conjunction	And	\wedge	$P \wedge Q$	P and Q
Disjunction	Or	\vee	$P \vee Q$	P or Q
Conditional	If then	\rightarrow	$P \rightarrow Q$	P implies Q i.e If P then Q
Bi-conditional	If and only if	\leftrightarrow	$P \leftrightarrow Q$	P double implies Q i.e. P if and only if Q
Negation	Not or No	\neg or \top	$\neg P$	Negation P (or) Not P

Models

- Model: an assignment of (True/False) values to each of the symbols. If the knowledge base is built from n symbols, there are 2^n possible models.

Example KB			
A	B	C	KB
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

KB:
 $A \vee B$
 $\neg C \vee A$

Models

- Sentence is represented as α
- We use the notation $M(\alpha)$ - set of all models of α
- If a sentence α is true in model m ,
we say that m satisfies α or sometimes m is a model of α .

Entailment

Entailment is a specific kind of relationship between two sentences.

Example

- **Statement A:** "I will turn 28 this year;"
- **Statement B:** "I am currently living."

Entailment is present here because the truth of A requires the truth of B: if I am not currently living, then I cannot age, and therefore I will not turn 28 this year.

Logical Entailment

In mathematical notation,

we write : $\alpha \models \beta$, sentence α entails the sentence β

The formal definition of entailment is this:

$\alpha \models \beta$ if and only if, in every model in which α is true, β is also true.

	A	B	C	KB	S
KB:	F	F	F	F	F
$A \vee B$	F	F	T	F	T
$\neg C \vee A$	F	T	F	T	T
$S:$	F	T	T	F	T
	T	F	F	T	T
	T	F	T	T	T
	T	T	F	T	T
	T	T	T	T	T

$KB \models S$
because S is true for all the assignments for which KB is true

	A	B	C	KB	S
KB:	F	F	F	F	F
$A \vee B$	F	F	T	F	F
$\neg C \vee A$	F	T	F	T	F
$S:$	F	T	T	F	F
	T	F	F	T	F
	T	F	T	T	F
	T	F	T	T	T
	T	T	F	T	F
	T	T	T	T	T

$KB \not\models S$
because KB is true but S is false

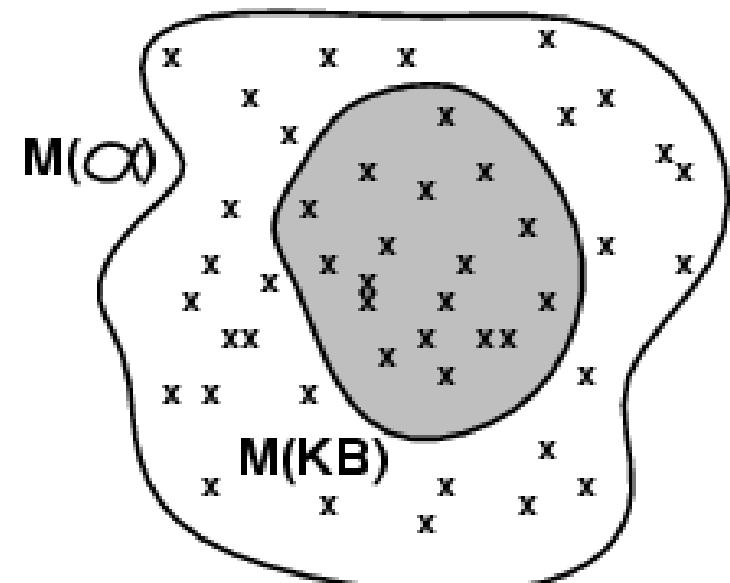
Models and Entailment

Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated

We say m is a model of a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ iff $M(KB) \subset M(\alpha)$



Inference

Inference is a procedure that allows new sentences to be derived from a knowledge base.

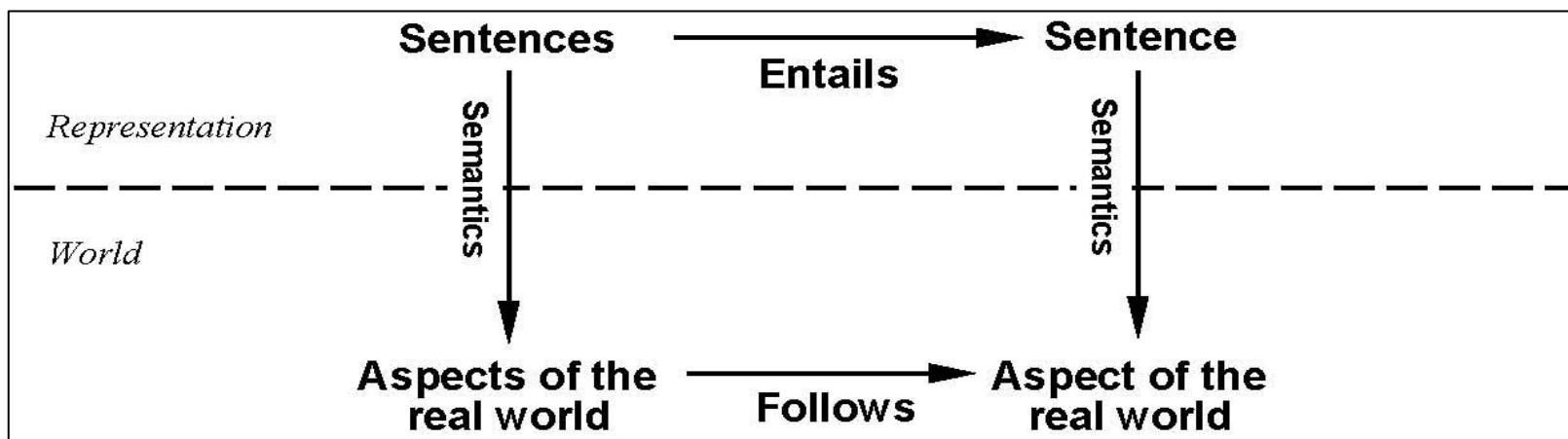
If an inference algorithm i can derive α from KB , we write

$$KB \vdash_i \alpha$$

Pronounced “ α is derived from KB by i ” or “ i derives α from KB ”.

Inference

- An inference algorithm that derives only entailed sentences is called **sound or TRUTH-PRESERVING**
- KB is true in the real world, then any sentence α derived from KB by a sound inference procedure is also true in the real world.



Propositional logic(Boolean Logic)

- Propositional logic is the **simplest logic**
- One of the **Knowledge representation scheme** called **propositional logic**.
- **Syntax and Semantics of Propositional Logic**

Propositional logic Syntax

- The **Atomic sentences** consist of a single proposition symbol.
- **Complex sentences** are constructed from simpler sentences, using parentheses and logical connectives.

Type	Connective	Symbol	Notation	Read as
Conjunction	And	\wedge	$P \wedge Q$	P and Q
Disjunction	Or	\vee	$P \vee Q$	P or Q
Conditional	If then	\rightarrow	$P \rightarrow Q$	P implies Q i.e If P then Q
Bi-conditional	If and only if	\leftrightarrow	$P \leftrightarrow Q$	P double implies Q i.e. P if and only if Q
Negation	Not or No	\sim or !	$\sim P$	Negation P (or) Not P

Propositional logic Syntax

There are five connectives in common use

Negation: A sentence such as $\neg P$ is called negation of P .

A literal can be either Positive literal or negative literal.

Conjunction: A sentence which has \wedge connective such as, $P \wedge Q$ is called a conjunction.

Example: Rohan is intelligent and hardworking.

It can be written as,

- $P = \text{Rohan is intelligent}$,
- $Q = \text{Rohan is hardworking}$. $\rightarrow P \wedge Q$.

Disjunction: A sentence which has \vee connective, such as $P \vee Q$. is called disjunction, where P and Q are the propositions.

Example: "Ritika is a doctor or Engineer",

Here $P = \text{Ritika is Doctor}$. $Q = \text{Ritika is Engineer}$, so we can write it as $P \vee Q$.

Propositional logic Syntax

Implication: A sentence such as $P \rightarrow Q$, is called an implication.

Implications are also known as if-then rules.

If it is raining, then the street is wet.

Let P = It is raining, and Q = Street is wet, so it is represented as $P \rightarrow Q$

Biconditional: A sentence such as $P \Leftrightarrow Q$ is a Biconditional sentence,

Example If I am breathing, then I am alive

P = I am breathing, Q = I am alive, it can be represented as $P \Leftrightarrow Q$.

Propositional logic Semantics

- All sentences are constructed from atomic sentences and the five connectives are expressed with truth tables

- $\neg P$ is true iff P is false in m .
- $P \wedge Q$ is true iff both P and Q are true in m .
- $P \vee Q$ is true iff either P or Q is true in m .
- $P \Rightarrow Q$ is true unless P is true and Q is false in m .
- $P \Leftrightarrow Q$ is true iff P and Q are both true or both false in m .

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

Propositional logic Syntax

Figure gives a formal grammar of propositional logic with the BNF notation

```
Sentence → AtomicSentence | ComplexSentence  
AtomicSentence → True | False | P | Q | R | ...  
ComplexSentence → ( Sentence ) | [ Sentence ]  
                  |  $\neg$  Sentence  
                  | Sentence  $\wedge$  Sentence  
                  | Sentence  $\vee$  Sentence  
                  | Sentence  $\Rightarrow$  Sentence  
                  | Sentence  $\Leftrightarrow$  Sentence
```

OPERATOR PRECEDENCE : \neg , \wedge , \vee , \Rightarrow , \Leftrightarrow

Figure 7.7 A BNF (Backus–Naur Form) grammar of sentences in propositional logic, along with operator precedences, from highest to lowest.

Propositional Theorem Proving

Entailment by theorem proving, so we need additional concepts

- Logical equivalence
- Validity
- Satisfiability

Propositional Theorem Proving

Logical equivalence

- Two sentences α and β are logically equivalent if they are true in the same set of models.
$$\vdash \alpha \equiv \beta$$
- For example, $P \wedge Q$ and $Q \wedge P$ are logically equivalent;
- Any two sentences α and β are equivalent only if each of them entails the other: $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

Propositional Theorem Proving

Laws of Logic

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$	commutativity of \wedge
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$	commutativity of \vee
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of \wedge
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$	associativity of \vee
$\neg(\neg \alpha) \equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$	contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg \alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$	De Morgan
$\neg(\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of \wedge over \vee
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of \vee over \wedge

Figure 7.11 Standard logical equivalences. The symbols α , β , and γ stand for arbitrary sentences of propositional logic.

Propositional Theorem Proving

- Valid: A sentence is valid if it is true for all models.
- Satisfiable: A sentence is satisfiable if it is true for some models.
- Unsatisfiable: A sentence is unsatisfiable if it is true for no models.

KB:
 $A \vee B$
 $\neg C \vee A$

Example KB			
A	B	C	KB
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

Propositional Theorem Proving

Validity

A sentence is valid if it is true in all models.

Example, the sentence $P \vee \neg P$ is valid.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Valid sentences are also known as tautologies—they are necessarily true.

α and β , $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid

Propositional Theorem Proving

Satisfiability

A Compound proposition is satisfiable if there is at least one TRUE results in Truth table

Example : $P \wedge Q$

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The problem of determining the satisfiability of sentences in propositional logic — **SAT problem**

Propositional Theorem Proving

Inference Rule

In AI , we need intelligent computers which can create new logic from old logic or by evidence, so generating the conclusions from evidence and facts is termed as Inference.

Inference : Deriving conclusion from evidences(Premises/Assumptions)

Rules of Inference : Are the templates for constructing valid arguments

Propositional Theorem Proving

Proof is a **sequence of sentences**, where each sentence is either a **premise** or a sentence derived from earlier sentences in the proof by one of the rules of inference.

The last sentence is the **theorem** (also called goal or query) that we want to prove.

Example for the “weather problem”.

1 Hu	Premise	“It is humid”
------	---------	---------------

Propositional Theorem Proving

Arguments in Propositional Logic

- A argument in propositional logic is a sequence of propositions.
- All the proposition are called premises. The last statement is the conclusion.
- The argument is valid if the premises imply the conclusion.

Propositional Theorem Proving

Types of Inference Rule :

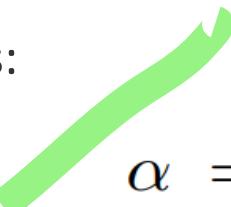
- Modus Ponens
- And-Elimination
- Resolution

Propositional Theorem Proving

Modus Ponens

The Modus Ponens rule states **that if α and $\alpha \rightarrow \beta$ is true, then we can infer that β will be true.**

It can be represented as:


$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

Any sentences of the form $\alpha \Rightarrow \beta$ and α are given, then the sentence β can be inferred.

Propositional Theorem Proving

And-Elimination

From a conjunction, any of the conjuncts can be inferred

$$\frac{\alpha \wedge \beta}{\alpha}$$

Example

"He studies very hard and he is the best boy in the class", $P \wedge Q$

Therefore – "He studies very hard" P

Propositional Theorem Proving

All of the logical equivalences in Figure 7.11 can be used as inference rules.

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$	commutativity of \wedge
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$	commutativity of \vee
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of \wedge
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$	associativity of \vee
$\neg(\neg\alpha) \equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of \wedge over \vee
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of \vee over \wedge

Figure 7.11 Standard logical equivalences. The symbols α , β , and γ stand for arbitrary sentences of propositional logic.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Determine the validity of the following argument using propositional logic: If there is a cream, then I will drink coffee. If there is a donut , then I will drink coffee. There is no cream and there is a donut. So, I drink coffee

P – If there is a cream

Q- I will drink coffee

R - If there is a donut

Premises	$P \rightarrow Q$	$R \rightarrow Q$	$\sim P \wedge R$	Conclusion
				Q

P	Q	$\sim P$	R	$P \rightarrow Q$	$R \rightarrow Q$	$\sim P \wedge R$	$(P \rightarrow Q) \wedge (R \rightarrow Q) \wedge (\sim P \wedge R)$	$(P \rightarrow Q) \wedge (R \rightarrow Q) \wedge (\sim P \wedge R) \rightarrow Q$
T	T	F	T	T	T	F	F	T
T	T	F	F	T	T	F	F	T
T	F	F	T	F	F	F	F	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	T	F	T	T	F	F	T
F	F	T	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T

$$(P \rightarrow Q) \wedge (R \rightarrow Q) \wedge (\sim P \wedge R) \rightarrow Q$$

Validity of 'verbal arguments'

In everyday life, we come across argument expressed in (English) sentences. We can represent the premises in symbols and verify the validity as in earlier examples.

W.E.11. Determine the validity of the following argument:

If two sides of a triangle are equal, then two opposite angles are equal.

Two sides of a triangle are not equal.

Therefore, the opposite angles are not equal.

Solution

Let us indicate the statements as follows :

P : Two sides of a triangle are equal.

Q : The two opposite angle are equal.

The given argument in the form

$$P \rightarrow Q, \neg P \Rightarrow \neg Q.$$

Let us now construct the truth table for $(P \rightarrow Q) \wedge (\neg P) \rightarrow \neg Q$.

P	Q	$P \rightarrow Q$	$\neg P$	$(P \rightarrow Q) \wedge (\neg P)$	$\neg Q$
T	T	T	F	F	F
T	F	F	F	F	T
F	T	T	T	T	F
F	F	T	T	T	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

This shows that $(P \rightarrow Q) \wedge (\neg P) \rightarrow \neg Q$ is not a tautology. Hence the conclusion $\neg Q$ is not valid.

W.E.12. Determine the validity of the following argument :

My father praises me only if I can be proud of myself.

Either I do well in sports or I cannot be proud of myself. If I study hard, then I cannot do well in sports. Therefore, if father praises me, then I do not study well.

Solution

Let us indicate the statements as follows :

P : My father praises me.

Q : I can be proud of myself.

R : I do well in sports.

S : I study hard.

The given argument is of the form

$$P \rightarrow Q, R \vee \neg Q, S \rightarrow \neg R \Rightarrow P \rightarrow \neg S.$$

As the desired result is $P \rightarrow \neg S$, assume that P is True.

It is enough to verify the validity of

$$P, P \rightarrow Q, R \vee \neg Q, S \rightarrow \neg R \Rightarrow \neg S.$$

From P and $P \rightarrow Q$, we have Q.

From Q and $R \vee \neg Q$, we have R.

From R and $S \rightarrow \neg R$, we have $\neg S$.

Thus the argument is valid.

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

Is the following argument valid, or is it a fallacy?

"If you wear an enormous cowboy hat, people will stare. If people stare at you, you get embarrassed. Therefore, if you wear an enormous cowboy hat, you will be embarrassed."

Premises:

- $p \rightarrow q$: If you wear an enormous cowboy hat, people will stare.
- $q \rightarrow r$: If people stare at you, you get embarrassed.

Conclusion:

- $p \rightarrow r$: If you wear an enormous cowboy hat, you will be embarrassed.

Argument:

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	F	T	F	T	T	F	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	F	T	F	T	F	T
F	F	F	T	T	T	T	T

Is the following argument valid, or is it a fallacy?

“A musician can play the guitar or the piano. She can play the guitar, so she can’t play the piano.”

Premises:

$p \vee q$: She can play the guitar or she can play the piano.
 p : She can play the guitar.

Conclusion:

$\sim q$: She can’t play the piano.

Argument:

$$[(p \vee q) \wedge p] \rightarrow \sim q.$$

p	q	$\sim q$	$p \vee q$	$(p \vee q) \wedge p$	$[(p \vee q) \wedge p] \rightarrow \sim q$
T	T	F	T	T	F
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	F	F	T

Propositional Theorem Proving

Monotonicity

- In monotonicity, once the conclusion is taken, then it will remain the same even if we add some other information to existing information in our knowledge base.

Example:

- “Earth revolves around the Sun”
- It is a true fact, and it cannot be changed even if we add another sentence in knowledge base like, “The moon revolves around the earth” Or “Earth is not round,” etc.

Propositional Theorem Proving

Monotonicity

- Monotonicity means that inference rules can be applied whenever suitable premises are found in the knowledge base—the conclusion of the rule must follow regardless of what else is in the knowledge base
- Set of entailed sentences can increase as information is added to the knowledge base.
- For any sentences α and β

if $\text{KB} \models \alpha$ then $\text{KB} \wedge \beta \models \alpha$

Propositional Theorem Proving

Literal : Is either a propositional variable P or a negated propositional variable $\neg P$.

Clause : is

- A literal Example: P
- Disjunction of two or more literals Example: $P \vee \neg Q \vee R$
- The empty clause
Example: {}

$$\text{CNFSentence} \rightarrow \text{Clause}_1 \wedge \dots \wedge \text{Clause}_n$$

$$\text{Clause} \rightarrow \text{Literal}_1 \vee \dots \vee \text{Literal}_m$$

$$\text{Literal} \rightarrow \text{Symbol} \mid \neg \text{Symbol}$$

$$\text{Symbol} \rightarrow P \mid Q \mid R \mid \dots$$

Propositional Theorem Proving

Resolution Method

Resolution method is an inference rule which is used in both propositional logic as well as First order predicate logic

This method is used for proving the satisfiability of a sentence

Propositional Theorem Proving

Resolution Rule

In PL, resolution method is the inference rule takes **two premises in the form of clauses** and gives the clause as a conclusion.

$$(\alpha \vee \neg\beta) \wedge (\gamma \vee \beta) \text{ premise}$$

$$(\alpha \vee \gamma) \text{ conclusion}$$

The resolution rule applies only to clauses

Propositional Theorem Proving

Two Normal form

Conjunctive Normal Form (CNF)

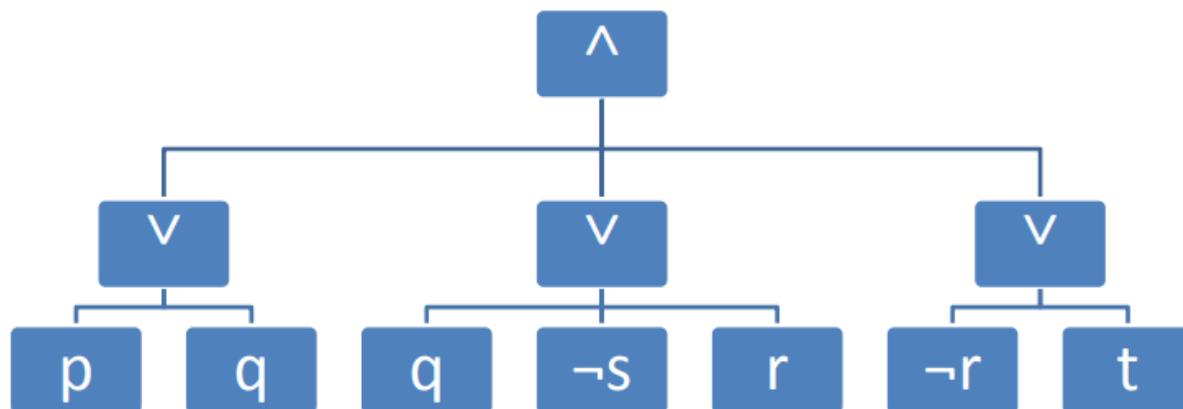
Disjunctive Normal Form (DNF)

Propositional Theorem Proving

Conjunctive Normal Form:

A sentence expressed as a conjunction of clauses is said to be in conjunctive normal form

$$(p \vee q) \wedge (q \vee \neg s \vee r) \wedge (\neg r \vee t)$$



<i>CNF Sentence</i>	\rightarrow	<i>Clause₁</i> $\wedge \dots \wedge$ <i>Clause_n</i>
<i>Clause</i>	\rightarrow	<i>Literal₁</i> $\vee \dots \vee$ <i>Literal_m</i>
<i>Literal</i>	\rightarrow	<i>Symbol</i> \neg <i>Symbol</i>
<i>Symbol</i>	\rightarrow	<i>P</i> <i>Q</i> <i>R</i> ...

