UNIT-IV

RESONANCE

All derivations go through
Remember all formulae

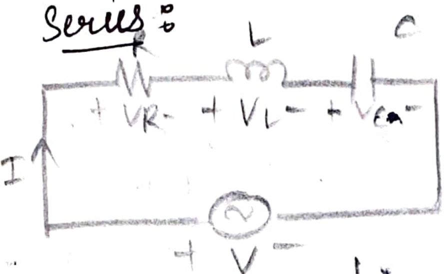
- Series
- Parallel circuits
- Concept of bandwidth and Q factor.
- Steady state response of a network to non-sinusoidal periodic inputs.

↳ similar model

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$\omega L - \frac{1}{\omega C} \to 0$

Series :



at ω_0

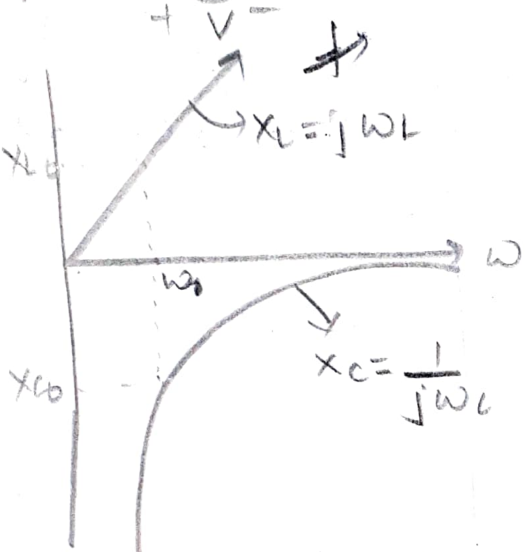
$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

$Z = R$ (Min)

$I_0 = \frac{V}{R}$ (Max)

$P.F = 1$



$|Z|_{at\ \omega=0} = |R + 0 - j\infty| = \infty \quad I = 0$

$|Z|_{at\ \omega=\infty} = |R + j\infty - j0| = \infty \quad I = 0$

$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$

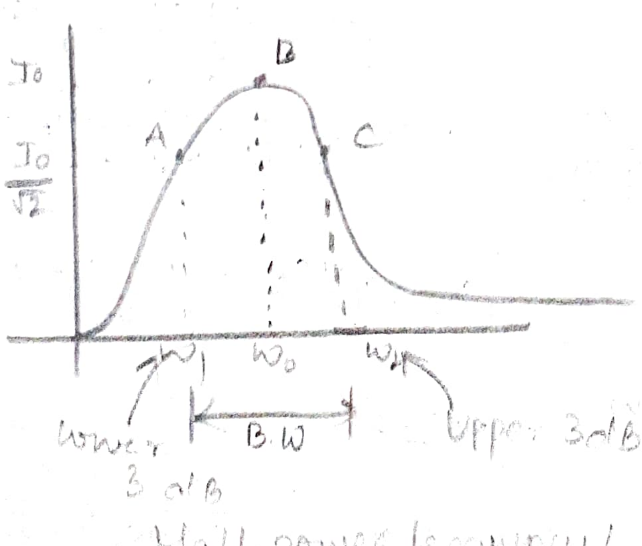
$$\boxed{\omega L - \frac{1}{\omega C} = \pm R}$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 - \omega_1 = \frac{R}{L} = B.W.$$



$$1\,dB = 10\log_{10}\frac{P_o}{P_i}$$

$$= 20\log_{10}\frac{V_o}{V_i}$$

$$= 20\log_{10}\frac{I_o}{I_i}$$

$= 0\,dB$ if $P_o = P_i$

$= -3\,dB$ if $P_o = \frac{P_i}{2}$ (OR) $V_o = \frac{V_i}{\sqrt{2}}$ $I_o = \frac{I_i}{\sqrt{2}}$

At point ~~A~~ or C:

$\omega_1 =$ lower 3dB freq.

$\omega_2 =$ upper 3dB freq

ω_1 & $\omega_2 \rightarrow$ half power frequencies

$$I = \frac{I_o}{\sqrt{2}} = \frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\Rightarrow R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

Nett reactance at half power frequencies is $\pm R$

$$\boxed{\omega L - \frac{1}{\omega C} = \pm R}$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 - \omega_1 = \boxed{\frac{R}{L} = B.W}.$$

$$Q_0 = \frac{\omega_0}{\frac{R}{L}} = \frac{\omega_0}{B.W} = Q_0$$

$$Q_0 = \frac{\omega_0 L}{R} \times \frac{I_0}{I_0} = \frac{V_{L_0}}{V}$$

$\therefore$ It works like Voltage Amplifier.

$$\omega_1 L - \frac{1}{\omega_1 C} + \omega_2 L - \frac{1}{\omega_2 C} = 0.$$

$$(\omega_1 + \omega_2) L = \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)$$

$$LC = \frac{1}{(\omega_1 + \omega_2)} \cdot \frac{\omega_1 + \omega_2}{\omega_1 \times \omega_2}$$

$$LC = \frac{1}{\omega_1 \omega_2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_1 \omega_2}$$

$$\boxed{\omega_0 = \sqrt{\omega_1 \omega_2}}$$

$$-\omega_1 L + \frac{1}{\omega_1 C} \quad \omega_2 L - \frac{1}{\omega_2 C} = 2R$$

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R$$

$$L(\omega_2 - \omega_1) + \frac{(\omega_2 - \omega_1)}{C\,\omega_1 \omega_2} = 2R$$

$$(\omega_2 - \omega_1)\left(L + \frac{1}{C\,\omega_1 \omega_2}\right) = 2R$$

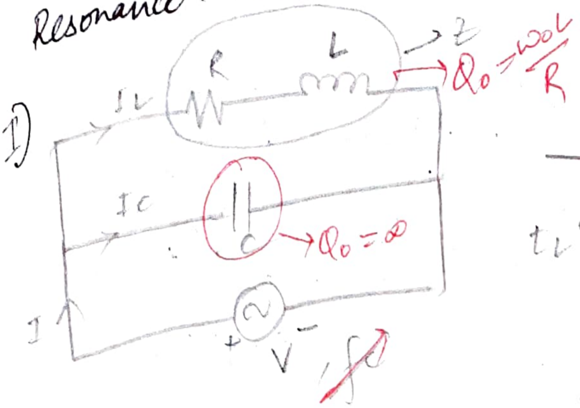
$$(\omega_2 - \omega_1) = \frac{2R}{L + \frac{1}{C\,\omega_1 \omega_2}} = \frac{2R}{L + \frac{1}{C} \times L} = \frac{2R}{2L}$$

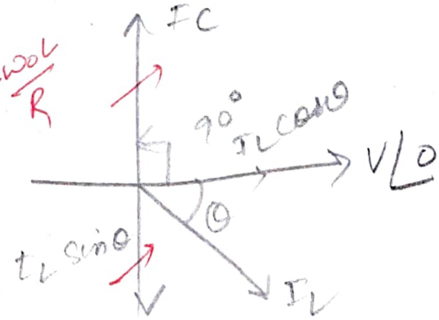
$$\boxed{\omega_2 - \omega_1 = \frac{R}{L}}$$

23/10/2019

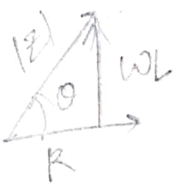
Resonance : Parallel.

1)





$Z = R + j\omega L$



$\cancel{\theta} = \tan^{-1} \frac{\omega L}{R}$

$\cos\theta = \frac{R}{|Z|}$

$\sin\theta = \frac{\omega L}{|Z|}$

at $\omega_0 \Rightarrow |I_C| = |I_L \sin\theta|$

$\frac{V}{X_C} = \frac{V}{|Z|} \frac{X_L}{|Z|}$

$|Z|^2 = X_L X_C$

★ if $R=0$

Dynamic Resistance $= \infty$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$I_0 = 0$

$|R + j\omega_0 L|^2 = \omega L \times \frac{1}{\omega C} = \frac{L}{C}$

$\therefore \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$I_0 = I_L \cos\theta = \frac{V}{|Z|} \cdot \frac{R}{|Z|} = \frac{VR}{|Z|^2}$

$I_0 = \frac{VR}{L/C} = \frac{V}{(L/CR)} \rightarrow$ Dynamic resistance (Ω) ★ (Max).

★ Min

★ $\boxed{P.F. = 1}$

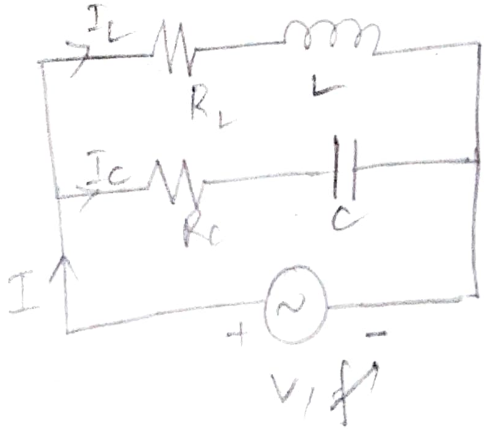
(OR)

$Y = Y_1 + Y_2 = \frac{1}{R + j\omega L} + j\omega C$

$= (\quad) + j(\quad) \rightarrow 0$ at ω_0

$$\boxed{\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

①



$$Y = Y_1 + Y_2$$

$$= \frac{1}{R_L + j\omega L} + \frac{1}{R_C + \frac{j}{\omega C}}$$

$$= (\ \ ) + j(\ \ )$$ at ω_0

$$\omega_0 = \sqrt{\frac{1}{LC} \cdot \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}}$$

(i) If $R_L = R_C = 0$

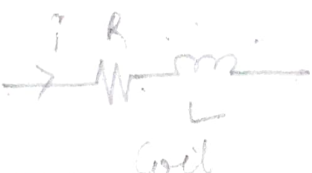
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

(ii) If $R_L^2 = R_C^2 = \frac{L}{C}$

The ckt is under resonance for entire frequency range

Series Resonance

$$Q = \text{Quality Factor} = \frac{2\pi \cdot E.S.\ max}{E.D. (OR)\ P.D/f}$$ $\frac{\text{Energy stored max}}{\text{Energy Dissipated}}$



$$= \frac{2\pi \frac{1}{2} L I_{max}^2}{(I_{rms}^2 R / f)} = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

$$\boxed{Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}} > 1$$

(worst) 0 to ∞ (Best)

$$Q_0 = \frac{\omega_0}{R/L} = \frac{\omega_0}{B.W} = Q_0.$$

$$Q_0 = \frac{\omega_0 L}{R} \times \frac{I_0}{I_0} = \frac{V_{L0}}{V}$$

Series Resonance 24/10/2019

$$\boxed{Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{Q_0}{BW} = \frac{V_{L0}}{V} = \frac{V_{C0}}{V} = \frac{\omega_0}{BW} = \frac{1}{R}\sqrt{\frac{L}{C}} > 1}$$

