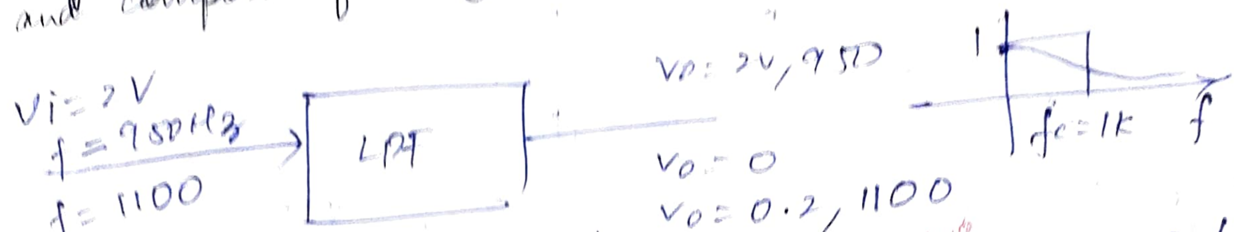
31st October, 2017

UNIT 5

FILTERS:

Ryder.

- Image Impedance
- Iterative impedance of T, π, L Sections (1)
- Characteristic impedance
- Image transfer constants
- Filter fundamentals.
- design of LP, HP and BP Filters using constant k, m-derived and composite filters. (1)



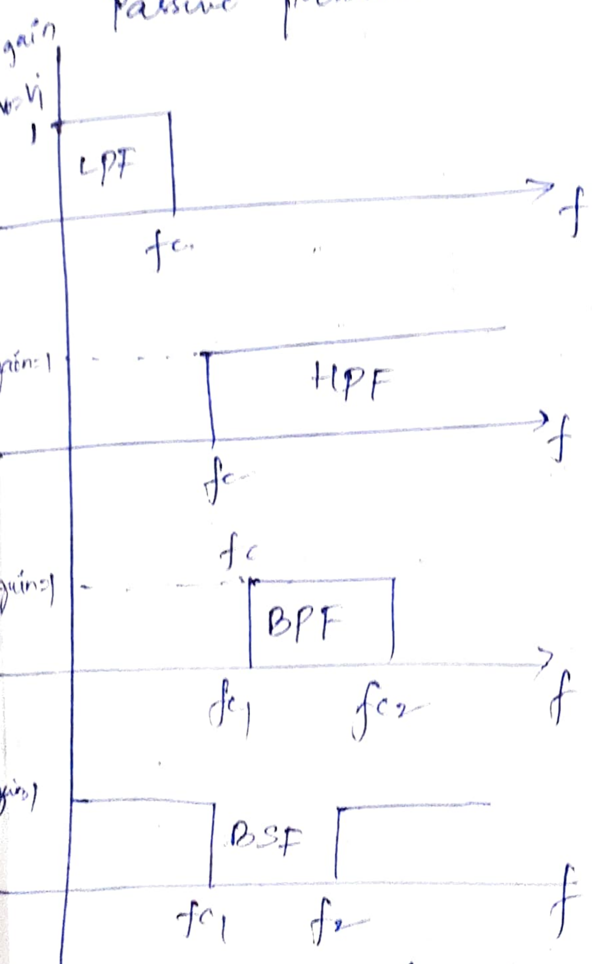
$\frac{Vo}{Vi} = gain$

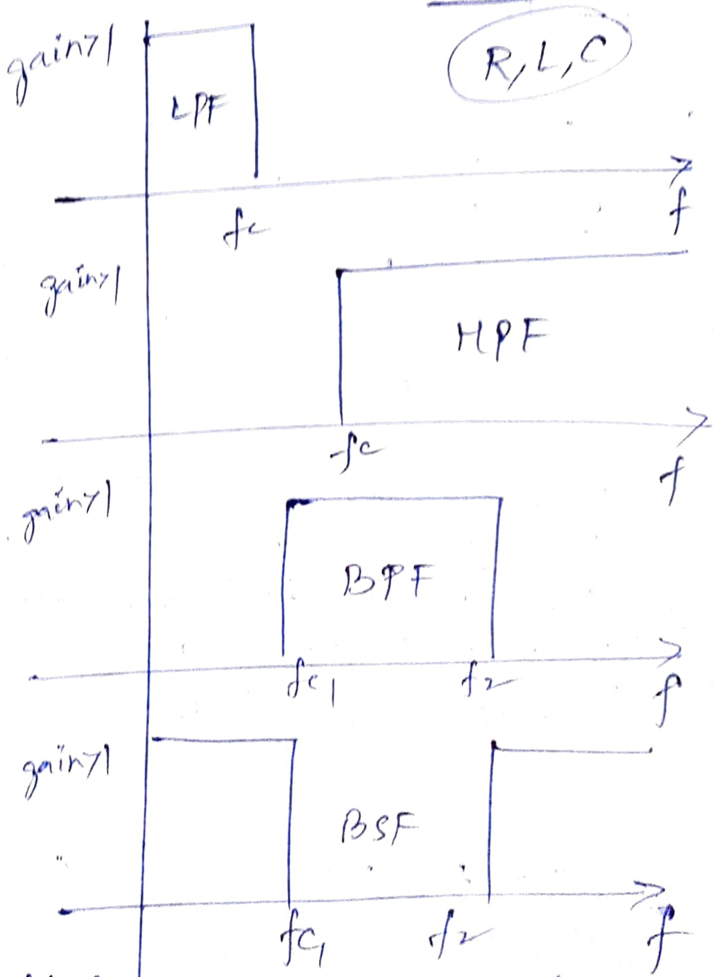
Real Domain / Time Domain

(R, L, C)

Passive   passive

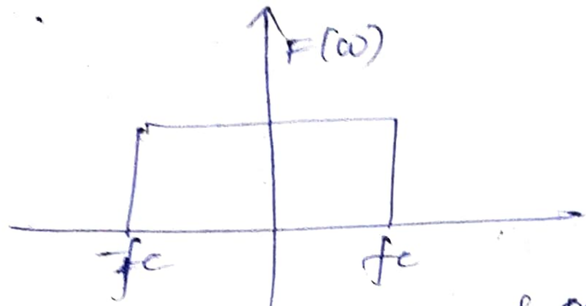
Active





Frequency Domain :- Theoretical.
Time " :- Practically possible.

