Time Series & Volatility Modeling

M&M Series

1. What is Time Series Analysis?

2. Some Forecasting Methods

Section 1

What is Time Series Analysis?

Just what is this?

General definition: A time series is a collection of observations made sequentially through time. The dynamics of said observations are often characterized by short-term/long-term fluctuations, long-term (direction) trends, and seasonal patterns.

Time Series Analysis is the process of analyzing time series data to extract meaningful statistics and other characteristics of the data.

Just what is this?

Our observations can be denoted by $y_1, y_2, \ldots, y_T$ where T is the total number of observations.

- y_1 is the observation at time $t = 1$,
- y_2 is the observation at time $t = 2$, and so on.
- The interval between observations can be any time interval (minute, hours, days, weeks, months, quarters, years, etc.) and we assume that these time periods are equally spaced.

Time Series vs Cross-Sectional Data

1. Cross-sectional data

- Multiple objects observed at a particular (single) point in time.
- Observations are *potentially* independent of each other.
- Examples:
  - Survey data for a single year
  - Census data for a single year
  - Farm Balance Sheets as at the end of the year, etc.

Time Series vs Cross-Sectional Data

2. Time series data

- Single object (GDP, stock price, temperature, etc.) observed at multiple **equally-spaced points** in time.
- Observations have *potentially* time dependence, generally called **SERIAL CORRELATION** OR **AUTOCORRELATION**.
- Examples:
  - Daily stock prices
  - Half-hourly temperature or CO2 levels
  - Monthly unemployment rates
  - Annual GDP growth rates, etc.

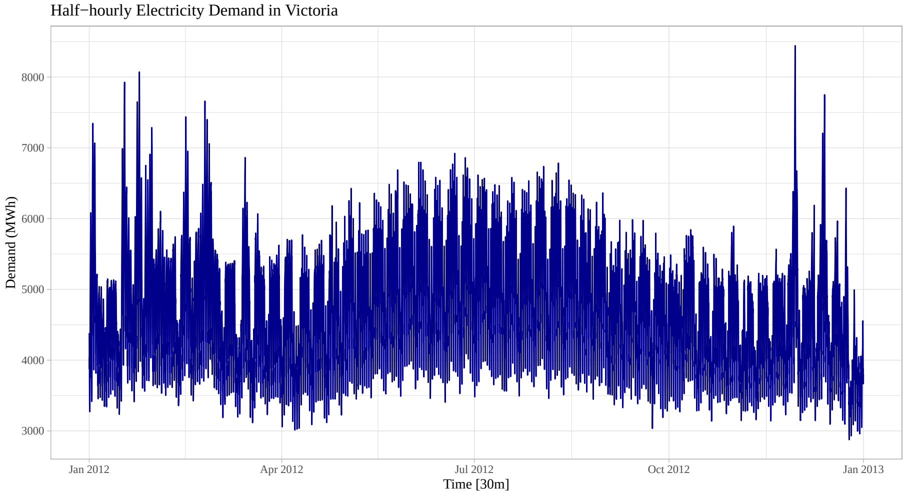
Time Series vs Cross-Sectional Data

3. Panel Data

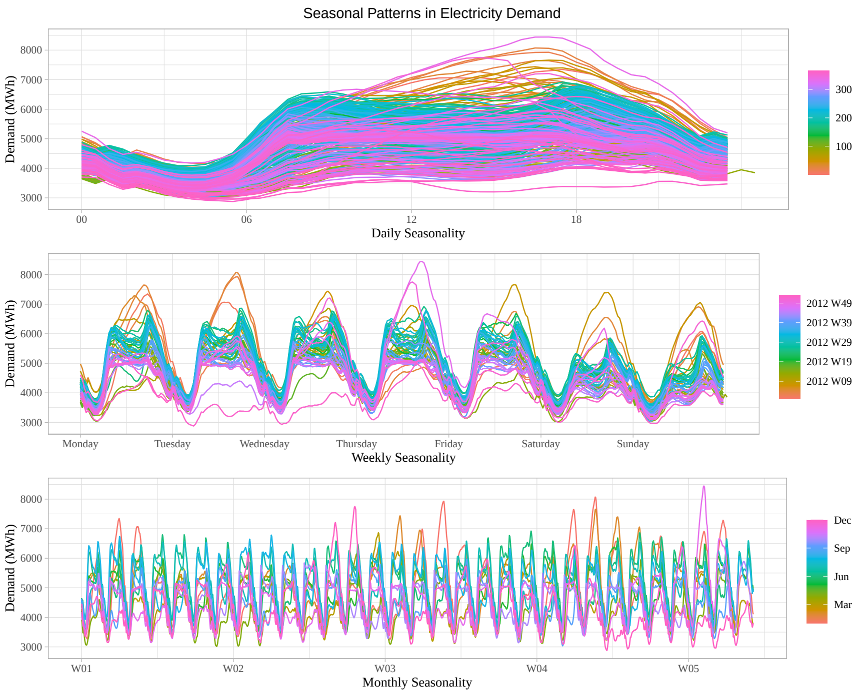
Panel data is a combination of both cross-sectional and time series data.

