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REINFORCEMENT LEARNING

Returns, Value-functions and MDPs

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Right, so I keep saying long term I said this multiple times in the earlier lectures also, but what really is long term right.

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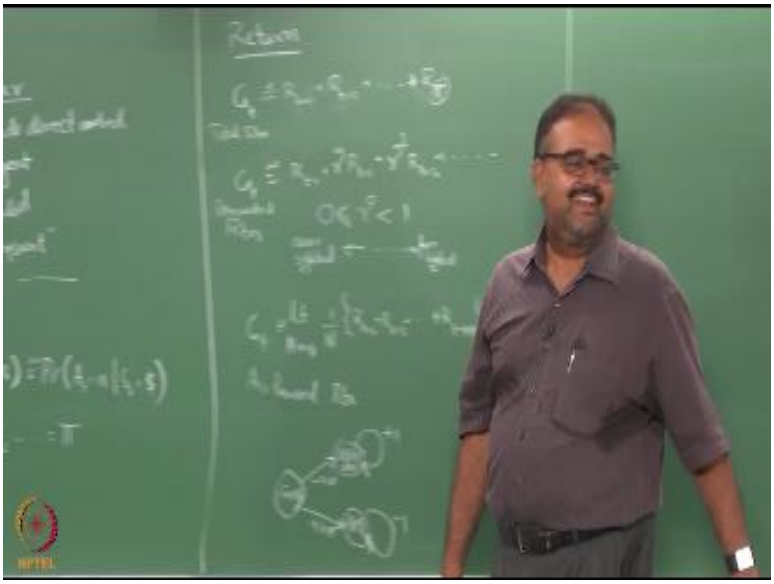

So we will try to quantify that now right, by introducing the notion of something called return right,

$$Total\ Return\ G_t = R_{t+1} + R_{t+2} + \cdots + R_T$$

so this is a common notation that I will use throughout where T indicates the timestamp of the end of the episode T means the end of the episode right essentially I have reached S^+ right, so this

could this is the most natural way of thinking about long term return correct, is the most natural way of thinking about long-term return and if there, if that T is actually finite right, so this is actually is not a bad idea and I will sum up all the reward I'm going to get in the future and I will try to optimize that, okay.

So again you can see that if I try to optimize each one of these individually what will happen I might get a very high R_{t+1} and R_{t+2} but it might put me in such a bad S_{t+2} from then on even if I behave optimally individually for each of this these numbers might be small, right. So what I really want to do is behave in such a way that even if some of these numbers are small right overall in the future the sum that I am going to get should be the maximum, right.

So overall sum should be maximum, so that is essentially the goal that I have here, okay so this is fine it works fine there is one circumstance where this would not work when T is I mean there is no end right, so I just keep doing this for forever if I keep solving a problem forever then it becomes a problem and not necessarily forever if I am also doing something over a very, very, very long horizon if T is very large not necessarily infinite but T is very large also I get into some issues, okay because this will become quantity with a large variance, okay. So one other thing I should point out this is something which people miss and get confused sometimes while reading on their own so T here is a random variable, right.

So every time I run the system it might stop at a different time right think of playing chess right, there is a finite endpoint chess will end, but every time I play the game I do not stop at the same number of moves, right even something like tic-tac-toe right. Even though there is an upper limit on the number of moves you can make right, it is not necessary that you will always end up playing five moves that you might end you might win in four moves you might win in well yeah, if you win in three moves well, okay.

The opponent is really bad but so you could win in three moves you could win in four moves or you could win in five moves, right or you cannot win at all I mean so you could still end up drawing the game so like that so T is a random variable so when I put T here does not mean it always runs for some T steps okay, so every time I run it will be a different value for T so remember that, right

and consequently G_t is also a random variable because all of these are random variables I am talking about sums of random variables, okay.

And a random number of those random variables are being summed up okay, so G_t is also a random variable that denotes the return okay, great. So another notion of return that people typically use is right,

$$Discounted\ Return\ G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots ;\ 0 \leq \gamma < 1$$

in fact I can even do that, okay. So go back and think about it purely from a mathematical perspective so if my sum is going to run till infinity and my γ is less than 1 and remember that each individually each r is bounded okay, so this sum will converge to something finite so I do not have to worry about what I had in this case this sum will not explode okay, so that is fine, right.

