Applied Economic Forecasting

2. Exploring & Visualizing Time series

Section 1

Time series in R

What is a Time Series?

- A time series can be thought of as a list of observations (the measurements), along with some information about what times those numbers were recorded (the index).
- A basic way of storing a time series is as a `ts` object.
- The `fpp3` package stores this information as a `tsibble` object.

`ts` objects and `ts` function

Let us get familiar with `ts` objects before tackling `tstibble` objects.

Example

Year	Obs.
2012	123
2013	39
2014	78
2015	52
2016	110

```r
y <- ts(c(123,39,78,52,110), start=2012)
```

`ts` objects and the `ts` function

For observations that are more frequent than once per year, we need to add a `frequency` argument. We usually think of the `frequency` argument as the number of time the series will be observed (collected) in a year.

E.g., monthly data (`frequency = 12`) stored as a numerical vector `z`:

```
y <- ts(z, frequency=12, start=c(2003, 1))
```

`ts` objects and `ts` function

'ts(data, frequency, start, end)'

Type of data	**frequency**	**start example**
Annual	1	1995
Quarterly	4	c(1995,2)
Monthly	12	c(1995,9)
Daily	7 *or* 365.25	*or* c(1995,234)
Weekly	52.18	c(1995,23)
Hourly	24 *or* 168 *or* 8,766	1
Half-hourly	48 *or* 336 *or* 17,532	1

We do not need to always specify both the `start` and `end` arguments. In many cases, just one will suffice.

Let's Practice!!!

1. Set a seed as 10ˆ3
2. Generate a random normal variable (`x`) with 200 observations, mean = 75, and sd = 5
3. Declare x as a *quarterly* `ts` object ending December 2018 (`x.ts`)
4. Now repeat step 3 with weekly, monthly and annual frequencies (*How about using a loop?*)

`tsibble` function

The `tsibble` function offers a nice mix between the `ts` objects/function and the `tidyverse` package.

Let's assume I have the following data:

Year	GDP
2010	155
2011	134
2012	80
2013	100
2014	120
2015	110

The tsibble function

We can record this as a **tsibble** object using the following syntax:

```
y <- tsibble(Year = 2010:2015,
             GDP = c(155,134,80,100,120,110),
             index = Year
             )
y
```

```
## # A tsibble: 6 x 2 [1Y]
##    Year   GDP
##   <int> <dbl>
## 1  2010   155
## 2  2011   134
## 3  2012    80
## 4  2013   100
## 5  2014   120
## 6  2015   110
```

The `tsibble()` function

Similar to the `ts()` function earlier, we have the ability to work with variables observed at higher frequencies, say quarterly, monthly, daily, etc. . .

For instance:

Month	Observation
2020 Jan	50
2020 Feb	23
2020 Mar	45
2020 Apr	67
2020 May	78
2020 Jun	98

The tsibble() function

We would first need to convert our dataset to a **tsibble** object.

```
z %>% mutate(Month = yearmonth(Month)) %>%
  as_tsibble(index = Month)
```

```
## # A tsibble: 6 x 2 [1M]
##      Month Observation
##      <mth>       <dbl>
## 1 2020 Jan          50
## 2 2020 Feb          23
## 3 2020 Mar          45
## 4 2020 Apr          67
## 5 2020 May          78
## 6 2020 Jun          98
```

Note:

It is worth mentioning the difference between the `as_tsibble` function in the code above and `tsibble`. The former is used when you would like to convert a dataframe or variable that *already exists*. `tsibble` is usually reserved for creating a new variable.

Other Classes we might encounter:

Frequency	Function
Annual	start:end
Quarterly	yearquarter()
Monthly	yearmonth()
Weekly	yearweek()
Daily	as_date(), ymd()
Sub-daily	as_datetime(), ymd_hms()

Working with tsibble objects

In many instances, we will be required to forecast a single (or subset of) variable(s) from our dataset. This is where `dplyr` functions such as `mutate()`, `filter()`, and `select()` come in handy.

For example, say we are interested in the winning times in the Olympic track events stored in the `olympic_running` data.

```
olympic_running
```

```
## # A tsibble: 312 x 4 [4Y]
## # Key:       Length, Sex [14]
##     Year Length Sex    Time
##    <int>  <int> <chr> <dbl>
##  1  1896    100 men    12
##  2  1900    100 men    11
##  3  1904    100 men    11
##  4  1908    100 men    10.8
##  5  1912    100 men    10.8
```

Working with tsibble objects

We can filter to see only the female races.

```
olympic_running %>% filter(Sex == "women")
```

```
## # A tsibble: 104 x 4 [4Y]
## # Key:       Length, Sex [7]
##     Year Length Sex    Time
##    <int>  <int> <chr> <dbl>
##  1  1928    100 women  12.2
##  2  1932    100 women  11.9
##  3  1936    100 women  11.5
##  4  1940    100 women  NA
##  5  1944    100 women  NA
##  6  1948    100 women  11.9
##  7  1952    100 women  11.5
##  8  1956    100 women  11.5
##  9  1960    100 women  11
## 10  1964    100 women  11.4
```

Exercise Time

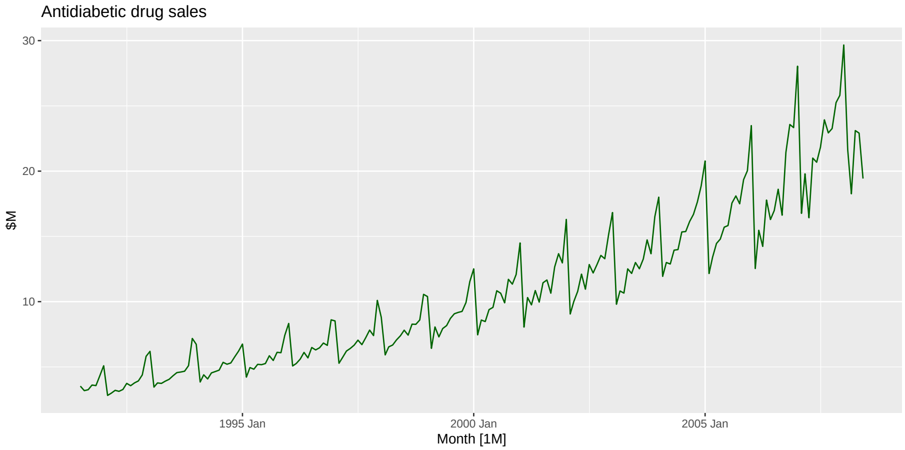
Let us explore a few exercises for selecting, mutating, and summarizing the data.

Section 2

Time plots

```r
melsyd <- ansett %>%
  filter(Airports == "MEL-SYD", Class == "Economy") %>%
  mutate(Passengers = Passengers/1000)

melsyd %>% autoplot(Passengers, col = "red") +
  labs(title = "Ansett airlines Economy class",
       subtitle = "Melbourne-Sydney", y = "Passengers ('000)")
```

```r
a10 <- PBS %>% filter(ATC2 == "A10") %>%
  summarise(TC = sum(Cost)/1e6)

autoplot(a10, TC, col = "darkgreen") +
  labs(y = "$M", title = "Antidiabetic drug sales")
```



Let's Practice

1. Using the data from our earlier exercise: Plot `x.ts` at the monthly, quarterly and yearly frequencies.
2. Create plots of the following time series: `Bricks` from `aus_production`, `Lynx` from `pelt`, `Close` from `gafa_stock`. Redo your plot for `Close` focusing in on `GOOG` instead.

