

Tutorial-1

Q1. What do you understand by the Asymptotic Notations. Define Different Asymptotic Notation with example.

→ Asymptotic Notations are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value/limiting value.

Using these analysis we can very well conclude best case, average case, and worst case of an algorithm.

If is input bound, if there is no i/p it will work in constant time.

Types -

① O Notation → Big O Notation represents the upper bound of the running time of an algorithm.

→ It gives the worst case complexity of an algorithm.

→ It tells us the no of operation an algorithm will make.

→ It tells us how fast an algorithm grows and let us compare it with others.

Omega Notation - It represents the lower bound of the running time of an algorithm.
→ It provides the best case complexity.

Example - If $T_c = 3n^2 + 5n$ for any algorithm, according to

→ It always indicates the minimum time required for any algorithm

Theta Notation - It represents the upper and lower bound of the running time.

→ It is used for analyzing the average case complexity.

Example: $T_c = 3n^2 + 5n$

$\Theta(n^2) = \Theta(n^2)$

→ Ignoring constant and remaining part

→ $k_1 + n^2 < n < k_2 + n^2$

$k_1, k_2 = \text{constants.}$

Q2. What should be the TC of
for $(i=1 \text{ to } n)$
 $\{ i = i + 2; \}$

$i=1;$

$i = 1, 2, 4, 8, \dots, n$

$i = 2^0, 2^1, 2^2, 2^3, \dots, 2^k$

$n = 2^k$

$k = \log n$

$T_n = O(\log n)$

3. $T(n) = \begin{cases} 3T(n-1) & \text{if } n > 0 \\ \text{else } 1. \end{cases}$

$T(n) = 3T(n-1) = 3T(n-1)$

$= 3(3T(n-1-1)) = 3(3T(n-2))$

$= 3^2 T(n-2)$

$= 3^3 T(n-3)$

\dots

$= 3^n T(n-n)$

$= 3^n T(0)$

$= 3^n \cdot 1$

$= 3^n$

(if $n \leq 0$ then 1)

→ $O(3^n)$

4. $T(n) = \begin{cases} 2T(n-1) - 1 & \text{if } n > 0 \\ T(n) = 1 & \text{if } n \leq 0 \end{cases}$

$T(n) = 2T(n-1) - 1$

$= 2(2T(n-2) - 1) - 1$

$= 2^2 T(n-2) - 2 - 1$

$$\begin{aligned}
 &= 2^2(2T(n-3)-1)-2-1 \\
 &= 2^3(2T(n-3)-2^2-2^1-2^0) \\
 &\vdots \\
 &= 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3} \dots - 2^0 \\
 &= 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} \dots - 2^0 \\
 &= 2^n - (2^n - 1) \\
 &= 2^n - 2^n + 1 \\
 &= 1
 \end{aligned}$$

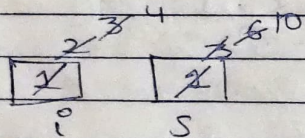
→ $O(1)$

5. What should be the TC of :-

```

int s=1, i=1;
while (s <= n) {
    i++;
    s = s+i;
    printf("%d\n", i);
}

```



$$s = 1 + 3 + 6 + 10 + \dots + k = n$$

$$\frac{k(k+1)(k+2)}{6} = n$$

$$O(k^3) = n$$

$$\begin{aligned}
 k &= \sqrt[3]{n} \\
 &= O(\sqrt[3]{n})
 \end{aligned}$$

Q6. void function (int n)
{
 int i, count=0;
 for (i=1; i*i <= n; i++)
 count++;
}

i	count
1	1
2	1
3	2
4	2

$$n = 10$$

$$4 \times n = 10 = 13$$

$$n = 20 = 4$$

$$n = 25 = 5$$

$$= O(\sqrt{n})$$

Q7. void function (int n) {
 int i, j, k, count=0;
 for (i=1; i*i <= n; i++)
 for (j=1; j*j <= n; j++)
 for (k=1; k*k <= n; k++)
 count++;
}

3

i will execute $n/2$ times = n times

since $j = j \times 2$, and $k = k \times 2$, both are increasing exponentially upto n so

$$\begin{aligned}
 TC &= O(\log n \times \log n \times \log n) = \\
 &= O(n \log^2 n)
 \end{aligned}$$

Q8. function (int n) {

if (n == 1)

return

for (i = 1; i < n; i++) {

for (j = 1; j < n; j++) {

printf(" + ");

}

}

function(n-3)

}

i-loop n times = $O(n^2)$

j-loop n times

Q9. void function (int n) {

for (i = 1 to n) {

for (j = 1; j < n; j = j + i)

printf(" + ");

}

}

Inner loop is dependent upon i

i = 1 \Rightarrow j = 1..2..3..4..5..6..7..8...n

i = 2 \Rightarrow j = 1..3..5..7..9..11..13...n/2

i = 3 \Rightarrow j = 1..4..7..10..13... n/3

$T(n) = n + n/2 + n/3 + \dots$

$O(n \log n)$