Applied Economic Forecasting

8. VAR Models

1. Vector Autoregessive (VAR) Models

Section 1

Vector Autoregessive (VAR) Models

VAR Models

What is it?

- Vector Autoregression (VAR) is a multivariate forecasting algorithm that is used when two or more time series influence each other.
- Each time series is modeled as a function of the past values of itself and past lags of the other variables.

How do VARs differ from other Autoregressive models?

Unlike the AR, ARMA, or ARIMA models, these models allow for bi-directional relationships.

- The predictors (**X**s) influence Y and are possibly affected by Y as well.

Introduction

- Use two random variables, $(y_{1t},\ y_{2t})'$ as an example.
- Then, a bivariate model of order 1 takes the following form:

$$\begin{cases} Y_{1t} = a_{10} - b_{12}Y_{2t} + a_{11}Y_{1,t-1} + a_{12}Y_{2,t-1} + \varepsilon_{1t} \\ Y_{2t} = a_{20} - b_{21}Y_{1t} + a_{21}Y_{1,t-1} + a_{22}Y_{2,t-1} + \varepsilon_{2t} \end{cases}$$

1. b_{12} and b_{21} represent ***contemporaneous/simultaneous/instantaneous effects of:***
   1. a unit change of Y_{2t} on Y_{1t}
   2. and a unit change of Y_{1t} on Y_{2t} , respectively.
2. a_{12} and a_{21} represent ***feedback effects of:***
   1. a unit change of $Y_{2,t-1}$ on Y_{1t}
   2. and a unit change of $Y_{1,t-1}$ on Y_{2t} , respectively.

VAR in matrix form

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{1}$$

- This can be rewritten as

$$\boldsymbol{A} Y_t = \boldsymbol{A_0} + \boldsymbol{A_1} Y_{t-1} + \boldsymbol{\varepsilon}_t \tag{2}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}, \quad Y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}, \quad Y_{t-1} = \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix}, \quad \boldsymbol{A_0} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix},$$
$$\boldsymbol{A_1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

Introduction

Note

The matrix form VAR (Eqn (1)) and the rewritten form (Eqn (2)) are referred to as a **Structural Vector Autoregressive model (SVAR)** if at least one of the off-diagonal elements of $\boldsymbol{A}$ is nonzero and $\boldsymbol{\varepsilon}_t$ has a diagonal covariance matrix, Σ_{ε}.

VAR Models

- Transform the SVAR to its reduced VAR form (RVAR) by premultiplying $\boldsymbol{A}^{-1}$,

$$Y_t = \boldsymbol{B_0} + \boldsymbol{B_1} Y_{t-1} + \boldsymbol{\upsilon}_t \tag{3}$$

where

$$\begin{aligned} \boldsymbol{B_0} &= \boldsymbol{A}^{-1}\boldsymbol{A_0} \\ \boldsymbol{B_1} &= \boldsymbol{A}^{-1}\boldsymbol{A_1} \\ \boldsymbol{\upsilon}_t &= \boldsymbol{A}^{-1}\boldsymbol{\varepsilon}_t \end{aligned}$$

Note

Y_t written this way is referred to VAR(1) where $\boldsymbol{\upsilon}_t$ is a White noise process.

- Suppose we have an initial value Y_0, then we can apply backward iteration such that

$$
\begin{aligned}
Y_t &= \boldsymbol{B_0} + \boldsymbol{B_1} Y_{t-1} + \boldsymbol{v}_t \\
&= \boldsymbol{B_0} + \boldsymbol{B_1}(\boldsymbol{B_0} + \boldsymbol{B_1} Y_{t-2} + \boldsymbol{v}_{t-1}) + \boldsymbol{v}_t \\
&= \boldsymbol{B_0} + \boldsymbol{B_0}\boldsymbol{B_1} + \boldsymbol{B_1^2} Y_{t-2} + \boldsymbol{B_1}\boldsymbol{v}_{t-1} + \boldsymbol{v}_t \\
&\vdots \\
&= \left(I + \boldsymbol{B_1} + \ldots + \boldsymbol{B_1}^{t-1}\right)\boldsymbol{B_0} + \boldsymbol{B_1^t} Y_0 + \sum_{i=0}^{t-1} \boldsymbol{B_1^i} \boldsymbol{v}_{t-i}
\end{aligned}
$$