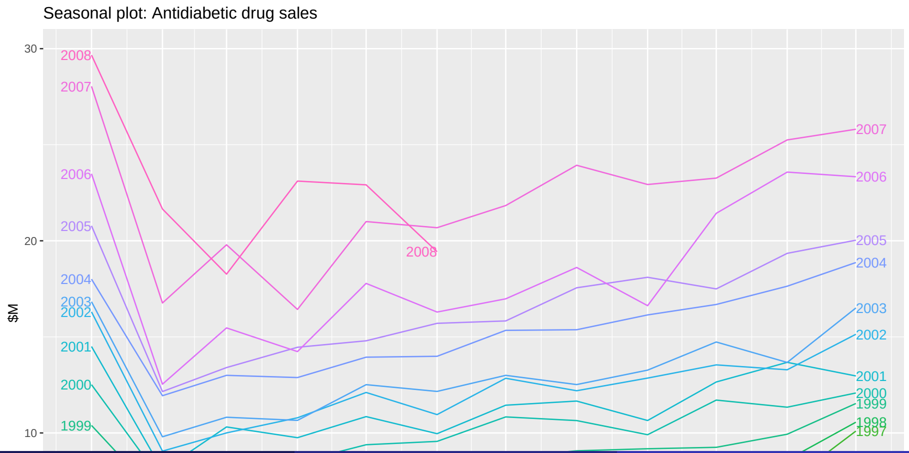
- Use `help()` or `?seriesName` to find out about the data in each series.
- For each of the plots, be sure to modify the axis labels and title.

Section 3

Seasonal plots

Seasonal plots

```r
a10 %>% gg_season(TC, labels = "both") +
  labs( y = "$M",
        title = "Seasonal plot: Antidiabetic drug sales") +
  #Add a bit of space on both sides of the graph.
  expand_limits(x = ymd(c("1972-12-28", "1973-12-05")))
```

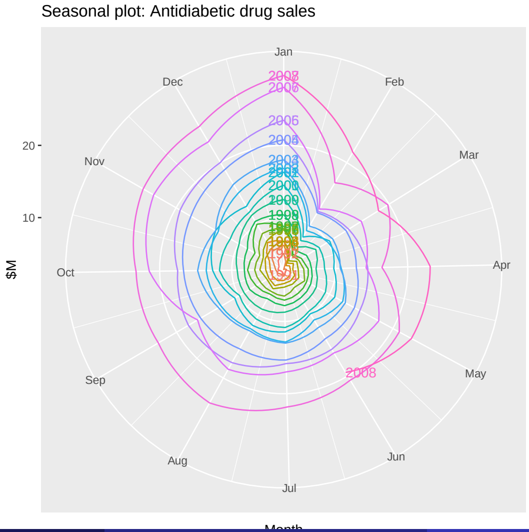


Seasonal plots

- Data plotted against the individual “seasons” in which the data were observed. (In this case a “season” is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
- In R: `gg_season()`

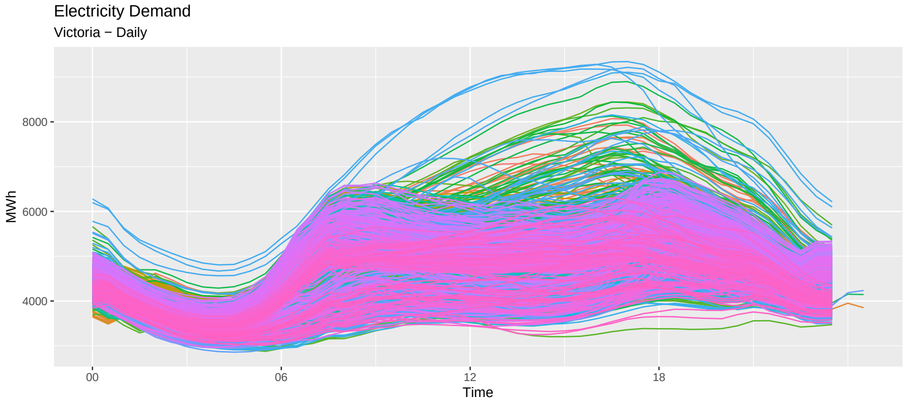
Seasonal polar plots

```r
a10 %>% gg_season(TC, labels = "both", polar = TRUE) +
  labs( y = "$M", title = "Seasonal plot: Antidiabetic drug sales")
```

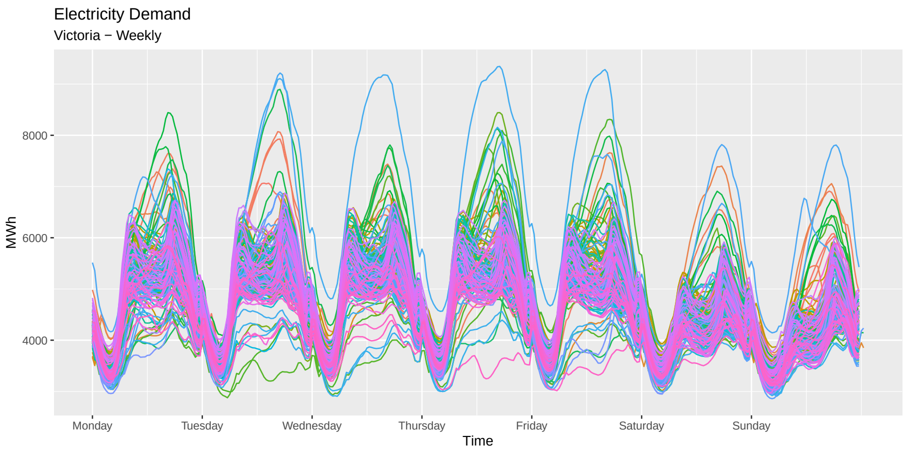


Multiple Seasonal Plots

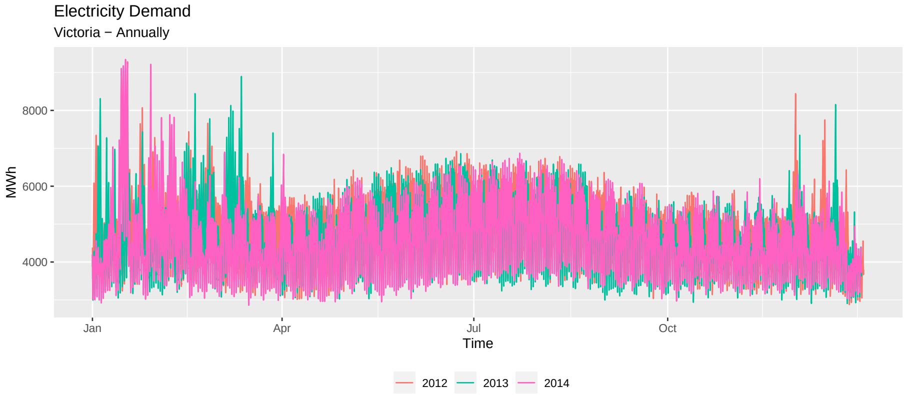
```
# Plot daily pattern
vic_elec %>% gg_season(Demand, period = "day") +
  theme(legend.pos = "none") +
  labs(title = "Electricity Demand", subtitle = "Victoria - Daily",
       y = "MWh")
```



```r
# Plot weekly pattern
vic_elec %>% gg_season(Demand, period = "week") +
  theme(legend.pos = "none") +
  labs(title = "Electricity Demand", subtitle = "Victoria - Weekly"
       y = "MWh")
```

