Scheduling, Map Coloring, and Graph Coloring

Scheduling via Graph Coloring: Final Exam Example

Suppose want to schedule some final exams for CS courses with following course numbers:

1007, 3137, 3157, 3203, 3261, 4115, 4118, 4156

Suppose also that there are no students in common taking the following pairs of courses:

1007-3137

1007-3157, 3137-3157

1007-3203

1007-3261, 3137-3261, 3203-3261

1007-4115, 3137-4115, 3203-4115, 3261-4115

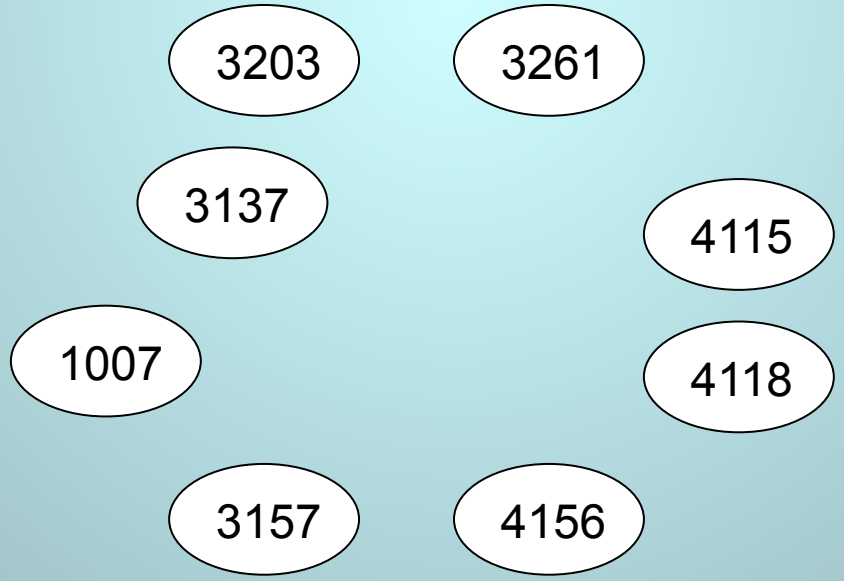
1007-4118, 3137-4118

1007-4156, 3137-4156, 3157-4156

How many exam slots are necessary to schedule exams?

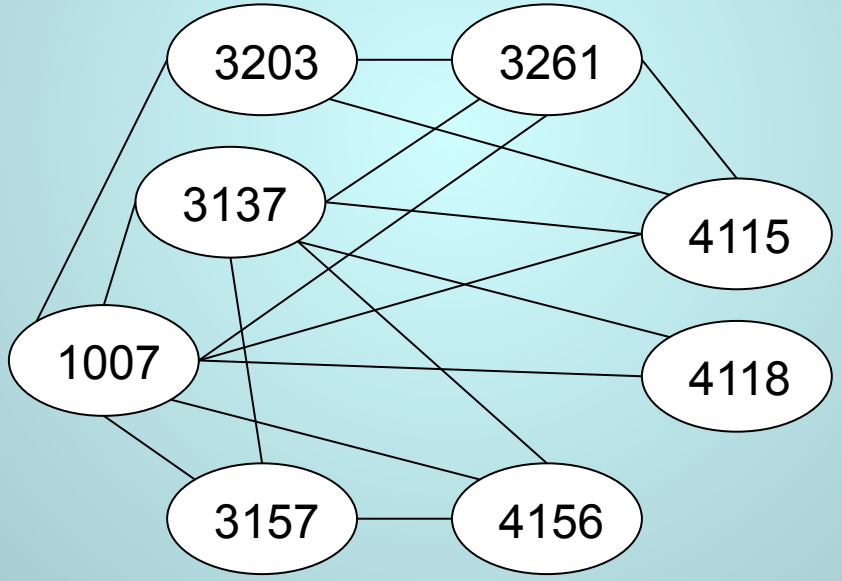
Graph Coloring and Scheduling

- Convert problem into a graph coloring problem.
- Courses are represented by vertices.
- Two vertices are connected with an edge if the corresponding courses have a student in common.