Propositional Theorem Proving

Procedure to convert sentence into CNF : Steps

1. Remove Bicondition

Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

2. Remove Implication

Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$

Propositional Theorem Proving

Procedure to convert sentence into CNF : Steps

3. Move Negation inwards

$$\neg(\neg\alpha) \equiv \alpha \quad (\text{double-negation elimination})$$

$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad (\text{De Morgan})$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad (\text{De Morgan})$$

4. Apply Distributive and/or Commutative law

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$

$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$

$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$

$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

Propositional Theorem Proving

Example : $A \Leftrightarrow (B \vee C)$

1. Remove \Leftrightarrow

$\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

$$[A \Rightarrow (B \vee C)] \wedge [(B \vee C) \Rightarrow A]$$

2. Remove Implication

$\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$

$$[\neg A \vee (B \vee C)] \wedge [\neg (B \vee C) \vee A]$$

Propositional Theorem Proving

$$[\neg A \vee (B \vee C)] \wedge [\neg (B \vee C) \vee A]$$

3. Move Negation inwards

$$[\neg A \vee (B \vee C)] \wedge [(\neg B \wedge \neg C) \vee A]$$

4. Apply Distributive and/or Commutative law

$$(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$$

$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$$

$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$$

$$\neg(\neg \alpha) \equiv \alpha \quad (\text{double-negation elimination})$$

$$\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta) \quad (\text{De Morgan})$$

$$\neg(\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta) \quad (\text{De Morgan})$$

This Sentence is now in CNF, as a conjunction of three clauses.

Propositional Theorem Proving

A resolution algorithm

To show that $KB \models \alpha$, we show that $(KB \wedge \neg\alpha)$ is unsatisfiable.

```
function PL-RESOLUTION( $KB, \alpha$ ) returns true or false
    inputs:  $KB$ , the knowledge base, a sentence in propositional logic
            $\alpha$ , the query, a sentence in propositional logic

     $clauses \leftarrow$  the set of clauses in the CNF representation of  $KB \wedge \neg\alpha$ 
     $new \leftarrow \{ \}$ 
    loop do
        for each pair of clauses  $C_i, C_j$  in  $clauses$  do
             $resolvents \leftarrow$  PL-RESOLVE( $C_i, C_j$ )
            if  $resolvents$  contains the empty clause then return true
             $new \leftarrow new \cup resolvents$ 
        if  $new \subseteq clauses$  then return false
         $clauses \leftarrow clauses \cup new$ 
```

Figure 7.12 A simple resolution algorithm for propositional logic. The function PL-RESOLVE returns the set of all possible clauses obtained by resolving its two inputs.

Two important properties for inference

Soundness: If $\text{KB} \vdash Q$ then $\text{KB} \models Q$

- If Q is derived from a set of sentences KB using a given set of rules of inference, then Q is entailed by KB .
- Hence, inference produces only real entailments, or any sentence that follows deductively from the premises is valid.

Completeness: If $\text{KB} \models Q$ then $\text{KB} \vdash Q$

- If Q is entailed by a set of sentences KB , then Q can be derived from KB using the rules of inference.
- Hence, inference produces all entailments, or all valid sentences can be proved from the premises.

Propositional Theorem Proving

Completeness of resolution

Resolution closure $RC(S)$ of a set of clauses S , which is the set of all clauses derivable by repeated application of the resolution rule to clauses in S or their derivatives.

The completeness theorem for resolution in propositional logic is called the **ground resolution theorem**

Propositional Theorem Proving

Completeness of resolution

Ground resolution theorem : If a set of clauses is unsatisfiable, then the resolution closure of those clauses contains the empty clause.

Construct a model for S with suitable truth values for P₁, ..., P_k.

The construction procedure is as follows:

For i from 1 to k,

- If a clause in RC(S) contains the literal $\neg P_i$ and all its other literals are false under the assignment chosen for P₁, ..., P_{i-1}, then assign false to P_i.
- Otherwise, assign true to P_i.

Propositional Theorem Proving

Horn clause

A clause which is a disjunction of literals of with at most one positive literal.

Example : $\neg A \vee \neg C \vee D$

Propositional Theorem Proving

Types of Horn Clauses :

- **Definite clause / Strict Horn clause**

It has exactly one positive literal.

Example: $\neg p \vee \neg q \vee \dots \vee \neg t \vee u$

- **Unit clause**

Definite clause with no negative literals.

Example: u

- **Goal clause**

Horn clause without a positive literal.

Example : $\neg p \vee \neg q \vee \dots \vee \neg t$

Propositional Theorem Proving

KB containing only definite clauses are interesting for three reasons:

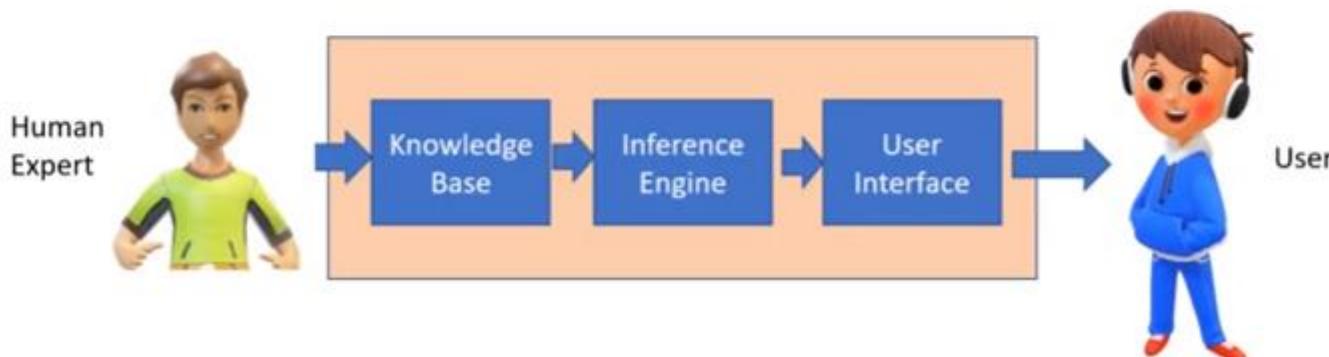
1. Every definite clause can be **written as an implication** whose premise is a conjunction of positive literals and whose conclusion is a single positive literal.
2. **Inference with Horn clauses** can be done through the forward-chaining and backward chaining algorithms
3. **Deciding entailment** with Horn clauses can be done in time that is linear in the size of the knowledge base

Propositional Theorem Proving

Forward and backward chaining

Inference engine

- It applies logical rules to the knowledge base to infer new information from known facts.
- Inference engine commonly proceeds in two modes:
 1. Forward chaining
 2. Backward chaining



Propositional Theorem Proving

Forward chaining

- Forward chaining **starts with the available data** and uses inference rules to extract more data until a goal is reached.
- **“What will happen next?”**
- It is a **bottom up** approach.
- Forward chaining is **data driven** because the reasoning starts from a set of data.



- Example

It is raining.
If it is raining, the street is wet.
The street is wet.

Propositional Theorem Proving

Backward chaining

- Backward chaining **starts from goal** and proceeds backward to determine the set of rules that match the goal.
- **“Why did this happen?”**
- It is **top down** approach.
- Backward chaining is **goal driven**
- **Example:** diagnose bacterial infections.



Example :

Patient -> Fever (Conclusion)

Temperature/Symptoms(Rules)

viral/bacterial Infection (fact)

• Example:

The street is wet.

If it is raining, the street is wet.

It is raining.

Propositional Theorem Proving

Forward chaining algorithm

function PL-FC-ENTAILS?(*KB*, *q*) **returns** *true* or *false*

inputs: *KB*, the knowledge base, a set of propositional definite clauses

q, the query, a proposition symbol

count \leftarrow a table, where *count*[*c*] is the number of symbols in *c*'s premise

inferred \leftarrow a table, where *inferred*[*s*] is initially *false* for all symbols

agenda \leftarrow a queue of symbols, initially symbols known to be true in *KB*

while *agenda* is not empty **do**

p \leftarrow POP(*agenda*)

if *p* = *q* **then return** *true*

if *inferred*[*p*] = *false* **then**

inferred[*p*] \leftarrow *true*

for each clause *c* in *KB* where *p* is in *c.PREMISE* **do**

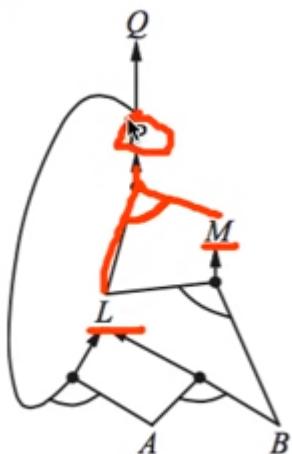
 decrement *count*[*c*]

if *count*[*c*] = 0 **then add** *c.CONCLUSION* to *agenda*

return *false*

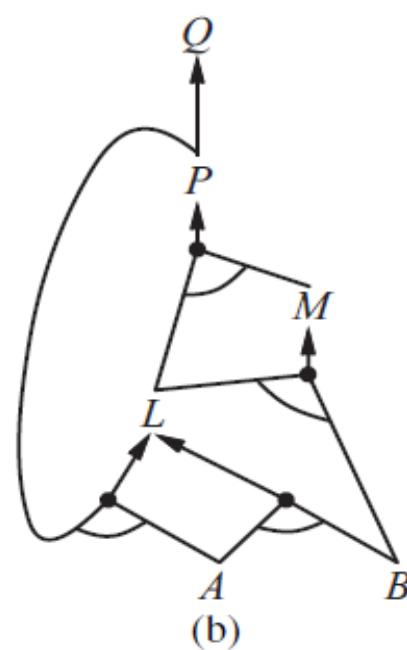
Propositional Theorem Proving

In **Forward chaining algorithm** knowledge base is drawn as an AND–OR graph



$$\begin{aligned}P &\Rightarrow Q \\L \wedge M &\Rightarrow P \\B \wedge L &\Rightarrow M \\A \wedge P &\Rightarrow L \\A \wedge B &\Rightarrow L \\A \\B\end{aligned}$$

(a)



(b)

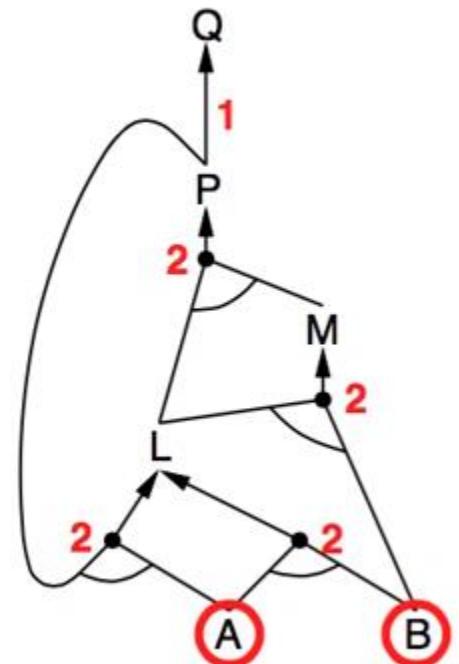
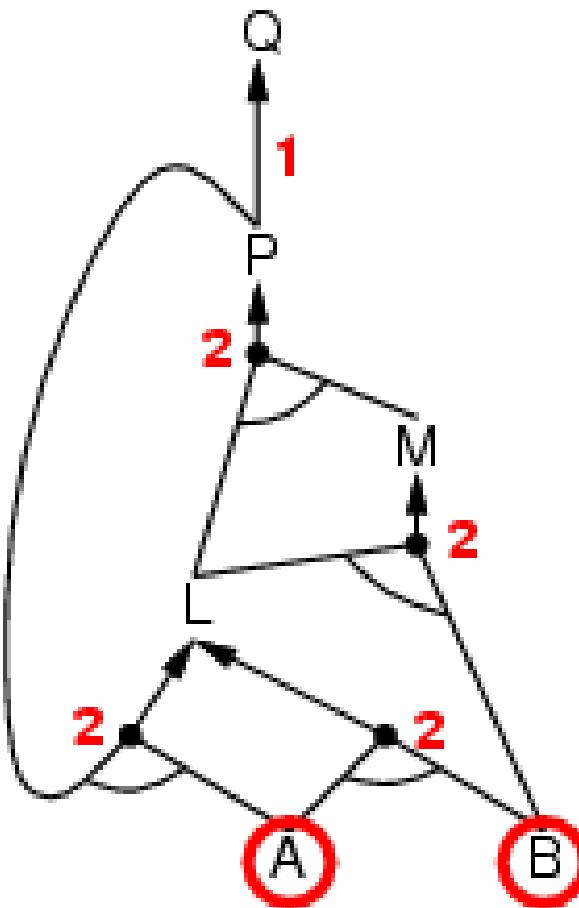
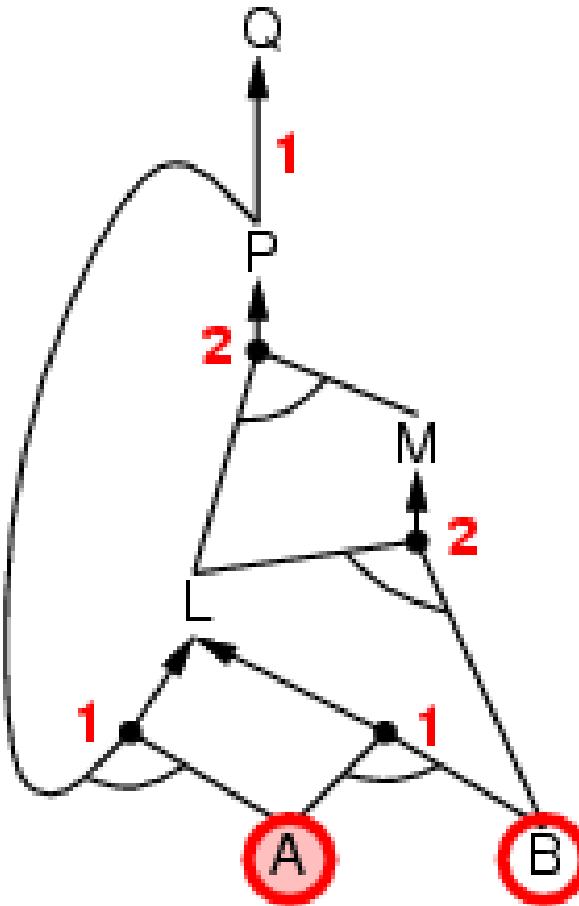


Figure 7.16 (a) A set of Horn clauses. (b) The corresponding AND–OR graph.