- Multiple objects observed at multiple points in time.
- Observations are potentially time and spatially dependent.
- Examples:
  - Household surveys over time
  - Firm-level profits and losses over time
  - The growth rate of GDP across countries over time, etc.

```r
v.elec <- vic_elec %>% filter(year(Date) == 2012)

v.elec %>% autoplot(Demand, col = "darkblue") +
  labs(title = "Half-hourly Electricity Demand in Victoria",
       y = "Demand (MWh)")
```



```r
p1 <- v.elec %>% gg_season(Demand, period = "day") +
  labs(x = "Daily Seasonality",
       y = "Demand (MWh)") + theme(legend.position = "right")

p2 <- v.elec %>% gg_season(Demand, period = "week") +
  labs(x = "Weekly Seasonality",
       y = "Demand (MWh)") + theme(legend.position = "right")

p3 <- v.elec %>% gg_season(Demand, period = "month") +
  labs(x = "Monthly Seasonality",
       y = "Demand (MWh)") + theme(legend.position = "right")

gridExtra::grid.arrange(p1, p2, p3, ncol = 1,
                        top = "Seasonal Patterns in Electricity Demand")
```


Seasonal Patterns in Electricity Demand
Demand (MWh)
Daily Seasonality
00
06
12
18
300
200
100
Weekly Seasonality
Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
Sunday
2012 W49
2012 W39
2012 W29
2012 W19
2012 W09
Monthly Seasonality
W01
W02
W03
W04
W05
Dec
Sep
Jun
Mar

Main Objectives of Time Series Analysis

1. Summary description (graphical and numerical) of data point vs. time
2. Interpretation of specific series features (e.g. seasonality, trend, relationship with other series)

- What is the average temperature in June over a 10-year period?
- What is the underlying trend in GDP growth over the last 20 years?
- Has technological advancements caused a change in the trend in corn yield over a 30 year period?

Main Objectives of Time Series Analysis

3. To estimate **dynamic** causal effects
   - If the Fed increases the Federal Funds rate now, what will be the effect on the rates of inflation and unemployment in 3 months? in 12 months?
   - What is the effect over time on cigarette consumption after a hike in the cigarette tax?
4. Forecasting (predicting the future values of the series).
   - What is the potential growth in yield in the next quarter?
   - What is the expected inflation rate in the next year?

Main Objectives of Time Series Analysis

5. Hypothesis testing and Simulation (comparing different scenarios)

- How does the GDP respond to a 1 standard deviation shock in oil prices?
- What is the spillover effect of a shock in the US stock market on the Nigerian stock market?

Section 2

Some Forecasting Methods

Volatility Modelling

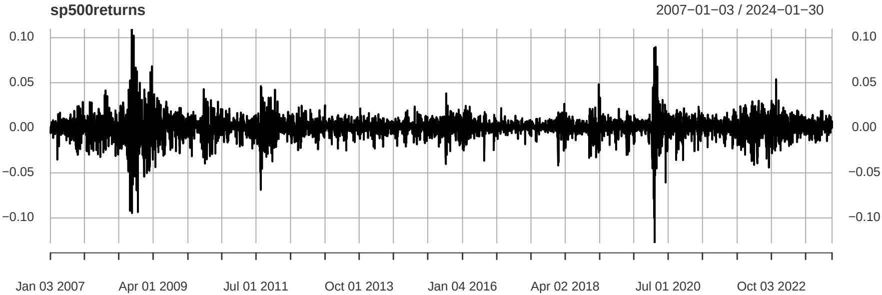
- ARCH/GARCH Models

$$returns = \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right) \approx \log\left(\frac{P_t}{P_{t-1}}\right)$$

Example in R for daily S&P 500 prices

```
library(PerformanceAnalytics)
library(quantmod)
#SP500 Prices
sp500prices <- getSymbols("^GSPC", from = "2007-01-01",
                          to = "2024-01-31",
                          auto.assign = FALSE) %>% Cl()
#SP500 Returns
sp500returns <- CalculateReturns(sp500prices, method = "log")
```

Volatility Modeling: S&P 500 Returns



Properties of the daily returns:

