Ramaiah Institute of Technology

V semester AL58 - Research Methodology and Intellectual Property Rights

Unit 3

Numerical problems

Note: Problems on testing of hypothesis on single sample are provided

If the level of significance is not given in the problem, assume it to be 5%.

1) Hypothesis testing on mean

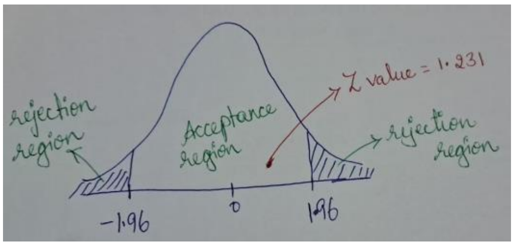
Problem 1: A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance.

Solution:
Taking the null hypothesis that the mean height of the population is equal to 67.39 inches. The hypothesized value is 67.39 inches.

$$H_0 : \mu_{H_0} = 67.39''$$
$$H_a : \mu_{H_0} \neq 67.39''$$

It is a two-tailed test

Sample size = 400, sample mean = 67.47
Population standard deviation = 1.3



$$\bar{X} = 67.47'', \sigma_p = 1.30'', \; n = 400.$$

We consider Z statistic

$$z = \frac{\bar{X} - \mu_{H_0}}{\sigma_p/\sqrt{n}} = \frac{67.47 - 67.39}{1.30/\sqrt{400}} = \frac{0.08}{0.065} = 1.231$$

Z statistic value = 1.231

To find the Z table value:
The level of significance given in the problem is 5%. It is a two – sided test and hence, each side will have 2.5% as rejection region.

For the probability of 0.025, the Z value from the normal distribution tables is -1.96. It’s a two-sided test and hence the critical region is at 1.96 on either tails of the normal distribution.

The observed value of Z is 1.231 which is in the acceptance region and thus *H*0 is accepted.

Conclusion:

We may conclude that the given sample (with mean height = 67.47″) can be regarded to have been taken from a population with mean height 67.39″ and standard deviation 1.30″ at 5% level of significance.

Problem 2: Suppose we are interested in a population of 20 industrial units of the same size, all of which are experiencing excessive labour turnover problems. The past records show that the mean of the distribution of annual turnover is 320 employees, with a standard deviation of 75 employees. A sample of 5 of these industrial units is taken at random which gives a mean of annual turnover as 300 employees. Is the sample mean consistent with the population mean? Test at 5% level.

Solution:

$$H_0 : \mu_{H_0} = 320 \text{ employees}$$
$$H_a : \mu_{H_0} \neq 320 \text{ employees}$$

$$\overline{X} = 300 \text{ employees}, \ \sigma_p = 75 \text{ employees}$$
$$n = 5; N = 20$$

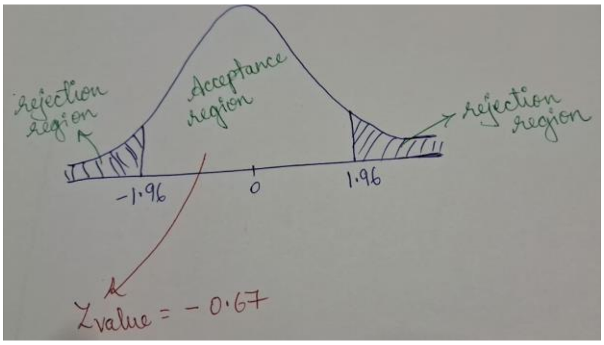
$$z^* = \frac{\overline{X} - \mu_{H_0}}{\sigma_p/\sqrt{n} \times \sqrt{(N-n)/(N-1)}}$$

$$= \frac{300 - 320}{75/\sqrt{5} \times \sqrt{(20-5)/(20-1)}} = -\frac{20}{(33.54)\,(.888)}$$

$$= -0.67$$

For the probability of 0.025, the Z value from the normal distribution tables is -1.96. It's a two-sided test and hence the critical region is at 1.96 on either tails of the normal distribution.