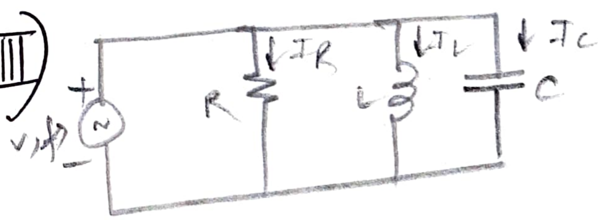
$$\frac{I}{I_0} = \frac{\dfrac{V}{\sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}}}{\dfrac{V}{R}} = \frac{R}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}} = \frac{1}{\sqrt{1+\left(\frac{\omega_0 L}{R}\right)^2\left[\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right]^2}}$$

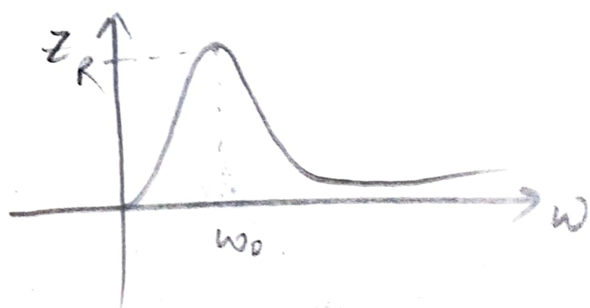
Let $\frac{I_0}{I} = N$

$$\frac{1}{N} = \frac{1}{\sqrt{1+Q_0^2\left[\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right]^2}}$$

$$\boxed{\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) = \pm\frac{\sqrt{N^2-1}}{Q_0}}$$

III)





$$Y = Y_1 + Y_2 + Y_3$$
$$= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$
$$= \frac{1}{R} + j\left[\omega C - \frac{1}{\omega L}\right]$$

Net susceptance at ω_0 0.

If $\omega_0 C - \frac{1}{\omega_0 L} = 0$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}.$$

units = rad/sec = Indep. of R.
same as series resonance.

$$Y = \frac{1}{R}$$

$Z = R$ (Maximum)

$I = \frac{V}{Z} = \frac{V}{R}$ (Min) Power Factor = 1.

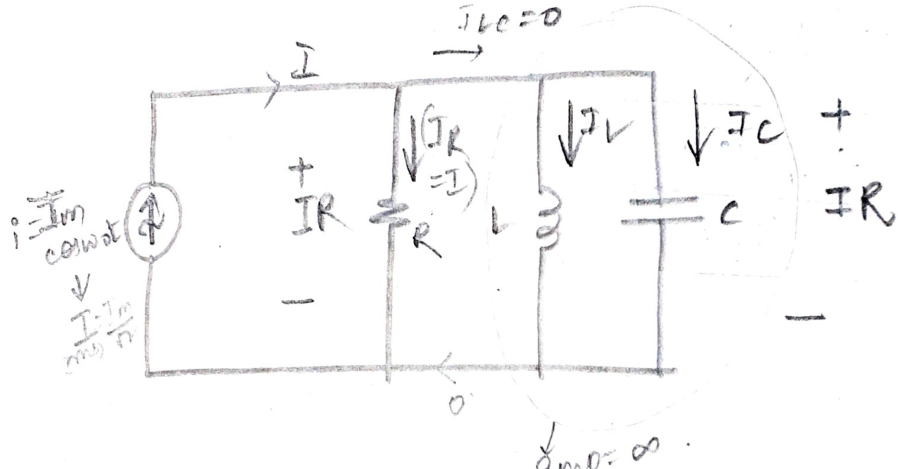
$$Q_T = 2\pi \frac{[E.S.L + E.S.C]_{max}}{E_D \text{ (or) } (PP/f)}$$

(Derivation)

$$Q_0 = \frac{R}{\omega_0 L} = \omega_0 CR \, (>1)$$

$$Q_0 = \frac{\omega_0}{\frac{1}{RC}} = \frac{\omega_0}{B.W}$$

$$\boxed{B.W = \frac{1}{RC}}$$ rad/sec Independent of 'L'



$$|I_{C,0}| = \frac{IR}{1/\omega_0 C} = \omega_0 CRI$$

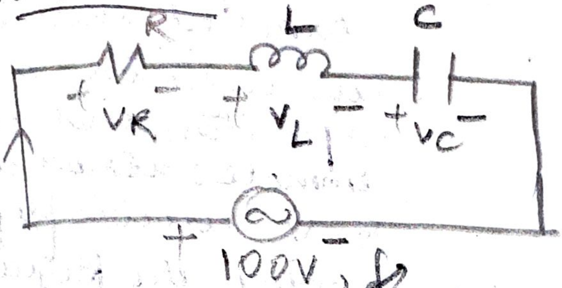
$$\left|\frac{I_{C,0}}{I}\right| = \omega_0 CR = Q_0 > 1$$

∴ It works like current Amplifier.

Summary

$$\boxed{Q_0 = \frac{R}{\omega_0 L} = \omega_0 CR = R\sqrt{\frac{C}{L}} = \left|\frac{I_{C0}}{I}\right| = \left|\frac{I_{L0}}{I}\right| = \frac{\omega_0}{B.W}} > 1$$

Problems



$R = 10\Omega$

$L = 0.2H$

$C = 40\mu F$

Find

a) $f_0 = 56.3Hz$

b) $I_0 = 10A$

c) $P.F = 1$

d) P at $f_0 = 100W$

e) $|V_{R0}| = 100V$

f) $|V_{L0}| = 707.10\,V$

g) $|V_{C0}| = 707.10\,V$

h) $Q_0 = 7.07$

i) $BW = 50.00Hz$

j) $\omega_1 = 330\,rad$

k) $\omega_2 = 380$

(i) $\omega_0 = \frac{1}{\sqrt{LC}} = 353.55$ $\quad f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times\pi\times\sqrt{0.2\times40\times10^{-6}}} = 56.269Hz$

(ii) $I_0 = \frac{V_0}{R} = \frac{100}{10} = 10$

(iii) Power Factor $= 1$

(iv) P at f_0 $= P = VI = I^2R = 10^2\times10 = 100.$

(v) $|V_{R0}| = I_0R = 10\times10 = 100$

(vi) $|V_{L0}| = I_0\,\omega_0 L = 10\times \omega_0 \times 0.2 = \cancel{2\times56.3} = 707.10$

(vii) $|V_{C0}| = \frac{I_0}{\omega_0 C} = \frac{10}{353.55\times40\times10^{-6}} = 707.11$

(viii) $Q_0 = \frac{\omega_0 L}{R} = \frac{353.55\times0.2}{10} = 7.07$

(ix) $BW = \frac{\omega_0}{Q_0} = \frac{353.55}{7.07} = 50.00$

(x) ~~$\omega_2 - \omega_1 = BW$~~

$\omega_1 L - \frac{1}{\omega_1 C} = -R$

$\omega_1(0.2) - \frac{1}{\omega_1(40\times10^{-6})} = -10$

$\omega_1^2(8\times10^{-6}) - 1 = -400\times10^{-6}\omega$

$\omega_1^2(8\times10^{-6}) + 400\times10^{-6}\omega - 1 = 0.$

$\omega_1^2(8\times10^{-6}) + 4\times10^{-4}\omega - 1 = 0.$

$\omega_1 = 330.$

$\omega_2(0.2) - \frac{1}{\omega_2(40\times10^{-6})} = 10.$

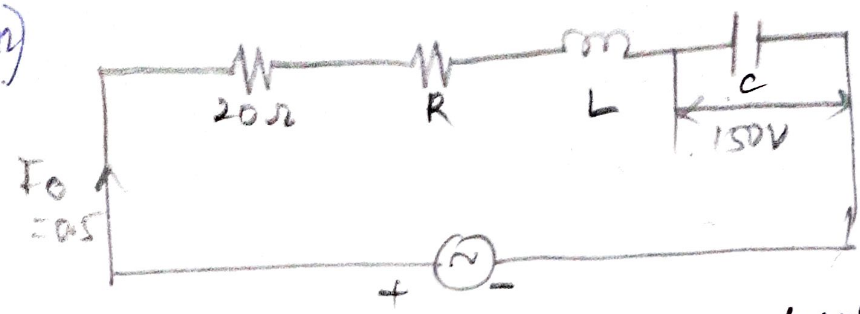
$\omega_2^2(8\times10^{-6}) - 1 = 400\times10^{-6}\omega_2$

$\omega_2^2(8\times10^{-6}) - 400\times10^{-6}\omega_2$

$\omega_2^2(8\times10^{-6}) - 4\times10^{-4}\omega - 1 = 0$

$\omega_2 = 380$

2)



A 20Ω resistor is connected in series an inductor, a capacitor and an ammeter across a 25V variable frequency 25 V, 400 Hz supply when the frequency is 400Hz, the current is at is max value of 0.5 and the potential diff across capacitor is 150 V.

Find

a) R = 30

b) L = 0.119H

c) C = 1.325 μF

$$X_{C0} = \frac{V_{C0}}{I_0} = \frac{150}{0.5} = 300\Omega$$

$$\frac{1}{\omega_0 C} = 300$$

$$C = \frac{1}{300 \times 2\pi \times 400} = 1.325 \times 10^{-6} = 1.325\ \mu F$$

$$X_{L0} = X_{C0} = 300$$

$$\omega_0 L = 300$$

$$L = \frac{300}{2\pi \times 400} = 0.119H$$

$$20 + R = \frac{25}{0.5} = 50$$

$$R = 50 - 20 = 30$$

(3) A series RLC circuit is excited by variable frequency source. The current in the circuit becomes maximum at a frequency of $\frac{600}{2\pi}$ Hz and falls to half the maximum at $\frac{400}{2\pi}$ Hz. If the resistance of the circuit is 3Ω, find Inductance and capacitance.

$I_0 \rightarrow \frac{600}{2\pi}$ Hz.

