Chapter 1

Introduction

Programming languages are notations for describing computations to people and to machines. The world as we know it depends on programming languages, because all the software running on all the computers was written in some programming language. But, before a program can be run, it first must be translated into a form in which it can be executed by a computer.

The software systems that do this translation are called *compilers*.

This book is about how to design and implement compilers. We shall discover that a few basic ideas can be used to construct translators for a wide variety of languages and machines. Besides compilers, the principles and techniques for compiler design are applicable to so many other domains that they are likely to be reused many times in the career of a computer scientist. The study of compiler writing touches upon programming languages, machine architecture, language theory, algorithms, and software engineering.

In this preliminary chapter, we introduce the different forms of language translators, give a high level overview of the structure of a typical compiler, and discuss the trends in programming languages and machine architecture that are shaping compilers. We include some observations on the relationship between compiler design and computer-science theory and an outline of the applications of compiler technology that go beyond compilation. We end with a brief outline of key programming-language concepts that will be needed for our study of compilers.

1.1 Language Processors

Simply stated, a compiler is a program that can read a program in one language — the *source* language — and translate it into an equivalent program in another language — the *target* language; see Fig. 1.1. An important role of the compiler is to report any errors in the source program that it detects during the translation process.

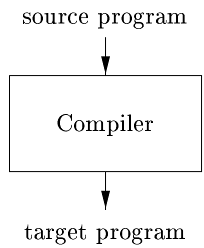


Figure 1.1: A compiler

If the target program is an executable machine-language program, it can then be called by the user to process inputs and produce outputs; see Fig. 1.2.

Figure 1.2: Running the target program

An *interpreter* is another common kind of language processor. Instead of producing a target program as a translation, an interpreter appears to directly execute the operations specified in the source program on inputs supplied by the user, as shown in Fig. 1.3.

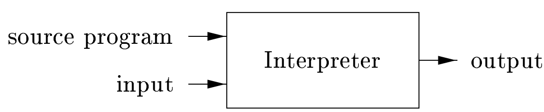


Figure 1.3: An interpreter

The machine-language target program produced by a compiler is usually much faster than an interpreter at mapping inputs to outputs . An interpreter, however, can usually give better error diagnostics than a compiler, because it executes the source program statement by statement.

Example 1.1: Java language processors combine compilation and interpretation, as shown in Fig. 1.4. A Java source program may first be compiled into an intermediate form called *bytecodes*. The bytecodes are then interpreted by a virtual machine. A benefit of this arrangement is that bytecodes compiled on one machine can be interpreted on another machine, perhaps across a network.

In order to achieve faster processing of inputs to outputs, some Java compilers, called *just-in-time* compilers, translate the bytecodes into machine language immediately before they run the intermediate program to process the input. □

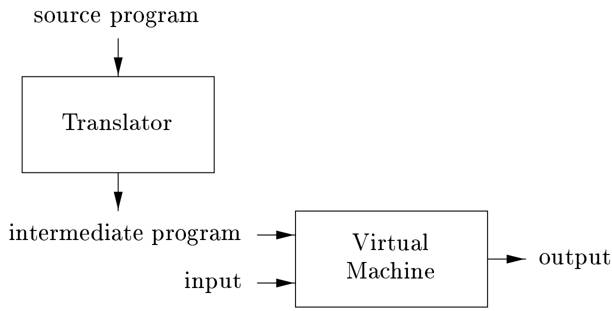


Figure 1.4: A hybrid compiler

In addition to a compiler, several other programs may be required to create an executable target program, as shown in Fig. 1.5. A source program may be divided into modules stored in separate files. The task of collecting the source program is sometimes entrusted to a separate program, called a *preprocessor*. The preprocessor may also expand shorthands, called macros, into source language statements.

The modified source program is then fed to a compiler. The compiler may produce an assembly-language program as its output, because assembly language is easier to produce as output and is easier to debug. The assembly language is then processed by a program called an *assembler* that produces relocatable machine code as its output.

Large programs are often compiled in pieces, so the relocatable machine code may have to be linked together with other relocatable object files and library files into the code that actually runs on the machine. The *linker* resolves external memory addresses, where the code in one file may refer to a location in another file. The *loader* then puts together all of the executable object files into memory for execution.

1.1.1 Exercises for Section 1.1

Exercise 1.1.1: What is the difference between a compiler and an interpreter?

Exercise 1.1.2: What are the advantages of (a) a compiler over an interpreter (b) an interpreter over a compiler?

Exercise 1.1.3: What advantages are there to a language-processing system in which the compiler produces assembly language rather than machine language?

Exercise 1.1.4: A compiler that translates a high-level language into another high-level language is called a *source-to-source* translator. What advantages are there to using C as a target language for a compiler?

Exercise 1.1.5: Describe some of the tasks that an assembler needs to perform.

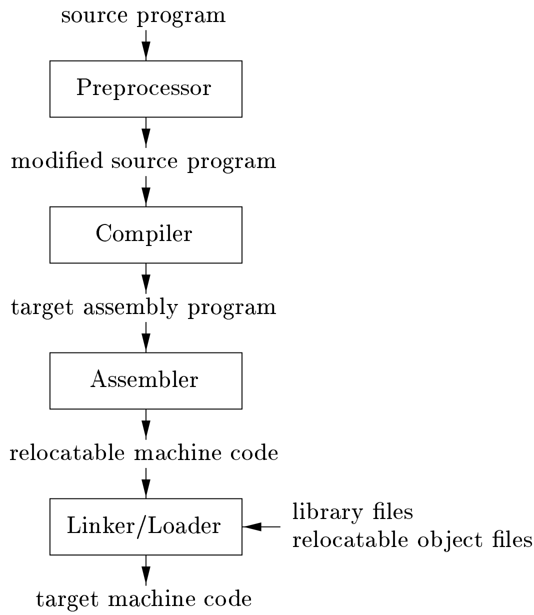


Figure 1.5: A language-processing system

1.2 The Structure of a Compiler

Up to this point we have treated a compiler as a single box that maps a source program into a semantically equivalent target program. If we open up this box a little, we see that there are two parts to this mapping: analysis and synthesis.

The *analysis* part breaks up the source program into constituent pieces and imposes a grammatical structure on them. It then uses this structure to create an intermediate representation of the source program. If the analysis part detects that the source program is either syntactically ill formed or semantically unsound, then it must provide informative messages, so the user can take corrective action. The analysis part also collects information about the source program and stores it in a data structure called a *symbol table*, which is passed along with the intermediate representation to the synthesis part.

The *synthesis* part constructs the desired target program from the intermediate representation and the information in the symbol table. The analysis part is often called the *front end* of the compiler; the synthesis part is the *back end*.

If we examine the compilation process in more detail, we see that it operates as a sequence of *phases*, each of which transforms one representation of the source program to another. A typical decomposition of a compiler into phases is shown in Fig. 1.6. In practice, several phases may be grouped together, and the intermediate representations between the grouped phases need not be constructed explicitly. The symbol table, which stores information about the

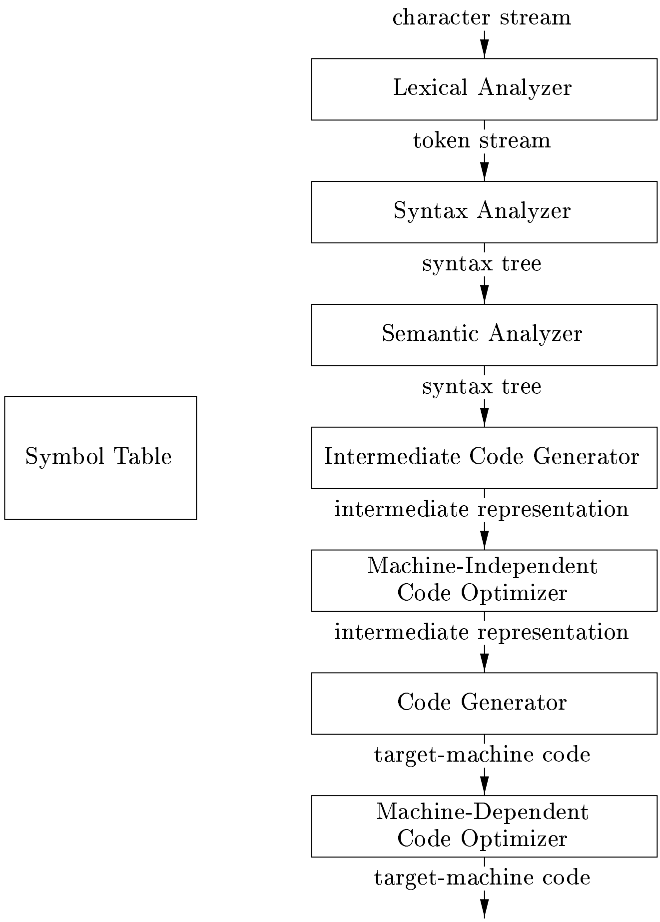


Figure 1.6: Phases of a compiler

entire source program, is used by all phases of the compiler.

Some compilers have a machine-independent optimization phase between the front end and the back end. The purpose of this optimization phase is to perform transformations on the intermediate representation, so that the back end can produce a better target program than it would have otherwise produced from an unoptimized intermediate representation. Since optimization is optional, one or the other of the two optimization phases shown in Fig. 1.6 may be missing.

1.2.1 Lexical Analysis

The first phase of a compiler is called *lexical analysis* or *scanning*. The lexical analyzer reads the stream of characters making up the source program and groups the characters into meaningful sequences called *lexemes*. For each lexeme, the lexical analyzer produces as output a *token* of the form

$$\langle \textit{token-name}, \textit{attribute-value} \rangle$$

that it passes on to the subsequent phase, syntax analysis. In the token, the first component *token-name* is an abstract symbol that is used during syntax analysis, and the second component *attribute-value* points to an entry in the symbol table for this token. Information from the symbol-table entry is needed for semantic analysis and code generation.

For example, suppose a source program contains the assignment statement

$$\texttt{position = initial + rate * 60} \tag{1.1}$$

The characters in this assignment could be grouped into the following lexemes and mapped into the following tokens passed on to the syntax analyzer:

1. `position` is a lexeme that would be mapped into a token $\langle \mathbf{id}, 1 \rangle$, where **id** is an abstract symbol standing for *identifier* and 1 points to the symbol-table entry for `position`. The symbol-table entry for an identifier holds information about the identifier, such as its name and type.

2. The assignment symbol `=` is a lexeme that is mapped into the token $\langle = \rangle$. Since this token needs no attribute-value, we have omitted the second component. We could have used any abstract symbol such as **assign** for the token-name, but for notational convenience we have chosen to use the lexeme itself as the name of the abstract symbol.

3. `initial` is a lexeme that is mapped into the token $\langle \mathbf{id}, 2 \rangle$, where 2 points to the symbol-table entry for `initial`.

4. `+` is a lexeme that is mapped into the token $\langle + \rangle$.

5. `rate` is a lexeme that is mapped into the token $\langle \mathbf{id}, 3 \rangle$, where 3 points to the symbol-table entry for `rate`.

6. `*` is a lexeme that is mapped into the token $\langle * \rangle$.

7. `60` is a lexeme that is mapped into the token $\langle 60 \rangle$.[1]

Blanks separating the lexemes would be discarded by the lexical analyzer.

Figure 1.7 shows the representation of the assignment statement (1.1) after lexical analysis as the sequence of tokens

$$\langle \mathbf{id}, 1 \rangle \; \langle = \rangle \; \langle \mathbf{id}, 2 \rangle \; \langle + \rangle \; \langle \mathbf{id}, 3 \rangle \; \langle * \rangle \; \langle 60 \rangle \tag{1.2}$$

In this representation, the token names $=$, $+$, and $*$ are abstract symbols for the assignment, addition, and multiplication operators, respectively.