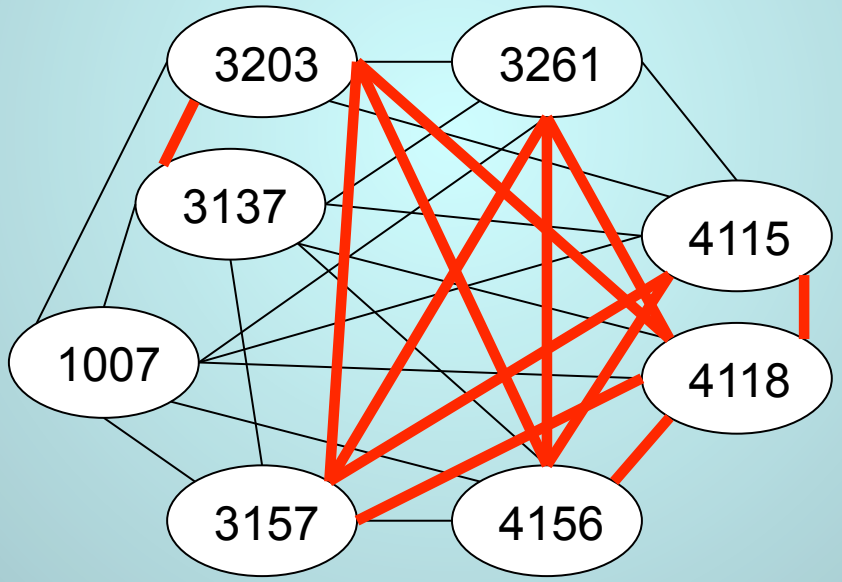
Graph Coloring and Scheduling

One way to do this is to put edges down where students mutually excluded…



Graph Coloring and Scheduling

…and then compute the complementary graph:



Graph Coloring and Scheduling

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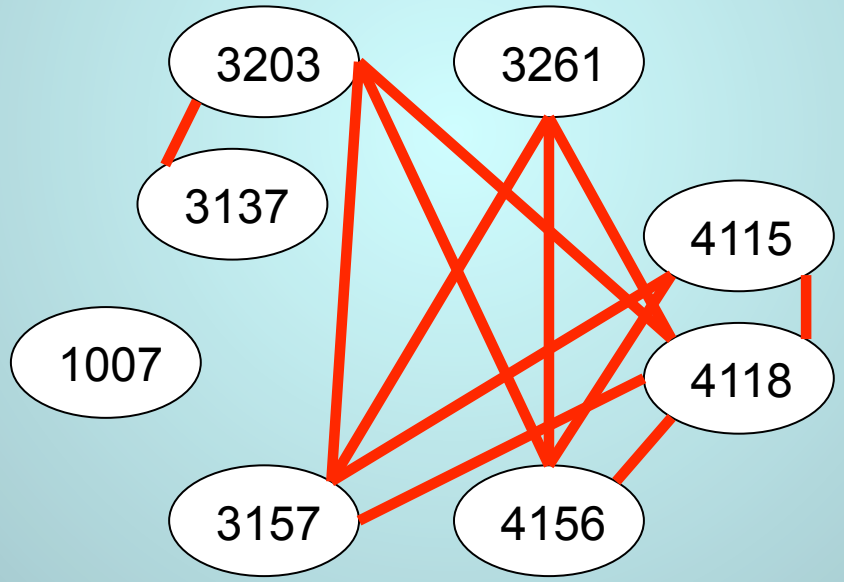
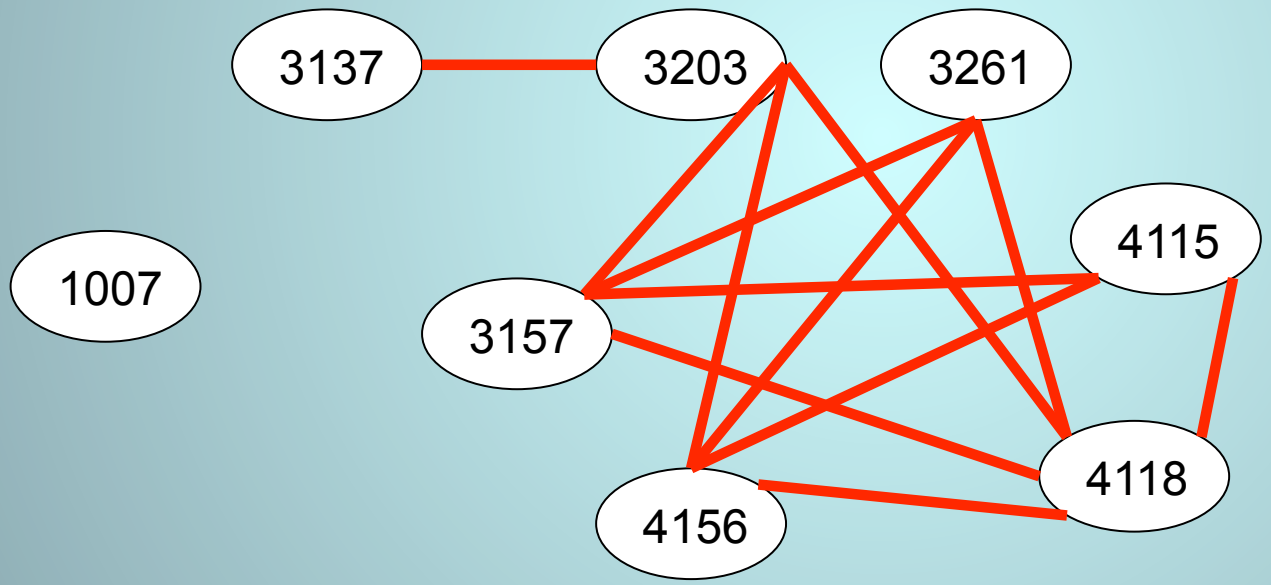


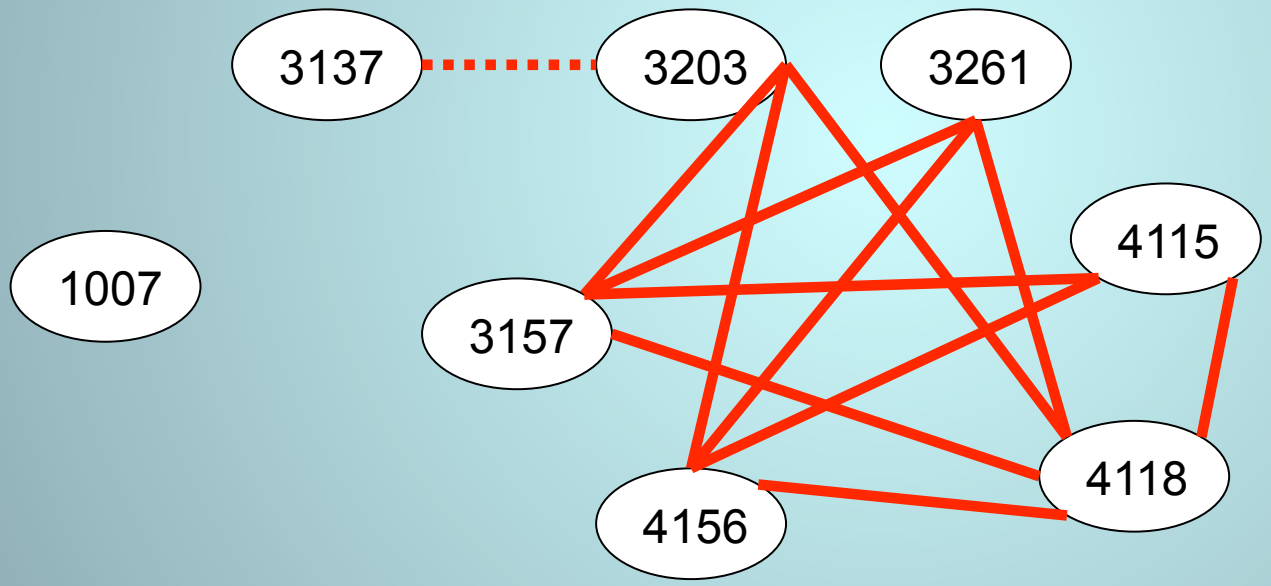
Graph Coloring and Scheduling

Redraw the graph for convenience:



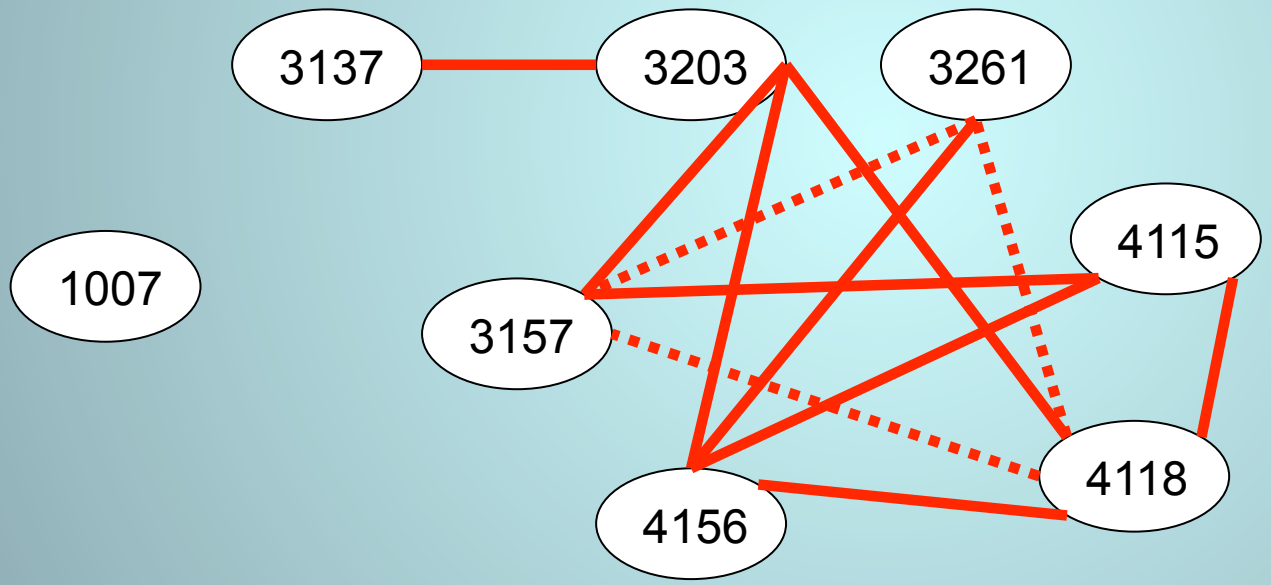
Graph Coloring and Scheduling

The graph is obviously not 1-colorable because there exist edges.



