

# AI1103-Assignment 4

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Download all python codes from

<https://github.com/Shambhu-K/Assignment-4/blob/main/Assignment-4.py>

and latex-tikz codes from

<https://github.com/Shambhu-K/Assignment-4/blob/main/Assignment-4.tex>

## QUESTION

(GATE 2019 CS Q-47)

Suppose  $Y$  is distributed uniformly in the open interval  $(1,6)$ . The probability that the polynomial  $3x^2+6xY+3Y+6$  has only real roots is (rounded off to 1 decimal place)

## SOLUTION

Given,  $Y$  has a uniform distribution in the interval  $(1,6)$ .

This implies, the probability density function of  $Y$ ,

$$f(y) = \begin{cases} \frac{1}{6-1} = \frac{1}{5} & (1 < y < 6) \\ 0 & \text{otherwise} \end{cases}$$

Given polynomial:  $3x^2+(6Y)x+(3Y+6)$   
Comparing it with the form:  $ax^2+bx+c$

Here,  $a=3$ ;  $b=6Y$ ;  $c=3Y+6$

Condition for real roots,

$$b^2 - 4ac \geq 0 \quad (0.0.1)$$

$$(6Y)^2 - 4(3)(3Y+6) \geq 0 \quad (0.0.2)$$

$$Y^2 - Y - 2 \geq 0 \quad (0.0.3)$$

$$(Y-2)(Y+1) \geq 0 \quad (0.0.4)$$

$$\therefore Y \leq -1, Y \geq 2 \quad (0.0.5)$$

We know that, for a given probability density function  $f(x)$ :

$$P(a < X < b) = \int_a^b f(x) dx \quad (0.0.6)$$

Probability that the given polynomial has real roots is,

$$P(Y \leq -1) + P(Y \geq 2) = \int_{-\infty}^{-1} f(y) dy + \int_2^{\infty} f(y) dy \quad (0.0.7)$$

$$= 0 + \int_2^6 f(y) dy + 0 \quad (0.0.8)$$

$$= \frac{1}{5} y \Big|_2^6 \quad (0.0.9)$$

$$= \frac{4}{5} \quad (0.0.10)$$

$$= 0.8 \quad (0.0.11)$$

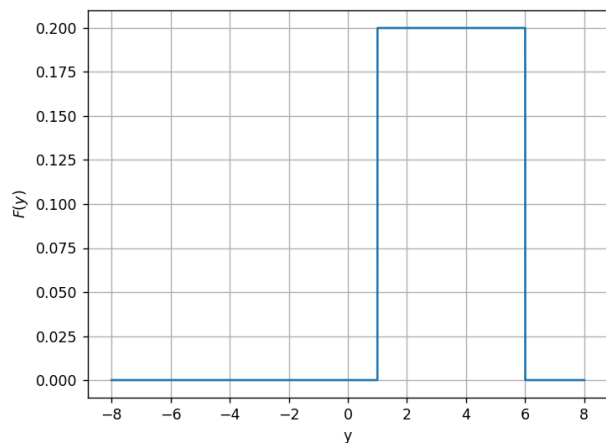


Figure 0: The figure depicts the PDF of  $Y$