[1]Technically speaking, for the lexeme `60` we should make up a token like $\langle \mathbf{number}, 4 \rangle$, where 4 points to the symbol table for the internal representation of integer 60 but we shall defer the discussion of tokens for numbers until Chapter 2. Chapter 3 discusses techniques for building lexical analyzers.

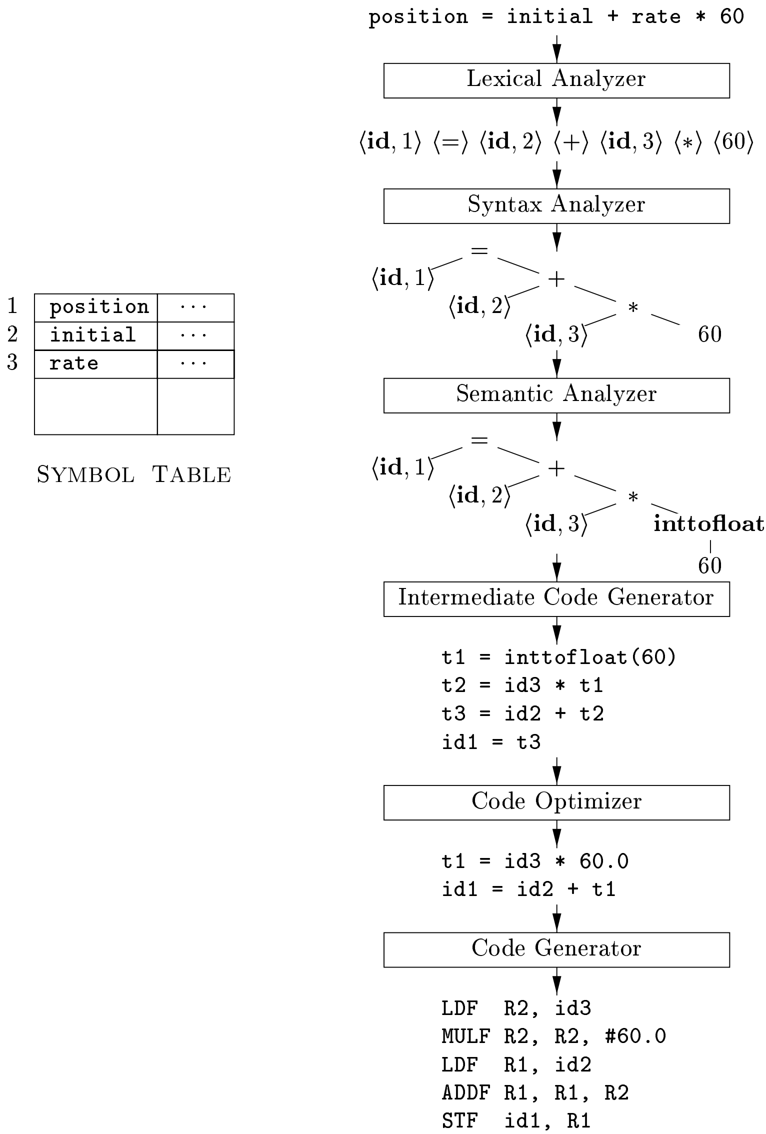


Figure 1.7: Translation of an assignment statement

1.2.2 Syntax Analysis

The second phase of the compiler is *syntax analysis* or *parsing*. The parser uses the first components of the tokens produced by the lexical analyzer to create a tree-like intermediate representation that depicts the grammatical structure of the token stream. A typical representation is a *syntax tree* in which each interior node represents an operation and the children of the node represent the arguments of the operation. A syntax tree for the token stream (1.2) is shown as the output of the syntactic analyzer in Fig. 1.7.

This tree shows the order in which the operations in the assignment

```
position = initial + rate * 60
```

are to be performed. The tree has an interior node labeled $*$ with $\langle \mathbf{id}, 3\rangle$ as its left child and the integer 60 as its right child. The node $\langle \mathbf{id}, 3\rangle$ represents the identifier `rate`. The node labeled $*$ makes it explicit that we must first multiply the value of `rate` by 60. The node labeled $+$ indicates that we must add the result of this multiplication to the value of `initial`. The root of the tree, labeled $=$, indicates that we must store the result of this addition into the location for the identifier `position`. This ordering of operations is consistent with the usual conventions of arithmetic which tell us that multiplication has higher precedence than addition, and hence that the multiplication is to be performed before the addition.

The subsequent phases of the compiler use the grammatical structure to help analyze the source program and generate the target program. In Chapter 4 we shall use context-free grammars to specify the grammatical structure of programming languages and discuss algorithms for constructing efficient syntax analyzers automatically from certain classes of grammars. In Chapters 2 and 5 we shall see that syntax-directed definitions can help specify the translation of programming language constructs.

1.2.3 Semantic Analysis

The *semantic analyzer* uses the syntax tree and the information in the symbol table to check the source program for semantic consistency with the language definition. It also gathers type information and saves it in either the syntax tree or the symbol table, for subsequent use during intermediate-code generation.

An important part of semantic analysis is *type checking*, where the compiler checks that each operator has matching operands. For example, many programming language definitions require an array index to be an integer; the compiler must report an error if a floating-point number is used to index an array.

The language specification may permit some type conversions called *coercions*. For example, a binary arithmetic operator may be applied to either a pair of integers or to a pair of floating-point numbers. If the operator is applied to a floating-point number and an integer, the compiler may convert or coerce the integer into a floating-point number.

Such a coercion appears in Fig. 1.7. Suppose that `position`, `initial`, and `rate` have been declared to be floating-point numbers, and that the lexeme `60` by itself forms an integer. The type checker in the semantic analyzer in Fig. 1.7 discovers that the operator $*$ is applied to a floating-point number `rate` and an integer 60. In this case, the integer may be converted into a floating-point number. In Fig. 1.7, notice that the output of the semantic analyzer has an extra node for the operator **inttofloat**, which explicitly converts its integer argument into a floating-point number. Type checking and semantic analysis are discussed in Chapter 6.

1.2.4 Intermediate Code Generation

In the process of translating a source program into target code, a compiler may construct one or more intermediate representations, which can have a variety of forms. Syntax trees are a form of intermediate representation; they are commonly used during syntax and semantic analysis.

After syntax and semantic analysis of the source program, many compilers generate an explicit low-level or machine-like intermediate representation, which we can think of as a program for an abstract machine. This intermediate representation should have two important properties: it should be easy to produce and it should be easy to translate into the target machine.

In Chapter 6, we consider an intermediate form called *three-address code*, which consists of a sequence of assembly-like instructions with three operands per instruction. Each operand can act like a register. The output of the intermediate code generator in Fig. 1.7 consists of the three-address code sequence

```
t1 = inttofloat(60)
t2 = id3 * t1
t3 = id2 + t2
id1 = t3
```
(1.3)

There are several points worth noting about three-address instructions. First, each three-address assignment instruction has at most one operator on the right side. Thus, these instructions fix the order in which operations are to be done; the multiplication precedes the addition in the source program (1.1). Second, the compiler must generate a temporary name to hold the value computed by a three-address instruction. Third, some "three-address instructions" like the first and last in the sequence (1.3), above, have fewer than three operands.

In Chapter 6, we cover the principal intermediate representations used in compilers. Chapter 5 introduces techniques for syntax-directed translation that are applied in Chapter 6 to type checking and intermediate-code generation for typical programming language constructs such as expressions, flow-of-control constructs, and procedure calls.

1.2.5 Code Optimization

The machine-independent code-optimization phase attempts to improve the intermediate code so that better target code will result. Usually better means faster, but other objectives may be desired, such as shorter code, or target code that consumes less power. For example, a straightforward algorithm generates the intermediate code (1.3), using an instruction for each operator in the tree representation that comes from the semantic analyzer.

A simple intermediate code generation algorithm followed by code optimization is a reasonable way to generate good target code. The optimizer can deduce that the conversion of 60 from integer to floating point can be done once and for all at compile time, so the **inttofloat** operation can be eliminated by replacing the integer 60 by the floating-point number 60.0. Moreover, `t3` is used only once to transmit its value to `id1` so the optimizer can transform (1.3) into the shorter sequence

```
t1 = id3 * 60.0
id1 = id2 + t1
```
(1.4)

There is a great variation in the amount of code optimization different compilers perform. In those that do the most, the so-called "optimizing compilers," a significant amount of time is spent on this phase. There are simple optimizations that significantly improve the running time of the target program without slowing down compilation too much. The chapters from 8 on discuss machine-independent and machine-dependent optimizations in detail.

1.2.6 Code Generation

The code generator takes as inpu[illegible]ediate representation of the source program and maps it into the targ[illegible]e. If the target language is machine code, registers or memory locations are selected for each of the variables used by the program. Then, the intermediate instructions are translated into sequences of machine instructions that perform the same task. A crucial aspect of code generation is the judicious assignment of registers to hold variables.

For example, using registers `R1` and `R2`, the intermediate code in (1.4) might get translated into the machine code

```
LDF  R2,  id3
MULF R2,  R2, #60.0
LDF  R1,  id2
ADDF R1,  R1, R2
STF  id1, R1
```
(1.5)

The first operand of each instruction specifies a destination. The `F` in each instruction tells us that it deals with floating-point numbers. The code in

(1.5) loads the contents of address id3 into register R2, then multiplies it with floating-point constant 60.0. The # signifies that 60.0 is to be treated as an immediate constant. The third instruction moves id2 into register R1 and the fourth adds to it the value previously computed in register R2. Finally, the value in register R1 is stored into the address of id1, so the code correctly implements the assignment statement (1.1). Chapter 8 covers code generation.

This discussion of code generation has ignored the important issue of storage allocation for the identifiers in the source program. As we shall see in Chapter 7, the organization of storage at run-time depends on the language being compiled. Storage-allocation decisions are made either during intermediate code generation or during code generation.

1.2.7 Symbol-Table Management

An essential function of a compiler is to record the variable names used in the source program and collect information about various attributes of each name. These attributes may provide information about the storage allocated for a name, its type, its scope (where in the program its value may be used), and in the case of procedure names, such things as the number and types of its arguments, the method of passing each argument (for example, by value or by reference), and the type returned.

The symbol table is a data structure containing a record for each variable name, with fields for the attributes of the name. The data structure should be designed to allow the compiler to find the record for each name quickly and to store or retrieve data from that record quickly. Symbol tables are discussed in Chapter 2.

1.2.8 The Grouping of Phases into Passes

The discussion of phases deals with the logical organization of a compiler. In an implementation, activities from several phases may be grouped together into a *pass* that reads an input file and writes an output file. For example, the front-end phases of lexical analysis, syntax analysis, semantic analysis, and intermediate code generation might be grouped together into one pass. Code optimization might be an optional pass. Then there could be a back-end pass consisting of code generation for a particular target machine.

Some compiler collections have been created around carefully designed intermediate representations that allow the front end for a particular language to interface with the back end for a certain target machine. With these collections, we can produce compilers for different source languages for one target machine by combining different front ends with the back end for that target machine. Similarly, we can produce compilers for different target machines, by combining a front end with back ends for different target machines.

1.2.9 Compiler-Construction Tools

The compiler writer, like any software developer, can profitably use modern software development environments containing tools such as language editors, debuggers, version managers, profilers, test harnesses, and so on. In addition to these general software-development tools, other more specialized tools have been created to help implement various phases of a compiler.

