Project Scheduling with Triangular Distributions

CSE412 : Simulation and Modeling Sessional

Assignment 3

Objective

PERT (Program Evaluation and Review Technique) Chart Analysis is a widely used project management tool for scheduling, organizing, and coordinating tasks within a certain project. The PERT Chart provides a graphical representation of a project's timeline, enabling project managers to break down each individual task within the project for analysis.

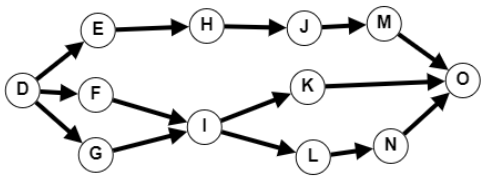
In this assignment, we will analyze two simulated projects using information from their corresponding PERT charts. We will use following 3 (three) probability distributions in this assignment to introduce stochasticity in the simulation.

1. Triangular Triang(a, b, m) distribution
2. Right-triangular RT(a, b) distribution
3. Left-triangular LT(a, b) distribution

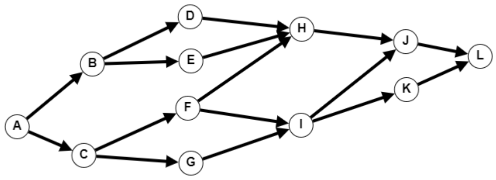
Here, a is location parameter, b-a is scaling parameter, and m is shape parameter. These distributions will be used to determine the duration of each task within a project.

Specifications

In this assignment, we will find out **average project duration** as well as **success rate** using Microsoft Excel, taking 1000 trials, for both projects. The success is defined as finishing the project on or before a fixed deadline. We will assume that the deadlines are **60 days** and **36 days** for Project-1 and Project-2 respectively. And, we will use the aforementioned 3 (three) triangular probability distributions to determine the duration of each task within a project.



(a) PERT 1



(b) PERT 2

Figure 1: PERT Diagrams

The left diagram depicts the PERT chart for Project-1 and the right diagram depicts the same for Project-2.

Project-1					Project-2				
Task	Immediate Ancestor(s)	a	m	b	Task	Immediate Ancestor(s)	a	m	b
D		5	6	7	A		3.5	4.75	6.25
E	D	7	7	7	B	A	5	6	7
F	D	5	8	13	C	A	4.3	4.3	4.3
G	D	3	8	14	D	B	3	4	6
H	E	8	9	10	E	B	7.4	8.7	9.6
I	F, G	6	8	10	F	C	7	9	11
J	H	10	13	15	G	C	3.2	4.4	5.6
K	I	5	5	5	H	D, E, F	5.1	7.3	9.9
L	I	11	13	16	I	F, G	8	9	10
M	J	9	10	12	J	H, I	2.4	2.4	2.4
N	L	3	4	7	K	I	2.2	2.8	5.2
O	K, M, N	12	14	17	L	J, K	3.3	5.5	7.8

Here, a is Optimistic value (in days), m is Most Likely value (in days), and b is Pessimistic value (in days) for the duration of a certain task.

Submission Guidelines

- Create a folder having the same name as your 7-digit student id. Put one or all your xlsx files inside the folder.
- Zip the folder and submit it in Moodle.

Please note that usage of any unfair means will be duly punished and will result in a -100% mark.

Submission Deadline : January 21, 2024 11:55 PM

Supporting Materials

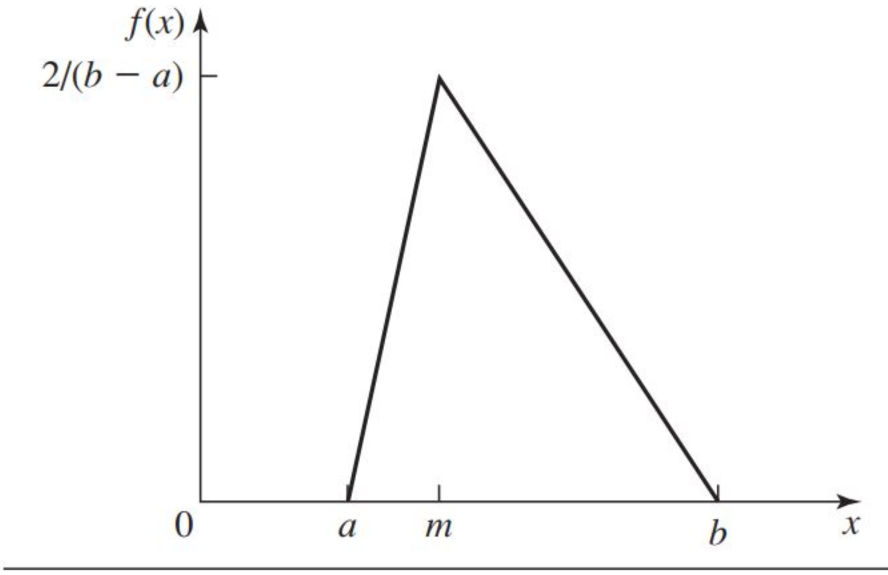


FIGURE 6.17
triang(a, b, m) density function

Figure 2: Triangular Distribution

Theorem: Generating Variates from Triangular Distribution

Random variates from the triangular distribution with minimum a, mode m, and maximum b can be generated in closed-form by inversion.

Proof

The triangular(a, m, b) distribution has probability density function

$$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(m-a)}, & \text{if } a < x < m \\ \frac{2(b-x)}{(b-a)(b-m)}, & \text{if } m \leq x < b \end{cases}$$

and cumulative distribution function

$$F(x) = \begin{cases} \frac{(x-a)^2}{2(b-a)(m-a)}, & \text{if } a < x < m \\ 1 - \frac{(x-b)^2}{2(b-a)(b-m)}, & \text{if } m \leq x < b \end{cases}$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \begin{cases} a + \sqrt{(b-a)(m-a)u} & \text{if } 0 < u < \frac{m-a}{b-a} \\ b - \sqrt{(b-a)(b-m)(1-u)} & \text{if } \frac{m-a}{b-a} \leq u < 1 \end{cases}$$

So a closed-form variate generation algorithm using inversion for the triangular(a, m, b) distribution is:

generate $U \sim U(0,1)$
if ($U < \frac{m-a}{b-a}$) then
$X \leftarrow a + \sqrt{(b-a)(m-a)U}$
else
$X \leftarrow b - \sqrt{(b-a)(b-m)(1-U)}$
endif
return(X)

Theorem: Generating Variates from Left and Right Triangular Distributions

$$\lim_{m \to b} \text{RT}(a,b) = \text{triang}(a,b,m)$$

$$\lim_{m \to a} \text{LT}(a,b) = \text{triang}(a,b,m)$$

Right Triangular Distribution (RT)

For the Right Triangular(a,m,b) distribution with minimum a, mode m, and maximum b, variates can be generated using the following algorithm:

Algorithm for Right Triangular Distribution

- Generate $U_1, U_2 \sim U(0,1)$
- Calculate $RT(0,1) = \max(U_1, U_2)$ (Refer to Prob 8.7 in Law's book)
- Calculate $RT(a,b) = (b-a) \times RT(0,1) + a$
- Return $RT(a,b)$

Left Triangular Distribution (LT)

For the Left Triangular(a,m,b) distribution with minimum a, mode m, and maximum b, variates can be generated using the following algorithm:

Algorithm for Left Triangular Distribution

- Generate $U_1, U_2 \sim U(0,1)$
- Calculate $LT(0,1) = 1 - \max(U_1, U_2)$
- Calculate $LT(a,b) = (b-a) \times LT(0,1) + a$
- Return $LT(a,b)$