The observed value of z is –0.67 which is in the acceptance region. H0 is accepted and we may conclude that the sample mean is consistent with population mean i.e., the population mean 320 is supported by sample results.



Problem 3: The mean of a certain production process is known to be 50 with a standard deviation of 2.5. The production manager may welcome any change is mean value towards higher side but would like to safeguard against decreasing values of mean. He takes a sample of 12 items that gives a mean value of 48.5. What inference should the manager take for the production process on the basis of sample results? Use 5 per cent level of significance for the purpose.

Solution:

$H_0 : \mu_{H_0} = 50$

$H_a : \mu_{H_0} < 50$

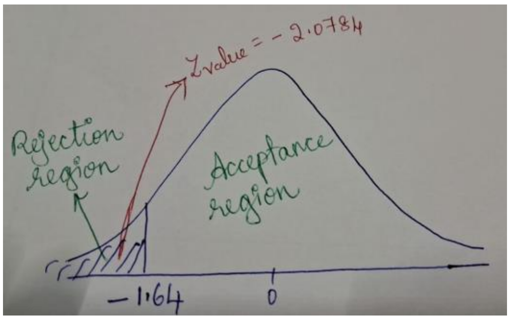
The production manager would like to safeguard against decreasing values of mean. So it is one sided test and it is left side test. The rejection region is on the left tail of the distribution. This is written in the alternate hypothesis.

$$\overline{X} = 48.5,\ \sigma_p = 2.5 \text{ and } n = 12$$

$$z = \frac{\overline{X} - \mu_{H_0}}{\sigma_p/\sqrt{n}} = \frac{48.5 - 50}{2.5/\sqrt{12}} = -\frac{1.5}{(2.5)/(3.464)} = -2.0784$$

It is one-sided test and level of significance is 5%. From normal distribution tables, for the probability of 0.05, the Z value is -1.64.

The observed value of *z* is – 2.0784 which is in the rejection region and thus, *H*0 is rejected at 5 per cent level of significance. We can conclude that the production process is showing mean which is significantly less than the population mean and this calls for some corrective action concerning the said process.



Problem 4: The specimen of copper wires drawn form a large lot have the following breaking strength (in kg. weight):

578, 572, 570, 568, 572, 578, 570, 572, 596, 544

Test whether the mean breaking strength of the lot may be taken to be 578 kg. weight (Test at 5 per cent level of significance).

Solution:

$$H_0 : \mu = \mu_{H_0} = 578 \text{ kg.}$$
$$H_a : \mu \neq \mu_{H_0}$$

As the sample size is mall (since n = 10) and the population standard deviation is not known, we shall use t-test assuming normal population

$$t = \frac{\bar{X} - \mu_{H_0}}{\sigma_s/\sqrt{n}}$$

Sample mean and sample standard deviation are calculated as follows:

$$\bar{X} = \frac{\Sigma X_i}{n} = \frac{5720}{10} = 572 \text{ kg.}$$

$$\sigma_s = \sqrt{\frac{\Sigma\left(X_i - \bar{X}\right)^2}{n-1}} = \sqrt{\frac{1456}{10-1}} = 12.72 \text{ kg.}$$

$$t = \frac{572 - 578}{12.72/\sqrt{10}} = -1.488$$

Calculation of standard deviation:

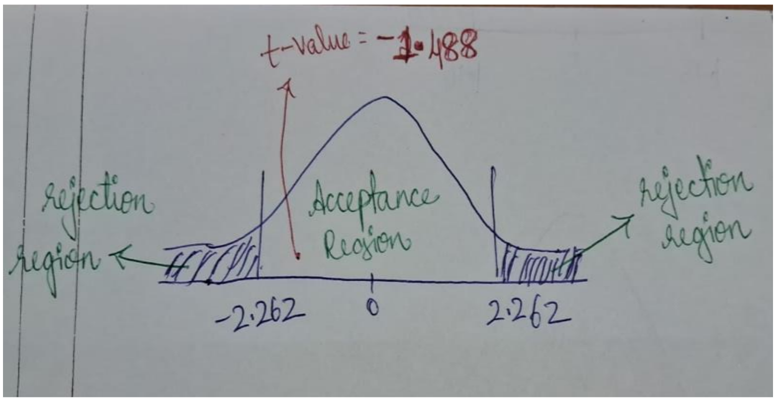
S. No.	X_i	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$
1	578	6	36
2	572	0	0
3	570	-2	4
4	568	-4	16
5	572	0	0
6	578	6	36
7	570	-2	4
8	572	0	0
9	596	24	576
10	544	-28	784
$n = 10$	$\Sigma X_i = 5720$		$\Sigma(X_i - \bar{X})^2 = 1456$