These tools use specialized languages for specifying and implementing specific components, and many use quite sophisticated algorithms. The most successful tools are those that hide the details of the generation algorithm and produce components that can be easily integrated into the remainder of the compiler. Some commonly used compiler-construction tools include

1. *Parser generators* that automatically produce syntax analyzers from a grammatical description of a programming language.

2. *Scanner generators* that produce lexical analyzers from a regular-expression description of the tokens of a language.

3. *Syntax-directed translation engines* that produce collections of routines for walking a parse tree and generating intermediate code.

4. *Code-generator generators* that produce a code generator from a collection of rules for translating each operation of the intermediate language into the machine language for a target machine.

5. *Data-flow analysis engines* that facilitate the gathering of information about how values are transmitted from one part of a program to each other part. Data-flow analysis is a key part of code optimization.

6. *Compiler-construction toolkits* that provide an integrated set of routines for constructing various phases of a compiler.

We shall describe many of these tools throughout this book.

1.3 The Evolution of Programming Languages

The first electronic computers appeared in the 1940's and were programmed in machine language by sequences of 0's and 1's that explicitly told the computer what operations to execute and in what order. The operations themselves were very low level: move data from one location to another, add the contents of two registers, compare two values, and so on. Needless to say, this kind of programming was slow, tedious, and error prone. And once written, the programs were hard to understand and modify.

Chapter 3

Lexical Analysis

In this chapter we show how to construct a lexical analyzer. To implement a lexical analyzer by hand, it helps to start with a diagram or other description for the lexemes of each token. We can then write code to identify each occurrence of each lexeme on the input and to return information about the token identified.

We can also produce a lexical analyzer automatically by specifying the lexeme patterns to a *lexical-analyzer generator* and compiling those patterns into code that functions as a lexical analyzer. This approach makes it easier to modify a lexical analyzer, since we have only to rewrite the affected patterns, not the entire program. It also speeds up the process of implementing the lexical analyzer, since the programmer specifies the software at the very high level of patterns and relies on the generator to produce the detailed code. We shall introduce in Section 3.5 a lexical-analyzer generator called *Lex* (or *Flex* in a more recent embodiment).

We begin the study of lexical-analyzer generators by introducing regular expressions, a convenient notation for specifying lexeme patterns. We show how this notation can be transformed, first into nondeterministic automata and then into deterministic automata. The latter two notations can be used as input to a "driver," that is, code which simulates these automata and uses them as a guide to determining the next token. This driver and the specification of the automaton form the nucleus of the lexical analyzer.

3.1 The Role of the Lexical Analyzer

As the first phase of a compiler, the main task of the lexical analyzer is to read the input characters of the source program, group them into lexemes, and produce as output a sequence of tokens for each lexeme in the source program. The stream of tokens is sent to the parser for syntax analysis. It is common for the lexical analyzer to interact with the symbol table as well. When the lexical analyzer discovers a lexeme constituting an identifier, it needs to enter that lexeme into the symbol table. In some cases, information regarding the kind of identifier may be read from the symbol table by the lexical analyzer to assist it in determining the proper token it must pass to the parser.

These interactions are suggested in Fig. 3.1. Commonly, the interaction is implemented by having the parser call the lexical analyzer. The call, suggested by the *getNextToken* command, causes the lexical analyzer to read characters from its input until it can identify the next lexeme and produce for it the next token, which it returns to the parser.

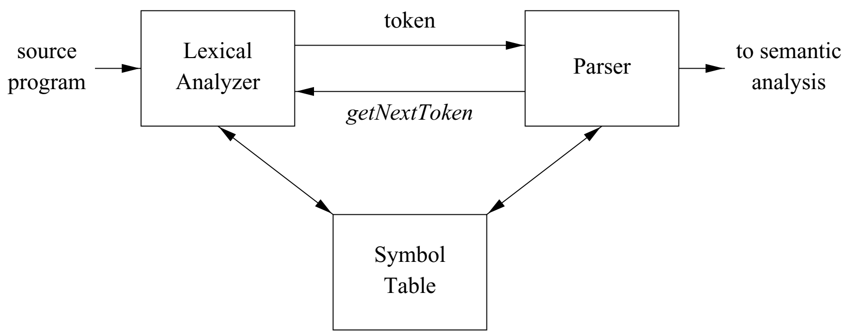


Figure 3.1: Interactions between the lexical analyzer and the parser

Since the lexical analyzer is the part of the compiler that reads the source text, it may perform certain other tasks besides identification of lexemes. One such task is stripping out comments and *whitespace* (blank, newline, tab, and perhaps other characters that are used to separate tokens in the input). Another task is correlating error messages generated by the compiler with the source program. For instance, the lexical analyzer may keep track of the number of newline characters seen, so it can associate a line number with each error message. In some compilers, the lexical analyzer makes a copy of the source program with the error messages inserted at the appropriate positions. If the source program uses a macro-preprocessor, the expansion of macros may also be performed by the lexical analyzer.

Sometimes, lexical analyzers are divided into a cascade of two processes:

a) *Scanning* consists of the simple processes that do not require tokenization of the input, such as deletion of comments and compaction of consecutive whitespace characters into one.

b) *Lexical analysis* proper is the more complex portion, which produces tokens from the output of the scanner.

3.1.1 Lexical Analysis Versus Parsing

There are a number of reasons why the analysis portion of a compiler is normally separated into lexical analysis and parsing (syntax analysis) phases.

1. Simplicity of design is the most important consideration. The separation of lexical and syntactic analysis often allows us to simplify at least one of these tasks. For example, a parser that had to deal with comments and whitespace as syntactic units would be considerably more complex than one that can assume comments and whitespace have already been removed by the lexical analyzer. If we are designing a new language, separating lexical and syntactic concerns can lead to a cleaner overall language design.

2. Compiler efficiency is improved. A separate lexical analyzer allows us to apply specialized techniques that serve only the lexical task, not the job of parsing. In addition, specialized buffering techniques for reading input characters can speed up the compiler significantly.

3. Compiler portability is enhanced. Input-device-specific peculiarities can be restricted to the lexical analyzer.

3.1.2 Tokens, Patterns, and Lexemes

When discussing lexical analysis, we use three related but distinct terms:

- A *token* is a pair consisting of a token name and an optional attribute value. The token name is an abstract symbol representing a kind of lexical unit, e.g., a particular keyword, or a sequence of input characters denoting an identifier. The token names are the input symbols that the parser processes. In what follows, we shall generally write the name of a token in boldface. We will often refer to a token by its token name.

- A *pattern* is a description of the form that the lexemes of a token may take. In the case of a keyword as a token, the pattern is just the sequence of characters that form the keyword. For identifiers and some other tokens, the pattern is a more complex structure that is *matched* by many strings.

- A *lexeme* is a sequence of characters in the source program that matches the pattern for a token and is identified by the lexical analyzer as an instance of that token.

Example 3.1 : Figure 3.2 gives some typical tokens, their informally described patterns, and some sample lexemes. To see how these concepts are used in practice, in the C statement

```
printf("Total = %d\n", score);
```

both `printf` and `score` are lexemes matching the pattern for token **id**, and `"Total = %d\n"` is a lexeme matching **literal**. □

In many programming languages, the following classes cover most or all of the tokens:

TOKEN	INFORMAL DESCRIPTION	SAMPLE LEXEMES
if	characters `i`, `f`	`if`
else	characters `e`, `l`, `s`, `e`	`else`
comparison	`<` or `>` or `<=` or `>=` or `==` or `!=`	`<=`, `!=`
id	letter followed by letters and digits	`pi`, `score`, `D2`
number	any numeric constant	`3.14159`, `0`, `6.02e23`
literal	anything but `"`, surrounded by `"`'s	`"core dumped"`

Figure 3.2: Examples of tokens

1. One token for each keyword. The pattern for a k[illegible]the same as the keyword itself.

2. Tokens for the operators, either individually or in classes such as the token `comparison` mentioned in Fig. 3.2.

3. One token representing all identifiers.

4. One or more tokens representing constants, such as numbers and literal strings.

5. Tokens for each punctuation symbol, such as left and right parentheses, comma, and semicolon.

3.1.3 Attributes for Tokens

When more than one lexeme can match a pattern, the lexical analyzer must provide the subsequent compiler phases additional information about the particular lexeme that matched. For example, the pattern for token **number** matches both 0 and 1, but it is extremely important for the code generator to know which lexeme was found in the source program. Thus, in many cases the lexical analyzer returns to the parser not only a token name, but an attribute value that describes the lexeme represented by the token; the token name influences parsing decisions, while the attribute value influences translation of tokens after the parse.

We shall assume that tokens have at most one associated attribute, although this attribute may have a structure that combines several pieces of information. The most important example is the token **id**, where we need to associate with the token a great deal of information. Normally, information about an identifier — e.g., its lexeme, its type, and the location at which it is first found (in case an error message about that identifier must be issued) — is kept in the symbol table. Thus, the appropriate attribute value for an identifier is a pointer to the symbol-table entry for that identifier.

Tricky Problems When Recognizing Tokens

Usually, given the pattern describing the lexemes of a token, it is relatively simple to recognize matching lexemes when they occur on the input. However, in some languages it is not immediately apparent when we have seen an instance of a lexeme corresponding to a token. The following example is taken from Fortran, in the fixed-format still allowed in Fortran 90. In the statement

```
DO 5 I = 1.25
```

it is not apparent that the first lexeme is `DO5I`, an instance of the identifier token, until we see the dot following the 1. Note that blanks in fixed-format Fortran are ignored (an archaic convention). Had we seen a comma instead of the dot, we would have had a do-statement

```
DO 5 I = 1,25
```

in which the first lexeme is the keyword `DO`.

Example 3.2: The token names and associated attribute values for the Fortran statement

```
E = M * C ** 2
```

are written below as a sequence of pairs.

<**id**, pointer to symbol-table entry for `E`>
<**assign_op**>
<**id**, pointer to symbol-table entry for `M`>
<**mult_op**>
<**id**, pointer to symbol-table entry for `C`>
<**exp_op**>
<**number**, integer value 2>

Note that in certain pairs, especially operators, punctuation, and keywords, there is no need for an attribute value. In this example, the token **number** has been given an integer-valued attribute. In practice, a typical compiler would instead store a character string representing the constant and use as an attribute value for **number** a pointer to that string. □

3.1.4 Lexical Errors

It is hard for a lexical analyzer to tell, without the aid of other components, that there is a source-code error. For instance, if the string `fi` is encountered for the first time in a C program in the context:

```
fi ( a == f(x)) ...
```

a lexical analyzer cannot tell whether `fi` is a misspelling of the keyword `if` or an undeclared function identifier. Since `fi` is a valid lexeme for the token **id**, the lexical analyzer must return the token **id** to the parser and let some other phase of the compiler — probably the parser in this case — handle an error due to transposition of the letters.

However, suppose a situation arises in which the lexical analyzer is unable to proceed because none of the patterns for tokens matches any prefix of the remaining input. The simplest recovery strategy is "panic mode" recovery. We delete successive characters from the remaining input, until the lexical analyzer can find a well-formed token at the beginning of what input is left. This recovery technique may confuse the parser, but in an interactive computing environment it may be quite adequate.

Other possible error-recovery actions are:

1. Delete one character from the remaining input.

2. Insert a missing character into the remaining input.

3. Replace a character by another character.

4. Transpose two adjacent characters.

Transformations like these may be tried in an attempt to repair the input. The simplest such strategy is to see whether a prefix of the remaining input can be transformed into a valid lexeme by a single transformation. This strategy makes sense, since in practice most lexical errors involve a single character. A more general correction strategy is to find the smallest number of transformations needed to convert the source program into one that consists only of valid lexemes, but this approach is considered too expensive in practice to be worth the effort.