But there is even something more interesting about this discounted reward formulation, in fact originally when people proposed a discounted reward formulation it was not really from the boundedness point of view it was more from a from a psychology point of view immediate gratification versus delayed gratification, right so if you let us look at what happens if γ is 0. All thing will go right, it is just R_{t+1} this goes back to just optimizing for the immediate payoff, right so this gives you essentially you are going to behave in a way so that the next time step what is the maximum reward that you are going to get so you are going to behave only in that way, right.

So if you think about what γ is doing so it is making suppose I get a 10 here versus getting a 10 here so 10 here is more valuable than 10 here right, of course 10 here is more valuable than 10 here, right. So let us go back to our robot grasping problem we are talking about having a -1 all the time and then giving you 100 when you actually grasp so instead of that I can say I am going to use the discounted rewards my γ is say 0.8 so what does it mean, so the sooner I grasp it the larger the value I am going to see, correct.

So the overall reward I am starting from here I should put myself in a trajectory so that I grasp this quickly. Otherwise I will see some $\gamma^{10000} \times 100$, so that is very, very small value so the shorter..

I know I can get rid of the -1 right. So I do not really have to do the -1 but then what becomes tricky here we have to figure out what is the right value of γ right, so that becomes tricky.

So this is, so they use this kind of γ to control for whether you are a near sighted agent will be worried about immediate gratification or whether you are okay delaying the gratification little bit so that you get a higher payoff further down the line. You have only 1, 1 payoff right, so just pull what we do not cumulative rewards so you are talking about regrets you are talking about regrets right, you are talking about the total reward case, right.

So yeah, having a discount there really does not make sense, right so anyway you are trying to optimize regret right, so in some case your γ is as small as possible right when you say you are optimizing regret that means you are trying to keep your γ as small as possible so essentially you want all your winnings to come in one pull right, one pull I want to be optimal, so that is your optimal regret case, right.

So when you are trying to do achieve something like this setting a γ does not make sense right, but I mean if you can come up with a different objective function maybe there is something that you can think over the entire learning process, okay. So one thing which I should point out just let me finish one thing I should point out is that classically the RL people right, look at bandit problems as immediate RL right, I mean the learning the the whole interaction is just that one arm pull, okay that is how we think of it while the, in the long-term case is the full RL problem.

But if you look at the people who work in the Bandit theory community right, they think of the interaction as the entire learning process, right that they do not think of it as one arm pull they think of it as the whole process itself is the learning process so for them it is like an episode, right so there is a very different mindset so the other people do not think of it that way and the bandit people do not think of discount factors and other things because that is something which the RL guys just came up with for looking at long-term payoffs so there is a mismatch of vocabulary there so maybe there is something to find that in the middle ground but I am not sure.

Return is a function of the time step, yeah here let us put it this way so I should optimize my policy in such a way that wherever I am starting from there and going forward I should get maximum possible return. So even if I am in the last step I do not want to goof up in the last step but if I am see, one thing you should remember is that if I know where I am going to end if that T is given to me before hand then it becomes a non-stationary problem.

Because I know okay, I have five more steps to go what should I do I take one more step I have only four more steps to go what should I do so immediately the problem changes right if I know the T then it is a non-stationary setting, right so we will not will not look at cases where the T is known, okay we will only look at cases where T is known to be finite but not deterministic, right it is not known apriori so it is not like I can say that I will optimize only at the last step or anything because I do not know what the last step is, right.

I might recognize it when I actually reach it but I do not know beforehand that okay, I am going to have only so many step steps to go, I mean this is the normal seeing that we assume but you can immediately see it is not realistic you know I mean when you are playing chess you can always say I am going to win in three more moves or well I am going to lose in three moves I mean depending on right, so how well the game is going. So in some cases you do have this kind of a little bit of a forecasting ability as to what the T is, but typical the basic setup that we will talk about will assume T is unknown.

Yeah so people understand why γ equal to 1, closer to 1 is far sighted, right and closer to 0 is near sighted clear.

