08th August 2019

UNIT 2: TRANSIENT ANALYSIS.

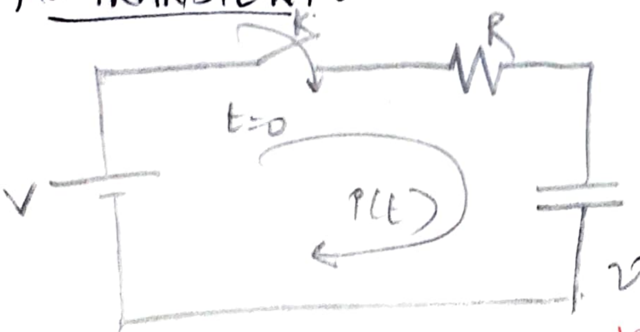
- RC
- RL
- RLC series and parallel circuits with DC and AC · response using
- step, impulse, ramp, exponential and AC · response using methods.
  Time Domain and Laplace transform methods.

I/p → N/w → $t_r + t_{ss}$

Even if current is send suddenly (C, L) it will behave peculiarly.

1st order – short ckt, 1st & 2nd → Mathematical, 3rd & above – Laplace any order.

RC TRANSIENT:



Find $i(t)$ for $t>0$ (0+ to ∞)

$$V = R\,i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

$V_c(0-) = 0$

$V_c(0+) = 0$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

$$= \underbrace{\boxed{\frac{1}{C}\int_{-\infty}^{0-} i(t)\,dt + \frac{1}{C}\int_{0-}^{0+} i(t)\,dt}}_{V_c(0+)} + \frac{1}{C}\int_{0+}^{t} i(t)\,dt$$

Not impulsive as $R \neq 0$

$\int_a^a f(t)\,dt = 0$

If it is impulse $\int_a^a f(t)\,dt = 1$

$= V_c(0-) + 0$

$V_c(0+) = V_c(0-)$

$i(0-) = 0$

$i(0+) = \frac{V}{R}$

i changing suddenly but not V, i.e. why we have transient nature in capacitor.

$$V = R\,i(t) + \frac{1}{C}\int_{0+}^{t} i(t)\,dt$$

(1) Voltage in Capacitor is a continuous function of time.
~~X~~ Current is a discontinuous function of time.

(2) Capacitor opposes sudden change of voltage.
Capacitor will not oppose sudden change of current.

$\sqcup\sqcap\sqcup\sqcap$ → continuously discontinuous

$v(t)$ → discontinuous

$v(0^-) = 0$ ↗ at zero.

$v(0^+) = 1$

value of discontinuity = 1.

(3) ~~Voltage is a continuous function~~
Capacitor smoothen the voltage wave, but it will not smoothen the current wave

→ $V = Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt$

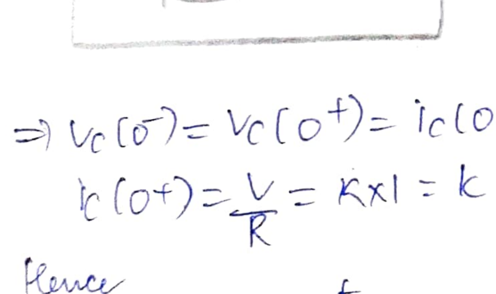
$0 = R\frac{di}{dt} + \frac{i}{C}$

$\left(D + \frac{1}{RC}\right) i = 0$. 1st order homogenous D.E

$i = i_c(t)$

14th August 2019

· RC TRANSIENT:



$V = Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt$

$0 = R\frac{di}{dt} + \frac{i}{C}$

$\left(D + \frac{1}{RC}\right) i = 0$

$i(t) = i_c(t) = k e^{-t/RC}$

⇒ $V_c(0^-) = V_c(0^+) = i_c(0^-) = 0$

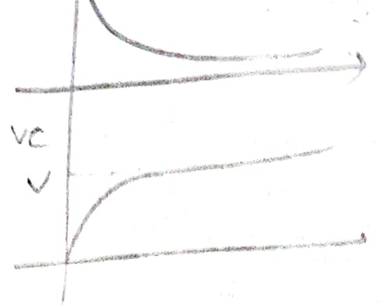
$i_c(0^+) = \frac{V}{R} = k \times 1 = k \Rightarrow k = \frac{V}{R}$

Hence

$i(t) = \frac{V}{R} e^{\frac{-t}{RC}}$ for $t > 0$

$V_R(t) = R.i(t) = Ve^{\frac{-t}{RC}}$

$V_c(t) = \frac{1}{C}\int_0^t i(t)\,dt = \frac{1}{C}\int_0^t \frac{V}{R} e^{\frac{-t}{RC}}\,dt$



$V_c(t) = V(1 - e^{\frac{-t}{RC}})$

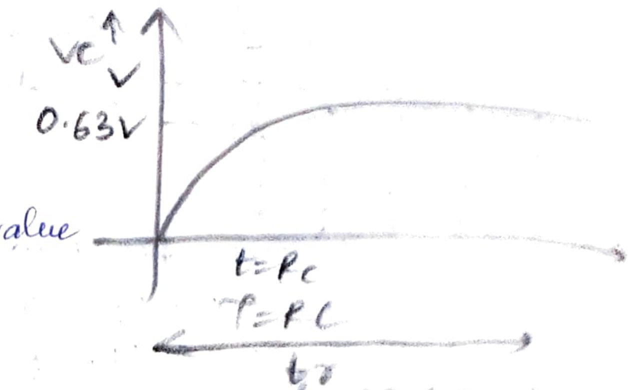
$i(\infty) = 0 \rightarrow$ In steady state capacitor behaves as Open

