Equivalence of two Finite Automata

Steps to identify equivalence

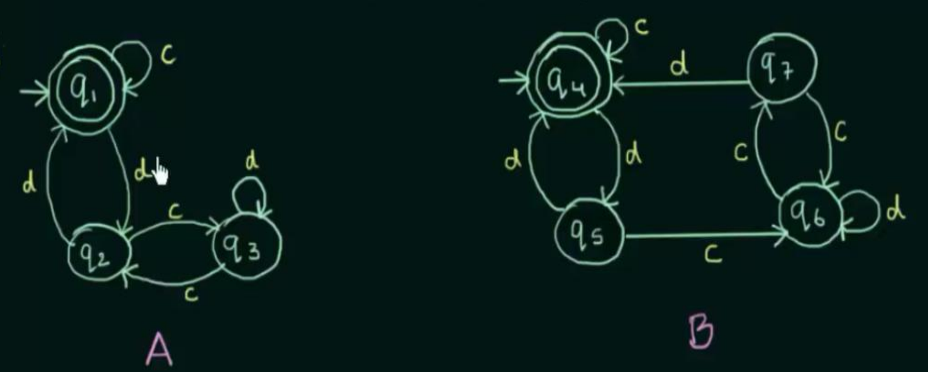
1) For any pair of states $\{q_i, q_j\}$ the transition for input $a \in \Sigma$ is defined by $\{q_a, q_b\}$ where $\delta\{q_i, a\} = q_a$ and $\delta\{q_j, a\} = q_b$

   The two automata are not equivalent if for a pair $\{q_a, q_b\}$ one is INTERMEDIATE State and the other is FINAL State.

2) If Initial State is Final State of one automaton, then in second automaton also Initial State must be Final State for them to be equivalent.

State must be Final State for them to be equivalent.

Eg:



States	c	d
(q_1, q_4)	(q_1, q_4) FS FS	(q_2, q_5) IS IS
(q_2, q_5)	(q_3, q_6) IS IS	(q_1, q_4)

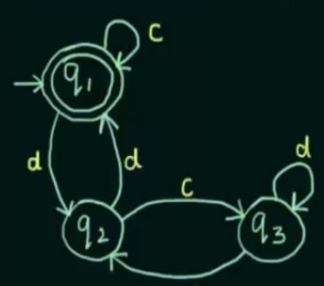
A B

States	c	d
(q_1, q_4)	(q_1, q_4) FS FS	(q_2, q_5) IS IS
(q_2, q_5)	(q_3, q_6) IS IS	(q_1, q_4) FS FS
(q_3, q_6)	(q_2, q_7) IS IS	(q_3, q_6) IS IS
(q_2, q_7)	(q_3, q_6) IS IS	(q_1, q_4) FS FS

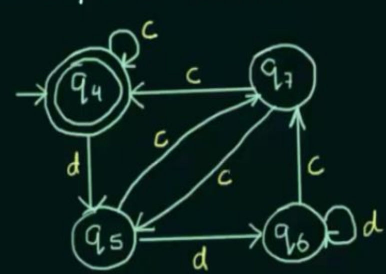
A and B are equivalent

Equivalence of two Finite Automata (Example)

Find out whether the following automata are equivalent or not



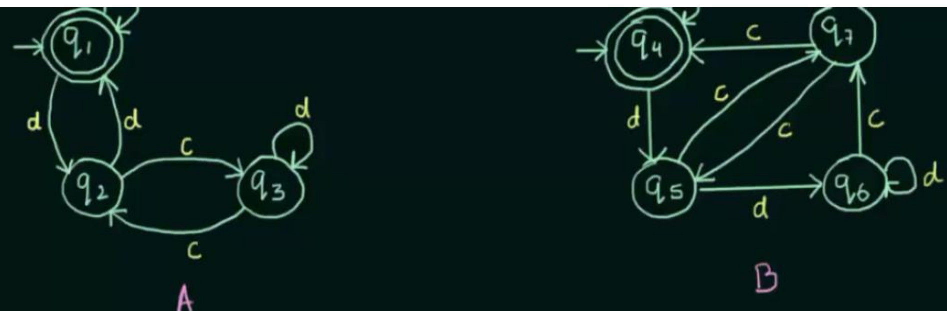
A



B

States	c	d
$\{q_1, q_4\}$	$\{q_1, q_4\}$ FS FS	$\{q_2, q_5\}$ IS IS

States	c	d
$\{q_1, q_4\}$	$\{q_1, q_4\}$ FS FS	$\{q_2, q_5\}$ IS IS
$\{q_2, q_5\}$	$\{q_3, q_7\}$ IS IS	$\{q_1, q_6\}$ FS IS


q_1
q_2
q_3
c
d
A
q_4
q_5
q_6
q_7
B

States	c	d
$\{q_1, q_4\}$	$\{q_1, q_4\}$ FS FS	$\{q_2, q_5\}$ IS IS
$\{q_2, q_5\}$	$\{q_3, q_7\}$ IS IS	$\{q_1, q_6\}$ FS IS