3.1.5 Exercises for Section 3.1

Exercise 3.1.1: Divide the following C++ program:

```
float limitedSquare(x) float x; {
    /* returns x-squared, but never more than 100 */
    return (x<=-10.0||x>=10.0)?100:x*x;
}
```

into appropriate lexemes, using the discussion of Section 3.1.2 as a guide. Which lexemes should get associated lexical values? What should those values be?

! **Exercise 3.1.2:** Tagged languages like HTML or XML are different from conventional programming languages in that the punctuation (tags) are either very numerous (as in HTML) or a user-definable set (as in XML). Further, tags can often have parameters. Suggest how to divide the following HTML document:

```
Here is a photo of <B>my house</B>:
<P><IMG SRC = "house.gif"><BR>
See <A HREF = "morePix.html">More Pictures</A> if you
liked that one.<P>
```

into appropriate lexemes. Which lexemes should get associated lexical values, and what should those values be?

3.2 Input Buffering

Before discussing the problem of recognizing lexemes in the input, let us examine some ways that the simple but important task of reading the source program can be speeded. This task is made difficult by the fact that we often have to look one or more characters beyond the next lexeme before we can be sure we have the right lexeme. The box on "Tricky Problems When Recognizing Tokens" in Section 3.1 gave an extreme example, but there are many situations where we need to look at least one additional character ahead. For instance, we cannot be sure we've seen the end of an identifier until we see a character that is not a letter or digit, and therefore is not part of the lexeme for **id**. In C, single-character operators like `-`, `=`, or `<` could also be the beginning of a two-character operator like `->`, `==`, or `<=`. Thus, we shall introduce a two-buffer scheme that handles large lookaheads safely. We then consider an improvement involving "sentinels" that saves time checking for the ends of buffers.

3.2.1 Buffer Pairs

Because of the amount of time taken to process characters and the large number of characters that must be processed during the compilation of a large source program, specialized buffering techniques have been developed to reduce the amount of overhead required to process a single input character. An important scheme involves two buffers that are alternately reloaded, as suggested in Fig. 3.3.

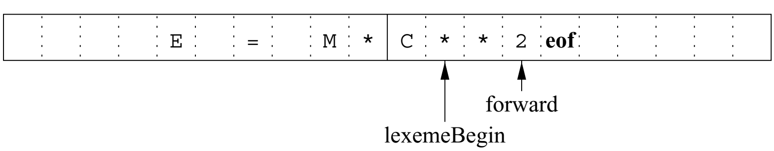


Figure 3.3: Using a pair of input buffers

Each buffer is of the same size N, and N is usually the size of a disk block, e.g., 4096 bytes. Using one system read command we can read N characters into a buffer, rather than using one system call per character. If fewer than N characters remain in the input file, then a special character, represented by **eof**, marks the end of the source file and is different from any possible character of the source program.

Two pointers to the input are maintained:

1. Pointer `lexemeBegin`, marks the beginning of the current lexeme, whose extent we are attempting to determine.

2. Pointer `forward` scans ahead until a pattern match is found; the exact strategy whereby this determination is made will be covered in the balance of this chapter.

Once the next lexeme is determined, `forward` is set to the character at its right end. Then, after the lexeme is recorded as an attribute value of a token returned to the parser, `lexemeBegin` is set to the character immediately after the lexeme just found. In Fig. 3.3, we see `forward` has passed the end of the next lexeme, `**` (the Fortran exponentiation operator), and must be retracted one position to its left.

Advancing `forward` requires that we first test whether we have reached the end of one of the buffers, and if so, we must reload the other buffer from the input, and move `forward` to the beginning of the newly loaded buffer. As long as we never need to look so far ahead of the actual lexeme that the sum of the lexeme's length plus the distance we look ahead is greater than N, we shall never overwrite the lexeme in its buffer before determining it.

3.2.2 Sentinels

If we use the scheme of Section 3.2.1 as described, we must check, each time we advance `forward`, that we have not moved off one of the buffers; if we do, then we must also reload the other buffer. Thus, for each character read, we make two tests: one for the end of the buffer, and one to determine what character is read (the latter may be a multiway branch). We can combine the buffer-end test with the test for the current character if we extend each buffer to hold a *sentinel* character at the end. The sentinel is a special character that cannot be part of the source program, and a natural choice is the character **eof**.

Figure 3.4 shows the same arrangement as Fig. 3.3, but with the sentinels added. Note that **eof** retains its use as a marker for the end of the entire input. Any **eof** that appears other than at the end of a buffer means that the input is at an end. Figure 3.5 summarizes the algorithm for advancing `forward`. Notice how the first test, which can be part of a multiway branch based on the character pointed to by `forward`, is the only test we make, except in the case where we actually are at the end of a buffer or the end of the input.

3.3 Specification of Tokens

Regular expressions are an important notation for specifying lexeme patterns. While they cannot express all possible patterns, they are very effective in spec-

> **Can We Run Out of Buffer Space?**
>
> In most modern languages, lexemes are short, and one or two characters of lookahead is sufficient. Thus a buffer size N in the thousands is ample, and the double-buffer scheme of Section 3.2.1 works without problem. However, there are some risks. For example, if character strings can be very long, extending over many lines, then we could face the possibility that a lexeme is longer than N. To avoid problems with long character strings, we can treat them as a concatenation of components, one from each line over which the string is written. For instance, in Java it is conventional to represent long strings by writing a piece on each line and concatenating pieces with a `+` operator at the end of each piece.
>
> A more difficult problem occurs when arbitrarily long lookahead may be needed. For example, some languages like PL/I do not treat keywords as *reserved*; that is, you can use identifiers with the same name as a keyword like `DECLARE`. If the lexical analyzer is presented with text of a PL/I program that begins `DECLARE ( ARG1, ARG2,...` it cannot be sure whether `DECLARE` is a keyword, and `ARG1` and so on are variables being declared, or whether `DECLARE` is a procedure name with its arguments. For this reason, modern languages tend to reserve their keywords. However, if not, one can treat a keyword like `DECLARE` as an ambiguous identifier, and let the parser resolve the issue, perhaps in conjunction with symbol-table lookup.

ifying those types of patterns that we actually need for tokens. In this section we shall study the formal notation for regular expressions, and in Section 3.5 we shall see how these expressions are used in a lexical-analyzer generator. Then, Section 3.7 shows how to build the lexical analyzer by converting regular expressions to automata that perform the recognition of the specified tokens.

3.3.1 Strings and Languages

An *alphabet* is any finite set of symbols. Typical examples of symbols are letters, digits, and punctuation. The set $\{0, 1\}$ is the *binary alphabet*. ASCII is an important example of an alphabet; it is used in many software systems. Uni-

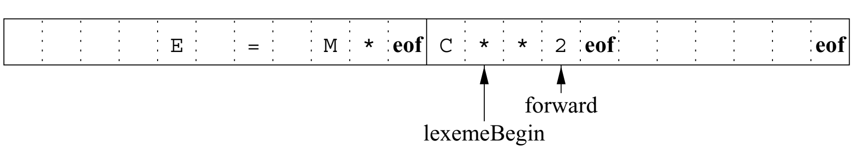


Figure 3.4: Sentinels at the end of each buffer

```
switch ( *forward++ ) {
    case eof:
        if (forward is at end of first buffer ) {
            reload second buffer;
            forward = beginning of second buffer;
        }
        else if (forward is at end of second buffer ) {
            reload first buffer;
            forward = beginning of first buffer;
        }
        else /* eof within a buffer marks the end of input */
            terminate lexical analysis;
        break;
    Cases for the other characters
}
```

Figure 3.5: Lookahead code with sentinels

Implementing Multiway Branches

We might imagine that the switch in Fig. 3.5 requires many steps to execute, and that placing the case **eof** first is not a wise choice. Actually, it doesn't matter in what order we list the cases for each character. In practice, a multiway branch depending on the input character is made in one step by jumping to an address found in an array of addresses, indexed by characters.

code, which includes approximately 100,000 characters from alphabets around the world, is another important example of an alphabet.

A *string* over an alphabet is a finite sequence of symbols drawn from that alphabet. In language theory, the terms "sentence" and "word" are often used as synonyms for "string." The length of a string s, usually written $|s|$, is the number of occurrences of symbols in s. For example, `banana` is a string of length six. The *empty string*, denoted ϵ, is the string of length zero.

A *language* is any countable set of strings over some fixed alphabet. This definition is very broad. Abstract languages like $\emptyset$, the *empty set*, or $\{\epsilon\}$, the set containing only the empty string, are languages under this definition. So too are the set of all syntactically well-formed C programs and the set of all grammatically correct English sentences, although the latter two languages are difficult to specify exactly. Note that the definition of "language" does not require that any meaning be ascribed to the strings in the language. Methods for defining the "meaning" of strings are discussed in Chapter 5.

Terms for Parts of Strings

The following string-related terms are commonly used:

1. A *prefix* of string s is any string obtained by removing zero or more symbols from the end of s. For example, `ban`, `banana`, and ϵ are prefixes of `banana`.

2. A *suffix* of string s is any string obtained by removing zero or more symbols from the beginning of s. For example, `nana`, `banana`, and ϵ are suffixes of `banana`.

3. A *substring* of s is obtained by deleting any prefix and any suffix from s. For instance, `banana`, `nan`, and ϵ are substrings of `banana`.

4. The *proper* prefixes, suffixes, and substrings of a string s are those, prefixes, suffixes, and substrings, respectively, of s that are not ϵ or not equal to s itself.

5. A *subsequence* of s is any string formed by deleting zero or more not necessarily consecutive positions of s. For example, `baan` is a subsequence of `banana`.

If x and y are strings, then the *concatenation* of x and y, denoted xy, is the string formed by appending y to x. For example, if x = `dog` and y = `house`, then xy = `doghouse`. The empty string is the identity under concatenation; that is, for any string s, $\epsilon s = s\epsilon = s$.

If we think of concatenation as a product, we can define the "exponentiation" of strings as follows. Define s^0 to be ϵ, and for all $i > 0$, define s^i to be $s^{i-1}s$. Since $\epsilon s = s$, it follows that $s^1 = s$. Then $s^2 = ss$, $s^3 = sss$, and so on.

3.3.2 Operations on Languages

In lexical analysis, the most important operations on languages are union, concatenation, and closure, which are defined formally in Fig. 3.6. Union is the familiar operation on sets. The concatenation of languages is all strings formed by taking a string from the first language and a string from the second language, in all possible ways, and concatenating them. The (*Kleene*) *closure* of a language L, denoted L^*, is the set of strings you get by concatenating L zero or more times. Note that L^0, the "concatenation of L zero times," is defined to be $\{\epsilon\}$, and inductively, L^i is $L^{i-1}L$. Finally, the positive closure, denoted L^+, is the same as the Kleene closure, but without the term L^0. That is, ϵ will not be in L^+ unless it is in L itself.

OPERATION	DEFINITION AND NOTATION
Union of L and M	$L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
Concatenation of L and M	$LM = \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$
Kleene closure of L	$L^* = \cup_{i=0}^{\infty} L^i$
Positive closure of L	$L^+ = \cup_{i=1}^{\infty} L^i$

Figure 3.6: Definitions of operations on languages

Example 3.3: Let L be the set of letters $\{\texttt{A}, \texttt{B}, \ldots, \texttt{Z}, \texttt{a}, \texttt{b}, \ldots, \texttt{z}\}$ and let D be the set of digits $\{\texttt{0}, \texttt{1}, \ldots \texttt{9}\}$. We may think of L and D in two, essentially equivalent, ways. One way is that L and D are, respectively, the alphabets of uppercase and lowercase letters and of digits. The second way is that L and D are languages, all of whose strings happen to be of length one. Here are some other languages that can be constructed from languages L and D, using the operators of Fig. 3.6:

1. $L \cup D$ is the set of letters and digits — strictly speaking the language with 62 strings of length one, each of which strings is either one letter or one digit.