Well, depends on many things right, depends on the value of γ and depends on the magnitude of the other rewards that you are getting right, so let us say γ is 0.9999999. Then the bias is low depending on how far sighted you want to be you have to pick your γ, right yeah they are unsatisfactory you have another free parameter really free parameter that you have no way how to tune but yeah it is there right so we will talk about another reward formulation shortly.
Which gets rid of the γ but makes it very hard to optimize anyway so any other questions on this. Doesn't matter, Yeah, I just add it up yeah sure, yeah but then that means I am fall, I am failing

after more time steps, right so I am okay whether that is positive or negative it does not matter as far as this return definitions are concerned, you are essentially interested in maximizing the return therefore if you are going to take an action that gives you a -100 reward the farther out in the future you take it is better, right.

So succeed for as long as you can and then die you know right, so yeah it does not matter so the rewards can be positive, negative. So reinforcement learning is really the community is little weird right, so we talk about positive rewards and negative rewards right if you look at the psychology literature they talk about rewards and punishments right, for some reason the RL community and if you look at the, you know other places they call it payoffs right or they call it cost negative rewards are called costs but the RL community somehow thinks of negative things also as rewarding so we have positive rewards and you have negative rewards but in the larger scheme of things you know negative things also improve your experience you know they enrich your life in some way so you can think of negative rewards also anyway.

So good so this is fine then we will move on to one more definition I have to give this right,

$$Average\ Reward\ Return\ G_t = \lim_{N\to\infty} \frac{1}{N}\{R_{t+1} + R_{t+2} + \cdots + R_{t+N}\}$$

so what does that do just gives me the average right, so the more the reward I accumulate the larger the average is going to be right so for a finite number of N this makes sense right, but what if I have an infinite number of rewards in the future. Under mild regularity assumptions and bounded-ness of the individual Rs this will be bounded, okay.

What is not clear is that it will have actually have a limit, might not right in general it might not have a limit so you have to impose additional constraints on the kind of problems that you are solving not all problems have this defined right or I mean you can make some small variations to this to make sure that all problems have this limit defined, okay. So there are slight different the limit can be we can play around with the limit.

For example one case where limit will not exceed exists is this if the sequence is periodic, right if the sequence is periodic then the limit will not exist right, because it will just keep oscillating all the time, right. So in this case we use something called the.. do people know how to handle limits of periodic sequences use something called the cesaro's limit which essentially looks at the average over periods and lets the number of periods run to infinity, right so in the reason the limit does not exist is because you cut the sequence in the middle of a period right so if it is going to be periodic so you only take a period as a unit and take the reward over the period, right and then you keep adding that over subsequent periods and take the average, right and limit as the number of such periods you consider it goes to infinity is called the cesaro's limit.

Then in such cases limit might exist of course there are bunch of other conditions you have to put on the actual structure of the problem itself for you to ensure that this limit exists but for a very broad class of problems you can talk about this limits as well. So the nice thing about this kind of the return definition is that it does not have a pesky γ parameter right so it makes it nice from a computational point of view not to have the γ parameter.

But it makes it little weaker from a, one of the original motivations of RL point of view because it does not have the γ parameter, okay. So γ parameter allows you to have you know some sense personality for your agent right is it a near sighted agent or a far sighted agent and so on so forth right so especially people in neuroscience when they are looking at modeling reinforcement learning humans right, so they actually have they can think of estimating what is your γ by running experiments with you right.

Because it is kind of you know it is a personality issue thing right, so some people will tend to be in fact there are studies that try to correlate the levels of serotonin in your head to γ, right so more the serotonin the larger the γ value you have in your decision making and so on so forth I mean it is like this is really weird how we seem to have all these parameters in our head tied up with the different kinds of neurotransmitters, right anyway.

okay so there is a very nice problem that people use for motivating why discounted return is bad, okay. So let me let me try and do that here, okay so I am implying no theological leanings here

okay this is just a fun example, so that is earth okay that is heaven and that is hell so the path to heaven has a -100 reward and the path to hell has a +1000 reward right but once you reach heaven you get a +1 infinitely once you reach hell you get a -1 infinitely I said I am not getting into theology here okay, this is just a fun example of course you can always think about other things here, no reward for ..nahh..anyway you are finished here right, it is a transient state we do not care.