VAR Stability and Stationarity

Side Note

- If $\{\lambda_1, \ldots, \lambda_k\}$ are the eigenvalues of $\boldsymbol{B_1}$, then $\{\lambda_1^t, \ldots, \lambda_k^t\}$ are the eigenvalues of $\boldsymbol{B_1^t}$.
- Also, if all eigenvalues of a matrix are 0, then the matrix must be $\emptyset$.
- So, if $\boldsymbol{B_1^t} \to 0$ as $t \to \infty$, then $\{\lambda_1^t, \ldots, \lambda_k^t\} \to 0$ as $t \to \infty$
- This implies that the absolute value of all eigenvalues of $\boldsymbol{B_1}$ must be less than 1.

VAR Stability and Stationarity

Side Note

The eigenvalues of $\boldsymbol{B_1}$ are solutions of the determinant equation

$$|\lambda I - \boldsymbol{B_1}| = 0$$

which can be written as

$$|I - \boldsymbol{B_1}\frac{1}{\lambda}| = 0$$

$$|I - \boldsymbol{B_1}z| = 0$$

where $z = \frac{1}{\lambda}$ and should be greater than 1 in absolute value.

VAR Stability and Stationarity

So, as $t \to \infty$, if

$$det(I - \boldsymbol{B_1} z) = 0, \text{ for } |z| > 1,$$

then

$$\begin{aligned} \left(I + \boldsymbol{B_1} + \ldots + \boldsymbol{B_1}^{t-1}\right)_{|t\to\infty} &\to (I - \boldsymbol{B_1})^{-1} \\ \boldsymbol{B_1^t}_{|t\to\infty} &\to 0 \\ &\sum_{i=0}^{t-1} \boldsymbol{B_1^i} \boldsymbol{v}_{t-i} \text{ exists} \end{aligned}$$

VAR Stability and Stationarity

- So,

$$\mathbf{E}(Y_t) = (I - \boldsymbol{B_1})^{-1}\boldsymbol{B_0}$$

and the autocovariance

$$\begin{aligned}
\Gamma_Y(h) &= \mathrm{Cov}(Y_t, Y_{t-h}) = \mathbf{E}(Y_t - \mu)(Y_{t-h} - \mu)' \\
&= \lim_{n\to\infty} \sum_{i=0}^{n} \sum_{j=0}^{n} \boldsymbol{B}^i \mathbf{E}(\boldsymbol{\upsilon}_{t-i}\boldsymbol{\upsilon}_{t-h-j})\boldsymbol{B}^{j'} \\
&= \lim_{n\to\infty} \sum_{j=0}^{n} \boldsymbol{B}^{h+j}\Sigma_\upsilon \boldsymbol{B}^j \\
&= \sum_{i=0}^{\infty} \boldsymbol{B}^{h+j}\Sigma_\upsilon \boldsymbol{B}^j
\end{aligned}$$

because $\mathbf{E}(\boldsymbol{\upsilon}_t\boldsymbol{\upsilon}_s') = 0$ for $s \neq t$, $\mathbf{E}(\boldsymbol{\upsilon}_t\boldsymbol{\upsilon}_t') = \Sigma_\upsilon$ for all t, and $\boldsymbol{B}^\infty \to 0$.

VAR Stability and Stationarity

A VAR(1) process is stable if all eigenvalues of $\boldsymbol{B_1}$ have modulus less than 1, that is

$$det(I - \boldsymbol{B_1} z) = 0, \text{ for } |z| > 1$$

where $z = \frac{1}{\lambda}$.

VAR(p) Model

- In general, a VAR(p) model is expressed as

$$Y_t = \boldsymbol{B_0} + \boldsymbol{B_1} Y_{t-1} + \ldots + \boldsymbol{B_p} + \boldsymbol{\upsilon}_t$$

where

$$Y_t = \begin{pmatrix} Y_{1t} \\ \vdots \\ Y_{Nt} \end{pmatrix}$$

and $\boldsymbol{\upsilon}_t$ is $N-$dimensional white noise or innovation process, ie,

$$\mathbf{E}(\boldsymbol{\upsilon}_t) = 0$$
$$\mathbf{E}(\boldsymbol{\upsilon}_t \boldsymbol{\upsilon}_t^{'}) = \Sigma_\upsilon$$
$$\mathbf{E}(\boldsymbol{\upsilon}_t \boldsymbol{\upsilon}_{t-s}^{'}) = 0, \text{ for } t \neq s$$

Intuition

Recall that a typical AR model is a linear combination of it's own lags. To that end, we use past values of the variable to predict current and future values.