Initially V_c is 0, $\Rightarrow$ short

$V_c(t=RC) = V[1 - e^{-}] = 0.63V$

$\tau \rightarrow$ time taken to reach final value

$\tau \rightarrow$ time const

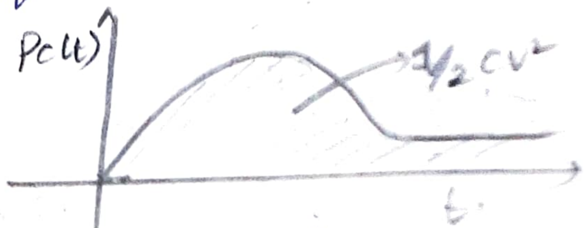


• Power in the Capacitor,

$P_c(t) = V_c(t) i(t) = V[1 - e^{\frac{-t}{RC}}][\frac{V}{R} e^{\frac{-t}{RC}}]$ Here $RC = \tau$

$P_c(t) = \frac{V^2}{R}[e^{\frac{-t}{\tau}} - e^{\frac{-2t}{\tau}}]$

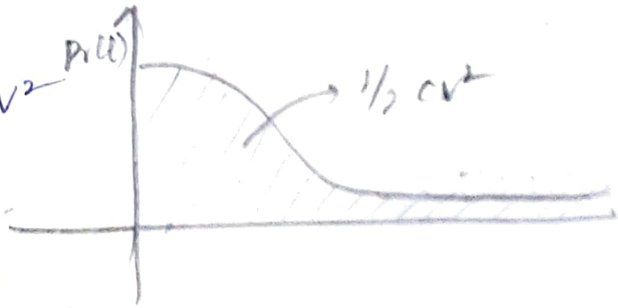
$E_c = \int_0^\infty P_c(t)dt = \frac{1}{2}CV^2$



$P_r(t) = V_r(t) \cdot i(t) = Ve^{-t/\tau} \cdot \frac{V}{R} e^{-t/\tau} = \frac{V^2}{R} e^{-2t/\tau}$

$E_r(t) = \int_0^\infty p_r(t)dt = \frac{1}{2}CV^2$

$V = V_R + V_C = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$



Efficiency, $\eta = \frac{\text{Energy stored}}{\text{Energy supplied}}$

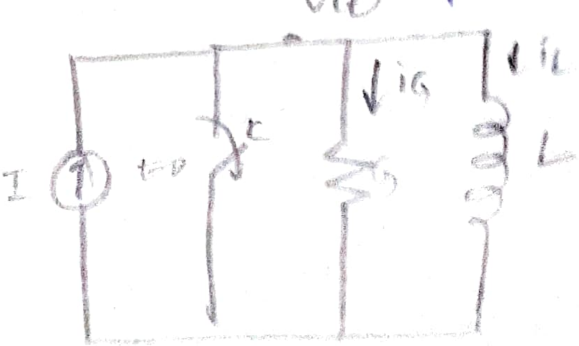
$= \frac{\frac{1}{2}CV^2}{CV^2} = 50\%$

$\eta \rightarrow$ independent of R & C

$\rightarrow \tau = RC$ If $R\downarrow C\downarrow \Rightarrow \tau\downarrow$

If transient time (t_r) $\uparrow$es, Capacitor charges slowly

Dual of RC:



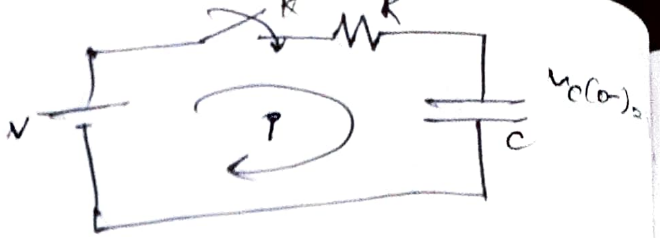
$$I = G v(t) + \frac{1}{L}\int_0^t v(t)\,dt$$

$$0 = G v(t) + \frac{1}{L}\int_0^t v(t)\,dt$$

21st August 2019

(Continuation of RC Transient)

