

AI1103-Assignment 4

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Download all python codes from

<https://github.com/Shambhu-K/Assignment-4/blob/main/Assignment-4.py>

and latex-tikz codes from

<https://github.com/Shambhu-K/Assignment-4/blob/main/Assignment-4.tex>

Probability that the given polynomial has real roots is,

$$P(Y \leq -1) + P(Y \geq 2) = F(-1) + 1 - P(Y < 2) \quad (0.0.6)$$

$$= 0 + 1 - F(2^-) \quad (0.0.7)$$

$$= 1 - \left(\frac{2-1}{5} \right) \quad (0.0.8)$$

$$= 1 - \frac{1}{5} \quad (0.0.9)$$

$$= \frac{4}{5} \quad (0.0.10)$$

$$= 0.8 \quad (0.0.11)$$

QUESTION

(GATE 2019 CS Q-47)

Suppose Y is distributed uniformly in the open interval $(1,6)$. The probability that the polynomial $3x^2+6xY+3Y+6$ has only real roots is (rounded off to 1 decimal place)

SOLUTION

Given, Y has a uniform distribution in the interval $(1,6)$.

This implies, the probability density function of Y ,

$$f(y) = \begin{cases} \frac{1}{6-1} = \frac{1}{5} & (1 < y < 6) \\ 0 & \text{otherwise} \end{cases}$$

From this, cumulative distribution function of Y ,

$$F(y) = \begin{cases} \frac{y-1}{5} & (1 < y < 6) \\ 0 & y \leq 1 \\ 1 & y \geq 6 \end{cases}$$

Given polynomial: $3x^2+(6Y)x+(3Y+6)$

Comparing it with the form: ax^2+bx+c

Here, $a=3$; $b=6Y$; $c=3Y+6$

Condition for real roots,

$$b^2 - 4ac \geq 0 \quad (0.0.1)$$

$$(6Y)^2 - 4(3)(3Y+6) \geq 0 \quad (0.0.2)$$

$$Y^2 - Y - 2 \geq 0 \quad (0.0.3)$$

$$(Y-2)(Y+1) \geq 0 \quad (0.0.4)$$

$$\therefore Y \leq -1, Y \geq 2 \quad (0.0.5)$$

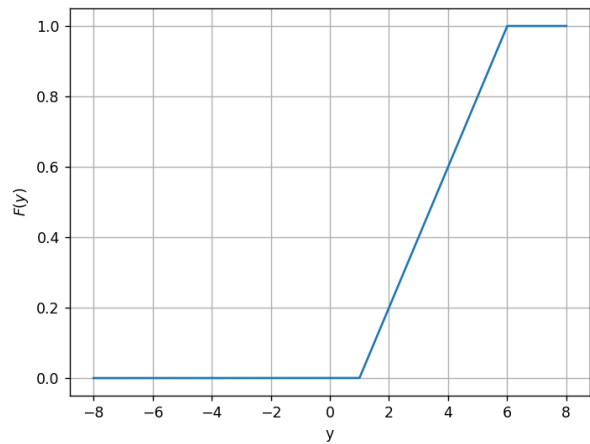


Figure 0: The figure depicts the CDF of Y