Selenocentric Coordinates and Transformations

This document describes several MATLAB functions and demonstration scripts that can be used to compute important selenocentric (moon-centered) coordinate information and transformations. Functions are provided for computing the orientation of the moon with respect to the Earth mean equator and equinox of J2000 (EME2000) system along with lunar libration angles extracted from a JPL binary ephemeris file.

Most of the information in this section was extracted from JPL D-32296, "Lunar Constants and Models Document" which is available at ssd.jpl.nasa.gov/dat/lunar_cmd_2005_jpl_d32296.pdf. Another useful reference is "Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2009", *Celestial Mechanics and Dynamical Astronomy*, **109**: 101-135, 2011.

The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this ECI inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time (TT).

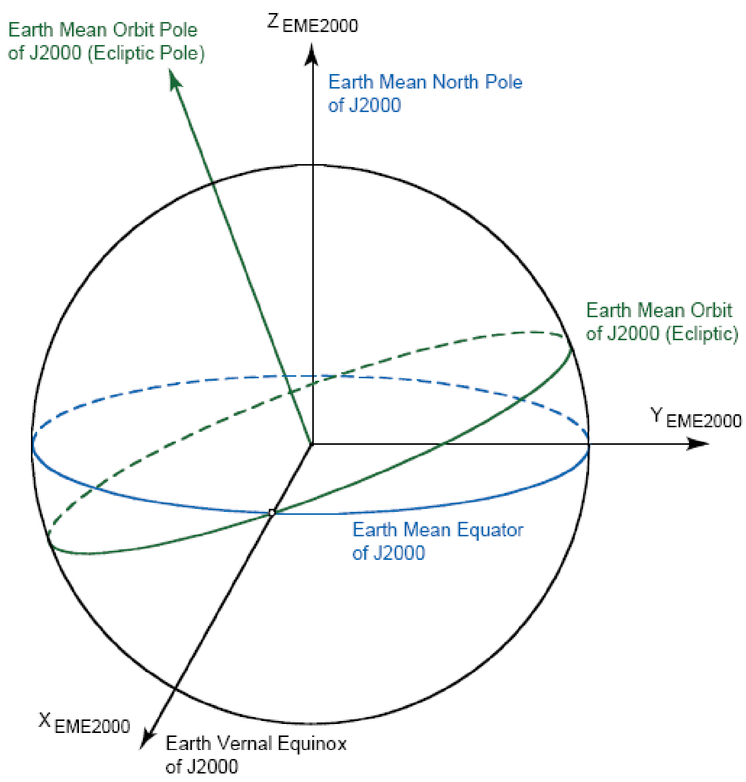

Figure 1. Earth mean equator and equinox of J2000 (EME2000) coordinate system

The following figure illustrates the orientation of the lunar mean equator and IAU node of epoch coordinate system relative to the Earth's mean equator and north pole of J2000. The x-axis or Q-vector is formed from the cross product of the Earth's mean pole of J2000 and the Moon's north pole relative to EME2000. The x-axis is aligned with the IAU node of epoch.

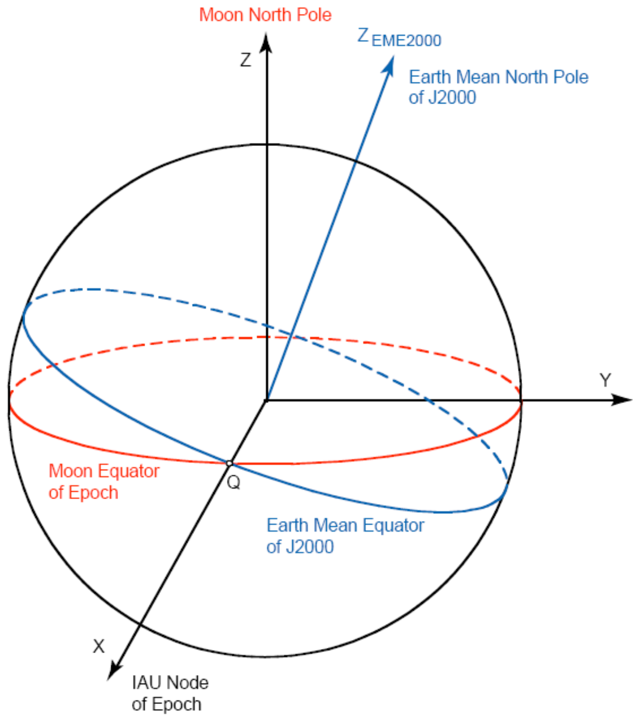

Figure 2. Moon mean equator and IAU node of epoch coordinate system

Lunar orientation with respect to EME2000

The following two equations describe the time evolution of the right ascension and declination of the moon's <u>mean</u> pole, in degrees, with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.