By Laplace Transform



$V = R i(t) + \frac{1}{C}\int_{0^+}^{t} i(t)dt + V'$

$\frac{V}{s} = R I(s) + \frac{I(s)}{Cs} + \cancel{\frac{V'}{s}} \rightarrow 0$

$I(s) = \_\_\_\_$

$\frac{1}{C}\int_{-\infty}^{t} i(t)dt$

$= \frac{1}{C}\int_{-\infty}^{0} i(t)dt + \frac{1}{C}\int_{0}^{t} i(t)dt$

$u(t) \longrightarrow \frac{1}{s}$

$\delta(t) \longrightarrow 1$

$t \rightarrow \frac{1}{s^2}$

$e^{-at} \longrightarrow \frac{1}{s+a}$

$A\sin\omega t \rightarrow \frac{A\omega}{s^2+\omega^2}$

$A\cos\omega t \rightarrow \frac{As}{s^2+\omega^2}$

$\frac{df}{dt} \rightarrow sF(s) - f(0)$

$\int_0^t f(t)dt \rightarrow \frac{F(s)}{s}$

$\int_{-\infty}^{t} f(t)dt \rightarrow \frac{F(s)}{s} + \frac{f(0)}{s}$

$i(t) = \frac{V}{R} e^{-t/RC}$

① With DC i/p

$i(t) = \frac{V}{R} e^{-t/\tau}$

② With unit step

$i(t) = \frac{1}{R} e^{-t/\tau}$

③ Impulse response

$= \frac{d}{dt}[\text{unit step response}]$

$\boxed{\frac{d\,u(t)}{dt} = 1\,\delta t}$

$= \frac{d}{dt}\left[\frac{1}{R} e^{-t/\tau}\right]$

$= \frac{-1}{R\tau} e^{-t/\tau}$

④ Ramp response $\boxed{\int u(t)dt = t}$

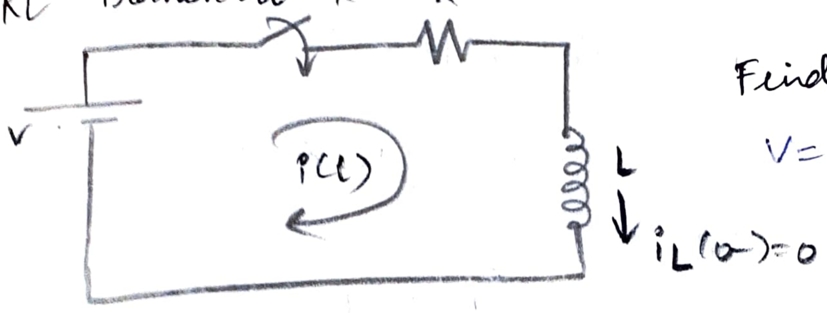
$= \int_0^t \frac{1}{R} e^{-t/\tau} dt$

⑤ Exponential response $e^{-t} = R i(t) + \frac{1}{C}\int_{0^+}^{t} i(t)dt$

$-e^{-t} = R\frac{di}{dt} + \frac{i}{C}$

$$\left(D+\frac{1}{RC}\right)i = -\frac{e^{-t}}{R}$$ First Order Non-homogeneous D.E

RL Transient



Find $i(t)$ for $t>0$

$V = Ri(t) + L\frac{di}{dt}.$

$$\left(D+\frac{R}{L}\right)i = \frac{V}{L}$$

1st Order H.D.E

$$i(t) = i_c(t) + i_p(t)$$

particular

complementary

$\tau = \frac{L}{R}$

$i(t) = ke^{-t/\tau}$ → complementary.

$+ e^{-t/\tau}\int e^{t/\tau}.\frac{V}{L}dt$

$$i(t) = ke^{-t/\tau} + e^{-t/\tau}\int e^{t/\tau}\frac{V}{L}dt = Ke^{-t/\tau} + \frac{V}{R}.$$