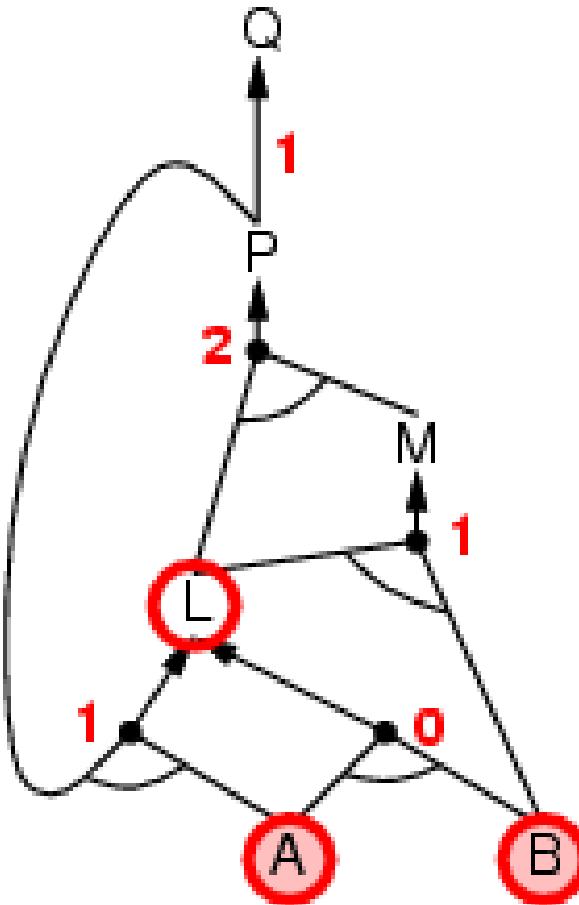
Forward chaining example



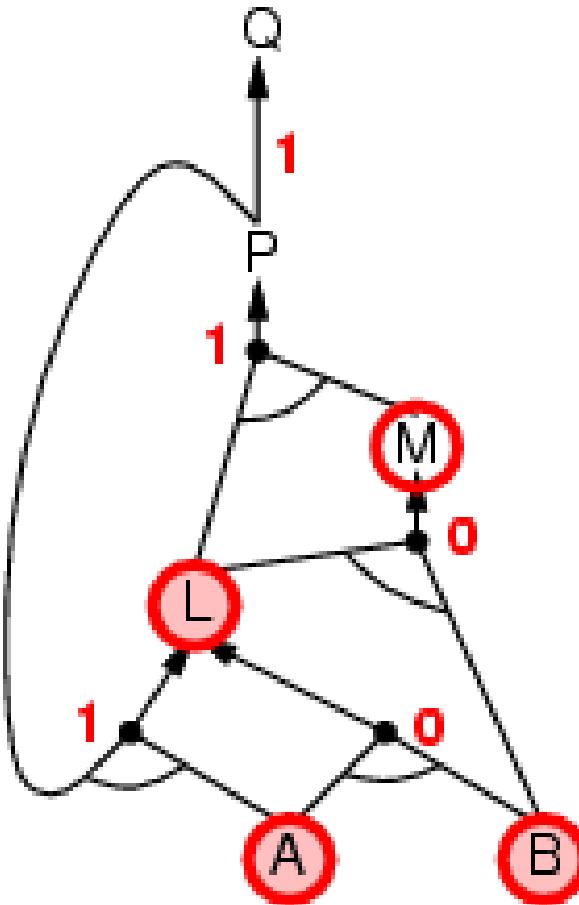
Forward chaining example



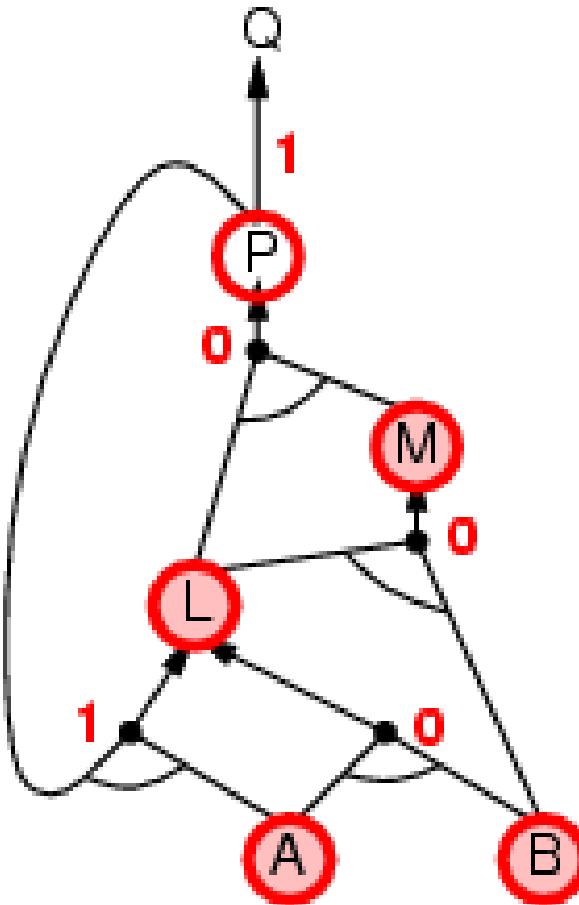
Forward chaining example



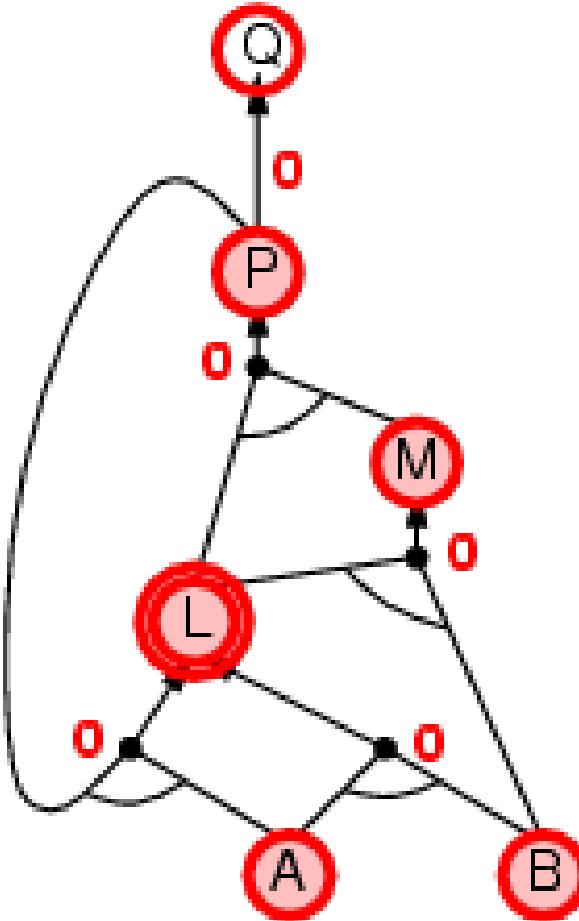
Forward chaining example



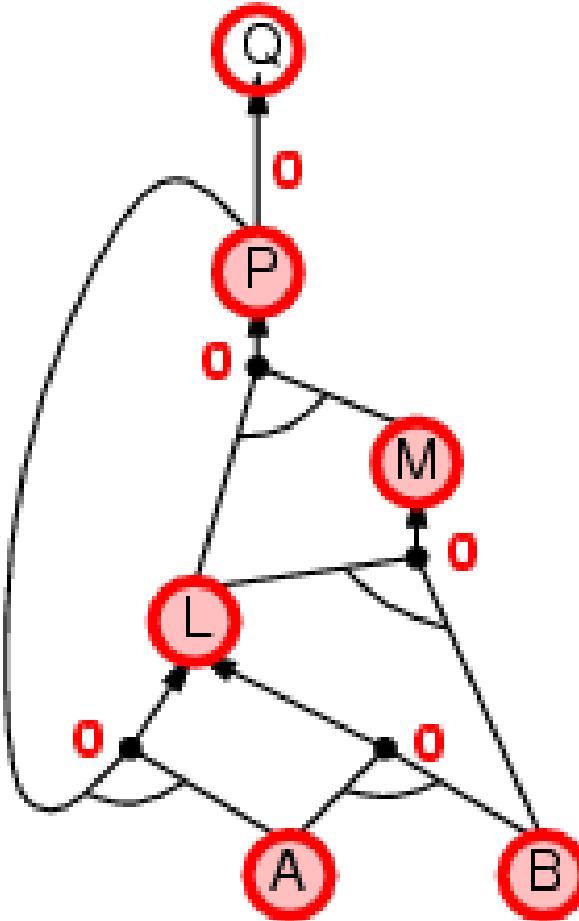
Forward chaining example



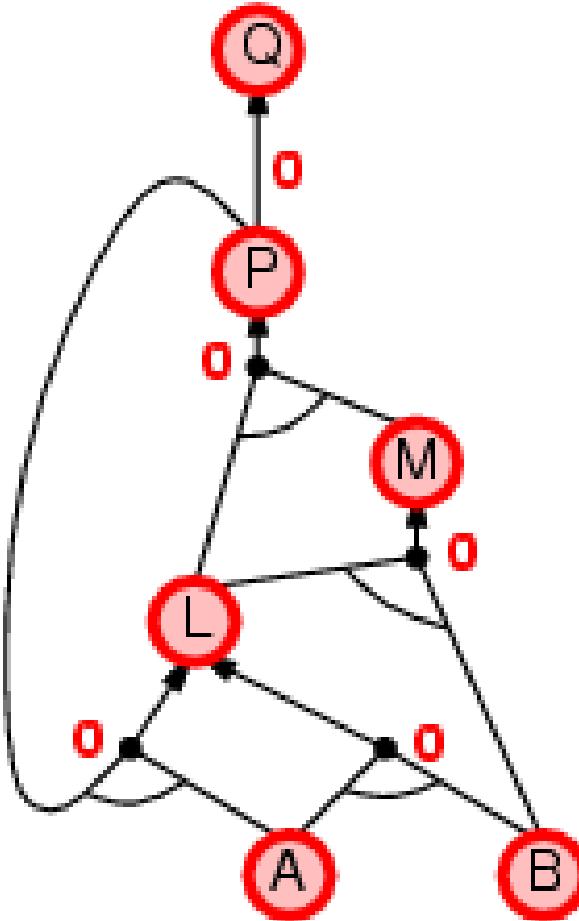
Forward chaining example



Forward chaining example



Forward chaining example



Outline

First Order logic

- Representation revisited
- Syntax and semantics of first-order logic
- Using first-order logic
- Knowledge engineering in first-order logic

Introduction

- In Propositional logic, we represent statements using propositional logic.
- But unfortunately, in propositional logic, we can only represent the facts, which are either true or false.
- PL is not sufficient to represent the complex sentences or natural language statements.
- The propositional logic has very limited expressive power.
- Consider the following sentence, which we cannot represent using PL logic.
"Some humans are intelligent"
"Sachin likes cricket."
- To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

If there is a cream, then I will drink coffee.

First Order logic

- First-order logic is another way of knowledge representation in artificial intelligence
- First order logic is an extension of PL
- It is also known as **Predicate Logic.**
- First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.

First Order logic

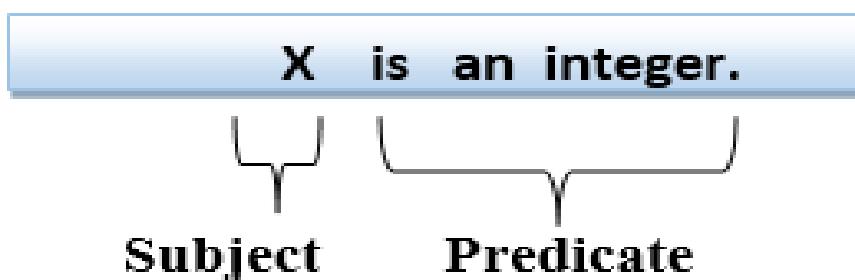
- FOL is a collection of objects, their attributes and relations among them to represent knowledge.
- **Objects:** houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .
- **Relations:** can be
 - unary relations or properties such as red, round, bogus, prime, multistoried
 - n-ary relations such as brother of, bigger than, part of, has color, occurred after, comes between, .
- **Function:** Father of, best friend, third inning of, end of,

First Order logic

First-order logic statements can be divided into two parts:

Subject: Subject is the main part of the statement.

Predicate: A predicate can be defined as a relation, which binds two atoms together in a statement



First Order logic

We write statements in short-hand notation in FOL.

x is an integer
Integer (x)

Pinky is a cat
cat(x)

First Order logic

- FOL has two main parts
 - Syntax
 - Semantics

The syntax of FOL determines which collection of symbols is a logical expression in first-order logic.

First Order logic

Syntax of FOL: Basic elements

Constant	1, 2, A, John, Mumbai, cat,....
Variables	x, y, z, a, b,....
Predicates	Brother, Father, >,....
Function	sqrt, LeftLegOf,....
Connectives	\wedge , \vee , \neg , \Rightarrow , \Leftrightarrow
Equality	$=$
Quantifier	\forall , \exists

```

Sentence → AtomicSentence | ComplexSentence
AtomicSentence → Predicate | Predicate(Term, ...) | Term = Term
ComplexSentence → ( Sentence ) | [ Sentence ]
                    |  $\neg$  Sentence
                    | Sentence  $\wedge$  Sentence
                    | Sentence  $\vee$  Sentence
                    | Sentence  $\Rightarrow$  Sentence
                    | Sentence  $\Leftrightarrow$  Sentence
                    | Quantifier Variable,... Sentence

Term → Function(Term, ...)
        | Constant
        | Variable

Quantifier →  $\forall$  |  $\exists$ 
Constant → A |  $X_1$  | John | ...
Variable → a | x | s | ...
Predicate → True | False | After | Loves | Raining | ...
Function → Mother | LeftLeg | ...

OPERATOR PRECEDENCE :  $\neg$ ,  $=$ ,  $\wedge$ ,  $\vee$ ,  $\Rightarrow$ ,  $\Leftrightarrow$ 

```

Figure 8.3 The syntax of first-order logic with equality, specified in Backus–Naur form

First Order logic

Atomic sentences:

Atomic sentences are the most basic sentences of first-order logic.

These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.

We can represent atomic sentences as **Predicate (term1, term2, , term n).**

Example: Richard and John are Brothers

Brother (Richard , John)

Complex Sentences:

Complex sentences are made by combining atomic sentences using connectives.

Example : King(Richard) V King(John)

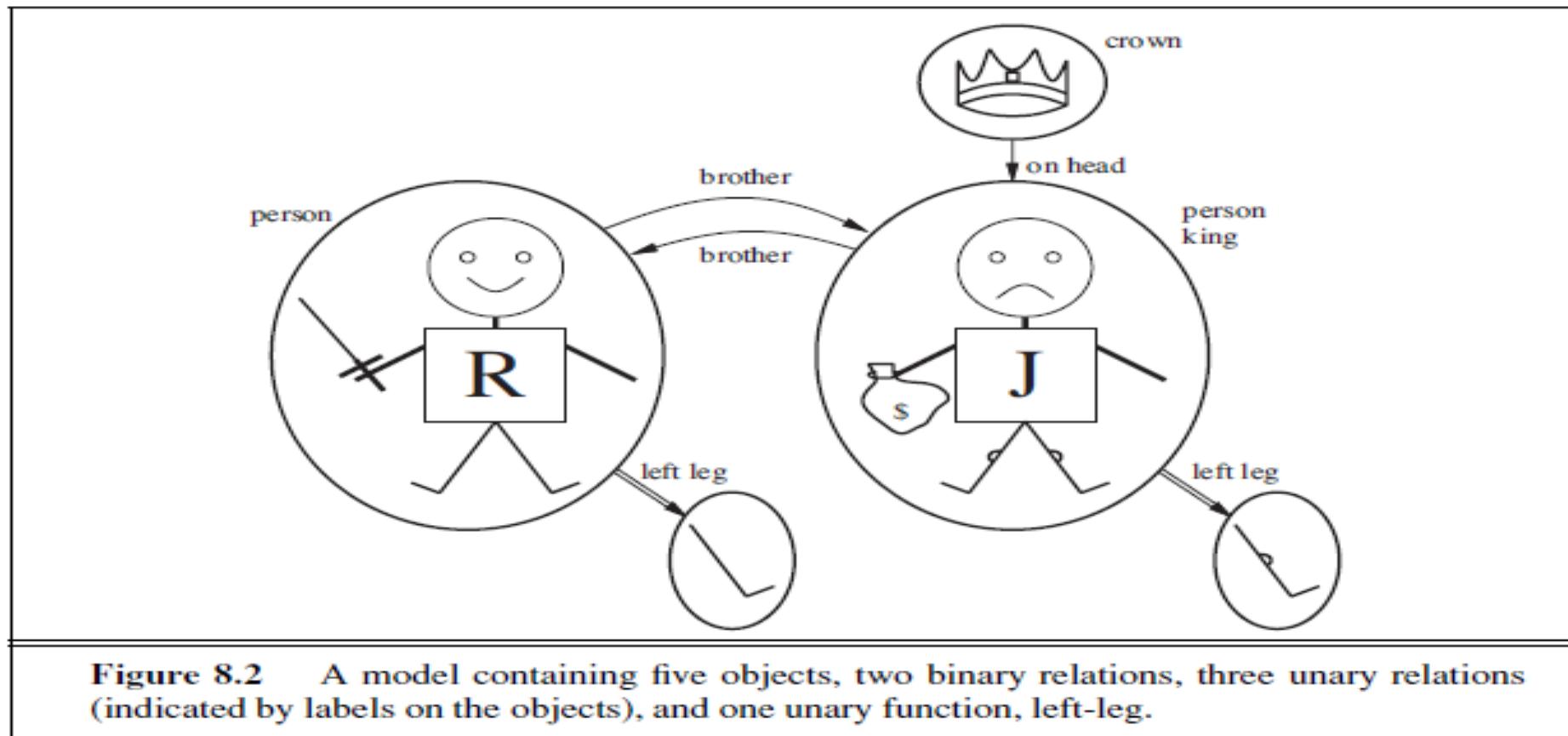
First Order logic

Terms

- A term is a logical expression that refers to an object.
- Constant symbols are therefore terms
- A complex term is formed by a function symbol followed by a parenthesized list of terms as arguments to the function symbol.
- $f(t_1, \dots, t_n)$, f refers to some function in the model

Example : $\text{LeftLeg}(\text{John})$

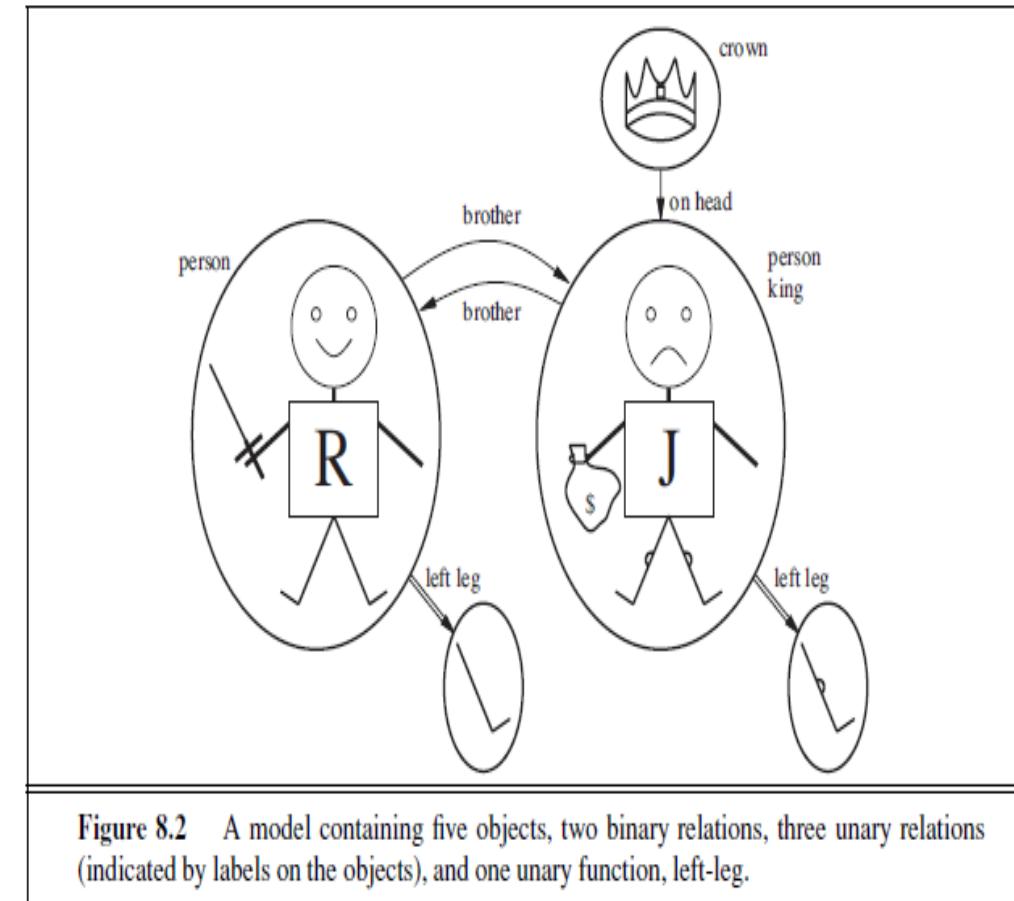
First Order logic - Example



First Order logic

Each model includes an interpretation that specifies exactly which objects, relations and functions are referred to by the constant, predicate, and function symbols.

- Richard refers to Richard the Lionheart and John refers to the evil King John.
- Brother refers to the brotherhood relation
- LeftLeg refers to the “left leg” function



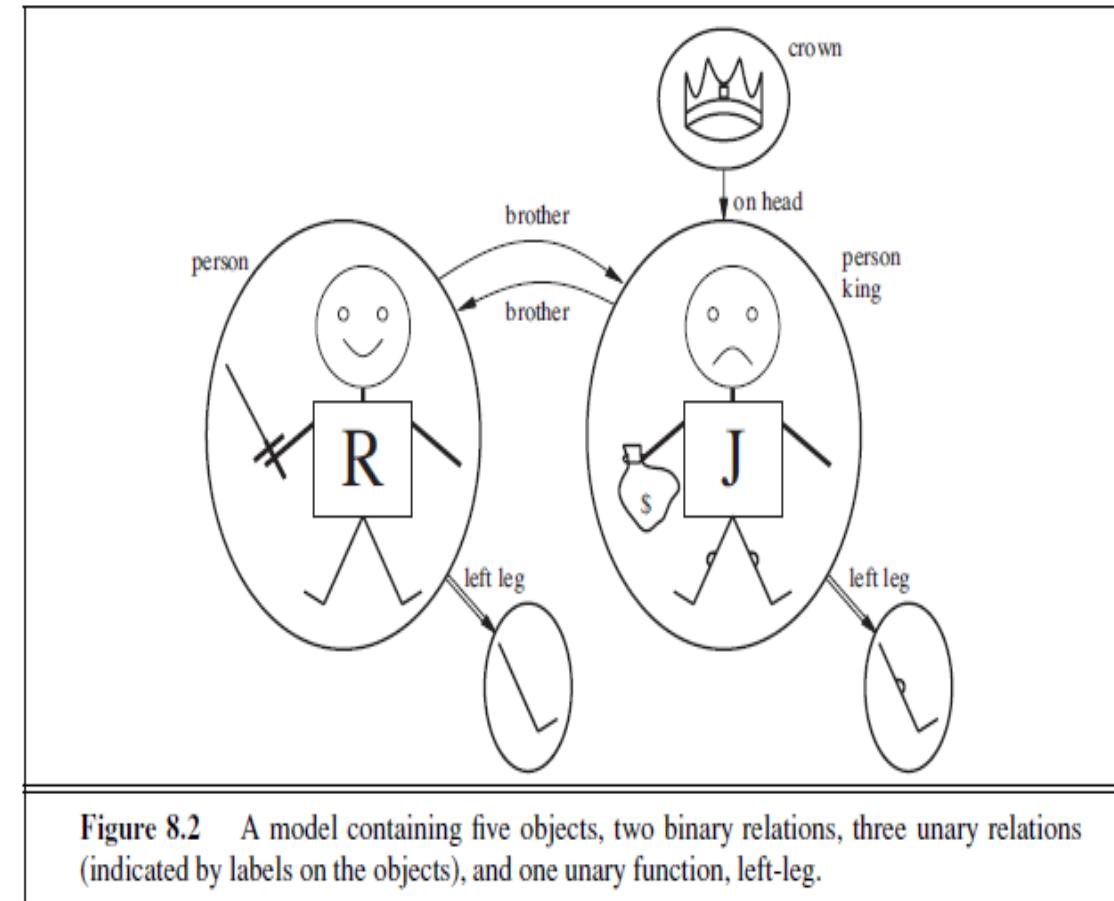
First Order logic - Example

Object

1. Richard the Lionheart, King of England from 1189 to 1199;
2. his younger brother, the evil King John, who ruled from 1199 to 1215;
3. the left leg of Richard
4. the left leg of John
5. Crown

A tuple is a collection of objects arranged in a fixed order and is written with angle brackets surrounding the objects.)

<Richard the Lionheart, King John>



First Order logic

The symbols come in three kinds:

1. Constant symbols which stand for objects
2. Predicate symbols which stand for relations
3. Function symbols which stand for functions

We adopt the convention that these symbols will begin with uppercase letters.

First Order logic

Constant symbols Richard and John

Predicate symbols Brother , OnHead,
Person, King, and Crown

Function symbol LeftLeg.