But then if I said γ is 0.1 or something, right you would prefer going to hell right if γ is like 0.9 in fact γ has to be pretty high for you to go to heaven right, so it is the same problem so now γ becomes part of the problem definition right, depending on what value I set for γ my optimal policy itself will change right, so this is not a desirable state of affairs I mean if you are thinking about saying okay here is a problem solve it and it turns out that if I use average reward scenario now which is better going to heaven is better always does not on depend on anything what does it depend on.

I am going to get an infinite number of plus 1s right that will thulp any my negative things I get initially I am getting a finite value for negative here I am going to get a $-\infty$ there and a $+\infty$ here so that trumps anything that happens here, so average reward will tell me that this is the best thing to do. But if I am going to use discounted returns so I will go up or down depending on what my γ is right. So now you can immediately see that discounted return is a more realistic return because not everybody is going to heaven right anyway that is the fun part.

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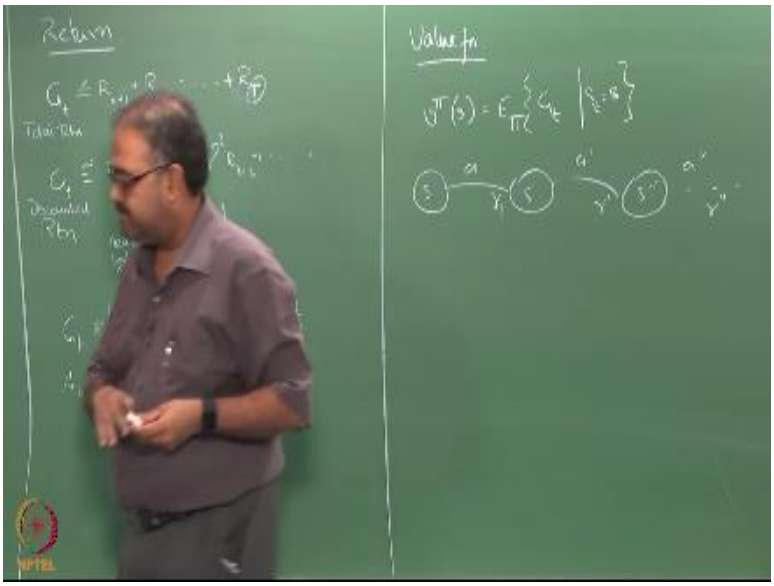

We come to the really one of the crux of reinforcement learning here, right in the full RL and a lot of the algorithms that we look at and the notion of a value function. Right we already looked at the value function right, but the value function we looked at was very simplistic right so what did we do there we said okay, average of all the rewards I obtained so far right that is the value function then we have another value which we looked at which is the expected true expected reward that I should get if I take an action, right so we denote that by q^* and then we denoted by Q the average that I am maintaining for this right.

But here we have something more complex, what do we have here we have a policy π which determines not just the current reward but also the sequence that I am going to see hence forth right, so when I am going to talk about a value function here I will talk about something called V^{π} right, so V^{π} is the value function associated with a policy π, right. So when you say $V^{\pi}(s)$, so this is essentially the expectation of, so what should I be taking the expectation of rewards or returns what do you think it is I should be taking the expectation of the returns, right.

$$V^{\pi}(s) = E_{\pi}\{G_t | S_t = s\}$$

Because the return is what I am looking to optimize this is what I should be predicting so in the bandit case I was only looking at the reward so I was just making it with the expectation of the reward right here I should make it out as the expectation of return so given anything. Anything else, I need to condition it on π right, because I already have a π here so why do I need to condition on π because my G_t is going to depend on π. Regardless of what formulation I am using the subsequent rewards I am going to get depend on the actions I take, right so I really need to know what my π is right.

So I need to condition it on π so we will denote it like this, right when I write the π here that means that is a policy I am conditioning on, okay the expectation with respect to the distribution generated by following π okay, of G_t given that at time t, I started with state s okay, does it make sense everyone on board, right so I am starting from state s I am following π I am generating one trajectory right like this right so I start from s okay, go to some s', go to s'' so on so every time I take an action so I will take a, a', a'' and so on and I will get some r, right this is one trajectory.