$$\begin{aligned}\alpha_p = {} & 269.9949 + 0.0031T - 3.8787\sin E1 - 0.1204\sin E2 + 0.0700\sin E3 \\ & -0.0172\sin E4 + 0.0072\sin E6 - 0.0052\sin E10 + 0.0043\sin E13\end{aligned}$$

$$\begin{aligned}\delta_p = {} & 66.5392 + 0.0130T + 1.5419\cos E1 + 0.0239\cos E2 - 0.0278\cos E3 \\ & +0.0068\cos E4 - 0.0029\cos E6 + 0.0009\cos E7 + 0.0008\cos E10 - 0.0009\cos E13\end{aligned}$$

The equation for the prime meridian of the Moon, in degrees, with respect to the IAU node vector is given by the following expression

$$\begin{aligned}W = W_p & + 3.5610\sin E1 + 0.1208\sin E2 - 0.0642\sin E3 + 0.0158\sin E4 \\ & +0.0252\sin E5 - 0.0066\sin E6 - 0.0047\sin E7 - 0.0046\sin E8 \\ & +0.0028\sin E9 + 0.0052\sin E10 + 0.0040\sin E11 + 0.0019\sin E12 - 0.0044\sin E13\end{aligned}$$

where $W_p = 38.3213 + \dot{W}D - 1.4 \bullet 10^{-12} D^2$ degrees, $\dot{W} = 13.17635815$ degrees/day and $D = JD - 2451545.0$.

In these equations, T is the time in Julian centuries given by $T = \left(JD - 2451545.0\right)/36525$ and JD is the Barycentric Dynamical Time (TDB) Julian Date.

The trigonometric arguments, in degrees, for these equations are

$$
\begin{aligned}
E1 &= 125.045 - 0.0529921d \\
E2 &= 250.089 - 0.1059842d \\
E3 &= 260.008 + 13.0120009d \\
E4 &= 176.625 + 13.3407154d \\
E5 &= 357.529 + 0.9856003d \\
E6 &= 311.589 + 26.4057084d \\
E7 &= 134.963 + 13.0649930d \\
E8 &= 276.617 + 0.3287146d \\
E9 &= 34.226 + 1.7484877d \\
E10 &= 15.134 - 0.1589763d \\
E11 &= 119.743 + 0.0036096d \\
E12 &= 239.961 + 0.1643573d \\
E13 &= 25.053 + 12.9590088d
\end{aligned}
$$

where $d = JD - 2451545$ is the number of days since January 1.5, 2000. These equations can also be found in "Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2009", *Celestial Mechanics and Dynamical Astronomy*, **109**: 101-135, 2011.

A unit vector in the direction of the pole of the moon can be determined from

$$
\hat{\mathbf{p}}_{Moon} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}
$$

The unit vector in the x-axis direction of this selenocentric coordinate system is given by

$$
\hat{\mathbf{x}} = \hat{\mathbf{z}} \times \hat{\mathbf{p}}_{Moon}
$$

where $\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. The unit vector in the y-axis direction can be determined using

$$
\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Moon} \times \hat{\mathbf{x}}
$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the moon-centered (selenocentric) mean equator and IAU node of epoch system are as follows:

$$
\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Moon} \end{bmatrix}
$$

moon_angles.m – orientation angles of the moon with respect to EME2000

This MATLAB function computes the orientation angles of the moon with respect to EME2000. It implements the polynomial equations described above.

The syntax of this MATLAB function is

```
function [rasc_pole, decl_pole, rasc_pm] = moon_angles (jdate)

% orientation angles of the moon with respect to EME2000

% input

%  jdate = julian date

% output

%  rasc_pole = right ascension of the lunar pole (radians)
%  decl_pole = declination of the lunar pole (radians)
%  rasc_pm   = right ascension of the lunar prime meridian (radians)
```

mm2000.m – EME2000-to-selenocentric coordinate transformation

This MATLAB function computes the matrix which transforms coordinates between the Earth mean equator and equinox of J2000 (EME2000) and lunar mean equator and IAU node of epoch coordinate systems. It implements the equations described above.

The syntax of this MATLAB function is

```
function tmatrix = mm2000 (xjdate)

% eme2000 to moon mean equator and IAU node of epoch

% input

%  xjdate = julian date

% output

%  tmatrix = transformation matrix
```

lunarlib.m – lunar libration angles

This MATLAB function computes lunar libration angles and rates using information available on modern JPL binary ephemeris files. Binary ephemeris files for Windows compatible computers can be downloaded from the Celestial and Orbital Mechanics web site located at www.cdeagle.com.

The syntax of this MATLAB function is

```
function [phi, theta, psi, phi_dot, theta_dot, psi_dot] = lunarlib(jdate)
```

```
% lunar libration angles and rates using a JPL binary ephemeris

% input

%  jdate = TDB julian date

% output

%  phi, theta, psi = libration angles (radians)
%  phi_dot, theta_dot, psi_dot = libration angle rates (radians/day)
```

This software suite includes a MATLAB script named `demo_llib` that demonstrates how to interact with this function. The following is a summary of the results computed by this script.

```
program demo_llib

lunar libration angles and rates

TDB Julian date          2451545.000000

phi             -3.10247126 degrees

theta           24.34245494 degrees

psi             41.17669108 degrees


phi_dot         -0.00000008 degrees/day

theta_dot        0.00000003 degrees/day

psi_dot          0.00015259 degrees/day
```

moon_pa1.m – lunar principal axis coordinate transformation – JPL binary ephemeris

This MATLAB function determines a matrix that can be used to transform coordinates from the lunar mean equator and IAU node of J2000 (which we'll call the moon_j2000 system) to the lunar principal axes (PA) system. The principal axis frame is aligned with the three maximum moments of inertia of the Moon.

The syntax of this MATLAB function is

```
function tmatrix = moon_pa1(jdate)

% transformation matrix from lunar mean equator and IAU node of j2000
% to the lunar principal axes system using JPL binary ephemeris

% input

%  jdate = TDB julian date

% output

%  tmatrix = transformation matrix
```

moon_pa2.m – lunar principal axis coordinate transformation – JPL approximation

This MATLAB function determines a matrix that can be used to transform coordinates from the lunar mean equator and IAU node of J2000 (which we'll call the moon_j2000 system) to the lunar principal axes (PA) system. The principal axis frame is aligned with the three maximum moments of inertia of the Moon.

The syntax of this MATLAB function is

```
function tmatrix = moon_pa2(jdate)

% transformation matrix from lunar mean equator and IAU node of j2000
% to the lunar principal axes system using JPL approximate equations

% input

%  jdate = TDB julian date

% output

%  tmatrix = transformation matrix
```

According to "Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements: 2009", *Celestial Mechanics and Dynamical Astronomy*, **109**: 101-135, 2011, the transformation matrix from the Earth mean equator and equinox of J2000 (EME2000) coordinate system to the moon-centered, body-fixed lunar principal axis (PA) system is given by the following (3-1-3) rotation sequence;

$$[M]_{EME2000}^{PA} = R_z(\psi)R_x(\theta)R_z(\varphi)$$

In this equation, φ is the angle along the International Celestial Reference Frame (ICRF) equator, from the ICRF x-axis to the ascending node of the lunar equator, θ is the inclination of the lunar equator to the ICRF equator, and ψ is the angle along the lunar equator from the node to the lunar prime meridian. These three Euler angles represent the numerically integrated physical librations of the Moon.

The relationship between these angles and the classical IAU orientation angles is

$$\alpha = \varphi - 90^\circ$$

$$\delta = 90^\circ - \theta$$

$$W = \psi$$

The transformation from the moon_j2000 system to the PA system is given by the following matrix multiplication;

$$[N]_{moon\_j2000}^{PA} = [M]_{EME2000}^{PA}[P]_{moon\_j2000}^{EME2000}$$

The numerical components of the constant moon_j2000-to-EME200 transformation matrix are as follows;

```
 0.998496505205088      4.993572939853833E-2   -2.260867140418499E-2
-5.481540926807404E-2   0.909610125238044      -0.411830900942612
 0.000000000000000      0.412451018902688       0.910979778593430
```

Approximate lunar pole right ascension, declination and prime meridian in the PA system

Page 7 of the JPL document also provides the following "tweaks" to the orientation of the moon in order to approximate the orientation in the PA system.