A standard AR(p) model would look like:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t$$

where ϕ_0 is the intercept and $\phi_j \, ; \; j \in \{1, p\}$ are the lagged coefficients of Y.

Intuition

In the case of the VAR model, we have several Y variable (for example the GDP of US and China) and each has its own equation. For simplicity, let us assume that we have a VAR(1) model and two GDP series (Y_1 and Y_2).

$$Y_{1,t} = \gamma_1 + \phi_{11} Y_{1,t-1} + \phi_{12} Y_{2,t-1} + \varepsilon_{1,t} \tag{4}$$
$$Y_{2,t} = \gamma_2 + \phi_{21} Y_{1,t-1} + \phi_{22,} Y_{2,t-1} + \varepsilon_{1,t} \tag{5}$$

Each variable is a linear function of the lag 1 values for all variables in the set.

If we had three variables we could easily add an additional equation and also an extra regressor to each.

$$Y_{1,t} = \gamma_1 + \phi_{11} Y_{1,t-1} + \phi_{12} Y_{2,t-1} + \phi_{13} Y_{3,t-1} + \varepsilon_{1,t} \tag{6}$$
$$Y_{2,t} = \gamma_2 + \phi_{21} Y_{1,t-1} + \phi_{22,} Y_{2,t-1} + \phi_{23} Y_{3,t-1} + \varepsilon_{1,t} \tag{7}$$
$$Y_{3,t} = \gamma_2 + \phi_{21} Y_{1,t-1} + \phi_{22,} Y_{2,t-1} + \phi_{33} Y_{3,t-1} + \varepsilon_{1,t} \tag{8}$$

VAR Estimation Steps

1. Visualize the Data.
2. Check for stationarity and make the time series stationary.
3. Select the order (p) of the VAR Model
4. Estimate the implied model
5. Check for Serial Correlation in the Residuals
6. Forecast VAR

Visualizing the Data: `uschange` data (Revisted)

```r
autoplot(uschange, facets = TRUE, colour = TRUE) +
  theme(legend.position = "none")
```

`uschange` data (Revisted)

- It is logical that the relationship between these variables is not contemporaneous as we assumed earlier in the semester.
- Instead, last quarter's personal disposable income could affect this quarter's consumption and so forth.
- We can therefore use a VAR to determine the relationship between these series.

2. Check for Stationarity

Visually:

- ACF

Formally:

- ur.df
- ur.kpss

Using the ADF Test:

	Conclusion
Consumption	Reject Null. The Series is stationary at $\alpha = 5\%$.
Income	Reject Null. The Series is stationary at $\alpha = 5\%$.
Production	Reject Null. The Series is stationary at $\alpha = 5\%$.
Savings	Reject Null. The Series is stationary at $\alpha = 5\%$.
Unemployment	Reject Null. The Series is stationary at $\alpha = 5\%$.

2. Check for Stationarity

- If the series are stationary, we forecast them by fitting a VAR to the data directly (known as a "VAR in levels").
- If the series are non-stationary, we take differences of the data in order to make them stationary, then fit a VAR model (known as a "VAR in differences").
- In both cases, the models are estimated equation by equation using the principle of least squares.
  - For each equation, the parameters are estimated by minimising the sum of squared $\varepsilon_{i,t}$ values.

3. Select the order (p) of the VAR Model

We will require the `vars` package for the analysis that follows.

The optimal lag length will be selected by iteratively fitting increasing orders of VAR model and pick the order that gives a model with least AIC or BIC value.

Caution!!!

Care should be taken when using the AIC as it tends to choose large numbers of lags.

```
VARselect(uschange[,1:2], lag.max = 8, type ="const")
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      5      1      1      5
##
## $criteria
##                  1          2          3          4          5          6
## AIC(n) -1.3787011 -1.374328 -1.4064675 -1.4171747 -1.4305682 -1.4033960
## HQ(n)  -1.3353784 -1.302123 -1.3053813 -1.2872066 -1.2717184 -1.2156644
```

3. Select the order (p) of the VAR Model

```r
VARselect(uschange[,1:2], lag.max = 8, type ="const")$selection
```

```
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      5      1      1      5
```

The AIC criteria selects a model with 5 lags and the BIC chooses a more parsimonious model, with 1 lag.