Initially

$L \Rightarrow$ open

$i(0+) = 0 = K\times 1 + \frac{V}{R}$

$K = -\frac{V}{R}$

On steady state

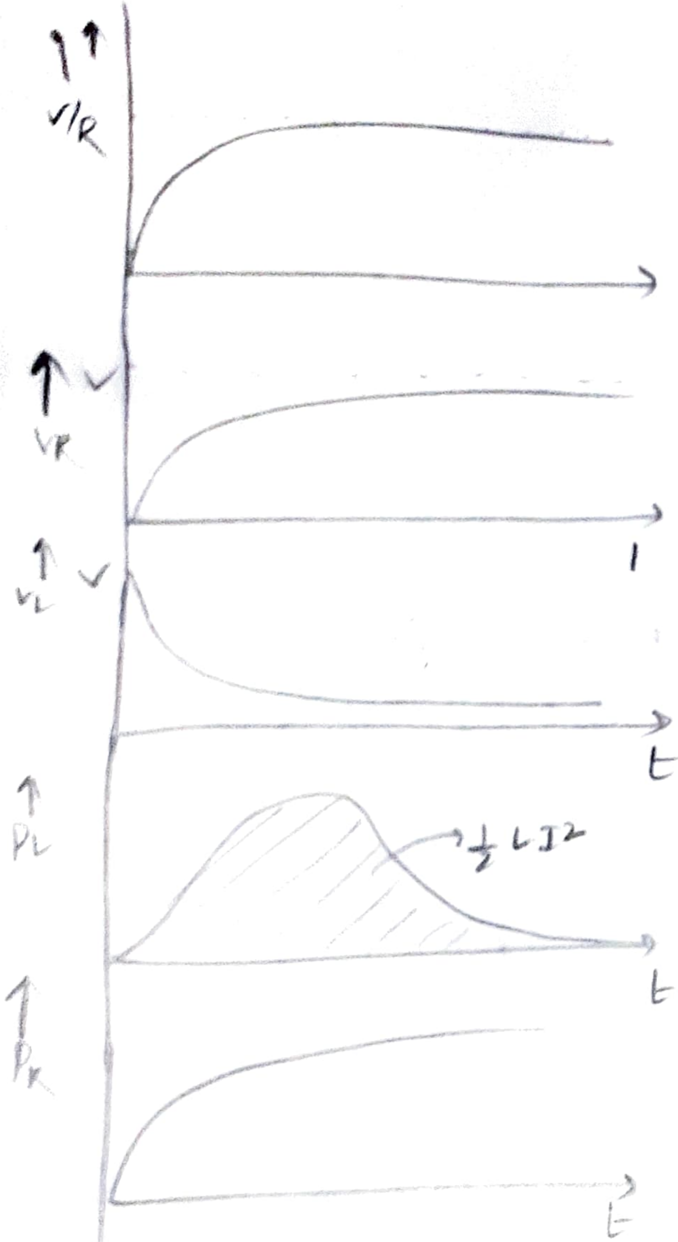
$L \to$ short circuit

$$i(t) = \frac{V}{R}\left[1-e^{-t/\tau}\right]$$

$$i(t) = \frac{V}{R}(1-e^{-t/\tau}) \quad | \quad i(\infty) = \frac{V}{R}.$$

$$V_R(t) = V(1-e^{-t/\tau})$$

$$V_L(t) = L\frac{di}{dt} = Ve^{-t/\tau}$$



$$P_L(t) = V e^{-t/\tau}\,\frac{V}{R}\left(1-e^{-t/\tau}\right)$$

$$= \frac{V^2}{R}\left(e^{-t/\tau} - e^{-2t/\tau}\right)$$

$$E_L = \int_0^\infty P_L(t)\,dt = \frac{1}{2}L\left(\frac{V}{R}\right)^2$$

$$V = Ri(t) + L\frac{di}{dt}$$

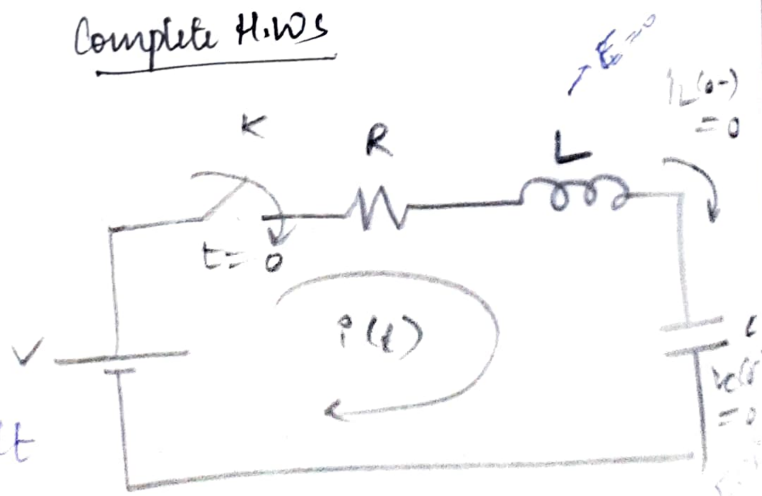
$$P_R = V\left(1-e^{-t/\tau}\right)\frac{V}{R}\left(1-e^{-t/\tau}\right)$$

$$= \frac{V^2}{R}\left(1-e^{-t/\tau}\right)^2$$

$$E_R = \int_0^\infty P_R(t)\,dt = \infty$$

$$\eta = \frac{\frac{1}{2}LI^2}{\infty} = 0\,\%$$

Complete H.W.s



22nd August 2019

Second order RLC series.

Find i(t) for t > 0

$$V = R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt$$

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}$$

$$\left(\cancel{D^2 + \frac{RD}{L}}\right)\cdot\left(D^2 + \frac{RD}{L} + \frac{1}{LC}\right)i = 0$$

Second order homogeneous D.E

$i(t) = i_c(t)$
depends on nature of roots.

$$D_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \times \frac{1}{LC}}}{2 \times 1}$$

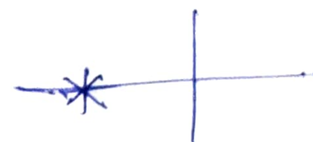
$$D_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Let $\alpha = \frac{-R}{2L}$; $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$D_1 = \alpha + \beta$.
$D_2 = \alpha - \beta$.

Case 1: if $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, two roots are real and not equal.
overdamping.

$$i(t) = k_1 e^{(\alpha+\beta)t} + k_2 e^{(\alpha-\beta)t}$$ $k_1, k_2 \to$ constant.



Case 2: if $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ $(\beta = 0)$

$D_1 = \alpha$, $D_2 = \alpha$, Two roots are ~~eq~~ real and equal.
Critically damped

$$i(t) = e^{\alpha t}(k_1 + k_2 t)$$

Case 3: if $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ two roots are complex and conjugate of each other
underdamped

$$i(t) = e^{\alpha t}[k_1 \cos\beta t + k_2 \sin\beta t]$$

I) $i(0+) = 0$

II) $V = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{0+}^{t} i(t)\,dt$; at $t = 0+$

$$V = R\,i(0+) + L\frac{di(0+)}{dt} + \frac{1}{C}\int_{0+}^{0+} i(t)\,dt$$

$$V = 0 + L\frac{di(0+)}{dt} + 0$$

$$\frac{di(0+)}{dt} = \frac{V}{L}$$

① $0 = k_1 + k_2$ —①

Differentiate $i(t) = k_1 e^{(\alpha+\beta)t} + k_2 e^{(\alpha-\beta)t}$

$$\frac{di(t)}{dt} = k_1(\alpha+\beta) e^{(\alpha+\beta)t} + k_2(\alpha-\beta) e^{(\alpha-\beta)t}$$

at $t=0$ $\frac{V}{L} = k_1(\alpha+\beta) + k_2(\alpha-\beta)$

$$\frac{V}{L} = \alpha(k_1+k_2) + \beta(k_1-k_2)$$

$$\frac{V}{L} = \beta(k_1-k_2)$$

$$k_1 - k_2 = \frac{V}{\beta L}$$

$$k_1 + k_2 = 0$$

$$2k_1 = \frac{V}{\beta L} \Rightarrow k_1 = \frac{V}{2\beta L}$$

$$k_2 = \frac{V}{2\beta L} - \frac{V}{\beta L} = \frac{-V}{2\beta L}$$

$$k_1 = \frac{V}{2\beta L}, \quad k_2 = \frac{-V}{2\beta L}$$

H.W

②.

