AI1103-Assignment 4

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Download all python codes from

```
https://github.com/Shambu−K/Assignment−4/blob/main/Assignment−4.py
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and latex-tikz codes from

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https://github.com/Shambu−K/Assignment−4/blob/main/Assignment−4.tex
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Question

(GATE 2019 MA Q-47)
Suppose Y is distributed uniformly in the open interval (1,6). The probability that the polynomial $3x^2$+$6xY$+$3Y$+6 has only real roots is (rounded off to 1 decimal place)

Solution

Given, Y has a uniform distribution in the interval (1,6).
This implies, the probability density function of Y,

$$\text{f}(y) = \begin{cases} \frac{1}{6-1} = \frac{1}{5} & (1 < y < 6) \\ 0 & \text{otherwise} \end{cases}$$

Given polynomial: $3x^2$+$(6Y)x$+$(3Y$+6)
Comparing it with the form: ax^2+bx+c

Here, a=3; b=6Y; c=$3Y + 6$
Condition for real roots,

$$\text{b}^2 - 4ac \geq 0 \tag{0.0.1}$$

$$(6\text{Y})^2 - 4(3)(3Y + 2) \geq 0 \tag{0.0.2}$$

$$\text{Y}^2 - Y - 2 \geq 0 \tag{0.0.3}$$

$$\text{(Y-2)(Y+1)} \geq 0 \tag{0.0.4}$$

$$\therefore \text{Y} \leq \text{-1, Y} \geq 2 \tag{0.0.5}$$

We know that, for a given probability density function f(x):

$$P(a < X < b) = \int_a^b f(x)\,dx \tag{0.0.6}$$

Probability that the given polynomial has real roots is,

$$P(Y \leq -1) + P(Y \geq 2) = \int_{-\infty}^{-1} f(y)\,dy + \int_2^{\infty} f(y)\,dy \tag{0.0.7}$$

$$= 0 + \int_2^6 f(y)\,dy + 0 \tag{0.0.8}$$

$$= \frac{1}{5}y\Big|_2^6 \tag{0.0.9}$$

$$= \frac{4}{5} \tag{0.0.10}$$

$$= 0.8 \tag{0.0.11}$$

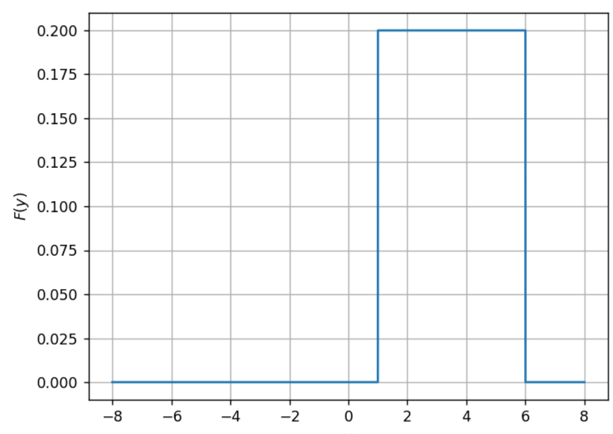

Figure 0: The figure depicts the PDF of Y