4. Estimate the implied model

We have two competing models. Let us start by estimating the more parsimonious model, VAR(1) as implied by the BIC criteria.

```
fit <-  VAR(uschange[,1:2], p = 1, type = "const")
fit
```

```
##
## VAR Estimation Results:
## =======================
##
## Estimated coefficients for equation Consumption:
## ============================================
## Call:
## Consumption = Consumption.l1 + Income.l1 + const
##
## Consumption.l1      Income.l1          const
##     0.29645664     0.09434292     0.45810920
##
```

5. Check for Serial Correlation in the Residuals

Portmanteau Test

```
serial.test(fit, lags.pt = 10, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fit
## Chi-squared = 49.102, df = 36, p-value = 0.07144
```

The Null of *No serial correlation* can be rejected at the 10% level of significance so we will need to proceed to higher order lags until we no longer have this issue.

5. Check for Serial Correlation in the Residuals

```
fit2 <-  VAR(uschange[,1:2], p = 2, type = "const")
serial.test(fit2, lags.pt = 10, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fit2
## Chi-squared = 47.741, df = 32, p-value = 0.03633
```

```
fit3 <-  VAR(uschange[,1:2], p = 3, type = "const")
serial.test(fit3, lags.pt = 10, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fit3
## Chi-squared = 33.617, df = 28, p-value = 0.2138
```

The VAR(3) model passes the test for serial correlation. We will therefore use this to generate forecasts.

Creating the Lagged Variables and appending to a new dataframe:

```
l.uschange <- data.frame(uschange[,1:2])

# Creating lags 1 - 3 for Consumption

l.uschange$Consumption_1 <- c(rep(NA,1),uschange[1:(nrow(uschange)-1),1])
l.uschange$Consumption_2 <- c(rep(NA,2),uschange[1:(nrow(uschange)-2),1])
l.uschange$Consumption_3 <- c(rep(NA,3),uschange[1:(nrow(uschange)-3),1])

# Creating lags 1 - 3 for Income

l.uschange$Income_1 <- c(rep(NA,1),uschange[1:(nrow(uschange)-1),2])
l.uschange$Income_2 <- c(rep(NA,2),uschange[1:(nrow(uschange)-2),2])
l.uschange$Income_3 <- c(rep(NA,3),uschange[1:(nrow(uschange)-3),2])
```

Estimation by Least Squares vs VAR

Pulling the VAR Coefficients (Method I):

```
fit3$varresult$Consumption$coefficients
```

```
## Consumption.l1      Income.l1 Consumption.l2      Income.l2 Consumption.
##     0.19100120     0.07836635     0.15953548    -0.02706495     0.226455
##      Income.l3          const
##    -0.01453688     0.29081124
```

```
fit3$varresult$Income$coefficients
```

```
## Consumption.l1      Income.l1 Consumption.l2      Income.l2 Consumption.
##     0.45349152    -0.27302538     0.02166532    -0.09004735     0.353766
##      Income.l3          const
##    -0.05375916     0.38749574
```

Pulling the VAR Coefficients (Method II):

```
Bcoef(fit3)
```

```
##             Consumption.l1   Income.l1 Consumption.l2   Income.l2
## Consumption      0.1910012  0.07836635     0.15953548 -0.02706495
## Income           0.4534915 -0.27302538     0.02166532 -0.09004735
##             Consumption.l3   Income.l3      const
```

Estimation by Least Squares vs VAR

OLS:

```
lm(Consumption ~ Consumption_1 + Income_1 +
     Consumption_2 + Income_2 +
     Consumption_3 + Income_3, data = l.uschange) %>% coef()
```

```
##   (Intercept) Consumption_1      Income_1 Consumption_2      Income_2
##    0.29081124    0.19100120    0.07836635    0.15953548   -0.02706495
## Consumption_3      Income_3
##    0.22645563   -0.01453688
```