$I = \frac{I_0}{2} \rightarrow \frac{400}{2\pi}$ Hz

R = 3Ω

L = 10.4 mH

C = 267 μF

$\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = \frac{\pm\sqrt{N^2-1}}{Q_0}$ $\qquad N = \frac{I_0}{I} = 2$

$\frac{400}{600} - \frac{600}{400} = \frac{\pm\sqrt{4-1}}{Q_0}$

$\frac{2}{3} - \frac{3}{2} = \frac{\pm\sqrt{3}}{Q_0}$

$\frac{4-9}{6} = \frac{\pm\sqrt{3}}{Q_0}$

$Q_0 = \frac{6}{-5} \times -\sqrt{3}$

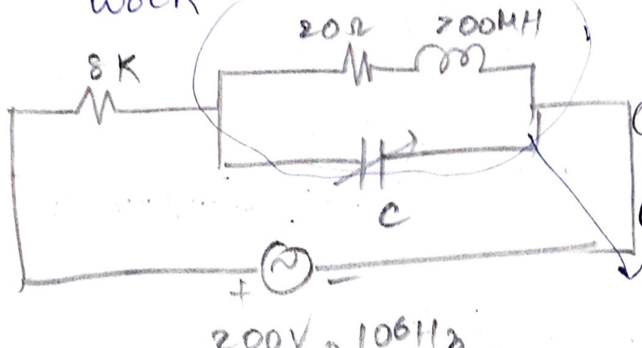
$Q_0 = 2.08$

$Q_0 = \frac{\omega_0 L}{R} \Rightarrow 2.08 = \frac{600 \times L}{3} \Rightarrow L = \frac{2.08}{200} = 10.4\,mH$

$Q_0 = \frac{1}{\omega_0 C R} \Rightarrow 2.08 = \frac{1}{600 \times C \times 3} \Rightarrow C = \frac{1}{2.08 \times 3 \times 600}$

$C = 267\,\mu F$

(04)



Ass-3

(a) For what value of C, circuit is under resonance.

(b) Find the Quality Factor of the coil at resonance

Dynamic resistance $= \frac{L}{CR}$

$= \frac{200\times10^{-6}}{12.5\times10^{-12}\times20}$

$= 80K$

(c) Find branch currents at resonance.

$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$2\pi\times10^6 = \sqrt{\frac{1}{200\times10^{-6}\,C} - \frac{400}{(200\times10^{-6})^2}}$

$2\pi\times10^6 = \sqrt{\frac{1}{2\times10^{-4}C} - \frac{400}{4\times10^{12}\times10^{4}}}$

$2\pi\times10^6 = \sqrt{\frac{1}{2\times10^{-4}C} - \frac{1}{10^{-10}}}$

$4\pi^2\times10^{12} = \frac{1}{2\times10^{-4}C} - \frac{10^{10}}{1} \Rightarrow \frac{1}{2\times10^{-4}C} = 3.95\times10^{13}$

$C = \frac{1}{2\times10^{-4}\times3.95\times10^{13}}$

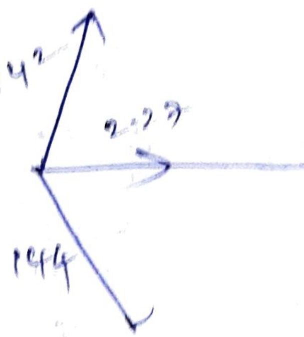
$C = 1.265 \times 10^{10}$

$= 126.5 \times 10^{12}$

$\therefore C = 126 \mu\mu F = 126\, pF$

$$Q_0 = \frac{\omega_0 L}{R}$$

$$= \frac{2 \times \pi \times 10^6 \times 200 \times 10^6}{20}$$



$$\boxed{Q_0 = 62.8}$$

$R_T = 8K + 80K = 88K$

$I_T = \frac{200}{88K} = 2.27\, ma$

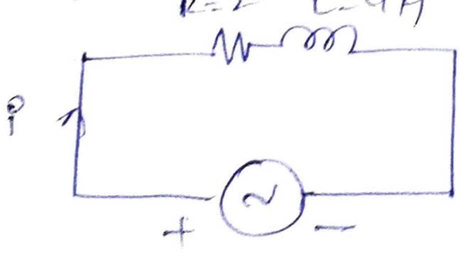
Voltage across Tuning circuit $= 2.27 \times 10^{-3} \times 80K = 181.1\, V$

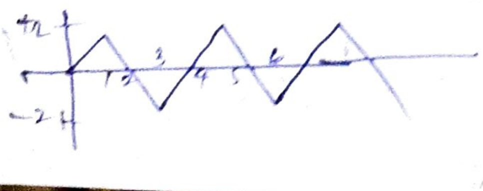
$$|I_L| = \frac{181.1}{\sqrt{R^2 + (\omega_0 L)^2}} = 144\, ma = \frac{181.1}{\sqrt{(80)^2 + (2\pi \times 10^6 \times 200 \times 10^{-6})^2}}$$

$$|I_c| = \frac{181.1}{\frac{1}{\omega C}} = 142\, ma. = \frac{181.1}{\frac{1}{2 \times \pi \times 10^6 \times 126 \times 10^{-12}}}$$

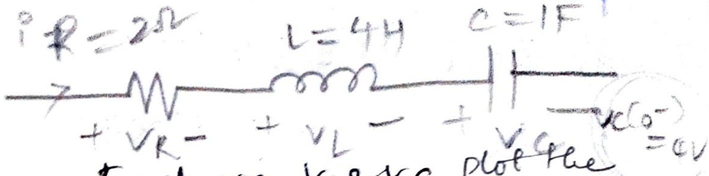
Steady state response of non sinusoidal inputs to N/w.

Triangular, square Trapezoidal, Sawtooth, rect.

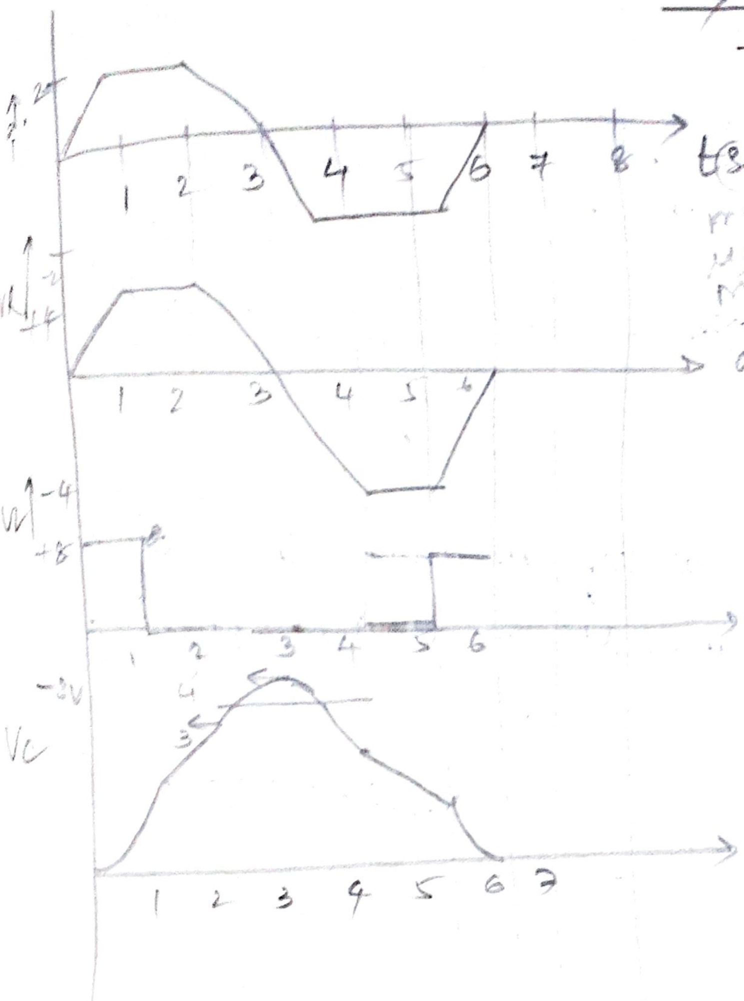




30th October, 2019



Find V_R, V_L & V_C plot the waveforms.



$i(t) = 2t$ $t = 0$ to 1

$= 2$ $t = 1$ to 2

$= \{-2t+6\}$ $t = 2$ to 4.

$= -2$ $t = 4$ to 5

$= 2t-12$ $t = 5$ to 6

1) $V_R = Ri = 2i$

2) $V_L = L\frac{di}{dt}$

$= 4\frac{d}{dt}(2t) = 8V$

$= 4\frac{d}{dt}(2) = 0.$

$= 4\frac{d}{dt}(-2t+6) = -8$

3) $V_C = \frac{1}{C}\int_0^t i(t)dt = \frac{1}{1}\int_0^1 2t\,dt = t^2$

$1 + 2(t)_1^2 = 3.$

$V_C = 1 + \frac{1}{1}\int^t 2dt = 2t-1$

$V_C = 3 + \frac{1}{1}\int_2^4 (-2t+6)dt = 3 + (-t^2)_2^4 + (6t)_2^4$

$= 3 - 16 + 4 + 24 - 12$

$= 7 + 24 - 28$

$= 7 - 4 = 3.$

4) a) $v_C(t=1) = \frac{1}{1}\int_0^1 2t\,dt = 1$

b) $v_C(t=3) = \frac{1}{1}\int_0^1 2t\,dt + \frac{1}{1}\int_1^2 dt + \frac{1}{1}\int_2^3 (-2t+6)dt = 4V$

c) $v_C(t=8) = v_C(t=1) = 1V.$

05) a) $E_C(t=1) = \frac{1}{2} \times 1 \times 1^2 = 0.5\,J$

b) $E_C(t=3) = \frac{1}{2} \times 1 \times 4^2 = 8\,J$

06) a) $E_L(t=1) = \frac{1}{2} \times 4 \times 2^2 = 8\,J$

b) $E_L(t=3.5) = \frac{1}{2} \times 4 \times (-1)^2 = 2\,J$

07) a) $E_R(t=0 \text{ to } 3) = \int_0^1 (2t)^2 \cdot 2\,dt + \int_1^2 2^2 \cdot 2\,dt + \int_2^3 (-2t+6)^2\, 2\,dt$

b) $E_R(t=0 \text{ to } 6) = 2 \times 7\,(a)$

* If $v_C(0) = 4V$; only v_C waveform move above by 4V
* If time is given in ms, μs, mnts, hrs, days convert into seconds.

UNIT-4 RESONANCE

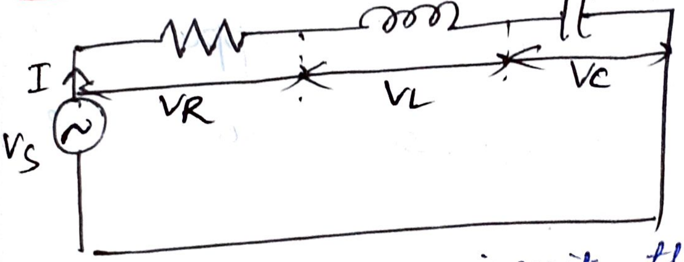
Series
parallel circuits
Concept of bandwidth & Q factor.
Steady state response of a network to non-sinusoidal periodic inputs.