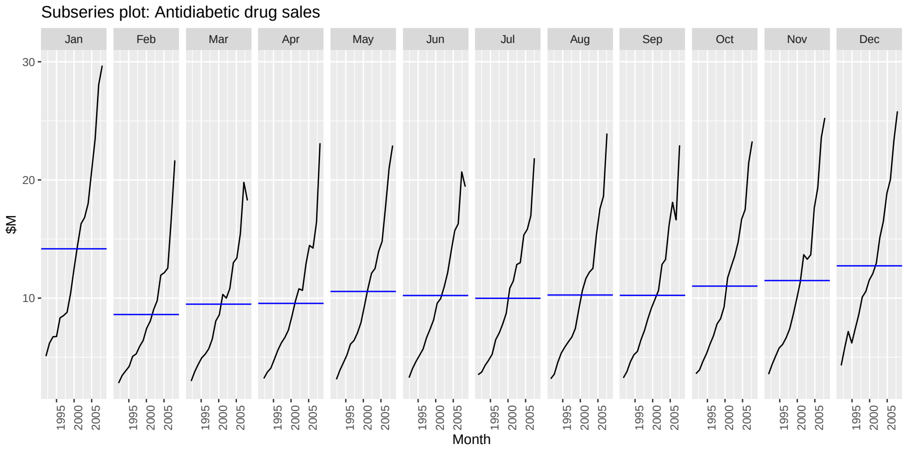


```r
# Plot yearly pattern
vic_elec %>% gg_season(Demand, period = "year") +
  theme(legend.pos = "bottom") +
  labs(title = "Electricity Demand",
       subtitle = "Victoria - Annually",
       y = "MWh")
```



Seasonal subseries plots

```r
a10 %>% gg_subseries(TC) +
  labs(y = "$M", title= "Subseries plot: Antidiabetic drug sales")
```

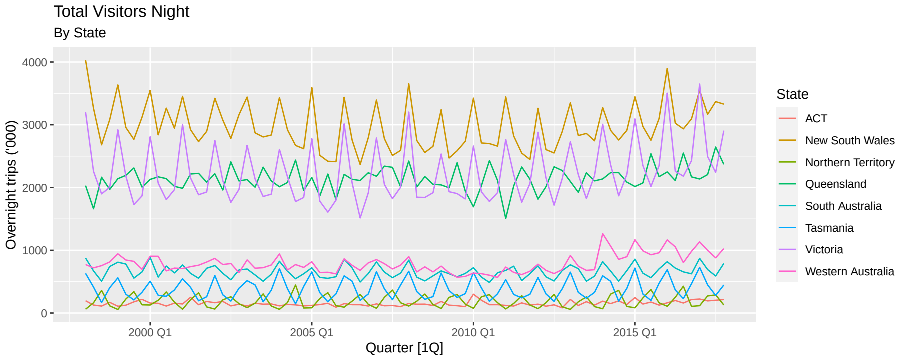


Seasonal subseries plots

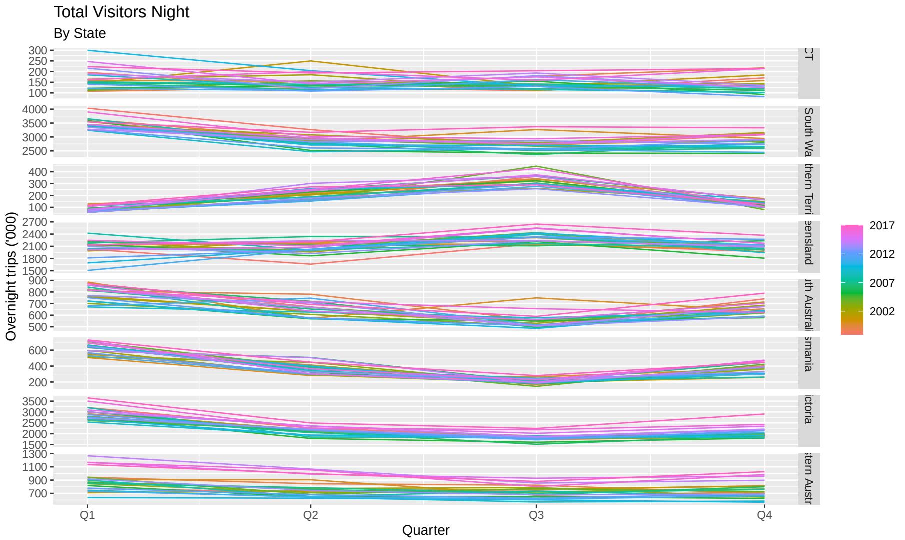
- Data for each season collected together in time plot as separate time series.
- Enables the underlying seasonal pattern to be seen clearly, and changes in seasonality over time to be visualized.
- In R: `gg_subseries()`

Australian Tourism Data

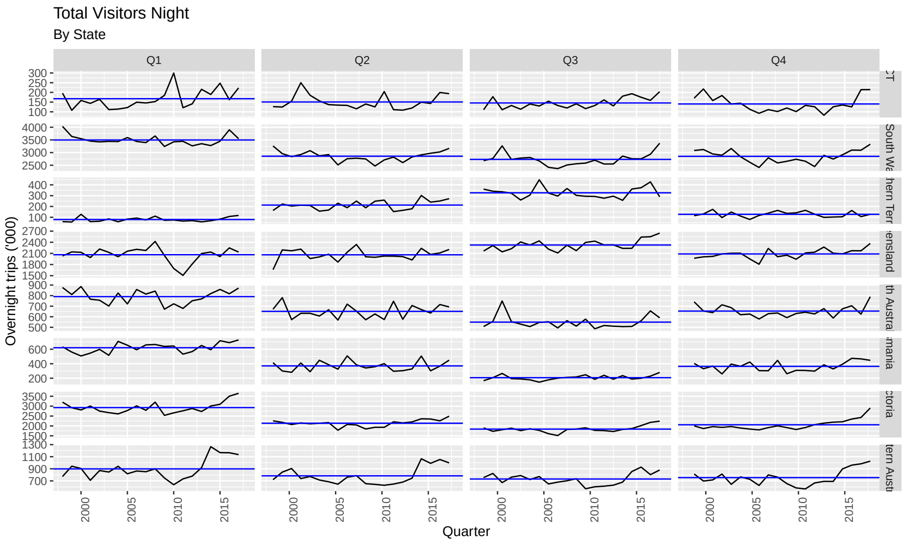
```r
holiday <- tourism %>% filter(Purpose == "Holiday") %>%
  #Group the data by state then aggregate
  group_by(State) %>% summarise(total = sum(Trips))

holiday %>% autoplot(total) +
  labs(title = "Total Visitors Night", subtitle = "By State",
       y = "Overnight trips ('000)")
```



```r
holiday %>% gg_season(total) +
  labs(title = "Total Visitors Night", subtitle = "By State",
       y = "Overnight trips ('000)")
```



```r
holiday %>% gg_subseries(total) +
  labs(title = "Total Visitors Night", subtitle = "By State",
       y = "Overnight trips ('000)")
```



Let's Practice!!!

The `aus_arrivals` data set reports quarterly international arrivals to Australia from Japan, New Zealand, UK, and the US.

- Using the `autoplot()` and `gg_season()`, and `gg_subseries()` functions, **discuss** and **compare** the arrivals from each of these.
- Can you identify any unusual observations?