$$i(t) = e^{\alpha t}(K_1 + K_2 t)$$

$$\boxed{0 = K_1}$$

Diff $i(t) = e^{\alpha t}(K_1 + K_2 t)$

$$\frac{di(t)}{dt} = e^{\alpha t}(K_2) + \alpha(K_1 + K_2 t) e^{\alpha t}$$

at $t=0$ $\frac{V}{L} = K_2 + \alpha K_1$

$K_1 = 0$.

$$K_2 = \frac{V}{L}$$

③ $i(t) = e^{\alpha t}[K_1 \cos\beta t + k_2 \sin\beta t]$

$\boxed{0 = K_1}$

$i'(t) = e^{\alpha t}[-K_1 \beta \sin\beta t + K_2 \cos\beta t] + [K_1 \cos\beta t + K_2 \sin\beta t]\alpha e^{\alpha t}$

at $t=0$
$i'(0)$

$\frac{V}{L} = [0 + k_2] + [K_1]\alpha$

$K_2 = \frac{V}{L}$

Apply Laplace transform

$V = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{0^+}^{t} i(t)dt$

$\frac{V}{S} = RI(S) + L[SI(S) - \cancel{I(0^-)}^{0}] + \frac{I(S)}{CS}$

$I(S)\left[R + LS + \frac{1}{CS}\right] = \frac{V}{S}$

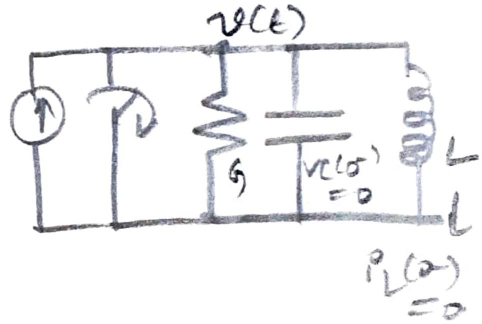
$I(S)\left[\frac{CRS + CLS^2 + 1}{C}\right] = V$

$I(S) = \frac{VC}{1 + CRS + CLS^2} = \frac{VC}{CLS^2 + CRS + 1} = \frac{VC}{S^2\left(LC + \frac{CR}{S} + \frac{1}{S^2}\right)}$

$I(S) = \frac{VC}{LC\left(S^2 + \frac{RS}{L} + \frac{1}{LC}\right)} = \frac{VC}{LC\left(S^2 + \left(\frac{R}{L}\right)S + \frac{1}{LC}\right)} = \frac{V/L}{S^2 + \left(\frac{R}{L}\right)S + \frac{1}{LC}}$

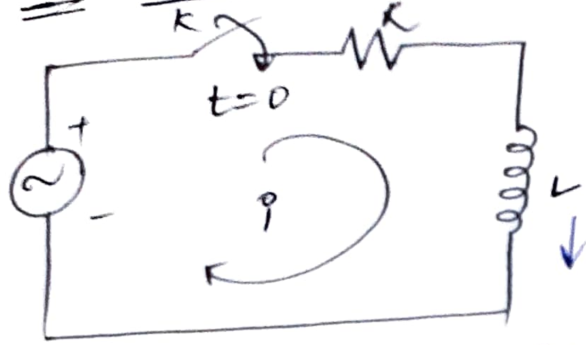
Dual of this

H.W.



23rd August 2019

AC TRANSIENT:-



$V = V_m \sin \omega t$

Find $i(t)$ for $t > 0$

(6 combinations)

$V = V_m \cos(\omega t \pm \theta)$

$\downarrow i_L(0^-) = 0$

$$V_m \sin \omega t = R i(t) + L \frac{di}{dt}$$

$$\left(D + \frac{R}{L}\right) i = \frac{V_m}{L} \sin \omega t$$

$$i(t) = i_c(t) + i_p(t) = K e^{-t/\tau}$$

Let $i_p = A \sin \omega t + B \cos \omega t$

$$V_m \sin \omega t = R[A \sin \omega t + B \cos \omega t] + L[A \omega \cos \omega t - B \omega \sin \omega t]$$

$V_m = AR - B\omega L \rightarrow (1) \times R$

$0 = RB + A\omega L \rightarrow (2) \times \omega L$

Solve for $A =$

$B =$

$R V_m = AR^2 - BR\omega L$

$0 = BR\omega L + A\omega^2 L^2$

$R V_m = A(R^2 + \omega^2 L^2)$

$$\boxed{A = \frac{R V_m}{R^2 + \omega^2 L^2}}$$

$V_m \omega L = AR\omega L - B\omega^2 L^2$

$0 = -R^2 B + AR\omega L$

$V_m \omega L = -B(R^2 + \omega^2 L^2)$

$$\boxed{B = -\frac{\omega L V_m}{R^2 + \omega^2 L^2}}$$

$\Rightarrow i_p = A \sin \omega t + B \cos \omega t = p \sin(\omega t + b)$

$i_p = p[\sin \omega t \cos b + \cos \omega t \sin b]$

Here $p \cos b = A$; $p \sin b = B$.

$$P = \sqrt{A^2 + B^2} = \sqrt{\frac{(R^2 + \omega^2 L^2) V_m^2}{(R^2 + \omega^2 L^2)^2}}$$

$$\boxed{P = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}}$$

$$b = \tan^{-1}\left(\frac{B}{A}\right) \Rightarrow \boxed{b = \tan^{-1}\left(\frac{2\omega L}{R}\right)}$$

$$\Rightarrow i(t) = Ke^{-t/\tau} + \frac{V_m \sin\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$i(0^-) = 0 = K \times 1 + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$K = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\tan^{-1}\left(\frac{\omega L}{R}\right)\right) e^{-t/\tau} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

i(t) using Laplace transforms.