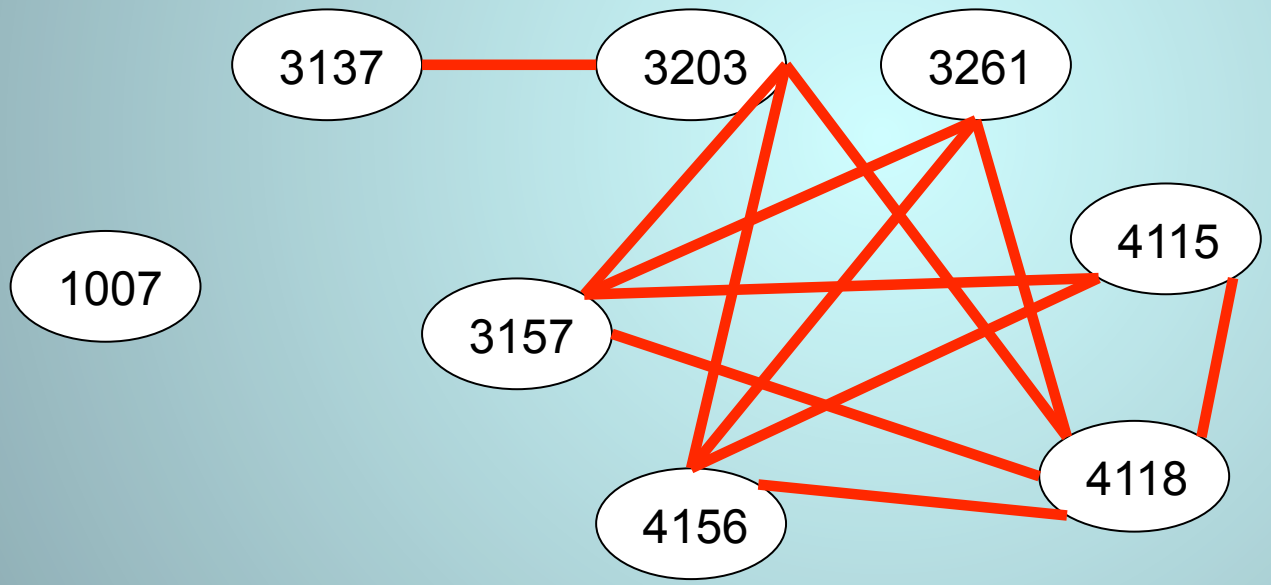
Graph Coloring and Scheduling

The graph is not 2-colorable because there exist triangles.



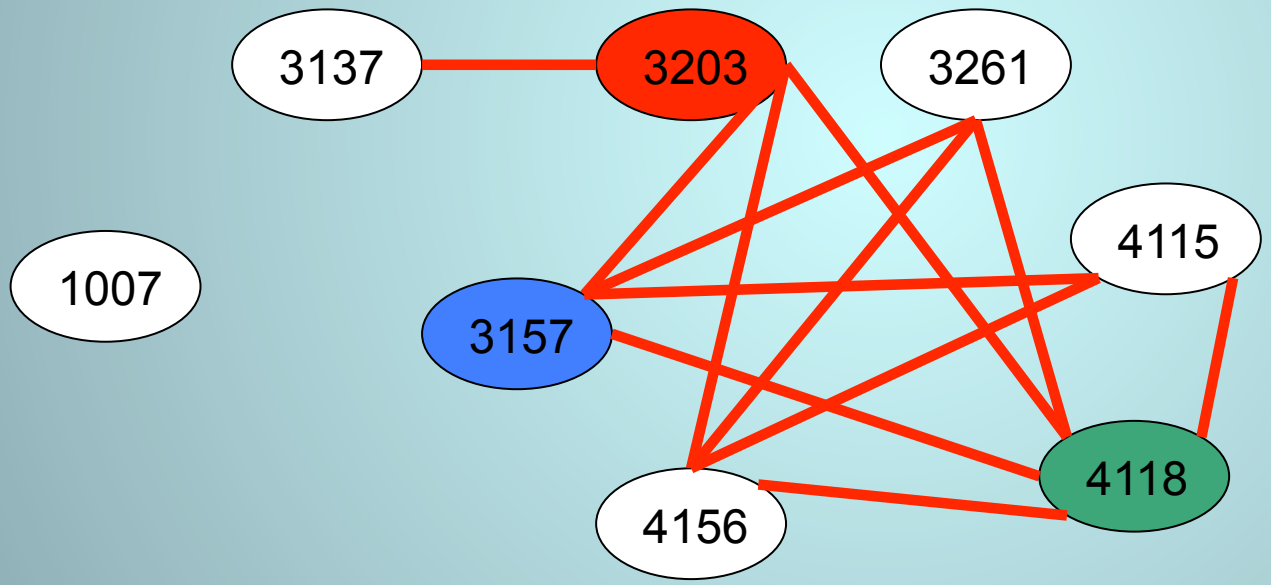
Graph Coloring and Scheduling

Is it 3-colorable? Try to color by Red, Green, Blue.