2. LD is the set of 520 strings of length two, each consisting of one letter followed by one digit.

3. L^4 is the set of all 4-letter strings.

4. L^* is the set of all strings of letters, including ϵ, the empty string.

5. $L(L \cup D)^*$ is the set of all strings of letters and digits beginning with a letter.

6. D^+ is the set of all strings of one or more digits.

□

3.3.3 Regular Expressions

Suppose we wanted to describe the set of valid C identifiers. It is almost exactly the language described in item (5) above; the only difference is that the underscore is included among the letters.

In Example 3.3, we were able to describe identifiers by giving names to sets of letters and digits and using the language operators union, concatenation, and closure. This process is so useful that a notation called *regular expressions* has come into common use for describing all the languages that can be built from these operators applied to the symbols of some alphabet. In this notation, if *letter_* is established to stand for any letter or the underscore, and *digit* is established to stand for any digit, then we could describe the language of C identifiers by:

$$letter\_ \ (\ letter\_ \mid digit \ )^*$$

The vertical bar above means union, the parentheses are used to group subexpressions, the star means "zero or more occurrences of," and the juxtaposition of *letter_* with the remainder of the expression signifies concatenation.

The regular expressions are built recursively out of smaller regular expressions, using the rules described below. Each regular expression r denotes a language $L(r)$, which is also defined recursively from the languages denoted by r's subexpressions. Here are the rules that define the regular expressions over some alphabet Σ and the languages that those expressions denote.

BASIS: There are two rules that form the basis:

1. ϵ is a regular expression, and $L(\epsilon)$ is $\{\epsilon\}$, that is, the language whose sole member is the empty string.

2. If a is a symbol in Σ, then $\mathbf{a}$ is a regular expression, and $L(\mathbf{a}) = \{a\}$, that is, the language with one string, of length one, with a in its one position. Note that by convention, we use italics for symbols, and boldface for their corresponding regular expression.[1]

INDUCTION: There are four parts to the induction whereby larger regular expressions are built from smaller ones. Suppose r and s are regular expressions denoting languages $L(r)$ and $L(s)$, respectively.

1. $(r)|(s)$ is a regular expression denoting the language $L(r) \cup L(s)$.

2. $(r)(s)$ is a regular expression denoting the language $L(r)L(s)$.

3. $(r)^*$ is a regular expression denoting $\big(L(r)\big)^*$.

4. (r) is a regular expression denoting $L(r)$. This last rule says that we can add additional pairs of parentheses around expressions without changing the language they denote.

As defined, regular expressions often contain unnecessary pairs of parentheses. We may drop certain pairs of parentheses if we adopt the conventions that:

a) The unary operator $*$ has highest precedence and is left associative.

b) Concatenation has second highest precedence and is left associative.

[1] However, when talking about specific characters from the ASCII character set, we shall generally use teletype font for both the character and its regular expression.

c) $|$ has lowest precedence and is left associative.

Under these conventions, for example, we may replace the regular expression $(\mathbf{a})|((\mathbf{b})^*(\mathbf{c}))$ by $\mathbf{a}|\mathbf{b}^*\mathbf{c}$. Both expressions denote the set of strings that are either a single a or are zero or more b's followed by one c.

Example 3.4: Let $\Sigma = \{a, b\}$.

1. The regular expression $\mathbf{a}|\mathbf{b}$ denotes the language $\{a, b\}$.

2. $(\mathbf{a}|\mathbf{b})(\mathbf{a}|\mathbf{b})$ denotes $\{aa, ab, ba, bb\}$, the language of all strings of length two over the alphabet Σ. Another regular expression for the same language is $\mathbf{aa}|\mathbf{ab}|\mathbf{ba}|\mathbf{bb}$.

3. $\mathbf{a}^*$ denotes the language consisting of all strings of zero or more a's, that is, $\{\epsilon, a, aa, aaa, \ldots\}$.

4. $(\mathbf{a}|\mathbf{b})^*$ denotes the set of all strings consisting of zero or more instances of a or b, that is, all strings of a's and b's: $\{\epsilon, a, b, aa, ab, ba, bb, aaa, \ldots\}$. Another regular expression for the same language is $(\mathbf{a}^*\mathbf{b}^*)^*$.

5. $\mathbf{a}|\mathbf{a}^*\mathbf{b}$ denotes the language $\{a, b, ab, aab, aaab, \ldots\}$, that is, the string a and all strings consisting of zero or more a's and ending in b.

□

A language that can be defined by a regular expression is called a *regular set*. If two regular expressions r and s denote the same regular set, we say they are *equivalent* and write $r = s$. For instance, $(\mathbf{a}|\mathbf{b}) = (\mathbf{b}|\mathbf{a})$. There are a number of algebraic laws for regular expressions; each law asserts that expressions of two different forms are equivalent. Figure 3.7 shows some of the algebraic laws that hold for arbitrary regular expressions r, s, and t.

LAW	DESCRIPTION
$r\|s = s\|r$	$\|$ is commutative
$r\|(s\|t) = (r\|s)\|t$	$\|$ is associative
$r(st) = (rs)t$	Concatenation is associative
$r(s\|t) = rs\|rt$; $(s\|t)r = sr\|tr$	Concatenation distributes over $\|$
$\epsilon r = r\epsilon = r$	ϵ is the identity for concatenation
$r^* = (r\|\epsilon)^*$	ϵ is guaranteed in a closure
$r^{**} = r^*$	* is idempotent

Figure 3.7: Algebraic laws for regular expressions

3.3.4 Regular Definitions

For notational convenience, we may wish to give names to certain regular expressions and use those names in subsequent expressions, as if the names were themselves symbols. If Σ is an alphabet of basic symbols, then a *regular definition* is a sequence of definitions of the form:

$$\begin{array}{ccc} d_1 & \to & r_1 \\ d_2 & \to & r_2 \\ & \cdots & \\ d_n & \to & r_n \end{array}$$

where:

1. Each d_i is a new symbol, not in Σ and not the same as any other of the d's, and

2. Each r_i is a regular expression over the alphabet $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$.

By restricting r_i to Σ and the previously defined d's, we avoid recursive definitions, and we can construct a regular expression over Σ alone, for each r_i. We do so by first replacing uses of d_1 in r_2 (which cannot use any of the d's except for d_1), then replacing uses of d_1 and d_2 in r_3 by r_1 and (the substituted) r_2, and so on. Finally, in r_n we replace each d_i, for $i = 1, 2, \ldots, n-1$, by the substituted version of r_i, each of which has only symbols of Σ.

Example 3.5 : C identifiers are strings of letters, digits, and underscores. Here is a regular definition for the language of C identifiers. We shall conventionally use italics for the symbols defined in regular definitions.

$$\begin{array}{rcl} \mathit{letter\_} & \to & \mathtt{A} \mid \mathtt{B} \mid \cdots \mid \mathtt{Z} \mid \mathtt{a} \mid \mathtt{b} \mid \cdots \mid \mathtt{z} \mid \mathtt{\_} \\ \mathit{digit} & \to & \mathtt{0} \mid \mathtt{1} \mid \cdots \mid \mathtt{9} \\ \mathit{id} & \to & \mathit{letter\_}\ (\ \mathit{letter\_} \mid \mathit{digit}\ )^* \end{array}$$

□

Example 3.6 : Unsigned numbers (integer or floating point) are strings such as `5280`, `0.01234`, `6.336E4`, or `1.89E-4`. The regular definition

$$\begin{array}{rcl} \mathit{digit} & \to & \mathtt{0} \mid \mathtt{1} \mid \cdots \mid \mathtt{9} \\ \mathit{digits} & \to & \mathit{digit}\ \mathit{digit}^* \\ \mathit{optionalFraction} & \to & \mathtt{.}\ \mathit{digits} \mid \epsilon \\ \mathit{optionalExponent} & \to & (\ \mathtt{E}\ (\ \mathtt{+} \mid \mathtt{-} \mid \epsilon\ )\ \mathit{digits}\ ) \mid \epsilon \\ \mathit{number} & \to & \mathit{digits}\ \mathit{optionalFraction}\ \mathit{optionalExponent} \end{array}$$

is a precise specification for this set of strings. That is, an *optionalFraction* is either a decimal point (dot) followed by one or more digits, or it is missing (the empty string). An *optionalExponent*, if not missing, is the letter E followed by an optional $+$ or $-$ sign, followed by one or more digits. Note that at least one digit must follow the dot, so *number* does not match `1.`, but does match `1.0`.
□

3.3.5 Extensions of Regular Expressions

Since Kleene introduced regular expressions with the basic operators for union, concatenation, and Kleene closure in the 1950s, many extensions have been added to regular expressions to enhance their ability to specify string patterns. Here we mention a few notational extensions that were first incorporated into Unix utilities such as `Lex` that are particularly useful in the specification lexical analyzers. The references to this chapter contain a discussion of some regular-expression variants in use today.

1. *One or more instances.* The unary, postfix operator $^+$ represents the positive closure of a regular expression and its language. That is, if r is a regular expression, then $(r)^+$ denotes the language $\big(L(r)\big)^+$. The operator $^+$ has the same precedence and associativity as the operator $*$. Two useful algebraic laws, $r^* = r^+|\epsilon$ and $r^+ = rr^* = r^*r$ relate the Kleene closure and positive closure.

2. *Zero or one instance.* The unary postfix operator ? means "zero or one occurrence." That is, $r?$ is equivalent to $r|\epsilon$, or put another way, $L(r?) = L(r) \cup \{\epsilon\}$. The ? operator has the same precedence and associativity as $*$ and $+$.

3. *Character classes.* A regular expression $a_1|a_2|\cdots|a_n$, where the a_i's are each symbols of the alphabet, can be replaced by the shorthand $[a_1a_2\cdots a_n]$. More importantly, when $a_1, a_2, \ldots, a_n$ form a logical sequence, e.g., consecutive uppercase letters, lowercase letters, or digits, we can replace them by a_1-a_n, that is, just the first and last separated by a hyphen. Thus, $[\mathbf{abc}]$ is shorthand for $\mathbf{a}|\mathbf{b}|\mathbf{c}$, and $[\mathbf{a}\text{-}\mathbf{z}]$ is shorthand for $\mathbf{a}|\mathbf{b}|\cdots|\mathbf{z}$.

Example 3.7: Using these shorthands, we can rewrite the regular definition of Example 3.5 as:

$$\begin{array}{rcl} letter\_ & \rightarrow & [\texttt{A-Za-z\_}] \\ digit & \rightarrow & [\texttt{0-9}] \\ id & \rightarrow & letter\_ \ (\ letter\_ \mid digit\ )^* \end{array}$$

The regular definition of Example 3.6 can also be simplified:

$$\begin{array}{rcl} digit & \rightarrow & [\texttt{0-9}] \\ digits & \rightarrow & digit^+ \\ number & \rightarrow & digits\ (\texttt{.}\ digits)?\ (\ \texttt{E}\ [\texttt{+-}]?\ digits\ )? \end{array}$$

□

3.3.6 Exercises for Section 3.3

Exercise 3.3.1: Consult the language reference manuals to determine (*i*) the sets of characters that form the input alphabet (excluding those that may only appear in character strings or comments), (*ii*) the lexical form of numerical constants, and (*iii*) the lexical form of identifiers, for each of the following languages: (a) C (b) C++ (c) C# (d) Fortran (e) Java (f) Lisp (g) SQL.