$$V_m \sin \omega t = R\, i(t) + L\frac{di}{dt}$$

Laplace Transforms

$$\frac{V_m \omega}{\omega^2 L^2 + s^2} = R I(s) + L s I(s)$$

$$I(s) = \frac{V_m \cdot \omega}{(s^2+\omega^2L^2)(R+Ls)} =$$

$$I(s) = \frac{A}{(Ls+R)} + \frac{Bs+C}{(s^2+\omega^2L^2)} = \frac{V_m \omega}{(s^2+\omega^2L^2)(R+Ls)}$$

$$A(s^2+\omega^2L^2) + (Ls+R)(Bs+C) = V_m \cdot \omega$$

$$s^2(A+BL) + s(CL+BR) + (A\omega^2 + RC) = V_m \cdot \omega.$$

$$A + BL = 0$$

$$B = -\frac{A}{L}$$

$$-\frac{A}{L} = -\frac{CL}{R}$$

$$AR = CL^2 \qquad \boxed{A = \frac{CL^2}{R}}$$

$$CL + BR = 0$$

$$B = -\frac{CL}{R}$$

$$A = \frac{CL}{R}$$

$$A\omega^2 + RC = V_m \omega$$

$$A = \frac{CL^2}{R}, \quad C = \frac{AR}{L^2}$$

$$B = -\frac{CL}{R}$$

$$\frac{CL^2}{R}\omega^2 + RC = V_m\omega$$

$$\boxed{C = \frac{V_m \cdot \omega R}{R^2+\omega^2L^2}}$$

$$A = \frac{V_m \cdot \omega R}{(R^2+\omega^2L^2)} \cdot \frac{L^2}{R} = \frac{V_m\omega L^2}{R^2+\omega^2L^2}$$

$$\boxed{A = \frac{V_m\omega L^2}{R^2+\omega^2L^2}}$$

$$B = \left(\frac{-V_m\omega R}{R^2+\omega^2L^2}\right)\frac{L}{R} \Rightarrow \boxed{B = \frac{-V_m\omega L}{R^2+\omega^2L^2}}$$

$$I(s) = \frac{V_m\omega L^2}{(R^2+\omega^2L^2)(Ls+R)} + \frac{\left(\frac{-V_m\omega L}{R^2+\omega^2L^2}\right)s + \left(\frac{V_m\omega R}{R^2+\omega^2L^2}\right)}{s^2+\omega^2}$$

Inverse Laplace transform

$$i(t) = \frac{V_m\omega L^2}{(R^2+\omega^2L^2)}\cdot\frac{1}{L}e^{-\frac{R}{L}} + \left[\left(\frac{-V_m\omega L}{R^2+\omega^2L^2}\right)\frac{s}{s^2+\omega^2} + \frac{V_m\omega R}{R^2+\omega^2L^2}\times\frac{1}{s^2+\omega^2}\right]$$

$$i(t) = \frac{V_m\omega L}{R^2+\omega^2L^2}e^{-\frac{Rt}{L}} - \frac{V_m\omega L}{R^2+\omega^2L^2}\cos\omega t + \frac{V_mR}{(R^2+\omega^2L^2)}\sin\omega t$$

$$i(t) = \frac{V_m\omega L e^{-\frac{Rt}{L}}}{(R^2+\omega^2L^2)} - \frac{V_m\omega L}{(R^2+\omega^2L^2)}\cos\omega t + \frac{V_mR\sin\omega t}{R^2+\omega^2L^2}$$

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\frac{\omega L}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left(\sin\omega t\cos\left(\tan^{-1}\left(\frac{\omega L}{R}\right)\right) - \cos\omega t\sin\left(\tan^{-1}\left(\frac{\omega L}{R}\right)\right)\right)$$

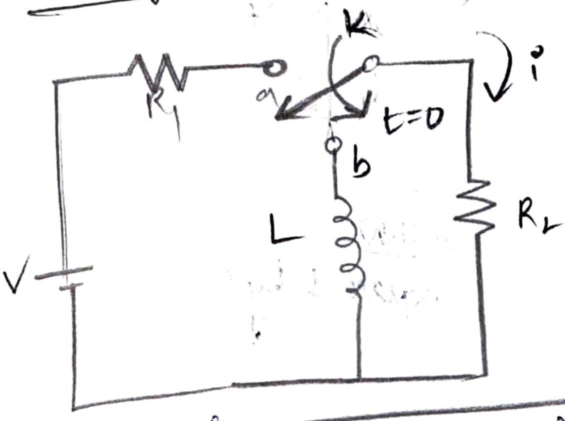
$$i(t) = \frac{V_m\omega L}{R^2+\omega^2L^2} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left(\sin\omega t\frac{R}{\sqrt{\omega^2L^2+R^2}}\right) - \cos\omega t\left(\frac{\omega L}{\sqrt{R^2+\omega^2L^2}}\right)$$

$$i(t) = \frac{V_m\omega L}{R^2+\omega^2L^2} + \frac{V_m}{R^2+\omega^2L^2}(R\sin\omega t - \omega L\cos\omega t)$$

$$i(t) = \frac{V_m \omega L e^{-\frac{Rt}{L}}}{R^2+\omega^2 L^2} + \frac{V_m R \sin\omega t}{R^2+\omega^2L^2} - \frac{V_m \omega L \cos\omega t}{R^2+\omega^2 L^2}$$

28th August 2019

* Single pole double throw switch



Switch K connected to a, until it reaches steady state and moves to b at t=0. Find $i(t)$ for $t > -\infty$.

