

Properties of Context Free Languages

Union: If L_1 and L_2 are two context free languages, their union $L_1 \cup L_2$ will also be context free. For example,

$L_1 = \{ a^n b^n c^m \mid m \geq 0 \text{ and } n \geq 0 \}$ and $L_2 = \{ a^n b^m c^m \mid n \geq 0 \text{ and } m \geq 0 \}$

$L_3 = L_1 \cup L_2 = \{ a^n b^n c^m \cup a^n b^m c^m \mid n \geq 0, m \geq 0 \}$ is also context free.

L_1 says number of a's should be equal to number of b's and L_2 says number of b's should be equal to number of c's. Their union says either of two conditions to be true. So it is also context free language.

Note: So CFL are closed under Union.

Concatenation: If L_1 and L_2 are two context free languages, their concatenation $L_1.L_2$ will also be context free. For example,

$L_1 = \{ a^n b^n \mid n \geq 0 \}$ and $L_2 = \{ c^m d^m \mid m \geq 0 \}$

$L_3 = L_1.L_2 = \{ a^n b^n c^m d^m \mid m \geq 0 \text{ and } n \geq 0 \}$ is also context free.

L_1 says number of a's should be equal to number of b's and L_2 says number of c's should be equal to number of d's. Their concatenation says first number of a's should be equal to number of b's, then number of c's should be equal to number of d's. So, we can create a PDA which will first push for a's, pop for b's, push for c's then pop for d's. So it can be accepted by pushdown automata, hence context free.

Note: So CFL are closed under Concatenation.

Kleene Closure: If L_1 is context free, its Kleene closure L_1^* will also be context free. For example,

$L_1 = \{ a^n b^n \mid n \geq 0 \}$

$L_1^* = \{ a^n b^n \mid n \geq 0 \}^*$ is also context free.

Note: So CFL are closed under Kleen Closure.

Intersection and complementation: If L_1 and L_2 are two context free languages, their intersection $L_1 \cap L_2$ need not be context free. For example,

$L_1 = \{ a^n b^n c^m \mid n \geq 0 \text{ and } m \geq 0 \}$ and $L_2 = \{ a^m b^n c^n \mid n \geq 0 \text{ and } m \geq 0 \}$

$L_3 = L_1 \cap L_2 = \{ a^n b^n c^n \mid n \geq 0 \}$ need not be context free.

L_1 says number of a's should be equal to number of b's and L_2 says number of b's should be equal to number of c's. Their intersection says both conditions need to be true, but push down automata can compare only two. So it cannot be accepted by pushdown automata, hence not context free.

Similarly, complementation of context free language L_1 which is $\Sigma^* - L_1$, need not be context free.

Note: So CFL are not closed under Intersection and Complementation.