

AI1103-Assignment 3

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Download all python codes from

<https://github.com/Shambhu-K/Assignment-3/blob/main/Assignment-3.py>

and latex-tikz codes from

<https://github.com/Shambhu-K/Assignment-3/blob/main/Assignment-3.tex>

QUESTION

(GATE 2010 MA Q-26)

Let X have a binomial distribution with parameters, n and p , $n=3$. For testing the hypothesis $H_0: p = \frac{2}{3}$ against $H_1: p = \frac{1}{3}$, let a test be: "Reject H_0 if $X \geq 2$ and accept H_0 if $X \leq 1$."

Then the probabilities of Type-1 and Type-2 errors respectively are

- | | |
|--|---------------------------------------|
| 1) $\frac{20}{27}$ and $\frac{20}{27}$ | 3) $\frac{20}{27}$ and $\frac{7}{27}$ |
| 2) $\frac{7}{27}$ and $\frac{20}{27}$ | 4) $\frac{7}{27}$ and $\frac{7}{27}$ |

SOLUTION

0.1 Type-1 Error

A type-1 error occurs when the null hypothesis(H_0) is true, but is rejected. For this reason, it is also called a false alarm.

$$P(X \geq 2) = P(X = 2) + P(X = 3) \quad (0.1.1)$$

$$= {}^3C_2 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right)^1 + {}^3C_3 \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^0 \quad (0.1.2)$$

$$= \frac{12}{27} + \frac{8}{27} \quad (0.1.3)$$

$$= \frac{20}{27} \quad (0.1.4)$$

0.2 Type-2 Error

A type-2 error occurs when the null hypothesis(H_0) is false, but it is accepted as true. Therefore, it is called a miss alarm. Alternatively, it can also be defined as the error which occurs when the alternate hypothesis(H_1) is true, but is rejected

$$P(X \leq 1) = P(X = 1) + P(X = 0) \quad (0.2.1)$$

$$= {}^3C_1 \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^2 + {}^3C_0 \times \left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^3 \quad (0.2.2)$$

$$= \frac{12}{27} + \frac{8}{27} \quad (0.2.3)$$

$$= \frac{20}{27} \quad (0.2.4)$$

\therefore Option-1 is correct