$-\infty$ to $0-$ (a)

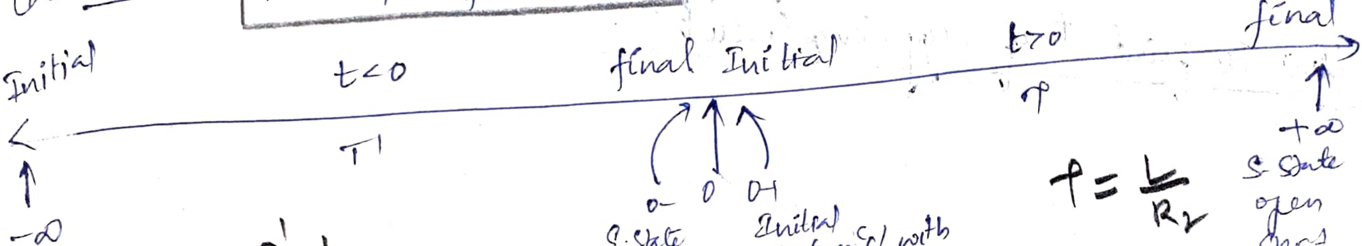
$0+$ to $+\infty$ (b)

$V_C(0+) = V_C(0-) = V_1$

$$\boxed{i(t) = i_f + (i_i - i_f)e^{-t/\tau}}$$

valid only to 1st order CKt

① Short-Cut $\boxed{i(t) = i_i + (i_f - i_i)\, e^{t/\tau'}}$



Initial
C ⟹ short
L ⟹ open

$\tau' = \frac{L}{R_1}$

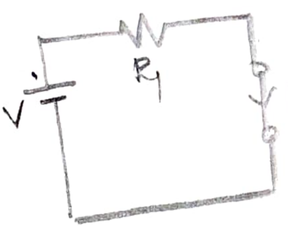
0- S. State open short

0+ Initial
Replace C with V.S $[V_C(0)]$
Replace L with I.S $[I_L(0^-)]$

$\tau = \frac{L}{R_2}$

$+\infty$ S. State open short

at $t = 0-$ (S.S)



at $t = 0+$

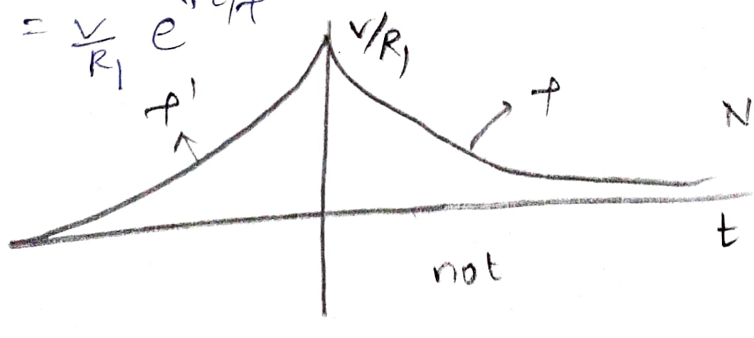
since $i(0+) = \frac{V}{R_1}$
[sudden change ~~of~~ current in L not allowed].

$i(\infty) = 0$.

$i(t) = 0 + \left(\frac{V}{R_1} - 0\right) e^{-t/\tau}$

$i(t) = \frac{V}{R_1} e^{-t/\tau}$

$$i(t) = i_i + (i_f - i_i)\, e^{t/\tau}$$
$$= 0 + \left(\frac{V}{R_1} - 0\right) e^{t/\tau}$$
$$= \frac{V}{R_1}\, e^{+t/\tau'}$$



Need not be symmetrical!

Mesh Analysis

$L\frac{di}{dt} + R_2 i = 0$

② Mathematical Approach

i

③ Laplace Transform

$\left(D + \frac{R_2}{L}\right) i = 0.$

$i = K e^{-t/\tau}$ where $\tau = \frac{L}{R_2}$

$i(0+) = i(0-) = \frac{V}{R_1} = K \times 1$

$\boxed{K = \frac{V}{R_1}}$

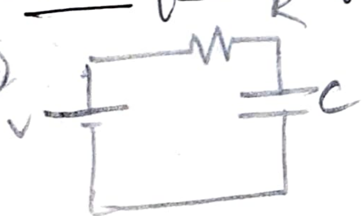
$i = \frac{V}{R_1} e^{-t/\tau}$

☆ H.W
Replace L by C

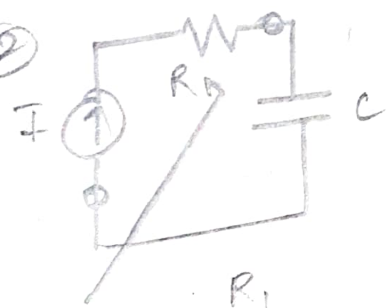
How to find τ of any given 1st order circuit?