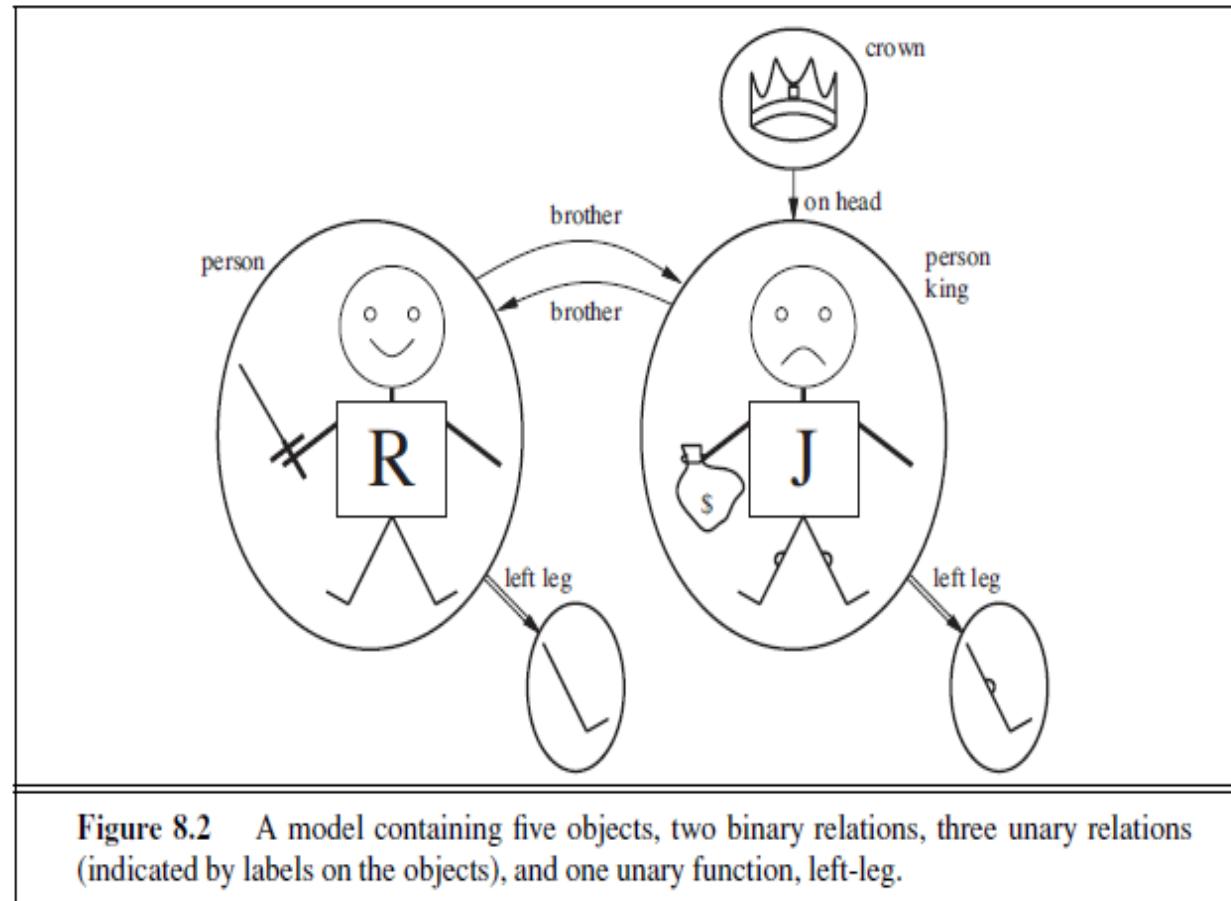


Figure 8.2 A model containing five objects, two binary relations, three unary relations (indicated by labels on the objects), and one unary function, left-leg.

Example: Write statement in PL

1. Marcus was a man

man(marcus)

2. X is an integer

integer(x)

3. Caesor was a rular

rular(caesor)

Example: Write statement in PL

4.Ravi and Rama are brothers

brothers(ravi,rama)

5. Marcus tried to assassinate Caesar

trytoassassinate(marcus,Caesar)

First Order logic

Quantifiers in First-order logic

A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.

These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression.

There are two types of quantifier:

- **Universal Quantifier(for all, everyone, everything)**
- **Existential quantifier(for some, at least one).**

First Order logic

Universal Quantifier

Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.

The Universal quantifier is represented by a **symbol \forall** , which resembles an inverted A.

The main connective for universal quantifier \forall is implication \Rightarrow .

Example : “All kings are persons”

$\forall x \text{ King}(x) \Rightarrow \text{Person}(x)$

For all x, if x is a king, then x is a person.

First Order logic

Universal Quantifier

Example : “All man drink coffee”

$$\forall x \text{ man}(x) \Rightarrow \text{drink}(x, \text{coffee})$$

For all x, if x is a man, then x drink coffee.

The symbol x is called a variable.

By convention, variables are lowercase letters.

A **variable is a term** all by itself, and as such can also serve as the argument of a function

First Order logic

Existential Quantifier

Existential quantifiers express that the statement within its scope is true for at least one instance of something.

- It is denoted by the logical operator \exists , which resembles as inverted E.
- When it is used with a predicate variable then it is called as an existential quantifier.
- The main **connective** for existential quantifier \exists is and \wedge .
- Example : King John has a crown on his head

$$\exists x \text{ Crown}(x) \wedge \text{OnHead}(x, \text{John})$$

$\exists x$ is pronounced “There exists an x such that . . .” or “For some x . . .”.

First Order logic

Nested quantifiers

- We will often want to express more complex sentences using multiple quantifiers.
- written $\exists!$ or $\exists!$, that means “**There exists exactly one**”

For example, “Brothers are siblings”

$$\forall x \forall y \text{Brother}(x, y) \Rightarrow \text{Sibling}(x, y)$$

First Order logic

Equality

First-order logic includes one more way to make atomic sentences

Equality symbol signify that two terms refer to the same object.

It can also be used with negation to insist that two terms are not the same object.

For example,

Father (John)=Henry

Using First Order logic

Assertions and queries in first-order logic

Sentences are added to a knowledge base using TELL, exactly as in propositional logic. Such sentences are called **assertions**.

For example, we can assert that

John is a king, Richard is a person, and all kings are persons

- TELL(KB, King(John)) .
- TELL(KB, Person(Richard)) .
- TELL(KB, $\forall x \text{ King}(x) \Rightarrow \text{Person}(x)$)

Using First Order logic

Assertions and queries in first-order logic

We can ask questions of the knowledge base using ASK.

Questions asked with ASK are called **queries or goals**.

For example, **ASK(KB, King(John))**

Example: Write statement in PL

1. All man drink coffee

$$\forall x : \text{man}(x) \rightarrow \text{drink}(x, \text{coffee})$$

2. Every may respects his parent

$$\forall x : \text{man}(x) \rightarrow \text{respects}(x, \text{parent})$$

3. Some boys play cricket

$$\exists x : \text{boys}(x) \wedge \text{play}(x, \text{cricket})$$

Example : Write statement in PL

4. Not all students like both maths and science

$$\sim \forall x : \text{students}(x) \rightarrow \text{like}(x, \text{maths}) \wedge \text{like}(x, \text{science})$$

5. Everyone is loyal to Someone

$$\forall x \exists y \text{loyal}(x, y)$$

6. Everyone loves everyone

$$\forall x \exists y \text{loves}(x, y)$$

Example : Write statement in PL

7. “All boys like cricket”

$\forall x : \text{boys}(x) \rightarrow \text{like}(x, \text{cricket})$

8. “Some boys like cricket”

$\exists x : \text{boys}(x) \wedge \text{like}(x, \text{cricket})$

Using First Order logic

In knowledge representation,

A **domain** is some part of the world about which we wish to express some knowledge.

Domains of

- Family relationships(The kinship domain)
- Wumpus world

Using First Order logic

Kinship domain(family relationship)

It consists of

- Object – People
- Unary Predicates - Male and Female
- Binary Predicates - Parent,Brother,Sister
- Functions – Father, Mother
- Relations – Brotherhood, sisterhood.

Using First Order logic

The kinship domain

- An example KB includes things like:
 - Fact:
 - "Elizabeth is the mother of Charles"
 - "Charles is the father of William"
 - Rules:
 - One's grandmother is the mother of one's parent"
- Object: people
- Unary predicate: Male, Female
- Binary predicate: Son, Spouse, Wife, Husband, Grandparent, Grandchild, Cousin, Aunt, and Uncle
- Function: Mother, Father

Using First Order logic

Examples

The kinship domain:

- Brothers are siblings
 $\forall x, y \ Brother(x, y) \Rightarrow \text{Sibling}(x, y)$
- Male and female are disjoint categories
 $\forall x, \text{Male}(x) \Leftrightarrow \neg \text{Female}(x)$
- Parent and child are inverse relations
 $\forall p, c \ \text{Parent}(p, c) \Leftrightarrow \text{Child}(c, p)$

Using First Order logic

The Wumpus World

- A percept is a binary predicate:
 - Percept([Stench, Breeze, Glitter, None, None], 5)
- Actions are logical terms:
 - Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb
- Query:
 - $\exists a \text{ BestAction}(a, 5)$
 - Answer: {a/Grab}
- Perception: percept implies facts:
 - $\forall t, s, b, m, c \text{ Percept}([s, b, \text{Glitter}, m, c], t) \Rightarrow \text{Glitter}(t)$
- Reflex behavior:
 - $\forall t \text{ Glitter}(t) \Rightarrow \text{BestAction}(\text{Grab}, t)$

Using First Order logic

The Wumpus World

- The Environment:
 - Objects: squares, pits and the wumpus
 - $\forall x,y,a,b \text{Adjacent}([x,y],[a,b]) \Leftrightarrow [a,b] \in \{[x+1,y], [x-1,y], [x,y+1], [x,y-1]\}$
 - A unary predicate *Pit*(x)
 - $\forall s,t \text{At}(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$, the agent infers properties of the square from properties of its current percept
 - *Breezy()* has no time argument
 - Having discovered which places are breezy or smelly, not breezy or not smelly, the agent can deduce where the pits are and where the wumpus is

Knowledge engineering in first-order logic

- Knowledge engineering is a field of study where we do the engineering of all thought processes for specific domains.
- The general process of constructing knowledge base is called knowledge engineering
- A knowledge engineer investigates a
 - Particular domain
 - learns what concepts are important in that domain
 - creates a formal representation of the objects and relations in the domain

Knowledge engineering in first-order logic

Steps in the knowledge-engineering process

1. Identify the task – Figure out the range of questions that can be asked
2. Assemble the relevant knowledge – requirement analysis
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

Knowledge engineering in first-order logic

Steps in the knowledge-engineering process

1. Identify the task

- Goal formulation step

e.g. In crime mystery solution , the task

- Find the motive
- Find all the suspects
- List all victims
- Find out all the lies the culprit has stated

- Does wumpus kb need to be able to choose actions? Answer questions about the environment? Provide for location of the agent?
- Analogous to the PEAS process.

Knowledge engineering in first-order logic

Steps in the knowledge-engineering process

2. Assemble the relevant knowledge

- Collect common sense knowledge of model
 - E.g. in crime mystery story
 - what are typical crime motives,
 - how the crime stories are solved,
 - how the world around the victims and culprits work etc.
- Get an idea of the KB scope.
 - What are the “rules” of the world?
 - In terms of wumpus world, easy. Other domains, much harder.

Knowledge engineering in first-order logic

3. Decide on a vocabulary of predicates, functions, and constants

E.g.

- `person(name, age, sex, occupation)`
- `had_affair(name, name)`
- `killed_with(name, object)`
- `killed(name)`
- `killer(name)`
- `motive(vice)`
- `smeared_in(name, substance)`
- `owns(name, object)`
- `operates_identically(object, object)`
- `owns_probably(name, object)`
- `suspect(name)`

- Should pits be objects or unary predicates?
- Should agent orientation be function or predicate?
- Should wumpus location depend on time?

Knowledge engineering in first-order logic

4. Encode general knowledge about the domain

- Record all axioms for all the vocabulary terms.
- Tries to specify meanings of terms, allowing the expert to verify content.
- Can elaborate on issues that need to be fixed by repeating the previous step.

Knowledge engineering in first-order logic

5. Encode a description of the specific problem instance

- Simply write simple atomic sentences about instances of concepts already part of the ontology.
- Problem instances supplied to logical agents through sensors.
- “Disembodied” knowledge bases are given additional sentences like other programs get input.

Knowledge engineering in first-order logic

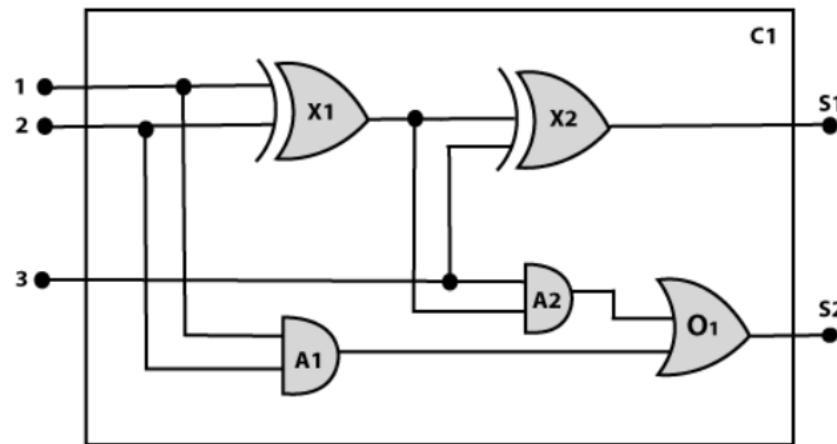
6. Pose queries to the inference procedure and get answers

7. Debug the knowledge base

Knowledge engineering in first-order logic

Steps in the knowledge-engineering process

Develop a knowledge base which will allow us to reason about digital circuit (One-bit full adder) which is given below



Knowledge engineering in first-order logic

Steps in the knowledge-engineering process

1. Identify the task

At the first level, we will examine the functionality of the circuit:

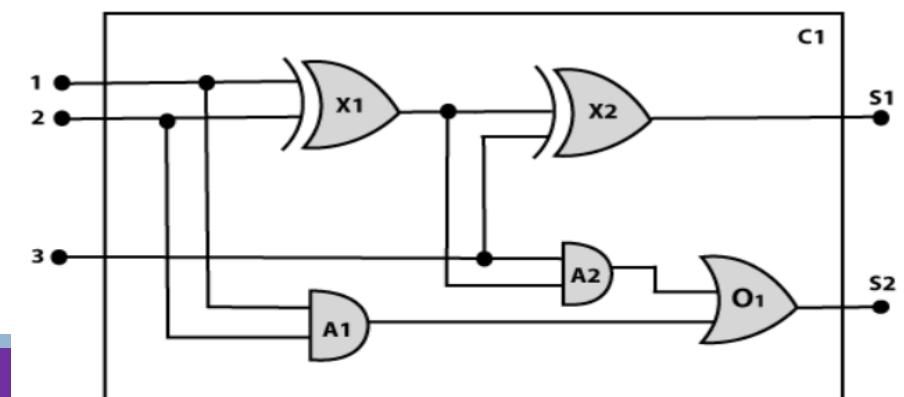
Does the circuit add properly?

What will be the output of gate A2, if all the inputs are high?

At the second level, we will examine the circuit structure details such as:

Which gate is connected to the first input terminal?

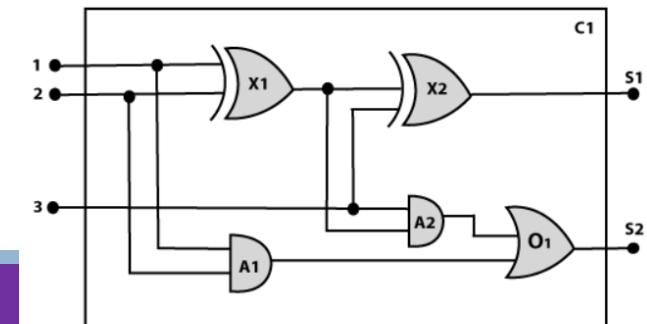
Does the circuit have feedback loops?



Knowledge engineering in first-order logic

2. Assemble the relevant knowledge

- Logic circuits are made up of wires and gates.
- Signal flows through wires to the input terminal of the gate, and each gate produces the corresponding output which flows further.
- In this logic circuit, there are four types of gates used: AND, OR, XOR, and NOT.
- All these gates have one output terminal and two input terminals (except NOT gate, it has one input terminal).



Knowledge engineering in first-order logic

3. Decide on a vocabulary of predicates, functions, and constants

Select functions, predicate, and constants to represent the circuits, terminals, signals, and gates.

Each **gate** is represented as an object which is named by a constant, **Gate(X1)**.

Circuits will be identified by a predicate: **Circuit (C1)**
 Terminal(x)

Objects two **signal values 1 and 0**, and a function **Signal(t)** that denotes the signal value for the terminal t.

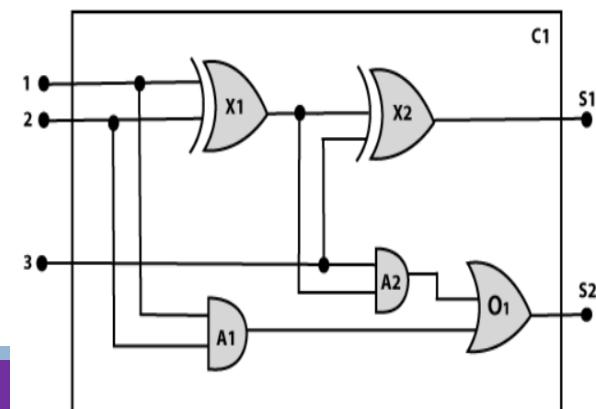
The **functionality of each gate is determined by its type**, which is taken as constants such as **AND, OR, XOR, or NOT**.

For gate input, we will use the function

In(1, X1) - first input terminal of the gate

Out (1, X1) - output terminal

The connectivity between gates can be represented by predicate **Connect(Out(1, X1), In(1, X1))**.



Knowledge engineering in first-order logic

4. Encode general knowledge about the domain

To encode the general knowledge about the logic circuit, we need some following rules:

- If two terminals are connected then they have the same input signal, it can be represented as:

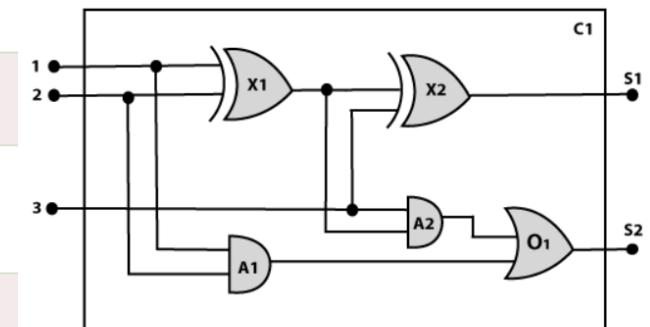
$$\forall t_1, t_2 \text{ Terminal}(t_1) \wedge \text{Terminal}(t_2) \wedge \text{Connect}(t_1, t_2) \rightarrow \text{Signal}(t_1) = \text{Signal}(t_2).$$

- Signal at every terminal will have either value 0 or 1, it will be represented as:

$$\forall t \text{ Terminal}(t) \rightarrow \text{Signal}(t) = 1 \vee \text{Signal}(t) = 0.$$

- Connect predicates are commutative:

$$\forall t_1, t_2 \text{ Connect}(t_1, t_2) \rightarrow \text{Connect}(t_2, t_1).$$



Knowledge engineering in first-order logic

5. Encode a description of the specific problem instance

This step write simple atomics sentences of instances of concepts

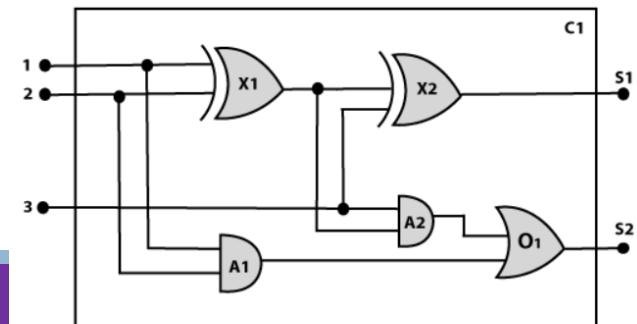
For the given circuit C1, we can encode the problem instance in atomic sentences :

In circuit there are two XOR, two AND, and one OR gate so atomic sentences for these gates will be:

For XOR gate: Type(x1)= XOR, Type(X2) = XOR

For AND gate: Type(A1) = AND, Type(A2)= AND

For OR gate: Type (O1) = OR.



Knowledge engineering in first-order logic

6.Pose queries to the inference procedure and get answers

Find all the possible set of values of all the terminal for the adder circuit.

The first query will be: What should be the combination of input which would generate the first output of circuit C1 as 0 and a second output to be 1?

$$\exists i_1, i_2, i_3 \text{ Signal (In}(1, C1)\text{)=}i_1 \wedge \text{Signal (In}(2, C1)\text{)=}i_2 \wedge \text{Signal (In}(3, C1)\text{)= } i_3 \\ \wedge \text{Signal (Out}(1, C1)\text{)=}0 \wedge \text{Signal (Out}(2, C1)\text{)=}1$$

Knowledge engineering in first-order logic

7.Debug the knowledge base

In this step, we will try to debug the issues of knowledge base.

Outline

Inference in First Order logic

- Propositional vs. First-order inference
- Unification
- Forward chaining
- Backward chaining
- Resolution

Propositional vs. First-order inference

<i>Ontology</i>	Facts (P, Q)	Objects, Properties, Relations
<i>Syntax</i>	Atomic sentences Connectives	Variables & quantification Sentences have structure: terms father-of(mother-of(X)))
<i>Semantics</i>	Truth Tables	Interpretations (Much more complicated)
<i>Inference Algorithm</i>	DPLL, GSAT Fast in practice	Unification Forward, Backward chaining Prolog, theorem proving
<i>Complexity</i>	NP-Complete	Semi-decidable

Propositional vs. First-order inference

Substitutions

SUBST(θ, α) denote the result of applying the substitution θ to the sentence α .

Substitution

A substitution is a Finite set $\{t_1/v_1, \dots, t_n/v_n\}$

where v_i is a variable

t_i is a term, different from v_i ,

and no two elements in the set have the same variable after the / symbol.