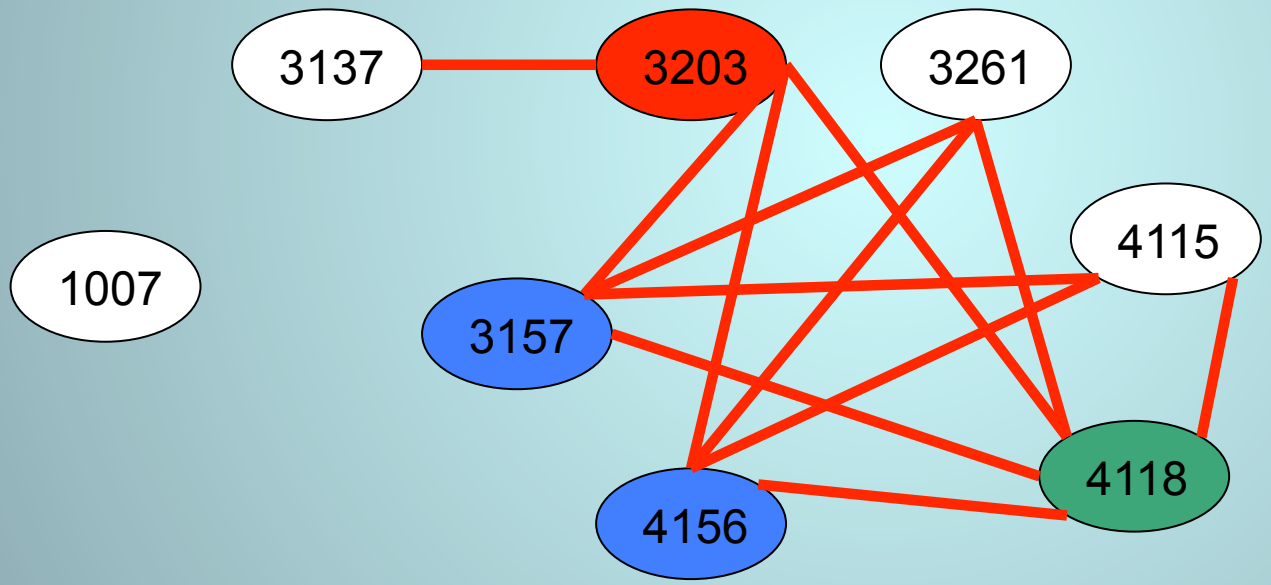
Graph Coloring and Scheduling

Pick a triangle and color the vertices 3203-Red, 3157-Blue and 4118-Green.