RESONANCE

→ very important phenomenon
→ very useful in communications.

Ex: the ability of a radio receiver to select a certain frequency, transmitted by a station and to eliminate frequencies from other stations is based on the principle of resonance.

SERIES RESONANCE:



In a series RLC circuit, the current lags behind, or leads the applied voltage depending upon the values of X_L and X_C

X_L causes the total current to lag behind the applied voltage, while X_C causes the total current to lead the applied voltage.

When $X_L > X_C$, the circuit is predominantly inductive,

When $X_C > X_L$, the circuit is predominantly capacitive.

However, if one of the parameters of the series RLC circuits is varied in such a way that the current in the circuit is in phase with the applied voltage, then the circuit is said to be in resonance.

The total impedance for the series RLC circuit is

$$Z = R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$I = \frac{V_S}{Z}.$$

* The circuit is said to be in resonance if the current is in phase with the applied voltage.

* In a series RLC circuit, series resonance occurs when $X_L = X_C$.

* The frequency at which the resonance occurs is called the resonant frequency.

* Since $X_L = X_C$, the impedance in a series RLC circuit is purely resistive.

* At the resonant frequency, f_0, the voltages across capacitance and inductance are equal in magnitude.

Since they are 180° out of phase with each other, they cancel each other and, 1st October, ...

hence zero voltage appears across the LC combination

At resonance,

$$X_L = X_C \quad \text{i.e.,} \quad \omega L = \frac{1}{\omega C}$$

$$2\pi f_R L = \frac{1}{2\pi f_R C}$$

$$f_R^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_R = \frac{1}{2\pi\sqrt{LC}}}$$

$$Z = R + j\left(\cancel{\omega L - \frac{1}{\omega C}}\right) \to 0$$

at ω_0

$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$Z = R$ (Min)

$I_0 = \frac{V}{R}$ (Max)

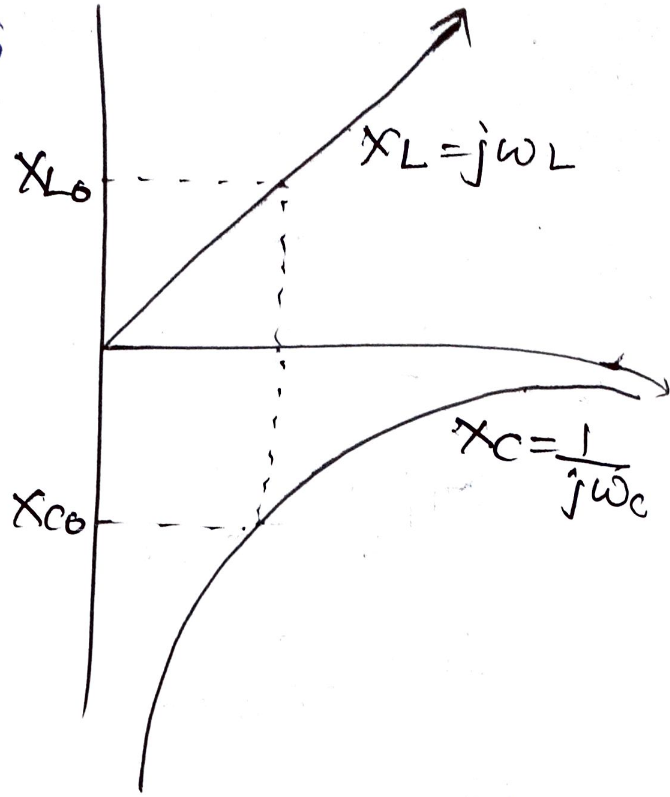
Power Factor $= 1$

$|Z|_{\text{at } \omega=0} = |R + j0 - j\infty| = \infty \rightarrow I = 0$

$|Z|_{\text{at } \omega=\infty} = |R + j\infty - j0| = \infty \rightarrow I = 0$

In a series RLC circuit, resonance may be produced by varying the frequency, keeping

L and C constant; otherwise, resonance may be produced by varying either L or C for a fixed frequency.



- Impedance and phase Angle of a Series resonant circuit:

The impedance of a series RLC circuit is

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The variation of X_C and X_L with frequency is shown.

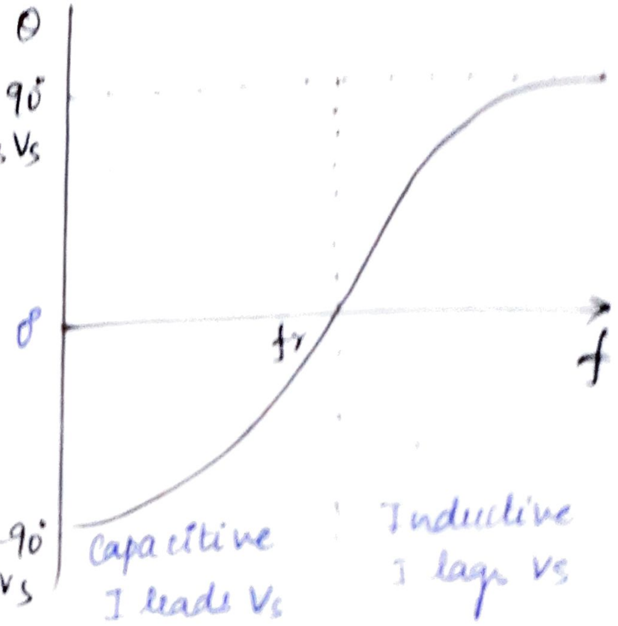
* At zero frequency, both X_C and Z are infinitely large, and X_L is zero because at zero frequency, the capacitor acts as an open circuit and the inductor acts as a short circuit.

* As the frequency increases, X_C decreases and X_L increases.

* Since X_C is larger than X_L, at frequencies below the resonant frequency f_0, Z decreases along with X_C.

* At resonant frequency f_r, $X_C = X_L$, and $Z = R$.
* At frequencies above the resonant frequency, f_r, X_L is larger than X_C, causing Z to increase.

The phase angle as a function of frequency is shown



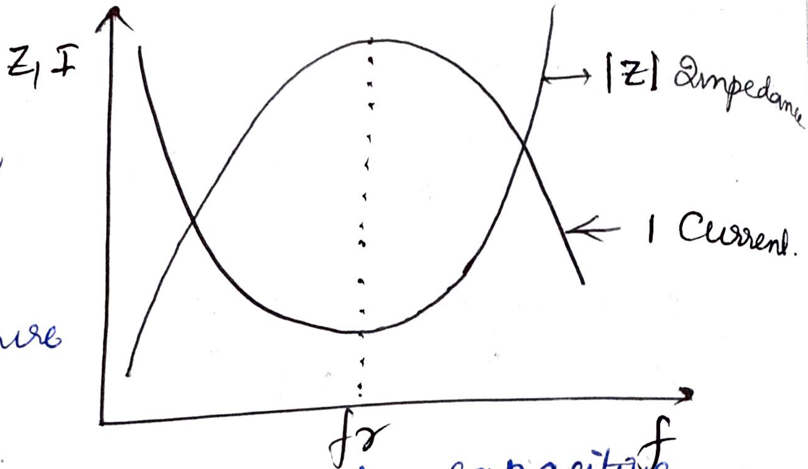
* At a frequency below the resonant frequency, current leads the source voltage because the capacitive reactances is greater than the inductive reactance.

* The phase angle decreases as the frequency approaches the resonant value, and is 0° at resonance.
* At frequencies above resonance, the current lags behind the source voltage, because the inductive reactance is greater than capacitive reactance. As the frequency goes higher, the phase angle approaches 90°.

f

• Voltages and Currents in a series resonant circuit.

The variation of impedance and current with frequency is shown in figure



At resonant frequency, the capacitive reactance is equal to the inductive reactance, and hence the impedance is minimum.

Because of minimum impedance, maximum current flows through the circuit.

The current variation with frequency is plotted.

The voltage drop across resistance, inductance and capacitance also varies with frequency.

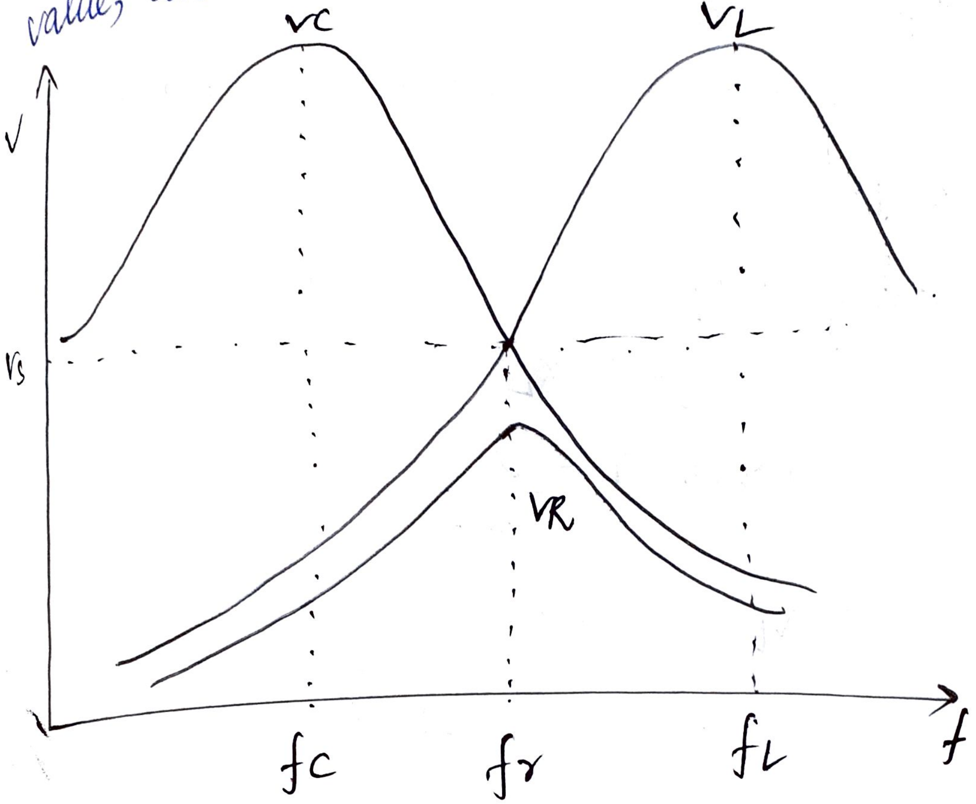
At f=0, the capacitor acts as an open circuit and blocks current. The complete source voltage appears across the capacitor.