Bandwidth $= fc_2 - fc_1$

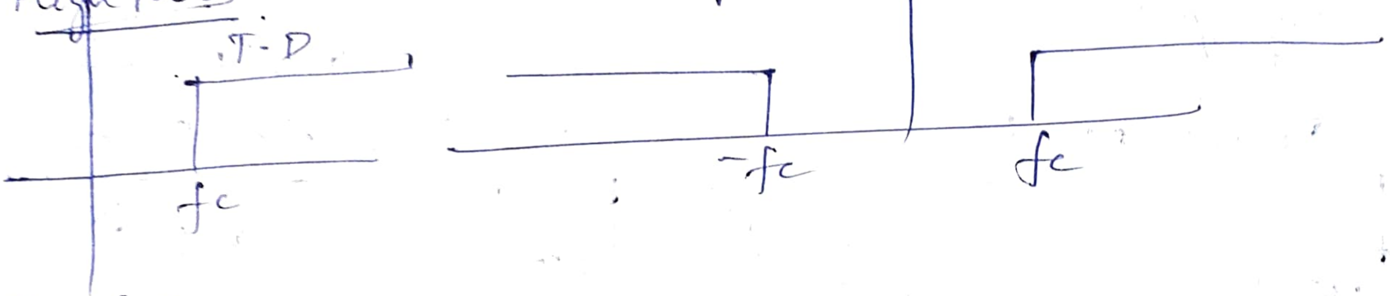


In Frequency, Bandwidth is $A * f_c$. only +ve frequencies should be taken as bandwidth

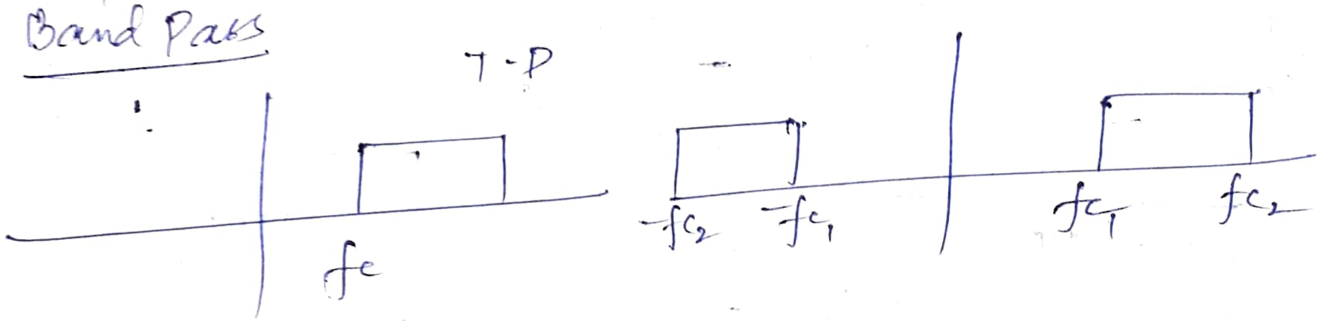
Practically it is not possible to obtain -ve frequencies

Low Pass

High Pass



Band Pass



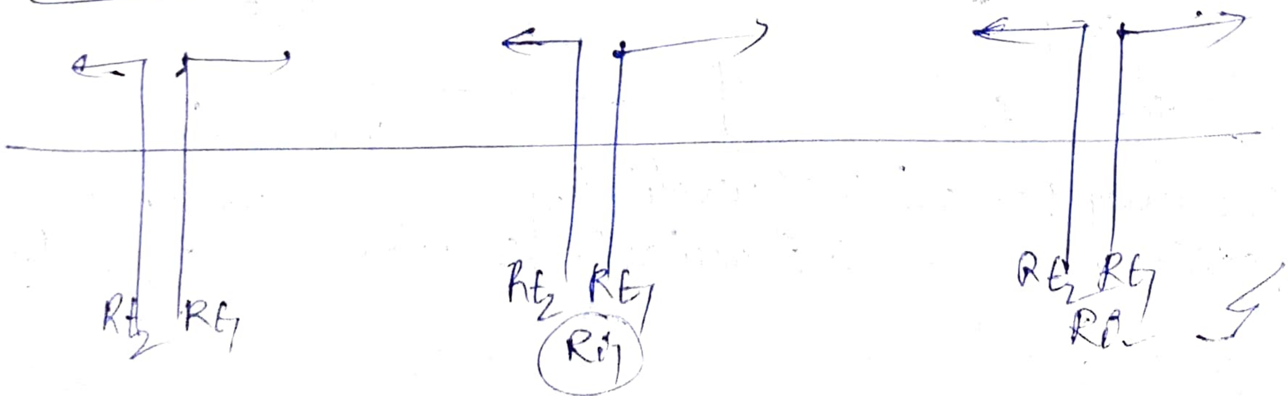
Asymmetrical N/w T, π, L

Symmetrical N/w T, π, L

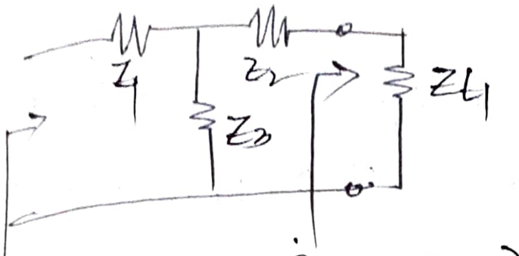
↳ Characteristic Impedance (If Iterative = image)

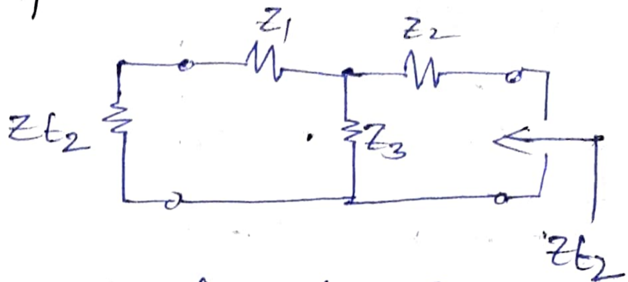
Iterative, Image Impedance.

↳ repetition of the Impedance



Assy. T. N/w: Iterative Imp.