$\{f(z)/x, y/z\}$ is a substitution

$\{a/x, g(y)/y, f(g(b))/z\}$ is a substitution

$\{y/x, g(b)/y\}$ is a substitution

$\{a/x, g(y)/x, f(g(b))/z\}$ is not a substitution

$\{g(y)/x, z/f(g(b))\}$ is not a substitution

Propositional vs. First-order inference

Inference rules for quantifiers

- Universal Generalization
- Universal Instantiation
- Existential Instantiation
- Existential introduction

Propositional vs. First-order inference

Universal Generalization

- It is a valid inference rule which states that if premise $P(c)$ is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as $\forall x P(x)$.

$$\frac{P(c)}{\forall x P(x)}$$

Example

Let's represent, $P(c)$: "**A byte contains 8 bits**",
so for $\forall x P(x)$ "**All bytes contain 8 bits.**", it will also be true.

Propositional vs. First-order inference

Universal Instantiation/elimination

It is a valid inference rule, **It can be applied multiple times** to add new sentences. As per UI, we can infer any sentence obtained by substituting a ground term for the variable.

The UI rule state that we can infer any sentence $P(c)$ by substituting a ground term c (a constant within domain x) from $\forall x P(x)$ for any object in the universe of discourse.

It can be represented as:

$$\frac{\forall x P(x)}{P(c)}$$

Propositional vs. First-order inference

Universal Instantiation/elimination

It can be represented as:

$$\frac{\forall x P(x)}{P(c)}$$

Example 1:

IF "Every person like ice-cream" $\Rightarrow \forall x P(x)$

so we can infer that "John likes ice-cream" $\Rightarrow P(c)$

Propositional vs. First-order inference

Universal Instantiation/elimination

Example 2:

$$\frac{\forall x P(x)}{P(c)}$$

"All kings who are greedy are Evil"

$$\forall x \text{ king}(x) \wedge \text{greedy}(x) \rightarrow \text{Evil}(x)$$

we can infer any of the following statements using Universal Instantiation:

$$\text{King(John)} \wedge \text{Greedy(John)} \rightarrow \text{Evil(John)}$$

$$\text{King(Richard)} \wedge \text{Greedy(Richard)} \rightarrow \text{Evil(Richard)}$$

Propositional vs. First-order inference

Existential Instantiation/Elimination

- It is a valid inference rule in first-order logic, It **can be applied only once to replace** the existential sentence.
- The new KB is not logically equivalent to old KB, but it will be satisfiable if old KB was satisfiable.
- This rule states that one can infer $P(c)$ from the formula given in the form of $\exists x P(x)$ for a new constant symbol c .
- The restriction with this rule is that c used in the rule must be a new term for which $P(c)$ is true.
- It can be represented as:

$$\frac{\exists x P(x)}{P(c)}$$

Propositional vs. First-order inference

Existential Instantiation/Elimination

$$\frac{\exists x P(x)}{P(c)}$$

Example

- From the given sentence: $\exists x \text{ Crown}(x) \wedge \text{OnHead}(x, \text{John})$
- So we can infer: $\text{Crown}(K) \wedge \text{OnHead}(K, \text{John})$, as long as K does not appear in the knowledge base.
- The above used K is a constant symbol, which is called Skolem constant.
- The Existential instantiation is a special case of Skolemization process.

Propositional vs. First-order inference

Existential introduction/generalization

- It is a valid inference rule in first-order logic.
- This rule states that if there is some element c in the universe of discourse which has a property P , then we can infer that there exists something in the universe which has the property P .
- It can be represented as:

$$\frac{P(c)}{\exists x P(x)}$$

Propositional vs. First-order inference

Existential introduction

$$\frac{P(c)}{\exists x P(x)}$$

Example

"Pinky got good marks in English."

c= pinky

"Therefore, someone got good marks in English."

Lifting

Lifting was first introduced by Alan Robinson.

It is the primary innovation of the original resolution procedure.

Lifting makes those substitutions that are required to allow particular inferences to proceed

- E.g. *generalised Modus Ponens*
 - To introduce substitutions different logical expressions have to be look identical
 - **Unification**

Unification

- Unification means making expression looks identical.
- Can be done with the process of Substitution.

Example : $p(x, F(y))$ ----- 1

$p(x, F(g(z)))$ ----- 2

1 and 2 are identical if x is replaced with a
y is replaced with g(z)

[$a/x , g(z)/y$] –substitution set

$p(a, F(g(z)))$ ----- 1

$p(a, F(g(z)))$ ----- 2

Unification

Example : $p(x, F(y))$ ----- 1
 $p(x, F(g(z)))$ ----- 2

Unification conditions

1. Predicate symbol must be same
2. Number of arguments in both expressions must be identical
3. If two similar variables present in same expression, then unification fails

Unification

$$\begin{array}{ll} \text{Example : } & Q(a, g(x,a), f(y)) \\ & Q(a, g(f(b) ,a),x) \end{array} \quad \begin{array}{l} \text{----- 1} \\ \text{----- 2} \end{array}$$

- Substitute $f(b)/x$

$$Q(a, g(f(b),a), f(y)) \quad Q(a, g(f(b) ,a), f(b))$$

- Substitute b/y

$$Q(a, g(f(b),a), f(b)) \quad Q(a, g(f(b) ,a), f(b))$$

Unification

function UNIFY(x, y, θ) returns a substitution to make x and y identical

inputs: x , a variable, constant, list, or compound expression

y , a variable, constant, list, or compound expression

θ , the substitution built up so far (optional, defaults to empty)

if $\theta = \text{failure}$ then return failure

else if $x = y$ then return θ

else if VARIABLE?(x) then return UNIFY-VAR(x, y, θ)

else if VARIABLE?(y) then return UNIFY-VAR(y, x, θ)

else if COMPOUND?(x) and COMPOUND?(y) then

return UNIFY($x.\text{ARGS}, y.\text{ARGS}, \text{UNIFY}(x.\text{OP}, y.\text{OP}, \theta)$)

else if LIST?(x) and LIST?(y) then

return UNIFY($x.\text{REST}, y.\text{REST}, \text{UNIFY}(x.\text{FIRST}, y.\text{FIRST}, \theta)$)

else return failure

function UNIFY-VAR(var, x, θ) returns a substitution

if $\{var/val\} \in \theta$ then return UNIFY(val, x, θ)

else if $\{x/val\} \in \theta$ then return UNIFY(var, val, θ)

else if OCCUR-CHECK?(var, x) then return failure

else return add $\{var/x\}$ to θ

Forward chaining

Start from the known facts, it triggers all the rules whose premises are satisfied, adding their conclusions to the known facts.

The process repeats until the query is answered or no new facts are added.

- Forward chaining **starts with the available data** and uses inference rules to extract more data until a goal is reached.
- **“What will happen next?”**
- It is a **bottom up** approach.
- Forward chaining is **data driven** because the reasoning starts from a set of data.



Example: Write statement in PL

1. Sky is blue

sky → blue

2. Knife is a weapon

knife → weapon

3. John is a king

king(john)

Example :Forward chaining

"The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by ColonelWest, who is American.

Prove that "West is criminal."

Forward chaining

Convert all the facts into first-order definite clauses, and then use a forward-chaining algorithm to reach the goal.

There are some facts give here:

- It is a crime for an American to sell weapons to an enemy of America
- Country A is the enemy of America
- A colonel of the army sells missiles to A
- Missiles are weapons
- The colonel is an American citizen

Forward chaining

It is a crime for an American to sell weapons to hostile nations. (Let's say x, y, and z are variables)

“... it is a crime for an American to sell weapons to hostile nations”:

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x) .$$

Forward chaining

Nono has some missiles

There exist something in Universe, that NONO owns, and That something is missile.

“Nono . . . has some missiles.” The sentence $\exists x \text{ } Owns(\text{Nono}, x) \wedge \text{Missile}(x)$ is transformed into two definite clauses by Existential Instantiation, introducing a new constant M_1 :

$Owns(\text{Nono}, M_1)$

$\text{Missile}(M_1)$

$$\frac{\exists x \text{ } P(x)}{P(c)}$$

Forward chaining

“All of its missiles were sold to it by Colonel West”:

$$\text{Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \Rightarrow \text{Sells}(\text{West}, x, \text{Nono}) .$$

We will also need to know that missiles are weapons:

$$\text{Missile}(x) \Rightarrow \text{Weapon}(x)$$

Forward chaining

and we must know that an enemy of America counts as “hostile”:

$$\textit{Enemy}(x, \textit{America}) \Rightarrow \textit{Hostile}(x).$$

“West, who is American . . .”:

$$\textit{American}(\textit{West}).$$

“The country Nono, an enemy of America . . .”:

$$\textit{Enemy}(\textit{Nono}, \textit{America}).$$

Forward chaining

In the first step, start with the known facts and choose the sentences which do not have implications,

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x) .$$

$$Owns(Nono, M_1)$$

$$Missile(M_1)$$

$$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono) .$$

$$Missile(x) \Rightarrow Weapon(x)$$

$$Enemy(x, America) \Rightarrow Hostile(x) .$$

$$\boxed{American(West)}$$

$$\boxed{\cancel{Missile(M_1)}}$$

$$\boxed{Owns(Nono, M_1)}$$

$$\boxed{\cancel{Enemy(Nono, America)}}$$

$$American(West) .$$

$$Enemy(Nono, America) .$$

Forward chaining

At the second step, see those facts which infer from available facts and with satisfied premises.

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x).$$

$$Owns(Nono, M_1)$$

$$Missile(M_1)$$

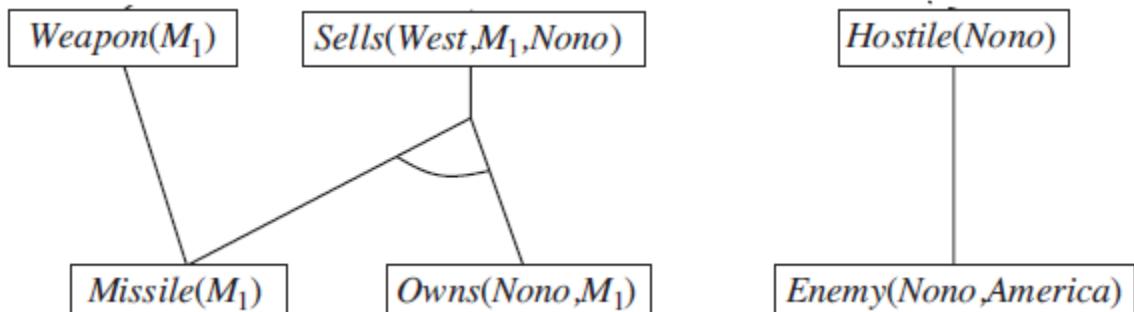
$$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono).$$

$$Missile(x) \Rightarrow Weapon(x)$$

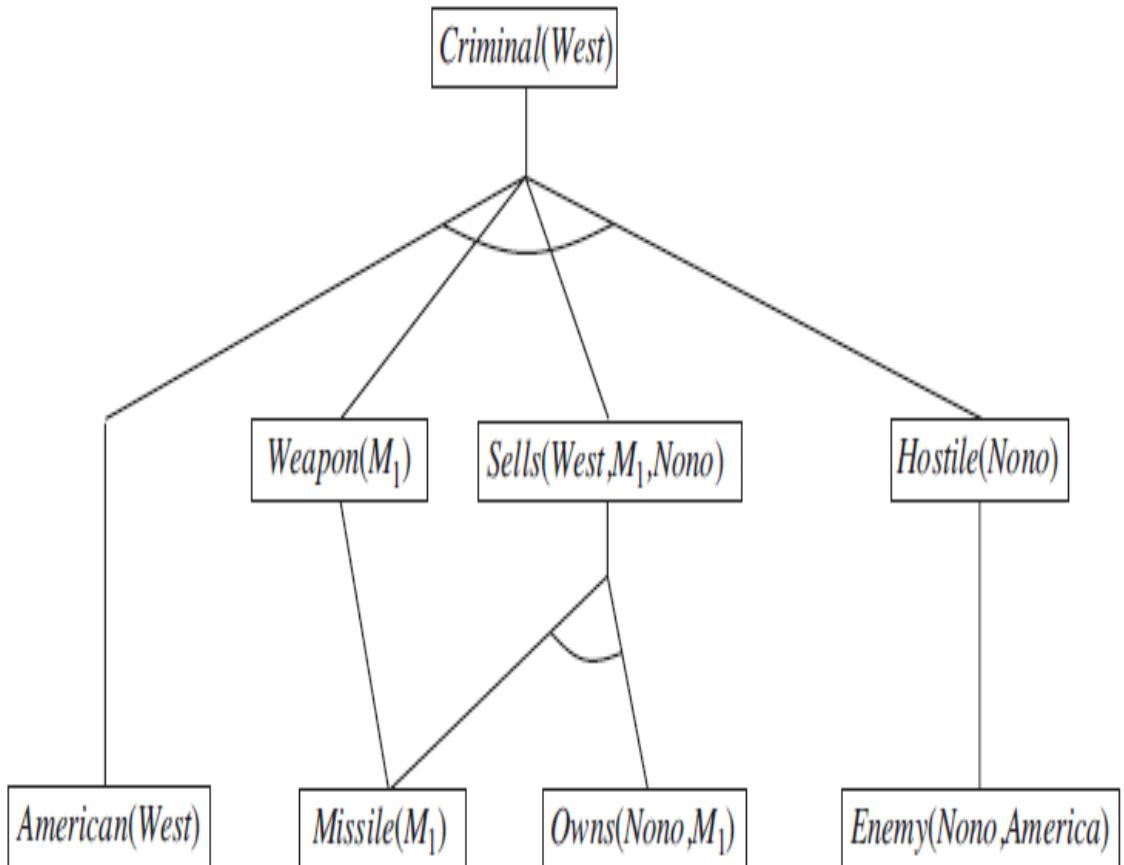
$$Enemy(x, America) \Rightarrow Hostile(x).$$

$$American(West).$$

$$Enemy(Nono, America).$$



Forward chaining



$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \wedge \text{Hostile}(z) \Rightarrow \text{Criminal}(x)$.

$\text{Owes}(\text{Nono}, M_1)$

$\text{Missile}(M_1)$

$\text{Missile}(x) \wedge \text{Owes}(\text{Nono}, x) \Rightarrow \text{Sells}(\text{West}, x, \text{Nono})$.

$\text{Missile}(x) \Rightarrow \text{Weapon}(x)$

$\text{Enemy}(x, \text{America}) \Rightarrow \text{Hostile}(x)$.

$\text{American}(\text{West})$.

$\text{Enemy}(\text{Nono}, \text{America})$.

Properties of Forward chaining

Sound and complete for first-order definite clauses

Datalog = first-order definite clauses + no functions
(e.g., crime KB)

Forward chaining

Consider the following sentences:

- 1.John likes all kinds of food
- 2.Apples are food
- 3.Chicken is food
- 4.Anything anyone eats and isn't killed by is food.
- 5.Bill eats peanut and is still alive
6. sue eats everything bill eats

- (i) Translate these sentences into formulas in predicate logic.
- (ii) Prove that john likes peanuts using forward chaining

Forward chaining

John likes all kinds of food

$\forall x : \text{food}(x) \rightarrow \text{likes}(\text{John}, x)$

Apples are food

$\text{Apples}(x) \rightarrow \text{food}(x)$

Chicken is food

$\text{Chicken}(x) \rightarrow \text{food}(x)$

Forward chaining

Anything anyone eats and isn't killed by is food

$\forall x \forall y \text{ eats}(x, y) \wedge \neg \text{killed}(y) \rightarrow \text{food}(x)$

Bill eats peanut and is still alive

$\text{eats}(\text{bill}, \text{peanut}) \wedge \neg \text{killed}(\text{bill})$

sue eats everything bill eats

$\forall x \text{ eats}(\text{bill}, x) \rightarrow \text{eat}(\text{sue}, x)$

Forward chaining

$\forall x : \text{food}(x) \rightarrow \text{likes}(\text{John}, x)$

$\text{Apples}(x) \rightarrow \text{food}(x)$ food(Apple)

$\text{Chicken}(x) \rightarrow \text{food}(x)$ food(Chicken)

$\forall x \forall y \text{ eats}(x, y) \wedge \sim \text{killed}(y) \rightarrow \text{food}(x)$

eats(bill, peanut) $\wedge \sim \text{killed}(\text{bill})$

$\forall x \text{ eats}(x, x) \rightarrow \text{eat}(\text{sue}, x)$

eats(bill, peanut)

$\sim \text{killed}(\text{bill})$

Forward chaining

$\forall x : \text{food}(x) \rightarrow \text{likes}(\text{John}, x)$

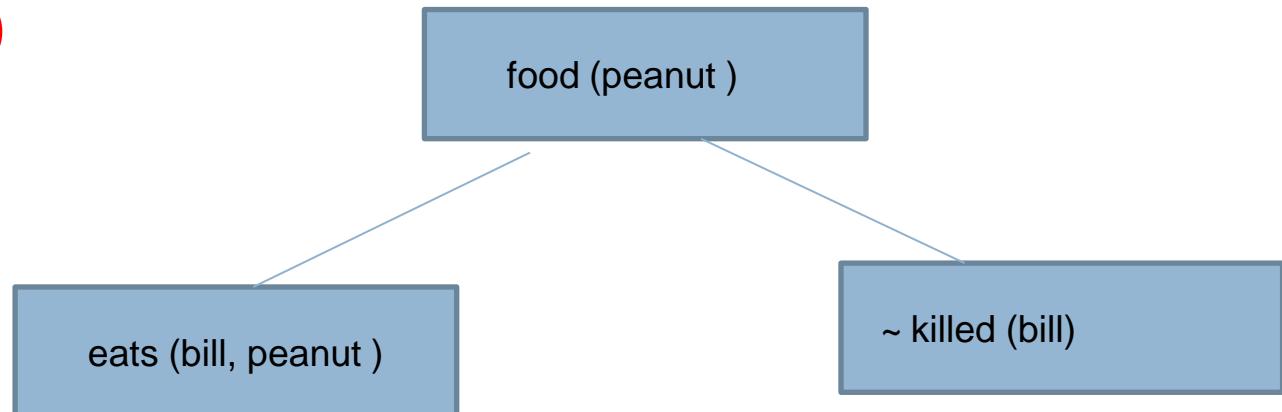
$\text{Apples}(x) \rightarrow \text{food}(x)$

$\text{Chicken}(x) \rightarrow \text{food}(x)$

$\forall x \forall y \text{ eats}(x, y) \wedge \sim \text{killed}(y) \rightarrow \text{food}(x)$

$\text{eats}(\text{bill}, \text{peanut}) \wedge \sim \text{killed}(\text{bill})$

$\forall x \text{ eats}(\text{bill}, x) \rightarrow \text{eat}(\text{sue}, x)$



Forward chaining

$\forall x : \text{food}(x) \rightarrow \text{likes}(\text{John}, x)$

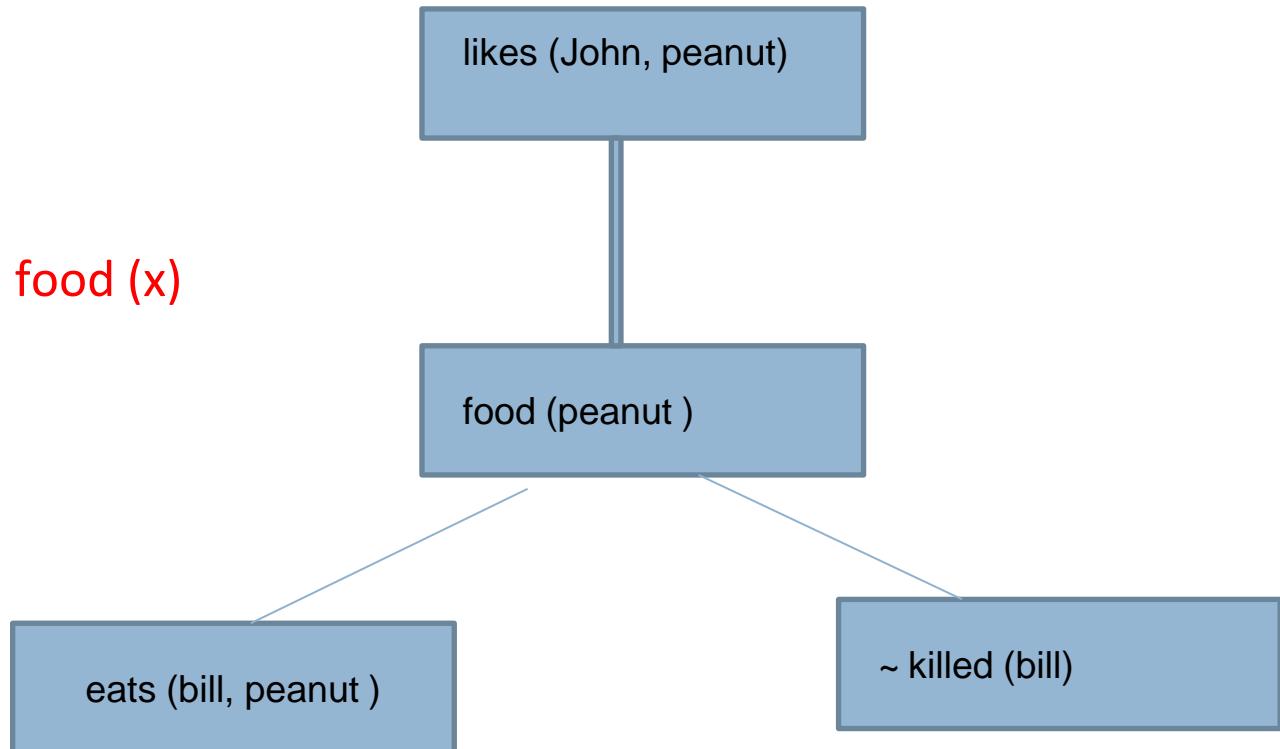
$\text{Apples}(x) \rightarrow \text{food}(x)$

$\text{Chicken}(x) \rightarrow \text{food}(x)$

$\forall x \forall y \text{ eats}(x, y) \wedge \sim \text{killed}(y) \rightarrow \text{food}(x)$

$\text{eats}(\text{bill}, \text{peanut}) \wedge \sim \text{killed}(\text{bill})$

$\forall x \text{ eats}(\text{bill}, x) \rightarrow \text{eat}(\text{sue}, x)$



Forward chaining algorithm

```
function FOL-FC-ASK( $KB, \alpha$ ) returns a substitution or false
    inputs:  $KB$ , the knowledge base, a set of first-order definite clauses
             $\alpha$ , the query, an atomic sentence
    local variables:  $new$ , the new sentences inferred on each iteration

    repeat until  $new$  is empty
         $new \leftarrow \{ \}$ 
        for each  $rule$  in  $KB$  do
             $(p_1 \wedge \dots \wedge p_n \Rightarrow q) \leftarrow \text{STANDARDIZE-VARIABLES}(rule)$ 
            for each  $\theta$  such that  $\text{SUBST}(\theta, p_1 \wedge \dots \wedge p_n) = \text{SUBST}(\theta, p'_1 \wedge \dots \wedge p'_n)$ 
                for some  $p'_1, \dots, p'_n$  in  $KB$ 
                 $q' \leftarrow \text{SUBST}(\theta, q)$ 
                if  $q'$  does not unify with some sentence already in  $KB$  or  $new$  then
                    add  $q'$  to  $new$ 
                     $\phi \leftarrow \text{UNIFY}(q', \alpha)$ 
                    if  $\phi$  is not fail then return  $\phi$ 
            add  $new$  to  $KB$ 
    return false
```

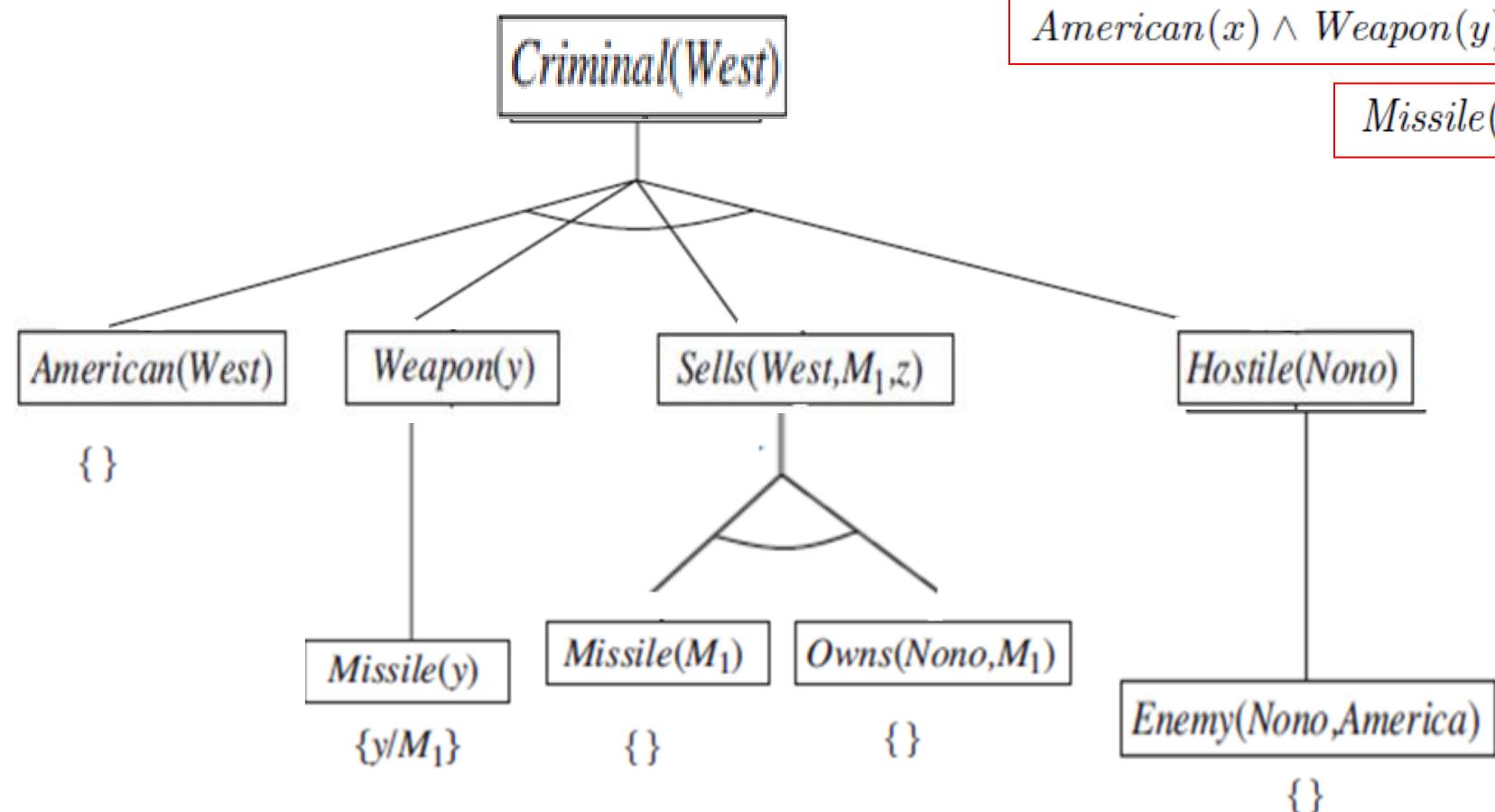
Backward-chaining algorithm

```
function FOL-BC-ASK(KB, query) returns a generator of substitutions  
  return FOL-BC-OR(KB, query, { })
```

```
generator FOL-BC-OR(KB, goal, θ) yields a substitution  
  for each rule (lhs  $\Rightarrow$  rhs) in FETCH-RULES-FOR-GOAL(KB, goal) do  
    (lhs, rhs)  $\leftarrow$  STANDARDIZE-VARIABLES((lhs, rhs))  
    for each  $\theta'$  in FOL-BC-AND(KB, lhs, UNIFY(rhs, goal, θ)) do  
      yield  $\theta'$ 
```

```
generator FOL-BC-AND(KB, goals, θ) yields a substitution  
  if  $\theta = \text{failure}$  then return  
  else if LENGTH(goals) = 0 then yield  $\theta$   
  else do  
    first, rest  $\leftarrow$  FIRST(goals), REST(goals)  
    for each  $\theta'$  in FOL-BC-OR(KB, SUBST(θ, first), θ) do  
      for each  $\theta''$  in FOL-BC-AND(KB, rest, θ') do  
        yield  $\theta''$ 
```

Backward chaining



$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

$Missile(x) \Rightarrow Weapon(x)$

$Enemy(x, America) \Rightarrow Hostile(x)$.

$Owns(Nono, M_1)$

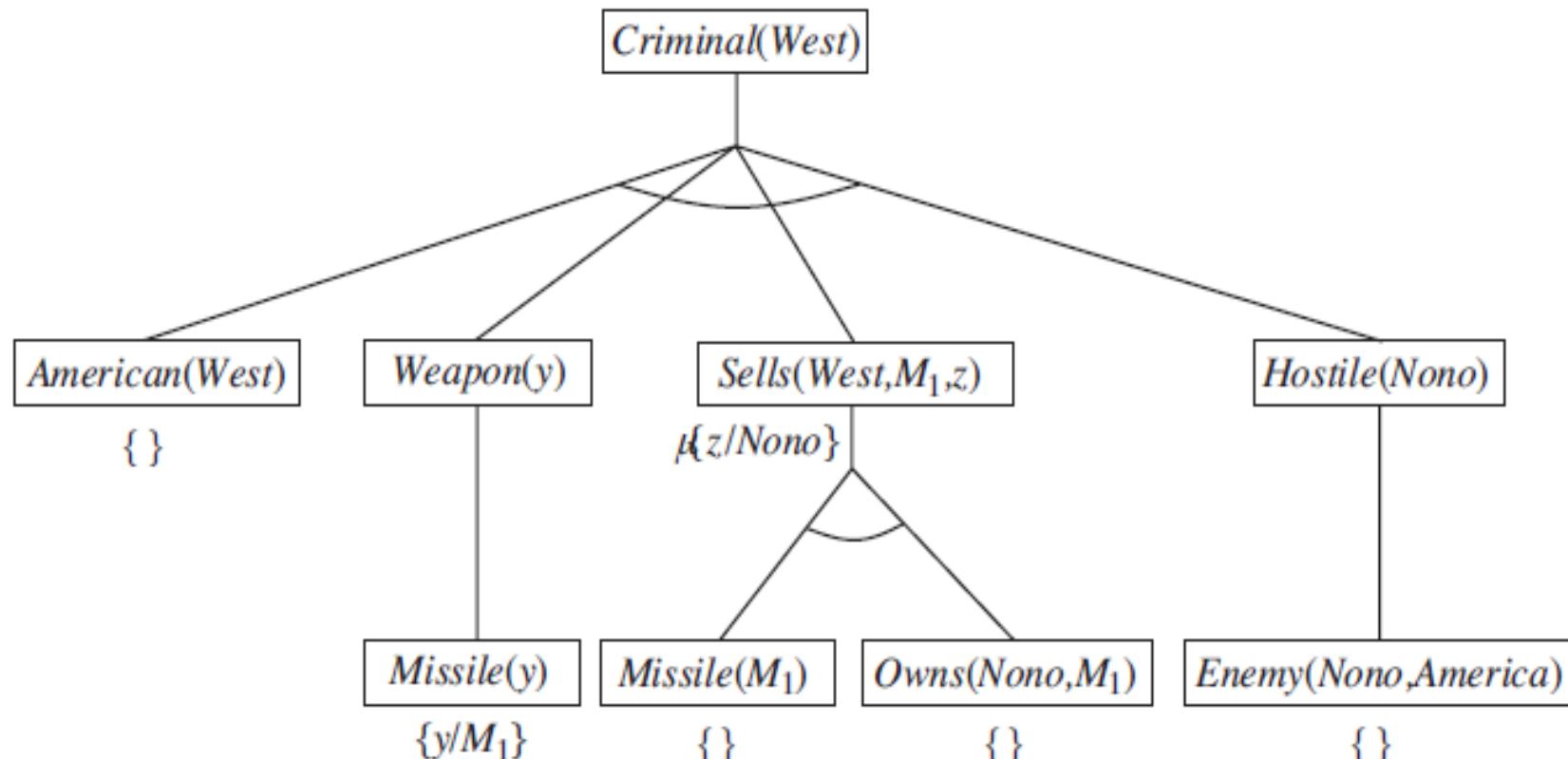
$Missile(M_1)$

$American(West)$.

$Enemy(Nono, America)$.

Backward-chaining algorithm

Backward chaining is clearly a depth-first search algorithm.



Backward chaining

Consider the following sentences:

- 1.John likes all kinds of food
- 2.Apples are food
- 3.Chicken is food
- 4.Anything anyone eats and isn't killed by is food.
- 5.Bill eats peanut and is still alive
6. sue eats everything bill eats

- (i) Translate these sentences into formulas in predicate logic.
- (ii) Prove that john likes peanuts using Backward chaining

Resolution

- First-order resolution requires that sentences to be in conjunctive normal form (CNF)
- The procedure for conversion to CNF
- Resolution Refutation

Steps to convert FOL to Clause form

1. Eliminate 
2. Reduce the scope of negation
3. Standardize variables apart
4. Move all quantifiers to left
5. Eliminate existential quantifiers
6. Drop all universal quantifiers
7. Convert to connect of disjunct form
8. Make each conjunct a separate clause
9. Standardize variables apart again

ALGORITHM TO CONVERT TO CLAUSAL FORM

1. Eliminate biconditionals and implications

$$(P \leftrightarrow Q) \Rightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P))$$

$$(P \rightarrow Q) \Rightarrow (\neg P \vee Q)$$

2. Move \neg inwards

$$\neg(\neg P) \Rightarrow P$$

$$\neg(P \vee Q) \Rightarrow \neg P \wedge \neg Q$$

$$\neg(P \wedge Q) \Rightarrow \neg P \vee \neg Q$$

$$\neg(\forall x)P \Rightarrow (\exists x)\neg P$$

$$\neg(\exists x)P \Rightarrow (\forall x)\neg P$$

ALGORITHM TO CONVERT TO CLAUSAL FORM

3. Standardize variables: rename all variables so that each quantifier has its own unique variable name

Example:

$$\forall x : \text{man}(x)$$
$$\forall x : \text{Happy}(x)$$
$$\exists x : \text{boys}(x)$$
$$\forall x : \text{man}(x)$$
$$\forall y : \text{Happy}(y)$$
$$\exists z : \text{boys}(z)$$

ALGORITHM TO CONVERT TO CLAUSAL FORM

Skolem constants and functions

4. Skolemization

Eliminate existential quantifier and replace it by skolem constants

Example:

$$\exists x : \text{Smile}(x)$$
$$\exists y : \text{Graduating}(y)$$

After Skolemization

$$\text{Smile}(A)$$
$$\text{Graduating}(B)$$

ALGORITHM TO CONVERT TO CLAUSAL FORM

5. Drop or Remove universal quantifiers by

- (1) moving them all to the left end;
- (2) making the scope of each the entire sentence;
- (3) dropping the “prefix” part

Example:

$$\begin{aligned} & \forall x : \text{Smile}(x) \\ & \forall y : \text{Graduating}(y) \end{aligned}$$

After Dropping

$\text{Smile}(x)$

$\text{Graduating}(y)$

ALGORITHM TO CONVERT TO CLAUSAL FORM

6. Apply distribute law / associative laws

$$(P \wedge Q) \vee R \Rightarrow (P \vee R) \wedge (Q \vee R)$$

$$(P \vee Q) \vee R \Rightarrow (P \vee Q \vee R)$$

7. Split conjuncts into separate clauses

8. Standardize variables so each clause contains only variable names that do not occur in any other clause

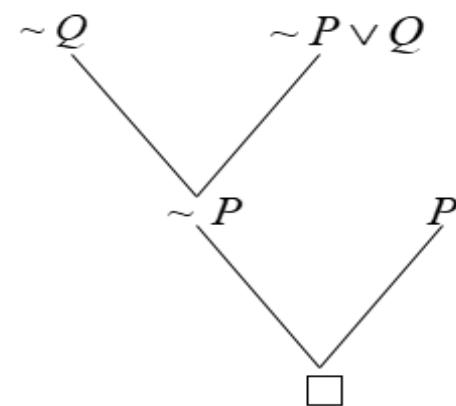
Resolution Refutation

- Resolution is a theorem proving technique that proceeds by building refutation proofs, i.e., proofs by contradictions.
- It was invented by a Mathematician John Alan Robinson in the year 1965.
- Resolution is used, if there are various statements given, and we need to prove a conclusion of those statements.

Resolution

- Pair of clauses being resolved is called the Resolvents.
- The resulting clause is called the **Resolute**.
- Choosing the correct pair of resolvents is a matter of search.

1. P
2. $\sim P \vee Q$
3. $\sim Q$



Resolution refutation proofs involve the following steps:

1. Put the premises or axioms into *clause form* (13.2.2).
2. Add the negation of what is to be proved, in clause form, to the set of axioms.
3. *Resolve* these clauses together, producing new clauses that logically follow from them (13.2.3).
4. Produce a contradiction by generating the empty clause.
5. The substitutions used to produce the empty clause are those under which the opposite of the negated goal is true (13.2.4).

Resolution refutation proofs steps:

1. Convert facts into First order logic (FOL)
2. Convert FOL into **clause form**
3. Negate the statement to be proved , and add the result to the knowledge base.
4. Draw Resolution graph .
5. If empty clause (NIL) is produced , stop and report that original theorem is true .

Example -1

Problem Statement:

Prove by resolution

1. If It is sunny and warm day you will enjoy
2. If it is raining you will get wet
3. It is warm day
4. It is raining
5. It is sunny

Goal: You will enjoy

Prove: enjoy

1. Convert to FOL

1. If It is sunny and warm day you will enjoy
2. If it is raining you will get wet
3. It is warm day
4. It is raining
5. It is sunny

Goal: You will enjoy

Prove: enjoy

Sunny \wedge warm \rightarrow enjoy

raining \rightarrow wet

warm

raining

Sunny

if a sentence has Conclusions then that statement is CONDITION -> CONCLUSION

2. Convert FOL to CNF

Sunny \wedge warm \rightarrow enjoy

raining \rightarrow wet

warm

raining

Sunny

- **Sunny \wedge Warm \rightarrow enjoy**
Eliminate implication:
 $\neg(\text{Sunny} \wedge \text{warm}) \vee \text{enjoy}$
Moving negation inside
 $\neg\text{Sunny} \vee \neg\text{warm} \vee \text{enjoy}$
- **raining \rightarrow wet**
Eliminate implication:
 $\neg\text{raining} \vee \text{wet}$
- **warm**
- **raining**
- **Sunny**

3. Negate the statement to be proved

1. If It is sunny and warm day you will enjoy
2. If it is raining you will get wet
3. It is warm day
4. It is raining
5. It is sunny

Goal: You will enjoy

Prove: enjoy

\neg enjoy

Draw resolution graph

1. Convert facts into First order logic (FOL)
2. Convert FOL into CNF (Conjuctive Normal Form)
3. Negate the statement to be proved , and add the result to the knowledge base.
4. Draw Resolution graph .
5. If empty clause (NIL) is produced , stop and report that original theorem is true .

$\neg \text{Sunny} \vee \neg \text{warm} \vee \text{enjoy}$

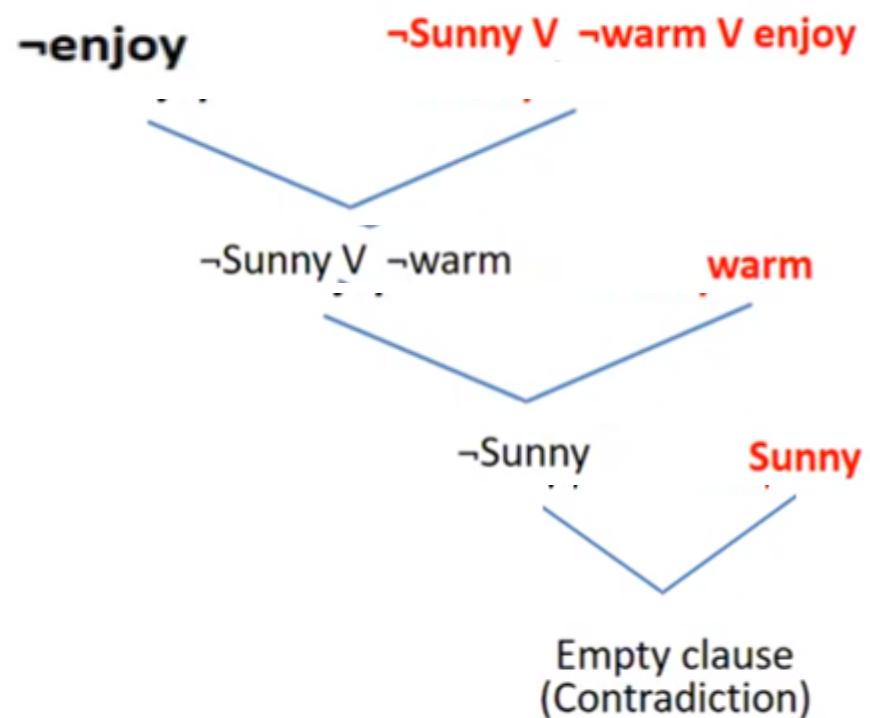
$\neg \text{raining} \vee \text{wet}$

warm

raining

Sunny

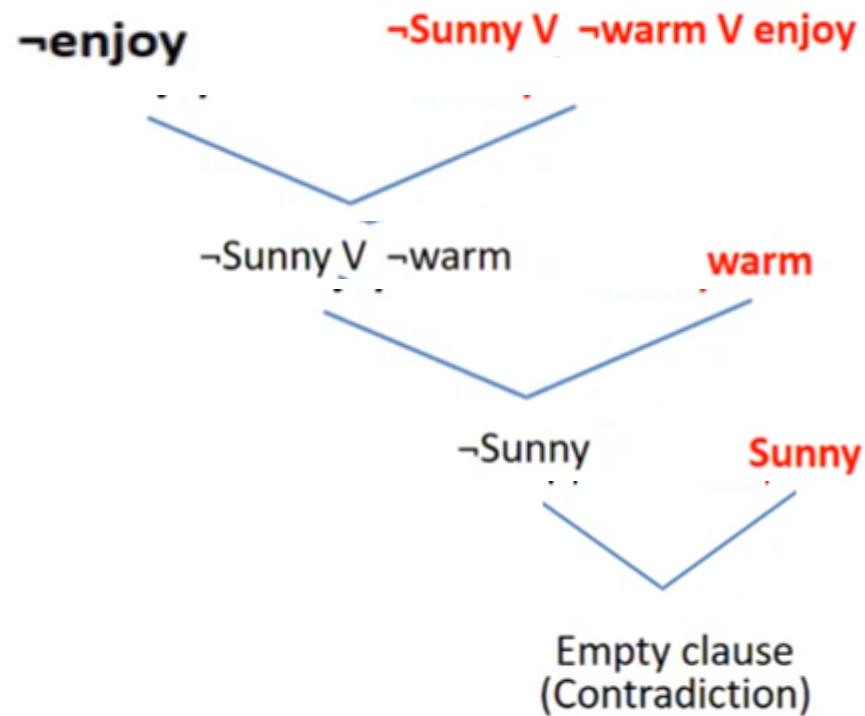
$\neg \text{enjoy}$



Draw resolution graph

Hence we see that the negation of the conclusion has been proved as a complete contradiction with the given set of facts.