As the frequency increases, X_C decreases and X_L increases, causing total reactance $X_C - X_L$ to decrease.

As a result, the impedance decreases and the current increases.

As the current increases, V_R also increases and both V_C and V_L increase.

when the frequency reaches its resonant value f_r, the impedance is equal to R, and hence the current reaches its maximum value, and V_R is at its maximum value.



As the frequency is increased above resonance, X_L continues to increase and X_C continues to decrease, causing the total reactance, $X_L - X_C$ to increase.

As a result there is an increase in impedance and a decrease in current.

As the current decreases, V_R also decreases, and both V_C and V_L decrease.

As the frequency becomes very high, the current approaches zero, both V_R and V_C approach zero, and V_L approaches V_S.

The response of different voltages with frequency is shown above.

The drop across the resistance reaches its maximum when $f = f_r$.

The maximum voltage across the capacitor occurs at $f = f_C$

the maximum voltage across the inductor occurs at $f = f_L$

The voltage drop across the inductor is

$$V_L = I X_L$$

where $I = \frac{V}{Z}$

$$\therefore V_L = \frac{\omega L V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

To obtain the condition for maximum voltage across the inductor.

$$\therefore \frac{dV_L}{d\omega} = 0$$

$$\frac{dV_L}{d\omega} = \frac{d}{d\omega}\left\{ \omega L V \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right]^{-1/2} \right\}$$

$$= \frac{LV\left(R^2 + \omega^2 L^2 - \frac{2L}{C} + \frac{1}{\omega^2 C^2}\right)^{-1/2} - \omega L V\left(R^2 + \omega^2 L^2 - \frac{2L}{C} + \frac{1}{\omega^2 C^2}\right)\left(2\omega L^2 - \frac{2}{\omega^3 C^2}\right)}{R^2 + \omega^2 L^2 - \frac{2L}{C} + \frac{1}{\omega^2 C^2}} = 0$$

$$R^2 - \frac{2L}{C} + \frac{2}{\omega^2 C^2} = 0$$

$$\omega L = \sqrt{\frac{2}{2LC - R^2C^2}}$$

$$= \frac{1}{\sqrt{LC}} \sqrt{\frac{2}{2 - \frac{R^2C}{L}}}$$

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{1 - \frac{R^2C}{2L}}}}$$

The voltage across the capacitor is

$$V_C = I X_C = \frac{1}{\omega C}$$

$$\therefore V_C = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \times \frac{1}{\omega C}$$

To get maximum value

$$\frac{dV_C}{d\omega} = 0$$

If we solve for ω, we obtain the value of ω when V_C is maximum.

$$\frac{dV_C}{d\omega} = \omega C \frac{1}{2} \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2} \left[2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)\right] +$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} C = 0$$

$$\omega_c^2 = \frac{1}{LC} - \frac{R^2}{2L}$$

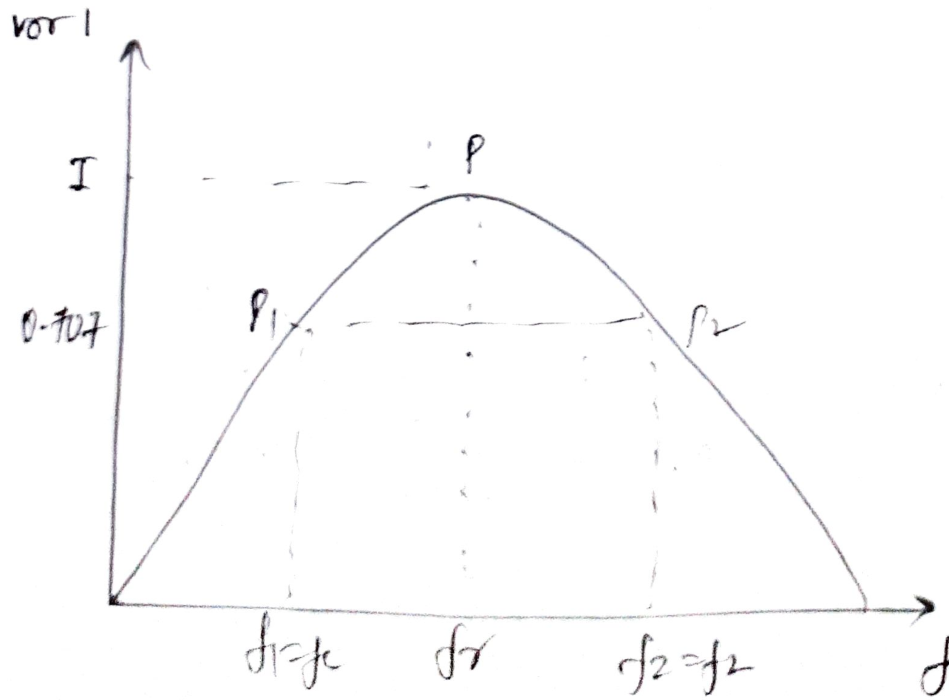
$$\omega_c = \sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$\therefore f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

The maximum voltage across the capacitor occurs below the resonant frequency; and the maximum voltage across the inductor occurs above the resonant frequency.

Bandwidth of an RLC Circuit:

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency and it is denoted by BW.



Response of a series RLC Circuit

The frequency f_1 is the frequency at which the current is 0.707 times the current at resonant value, and it is called the lower cut-off frequency.

The frequency f_2 is the frequency at which the current is 0.707 times the current at resonant value (i.e. maximum value), and is called the upper cut-off frequency.

The bandwidth, is defined as the frequency difference between f_2 and f_1

$$\therefore BW = f_2 - f_1 \quad \text{Unit :- Hertz (Hz)}$$

If the current at P_1 is $0.707\, I_{max}$, the impedance of the circuit at this point is $\sqrt{2}\, R$, and hence

$$\frac{1}{\omega_1 C} - \omega_1 L = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\therefore \omega_r^2 = \omega_1 \omega_2$$

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R$$

$$(\omega_2 - \omega_1)L + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = 2R$$

Since $C = \frac{1}{\omega_r^2 L}$

$$\omega_1 \omega_2 = \omega_r^2$$

$$(\omega_2 - \omega_1)L + \frac{\omega_r^2 L(\omega_2 - \omega_1)}{\omega_r^2} = 2R$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$\therefore f_2 - f_1 = \frac{R}{2\pi L}$$

$$BW = \frac{R}{2\pi L}$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

$$\therefore f_r - f_1 = \frac{R}{4\pi L}$$

$$f_2 - f_r = \frac{R}{4\pi L}$$

The lower frequency limit $f_1 = f_r - \frac{R}{4\pi L}$

The upper frequency limit $f_2 = f_r + \frac{R}{4\pi L}$

31st October, ...

If we divide the equation on both sides by f_r, we get

$$\frac{f_2 - f_1}{f_r} = \frac{R}{2\pi f_r L}$$

Here, an important property of a coil is defined.
It is the ratio of the reactance of the coil to its resistance.
This ratio is defined as the Q of the coil.
Q is known as a figure of merit, it is also called quality factor and is an indication of the quality of a coil

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$$

$$Q = \frac{f_r}{f_2 - f_1} = \frac{\text{Reactance}}{\text{Resistance}}$$

$$\frac{f_2 - f_1}{f_r} = \frac{1}{Q}.$$

The upper and lower cut-off frequencies are sometimes called the half-power frequencies.

At these frequencies, the power from the source is half of the power delivered at the resonant frequency.

At resonant frequency, the power is

$$P_{max} = I^2_{max} R$$

At frequency f_1, the power is $P_1 = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$

$$= \frac{I^2_{max} R}{2}$$

At frequency f_2, the power is

$$P_2 = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$$

$$= \frac{I^2_{max} R}{2}$$

The response curve is also called the selectivity curve of the circuit.

Selectivity indicates how well a resonant circuit responds to a certain frequency and eliminates all other frequencies.

The narrower the bandwidth, the greater the selectivity.

- **The Quality factor (Q) and its effect on Bandwidth:**

The quality factor, Q is the ratio of the reactive power in the inductor or capacitor to the true power in the resistance in series with the coil or capacitor.

$$Q = 2\pi \times \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

In an inductor,

the maximum energy stored is given by $\frac{LI^2}{2}$

Energy dissipated per cycle $= \left(\frac{I}{\sqrt{2}}\right)^2 RT = \frac{I^2RT}{2}$

∴ Quality factor of the coil, Q

$$Q = 2\pi \times \frac{\frac{1}{2} L I^2}{\frac{I^2 R}{2} \times \frac{1}{f}}$$

$$\boxed{Q = \frac{2\pi f L}{R} = \frac{\omega L}{R}}$$

Similarly,
In capacitor, the maximum energy stored is given by $\frac{CV^2}{2}$.

The energy dissipated per cycle $= \left(\frac{I}{\sqrt{2}}\right)^2 R \times T$.

The Quality factor of the capacitance circuit

$$Q = 2\pi \times \frac{\frac{1}{2} C \left(\frac{1}{\omega C}\right)^2}{\frac{I^2}{2} R \times \frac{1}{f}} = \frac{1}{\omega C R}$$

In series circuits,

the Quality factor, $Q = \frac{\omega L}{R} = \frac{1}{\omega C R}$.

$$Q = \frac{f_r}{BW}.$$

A higher value of the ~~circuit~~ Q results in a smaller bandwidth.

A lower value of Q causes larger Bandwidth

• Magnification in Resonance:

If we assume that the voltage applied to the series RLC circuit is V, and the current at resonance is I, then the voltage across L is $V_L = I X_L = \left(\frac{V}{R}\right) \omega_0 L$

Similarly, the voltage across C

$$V_C = I X_C = \frac{V}{R \omega_0 C}$$

Since $Q = \frac{1}{\omega_0 C R} = \frac{\omega_0 L}{R}$

where ω_0 is the frequency at resonance

Therefore $V_L = VQ$

$V_C = VQ$

The ratio of voltage across either L or C to the voltage applied at resonance can be defined as magnification.