A and B are not equivalent

Pumping Lemma (For Regular Languages)

>> Pumping Lemma is used to prove that a Language is NOT REGULAR

>> It cannot be used to prove that a Language is Regular

If A is a Regular Language, then A has a Pumping Length 'P' such that any string 'S' where $|S| \geqslant P$ may be divided into 3 parts $S = x\,y\,z$ such that the following conditions must be true:

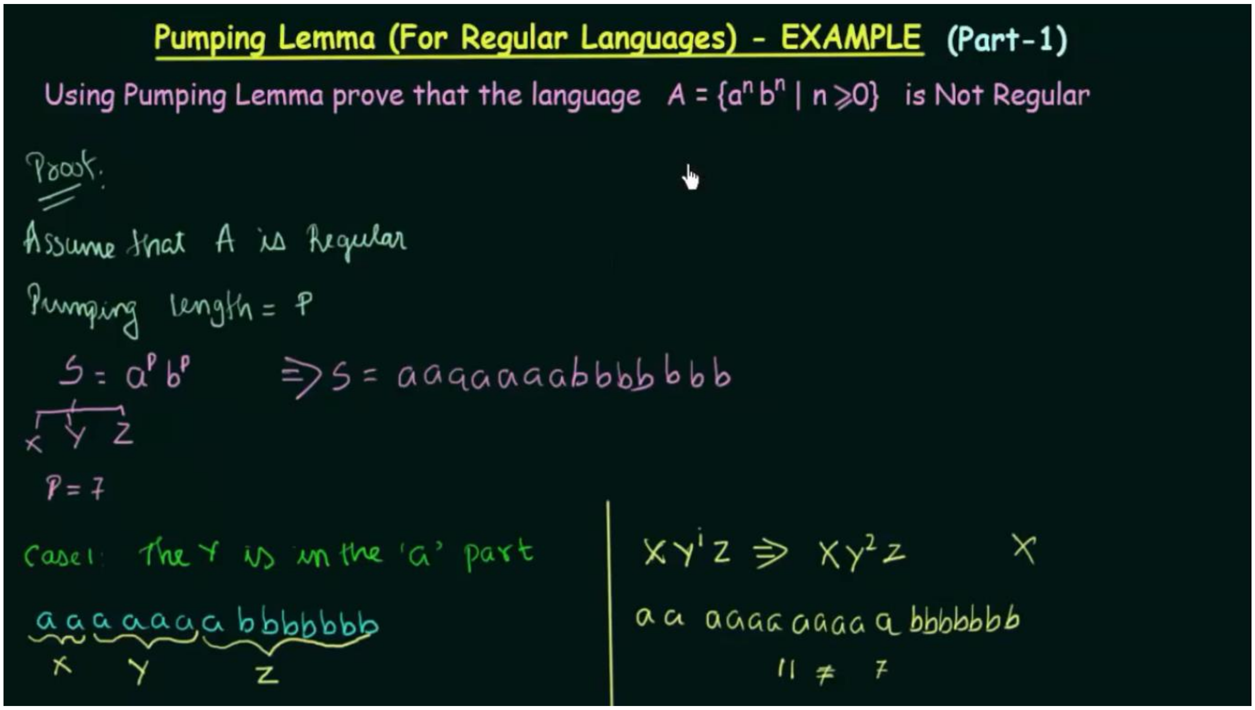
(1) $x\,y^i z \in A$ for every $i \geqslant 0$
(2) $|y| > 0$
(3) $|xy| \leqslant P$