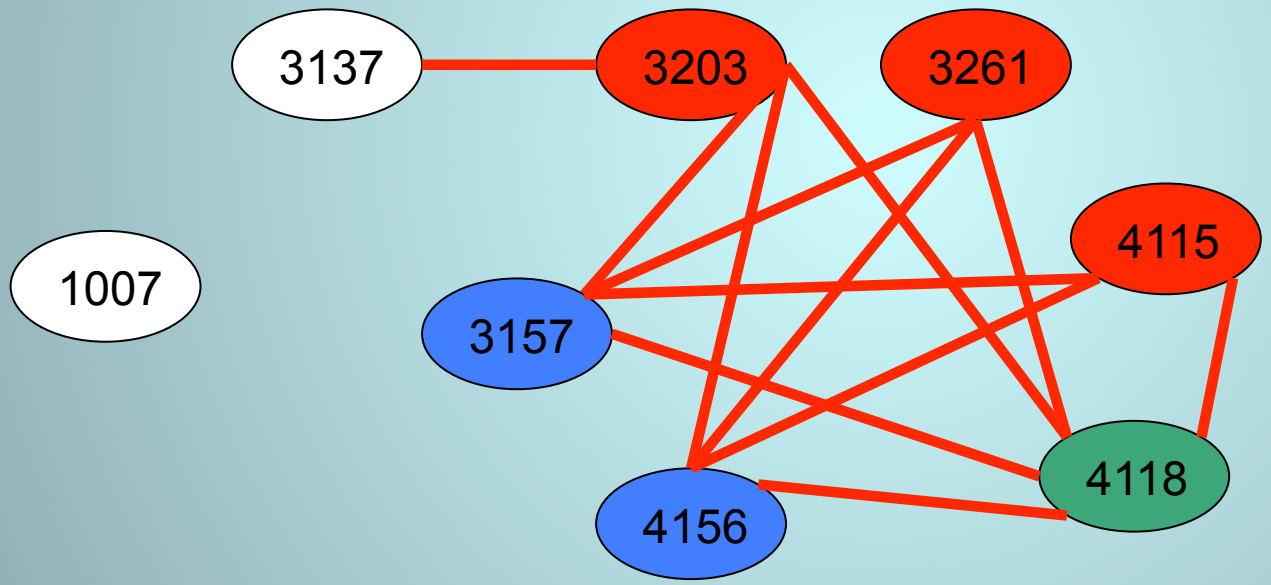
Graph Coloring and Scheduling

So 4156 must be Blue:



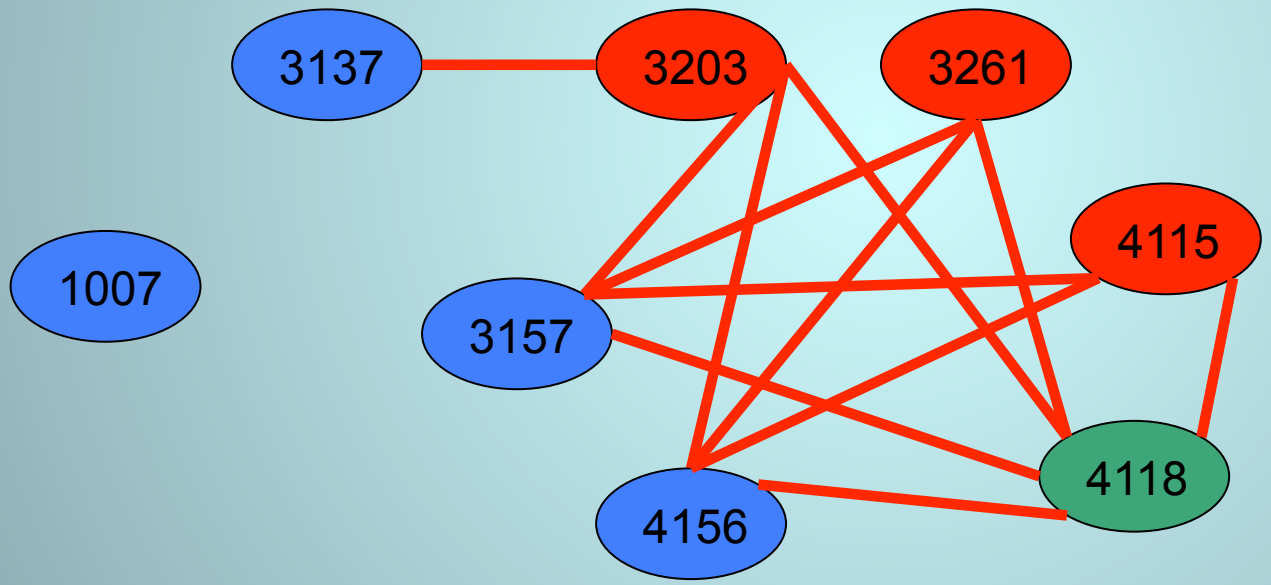
Graph Coloring and Scheduling

So 3261 and 4115 must be Red.



Graph Coloring and Scheduling

3137 and 1007 easy to color – pick Blue.



Graph Coloring and Scheduling

Therefore we need 3 exam slots: