

Discussion on Domain and Range

Differential Calculus

In this section we will define and develop the concept of a “function,” which is the basic mathematical object that scientists and mathematicians use to describe relationships between variable quantities. Functions play a central role in calculus and its applications.

SET: A set is a well defined collection of data or objects. There are two types of set

- i) Finite Set ii) Infinite Set

Finite Set: In mathematics, a finite set is a set that has a finite number of elements.

For example,

$$A = \{a, b, c\}, B = \{p, q, s\}$$

is a finite set with three elements. The number of elements of a finite set is a natural number (a non-negative integer) and is called the cardinality of the set.

Infinite Set: A set that is not finite is called infinite Set.

For Example: Integer $\mathbb{Z} = \{ \dots -2, -1, 0, 1, 2, 3 \dots \}$

$$\text{Natural Number } \mathbb{N} = \{ 1, 2, 3, 4, \dots \}$$

$$\text{Real Number } \mathbb{R} = \{ \dots, -3, -2, -1.5, -1, 0, 1, 2, 3, 4, \dots \}$$

DEFINITION OF A FUNCTION

Many scientific laws and engineering principles describe how one quantity depends on another. This idea was formalized in 1673 by Gottfried Wilhelm Leibniz who coined the term *function* to indicate the dependence of one quantity on another, as described in the following definition.

Definition If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y , then we say that y is a *function of x* .

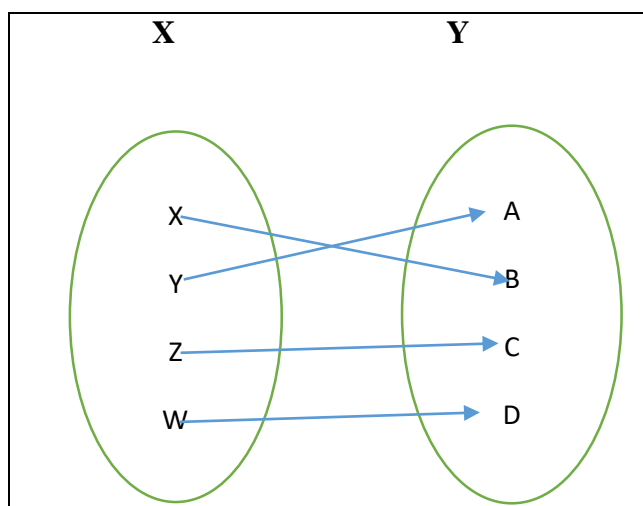
Definition A *function* f is a rule that associates a unique output with each input. If the input is denoted by x , then the output is denoted by $f(x)$ (read “ f of x ”).

Function: If X and Y are two non-empty sets and f is a such rule that gives a unique $y \in Y$ for each $x \in X$ then f is called a function from the set X to the set Y . Here X is independent variable and Y is dependent variable. A function is a special relationship where each input has a single output.

Discussion on Domain and Range

We will see many ways to think about functions, but there are always three main parts:

- The input
- The relationship
- The output.



Properties of Function:

- 1) If X and Y are two sets then Must uses all elements of X.
- 2) If X and Y are two sets then one element of X and many elements of Y does not relation each others.
- 3) If X and Y are two sets then many elements of X and one element of Y relation.

Domain: If A is the set of all real values of x such that the formula or equation $y = f(x)$ be defined or satisfied, then A is called domain of the formula.

**** $f: A \rightarrow B$, A is domain of f and B is codomain of f ****

Let, $x = \{1,2,3\}$ and $y = \{6,7,8,9,10\}$

$$y = f(x) = x + 5$$

Range= $\{6,7,8\}$

$x = \{1,2,3\}$ and $z = \{6,7,8\}$

Range: If B is the set of all real values of y or $f(x)$ corresponding each of the values or points x in domain A of the formula $y = f(x)$ then B is called Range of the formula.

Discussion on Domain and Range

Undefined form

i. $\frac{\text{some thing}}{0} = \text{undefined}$

ii. $\frac{1}{0} = \text{undefined}$

iii. $\sqrt{-ve} = \text{undefined}$

Example

* $f(x) = \frac{1}{x+2}, x+2=0, x=-2$

* $f(x) = \frac{1}{x-2}, x-2=0, x=2$

* $f(x) = \frac{1}{3x-2}, 3x-2=0, x=2/3$

* $f(x) = 2x+1$

1. Find the domain & range of $f(x) = ax + b$ ($a \neq 0$, a not equal zero).

Solution: Here $f(x)$ gives real values for all real values of x .

So, Domain = $D_f = \mathbb{R}$ (All real number) and

Again, $y = ax + b$

$$ax = y - b$$

$$\Rightarrow x = \frac{y-b}{a}$$

Here x gives real values for all real values of y .

So Range $R_f = \mathbb{R}$ (All real number)

2. Find the domain and range of $f(x) = \frac{1}{ax+b}, a \neq 0$

Solution:

Here, $f(x)$ is not defined for $ax + b = 0$ or $x = -\frac{b}{a}$ and $f(x)$ gives real values that is defined

for all real values for x except, $x = -\frac{b}{a}$ of domain of f or $D_f = \mathbb{R} - \left\{-\frac{b}{a}\right\}$

Again,

$$y = \frac{1}{ax+b}, a \neq 0$$

$$\Rightarrow ax + b = \frac{1}{y}$$

Discussion on Domain and Range

$$\Rightarrow ax = \frac{1}{y} - b$$

$$\Rightarrow x = \frac{1}{a} \left(\frac{1}{y} - b \right)$$

Here, x gives real values for all real values of y , except $y = 0$

So range of f ,

$$R_f = \mathbb{R} - \{0\}$$

3. Find the domain and range of $f(x) = \frac{3}{2x+7}$.

Solution: Here $f(x)$ is not defined for $2x + 7 = 0$, or $x = -\frac{7}{2}$ and $f(x)$ gives real values that is defined for all real values of x , except $x = -\frac{7}{2}$, for Domain of f

$$D_f = \mathbb{R} - \left\{ -\frac{7}{2} \right\}$$

Again,

$$y = \frac{3}{2x+7}$$

$$\Rightarrow 2x + 7 = \frac{3}{y}$$

$$\Rightarrow 2x = \left(\frac{3}{y} - 7 \right)$$

$$\Rightarrow x = \frac{1}{2} \left(\frac{3}{y} - 7 \right)$$

Here, x gives real values for all real values of y , except $y = 0$

So range of f ,

$$R_f = \mathbb{R} - \{0\}.$$

4. Find the domain and range of $f(x) = \frac{ax+b}{cx+d}$, $[c \neq 0, a \neq 0]$

Solution: Here $f(x)$ is not defined for $cx + d = 0$, or $x = -\frac{d}{c}$ and $f(x)$ gives real values that is defined for all real values of x , except $x = -\frac{d}{c}$, for Domain of f

$$D_f = \mathbb{R} - \left\{ -\frac{d}{c} \right\}$$

Discussion on Domain and Range

Again,

$$y = \frac{ax + b}{cx + d}$$

$$\Rightarrow y(cx + d) = ax + b$$

$$\Rightarrow cxy + dy = ax + b$$

$$\Rightarrow cxy - ax = b - dy$$

$$\Rightarrow (cy - a)x = b - dy$$

$$\Rightarrow x = \frac{b - dy}{cy - a}$$

Here, x gives real values for all real values of y , except $cy - a = 0$ or $y = \frac{a}{c}$

So range of f ,

$$R_f = \mathbb{R} - \left\{\frac{a}{c}\right\}.$$

5. Find the domain and range of $f(x) = \frac{3x+2}{5x+3}$.

Solution: Here $f(x)$ is not defined for $5x + 3 = 0$, or $x = -\frac{3}{5}$ and $f(x)$ gives real values

that is defined for all real values of x , except, $x = -\frac{3}{5}$, for Domain of f

$$D_f = \mathbb{R} - \left\{-\frac{3}{5}\right\}$$

$$\text{Again, } y = \frac{3x+2}{5x+3}$$

$$\Rightarrow y(5x + 3) = 3x + 2$$

$$\Rightarrow 5xy + 3y = 3x + 2$$

$$\Rightarrow 5xy - 3x = 2 - 3y$$

$$\Rightarrow x(5y - 3) = 2 - 3y$$

$$\Rightarrow x = \frac{2 - 3y}{5y - 3}$$

Here, x gives real values for all real values of y , except $5y - 3 = 0$ or $y = \frac{3}{5}$

So range of f ,

$$R_f = \mathbb{R} - \left\{\frac{3}{5}\right\}.$$

Discussion on Domain and Range

6. Find the domain and range of $f(x) = \frac{x^2 - a^2}{x - a}$

Solution: Given, $f(x) = \frac{x^2 - a^2}{x - a}$

Here, $f(x)$ gives real values for all real values of x except, $x = a$.

So, the domain of this function, $D_f = R - \{a\}$

Again,

$$y = f(x) = \frac{x^2 - a^2}{x - a}$$

$$\Rightarrow y = x + a, \text{ where } x \neq a$$

$$\Rightarrow x = y - a, \text{ where } y \neq 2a \text{ and } x \neq a$$

Here, x gives all real values for all real values of y except $y = 2a$

So, the range of this function, $R_f = R - \{2a\}$.

7. Find the domain and range of $f(x) = \frac{x^2 - 9}{x - 3}$

Solution: Given, $f(x) = \frac{x^2 - 9}{x - 3}$

Here, $f(x)$ gives real values for all real values of x except, $x = 3$.

So, the domain of this function, $D_f = R - \{3\}$

Again,

$$y = f(x) = \frac{x^2 - 9}{x - 3}$$

$$\Rightarrow y = x + 3, \text{ where } x \neq 3$$

$$\Rightarrow x = y - 3, \text{ where } y \neq 6 \text{ and } x \neq 3$$

Here, x gives all real values for all real values of y except $y = 6$

So, the range of this function, $R_f = R - \{6\}$.

Discussion on Domain and Range

H. W. Find the domain and range for the following functions

$$(i) f(x) = \frac{x+2}{5x+1}, \quad (ii) f(x) = \frac{3x}{2x+3}; \quad (iii) f(x) = \frac{x}{9x+3}$$

$$(iv) f(x) = \frac{5}{\frac{3}{2}x+1} \quad (v) f(x) = \frac{x^2-4}{x-2}; \quad (vi) f(x) = \frac{x-3}{2x+1}$$

Answer:

$$(i) D_f = \mathbb{R} - \left\{-\frac{1}{5}\right\}, R_f = \mathbb{R} - \left\{\frac{1}{5}\right\}; (ii) D_f = \mathbb{R} - \left\{-\frac{3}{2}\right\}, R_f = \mathbb{R} - \left\{\frac{3}{2}\right\};$$

$$(iii) D_f = \mathbb{R} - \left\{-\frac{1}{3}\right\}, R_f = \mathbb{R} - \left\{\frac{1}{9}\right\}$$