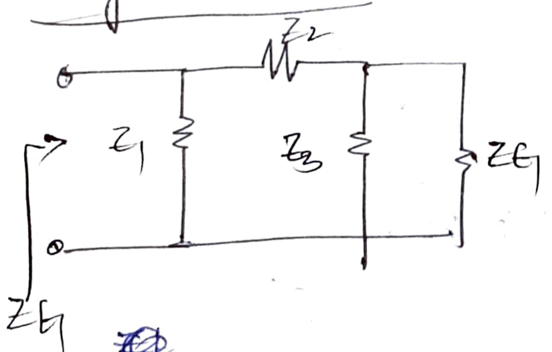
$$Z_{t_1} = Z_1 + Z_3 \,||\, (Z_2 + Z_{t_1}) \quad \text{— (1)}$$ solve for $Z_{t_1} = ?$

$$Z_{t_1} = \left(\frac{Z_1 - Z_2}{2}\right) \pm \sqrt{\frac{(Z_1+Z_2)^2}{4} + Z_3(Z_1+Z_2)}$$

$$Z_{t_2} = \left(\frac{Z_2 - Z_1}{2}\right) \pm \sqrt{\frac{(Z_1+Z_2)^2}{4} + Z_3(Z_1+Z_3)}$$

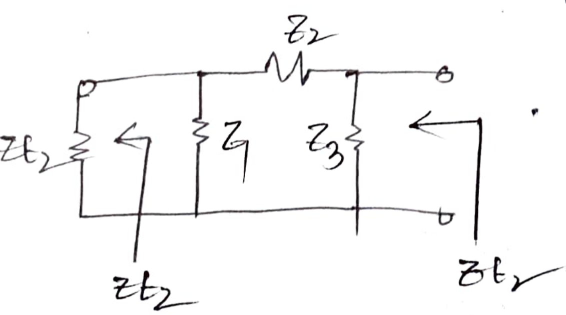
$$Z_{t_1} = Z_{t_2} = Z_2 + Z_3 \,||\, (Z_1 + Z_{t_2})$$

Assy. π. N/w:



$$Z_{t_1} = Z_1 \,||\, [Z_2 + (Z_3 \,||\, Z_{t_1})]$$

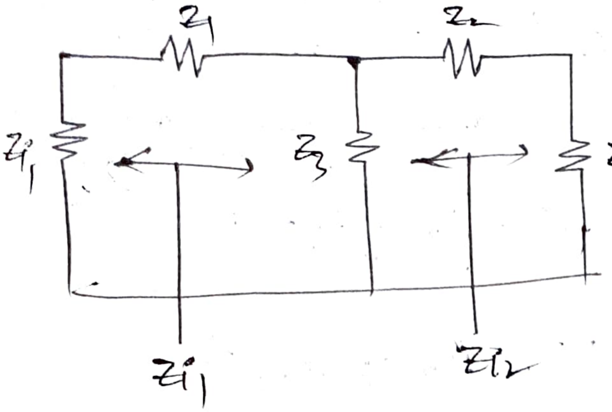
$$Z_{t_1} = \frac{Z_1 Z_2 - Z_2 Z_3 \pm \sqrt{(Z_1 Z_2 - Z_2 Z_3)^2 + 4 Z_1 Z_2 Z_3 (Z_1 + Z_2 + Z_3)}}{2(Z_1 + Z_2 + Z_3)}$$



$$Z_{t_2} = Z_3 \,||\, [Z_2 + (Z_1 \,||\, Z_{t_2})]$$

$$Z_{t_2} = \frac{(Z_2 Z_3 - Z_1 Z_2) \pm \sqrt{(Z_1 Z_2 - Z_2 Z_3)^2 + 4 Z_1 Z_2 Z_3 (Z_1 + Z_2 + Z_3)}}{2(Z_1 + Z_2 + Z_3)}$$

Image impl: to Asy. T N/W



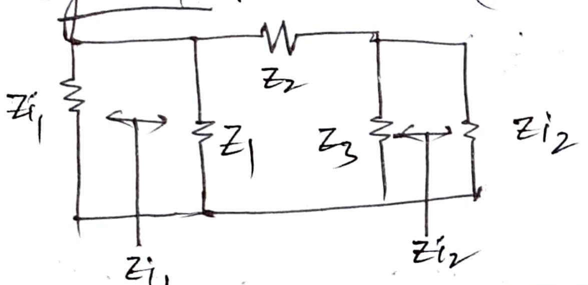
$$Z_{i1} = Z_1 + [Z_3 \,||\, (Z_2 + Z_{i2})] \quad (1)$$

$$Z_{i2} = Z_2 + [Z_3 \,||\, (Z_1 + Z_{i1})] \quad (2)$$

Solve for $Z_{i1} = \sqrt{(Z_1+Z_3)(Z_1+Z_2||Z_3)}$

$Z_{i2} = \sqrt{(Z_2+Z_3)(Z_2+Z_1||Z_3)}$

Asy. π N/W.



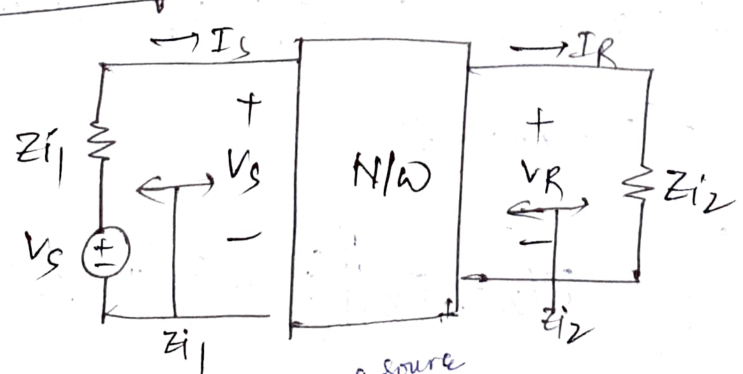
$$Z_{i1} = Z_1 \,||\, \{Z_2 + (Z_3 || Z_{i2})\} \quad (1)$$

$$Z_{i2} = Z_3 \,||\, \{Z_2 + (Z_1 || Z_{i1})\} \quad (2)$$

Solve for $Z_{i1} = \sqrt{Z_1||(Z_2+Z_3)(Z_1||Z_2)}$

$Z_{i2} = \sqrt{Z_3||(Z_2+Z_1)(Z_2||Z_3)}$

Image transfer const = θ_i



$$\theta_i = \frac{1}{2}\log_e\left(\frac{V_S I_S}{V_R I_R}\right) = \frac{1}{2}\log_e \frac{Z_{i1} \cdot I_S^2}{Z_{i2} I_R^2}$$

S-source, R-reciever

$$= \log_e \sqrt{\frac{Z_{i1}}{Z_{i2}}} \cdot \frac{I_S}{I_R}$$

For symmetrical Network $Z_{i1} = Z_{i2}$

$$\theta_i = \log_e \frac{I_S}{I_R}$$

$$N = e^{\theta_i} = \frac{I_S}{I_R} = e^{\gamma} = e^{(\alpha + j\beta)}$$

$$\ln N = \ln\left(\frac{I_S}{I_R}\right) = \alpha$$

γ = propagation constant
α = Attenuation constant in Nepers.