Degree of freedom = (n – 1) = (10 – 1) = 9
As *Ha* is two-sided, we shall determine the rejection region applying two-tailed test at 5 per cent level of significance (α = 5%), from t distribution tables.

$\alpha/2 = 0.025$
degrees of freedom = 9

t-value from the tables is 2.262

As the observed value of t (i.e., – 1.488) is in the acceptance region, we accept $H0$ at 5 per cent level and conclude that the mean breaking strength of copper wires lot may be taken as 578 kg. weight.



Problem 5: Raju Restaurant near the railway station at Falna has been having average sales of 500 tea cups per day. Because of the development of bus stand nearby, it expects to increase its sales. During the first 12 days after the start of the bus stand, the daily sales were as under:

550, 570, 490, 615, 505, 580, 570, 460, 600, 580, 530, 526

On the basis of this sample information, can one conclude that Raju Restaurant's sales have increased? Use 5 per cent level of significance.

Solution:

$H_0 : \mu = 500$ cups per day

$H_a : \mu > 500$ (as we want to conclude that sales have increased).

As the sample size is small and the population standard deviation is not known, we shall use *t*-test assuming normal population

$$t = \frac{\bar{X} - \mu}{\sigma_s / \sqrt{n}}$$

$$\bar{X} = \frac{\sum X_i}{n} = \frac{6576}{12} = 548$$

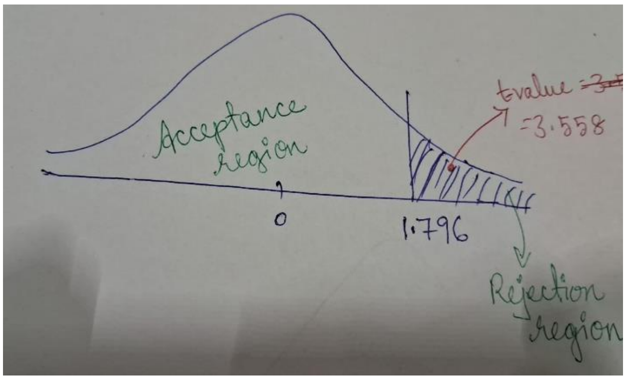
$$\sigma_s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n - 1}} = \sqrt{\frac{23978}{12 - 1}} = 46.68$$

$$t = \frac{548 - 500}{46.68/\sqrt{12}} = \frac{48}{13.49} = 3.558$$

Degree of freedom $= n - 1 = 12 - 1 = 11$

From the t-distribution tables, the t value is 1.796

The observed value of *t* is 3.558 which is in the rejection region and thus *H*0 is rejected at 5 per cent level of significance and we can conclude that the sample data indicate that Raju restaurant's sales have increased.



2) Hypothesis Testing of Proportion

Problem 6: A sample survey indicates that out of 3232 births, 1705 were boys and the rest were girls. Do these figures confirm the hypothesis that the gender ratio is 50 : 50? Test at 5 per cent level of significance.