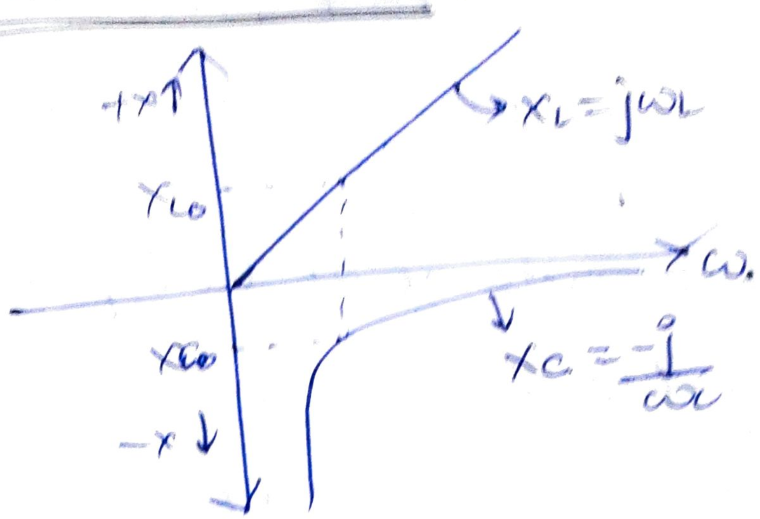
$\therefore$ Magnification $= Q = \frac{V_L}{V}$

(or) $\frac{V_C}{V}$

SERIES RESONANCE SUMMARY

01) $\omega_0 L - \frac{1}{\omega_0 C} = 0$

02) $\omega_0 = \frac{1}{\sqrt{LC}}$



03) $Z = R$ (Min)

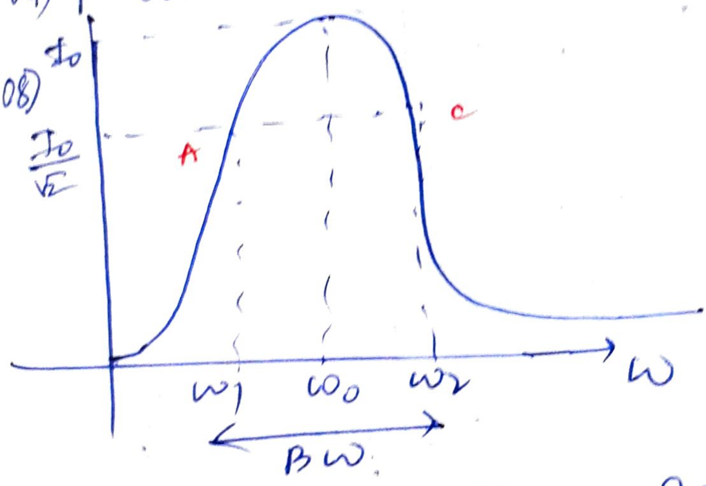
04) $I_0 = \frac{V}{R}$ (Max)

05) Power Factor = 1

06) $|Z|$ at $\omega = 0$ $|R + j0 - j\infty| = \infty \Rightarrow I = 0$

07) $|Z|$ at $\omega = \infty = |R + j\infty - j0| = \infty \Rightarrow I = 0$

08)



09) $1\,dB = 10 \log_{10} \frac{P_o}{P_i} = 20 \log \frac{I_o}{I_1}$

If $P_o = P_i \Rightarrow 0\,dB$

If $P_o = \frac{P_i}{2} \Rightarrow -3\,dB$

If $I_o = \frac{I_1}{\sqrt{2}} \Rightarrow -3\,dB$

ω_1 = Lower 3dB freq.
ω_2 = upper 3dB freq.
ω_1 & ω_2 → half power frequencies

10) At point A (or) C

$$I = \frac{I_0}{\sqrt{2}} = \frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\boxed{\left(\omega L - \frac{1}{\omega C}\right) = \pm R}$$

Net reactance at half power frequencies is $\pm R$

11) $\omega_1 L - \frac{1}{\omega_1 C} = -R$

12) $\omega_2 L - \frac{1}{\omega_2 C} = +R$

(+) $\Rightarrow L(\omega_2 + \omega_1) - \frac{1}{C}\left[\frac{\omega_2 + \omega_1}{\omega_1 \omega_2}\right] = 0$

$$(\omega_2 + \omega_1)\left[L - \frac{1}{C\omega_1 \omega_2}\right] = 0$$

$$\omega_1 \omega_2 = \frac{1}{LC} = \omega_0^2$$

13) $\boxed{\omega_0^2 = \sqrt{\omega_1 \omega_2}}$

14) $\omega_2 - \omega_1 = \boxed{\frac{R}{L} = B.W}$ [Independent of c]

rad/sec

15) Quality factor $= Q = \dfrac{2\pi \times (\text{Energy stored})_{Max}}{\dfrac{\text{Power/ Energy dissipated}}{f}}$

$$Q_L = \frac{2\pi \frac{1}{2} \times L I_{max}^2}{\frac{I_{rms}^2 R}{f}} = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

16) $\boxed{Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} > 1}$

(greater) (Best)

(17) $Q_0 = \frac{\omega_0 L}{R} = \frac{\omega_0}{\left(\frac{R}{L}\right)} = \frac{\omega_0}{B.W}$.

(18) $\boxed{Q_0 = \frac{\omega_0}{BW}}$

18) $Q_0 = \frac{\omega_0 L}{R} \times \frac{I_0}{I_0} = \frac{V_{L0}}{V} > 1$

It works like Voltage Amplifier.

19) $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\therefore Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{Q_0}{B.W} = \frac{V_{L0}}{V} = \frac{V_{C0}}{V} = \frac{1}{R}\sqrt{\frac{L}{C}} > 1$

20) $\frac{I}{I_0} = \frac{\frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}}{\frac{V}{R}}$

$= \frac{1}{\sqrt{1 + \left(\frac{\omega_0 L}{R}\right)^2 \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]^2}}$

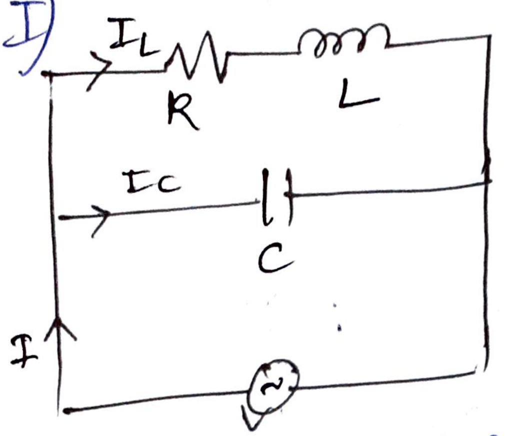
At resonance
- Net reactance is zero
- Impedance is min
→ Current is max
→ P.F = 1
→ Works like voltage amplifier not power amplifier

Let $\frac{I_0}{I} = N$

$\frac{1}{N} = \frac{1}{\sqrt{1 + Q_0^2 \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]^2}}$

$\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{\pm\sqrt{N^2 - 1}}{Q_0}$.

PARALLEL RESONANCE:

I) 

The parallel resonant circuit is generally called a tank circuit because of the fact that the circuit stores energy in the magnetic field of the coil and in the electric field of the capacitor.

The stored energy is transferred back and forth between the capacitor and coil and vice-versa.

The circuit is said to be in resonant condition when the susceptance part of admittance is zero.

The total admittance is

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{-jX_C}$$

$$Y = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R_L^2 + X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

To satisfy the condition for resonance, the susceptance part is zero

$$\frac{1}{X_C} = \frac{X_L}{R_L^2 + X_L^2}$$

$X_C = \frac{1}{j\omega C}$

$X_L = j\omega L$

$$\omega C = \frac{\omega L}{R_L^2 + \omega^2 L^2}$$

$$R_L^2 + \omega^2 L^2 = \frac{L}{C}$$

$$\omega^2 L^2 = \frac{L}{C} - R_L^2$$

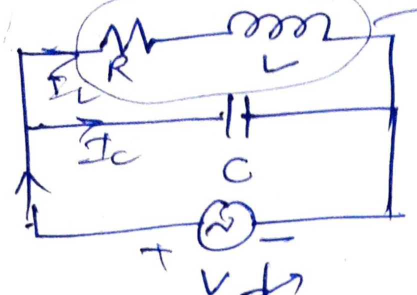
$$\omega^2 = \frac{1}{LC} - \frac{R_L^2}{L^2}$$

$$\therefore \omega = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

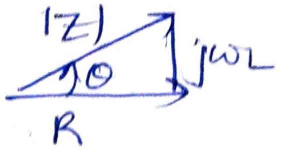
The resonant frequency for the tank circuit is

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}}$$

Summary of Type-1 of Parallel Resonance



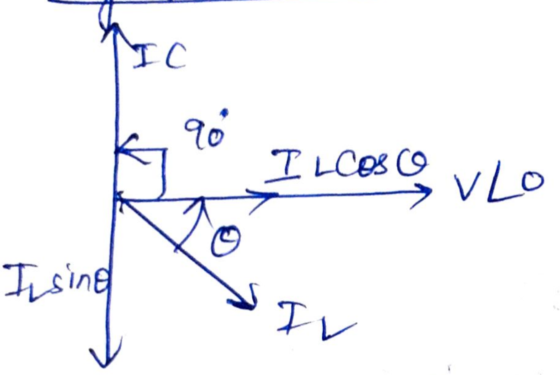
$\rightarrow Z = R + j\omega L$



$\theta = \tan^{-1}\frac{\omega L}{R}$

$\cos\theta = \frac{R}{|Z|}$, $\sin\theta = \frac{X_L}{|Z|}$

Using phasor method:



At $\omega_0 \Rightarrow (I_C) = [I_L \sin\theta)$

$$\left|\frac{V}{X_C}\right| = \left|\frac{V}{|Z|}\cdot\frac{X_L}{|Z|}\right|$$

$$= |Z|^2 = X_L X_C = \frac{L}{C}$$

$$(R + j\omega_0 L)^2 = \frac{L}{C}$$

$$R^2 - \omega_0^2 L^2 + 2jR\omega_0 L = \frac{L}{C}$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

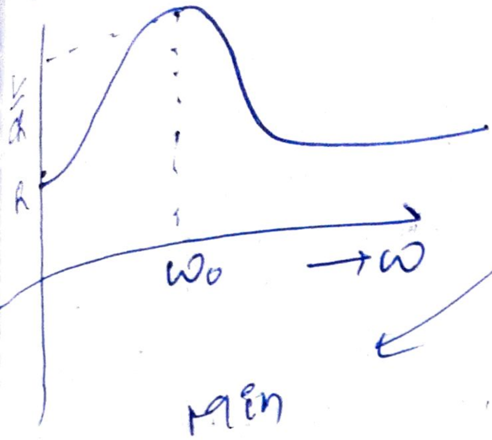
$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\boxed{\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

at ω_0 $\quad I = I_0 = I_L \cos\theta$

$$= \frac{V}{|Z|}\cdot\frac{R}{|Z|} = \frac{VR}{|Z|^2} = \frac{VR}{\frac{L}{C}}$$



$$\boxed{I_0 = \frac{V}{\left(\frac{L}{CR}\right)}}$$

Dynamic resistance (Max) (Ω)

min

$\boxed{P.F = 1}$

If $R = 0$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

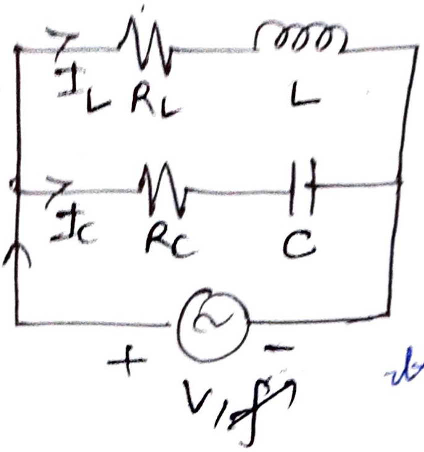
$$Y = Y_1 + Y_2 = \frac{1}{R + j\omega L} + j\omega C$$

$$= (\ ) + j(\ ) \to 0 \text{ at } \omega_0$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

→ Resonant frequency is the frequency at which net reactive current is zero, net susceptance (reciprocal of reactance) is zero, impedance is maximum, current is minimum, P.F = 1.

