AI1103-Assignment 4

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Download all python codes from

```
https://github.com/Shambu-K/Assignment-4/blob/main/Assignment-4.py
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and latex-tikz codes from

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https://github.com/Shambu-K/Assignment-4/blob/main/Assignment-4.tex
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Question

(GATE 2019 CS Q-47)
Suppose Y is distributed uniformly in the open interval (1,6). The probability that the polynomial $3x^2+6xY+3Y+6$ has only real roots is (rounded off to 1 decimal place)

Solution

Given, Y has a uniform distribution in the interval (1,6).
This implies, the probability density function of Y,

$$f(y) = \begin{cases} \frac{1}{6-1} = \frac{1}{5} & (1 < y < 6) \\ 0 & \text{otherwise} \end{cases} \tag{0.0.1}$$

From this, cumulative distribution function of Y,

$$F_Y(y) = \begin{cases} \frac{y-1}{5} & (1 < y < 6) \\ 0 & y \le 1 \\ 1 & y \ge 6 \end{cases} \tag{0.0.2}$$

Given polynomial: $3x^2+(6Y)x+(3Y+6)$
Comparing it with the form: ax^2+bx+c

Here, a=3; b=6Y; c=$3Y+6$
Condition for real roots,

$$b^2 - 4ac \ge 0 \tag{0.0.3}$$
$$(6Y)^2 - 4(3)(3Y+6) \ge 0 \tag{0.0.4}$$
$$Y^2 - Y - 2 \ge 0 \tag{0.0.5}$$
$$(Y-2)(Y+1) \ge 0 \tag{0.0.6}$$
$$\therefore Y \le -1, Y \ge 2 \tag{0.0.7}$$

Probability that the given polynomial has real roots is,

$$P(Y \le -1) + P(Y \ge 2) = F_Y(-1) + 1 - F_Y(2^-) \tag{0.0.8}$$
$$= 0 + 1 - \left(\frac{2-1}{5}\right) \tag{0.0.9}$$
$$= 0.8 \tag{0.0.10}$$

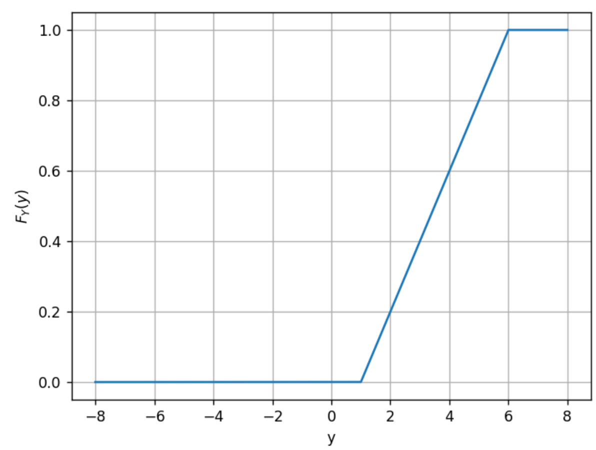

Figure 0: The figure depicts the CDF of Y