! **Exercise 3.3.2:** Describe the languages denoted by the following regular expressions:

a) $\mathbf{a}(\mathbf{a}|\mathbf{b})^*\mathbf{a}$.

b) $((\epsilon|\mathbf{a})\mathbf{b}^*)^*$.

c) $(\mathbf{a}|\mathbf{b})^*\mathbf{a}(\mathbf{a}|\mathbf{b})(\mathbf{a}|\mathbf{b})$.

d) $\mathbf{a}^*\mathbf{b}\mathbf{a}^*\mathbf{b}\mathbf{a}^*\mathbf{b}\mathbf{a}^*$.

!! e) $(\mathbf{aa}|\mathbf{bb})^*((\mathbf{ab}|\mathbf{ba})(\mathbf{aa}|\mathbf{bb})^*(\mathbf{ab}|\mathbf{ba})(\mathbf{aa}|\mathbf{bb})^*)^*$.

Exercise 3.3.3: In a string of length n, how many of the following are there?

a) Prefixes.

b) Suffixes.

c) Proper prefixes.

! d) Substrings.

! e) Subsequences.

Exercise 3.3.4: Most languages are *case sensitive*, so keywords can be written only one way, and the regular expressions describing their lexemes are very simple. However, some languages, like SQL, are *case insensitive*, so a keyword can be written either in lowercase or in uppercase, or in any mixture of cases. Thus, the SQL keyword `SELECT` can also be written `select`, `Select`, or `sElEcT`, for instance. Show how to write a regular expression for a keyword in a case-insensitive language. Illustrate the idea by writing the expression for "select" in SQL.

! **Exercise 3.3.5:** Write regular definitions for the following languages:

a) All strings of lowercase letters that contain the five vowels in order.

b) All strings of lowercase letters in which the letters are in ascending lexicographic order.

c) Comments, consisting of a string surrounded by `/*` and `*/`, without an intervening `*/`, unless it is inside double-quotes (`"`).

!! d) All strings of digits with no repeated digits. *Hint*: Try this problem first with a few digits, such as $\{0, 1, 2\}$.

!! e) All strings of digits with at most one repeated digit.

!! f) All strings of a's and b's with an even number of a's and an odd number of b's.

g) The set of Chess moves, in the informal notation, such as `p-k4` or `kbp`$\times$`qn`.

!! h) All strings of a's and b's that do not contain the substring abb.

i) All strings of a's and b's that do not contain the subsequence abb.

Exercise 3.3.6: Write character classes for the following sets of characters:

a) The first ten letters (up to "j") in either upper or lower case.

b) The lowercase consonants.

c) The "digits" in a hexadecimal number (choose either upper or lower case for the "digits" above 9).

d) The characters that can appear at the end of a legitimate English sentence (e.g., exclamation point).

The following exercises, up to and including Exercise 3.3.10, discuss the extended regular-expression notation from `Lex` (the lexical-analyzer generator that we shall discuss extensively in Section 3.5). The extended notation is listed in Fig. 3.8.

Exercise 3.3.7: Note that these regular expressions give all of the following symbols (*operator characters*) a special meaning:

```
\ " . ^ $ [ ] * + ? { } | /
```

Their special meaning must be turned off if they are needed to represent themselves in a character string. We can do so by quoting the character within a string of length one or more; e.g., the regular expression `"**"` matches the string `**`. We can also get the literal meaning of an operator character by preceding it by a backslash. Thus, the regular expression `\*\*` also matches the string `**`. Write a regular expression that matches the string `"\`.

Exercise 3.3.8: In `Lex`, a *complemented character class* represents any character except the ones listed in the character class. We denote a complemented class by using ^ as the first character; this symbol (caret) is not itself part of the class being complemented, unless it is listed within the class itself. Thus, `[^A-Za-z]` matches any character that is not an uppercase or lowercase letter, and `[^\^]` represents any character but the caret (or newline, since newline cannot be in any character class). Show that for every regular expression with complemented character classes, there is an equivalent regular expression without complemented character classes.

EXPRESSION	MATCHES	EXAMPLE
c	the one non-operator character c	`a`
$\backslash c$	character c literally	`\*`
"s"	string s literally	`"**"`
.	any character but newline	`a.*b`
^	beginning of a line	`^abc`
$	end of a line	`abc$`
$[s]$	any one of the characters in string s	`[abc]`
$[\hat{}s]$	any one character not in string s	`[^abc]`
$r*$	zero or more strings matching r	`a*`
$r+$	one or more strings matching r	`a+`
$r?$	zero or one r	`a?`
$r\{m,n\}$	between m and n occurrences of r	`a{1,5}`
r_1r_2	an r_1 followed by an r_2	`ab`
$r_1 \mid r_2$	an r_1 or an r_2	`a|b`
(r)	same as r	`(a|b)`
r_1/r_2	r_1 when followed by r_2	`abc/123`

Figure 3.8: `Lex` regular expressions

! **Exercise 3.3.9:** The regular expression $r\{m,n\}$ matches from m to n occurrences of the pattern r. For example, `a{1,5}` matches a string of one to five a's. Show that for every regular expression containing repetition operators of this form, there is an equivalent regular expression without repetition operators.

! **Exercise 3.3.10:** The operator ^ matches the left end of a line, and $ matches the right end of a line. The operator ^ is also used to introduce complemented character classes, but the context always makes it clear which meaning is intended. For example, `^[^aeiou]*$` matches any complete line that does not contain a lowercase vowel.

a) How do you tell which meaning of ^ is intended?

b) Can you always replace a regular expression using the ^ and $ operators by an equivalent expression that does not use either of these operators?

! **Exercise 3.3.11:** The UNIX shell command `sh` uses the operators in Fig. 3.9 in filename expressions to describe sets of file names. For example, the filename expression `*.o` matches all file names ending in `.o`; `sort1.?` matches all filenames of the form `sort1.`c, where c is any character. Show how `sh` filename

EXPRESSION	MATCHES	EXAMPLE
$'s'$	string s literally	`'\'`
$\backslash c$	character c literally	`\'`
$*$	any string	`*.o`
$?$	any character	`sort1.?`
$[s]$	any character in s	`sort1.[cso]`

Figure 3.9: Filename expressions used by the shell command `sh`

expressions can be replaced by equivalent regular expressions using only the basic union, concatenation, and closure operators.

! **Exercise 3.3.12:** SQL allows a rudimentary form of patterns in which two characters have special meaning: underscore (_) stands for any one character and percent-sign (%) stands for any string of 0 or more characters. In addition, the programmer may define any character, say e, to be the escape character, so an e preceding _, %, or another e gives the character that follows its literal meaning. Show how to express any SQL pattern as a regular expression, given that we know which character is the escape character.

3.4 Recognition of Tokens

In the previous section we learned how to express patterns using regular expressions. Now, we must study how to take the patterns for all the needed tokens and build a piece of code that examines the input string and finds a prefix that is a lexeme matching one of the patterns. Our discussion will make use of the following running example.

$$
\begin{array}{rcl}
stmt & \to & \textbf{if } expr \textbf{ then } stmt \\
 & | & \textbf{if } expr \textbf{ then } stmt \textbf{ else } stmt \\
 & | & \epsilon \\
expr & \to & term \textbf{ relop } term \\
 & | & term \\
term & \to & \textbf{id} \\
 & | & \textbf{number}
\end{array}
$$

Figure 3.10: A grammar for branching statements

Example 3.8: The grammar fragment of Fig. 3.10 describes a simple form of branching statements and conditional expressions. This syntax is similar to that of the language Pascal, in that **then** appears explicitly after conditions.

For **relop**, we use the comparison operators of languages like Pascal or SQL, where `=` is "equals" and `<>` is "not equals," because it presents an interesting structure of lexemes.

The terminals of the grammar, which are **if**, **then**, **else**, **relop**, **id**, and **number**, are the names of tokens as far as the lexical analyzer is concerned. The patterns for these tokens are described using regular definitions, as in Fig. 3.11. The patterns for *id* and *number* are similar to what we saw in Example 3.7.

$$
\begin{array}{rcl}
\textit{digit} & \rightarrow & [\texttt{0-9}] \\
\textit{digits} & \rightarrow & \textit{digit}^+ \\
\textit{number} & \rightarrow & \textit{digits}\ (\texttt{.}\ \textit{digits})?\ (\ \texttt{E}\ [\texttt{+-}]?\ \textit{digits}\ )? \\
\textit{letter} & \rightarrow & [\texttt{A-Za-z}] \\
\textit{id} & \rightarrow & \textit{letter}\ (\ \textit{letter} \mid \textit{digit}\ )^* \\
\textit{if} & \rightarrow & \texttt{if} \\
\textit{then} & \rightarrow & \texttt{then} \\
\textit{else} & \rightarrow & \texttt{else} \\
\textit{relop} & \rightarrow & \texttt{<} \mid \texttt{>} \mid \texttt{<=} \mid \texttt{>=} \mid \texttt{=} \mid \texttt{<>}
\end{array}
$$

Figure 3.11: Patterns for tokens of Example 3.8

For this language, the lexical analyzer will recognize the keywords `if`, `then`, and `else`, as well as lexemes that match the patterns for *relop*, *id*, and *number*. To simplify matters, we make the common assumption that keywords are also *reserved words*: that is, they are not identifiers, even though their lexemes match the pattern for identifiers.

In addition, we assign the lexical analyzer the job of stripping out whitespace, by recognizing the "token" *ws* defined by:

$$\textit{ws} \rightarrow (\ \textbf{blank} \mid \textbf{tab} \mid \textbf{newline}\ )^+$$

Here, **blank**, **tab**, and **newline** are abstract symbols that we use to express the ASCII characters of the same names. Token *ws* is different from the other tokens in that, when we recognize it, we do not return it to the parser, but rather restart the lexical analysis from the character that follows the whitespace. It is the following token that gets returned to the parser.

Our goal for the lexical analyzer is summarized in Fig. 3.12. That table shows, for each lexeme or family of lexemes, which token name is returned to the parser and what attribute value, as discussed in Section 3.1.3, is returned. Note that for the six relational operators, symbolic constants `LT`, `LE`, and so on are used as the attribute value, in order to indicate which instance of the token **relop** we have found. The particular operator found will influence the code that is output from the compiler. □

LEXEMES	TOKEN NAME	ATTRIBUTE VALUE
Any *ws*	–	–
`if`	**if**	–
`then`	**then**	–
`else`	**else**	–
Any *id*	**id**	Pointer to table entry
Any *number*	**number**	Pointer to table entry
`<`	**relop**	`LT`
`<=`	**relop**	`LE`
`=`	**relop**	`EQ`
`<>`	**relop**	`NE`
`>`	**relop**	`GT`
`>=`	**relop**	`GE`

Figure 3.12: Tokens, their patterns, and attribute values

3.4.1 Transition Diagrams

As an intermediate step in the construction of a lexical analyzer, we first convert patterns into stylized flowcharts, called "transition diagrams." In this section, we perform the conversion from regular-expression patterns to transition diagrams by hand, but in Section 3.6, we shall see that there is a mechanical way to construct these diagrams from collections of regular expressions.

Transition diagrams have a collection of nodes or circles, called *states*. Each state represents a condition that could occur during the process of scanning the input looking for a lexeme that matches one of several patterns. We may think of a state as summarizing all we need to know about what characters we have seen between the *lexemeBegin* pointer and the *forward* pointer (as in the situation of Fig. 3.3).