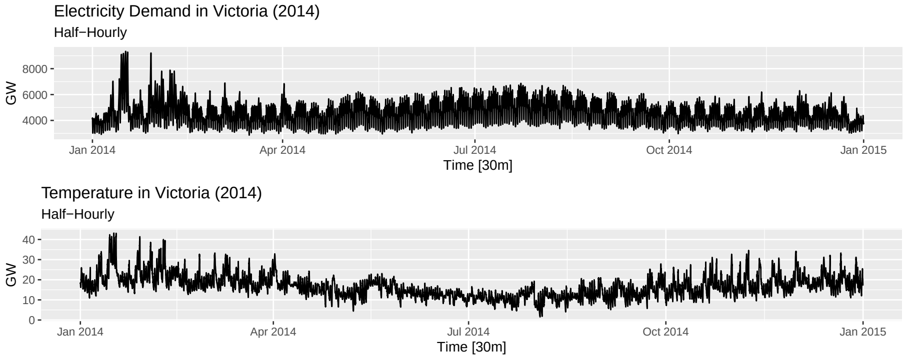
Section 4

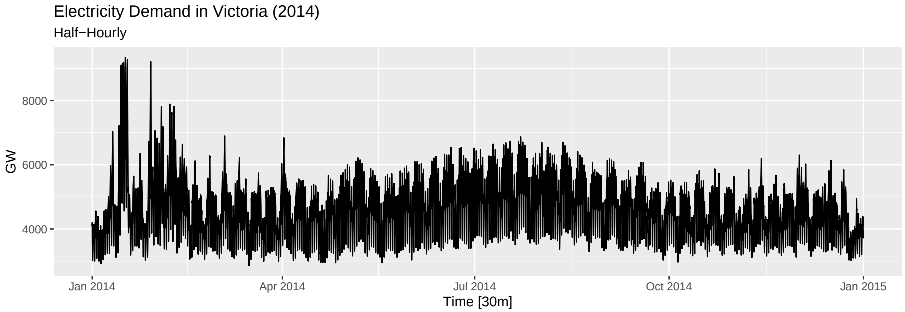
Scatterplots

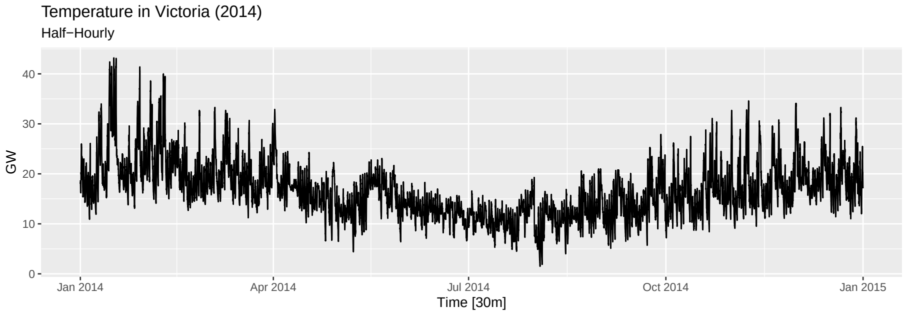
Scatterplots: Relationship *Between* Time Series

Scatterplots are commonly used tools for visualizing the relationship between 2 variables. Take the text example of Electricity Demand and Temperature in Victoria, Australia:

```
p1 <- vic_elec %>% filter(year(Date) == 2014) %>% autoplot(Demand) +
  labs(title = "Electricity Demand in Victoria (2014)",
       subtitle = "Half-Hourly", y = "GW")
p2 <- vic_elec %>% filter(year(Date) == 2014) %>% autoplot(Temperature) +
  labs(title = "Temperature in Victoria (2014)", subtitle = "Half-Hourly",
       y = "GW")
gridExtra::grid.arrange(p1,p2, ncol = 1)
```

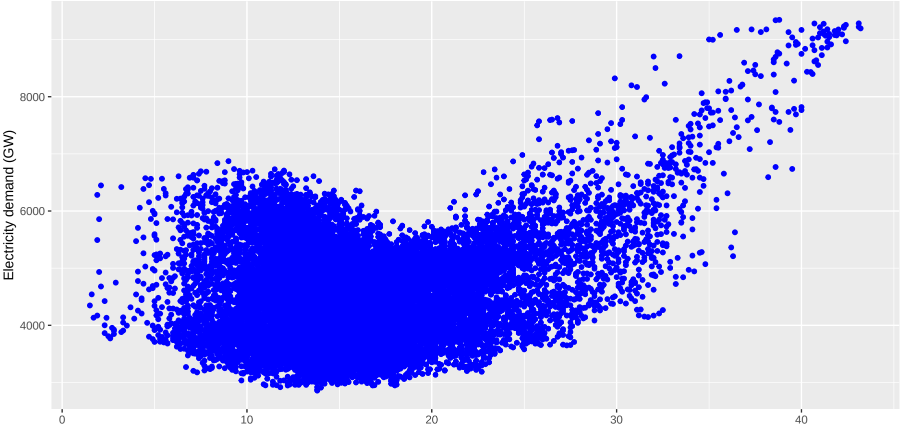



Electricity Demand in Victoria (2014)
Half−Hourly
GW
8000
6000
4000
Jan 2014
Apr 2014
Jul 2014
Oct 2014
Jan 2015
Time [30m]


Temperature in Victoria (2014)
Half−Hourly
GW
40
30
20
10
0
Jan 2014
Apr 2014
Jul 2014
Oct 2014
Jan 2015
Time [30m]

What would you make of this relationship?

```
vic_elec %>% filter(year(Date) == 2014) %>%
  ggplot(aes(x = Temperature, y = Demand)) +
  geom_point(col = "blue") +
  labs(x = "Temperature (degrees Celsius)",
       y = "Electricity demand (GW)")
```



Scatterplots: Relationship *Between* Time Series

- Higher temperatures are associated with higher demand for electricity.
- Some high demand for heating on the lower end of the graph.

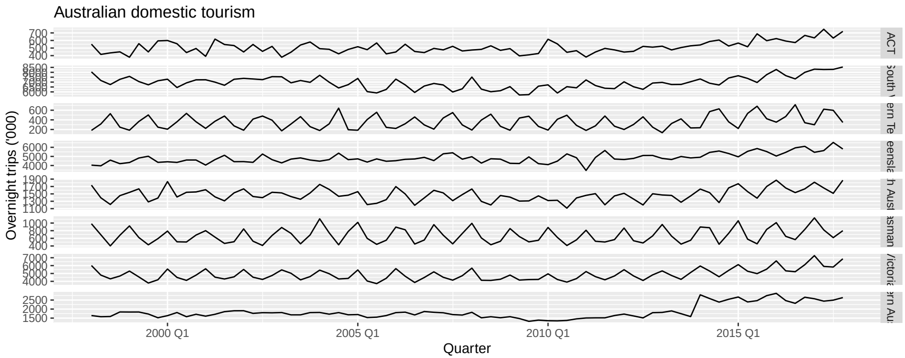
Measuring the strength of this relationship?

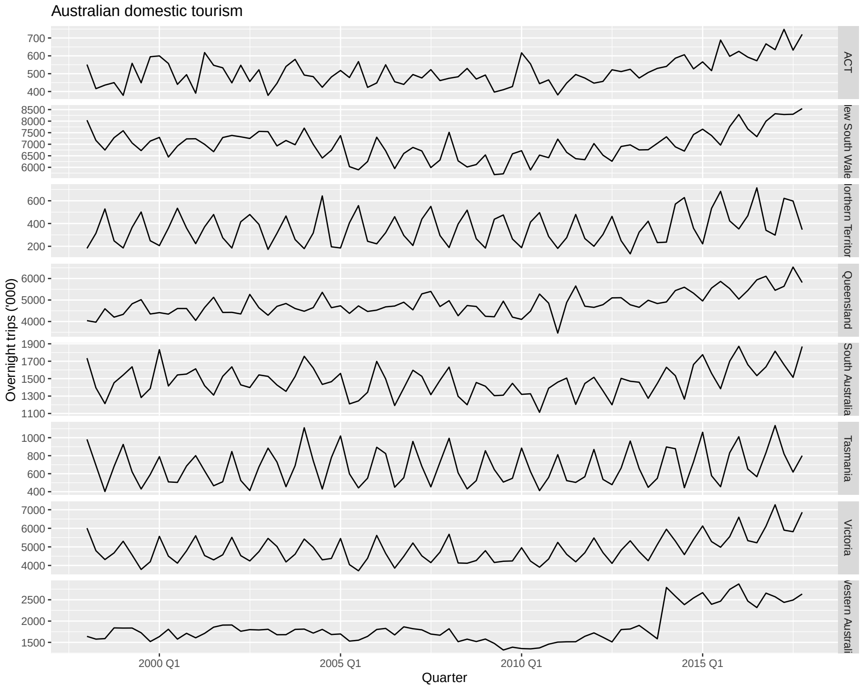
- For this, we will use the **correlation coefficient**:

$$\rho_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y - \bar{y})^2}} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sigma_x^2}\sqrt{\sigma_y^2}}, \quad -1 \leq \rho_{xy} \leq 1$$