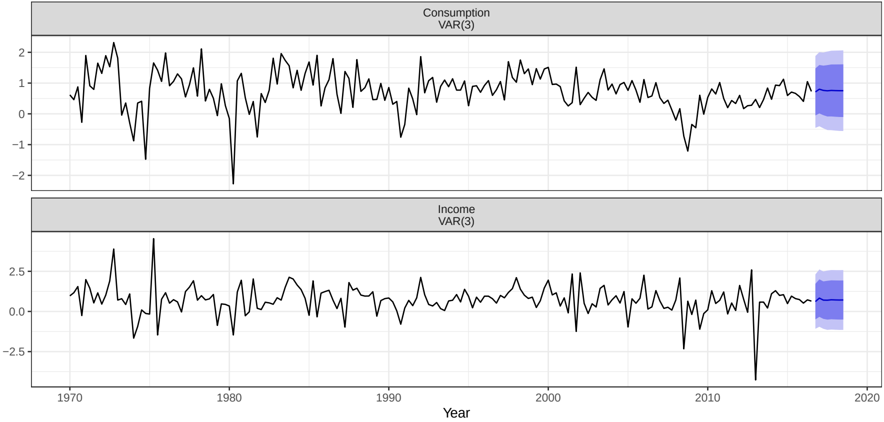
```
lm(Income ~ Consumption_1 + Income_1 +
     Consumption_2 + Income_2 +
     Consumption_3 + Income_3, data = l.uschange) %>% coef()
```

```
##   (Intercept) Consumption_1      Income_1 Consumption_2      Income_2
##    0.38749574    0.45349152   -0.27302538    0.02166532   -0.09004735
## Consumption_3      Income_3
##    0.35376691   -0.05375916
```

6. Forecast VAR

$$\hat{y}_{1,T+1|T} = \hat{\gamma}_1 + \hat{\phi}_{11} y_{1,T} + \hat{\phi}_{12} y_{2,T}$$
$$\hat{y}_{2,T+1|T} = \hat{\gamma}_2 + \hat{\phi}_{21} y_{1,T} + \hat{\phi}_{22} y_{2,T}.$$

```
fit3 %>% forecast(h = 8) %>% autoplot() + xlab("Year")
```



Remarks

A criticism that VARs face is that they are atheoretical; that is, they are not built on some economic theory that imposes a theoretical structure on the equations. Every variable is assumed to influence every other variable in the system, which makes a direct interpretation of the estimated coefficients difficult.

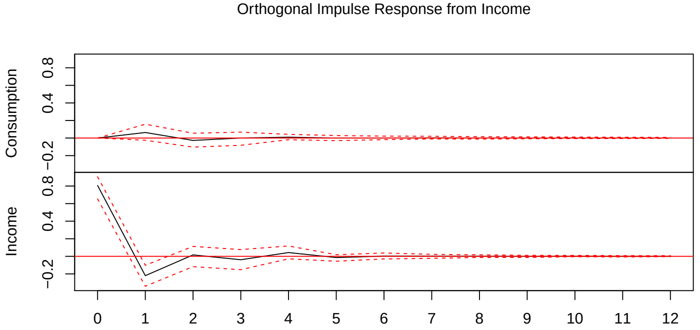
Despite this, VARs are useful in several contexts:

1. forecasting a collection of related variables where no explicit interpretation is required;
2. testing whether one variable is useful in forecasting another (the basis of Granger causality tests);
3. impulse response analysis, where the response of one variable to a sudden but temporary change in another variable is analysed;
4. forecast error variance decomposition, where the proportion of the forecast variance of each variable is attributed to the effects of the other variables.

Impulse Response Functions (IRF)

The `irf` function returns the dynamic response, or the impulse response function (IRF), to a one-standard-deviation shock to each variable in a VAR(p) model.

```
irf(fit3, impulse = "Income", n.ahead = 12) %>% plot()
```



Impulse Response Analysis

Consider a VAR(1) model $Y_t = \mathbf{B}_1 Y_{t-1} + \boldsymbol{v}_t$ for investment, income, and consumption

$$\begin{pmatrix} \text{Investment}_t \\ \text{Income}_t \\ \text{Consumption}_t \end{pmatrix} = \mathbf{c} + \begin{pmatrix} 0.5 & 0 & 0 \\ 0.1 & 0.1 & 0.3 \\ 0 & 0.2 & 0.3 \end{pmatrix} \begin{pmatrix} \text{Investment}_{t-1} \\ \text{Income}_{t-1} \\ \text{Consumption}_{t-1} \end{pmatrix} + \boldsymbol{v}_t$$