☆☆ $\tau = R_{T(eq)} C_{T(eq)}$

source
V → short
I → Open

① 

$\tau = RC$

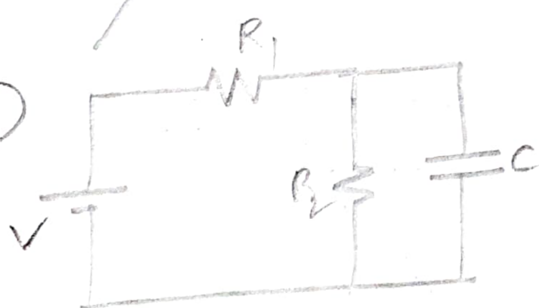
② 

$\tau = RC$

$\tau = \infty \times C$

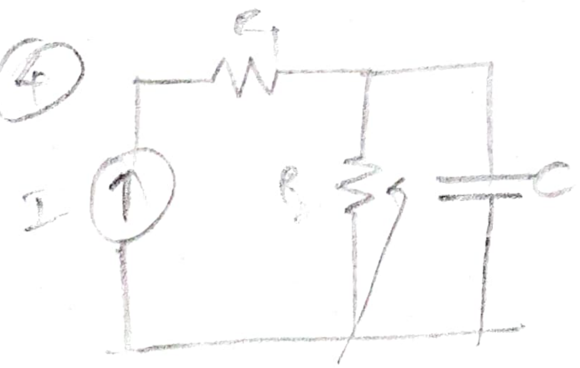
$\tau = \infty$, $\Rightarrow \tau = \infty$, $t_r = \infty$, Takes Infinite time to reach steady state

R equivalent looking from C

③ 

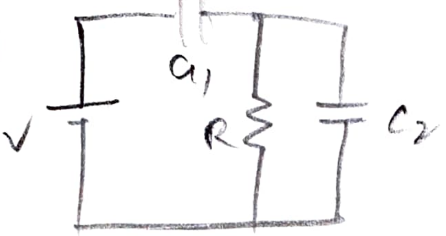
$\tau = (R_1 \| R_2)\, C$

$\tau = \left(\frac{R_1 R_2}{R_1 + R_2}\right) C$

④ 

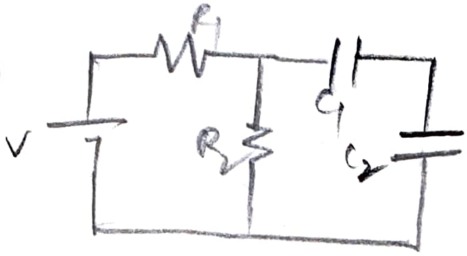
$\tau = R_2 C$

(5)



$\tau = R(C_1 + C_2)$

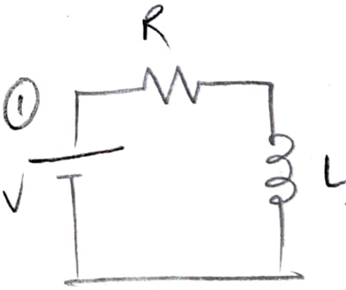
(6)



$\tau = (R_1 \| R_2) \dfrac{C_1 C_2}{C_1 + C_2}$

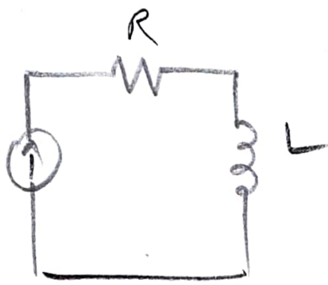
* H.W
Replace C by L
Find τ.

(1)



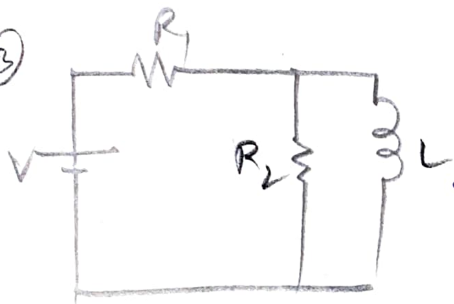
$\tau = \dfrac{L}{R}$

(2)



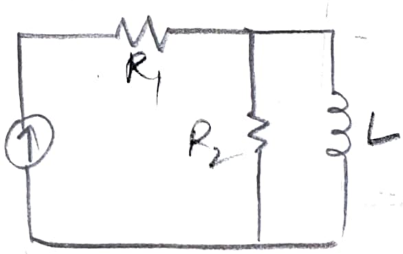
$\tau = \dfrac{L}{R} = \dfrac{L}{\infty} = 0$

(3)



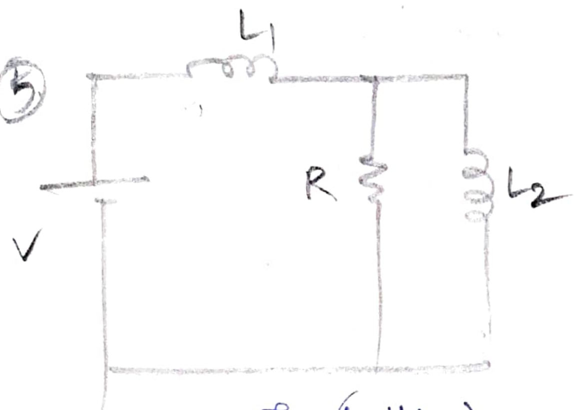
$\tau = \dfrac{L}{R_1 \| R_2}$

(4)



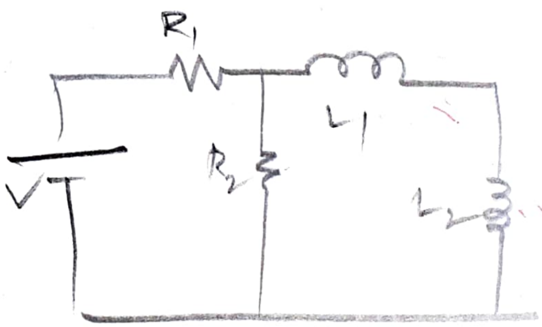
$\tau = \dfrac{L}{R_2}$

(5)



$\tau = \dfrac{(L_1 \| L_2)}{R}$

(6)



$\tau = \dfrac{(L_1 + L_2)}{(R_1 \| R_2)}$