Solution: The gender ratio is 50 : 50, that is 50% for each gender

$$H_0: p = p_{H_0} = \frac{1}{2}$$

$$H_a: p \neq p_{H_0}$$

the proportion success or $p = \frac{1}{2}$

the proportion of failure or $q = \frac{1}{2}$

$n = 3232$

The standard error of proportion of success.

$$= \sqrt{\frac{p \cdot q}{n}} = \sqrt{\frac{\frac{1}{2} \times \frac{1}{2}}{3232}} = 0.0088$$

Observed sample proportion of success, or

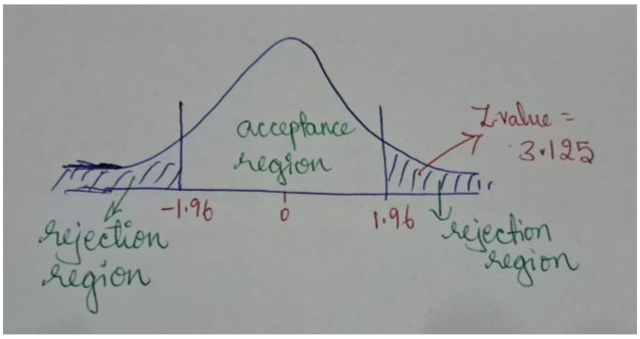
$$\hat{p} = 1705/3232 = 0.5275$$

and the test statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{0.5275 - .5000}{.0088} = 3.125$$

As *Ha* is two-sided in the given question, we shall be applying the two-tailed test for determining the rejection regions at 5 per cent level.
Z table value is 1.96.

The observed value of *z* is 3.125 which comes in the rejection region and thus, *H*0 is rejected; we conclude that the given figures do not conform the hypothesis of gender ratio being 50 : 50.



Problem 7: The null hypothesis is that 20 per cent of the passengers go in first class, but management recognizes the possibility that this percentage could be more or less. A random sample of 400 passengers includes 70 passengers holding first class tickets. Can the null hypothesis be rejected at 10 per cent level of significance?

Solution:

$H_0 : p = 20\%$ or 0.20

$H_a : p \neq 20\%$

$p = 0.20$

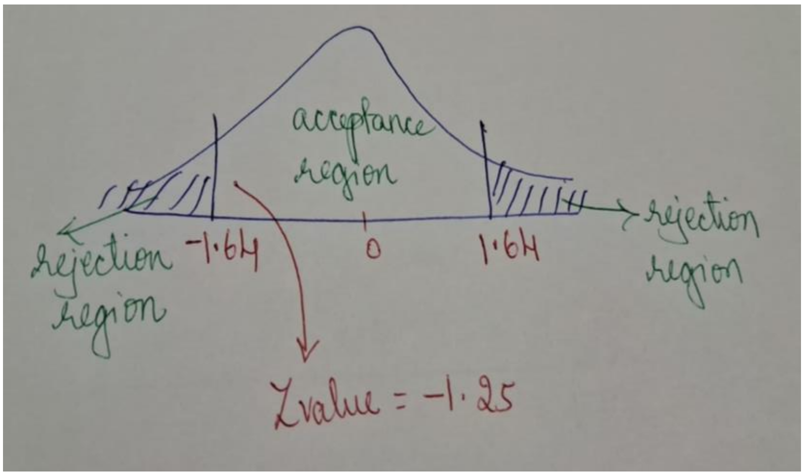
$q = 0.80$

Observed sample proportion $(\hat{p}) = 70/400 = 0.175$

and the test statistic $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \dfrac{0.175 - .20}{\sqrt{\dfrac{.20 \times .80}{400}}} = -1.25$

Here level of significance is 10% and its two-tailed test. Hence, on each side 5% is considered. For the probability of 0.05, from normal distribution tables, Z value is 1.64.

The observed value of z is –1.25 which is in the acceptance region and $H0$ is accepted.



Problem 8: A certain process produces 10 per cent defective articles. A supplier of new raw material claims that the use of his material would reduce the proportion of defectives. A random sample of 400 units using this new material was taken out of which 34 were defective units. Can the supplier's claim be accepted? Test at 1 per cent level of significance.