II)



Type - II

Parallel resonance occurs when $X_C = X_L$.

The frequency at which resonance occurs is called the resonant frequency.

When $X_C = X_L$, the two branch currents are equal in magnitude and 180° out of phase with each other. Therefore, the two currents cancel each other out, and the total current is zero.

The condition for resonance occurs when

$$X_L = X_C$$

The total admittance

$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - \left(\frac{j}{\omega C}\right)}$$

$$= \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + \left(\frac{j}{\omega C}\right)}{R_C^2 + \frac{1}{\omega C^2}}$$

$$= \frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} + j\left\{\left[\frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}\right] - \left[\frac{\omega L}{R_L^2 + \omega^2 L^2}\right]\right.$$

At resonance, the susceptance part becomes zero

$$\frac{\omega_r L}{R_L^2 + \omega_r^2 L^2} = \frac{\frac{1}{\omega_r C}}{R_C^2 + \frac{1}{\omega_r^2 C^2}}$$

$$\omega_r L \left[R_C^2 + \frac{1}{\omega_r^2 C^2} \right] = \frac{1}{\omega_r C} \left[R_L^2 + \omega_r^2 L^2 \right]$$

$$\omega_r^2 \left[R_C^2 + \frac{1}{\omega_r^2 C^2} \right] = \frac{1}{LC} \left[R_L^2 + \omega_r^2 L^2 \right]$$

$$\omega_r^2 R_C^2 - \frac{\omega_r^2 L}{C} = \frac{1}{LC} R_L^2 - \frac{1}{C^2}$$

$$\omega_r^2 \left[R_C^2 - \frac{L}{C} \right] = \frac{1}{LC} \left[R_L^2 - \frac{L}{C} \right]$$

$$\omega_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - \left(\frac{L}{C}\right)}{R_C^2 - \left(\frac{L}{C}\right)}}$$

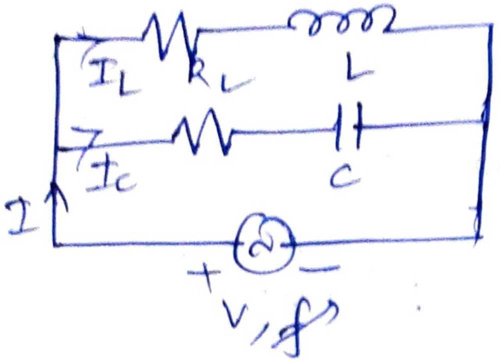
The condition for resonant frequency is given by above equation.

If $R_L = R_C$, then

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

Summary of Type-II



$$Y = Y_1 + Y_2$$

$$= \frac{1}{R_L + j\omega L} + \frac{1}{R_C + \frac{1}{j\omega C}}$$

$$= (\;) + j(\;) \rightarrow 0$$

at ω_0

$$\omega_0 = \sqrt{\frac{1}{LC}\left[\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}\right]}$$

(i) If $R_L = R_C = 0$

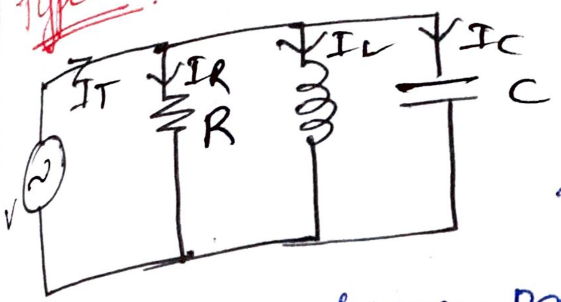
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

Same as Series Resonance.

(ii) If $R_L^2 = R_C^2 = \frac{L}{C}$;

The ckt is under resonance for entire frequency range.

Type-3



Consider the parallel RLC circuit

In the circuit shown, the condition for resonance occurs when the susceptance part is zero.

Admittance, $Y = G + jB$

$$= \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

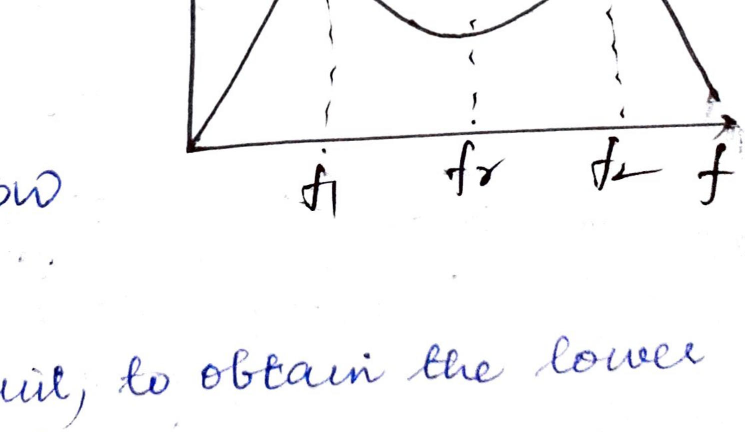
$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

The frequency at which resonance occurs is

$$\omega_r C - \frac{1}{\omega_r L} = 0$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

The voltage and current variation with frequency is shown.



At resonant frequency, the current is minimum.

The Bandwidth, BW $= f_2 - f_1$

For parallel circuit, to obtain the lower half power frequency,

$$\omega_1 C - \frac{1}{\omega_1 L} = \frac{-1}{R}$$

$$\omega_1^2 + \frac{\omega_1}{RC} - \frac{1}{LC} = 0$$

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

To obtain the upper half-power frequency

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R}$$

$$\omega_2^2 - \frac{1}{LC} - \frac{\omega}{RC} = 0$$

$$\omega_2^2 - \frac{\omega}{RC} - \frac{1}{LC} = 0$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Bandwidth, $BW = \omega_2 - \omega_1 = \frac{1}{RC}$

The Quality factor, $Q_r = \frac{\omega_r}{\omega_2 - \omega_1}$

$$Q_r = \frac{\omega_r}{\frac{1}{RC}} = \omega_r RC$$

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\frac{\text{Energy dissipated}}{\text{cycle}}}$$

In case of an inductor,
the maximum energy stored $= \frac{1}{2} L I^2$

Energy dissipated per cycle $= \left(\frac{I}{\sqrt{2}}\right)^2 \times R \times T$

$$Q_1 = 2\pi \times \frac{\frac{1}{2} L I^2}{\frac{I^2}{\cancel{2}} R \times \frac{1}{f}}$$

$$= 2\pi \times \frac{\frac{1}{2} \times L \times \left(\frac{V}{\omega L}\right)^2 R}{\frac{V^2}{\cancel{R}} \times \frac{1}{f}}$$

$$= \frac{2\pi f L R}{\omega^2 L^2} = \frac{R}{\omega L}$$

For a capacitor, maximum energy stored $= \frac{1}{2} C V^2$

Energy dissipated per cycle $= P \times T$
$$= \frac{V^2}{2 \times R} \times \frac{1}{f}$$

$$Q = 2\pi \times \frac{\frac{1}{2} C V^2}{\frac{V^2}{2R} \times \frac{1}{f}}$$

$$Q = 2\pi f C R = \omega C R$$

Magnification:
Current magnification occurs in a parallel resonant circuit. The voltage applied to the parallel circuit, $V = IR$

Since $I_L = \frac{V}{\omega_r L} = \frac{IR}{\omega_r L} = IQ_r$

For the capacitor,

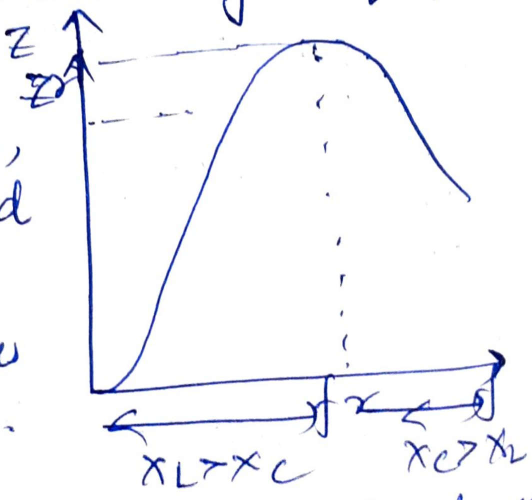
$I_C = \frac{V}{\frac{1}{\omega_r C}} = IR\omega_r C = IQ_r$

Therefore, the Quality factor

$Q_r = \frac{I_L}{I}$ or $\frac{I_C}{I}$

• Variation of impedance with frequency:

The impedance of a parallel resonant circuit is maximum at the resonant frequency and decreases at lower and higher frequencies as shown.



At very low frequencies, X_L is very small and X_C is very large, so the total impedance is essentially inductive.