Hence Proved



Example-2

1. The humidity is high or the sky is cloudy.
2. If the sky is cloudy, then it will rain.
3. If the humidity is high, then it is hot.
4. It is not hot.

Goal: It will rain.

- Let P : The humidity is high
- Let Q: sky is cloudy

The humidity is high or the sky is cloudy

$$P \vee Q$$

- Let Q: sky is cloudy
- Let R: it will rain

If the sky is cloudy, then it will rain

$$Q \rightarrow R$$

- Let P : The humidity is high
- Let S: it is hot

If the humidity is high, then it is hot

$$P \rightarrow S$$

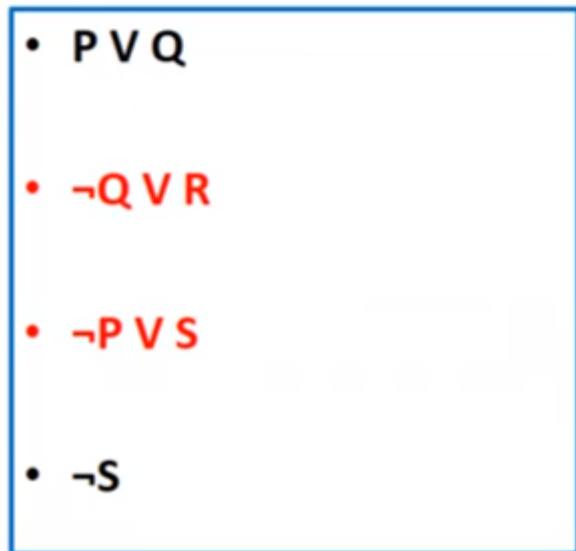
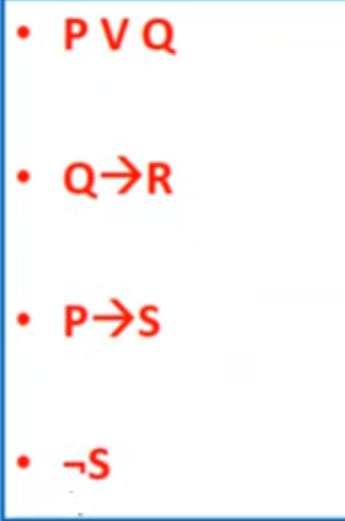
- Let S: it is hot

It is not hot
 $\neg S$

Example-2

CNF Form

Goal: It will rain. (**R**)



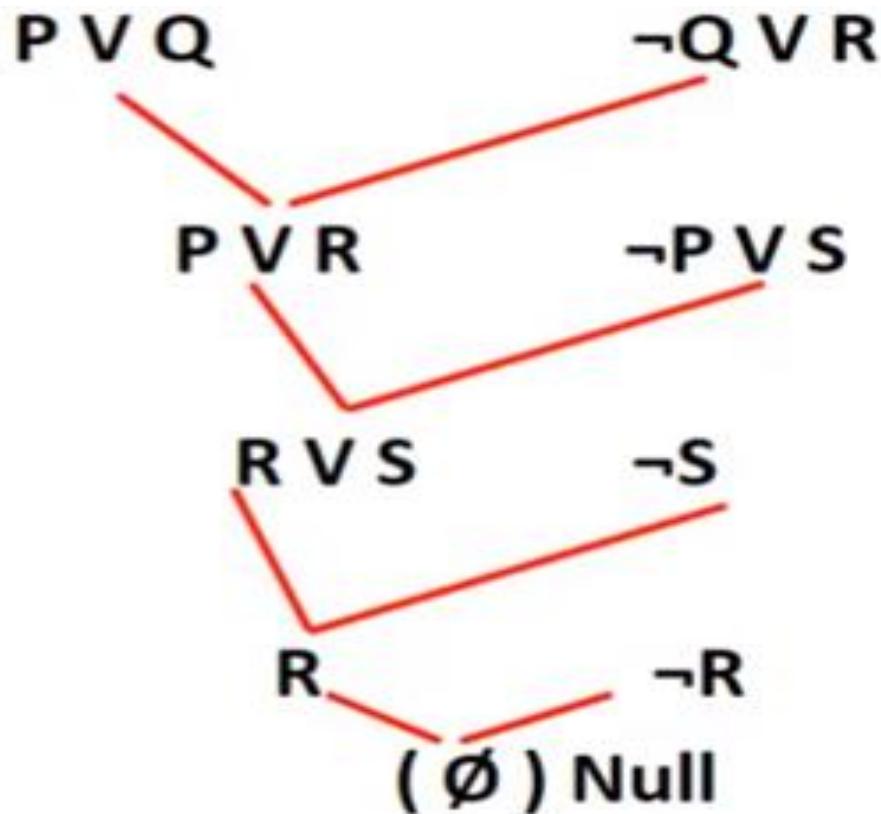
Example-2

Goal: It will rain. (R)

Negation of Goal ($\neg R$): It will not rain.

Example-2

- $P \vee Q$
- $\neg Q \vee R$
- $\neg P \vee S$
- $\neg S$
- $\neg R$ (Goal)



Hence we see that the negation of the conclusion has been proved as a complete contradiction with the given set of facts.

Hence Proved

Problem using Resolution Refutation

Consider the below argument and using Resolution Refutation Procedure check whether 'John is happy'

"Anyone passing his history exams and winning the lottery is happy. But anyone who studies or is lucky can pass all his exams. John did not study, but John is lucky. Anyone who is lucky wins the lottery."

if a sentence has Conclusions then that statement is
CONDITION -> CONCLUSION

Problem using Resolution Refutation

Anyone passing his history exams and winning the lottery is happy.

But anyone who studies or is lucky can pass all his exams.

John did not study, but John is lucky.

Anyone who is lucky wins the lottery

Anyone passing his history exams and winning the lottery is happy.

$$\forall X \text{ (pass (X,history) } \wedge \text{ win (X,lottery)} \rightarrow \text{ happy (X)})$$

Anyone who studies or is lucky can pass all his exams.

$$\forall X \vee Y \text{ (study (X) } \vee \text{ lucky (X)} \rightarrow \text{ pass (X,Y)})$$

John did not study but he is lucky.

$$\neg \text{ study (john) } \wedge \text{ lucky (john)}$$

Anyone who is lucky wins the lottery.

$$\forall X \text{ (lucky (X)} \rightarrow \text{ win (X,lottery)})$$

These four predicate statements are now changed to clause form (Section 12.2.2):

1. $\neg \text{ pass (X, history) } \vee \neg \text{ win (X, lottery)} \vee \text{ happy (X)}$
2. $\neg \text{ study (Y) } \vee \text{ pass (Y, Z)}$
3. $\neg \text{ lucky (W) } \vee \text{ pass (W, V)}$
4. $\neg \text{ study (john)}$
5. lucky (john)
6. $\neg \text{ lucky (U) } \vee \text{ win (U, lottery)}$

Into these clauses is entered, in clause form, the negation of the conclusion:

7. $\neg \text{ happy (john)}$

if a sentence has Conclusions then that statement is CONDITION -> CONCLUSION

Anyone passing his history exams and winning the lottery is happy.

$\forall X (\text{pass}(X, \text{history}) \wedge \text{win}(X, \text{lottery}) \rightarrow \text{happy}(X))$

Anyone who studies or is lucky can pass all his exams.

$\forall X \forall Y (\text{study}(X) \vee \text{lucky}(X) \rightarrow \text{pass}(X, Y))$

John did not study but he is lucky.

$\neg \text{study}(\text{john}) \wedge \text{lucky}(\text{john})$

Anyone who is lucky wins the lottery.

$\forall X (\text{lucky}(X) \rightarrow \text{win}(X, \text{lottery}))$

$$\underline{(P \rightarrow Q) \Rightarrow (\neg P \vee Q)}$$

$$\neg(P \wedge Q) \Rightarrow \neg P \vee \neg Q$$

1) $\forall x (\text{pass}(x, \text{history}) \wedge \text{win}(x, \text{lottery}) \rightarrow \text{happy}(x))$

$\forall x \neg [\text{pass}(x, \text{history}) \wedge \text{win}(x, \text{lottery})] \vee \text{happy}(x)$

$\forall x \neg \text{pass}(x, \text{history}) \vee \neg \text{win}(x, \text{lottery}) \vee \text{happy}(x)$

① $\neg \text{pass}(x, \text{history}) \vee \neg \text{win}(x, \text{lottery}) \vee \text{happy}(x)$

Remove
universal
quantifiers

2) $\forall x \forall y (\text{study}(x) \vee \text{lucky}(x) \rightarrow \text{Pass}(x, y))$

Remove implication

$$(P \rightarrow Q) \Rightarrow (\neg P \vee Q)$$

$\forall x \forall y \neg [\text{study}(x) \vee \text{lucky}(x)] \vee \text{Pass}(x, y)$

Move Negation inward

$$\neg(P \vee Q) \Rightarrow \neg P \wedge \neg Q$$

$\forall x \forall y [\neg \text{study}(x) \wedge \neg \text{lucky}(x)] \vee \text{Pass}(x, y)$

$(\neg \text{study}(x) \wedge \neg \text{lucky}(x)) \vee \text{Pass}(x, y)$

Remove Universal Quantifier

$$(P \wedge Q) \vee R \Rightarrow (P \vee R) \wedge (Q \vee R)$$

$[\neg \text{study}(x) \vee \text{Pass}(x, y)] \wedge [\neg \text{lucky}(x) \vee \text{Pass}(x, y)]$

Apply Distributive Law

Split Conjunction

② $\neg \text{study}(x) \vee \text{Pass}(x, y)$

③ $\neg \text{lucky}(x) \vee \text{Pass}(x, y)$

② $\neg \text{study}(y) \vee \text{Pass}(y, z)$

③ $\neg \text{lucky}(w) \vee \text{Pass}(w, v)$

3) $\neg \text{study}(\text{join}) \wedge \text{lucky}(\text{john})$

Split Conjunt into clauses

- ④ $\neg \text{study}(\text{join})$
- ⑤ $\text{lucky}(\text{john})$

$$4) \forall x (\text{lucky}(x) \rightarrow \text{win}(x, \text{lottery}))$$
$$\forall x (\neg \text{lucky}(x) \vee \text{win}(x, \text{lottery}))$$

$$\boxed{\textcircled{6} \quad \neg \text{lucky}(U) \vee \text{win}(U, \text{lottery})}$$

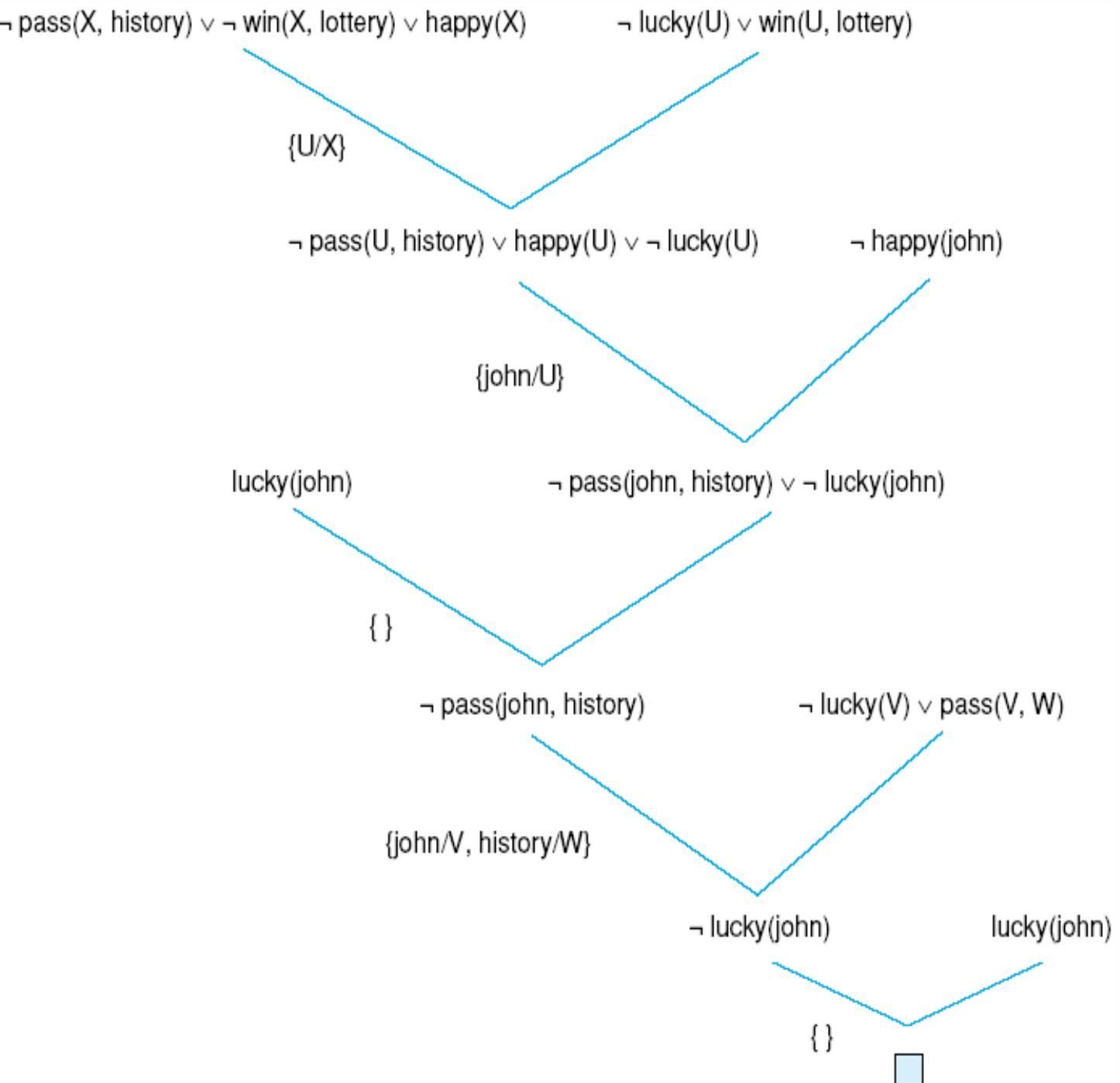
These four predicate statements are now changed to clause form (Section 12.2.2):

1. $\neg \text{pass}(X, \text{history}) \vee \neg \text{win}(X, \text{lottery}) \vee \text{happy}(X)$
2. $\neg \text{study}(Y) \vee \text{pass}(Y, Z)$
3. $\neg \text{lucky}(W) \vee \text{pass}(W, V)$
4. $\neg \text{study}(\text{john})$
5. $\text{lucky}(\text{john})$
6. $\neg \text{lucky}(U) \vee \text{win}(U, \text{lottery})$

John is Happy

Happy (John)

① \rightarrow happy (John)



1. $\neg \text{pass}(X, \text{history}) \vee \neg \text{win}(X, \text{lottery}) \vee \text{happy}(X)$
2. $\neg \text{study}(Y) \vee \text{pass}(Y, Z)$
3. $\neg \text{lucky}(W) \vee \text{pass}(W, V)$
4. $\neg \text{study}(\text{john})$
5. $\text{lucky}(\text{john})$
6. $\neg \text{lucky}(U) \vee \text{win}(U, \text{lottery})$
7. $\neg \text{happy}(\text{john})$

Problem – Dead Dog Problem

Axioms :

1. All dogs are animals
2. Fido is a dog
3. All animals will die

Theorem to proof : Fido will die

Problem – Dead Dog Problem

Axioms :

1. All dogs are animals : $\forall X [dog(X) \Rightarrow animal(X)]$
2. Fido is a dog : $dog(fido).$
3. All animals will die : $\forall Y [animal(Y) \Rightarrow die(Y)]$

Theorem to proof :

Fido will die : $die(fido).$

Problem – Dead Dog Problem

1. $\forall X [dog(X) \Rightarrow animal(X)]$

$\forall X \neg dog(X) \vee animal(X)$

$\neg dog(X) \vee animal(X)$

2. $dog(fido)$

3. $\forall Y [animal(Y) \Rightarrow die(Y)]$

$\forall Y \neg animal(Y) \vee die(Y)$

$\neg animal(Y) \vee die(Y)$

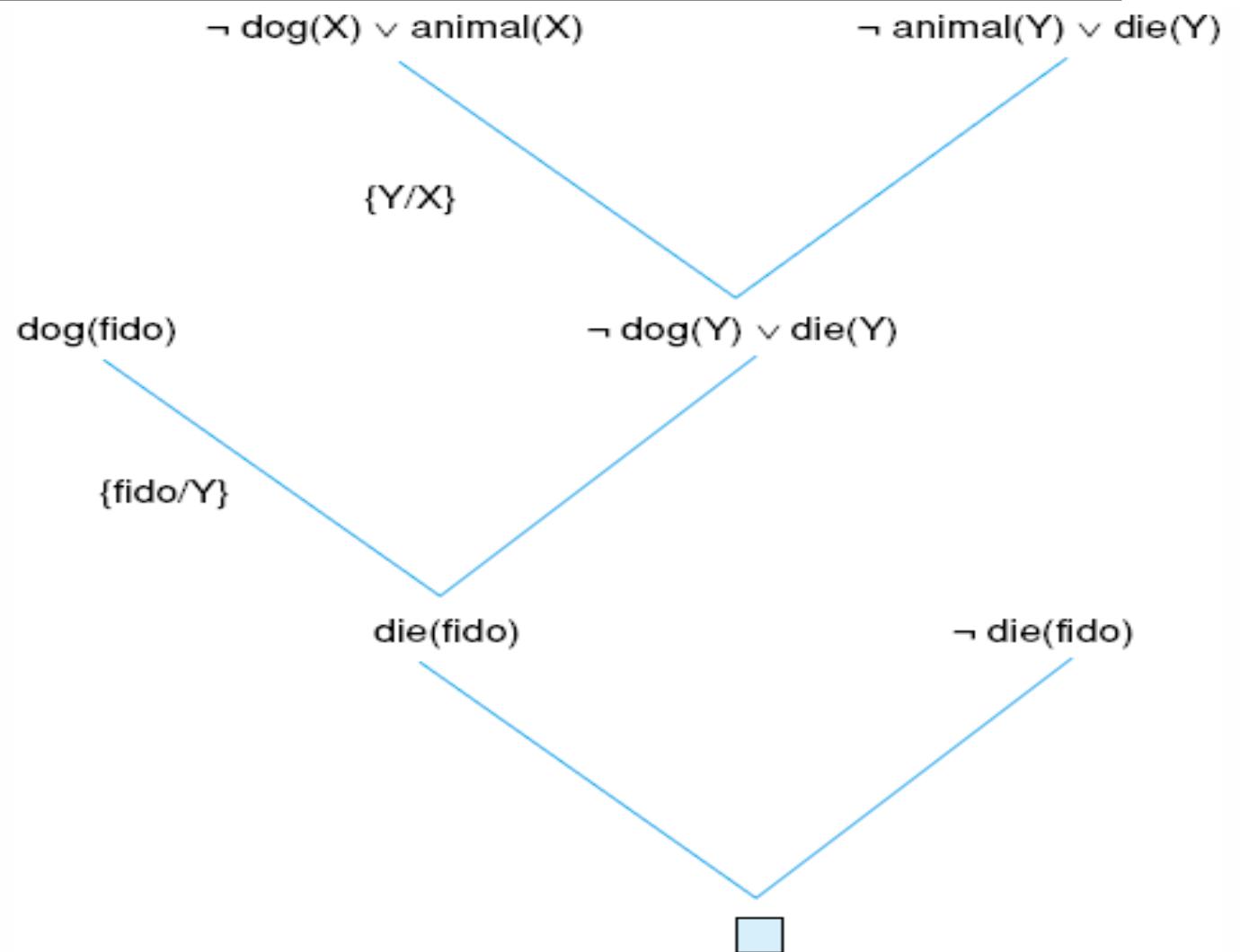
Problem – Dead Dog Problem

die(fido)

$\neg \text{die(fido)}$

Problem – Dead Dog Problem

- $\neg \text{dog}(X) \vee \text{animal}(X)$
- dog(fido)
- $\text{animal}(Y) \vee \text{die}(Y)$
- $\neg \text{die(fido)}$



All people who are not poor and are smart are happy. Those people who read are not stupid. John can read and is wealthy. Happy people have exciting lives. Can anyone be found with an exciting life?