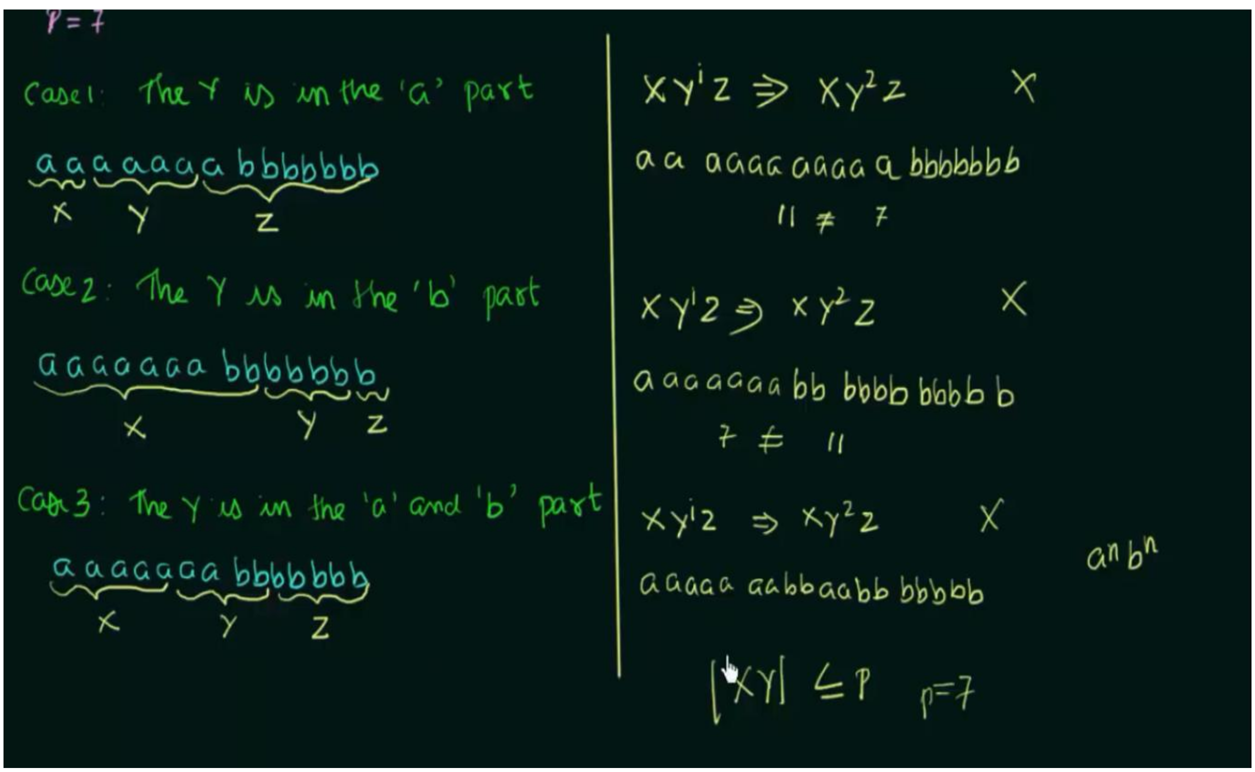
(1) $x\,y^i z \in A$ for every $i \geqslant 0$
(2) $|y| > 0$
(3) $|xy| \leqslant P$

To prove that a language is not Regular using PUMPING LEMMA, follow the below steps:

(We prove using Contradiction)

-> Assume that A is Regular
-> It has to have a Pumping Length (say P)
-> All strings longer than P can be pumped $|S| \geqslant P$
-> Now find a string 'S' in A such that $|S| \geqslant P$
-> Divide S into x y z
-> Show that $x\,y^i z \notin A$ for some i
-> Then consider all ways that S can be divided into x y z
-> Show that none of these can satisfy all the 3 pumping conditions at the same time
-> S cannot be Pumped == CONTRADICTION


Pumping Lemma (For Regular Languages) - EXAMPLE (Part-1)
Using Pumping Lemma prove that the language $A = \{a^n b^n \mid n \geqslant 0\}$ is Not Regular
Proof:
Assume that A is Regular
Pumping length = P
$S = a^P b^P$ $\Rightarrow S = aaaaaaabbbbbbb$
x y z
P = 7
Case 1: The Y is in the 'a' part
aaaaaaabbbbbbb
x y z
$xy^iz \Rightarrow xy^2z$ ✗
aa aaaa aaaa a bbbbbbb
$11 \neq 7$


P = 7
Case 1: The Y is in the 'a' part
aaaaaaabbbbbbb
x y z
$xy^iz \Rightarrow xy^2z$ ✗
aa aaaa aaaa a bbbbbbb
$11 \neq 7$
Case 2: The Y is in the 'b' part
aaaaaaabbbbbbb
x y z
$xy^iz \Rightarrow xy^2z$ ✗
aaaaaaa bb bbbb bbbb b
$7 \neq 11$
Case 3: The Y is in the 'a' and 'b' part
aaaaaaabbbbbbb
x y z
$xy^iz \Rightarrow xy^2z$ ✗
aaaaa aabbaabb bbbbb
$a^n b^n$
$|xy| \leq P$ $p=7$

Pumping Lemma (For Regular Languages) EXAMPLE (Part-2)

Using Pumping Lemma prove that the language $A = \{yy \mid y \in \{0,1\}^*\}$ is Not Regular

$\underbrace{01}\underbrace{01}$

Proof:

Assume that A is Regular

Then it must have a Pumping Length = P

$S = 0^P 1 0^P 1$

x y z

p = 7

$\underbrace{000}_{x}\underbrace{0000}_{y}\underbrace{100000001}_{z}$

$xy^iz \Rightarrow xy^2z$

00000000000100000001

$\notin A$

$|y| > 0$

$|xy| \leq P = 7$

A is not Regular