So like that I will start from s, I will do another trajectory then I will start from s I will do another trajectory, I will start from s I will do another trajectory then I will be getting all these rewards like we have r, r', r'' like that I will get some r_1', r_1'' and so on so forth.. each trajectory I will get one such sequence of rewards right, so I will add up all of these or I will take a discounted sum of all of this right, so I will do that right and I will take the average that is that essentially whatever that average will converge to is my expected value for this, this is essentially what I mean.

When i say G_t such that S_t equal to s, that means I will start with s okay, and compute this rewards okay, right. So is it clear why we need to condition on the π as well right the future is going to depend on π yeah, yeah I am talking about the expectation here I am just explaining to you what the expectation is so every time I generate the trajectory through π right, two things could happen A) I could pick different actions because π is stochastic, B) the world could change in a different way because the world is stochastic, so in the same s I could take the same a, I might end up in a different s' every time right, so there are multiple ways in which this stochastic can operate so one π itself is stochastic therefore in the same s I need not take the same action again.

Right first time I can take a_1, next time I can take a_2 and so on so forth right that will completely change the trajectory that will come later or I will take the same a right, I might end up in a different s' because of the transition probability so every time I do this trajectory I will get some different trajectory, I mean every time I generate a trajectory I will get a different sequence right, and the expectation is over all such sequences you know what is the total reward I will get so that is essentially what is the value function is, okay is it clear great.

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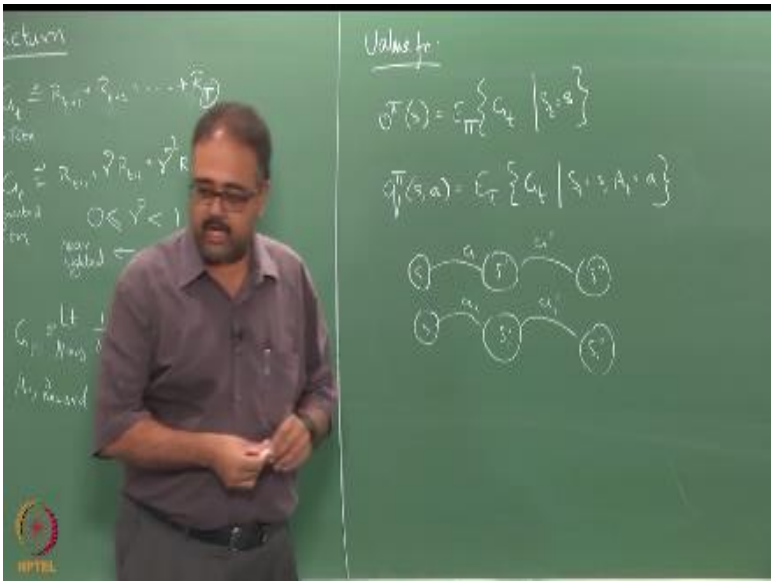

Let us confuse people some more so we had Q earlier right so what was Q(a), $q_*(a)$ was the true thing, Q(a) is the expectation, the expected estimate for taking action a, right but then here we have to worry about taking action a in a specific state because every state it is going to be different, correct every state it will be different so what this will be, I will have to do this I still have to do this, so what is the big difference between the V and q hmm..sorry,

$$q^{\pi}(s,a) = E_{\pi}[G_t|S_t = s, A_t = a]$$

yeah action a is fixed so earlier when you are generating trajectories for V^{π}, I said you start with s right, then you go to s' then you go to s'' and so on so forth and then you could I said you could pick any action.

So first time I will pick a, then a' the next time you put some a_1 so like this right, so you could pick different actions every time we generated the trajectory. So now I'm saying you cannot do that right, so every time I generate the trajectory you have to pick a first right, but subsequently you pick actions according to π, right the first action is something that is fixed now, the first action will be a all the time so what does this tell you it tells you how good is it to perform action a in a given state. So earlier you did not have that information but now you have the information about how good is it to perform action a in a given state, okay is it clear going back and thinking about the problem itself right. So there is a problem that we are trying to solve right, so how do you define the problem how do you characterize the problem right, so I am in a state s I take an action okay I go to a next state and I also get a reward right.