Assume $Y_0 = \boldsymbol{v}_0$ and $v_t = 0$ for $t > 0$

Impulse Response Analysis

We can trace a unit shock in investment in $t = 0$ in the system,

$$Y_0 = \begin{pmatrix} \text{Investment}_{t=0} \\ \text{Income}_{t=0} \\ \text{Consumption}_{t=0} \end{pmatrix} = \begin{pmatrix} \varepsilon_{10} \\ \varepsilon_{20} \\ \varepsilon_{30} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$Y_{1|0} = \begin{pmatrix} \text{Investment}_{t=1} \\ \text{Income}_{t=1} \\ \text{Consumption}_{t=1} \end{pmatrix} = \boldsymbol{B}_1 Y_0 = \begin{pmatrix} 0.5 \\ 0.1 \\ 0 \end{pmatrix}$$

$$Y_{2|0} = \begin{pmatrix} \text{Investment}_{t=2} \\ \text{Income}_{t=2} \\ \text{Consumption}_{t=2} \end{pmatrix} = \boldsymbol{B}_1 Y_{1|0} = \boldsymbol{B}_1^2 Y_0 = \begin{pmatrix} 0.25 \\ 0.06 \\ 0.02 \end{pmatrix}$$

$$Y_{3|0} = \begin{pmatrix} \text{Investment}_{t=3} \\ \text{Income}_{t=3} \\ \text{Consumption}_{t=3} \end{pmatrix} = \boldsymbol{B}_1 Y_{2|0} = \boldsymbol{B}_1^3 Y_0 = \begin{pmatrix} 0.125 \\ 0.037 \\ 0.018 \end{pmatrix}$$

Impulse Response Analysis

- More general, for a VAR(p) model in a compact form

$$\begin{aligned}\boldsymbol{Y}_t &= \boldsymbol{B}\boldsymbol{Y}_{t-1} + \boldsymbol{\Upsilon}_t \\ &= \sum_{i=0}^{t-1} \boldsymbol{B}^i \boldsymbol{\Upsilon}_{t-i} + \boldsymbol{B}^t Y_0\end{aligned}$$

- Assume $Y_0 = \boldsymbol{v}_0$ and $v_t = 0$ for $t > 0$
- If there is a shock to Y_0 at $t = 0$, $Y_0 = (\boldsymbol{v}_k, 0, \ldots, 0)'$

Impulse Response Analysis

- Then the shock is transmitted by $\boldsymbol{B}^t$,

$$Y_t = \sum_{i=0}^{t-1} J\boldsymbol{B}^i J' \boldsymbol{v}_{t-i} + J\boldsymbol{B}^t J' Y_0$$

where $J = (I_N, : 0, \ldots, 0)$

- Let $\boldsymbol{\Phi}_i = J\boldsymbol{B}^i J'$
- So, $\phi_{jk,t}$ is the response of the j^{th} variable to an impulse of the k^{th} variable in Y_t in the j^{th} row and the k^{th} column of

$$\boldsymbol{\Phi}_t = J\boldsymbol{B}^t J'$$

$$\boldsymbol{\Phi}_t = J\boldsymbol{B}^t J' = \begin{array}{r} \\ IRF_{inv} \\ IRF_{inc} \\ IRF_{cons} \end{array} \begin{array}{c} \begin{array}{ccc} \varepsilon_{inv} & \varepsilon_{inc} & \varepsilon_{cons} \end{array} \\ \begin{pmatrix} 0.001 & 0.002 & 0.003 \\ 0.004 & 0.005 & 0.006 \\ 0.007 & 0.008 & 0.009 \end{pmatrix} \end{array}$$

Forecast Error Variance Decomposition (FEVD)

The forecast error variance decomposition (FEVD) provides information about the relative importance of each innovation in affecting all variables in the system at each forecast horizon.

```
knitr::kable(fevd(fit3)$Consumption, digits = 2,
             align = "c" , caption = "Contribution of Consumption")
```

Table 2: Contribution of Consumption

Consumption	Income
1.00	0.00
0.99	0.01
0.99	0.01
0.99	0.01
0.99	0.01
0.99	0.01
0.99	0.01
0.99	0.01
0.99	0.01
0.99	0.01

Forecast Error Variance Decomposition (FEVD)

```r
knitr::kable(fevd(fit3)$Income, digits = 2,
             align = "c" , caption = "Contribution of Income")
```

Table 3: Contribution of Income

Consumption	Income
0.13	0.87
0.16	0.84
0.16	0.84
0.21	0.79
0.21	0.79
0.22	0.78
0.22	0.78
0.22	0.78
0.22	0.78
0.22	0.78