- Returns are mean zero
- Return variability/volatility (σ_t) changes through time
- Volatility clustering

Volatility Modeling: S&P 500 Returns

Daily vs Annualized Volatility

```r
# Daily Volatility
cat("Daily Volatility =", sd(sp500returns, na.rm = TRUE), "\n")
```

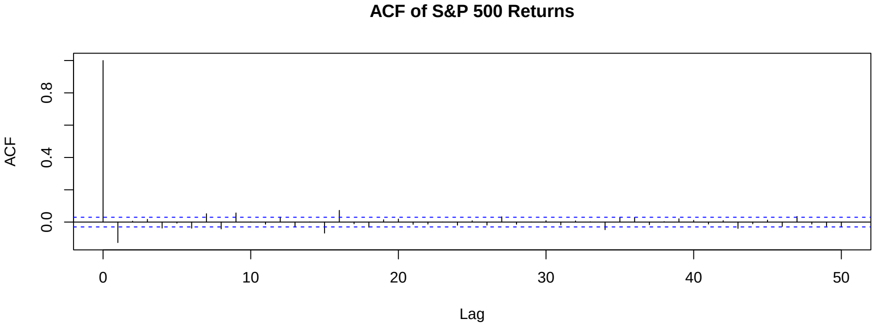
```
## Daily Volatility = 0.01278
```

```r
# Annualized Volatility
cat("Annualized Volatility =", sqrt(252)*sd(sp500returns, na.rm = TRUE))
```

```
## Annualized Volatility = 0.2028
```

Volatility Modeling: S&P 500 Returns

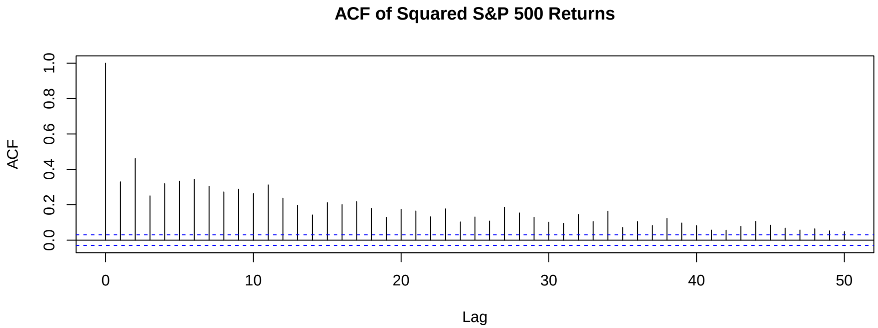
Autocorrelation of Returns




```
##
##  Box-Ljung test
##
## data:  sp500returns[-1]
## X-squared = 174, df = 20, p-value <2e-16
```

Volatility Modeling: S&P 500 Returns

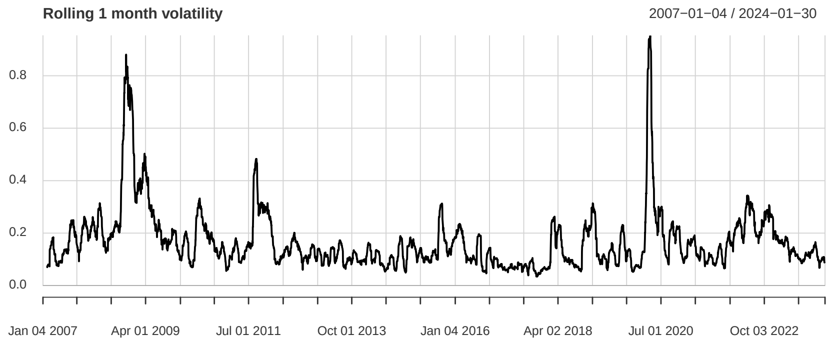
Autocorrelation of Squared Returns




```
##
##  Box-Ljung test
##
## data:  sp500returns[-1]^2
## X-squared = 6287, df = 20, p-value <2e-16
```

Volatility Modeling: S&P 500 Returns
Rolling Volatility

```r
chart.RollingPerformance(sp500returns ,
width = 22, # 22 trading days in a month
FUN = "sd.annualized",
scale = 252, # 252 trading days in a year
main = "Rolling 1 month volatility")
```



GARCH Models

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are used to model the conditional variance of a time series.

GARCH(1,1) model is given by:

$$r_t = \mu + \varepsilon_t;\ \varepsilon_t \sim \mathcal{N}(0, \sigma_t^2) \tag{1}$$
$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

where ε_t is the residual at time t and σ_t^2 is the conditional variance at time t.