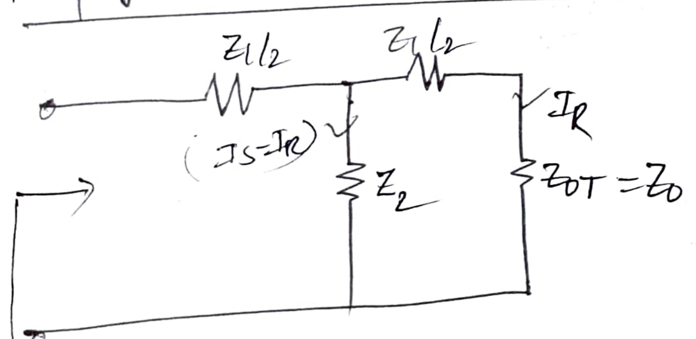
$$\text{Attenuation in dB} = 10 \log_{10} \frac{P_S}{P_R}$$

$$= 20 \log_{10} \frac{I_S}{I_R}$$

$$= 2 \log_{10} e \; \log_e \frac{I_S}{I_R}$$

$$\boxed{\text{Attenuation in dB} = 8.686 [\text{Attenuation in Nepers}]}$$

Propagation const. for sym/: T. Nw



$\to$ Characteristic Impedance

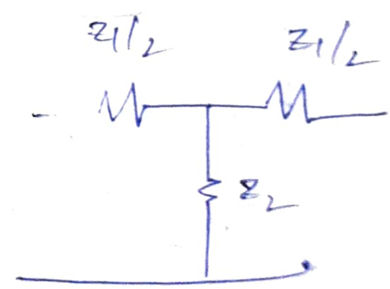
$$Z_{0T} + Z_2(I_S - I_R) = I_R\left(\frac{Z_1}{2} + Z_0\right)$$

$$Z_2 I_S = I_R\left[\frac{Z_1}{2} + Z_0 + Z_2\right]$$

$$\frac{I_S}{I_R} = e^{\gamma} = \frac{\frac{Z_1}{2} + Z_0 + Z_2}{Z_2}$$

$$Z_0 = Z_2(e^{\gamma} - 1) - \frac{Z_1}{2}$$

$$\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = Z_2(e^{\gamma} - 1) - \frac{Z_1}{2}$$



$$Z_{t1} = Z_{t2} = Z_{i1} = Z_{i2} = Z_0$$

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + Z_1/4Z_2}}$$

$$Z_{0T} = \sqrt{\left(\frac{Z_1}{2} + Z_2\right)\left(\frac{Z_1}{2} + \left(Z_2 \| \frac{Z_1}{2}\right)\right)}$$

$$= \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{Z_1 Z_2 + (Z_1^2/4)}$$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = Z_2^2(e^{2\gamma} + 1 - 2e^{\gamma}) + \frac{Z_1^2}{4} - Z_1 Z_2 (e^{\gamma} - 1)$$

$$\frac{e^{2\gamma} + 1 - 2e^{\gamma}}{e^{\gamma}} = \frac{Z_1}{Z_2}$$

$$e^{\gamma} + e^{-\gamma} - 2 = \frac{Z_1}{Z_2}$$

$$\frac{e^{\gamma} + e^{-\gamma}}{2} = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{1}{2}[\cosh\gamma - 1]}$$

$$\boxed{\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}}$$

$= \cancel{\sinh\frac{\gamma}{2}}$

06/11/2019

FILTER FUNDAMENTALS

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\left(\frac{\alpha + j\beta}{2}\right) = \sinh\frac{\alpha}{2}\cos\frac{\beta}{2} + j\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

I) If Z_1 & Z_2 are same type of element; $\frac{Z_1}{4Z_2} > 0$;

$\sqrt{\frac{Z_1}{4Z_2}}$ is real.

$\sqrt{\frac{Z_1}{4Z_2}}$ is real.

$\cosh\frac{\alpha}{2}\sin\beta = 0$

$\sin\beta/2 = 0$

$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

$\cos\beta/2 = 1$

if $\frac{\beta}{2} = n\pi$, $n = 0, 1, 2 \dots$

$\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

II) If Z_1 & Z_2 opposite type of elements

$\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = 0$

$\cosh\frac{\alpha}{2}\sin\beta/2 = \sqrt{\frac{Z_1}{4Z_2}}$

a) $\sinh\frac{\alpha}{2} = 0$

$\alpha = 0$

$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

(b) $\cos\beta/2 = 0$ $\sin\beta/2 = \pm 1$

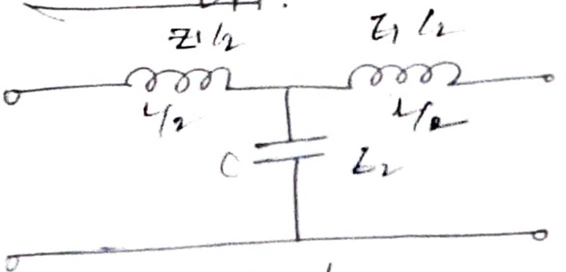
$\beta = (2n+1)\pi$

$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$

10m

$\frac{Z_1}{4Z_2}$	$+\infty$ to 0	0 to -1	-1 to $-\infty$
Reactance type	same	opp	oppo
Band	Stop	pass	stop
α	$2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	0	$2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$
β	π	$2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$	π

Const K-LPF:



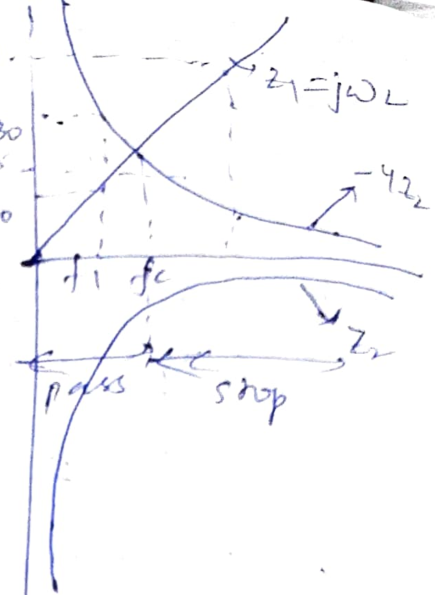
Sym. T N/W

$\frac{Z_1}{2} = j\omega\left(\frac{L}{2}\right)$

$\frac{Z_1}{4Z_2} = -1$

$Z_1 = -4Z_2$

$j\omega_c L = -4 \times \frac{1}{j\omega_c C}$



at f_1 $\frac{Z_1}{4Z_2} = \frac{j10}{-j30} = -\frac{1}{3}$ (0 to −1)

at $f_2 = \frac{Z_1}{4Z_2} = \frac{j40}{-j15} = -3.$ (−1 to −∞)

$4Z_2 = j\omega L \times \frac{1}{j\omega C} = \frac{L}{C} = R_K^2$ $\therefore \boxed{R_K = \sqrt{\frac{L}{C}}}$ ①

$R_L = R_0 = R_K = \sqrt{\frac{L}{C}}$

$\frac{Z_1}{4Z_2} = -1$

$Z_1 = -4Z_2$

$j\omega_c L = -4 \times \frac{1}{j\omega C}$

$j2\pi f_c L = \frac{-4 \times -j}{2\pi f_c C}$

R_K, f_c

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$

Ass-3

Q) Design a constant K-t and π type Low pass filter for the following specification Given f_c = cut off frequency 2KHz

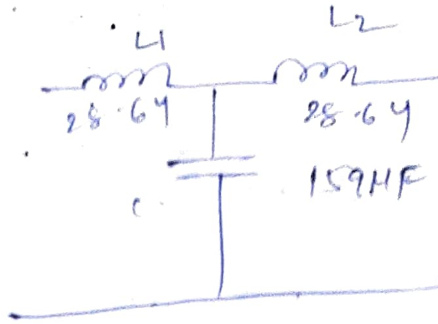
$R_0 = 600\Omega$

$\sqrt{\frac{L}{C}} = 600,\quad \frac{1}{\pi\sqrt{LC}} = 2 \times 10^3$

$\frac{1}{\pi C} = 2 \times 10^3 \Rightarrow C = \frac{1}{2\pi \times 10^3} = 1.59 \times 10^{-4} F$

$= 159 \mu F$

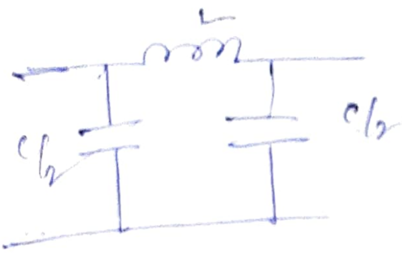
$\sqrt{\frac{L}{C}} = 600$

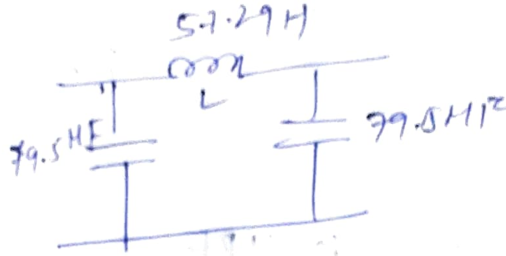


$\frac{L}{C} = 600\times600$

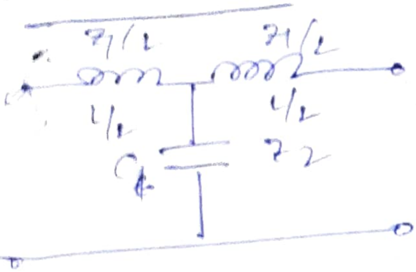
$L = 600\times600\times159\times10^{6}$

$L = 57.29H.$





Const-k-LPF



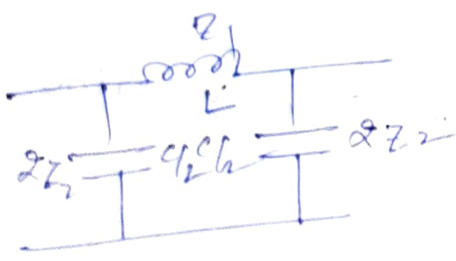
T-type

$Z_1 . Z_2 = \frac{L}{C} = R_k^2$

$R_k = \sqrt{\frac{L}{C}}$ ①

$Z_1 = -4Z_2$

$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$



$f_c = \frac{1}{\pi\sqrt{LC}}$ ②

$L =$

$C =$

$\pi^2 LC = \frac{1}{f_c^2}$

$\frac{Z_1}{4Z_2} = \frac{j\omega L}{4 * \frac{-1}{\omega C}} = \frac{-\omega^2 LC}{4}$

$= -\frac{(2\pi f)(2\pi f)LC}{4}$

$= -\frac{f^2}{f_c} = -\left(\frac{f}{f_c}\right)^2$

$\boxed{\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right)}$

$$Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{R_K^2 \left(1 - \frac{f}{f_c}\right)^2}$$

$$\frac{Z_{OT}}{R_K} = \sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

Ass-3
Q

(01) Design constant-K HPF [T & π] for following specifications

a) $f_c = 2k$

$R_0 = 600\Omega$

b) draw α, β; NOT (Normalised characteristic Impedance)

c) find α at 1KHz

β at 3.5 KHz

$Z_{0\pi}$ at 1.5K & 3K

Ass-3

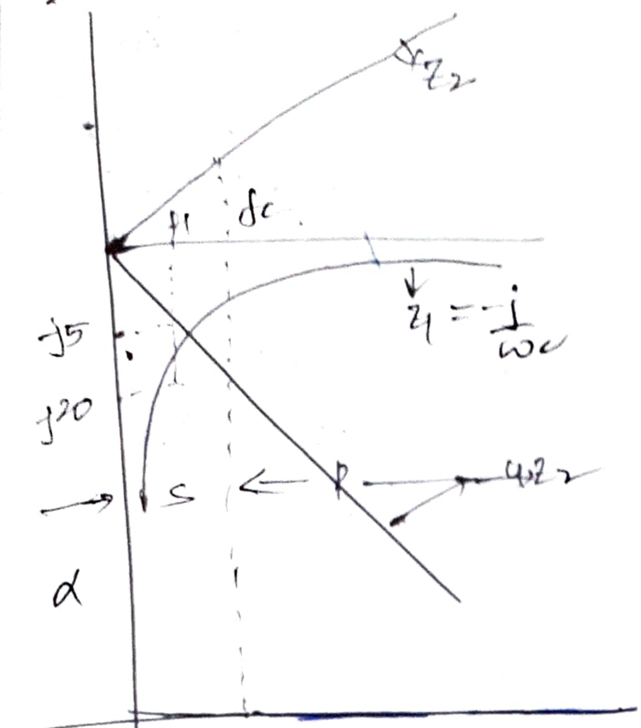
(b) Draw the α, β, Z_{OT} characteristic impedance variations in both the bands. (pass & stop)