- Negative values indicate a negative *linear* relationship between `x` and `y`.
- Positive values indicate a positive *linear* relationship between `x` and `y`.

```r
visitors <- tourism %>% group_by(State) %>%
  summarise(Trips = sum(Trips))

visitors %>% ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  #plot grid by state and with indep. y-axis scalings
  facet_grid(vars(State), scales = "free_y") +
  labs(title = "Australian domestic tourism", y = "Overnight trips ('000)")
```

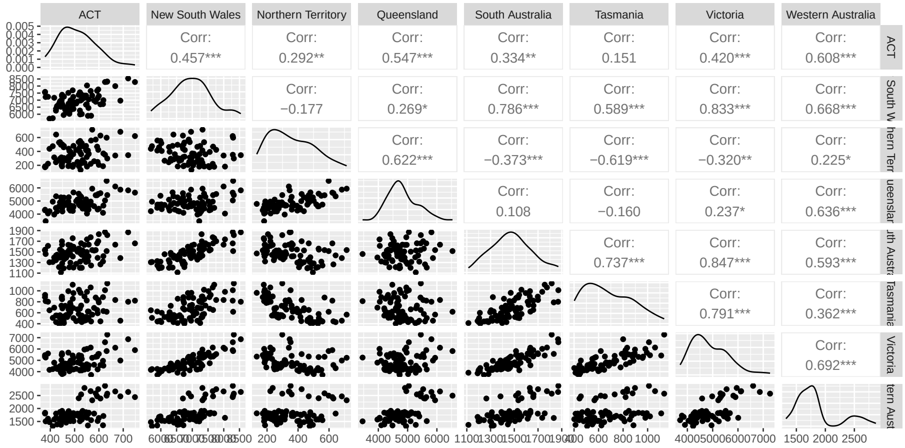



Australian domestic tourism
ACT
New South Wales
Northern Territory
Queensland
South Australia
Tasmania
Victoria
Western Australia
Overnight trips ('000)
Quarter
2000 Q1
2005 Q1
2010 Q1
2015 Q1

Scatterplots (Multiple Series){plain}

We can visualize the correlation between multiple series using a scatterplot matrix. In `R`, we will use the `ggpairs()` command from the `GGally` package.

```
visitors %>% pivot_wider(values_from = Trips, names_from = State) %>%
GGally::ggpairs(col = 2:ncol(.))
```

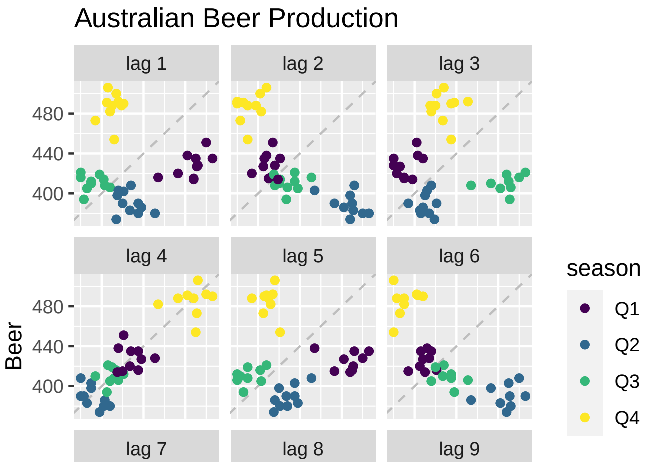


Section 5

Lag plots and autocorrelation

Example: Beer production

```r
recent_production <- aus_production %>%
  filter(year(Quarter) >= 2000)
recent_production %>% gg_lag(Beer, geom = "point") +
  labs(x = "lag(Beer, k)", title = "Australian Beer Production")
```



Lagged scatterplots

- Each graph shows y_t plotted against y_{t-k} for different values of k.
- Strongly positive relationships at lags 4 and 8, reflecting the strong seasonality in the data.
- The negative relationship at lags 2 and 6 occurs because peaks (in Q4) are plotted against troughs (in Q2).
- The autocorrelations are the correlations associated with these scatterplots.

Autocorrelation

Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

Autocovariance and **autocorrelation**: measure **linear relationship** between **lagged values** of a time series y.

We measure the relationship between:

- y_t and y_{t-1}
- y_t and y_{t-2}
- y_t and y_{t-3}
- etc.

Autocorrelation

Let us denote the sample autocovariance at lag k as γ_k and the sample autocorrelation at lag k by ρ_k. Then define

$$\gamma_k = \frac{1}{T}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

$$\text{and} \qquad \rho_k = \frac{\gamma_k}{\gamma_0}$$

- It is easy to see that γ_0 is actually the variance of y. Let `k = 0` then:

$$\gamma_0 = \frac{1}{T}\sum_{t=1}^{T}(y_t - \bar{y})(y_t - \bar{y}) = \frac{1}{T}\sum_{t=1}^{T}(y_t - \bar{y})^2$$

- ρ_1 indicates how successive values of y relate to each other
- ρ_2 indicates how y values two periods apart relate to each other
- ρ_k is *almost* the same as the sample correlation between y_t and y_{t-k}.

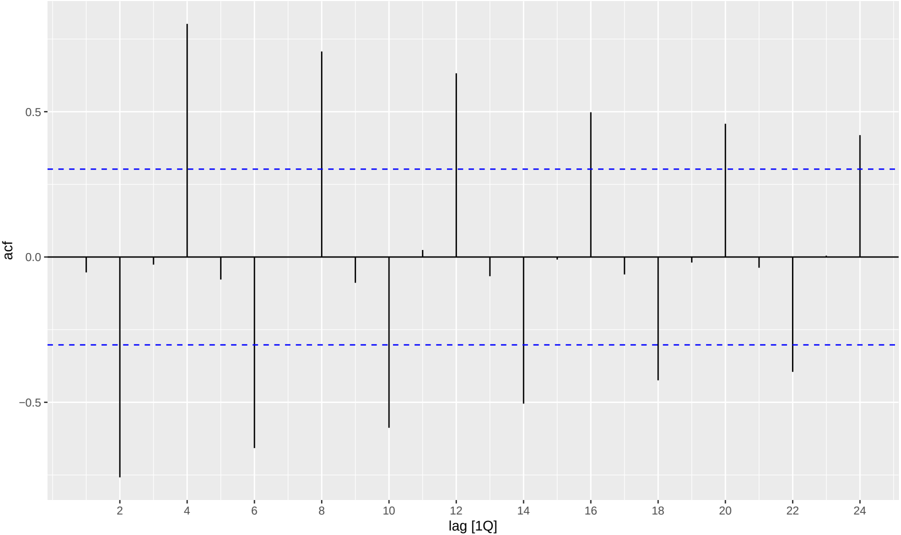
Results for first 9 lags for the Beer production data:

```
recent_production %>% ACF(Beer, lag_max = 9)
```

```
## # A tsibble: 9 x 2 [1Q]
##        lag      acf
##   <cf_lag>    <dbl>
## 1       1Q -0.0530
## 2       2Q -0.758
## 3       3Q -0.0262
## 4       4Q  0.802
## 5       5Q -0.0775
## 6       6Q -0.657
## 7       7Q  0.00119
## 8       8Q  0.707
## 9       9Q -0.0888
```