- $\omega, \alpha, \beta > 0$: ensures that the σ_t^2 is always positive.
- $\alpha + \beta < 1$: ensures that the model is mean-reverting/stationary (returns to the longrun variance).
  - LR variance $= \frac{\omega}{1-\alpha-\beta}$
- Estimate via Maximum Likelihood Estimation (MLE).

GARCH(1,1) Model

Manual Example

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Step I: Specify parameter values

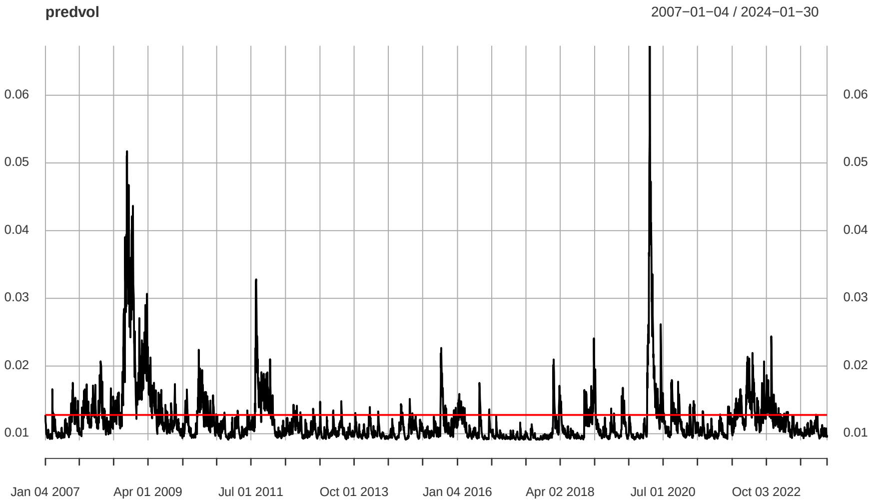
```
# Remove the first observation
sp500ret <- sp500returns[-1]
# Set parameter values
alpha <- 0.15
beta <- 0.7
omega <- var(sp500ret)*(1-alpha-beta)
# Then: var(sp500ret) = omega/(1-alpha-beta)

# Set series of prediction error
e <- sp500ret - mean(sp500ret) # Constant mean
e2 <- e^2
```

Step II: Calculate the conditional variance

```
# We predict for each observation its variance.
nobs <- length(sp500ret)
predvar <- rep(NA, nobs)

# Initialize the process at the sample variance
predvar[1] <- var(sp500ret)

# Loop through the rest of the observations
for (t in 2:nobs) {
  predvar[t] <- omega + alpha*e2[t-1] + beta*predvar[t-1]
}
```

Step III: Plot the predicted conditional variance

```r
# Volatility is sqrt of predicted variance
predvol <- sqrt(predvar)
predvol <- xts(predvol, order.by = time(sp500ret))

# We compare with the unconditional volatility
uncvol <- sqrt(omega / (1 - alpha-beta))
uncvol <- xts(rep(uncvol, nobs), order.by = time(sp500ret))

# Plot
pp <- plot(predvol)
(pp <- lines(uncvol, col = "red", lwd = 2))
```


predvol
2007−01−04 / 2024−01−30
0.06
0.05
0.04
0.03
0.02
0.01
Jan 04 2007
Apr 01 2009
Jul 01 2011
Oct 01 2013
Jan 04 2016
Apr 02 2018
Jul 01 2020
Oct 03 2022

GARCH(1,1) Model Optimized

```r
library(rugarch)
# Fit the GARCH(1,1) model
garch11 <- ugarchspec(variance.model = list(model = "sGARCH"),
                      distribution.model = "norm")
garchfit <- ugarchfit(garch11, sp500ret)
garchfit@fit$matcoef
```

```
##            Estimate  Std. Error  t value  Pr(>|t|)
## mu        7.499e-04   1.039e-04    7.220 5.205e-13
## ar1       6.612e-01   1.526e-01    4.334 1.464e-05
## ma1      -7.140e-01   1.423e-01   -5.018 5.209e-07
## omega     2.943e-06   8.854e-07    3.324 8.862e-04
## alpha1    1.439e-01   1.147e-02   12.549 0.000e+00
## beta1     8.369e-01   1.196e-02   69.956 0.000e+00
```

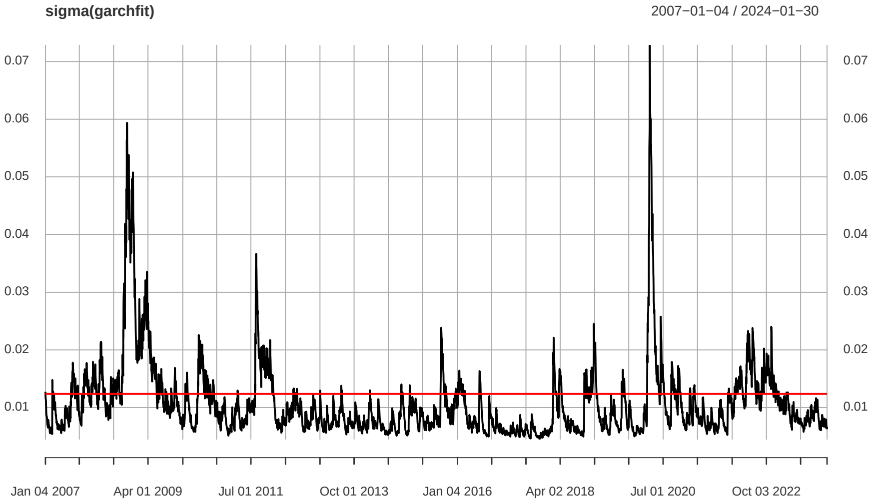
Estimated Model:

$$r_t = 7.236 \times 10^{-4} + \varepsilon_t \ \varepsilon_t \sim \mathcal{N}(0, \hat{\sigma}_t^2) \tag{3}$$

$$\sigma_t^2 = 2.996 \times 10^{-6} + 0.144\varepsilon_{t-1}^2 + 0.836\sigma_{t-1}^2; \tag{4}$$

GARCH(1,1) Model Optimized

Estimated Volatility



GARCH(1,1) Model Optimized

Forecasting Volatility

```r
garchforecast <- ugarchforecast(garchfit, n.ahead = 10)
(forecast_vol <- sigma(garchforecast))
```

```
##       2024-01-30
## T+1     0.006311
## T+2     0.006481
## T+3     0.006644
## T+4     0.006800
## T+5     0.006949
## T+6     0.007092
## T+7     0.007230
## T+8     0.007363
## T+9     0.007491
## T+10    0.007614
```

EGARCH Models: Allowing for Asymmetric Effects

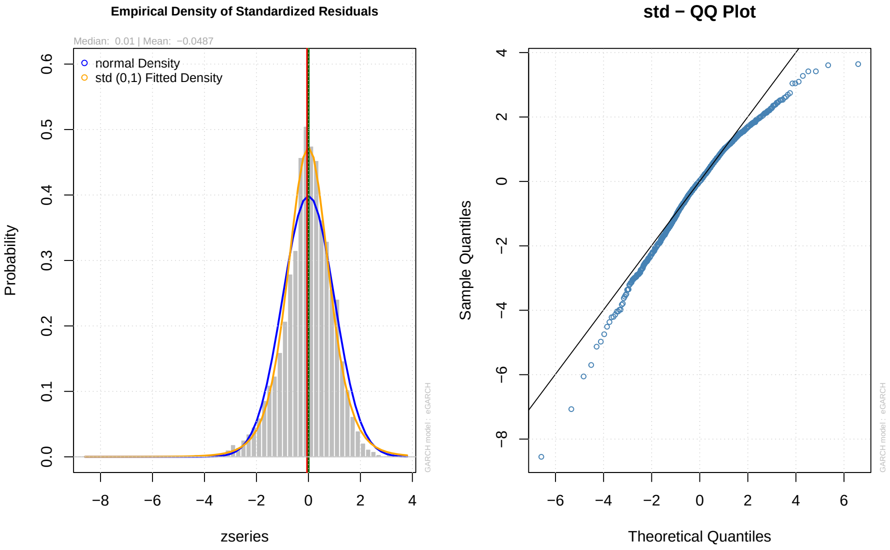
- Does the market respond the same to positive and negative news?
- Exponential GARCH (EGARCH) models allow for asymmetric effects in the volatility.
- The *leverage effect* describes news impacts volatility asymmetrically. ($\alpha < 0$)
- The EGARCH(1,1) model is given by:

$$r_t = \mu + \sigma\varepsilon_t;\ \varepsilon_t \tilde{?} \mathcal{N}(0, \sigma_t^2) \tag{5}$$

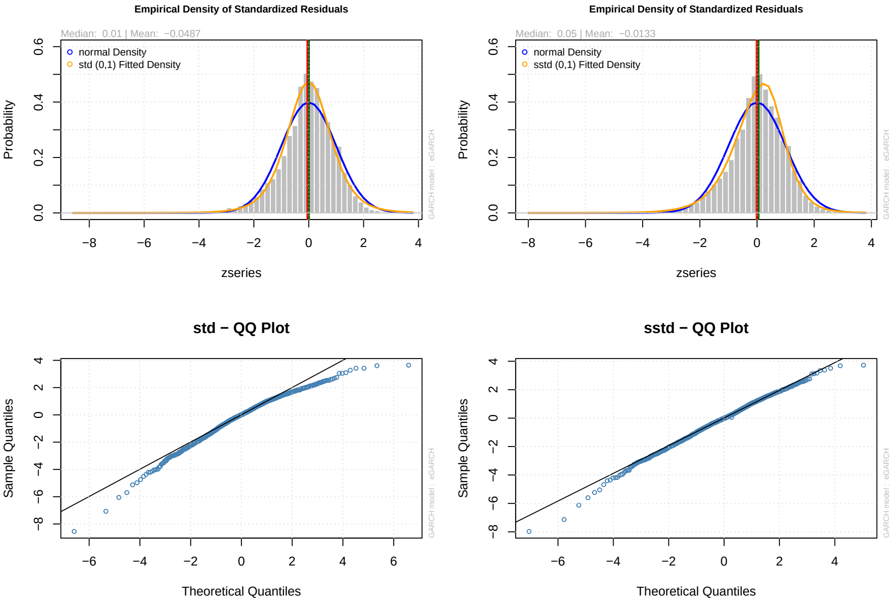
$$\log(\sigma_t^2) = \omega + \alpha\varepsilon_{t-1}^2 + \beta\log(\sigma_{t-1}^2) + \gamma\left|\varepsilon_{t-1}\right| \tag{6}$$

```r
egarch11 <- ugarchspec(variance.model = list(model = "eGARCH"),
                       distribution.model = "std")
egarchfit <- ugarchfit(egarch11, sp500ret)
```

```
##          Estimate  Std. Error  t value  Pr(>|t|)
## mu      0.0006529  9.588e-05    6.810 9.778e-12
## ar1     0.2374131  2.681e-02    8.855 0.000e+00
## ma1    -0.2874032  2.702e-02  -10.637 0.000e+00
## omega  -0.2138488  1.329e-02  -16.096 0.000e+00
## alpha1 -0.1702947  1.176e-02  -14.476 0.000e+00
## beta1   0.9779075  1.428e-03  684.839 0.000e+00
## gamma1  0.1719170  1.792e-02    9.593 0.000e+00
## shape   5.7064263  5.329e-01   10.708 0.000e+00
```

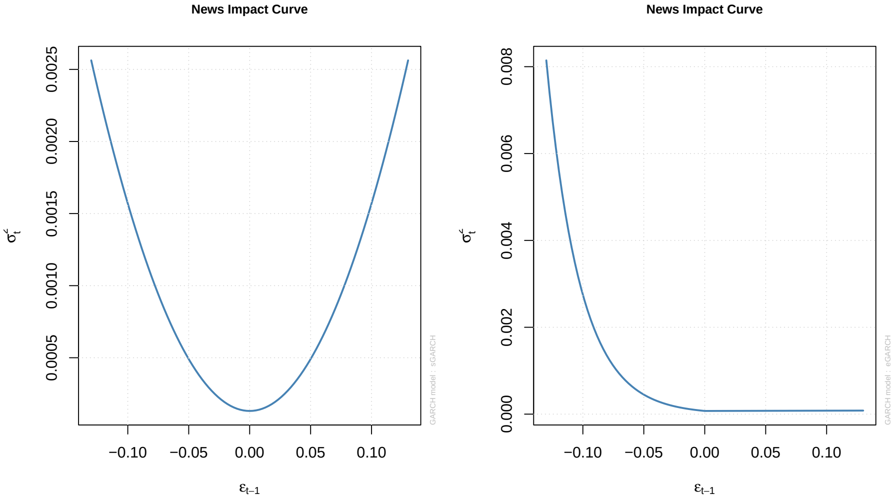


Empirical Density of Standardized Residuals
Median: 0.01 | Mean: −0.0487
normal Density
std (0,1) Fitted Density
Probability
zseries
GARCH model : eGARCH
std − QQ Plot
Sample Quantiles
Theoretical Quantiles
GARCH model : eGARCH


Empirical Density of Standardized Residuals
Median: 0.01 | Mean: −0.0487
normal Density
std (0,1) Fitted Density
Probability
zseries
GARCH model : eGARCH
Empirical Density of Standardized Residuals
Median: 0.05 | Mean: −0.0133
normal Density
sstd (0,1) Fitted Density
Probability
zseries
GARCH model : eGARCH
std − QQ Plot
Sample Quantiles
Theoretical Quantiles
GARCH model : eGARCH
sstd − QQ Plot
Sample Quantiles
Theoretical Quantiles
GARCH model : eGARCH

News Impact Curve

The news impact curve is a plot of the impact of a surprise (think shock) on the volatility of the series.



```r
par(mfrow=c(1,2))
plot(garchfit, which =12) # Standard GARCH
plot(skew.egarchfit, which =12) # EGARCH - Skewed Student-t
```

GJR-GARCH Models: Allowing for Asymmetric Effects

Again, we believe that negative shocks have greater impacts than positive shocks.

The GJR-GARCH(1,1) model is given by:

$$\log(\sigma_t^2) = \omega + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2\mathbf{I}\left(\varepsilon_{t-1} < 0\right) + \beta\sigma_{t-1}^2$$

where $I_{t-1}(\bullet) = 1$ if $\varepsilon_{t-1} < 0$ and 0 otherwise, and $\gamma > 0$.

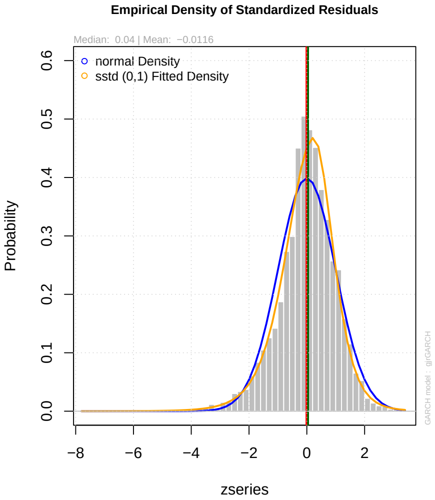
Therefore:

$$\sigma_t^2 = \begin{cases} \omega + (\alpha+\gamma)\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 & \text{if } \varepsilon_{t-1} < 0 \\ \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 & \text{if } \varepsilon_{t-1} \geq 0 \end{cases}$$

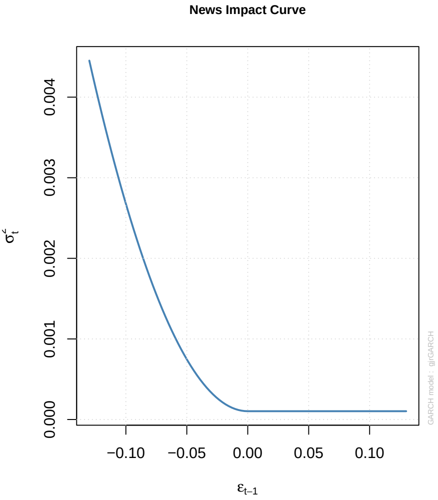
GJRGARCH Models: Allowing for Asymmetric Effects

```
##          Estimate  Std. Error   t value  Pr(>|t|)
## mu     3.390e-04   1.602e-04 2.117e+00 3.429e-02
## omega  2.319e-06   2.294e-06 1.011e+00 3.120e-01
## alpha1 4.454e-07   2.560e-02 1.740e-05 1.000e+00
## beta1  8.593e-01   1.463e-02 5.872e+01 0.000e+00
## gamma1 2.574e-01   5.942e-02 4.332e+00 1.477e-05
## skew   8.534e-01   1.859e-02 4.591e+01 0.000e+00
## shape  6.324e+00   6.156e-01 1.027e+01 0.000e+00
```

```r
gjrgarch11 <- ugarchspec(
  mean.model = list(armaOrder = c(0,0)),
  variance.model = list(model = "gjrGARCH",garchOrder = c(1,1)),
                       distribution.model = "sstd")
gjrgarchfit <- ugarchfit(gjrgarch11, sp500ret)
gjrgarchfit@fit$matcoef
```

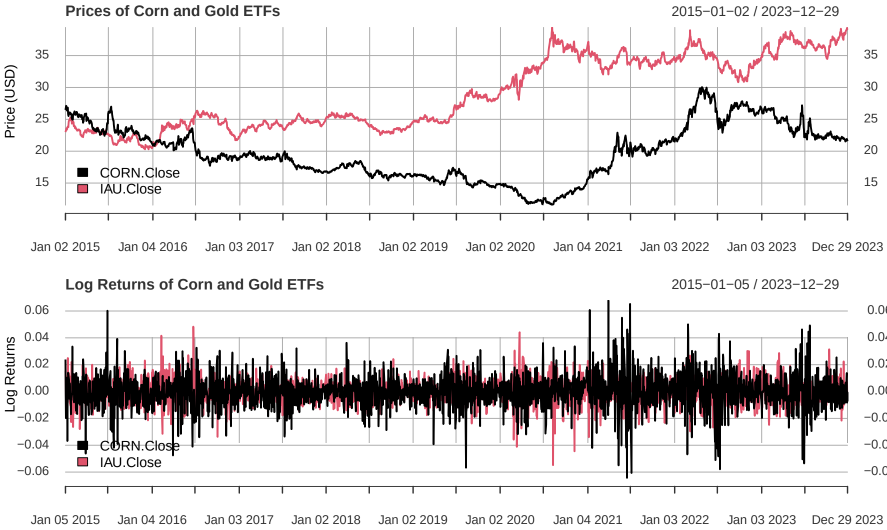


Empirical Density of Standardized Residuals
Median: 0.04 | Mean: −0.0116
normal Density
sstd (0,1) Fitted Density
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−8
−6
−4
−2
0
2
zseries
GARCH model : gjrGARCH



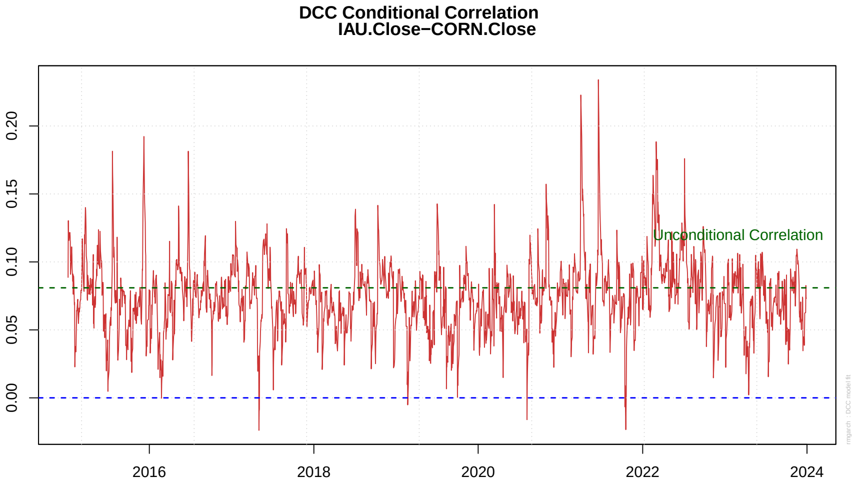
News Impact Curve
σ_t^2
0.000
0.001
0.002
0.003
0.004
−0.10
−0.05
0.00
0.05
0.10
ε_{t-1}
GARCH model : gjrGARCH

DCC-GARCH: Capturing Spillover Effects



```
##                         Estimate  Std. Error  t value  Pr(>|t|)
## [CORN.Close].mu       -1.658e-04   2.374e-04  -0.6986 4.848e-01
## [CORN.Close].omega     3.824e-06   8.632e-06   0.4430 6.578e-01
## [CORN.Close].alpha1    7.607e-02   2.170e-02   3.5048 4.569e-04
## [CORN.Close].beta1     9.006e-01   2.528e-02  35.6182 0.000e+00
## [IAU.Close].mu         9.754e-05   1.756e-04   0.5554 5.786e-01
## [IAU.Close].omega      1.112e-06   4.834e-06   0.2301 8.180e-01
## [IAU.Close].alpha1     4.144e-02   5.294e-02   0.7828 4.337e-01
## [IAU.Close].beta1      9.444e-01   6.145e-02  15.3682 0.000e+00
## [Joint]dcca1           1.529e-02   1.349e-02   1.1331 2.572e-01
## [Joint]dccb1           8.252e-01   1.229e-01   6.7117 1.923e-11
```

Spillover Effects (Corn to Gold)



```
##                        Estimate  Std. Error  t value  Pr(>|t|)
## [IAU.Close].mu         9.754e-05   1.756e-04   0.5554 5.786e-01
## [IAU.Close].omega      1.112e-06   4.834e-06   0.2301 8.180e-01
## [IAU.Close].alpha1     4.144e-02   5.294e-02   0.7828 4.337e-01
## [IAU.Close].beta1      9.444e-01   6.145e-02  15.3684 0.000e+00
## [CORN.Close].mu       -1.658e-04   2.374e-04  -0.6985 4.848e-01
## [CORN.Close].omega     3.824e-06   8.632e-06   0.4430 6.578e-01
## [CORN.Close].alpha1    7.607e-02   2.170e-02   3.5048 4.570e-04
## [CORN.Close].beta1     9.006e-01   2.528e-02  35.6212 0.000e+00
## [Joint]dcca1           1.529e-02   1.349e-02   1.1335 2.570e-01
## [Joint]dccb1           8.252e-01   1.221e-01   6.7567 1.412e-11
```

Thank you!