$$\alpha_{PA} = \alpha_{IAU} + 0.0553\cos W_p + 0.0034\cos\left(W_p + E1\right)$$

$$\delta_{PA} = \delta_{IAU} + 0.0220\sin W_p + 0.0007\sin\left(W_p + E1\right)$$

$$W_{PA} = W_{IAU} + 0.01775 - 0.0507\cos W_p - 0.0034\cos\left(W_p + E1\right)$$

where W_p is the polynomial part of the prime meridian equation given by

$$W_p = 38.3213 + \dot{W}d - 1.4 \bullet 10^{-12} d^2$$

and

$$\begin{aligned} \alpha_{IAU} &= 269.9949 + 0.0031T - 3.8787\sin E1 - 0.1204\sin E2 \\ &+0.0700\sin E3 - 0.0172\sin E4 + 0.0072\sin E6 \\ &-0.0052\sin E10 + 0.0043\sin E13 \end{aligned}$$

$$\begin{aligned} \delta_{IAU} &= 66.5392 + 0.0130T + 1.5419\cos E1 + 0.0239\cos E2 \\ &-0.0278\cos E3 + 0.0068\cos E4 - 0.0029\cos E6 \\ &+0.0009\cos E7 + 0.0008\cos E10 - 0.0009\cos E13 \end{aligned}$$

$$\begin{aligned} W_{IAU} &= W_p + 3.5610\sin E1 + 0.1208\sin E2 + \\ &-0.0642\sin E3 + 0.0158\sin E4 + 0.0252\cos E5 \\ &-0.0066\sin E6 - 0.0047\sin E7 - 0.0046\cos E8 \\ &+0.0028\sin E9 + 0.0052\sin E10 + 0.0040\sin E11 \\ &+0.0019\sin E12 - 0.0044\sin E13 \end{aligned}$$

moon_me2pa.m – lunar Mean Earth/polar axis to principal axis coordinate transformation

This MATLAB function determines a matrix that can be used to transform coordinates from the lunar Mean Earth/polar axis (ME) to the lunar principal axes (PA) system. The principal axis frame is aligned with the three maximum moments of inertia of the Moon.

The syntax of this MATLAB function is

```
function tmatrix = moon_me2pa

% transformation matrix from the lunar Mean Earth/polar axis (ME)
% system to the lunar principal axes (PA) system

% output

%  tmatrix = me-to-pa transformation matrix
```

According to JPL D-32296, "Lunar Constants and Models Document" and the IAU 2000 resolutions, the constant transformation matrix from the lunar Mean Earth/polar axis (ME) system to the lunar Principal axis (PA) system is given by the following (1-2-3) rotation sequence;

$$[PA] = R_z\left(63.8986''\right) R_y\left(79.0768''\right) R_x\left(0.1462''\right)[ME]$$

The transformation matrix from the PA system to the ME system is given by

$$R_x\left(-0.1462''\right) R_y\left(-79.0768''\right) R_z\left(-63.8986''\right)$$

The numerical components of the ME-to-PA transformation matrix are as follows;

```
  0.999999878527094       3.097894216177013E-004 -3.833748976184077E-004
 -3.097891271165531E-004  0.999999952015005       8.275630251118771E-007
  3.833751355924360E-004 -7.087975496937868E-007  0.999999926511499
```

The PA-to-ME transformation is the transpose of this matrix.

This software suite includes a MATLAB script named `demo_moon` that demonstrates how to interact with several of these coordinate transformation functions. The following is a summary of the results computed by this script.

```
program demo_moon

TDB Julian date          2451545.000000

orientation angles of the moon with respect to EME2000

rasc_pole           266.85773344 degrees

decl_pole            65.64110275 degrees

rasc_pm              41.19526398 degrees


eme2000 to moon mean equator and IAU node
of epoch transformation matrix

   +9.98496505205088e-001  -5.48154092680678e-002  +0.00000000000000e+000
   +4.99357293985326e-002  +9.09610125238044e-001  +4.12451018902689e-001
   -2.26086714041825e-002  -4.11830900942612e-001  +9.10979778593429e-001
```

```
lunar mean equator and IAU node of J2000 (moon_j2000)
to lunar principal axis (PA) transformation matrix

moon_pa1 function

   +7.52265999003059e-001  +6.58859395564263e-001  -4.04500463000584e-004
   -6.58859457533997e-001  +7.52266052983559e-001  -2.73229941726294e-005
   +2.86289955305899e-004  +2.87063115131547e-004  +9.99999917816412e-001

moon_pa2 function

   +7.52264777076062e-001  +6.58860807363059e-001  -3.76419448610194e-004
   -6.58860851635045e-001  +7.52264832430686e-001  +8.41278651081412e-006
   +2.88709968745162e-004  +2.41679395513839e-004  +9.99999929118810e-001


lunar mean Earth/polar axis (ME) to lunar
principal axis (PA) transformation matrix

   +9.99999878527094e-001  +3.09789421617701e-004  -3.83374897618408e-004
   -3.09789127116553e-004  +9.99999952015005e-001  +8.27563025111877e-007
   +3.83375135592436e-004  -7.08797549693787e-007  +9.99999926511499e-001
```