Autocorrelation

```
recent_production %>% ACF(Beer, lag_max = 24) %>% autoplot()
```



Autocorrelation

We again have confirmation that:

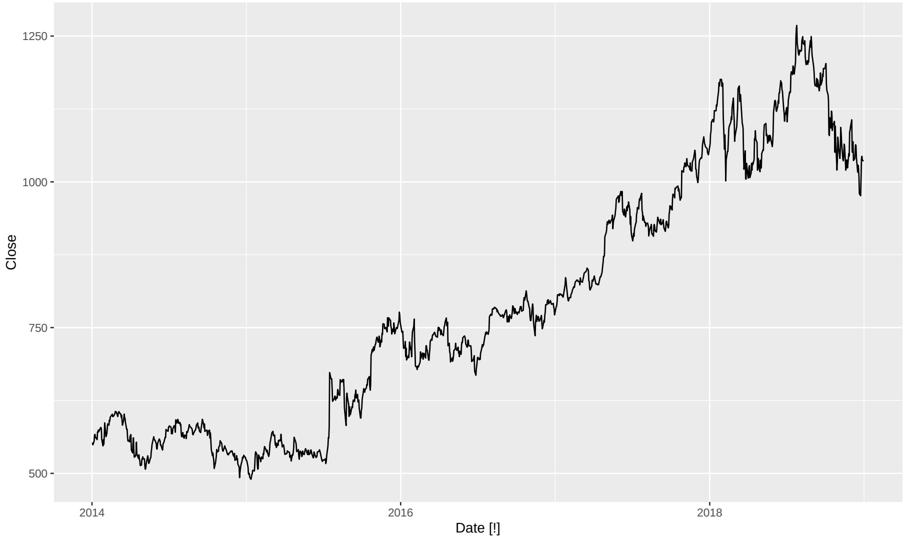
- ρ_4 higher than for the other lags. This is due to **the seasonal pattern in the data**: the peaks tend to be **4 quarters** apart and the troughs tend to be **2 quarters** apart.
- ρ_2 is more negative than for the other lags because troughs tend to be 2 quarters behind peaks.
- Together, the autocorrelations at lags 1, 2, . . . , make up the **autocorrelation function** or ACF.
- The plot is known as a **correlogram**

Trend and seasonality in ACF plots

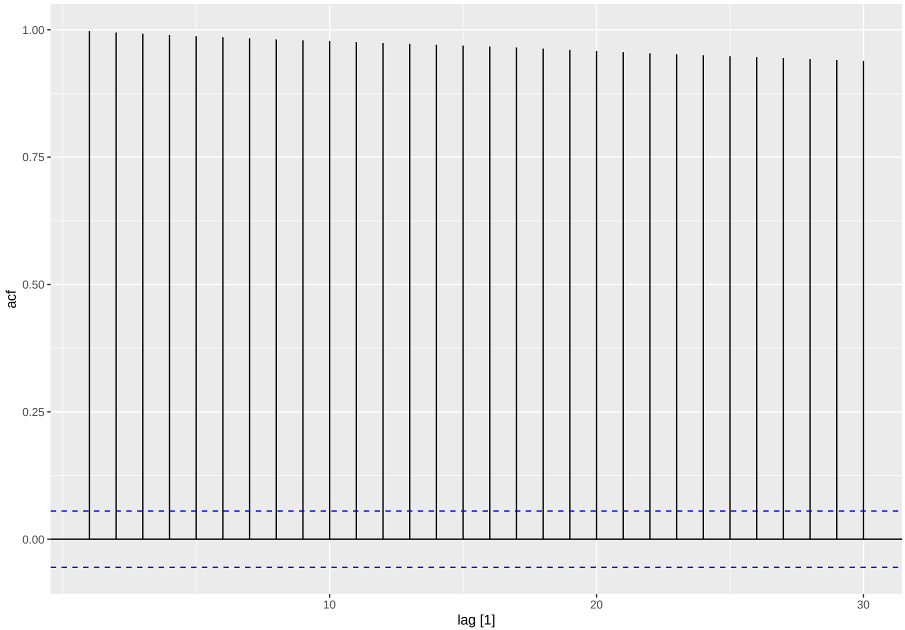
- When data have a trend, the autocorrelations for small lags tend to be large (near 1) and positive.
- When data are seasonal, the autocorrelations will be larger at the seasonal lags (i.e., at multiples of the seasonal frequency)
- When data are trended and seasonal, you see a combination of these effects.

Google Stock Prices

```r
gafa_stock %>% filter(Symbol == "GOOG") %>% autoplot(Close)
```

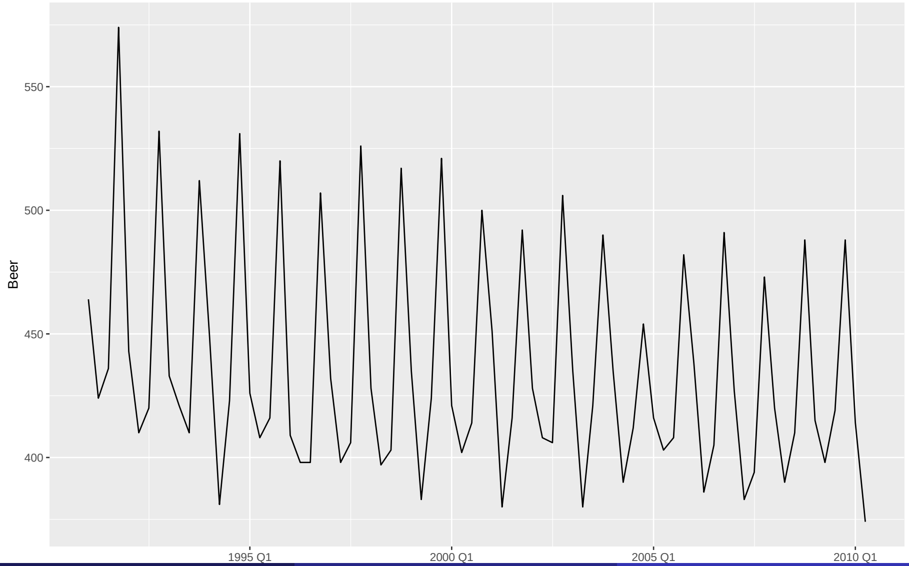


```r
gafa_stock %>% filter(Symbol == "GOOG") %>% ACF(Close) %>% autoplot()
```