Edges are directed from one state of the transition diagram to another. Each edge is *labeled* by a symbol or set of symbols. If we are in some state s, and the next input symbol is a, we look for an edge out of state s labeled by a (and perhaps by other symbols, as well). If we find such an edge, we advance the *forward* pointer and enter the state of the transition diagram to which that edge leads. We shall assume that all our transition diagrams are *deterministic*, meaning that there is never more than one edge out of a given state with a given symbol among its labels. Starting in Section 3.5, we shall relax the condition of determinism, making life much easier for the designer of a lexical analyzer, although trickier for the implementer. Some important conventions about transition diagrams are:

1. Certain states are said to be *accepting*, or *final*. These states indicate that a lexeme has been found, although the actual lexeme may not consist of all positions between the *lexemeBegin* and *forward* pointers. We always

indicate an accepting state by a double circle, and if there is an action to be taken — typically returning a token and an attribute value to the parser — we shall attach that action to the accepting state.

2. In addition, if it is necessary to retract the *forward* pointer one position (i.e., the lexeme does not include the symbol that got us to the accepting state), then we shall additionally place a * near that accepting state. In our example, it is never necessary to retract *forward* by more than one position, but if it were, we could attach any number of *'s to the accepting state.

3. One state is designated the *start state*, or *initial state*; it is indicated by an edge, labeled "start," entering from nowhere. The transition diagram always begins in the start state before any input symbols have been read.

Example 3.9: Figure 3.13 is a transition diagram that recognizes the lexemes matching the token **relop**. We begin in state 0, the start state. If we see `<` as the first input symbol, then among the lexemes that match the pattern for **relop** we can only be looking at `<`, `<>`, or `<=`. We therefore go to state 1, and look at the next character. If it is `=`, then we recognize lexeme `<=`, enter state 2, and return the token **relop** with attribute `LE`, the symbolic constant representing this particular comparison operator. If in state 1 the next character is `>`, then instead we have lexeme `<>`, and enter state 3 to return an indication that the not-equals operator has been found. On any other character, the lexeme is `<`, and we enter state 4 to return that information. Note, however, that state 4 has a * to indicate that we must retract the input one position.

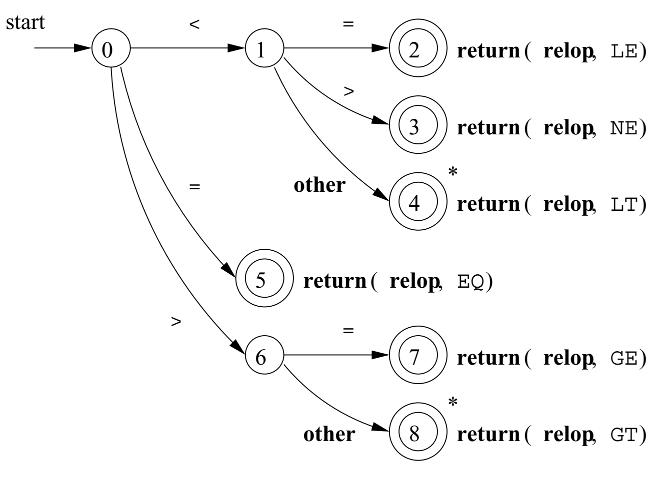


Figure 3.13: Transition diagram for **relop**

On the other hand, if in state 0 the first character we see is `=`, then this one character must be the lexeme. We immediately return that fact from state 5.

The remaining possibility is that the first character is `>`. Then, we must enter state 6 and decide, on the basis of the next character, whether the lexeme is `>=` (if we next see the `=` sign), or just `>` (on any other character). Note that if, in state 0, we see any character besides `<`, `=`, or `>`, we can not possibly be seeing a `relop` lexeme, so this transition diagram will not be used. □

3.4.2 Recognition of Reserved Words and Identifiers

Recognizing keywords and identifiers presents a problem. Usually, keywords like `if` or `then` are reserved (as they are in our running example), so they are not identifiers even though they *look* like identifiers. Thus, although we typically use a transition diagram like that of Fig. 3.14 to search for identifier lexemes, this diagram will also recognize the keywords `if`, `then`, and `else` of our running example.

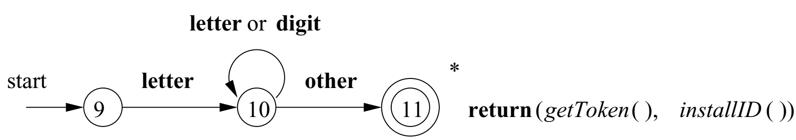


Figure 3.14: A transition diagram for **id**'s and keywords

There are two ways that we can handle reserved words that look like identifiers:

1. Install the reserved words in the symbol table initially. A field of the symbol-table entry indicates that these strings are never ordinary identifiers, and tells which token they represent. We have supposed that this method is in use in Fig. 3.14. When we find an identifier, a call to *installID* places it in the symbol table if it is not already there and returns a pointer to the symbol-table entry for the lexeme found. Of course, any identifier not in the symbol table during lexical analysis cannot be a reserved word, so its token is **id**. The function *getToken* examines the symbol table entry for the lexeme found, and returns whatever token name the symbol table says this lexeme represents — either **id** or one of the keyword tokens that was initially installed in the table.

2. Create separate transition diagrams for each keyword; an example for the keyword `then` is shown in Fig. 3.15. Note that such a transition diagram consists of states representing the situation after each successive letter of the keyword is seen, followed by a test for a "nonletter-or-digit," i.e., any character that cannot be the continuation of an identifier. It is necessary to check that the identifier has ended, or else we would return token **then** in situations where the correct token was **id**, with a lexeme like `thenextvalue` that has `then` as a proper prefix. If we adopt this approach, then we must prioritize the tokens so that the reserved-word

tokens are recognized in preference to **id**, when the lexeme matches both patterns. We *do not* use this approach in our example, which is why the states in Fig. 3.15 are unnumbered.

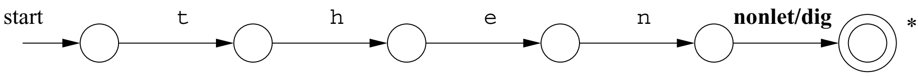


Figure 3.15: Hypothetical transition diagram for the keyword `then`

3.4.3 Completion of the Running Example

The transition diagram for **id**'s that we saw in Fig. 3.14 has a simple structure. Starting in state 9, it checks that the lexeme begins with a letter and goes to state 10 if so. We stay in state 10 as long as the input contains letters and digits. When we first encounter anything but a letter or digit, we go to state 11 and accept the lexeme found. Since the last character is not part of the identifier, we must retract the input one position, and as discussed in Section 3.4.2, we enter what we have found in the symbol table and determine whether we have a keyword or a true identifier.

The transition diagram for token **number** is shown in Fig. 3.16, and is so far the most complex diagram we have seen. Beginning in state 12, if we see a digit, we go to state 13. In that state, we can read any number of additional digits. However, if we see anything but a digit, dot, or `E`, we have seen a number in the form of an integer; `123` is an example. That case is handled by entering state 20, where we return token **number** and a pointer to a table of constants where the found lexeme is entered. These mechanics are not shown on the diagram but are analogous to the way we handled identifiers.

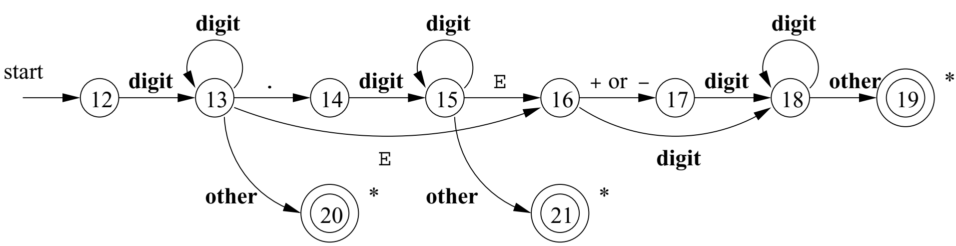


Figure 3.16: A transition diagram for unsigned numbers

If we instead see a dot in state 13, then we have an "optional fraction." State 14 is entered, and we look for one or more additional digits; state 15 is used for that purpose. If we see an `E`, then we have an "optional exponent," whose recognition is the job of states 16 through 19. Should we, in state 15, instead see anything but `E` or a digit, then we have come to the end of the fraction, there is no exponent, and we return the lexeme found, via state 21.

The final transition diagram, shown in Fig. 3.17, is for whitespace. In that diagram, we look for one or more "whitespace" characters, represented by **delim** in that diagram — typically these characters would be blank, tab, newline, and perhaps other characters that are not considered by the language design to be part of any token.

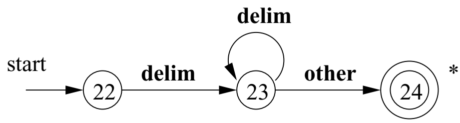


Figure 3.17: A transition diagram for whitespace

Note that in state 24, we have found a block of consecutive whitespace characters, followed by a nonwhitespace character. We retract the input to begin at the nonwhitespace, but we do not return to the parser. Rather, we must restart the process of lexical analysis after the whitespace.

3.4.4 Architecture of a Transition-Diagram-Based Lexical Analyzer

There are several ways that a collection of transition diagrams can be used to build a lexical analyzer. Regardless of the overall strategy, each state is represented by a piece of code. We may imagine a variable `state` holding the number of the current state for a transition diagram. A switch based on the value of `state` takes us to code for each of the possible states, where we find the action of that state. Often, the code for a state is itself a switch statement or multiway branch that determines the next state by reading and examining the next input character.

Example 3.10: In Fig. 3.18 we see a sketch of `getRelop()`, a C++ function whose job is to simulate the transition diagram of Fig. 3.13 and return an object of type `TOKEN`, that is, a pair consisting of the token name (which must be **relop** in this case) and an attribute value (the code for one of the six comparison operators in this case). `getRelop()` first creates a new object `retToken` and initializes its first component to `RELOP`, the symbolic code for token **relop**.

We see the typical behavior of a state in case 0, the case where the current state is 0. A function `nextChar()` obtains the next character from the input and assigns it to local variable c. We then check c for the three characters we expect to find, making the state transition dictated by the transition diagram of Fig. 3.13 in each case. For example, if the next input character is `=`, we go to state 5.

If the next input character is not one that can begin a comparison operator, then a function `fail()` is called. What `fail()` does depends on the global error-recovery strategy of the lexical analyzer. It should reset the `forward` pointer to `lexemeBegin`, in order to allow another transition diagram to be applied to

```
TOKEN getRelop()
{
    TOKEN retToken = new(RELOP);
    while(1) { /* repeat character processing until a return
                  or failure occurs */
        switch(state) {
            case 0: c = nextChar();
                    if ( c == '<' ) state = 1;
                    else if ( c == '=' ) state = 5;
                    else if ( c == '>' ) state = 6;
                    else fail(); /* lexeme is not a relop */
                    break;
            case 1: ...
            ...
            case 8: retract();
                    retToken.attribute = GT;
                    return(retToken);
        }
    }
}
```

Figure 3.18: Sketch of implementation of **relop** transition diagram

the true beginning of the unprocessed input. It might then change the value of `state` to be the start state for another transition diagram, which will search for another token. Alternatively, if there is no other transition diagram that remains unused, `fail()` could initiate an error-correction phase that will try to repair the input and find a lexeme, as discussed in Section 3.1.4.

We also show the action for state 8 in Fig. 3.18. Because state 8 bears a *, we must retract the input pointer one position (i.e., put c back on the input stream). That task is accomplished by the function `retract()`. Since state 8 represents the recognition of lexeme `>`, we set the second component of the returned object, which we suppose is named `attribute`, to `GT`, the code for this operator. □

To place the simulation of one transition diagram in perspective, let us consider the ways code like Fig. 3.18 could fit into the entire lexical analyzer.