We assume $\forall X (\text{smart}(X) \equiv \neg \text{stupid}(X))$ and $\forall Y (\text{wealthy}(Y) \equiv \neg \text{poor}(Y))$, and get:

$\forall X (\neg \text{poor}(X) \wedge \text{smart}(X) \rightarrow \text{happy}(X))$

$\forall Y (\text{read}(Y) \rightarrow \text{smart}(Y))$

$\text{read}(\text{john}) \wedge \neg \text{poor}(\text{john})$

$\forall Z (\text{happy}(Z) \rightarrow \text{exciting}(Z))$

The negation of the conclusion is:

$\neg \exists W (\text{exciting}(W))$

These predicate calculus expressions for the “exciting life” problem are transformed into the following clauses:

$\text{poor}(X) \vee \neg \text{smart}(X) \vee \text{happy}(X)$

$\neg \text{read}(Y) \vee \text{smart}(Y)$

$\text{read}(\text{john})$

$\neg \text{poor}(\text{john})$

$\neg \text{happy}(Z) \vee \text{exciting}(Z)$

$\neg \text{exciting}(W)$

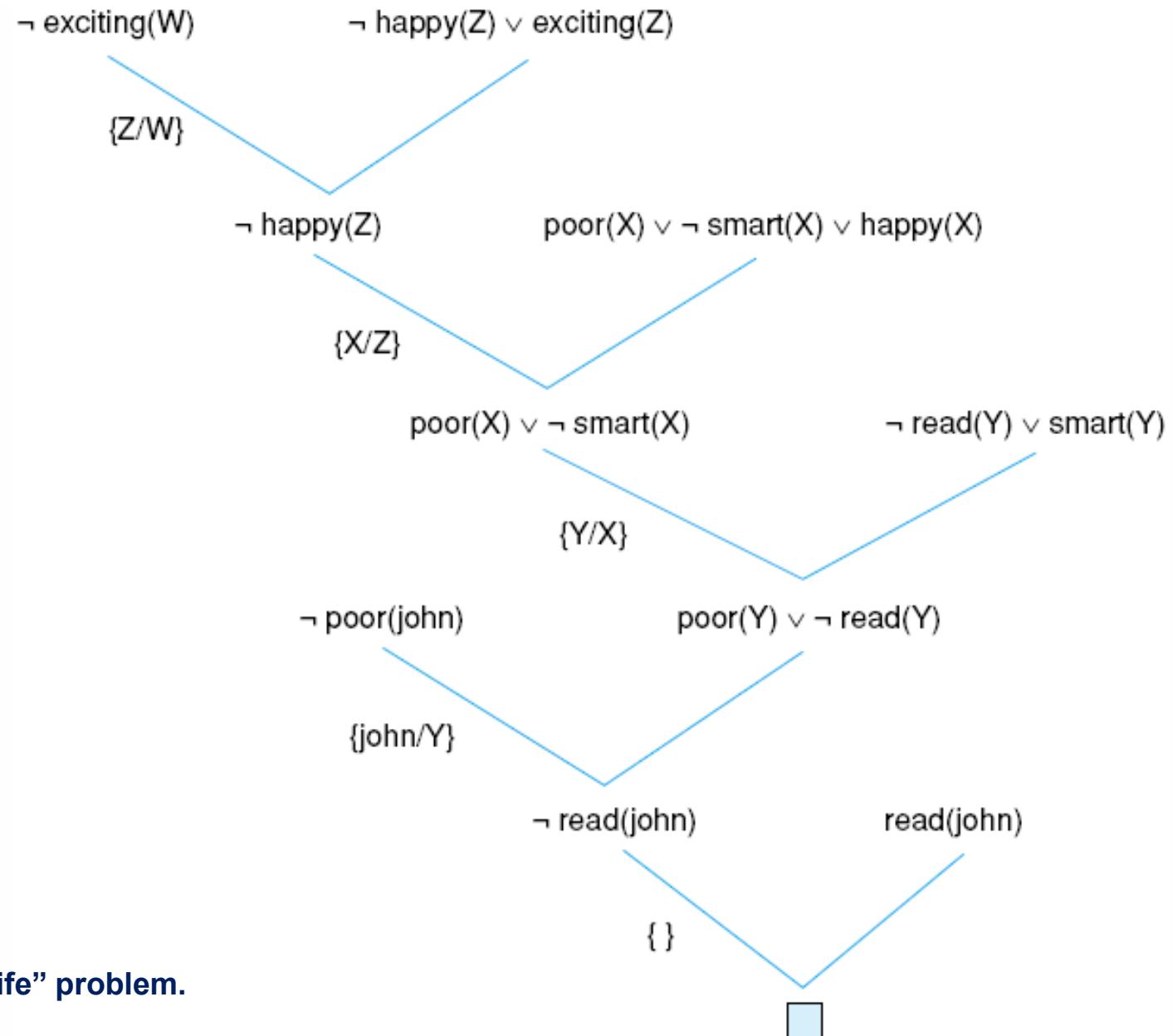
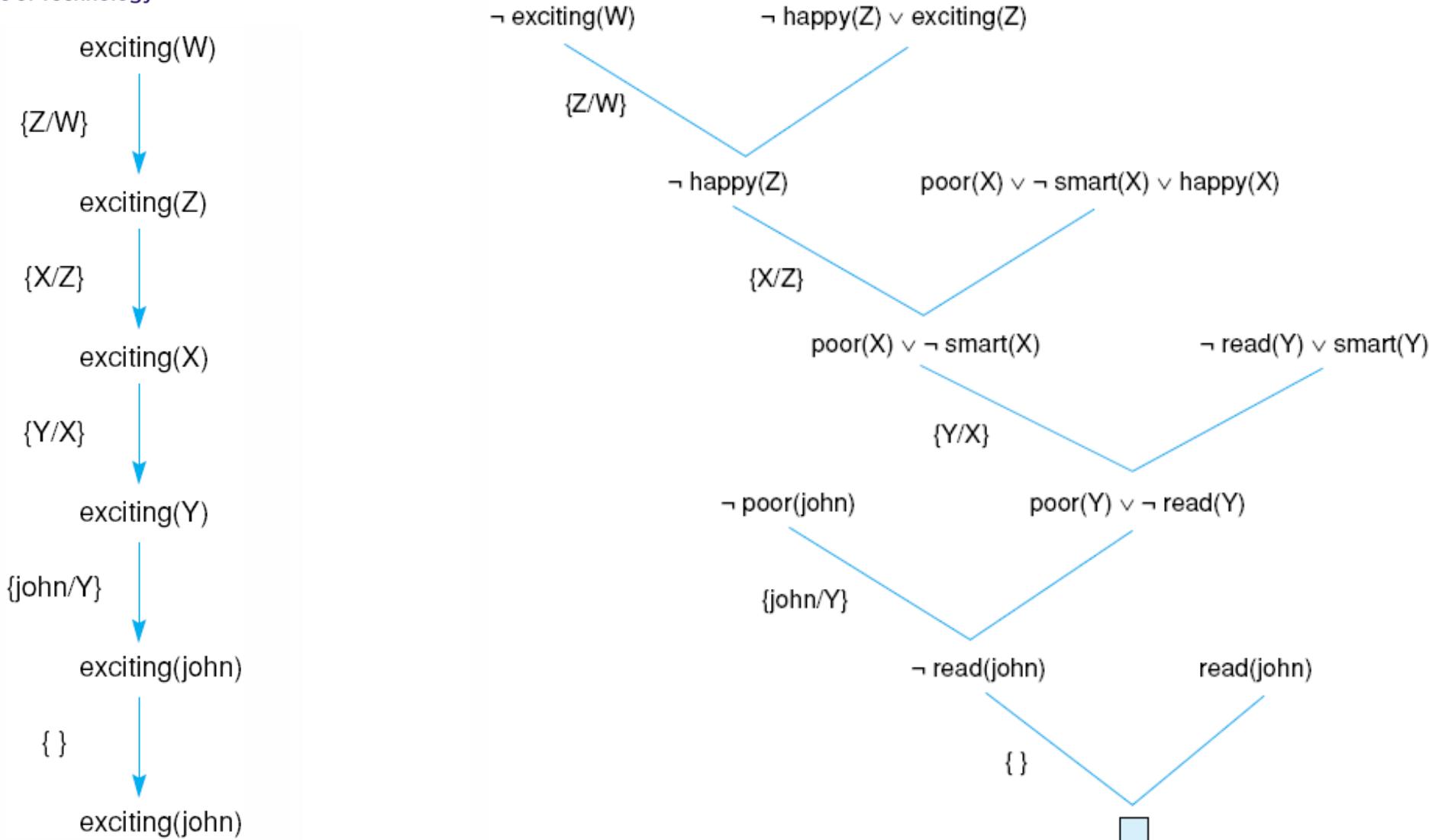


Fig. One refutation for the “exciting life” problem.

Fig Unification substitutions applied to the original query.

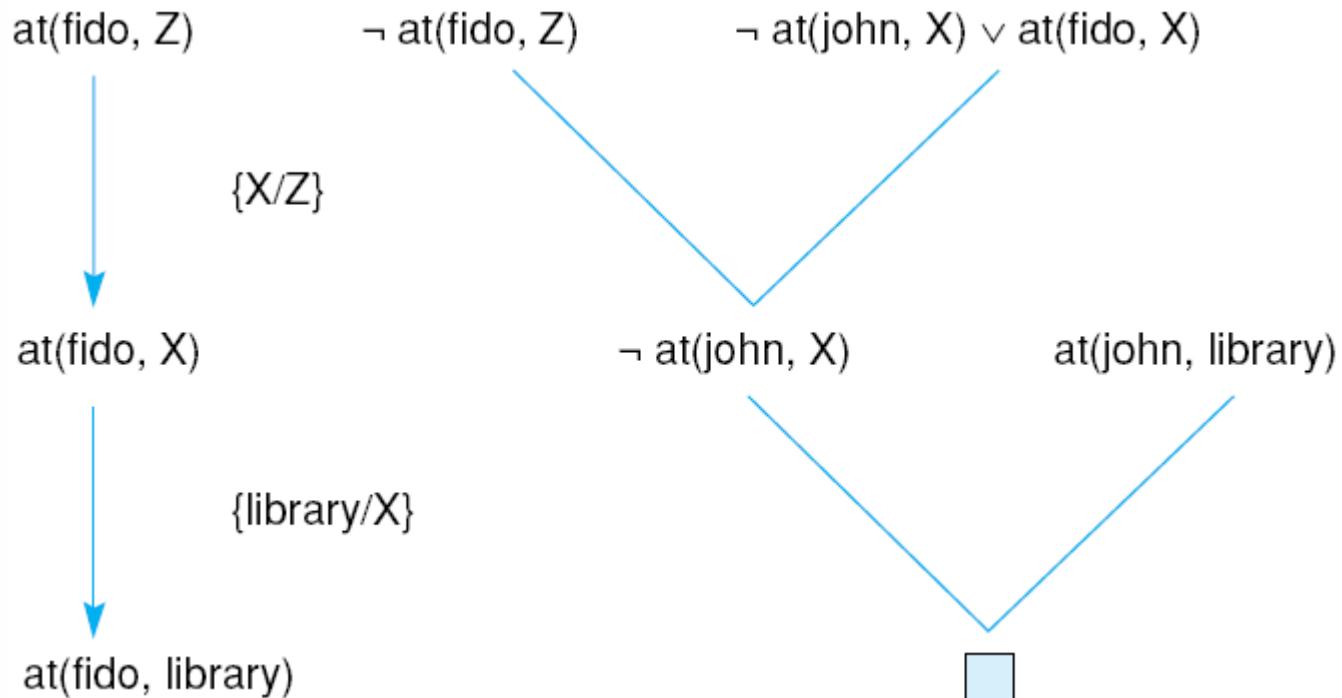


Extract the answer for the question “**Where is Fido?**”, given the below axioms in the KB.

- 1. Dog Fido goes wherever John goes.**
- 2. John is at library.**

Fig. Answer extraction process on the “finding fido” problem.

Dog Fido goes wherever John goes. John is at library. Where is Fido?



" If tom eats whatever Jade eats , and Tom eats banana , then what does Jade eat ? "

1. $\forall x \text{ eat(tom, } x) \rightarrow \text{eat(jade, } x)$

2. eat(tom, banana)

Conclusion:

$\exists x \text{ eat(jade, } x)$ or $\text{eat(jade, } x)$

Negation of Conclusion

3. $\neg \text{eat(jade, } x)$

Clausal forms.

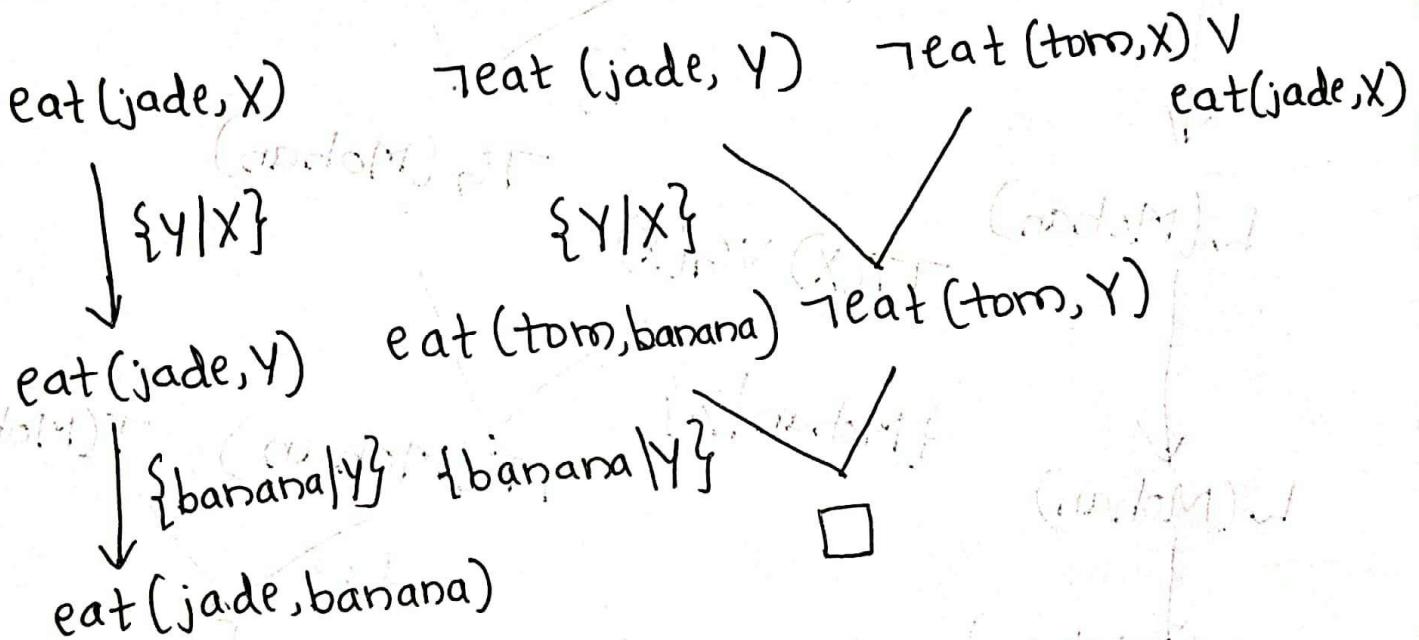
1. $\forall x \text{ eat(tom, } x) \rightarrow \text{eat(jade, } x)$

$\Rightarrow \forall x \neg \text{eat(tom, } x) \vee \text{eat(jade, } x)$

$\Rightarrow \neg \text{eat(tom, } x) \vee \text{eat(jade, } x)$

2. eat(tom, banana)

3. $\neg \text{eat(jade, } Y)$



Resolution Problem

All teachers are good. Anyone who is good and intelligent will deliver excellent lecture. Mohan is an intelligent teacher. Show that Mohan will deliver an excellent lecture.

- All teachers are good.
- Anyone who is good and intelligent will deliver excellent lecture.
- Mohan is an intelligent teacher.
- Show that Mohan will deliver an excellent lecture.

Predicates:

$T(x)$: x is a teacher

$G(x)$: x is good

$I(x)$: x is intelligent

$L(x)$: x delivers an excellent lecture

Predicate forms for given statements:

$$1. \forall x T(x) \rightarrow G(x)$$

$$2. \forall y G(y) \wedge I(y) \rightarrow L(y)$$

$$3. I(\text{Mohan}) \wedge T(\text{Mohan})$$

Conclusion:

$L(\text{Mohan})$

Negation of Conclusion:

$$4. \neg L(\text{Mohan})$$

Predicate form to Clausal form :

$$1. \neg T(x) \vee G(x)$$

$$2. \forall y (G(y) \wedge I(y)) \rightarrow L(y)$$

$$\Rightarrow \forall y \neg(G(y) \wedge I(y)) \vee L(y)$$

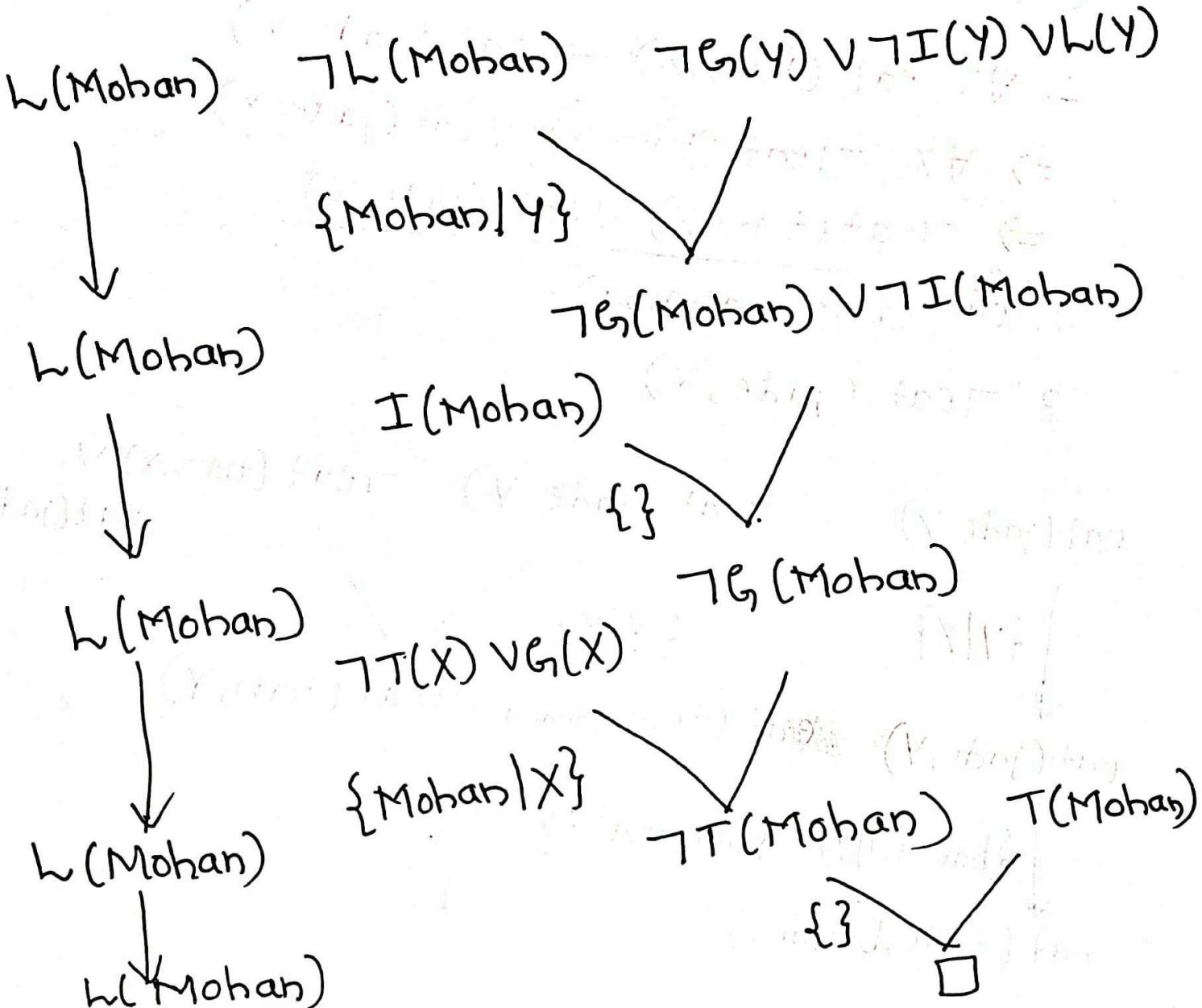
$$\Rightarrow \forall y \neg G(y) \vee \neg I(y) \vee L(y)$$

$$\Rightarrow \underline{\neg G(y) \vee \neg I(y) \vee L(y)}$$

$$3. I(Mohan)$$

$$4. T(Mohan)$$

$$5. \neg L(Mohan)$$



Thank you