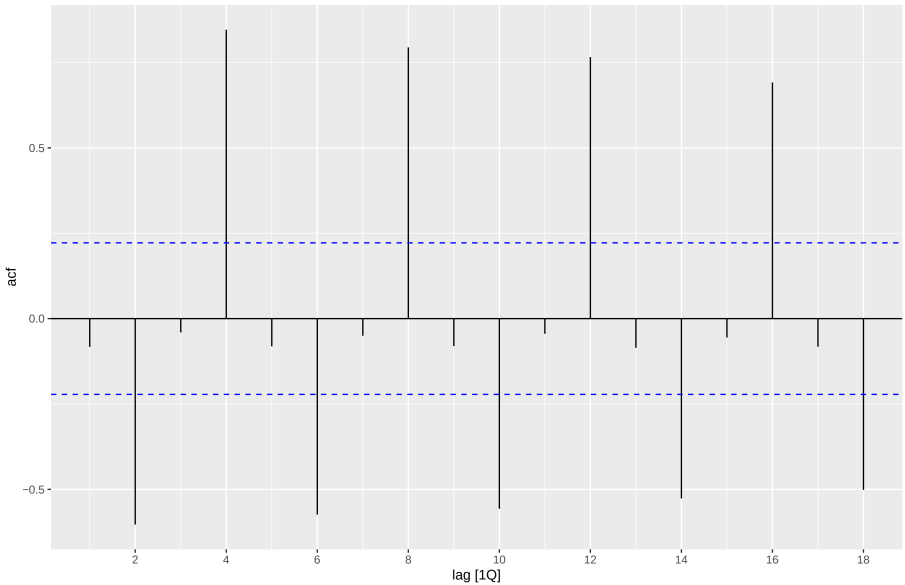


Aus Beer production

```
aus_production %>% filter(year(Quarter)> 1990) %>% autoplot(Beer)
```

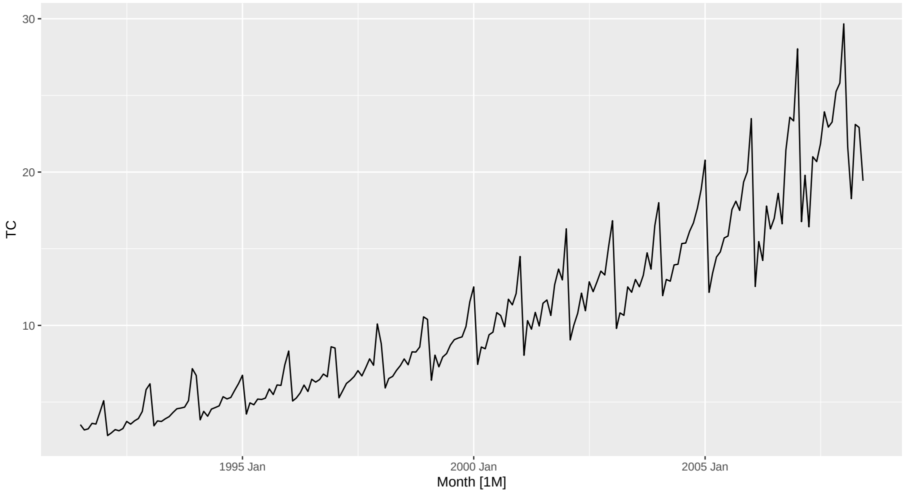


```r
aus_production %>% filter(year(Quarter)> 1990) %>%
  ACF(Beer) %>% autoplot()
```

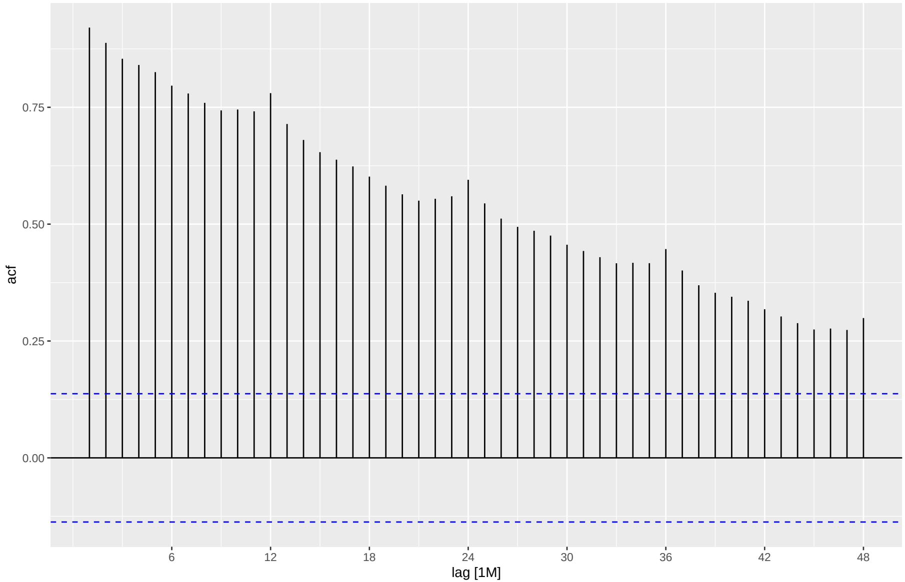


Aus Antidiabetic Sales

```
a10 %>% autoplot(TC)
```



```r
a10 %>% ACF(TC, lag_max = 48) %>% autoplot()
```



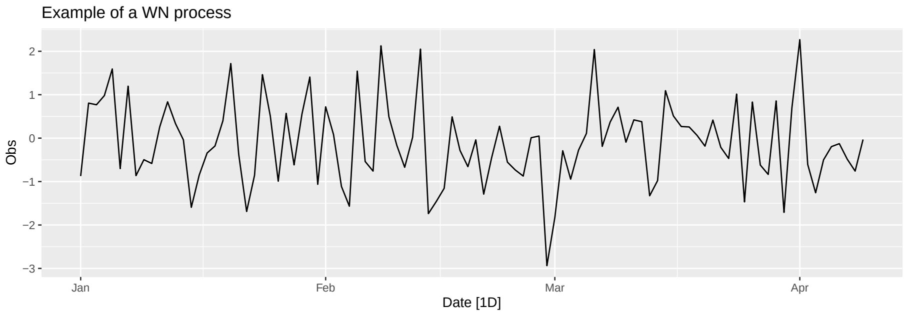
Section 6

White Noise Process

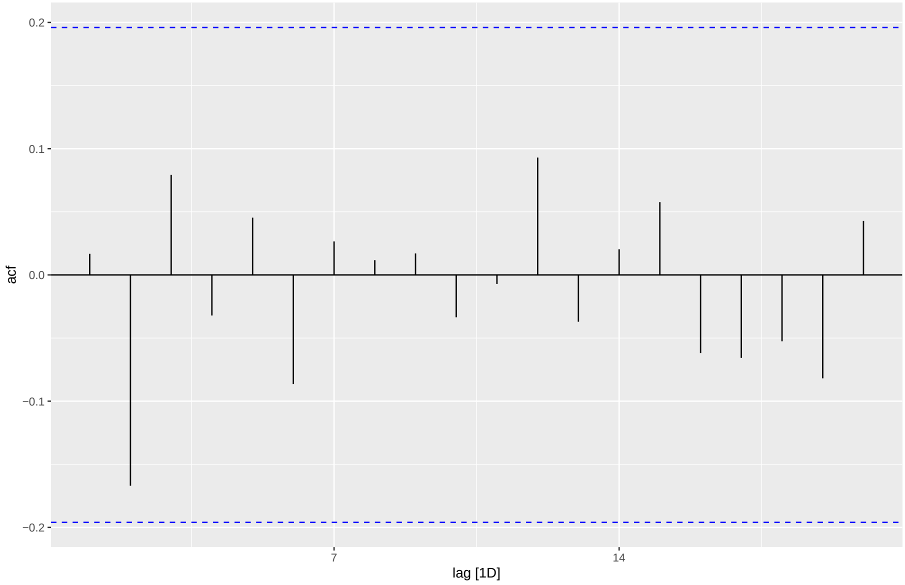
White Noise Process

A time series that displays no autocorrelation is called **white noise**.

```
set.seed(1256)
x <- tsibble(Date = as.Date("2020-01-01") + 0:99,
             Obs = rnorm(100), index = Date)
x %>% autoplot(Obs) + labs(title = "Example of a WN process")
```