So for me to characterize the problem I need to tell you how this transitions happen, how the reward generation happens right, so I need to really look at this following question what is the probability of given, sorry.

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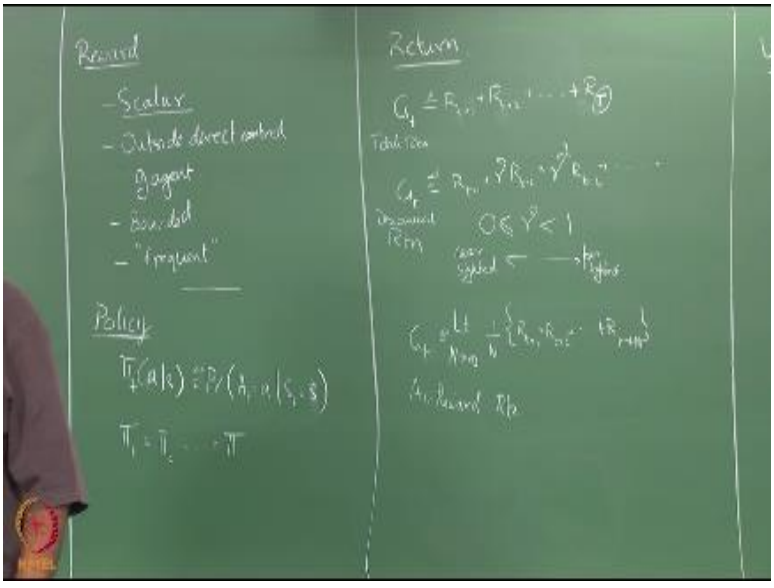

R_{t+1} given is this sufficient I mean think about it right, riding a bicycle so you probably need to know what is the velocity momentum that you had before and it is not enough to know where you are on the road and how you are tilting you might want to know something more about the history of how you got there, right. So you really need that quantity so you need to know what is the probability of S_{t+1}, R_{t+1} given everything that went before that, correct. This becomes a little complex to model because the number of parameters you will need for modeling something like this becomes pretty huge.

$$Pr(S_{t+1}, R_{t+1}|S_t, A_t, S_{t-1}, A_{t-1} \dots S_o) = Pr(S_{t+1}, R_{t+1}|S_t, A_t)$$

So what we typically end up doing is make a very strong assumption that this guy is equal to so what you truly need is the first quantity right, because it becomes very hard to estimate very hard to define you make the assumption that that quantity is really equal to this right so what is this assumption called typically Markov assumption it is essentially a first order Markov assumption where the history does not matter right only the current state and current action matters that will determine what the next state and next action will be right.

$$Pr(S_{t+1} = s', R_{t+1} = r|S_t = s, A_t = a) = p(s', r|s, a)$$

So we make a further assumption which is, what is this assumption I made.. stationarity, right so we will typically operate in markov environments right and will typically assume they are stationary, okay. So for me to characterize a problem now what should I give you what I need is what is the question yeah, to specify the problem what are the things I should give you okay, I should give you the states over which the problem is defined right.

The problem is defined by

$$S, A, p(s', r|s, a), \gamma$$

Also, sometimes defined alternatively by

$$S, A, p(s'|s, a), E[r|s, a, s'], \gamma$$

This is how you describe a Markov Decision Process.

I need to do that right I need to define the state space over which the problem is defined and then the set of actions that I can take and right, the P function right, so I need all of this to define the problem completely anything else I need, depends on what is the return I am using, I might also

need a γ right if my return, if I am using a discounted return depending on what the value of γ is the problem changes right that is what we discussed.

So if I am going to use discounted return I should also specify γ as part of the problem, okay is it clear so I need all of this to define my problem right in some cases we use a slightly simplified definition where instead of having the single joint distribution s over s' and r right, we also give it as two separate quantities.

