AI1103-Assignment 3

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QUESTION

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Let X have a binomial distribution with parameters, n and p, n=3. For testing the hypothesis H_0: p = $\frac{2}{3}$ against H_1: p = $\frac{1}{3}$, let a test be: "Reject H_0 if $X \geq 2$ and accept H_0 if $X \leq 1$."

Then the probabilities of Type-1 and Type-2 errors respectively are

1) $\frac{20}{27}$ and $\frac{20}{27}$ 3) $\frac{20}{27}$ and $\frac{7}{27}$

2) $\frac{7}{27}$ and $\frac{20}{27}$ 4) $\frac{7}{27}$ and $\frac{7}{27}$

SOLUTION

Clearly, H_0 and H_1 are mutually exclusive hypothesis.

For Type-1 error: Occurs when one rejects the null hypothesis(H_0) when it is true

$$P(X \geq 2) = P(X = 2) + P(X = 3) \qquad (0.0.1)$$

$$= {}^3C_2 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right)^1 + {}^3C_3 \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^0$$
$$(0.0.2)$$

$$= \frac{12}{27} + \frac{8}{27} \qquad (0.0.3)$$

$$= \frac{20}{27} \qquad (0.0.4)$$

For Type-2 error: Occurs when one rejects the alternate hypothesis(H_1) when it is true

$$P(X \leq 1) = P(X = 1) + P(X = 0) \qquad (0.0.5)$$

$$= {}^3C_1 \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^2 + {}^3C_0 \times \left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^3$$
$$(0.0.6)$$

$$= \frac{12}{27} + \frac{8}{27} \qquad (0.0.7)$$

$$= \frac{20}{27} \qquad (0.0.8)$$

$\therefore$ Option-1 is correct