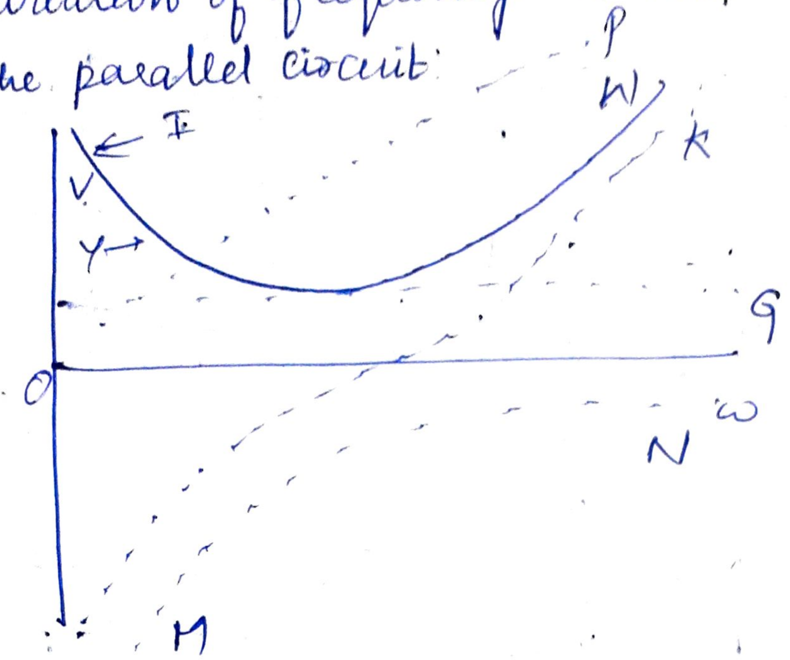
- As the frequency increases, the impedance also increases, and the inductive reactance dominates until the resonant frequency is reached.
- At this point $X_L = X_C$, and the impedance is at its maximum. As the frequency goes above resonance, capacitive reactance dominates and the impedance decreases.

. Reactance Curves in Parallel Resonance:

The effect of variation of frequency on the reactance of the parallel circuit:

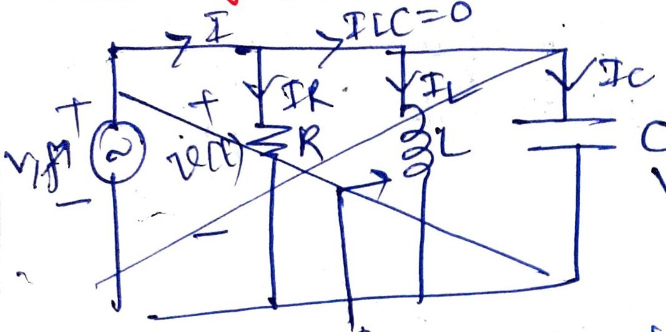
The effect of inductive susceptance,

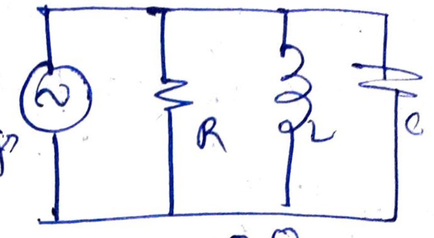
$$B_L = \frac{-1}{2\pi f L}$$



- Inductive susceptance is inversely proportional to the frequency ω.
  Hence, it is represented by a rectangular hyperbola, MN. It is drawn in fourth quadrant, since B_L is negative.
- Capacitive susceptance, $B_C = 2\pi f C$.
  It is directly proportional to the frequency f or ω. Hence it is represented by OP, passing through the origin.
- Net susceptance $B = B_C - B_L$. It is represented by the curve JK, which is a hyperbola.
- At the point ω₀, the total susceptance is zero, and the resonance takes place
- The variation of the admittance Y and the current I is represented by curve VW. the current will be minimum at resonant frequency.

Summary of Type-3





$$Y = \frac{1}{R} + \frac{1}{jwL} + jwC = \frac{1}{R} + j\left[wC - \frac{1}{wL}\right]$$

at w_0 $\left[wC - \frac{1}{wL}\right] = 0$

$$w_0C - \frac{1}{w_0 C} = 0$$

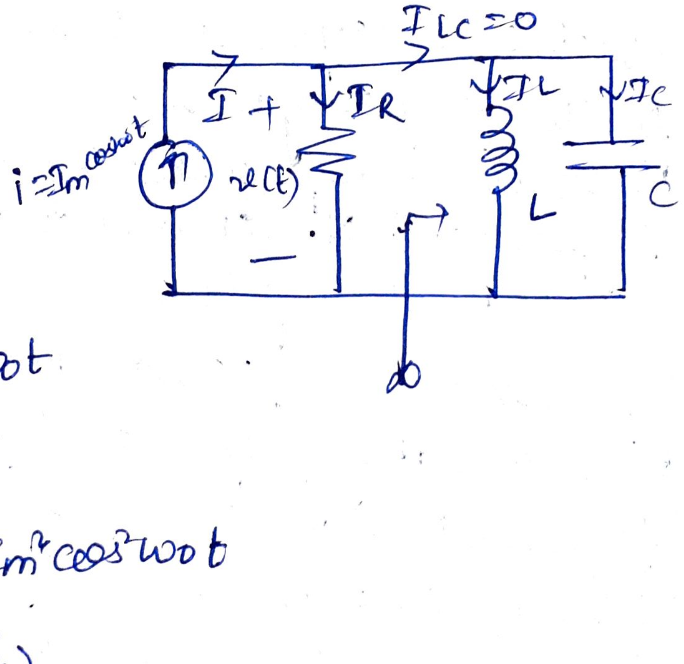
$$\boxed{w_0 = \frac{1}{\sqrt{LC}}}$$

Same as series resonance and it is independent of 'R'

$Z = R$ (max)

$I_0 = \frac{V}{R}$ (min)

P.F = 1



$$v(t) = R I_m \cos w_0 t$$

$$w_C(t) = \frac{1}{2} C v^2$$

$$= \frac{1}{2} C R^2 I_m^2 \cos^2 w_0 t$$

$$w_L(t) = \frac{1}{2} L i^2(t)$$

$$= \frac{1}{2} L \left[\frac{1}{L} \int v\,dt\,dt\right]^2$$

$$= \frac{I_m^2 R^2 C \sin^2 w_0 t}{2}$$

$$Q = 2\pi \frac{\text{Max Energy stored}}{\text{Total energy lost per period}}$$

$$= \frac{2\pi [w_L(t) + w_C(t)]_{max}}{P_R T}$$

$$= \frac{2\pi \left[\frac{I_m^2 R C^2}{2} \right]}{\frac{I_m^2 R}{2} / f_0}$$

$$Q_0 = 2\pi f_0 RC = \omega_0 RC = \frac{R}{\omega_0 L} > 1$$

$$Q_{0s} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} > 1$$

$i = I_m \cos \omega_0 t$

$I_{rms} = I$

$V_{rms} = IR$

$$Q_{0p} = \frac{R}{\omega_0 L} = \omega_0 CR > 1$$

$$|I_{C,0}| = \frac{IR}{\frac{1}{\omega_0 C}} = \omega_0 CRI$$

$$Q_0 = \omega_0 CR = \frac{\omega_0}{\frac{1}{RC}} = \frac{\omega_0}{BW}$$

$$\boxed{B \cdot W = \frac{1}{RC}}$$

Independent of 'L'

$$\left| \frac{I_{C,0}}{I} \right| = \omega_0 CR = Q_0 > 1$$

$\therefore$ It works like Current Amplifier

$$Q_0 = \omega_0 CR = \frac{1}{\sqrt{LC}} \cdot CR = R\sqrt{\frac{C}{L}}$$

$$\boxed{Q_0 = \frac{R}{\omega_0 L} = \omega_0 CR = \frac{\omega_0}{B \cdot W} = \frac{I_{C,0}}{I} = \frac{I_{L,0}}{I} = R\sqrt{\frac{C}{L}} > 1}$$

In s-domain:

$$Y(s) = \frac{1}{R} + \frac{1}{sL} + \frac{sC}{1}$$

$sL = j\omega L$, $\omega = 2\pi f$

$\boxed{s = j\omega}$

$$= \frac{SL + R + S^2LCR}{SLR}$$

$$= \frac{LCR\left[s^2 + \frac{s}{RC} + \frac{1}{LC}\right]}{SLR}$$

$$Z(S) = \frac{s/c}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \quad = \frac{\omega_0^2}{s^2 + 2\delta\omega_0 s + \omega_0^2}$$

Comparing denominator only

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\delta\omega_0 = \frac{1}{RC}$$

$$\delta = \frac{1}{2RC \times \frac{1}{\sqrt{LC}}}$$

$$\delta = \frac{1}{2R}\sqrt{\frac{L}{C}} = \frac{1}{2Q_0} = \delta$$

Damping factor, $\alpha = \delta\omega_0 = \frac{1}{2RC}$

$$\alpha = \frac{B.W}{2}$$

$\alpha = \frac{BW}{2}$

$$R_c = \frac{1}{2}\sqrt{\frac{L}{C}}$$

$\downarrow$

Critical resistance at which $\delta = 1$

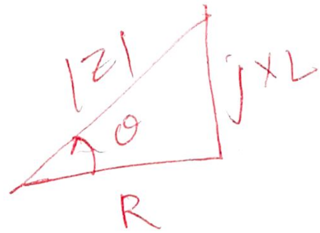
Resonance

Problems

Q) A choke coil when connected across a 500 V, 50 Hz supply takes 1A at 0.8 P.F. What capacitance must be placed in parallel with it so as to make the PF of combination unity?

Impedance of coil $= |Z| = \frac{500}{1} = 500\Omega$

P.F $= \cos\theta$



$\cos\theta = 0.8$

$\sin\theta = \sqrt{1-(0.8)^2}$

$= \sqrt{1-0.64}$

$= \sqrt{0.36} = 0.6$

$\sin\theta = \frac{X_L}{|Z|}$

$\sin\theta = \frac{X_L}{|Z|}$

$X_L = |Z| \sin\theta = 500 \times 0.6 = 300\Omega$

$2\pi f L = 300$

$L = \frac{300}{2\pi \times 50} = 0.955\ h$

To get PF=1 means to get resonance

At resonance, $Z^2 = \frac{L}{C}$

$C = \frac{L}{Z^2} = \frac{0.955}{500^2}$

$= 3.82\ \mu F.$ //

Q) Find the element values on a parallel resonant circuit having $Q_0 = 40$, $f_0 = 440$ Hz and an admittance of 500 μ℧ at resonance.

$$R = \frac{1}{500\mu} = \frac{10^6}{500} = \frac{10}{5} \times 10^{5-2} = 2 \times 10^3 = 2k\Omega$$

$$Q_0 = \omega_0 C R$$

$$C = \frac{Q_0}{\omega_0 R} = \frac{40}{2\pi \times 440 \times 2 \times 10^3} = 7.23\mu F$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\frac{1}{L} = f_0^2 (2\pi)^2 C$$

$$L = \frac{1}{f_0^2 (2\pi)^2 C} = 18.09 mH.$$