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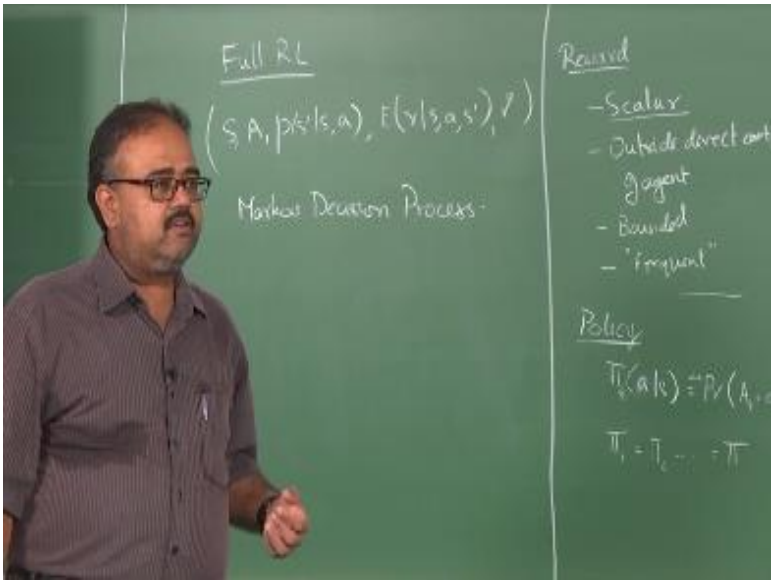

So I will suggest S I mean I have to specify S, A then I will specify that so what is that called, sometimes called the transition function or transition probabilities okay, and then I will specify the expected value of the return given that at t, I was in s, I did action a and I went to s' right, so essentially I am writing this out splitting this up right, so instead of giving the joint distribution I

am saying okay no, no I will specify the s' separately and for the R, I will condition it on s' also right.

So I can we can think in some sense I am using the chain rule and splitting that joint probability as $p(s') \times p(r|s,a,s')$, but what is the difference here instead of specifying $p(r|s,a,s')$, I am giving you only the first moment or zeroth moment, giving the first moment of the distribution right, so why is this enough why is this enough because I am going to be optimizing only the expected value of G_t, okay and then G_t is composed of these guys right.

So essentially I will be taking the individual expectations of each one of these right, and that is enough for me, I do not really need to know the distribution which generated these rewards because individually I will be taking the expectations of each one of these rewards and therefore knowing the expectation alone is enough so sometimes you specify the problem like this right, and this is typically how you would I would specify a what is called a Markov decision process.

So it is a system is a decision system that you have to for every state you have to actually keep giving a decision right, and it follows Markovian dynamics right, both the transition and the rewards follow, I mean satisfy the Markov property right, so such a system is called a Markov decision process right so we have states you have set of decisions right and the evolution of the states follows the Markov process okay so they are called Markov Decision Process, okay and what we will assume is that the reinforcement learning problems that we are trying to solve can be modeled as MDPs, okay.

We will assume that the reinforcement problems you are trying to solve can be modeled as MDPs so in some sense a value function really you know is useful only if you have the Markov property, why because I am assigning a value to a state regardless of how I got to the state right, so if the Markov property is not satisfied how I got to the state will influence what happens in the future right.

Since I am saying that no I do not care about what happened in the past I am saying I am starting in state s okay, I am going to be following policy π here after what is the expected return therefore

I am saying history is irrelevant right, when I am defining a value function another way of thinking about it is here the value function is marginalizing over history so it is essentially taking the expectation regardless of what the history was that is one way of looking at it I mean in some cases that is how you make sense out of the value functions.

Because as we go along later we will see that we actually apply RL blindly to problems that are non-Markov right and we will be using the V function and the q function in such cases. So the only way to make sense out of what is happening in such cases is to say that hey we are essentially marginalizing the history say we are just doing the taking the expectation over all possible histories as well, it may or may not be a sensible quantity.

But if it works hey..you just use it right, but normally your value function makes sense only if you have the Markov property forget about the future right, then the because you are ignoring the past it makes sense only if you have the Markov property right, does it make sense right so that is why I was saying that I should have probably defined value functions after telling about Markov decision processes but it is okay, you can still understand it in the general sense as well great. So the next thing we have to talk about is optimal value functions, right that I will start that in the next class.

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