Solution:

$H0 : p = 10\%$ or 0.10
$Ha : p < 0.10$

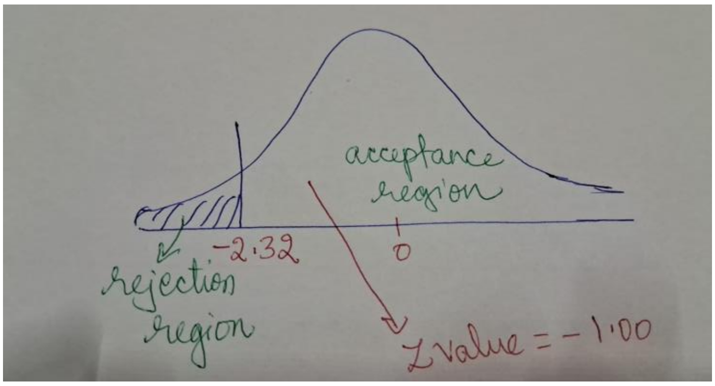
$p = 0.10$ and $q = 0.90$

$p = 34/400 = 0.085$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{.085 - .10}{\sqrt{\frac{.10 \times .90}{400}}} = \frac{.015}{.015} = -1.00$$

As *Ha* is one-sided, we shall determine the rejection region applying one-tailed test (in the left tail because *Ha* is of less than type) at 1% level of significance and table value of z is -2.32

As the computed value of *z* does not fall in the rejection region, *H*0 is accepted at 1% level of significance and we can conclude that on the basis of sample information, the supplier's claim cannot be accepted at 1% level.



3) Hypothesis Testing of Variance

Problem 9: A study on the average force transmitted by helmet to the construction workers is 800 pounds (or less) with a standard deviation to be less than 40 pounds. Tests were run on a random sample of n = 30 helmets, and the sample mean and sample standard deviation were found to be 825 pounds and 48.5 pounds, respectively.

Do the data provide sufficient evidence, at the α=0.05 level, to conclude that the population standard deviation exceeds 40 pounds?

Solution:

$$H_0: \sigma^2 = 40^2 = 1600$$

$$H_A: \sigma^2 > 1600$$

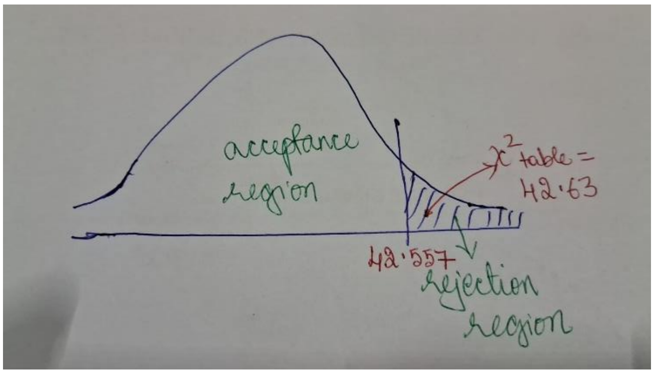
$$\chi^2 = \frac{\sigma_s^2}{\sigma_p^2}(n-1)$$

$$\chi^2 = \frac{(30-1)\,48.5^2}{40^2}$$

$$\chi^2 = 42.63$$

From Chi-square distribution tables, for degrees of freedom n-1 of 29 and significance level of 0.05, the critical value is 42.557

Since the calculated value of Chi-square is greater than the table value, null hypothesis is rejected.



Problem 10:

A sample of 24 donuts reveals a mean amount of filling equal to 0.05 cups, and the sample standard deviation is 0.11 cups. Test the hypothesis that the variance is equal to 0.04 cups.

Solution:

Ho: $\sigma^2 = 0.04$

Ha: $\sigma^2 \neq 0.04$

$$\chi^2 = \frac{\sigma_s^2}{\sigma_p^2}(n-1)$$

$$\chi^2 = \frac{(24-1)\ 0.11^2}{0.04}$$

$$\chi^2 = 6.95$$

Since it is two sided test, for 0.025 significance and 23 degrees of freedom, the Chi-square table value is 38.076

Since the calculated value is less than table value, Ho cannot be rejected. The variance is equal to 0.04 cups.