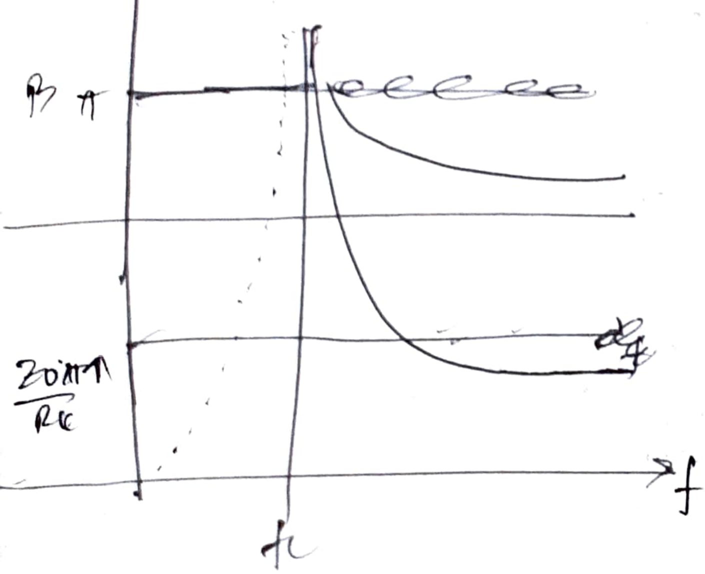
c) Find the value of α at $f = 3K$

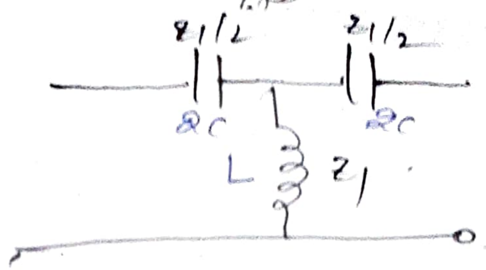
d) What is the value of phase constant at $f = 1.5K$

(e) Calculate β at $f = 1.5k$

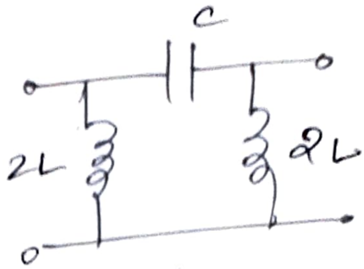
Const - k - HPF







$$\frac{Z_1}{2} = \frac{-j}{\frac{\omega C}{2}} = \frac{-j}{\omega(2C)}$$



(Π)

$Z_1 = -4Z_2$

$\frac{Z_1}{4Z_2} = 0$ to -1 pass.

-1 to $-\infty$ stop

$$\frac{Z_1}{4Z_2} = \frac{-j20}{j5} = -4$$

$$Z_1 Z_2 = \frac{L}{C} = R_K^2$$

$$R_K = \sqrt{\frac{L}{C}} \quad (1)$$

$$Z_1 = -4Z_2$$

$$\frac{-j}{\omega C} = -4 \times j\omega L$$

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} = \frac{\frac{-j}{\omega 2C}}{4 \times j\omega L}$$

$$= \frac{-1}{8\omega^2 LC}$$

$$= \frac{-1}{8 \times 2\pi f \times 2\pi f \times LC}$$

$= 16\pi^2$ [illegible]

$4\omega^2 LC = 1$

$\omega^2 = \frac{1}{4LC}$

$\omega = \frac{1}{2\sqrt{LC}}$

$f_c = \frac{1}{4\pi\sqrt{LC}}$

$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$ (2)

$\sqrt{LC} = \frac{1}{4\pi f_c}$

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

$$LC = \frac{1}{16\pi^2 f_c^2}$$

$$\frac{Z_1}{4Z_2} = \frac{\frac{-j}{\omega C}}{4j\omega L} = \frac{-1}{4\omega^2 LC} = \frac{-1}{\cancel{4}\omega^2} \times \cancel{16}^{\,2}\pi^2 f_c^2$$

$$= \frac{-\cancel{2}\times\cancel{\pi^2} f_c^2}{\cancel{2\pi} f \times \cancel{2\pi} f}$$

$$= \frac{-f_c^2}{f^2}$$

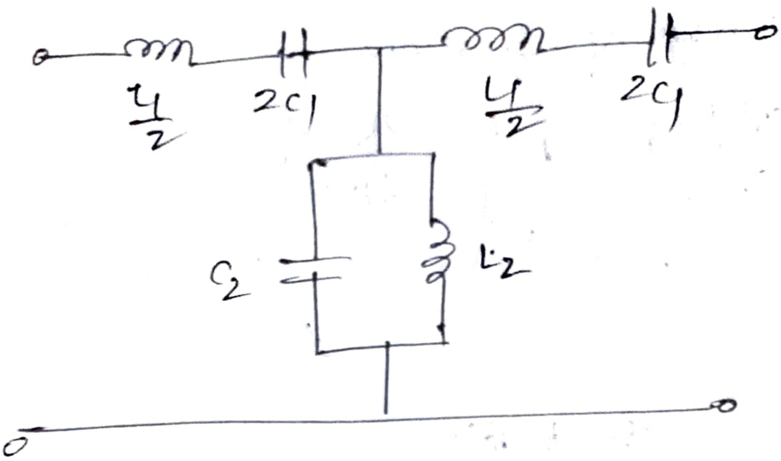
$$2\cosh^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) = 2\cosh^{-1}\left(\sqrt{\frac{-f_c^2}{f^2}}\right)$$

$$= 2\cosh^{-1}\left(\frac{f_c}{f}\right)$$

$$\frac{Z_{0T}}{R_K} = \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$Z_{0T} = \sqrt{\frac{Z_1 Z_2}{\left(1+\frac{Z_1}{4Z_2}\right)}}$$

const-k-BPF:



$f_s = f_p$

$\omega_{0s} = \frac{1}{\sqrt{L_1 C_1}}$

$\omega_{0p} = \frac{1}{\sqrt{L_2 C_2}}$

① $\boxed{L_1 C_1 = L_2 C_2}$

② $Z_1 Z_2 = \left(j\omega L_1 - \frac{j}{\omega C_1}\right)\left(j\omega L_2 \parallel \frac{-j}{\omega C_2}\right)$

$\boxed{\frac{L_2}{C_1} = \frac{L_1}{C_2} = R_k^2}$

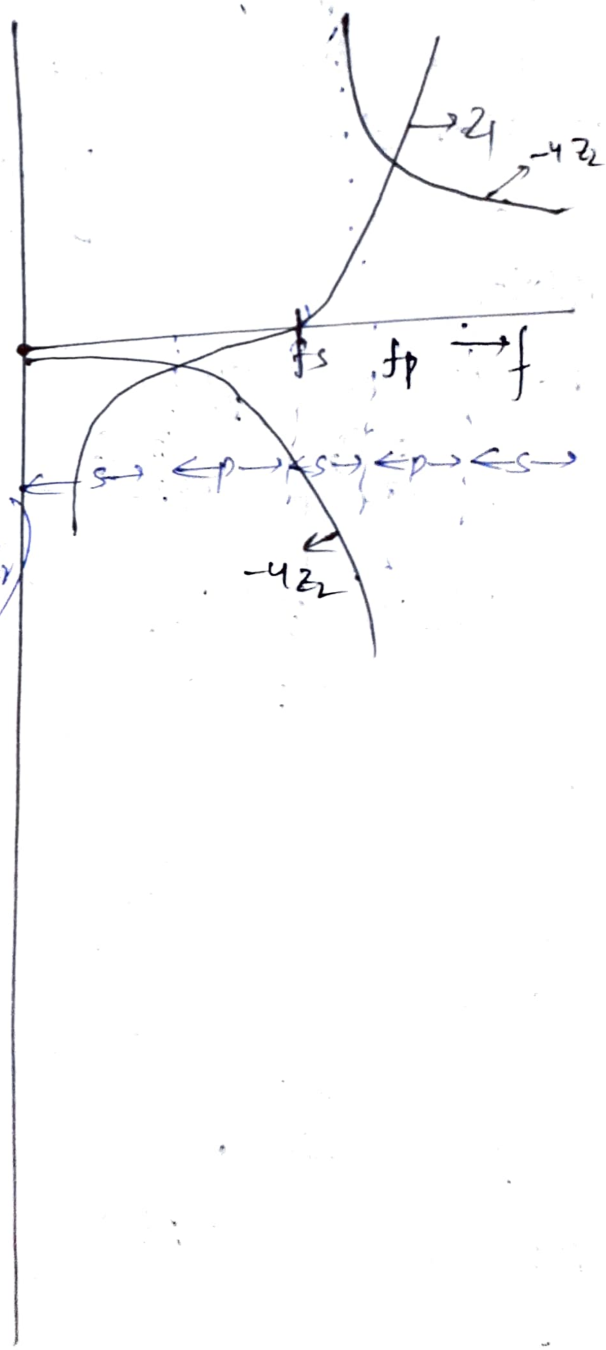
③ $f_0 = \sqrt{f_1 f_2}$

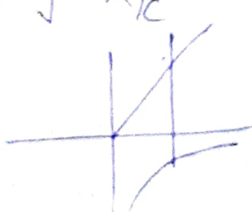
$Z_1 = -4Z_2$

$Z_1^2 = -4R_k^2$

$Z_1 = \pm j\,2R_k$

$j\omega_1 L_1 - \frac{j}{\omega_1 C_1} = -j\,2R_k$





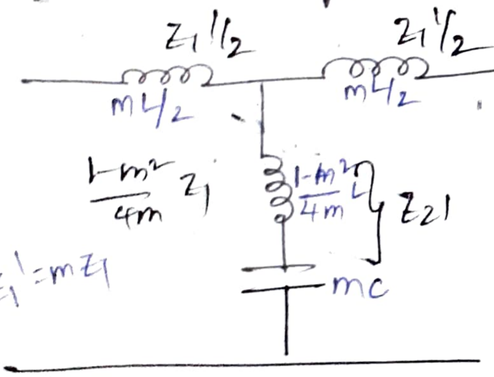
Q) Design a constant k band pass filter, T and π type
(1) for the following specifications
(i) Lower cut off frequency, $f_{c1} = 2K\ Hz$
(ii) Higher cut off frequency, $f_2 = 4K\ Hz$
(iii) Characteristic resistance $(K_0) = R_k = 500\Omega$

theory
Disadvantage of constant-K:

(1) Attenuation constant varying slowly in stop band.
(2) Characteristic impedance is varying in pass band due to which impedance matching not done

13/11/2019

M-derived T-type



at f_∞

$$\left|\frac{1-m^2}{4m} Z_1\right| = \left|\frac{Z_2}{m}\right|$$

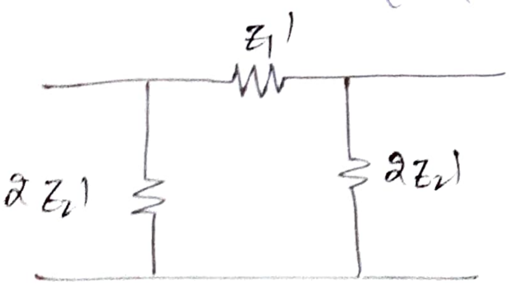
$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} \text{ for LPF}$$

$$f_\infty = f_c\sqrt{1-m^2} \text{ for HPF}$$

$Z_1 = j2\pi f_\infty L$
$Z_2 = \frac{1}{j2\pi f_\infty C}$
$f_c = \frac{1}{\pi\sqrt{LC}}$

$$Z_{OT} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{mZ_1 Z_2'\left(1 + \frac{mZ_1}{4Z_2'}\right)}$$

Solve for $Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m}$.