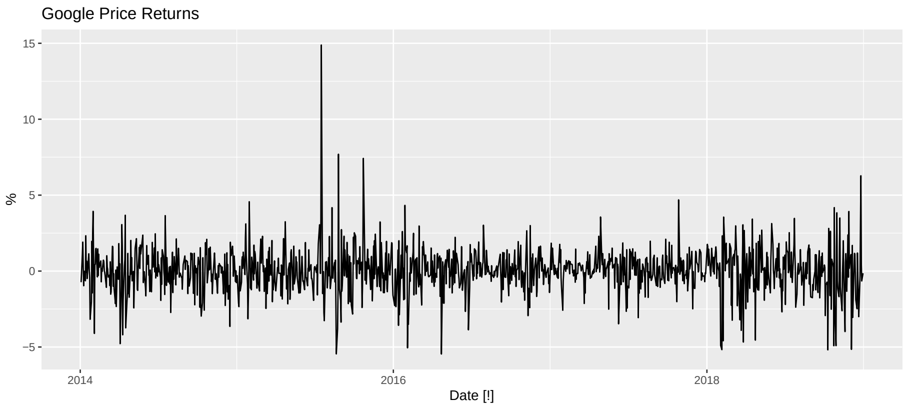


```r
x %>% ACF(Obs) %>% autoplot()
```

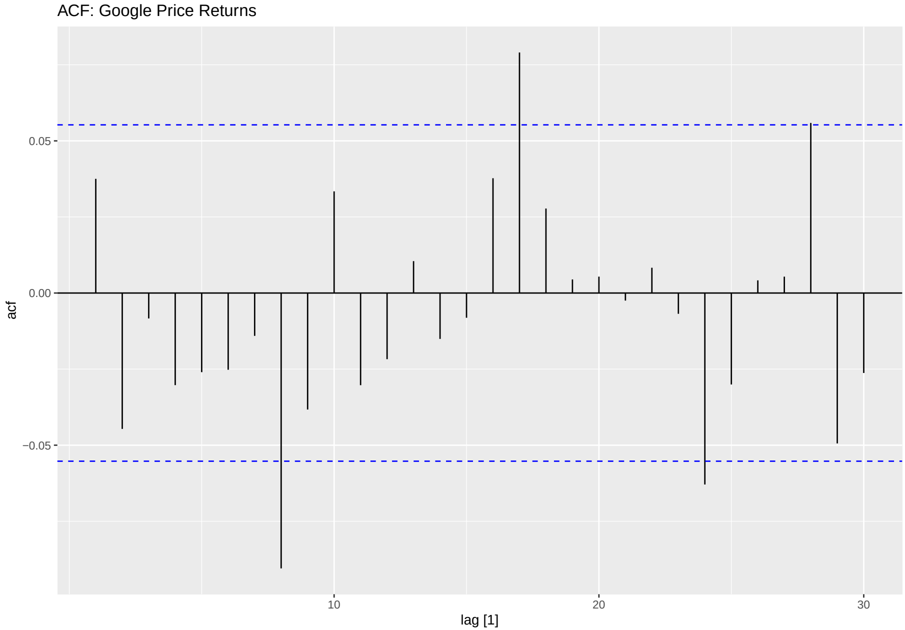


Google Returns

```r
dgoog <- gafa_stock %>% filter(Symbol == "GOOG") %>%
  select(Date,Close) %>%
   mutate(l.price = log(Close),
          return = c(NA,diff(l.price))*100)

dgoog %>% autoplot(return) + labs(title = "Google Price Returns",
                                   y = "%")
```



```r
dgoog %>% ACF(return) %>% autoplot() +
  labs(title = "ACF: Google Price Returns")
```



WN: What to look for

- We expect each autocorrelation to be **close to zero**.
  - They will not be exactly equal to zero as there is some random variation.
- We expect 95% of the spikes in the ACF to lie within $\pm 2/\sqrt{T}$, where T is the length of the time series (number of observations).
- These bounds are represented as the blue dashed lines above. If

i. one or more large spikes are outside these bounds, or
ii. if substantially more than 5% of spikes are outside these bounds,

then the series is probably not white noise.

White Noise Processes: Formally

A sequence $\{y_t\}$ is a **white noise** process if each value in the sequence has:

1. A mean of zero, such that $\mathbf{E}(y_t) = 0, \forall\, t$
2. A constant (finite) variance, such that $\mathbf{var}(y_t) = \sigma^2, \forall\, t$
3. Is uncorrelated with all other realizations (past values), i.e. There is no serial correlation. Such that $\rho_k = 0, \forall\, k \neq 0$

A slightly stronger condition is that they are independent from one another; this is an "independent white noise process."

Note: 1. and 2. are under the assumption that $\{y_t\}$ is normally distributed. **Otherwise, each value in the sequence should have a finite mean and variance.**

Do you Agree??

A linear combination of white noises **IS necessarily** also a White noise sequence.

Example: White noise

Assume that

$$y_t = \varepsilon_t + c_1 \varepsilon_{t-1},\ \varepsilon_t \sim WN(0, \sigma^2),\ c_1 \neq 0$$

Show that:

- $\mathbf{E}(y_t) = 0$ (Constant Mean)
- $\mathbf{var}(y_t) = (1 + c_1^2) \cdot \sigma^2$ (Constant Variance)
- Only γ_0 and γ_1 are non-zero, and $\rho_1 = \frac{c_1}{(1+c_1^2)}$
    - That is, there is autocorrelation at lag 1, and none at any other lag.

Final Words: ACF

The *kth* order of the autocovariance is the covariance of y_{t} with its own kth lag:

$$
\begin{aligned}
Cov(y_t, y_{t-1}) &= \gamma(1), \quad k = 1 \\
Cov(y_t, y_{t-2}) &= \gamma(2), \quad k = 2 \\
&\vdots \\
Cov(y_t, y_{t-k}) &= \gamma(k), \quad k = k
\end{aligned}
$$

Note that

$$Cov(y_t, y_{t-0}) = Var(y_t) = \gamma(0), \quad k = 0$$

$$ACF = \frac{Cov(y_t, y_{t-k})}{\sqrt{Var(y_t)Var(y_{t-k})}} = \frac{\gamma(k)}{\gamma(0)} = \rho(k)$$

Final Words: ACF

$$\rho_0 = \frac{Cov\left(Y_t, Y_{t-0}\right)}{\sqrt{Var(y_t)Var(y_{t-0})}} = \frac{\gamma(0)}{\gamma(0)} = 1$$

$$\rho_1 = \frac{Cov\left(Y_t, Y_{t-1}\right)}{\sqrt{Var(y_t)Var(y_{t-1})}} = \frac{\gamma(1)}{\gamma(0)}$$

$$\rho_2 = \frac{Cov\left(Y_t, Y_{t-2}\right)}{\sqrt{Var(y_t)Var(y_{t-2})}} = \frac{\gamma(2)}{\gamma(0)}$$

$$\vdots$$

$$\rho_k = \frac{Cov\left(Y_t, Y_{t-k}\right)}{\sqrt{Var(y_t)Var(y_{t-k})}} = \frac{\gamma(k)}{\gamma(0)}$$

$$\vdots$$

Let's Practice!!!

In this lesson, we explored the following graphics functions:

- `autoplot`
- `gg_season`
- `gg_subseries`
- `gg_lag`
- `ACF`

Using these functions, as appropriate, explore the meat production in Australia dataset, `aus_livestock`. Your interest is in the `Pigs` slaughtered in `Western Australia`.

- You might need to first view the data to get familiar with the structure.
- The `distinct()` function should come in handy.
- Can you spot any seasonality, cycle, and trend? What do you learn about the series?