1. We could arrange for the transition diagrams for each token to be tried sequentially. Then, the function `fail()` of Example 3.10 resets the pointer `forward` and starts the next transition diagram, each time it is called. This method allows us to use transition diagrams for the individual keywords, like the one suggested in Fig. 3.15. We have only to use these before we use the diagram for **id**, in order for the keywords to be reserved words.

2. We could run the various transition diagrams "in parallel," feeding the next input character to all of them and allowing each one to make whatever transitions it required. If we use this strategy, we must be careful to resolve the case where one diagram finds a lexeme that matches its pattern, while one or more other diagrams are still able to process input. The normal strategy is to take the longest prefix of the input that matches any pattern. That rule allows us to prefer identifier `thenext` to keyword `then`, or the operator `->` to `-`, for example.

3. The preferred approach, and the one we shall take up in the following sections, is to combine all the transition diagrams into one. We allow the transition diagram to read input until there is no possible next state, and then take the longest lexeme that matched any pattern, as we discussed in item (2) above. In our running example, this combination is easy, because no two tokens can start with the same character; i.e., the first character immediately tells us which token we are looking for. Thus, we could simply combine states 0, 9, 12, and 22 into one start state, leaving other transitions intact. However, in general, the problem of combining transition diagrams for several tokens is more complex, as we shall see shortly.

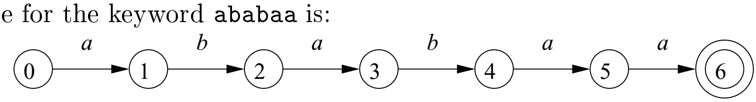
3.4.5 Exercises for Section 3.4

Exercise 3.4.1: Provide transition diagrams to recognize the same languages as each of the regular expressions in Exercise 3.3.2.

Exercise 3.4.2: Provide transition diagrams to recognize the same languages as each of the regular expressions in Exercise 3.3.5.

The following exercises, up to Exercise 3.4.12, introduce the Aho-Corasick algorithm for recognizing a collection of keywords in a text string in time proportional to the length of the text and the sum of the length of the keywords. This algorithm uses a special form of transition diagram called a *trie*. A trie is a tree-structured transition diagram with distinct labels on the edges leading from a node to its children. Leaves of the trie represent recognized keywords.

Knuth, Morris, and Pratt presented an algorithm for recognizing a single keyword $b_1 b_2 \cdots b_n$ in a text string. Here the trie is a transition diagram with $n+1$ states, 0 through n. State 0 is the initial state, and state n represents acceptance, that is, discovery of the keyword. From each state s from 0 through $n-1$, there is a transition to state $s+1$, labeled by symbol b_{s+1}. For example, the trie for the keyword `ababaa` is:



In order to process text strings rapidly and search those strings for a keyword, it is useful to define, for keyword $b_1 b_2 \cdots b_n$ and position s in that keyword (corresponding to state s of its trie), a *failure function*, $f(s)$, computed as in

Fig. 3.19. The objective is that $b_1 b_2 \cdots b_{f(s)}$ is the longest proper prefix of $b_1 b_2 \cdots b_s$ that is also a suffix of $b_1 b_2 \cdots b_s$. The reason $f(s)$ is important is that if we are trying to match a text string for $b_1 b_2 \cdots b_n$, and we have matched the first s positions, but we then fail (i.e., the next position of the text string does not hold b_{s+1}), then $f(s)$ is the longest prefix of $b_1 b_2 \cdots b_n$ that could possibly match the text string up to the point we are at. Of course, the next character of the text string must be $b_{f(s)+1}$, or else we still have problems and must consider a yet shorter prefix, which will be $b_{f(f(s))}$.

```
1)  t = 0;
2)  f(1) = 0;
3)  for (s = 1; s < n; s++) {
4)          while (t > 0 && b_{s+1} != b_{t+1}) t = f(t);
5)          if (b_{s+1} == b_{t+1}) {
6)                  t = t + 1;
7)                  f(s + 1) = t;
            }
8)          else f(s + 1) = 0;
    }
```

Figure 3.19: Algorithm to compute the failure function for keyword $b_1 b_2 \cdots b_n$

As an example, the failure function for the trie constructed above for `ababaa` is:

s	1	2	3	4	5	6
$f(s)$	0	0	1	2	3	1

For instance, states 3 and 1 represent prefixes `aba` and `a`, respectively. $f(3) = 1$ because `a` is the longest proper prefix of `aba` that is also a suffix of `aba`. Also, $f(2) = 0$, because the longest proper prefix of `ab` that is also a suffix is the empty string.

Exercise 3.4.3: Construct the failure function for the strings:

a) `abababaab`.

b) `aaaaaa`.

c) `abbaabb`.

! **Exercise 3.4.4:** Prove, by induction on s, that the algorithm of Fig. 3.19 correctly computes the failure function.

!! **Exercise 3.4.5:** Show that the assignment $t = f(t)$ in line (4) of Fig. 3.19 is executed at most n times. Show that therefore, the entire algorithm takes only $O(n)$ time on a keyword of length n.

Having computed the failure function for a keyword $b_1 b_2 \cdots b_n$, we can scan a string $a_1 a_2 \cdots a_m$ in time $O(m)$ to tell whether the keyword occurs in the string. The algorithm, shown in Fig. 3.20, slides the keyword along the string, trying to make progress by matching the next character of the keyword with the next character of the string. If it cannot do so after matching s characters, then it "slides" the keyword right $s - f(s)$ positions, so only the first $f(s)$ characters of the keyword are considered matched with the string.

```
1)  s = 0;
2)  for (i = 1; i <= m; i++) {
3)          while (s > 0 && a_i != b_{s+1}) s = f(s);
4)          if (a_i == b_{s+1}) s = s + 1;
5)          if (s == n) return "yes";
    }
6)  return "no";
```

Figure 3.20: The KMP algorithm tests whether string $a_1 a_2 \cdots a_m$ contains a single keyword $b_1 b_2 \cdots b_n$ as a substring in $O(m + n)$ time

Exercise 3.4.6: Apply Algorithm KMP to test whether keyword `ababaa` is a substring of:

a) `abababaab`.

b) `abababbaa`.

!! **Exercise 3.4.7:** Show that the algorithm of Fig. 3.20 correctly tells whether the keyword is a substring of the given string. *Hint*: proceed by induction on i. Show that for all i, the value of s after line (4) is the length of the longest prefix of the keyword that is a suffix of $a_1 a_2 \cdots a_i$.

!! **Exercise 3.4.8:** Show that the algorithm of Fig. 3.20 runs in time $O(m + n)$, assuming that function f is already computed and its values stored in an array indexed by s.

Exercise 3.4.9: The *Fibonacci strings* are defined as follows:

1. $s_1 =$ `b`.

2. $s_2 =$ `a`.

3. $s_k = s_{k-1} s_{k-2}$ for $k > 2$.

For example, $s_3 =$ `ab`, $s_4 =$ `aba`, and $s_5 =$ `abaab`.

a) What is the length of s_n?

b) Construct the failure function for s_6.

c) Construct the failure function for s_7.

!! d) Show that the failure function for any s_n can be expressed by $f(1) = f(2) = 0$, and for $2 < j \le |s_n|$, $f(j)$ is $j - |s_{k-1}|$, where k is the largest integer such that $|s_k| \le j + 1$.

!! e) In the KMP algorithm, what is the largest number of consecutive applications of the failure function, when we try to determine whether keyword s_k appears in text string s_{k+1}?

Aho and Corasick generalized the KMP algorithm to recognize any of a set of keywords in a text string. In this case, the trie is a true tree, with branching from the root. There is one state for every string that is a prefix (not necessarily proper) of any keyword. The parent of a state corresponding to string $b_1 b_2 \cdots b_k$ is the state that corresponds to $b_1 b_2 \cdots b_{k-1}$. A state is accepting if it corresponds to a complete keyword. For example, Fig. 3.21 shows the trie for the keywords `he`, `she`, `his`, and `hers`.

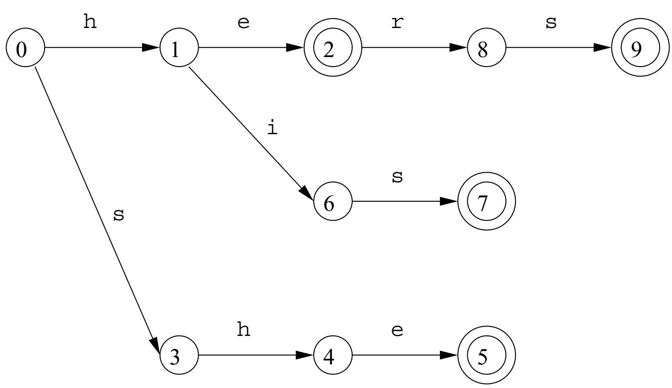


Figure 3.21: Trie for keywords `he`, `she`, `his`, `hers`

The failure function for the general trie is defined as follows. Suppose s is the state that corresponds to string $b_1 b_2 \cdots b_n$. Then $f(s)$ is the state that corresponds to the longest proper suffix of $b_1 b_2 \cdots b_n$ that is also a prefix of *some* keyword. For example, the failure function for the trie of Fig. 3.21 is:

s	1	2	3	4	5	6	7	8	9
$f(s)$	0	0	0	1	2	0	3	0	3

! **Exercise 3.4.10:** Modify the algorithm of Fig. 3.19 to compute the failure function for general tries. *Hint*: The major difference is that we cannot simply test for equality or inequality of b_{s+1} and b_{t+1} in lines (4) and (5) of Fig. 3.19. Rather, from any state there may be several transitions out on several characters, as there are transitions on both `e` and `i` from state 1 in Fig. 3.21. Any of those transitions could lead to a state that represents the longest suffix that is also a prefix.

Exercise 3.4.11: Construct the tries and compute the failure function for the following sets of keywords:

a) `aaa`, `abaaa`, and `ababaaa`.

b) `all`, `fall`, `fatal`, `llama`, and `lame`.

c) `pipe`, `pet`, `item`, `temper`, and `perpetual`.

! **Exercise 3.4.12:** Show that your algorithm from Exercise 3.4.10 still runs in time that is linear in the sum of the lengths of the keywords.

3.5 The Lexical-Analyzer Generator `Lex`

In this section, we introduce a tool called `Lex`, or in a more recent implementation `Flex`, that allows one to specify a lexical analyzer by specifying regular expressions to describe patterns for tokens. The input notation for the `Lex` tool is referred to as the *Lex language* and the tool itself is the *Lex compiler*. Behind the scenes, the Lex compiler transforms the input patterns into a transition diagram and generates code, in a file called `lex.yy.c`, that simulates this transition diagram. The mechanics of how this translation from regular expressions to transition diagrams occurs is the subject of the next sections; here we only learn the Lex language.

3.5.1 Use of `Lex`

Figure 3.22 suggests how `Lex` is used. An input file, which we call `lex.l`, is written in the Lex language and describes the lexical analyzer to be generated. The Lex compiler transforms `lex.l` to a C program, in a file that is always named `lex.yy.c`. The latter file is compiled by the C compiler into a file called `a.out`, as always. The C-compiler output is a working lexical analyzer that can take a stream of input characters and produce a stream of tokens.

The normal use of the compiled C program, referred to as `a.out` in Fig. 3.22, is as a subroutine of the parser. It is a C function that returns an integer, which is a code for one of the possible token names. The attribute value, whether it be another numeric code, a pointer to the symbol table, or nothing, is placed in a global variable `yylval`,[2] which is shared between the lexical analyzer and parser, thereby making it simple to return both the name and an attribute value of a token.

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[2]Incidentally, the `yy` that appears in `yylval` and `lex.yy.c` refers to the `Yacc` parser-generator, which we shall describe in Section 4.9, and which is commonly used in conjunction with `Lex`.