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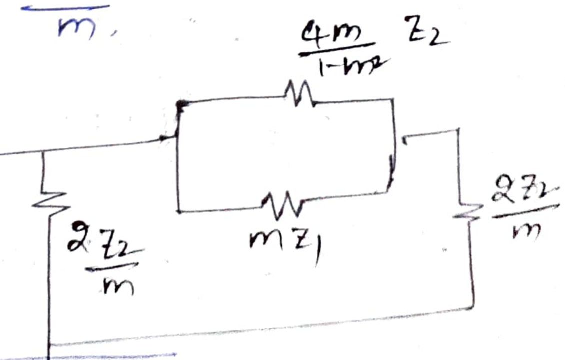


$$Z_{O\pi} = \sqrt{\frac{Z_1 Z_2}{1 + Z_1/4Z_2}} = \sqrt{\frac{Z_1' Z_2/m}{1 + \frac{Z_1'}{4Z_2/m}}}$$

Solve for $Z_1' = \dfrac{1}{\dfrac{1}{mZ_1} + \dfrac{1}{\left(\dfrac{4m}{1-m^2}\right) Z_2}}$

① $f_c = \dfrac{1}{\pi\sqrt{LC}}$

② $R_0 = \sqrt{\dfrac{L}{C}}$

③ $f_\infty = \dfrac{f_c}{\sqrt{1-m^2}}$

how to design
design m- derived T-type.

Theory of

Disadvantage of m-derived:
(i) characteristic impedance is still changing in m-derived in pass band.
It can be eliminated by taking $m = 0.6$ and bisection

Q) Design a composite LPF T-type filter with the following specification.

(i) $f_c = 2K$, (ii) $R_0 = 600\Omega$, (iii) $f_\infty = 2.2K$

Composite LPF T-type
" π
HPF T
" π

Constant LPF – T/π
HPF – T/π
BP – T/π

$f_c = \dfrac{1}{\pi\sqrt{LC}} = 2\times10^3$

$R_0 = \sqrt{\dfrac{L}{C}} = 600$

$\dfrac{1}{\pi C} = 2\times10^3 \times 600$

$C = \dfrac{1}{\pi\times12\times10^5} = 0.26\ \mu F$

$\sqrt{\dfrac{L}{C}} = 600$

$\dfrac{L}{C} = 36\times10^4$

$L = 0.26\times10^{-6}\times36\times10^4$
$= 9.36\times10^{-2}$
$L = 93.6\times10^{-3} = 93.6\ mF$

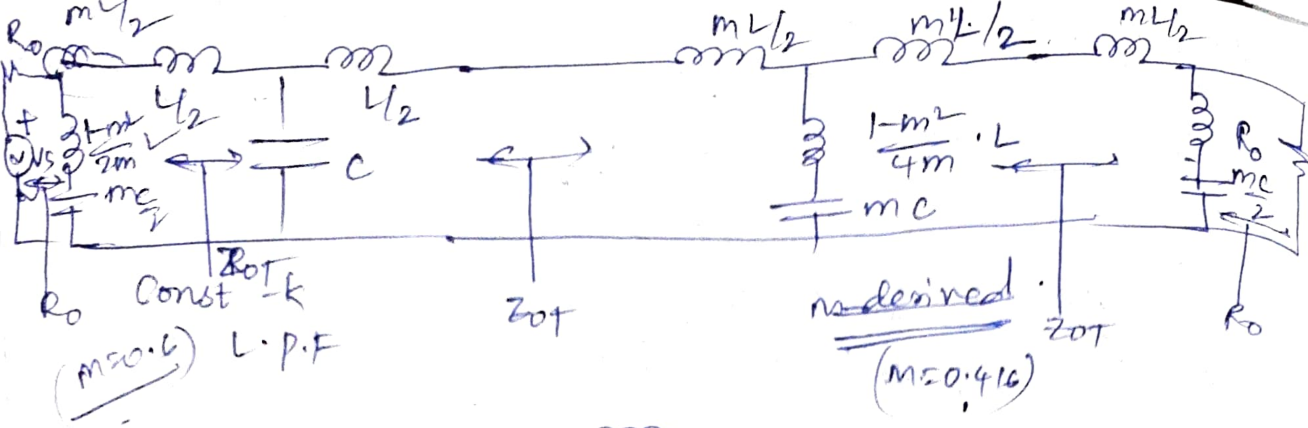
$f_\infty = \dfrac{f_c}{\sqrt{1-m^2}}$

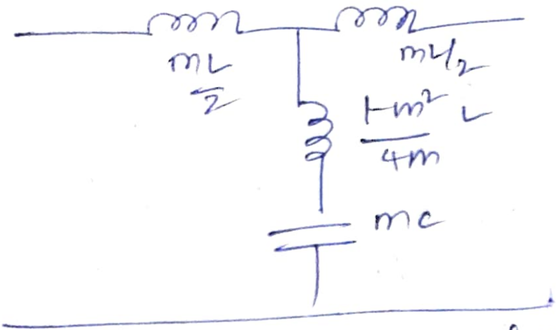
$2.2\times10^3 = \dfrac{2\times10^3}{\sqrt{1-m^2}}$

$\sqrt{1-m^2} = \dfrac{1}{1.1}$

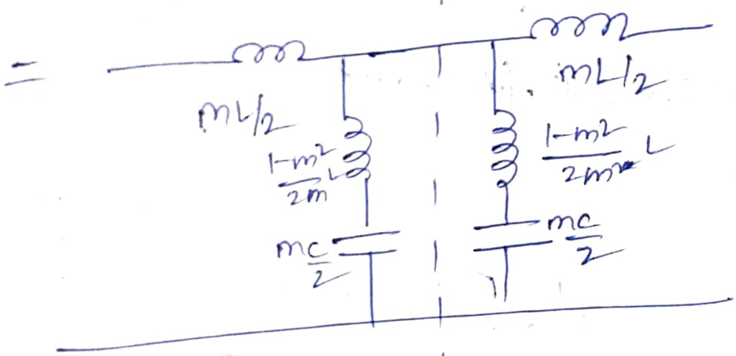
$\sqrt{1-m^2} = 0.8264$

$m^2 = \dfrac{21}{121} = \dfrac{\sqrt{21}}{11} = 0.416.$





With m=0.6) ← compulsory → characteristic impedance will match.



$$\frac{\frac{L}{2}\times\frac{L}{2}}{L} \quad \frac{L}{4}$$