

Cheat Sheet - Counting Principles

Permutation

- Permutation is the arrangement of items in which **order matters**
- Number of ways of **selection and arrangement of items** in which Order Matters

$${}^n P_r = \frac{n!}{(n-r)!}$$

Combination

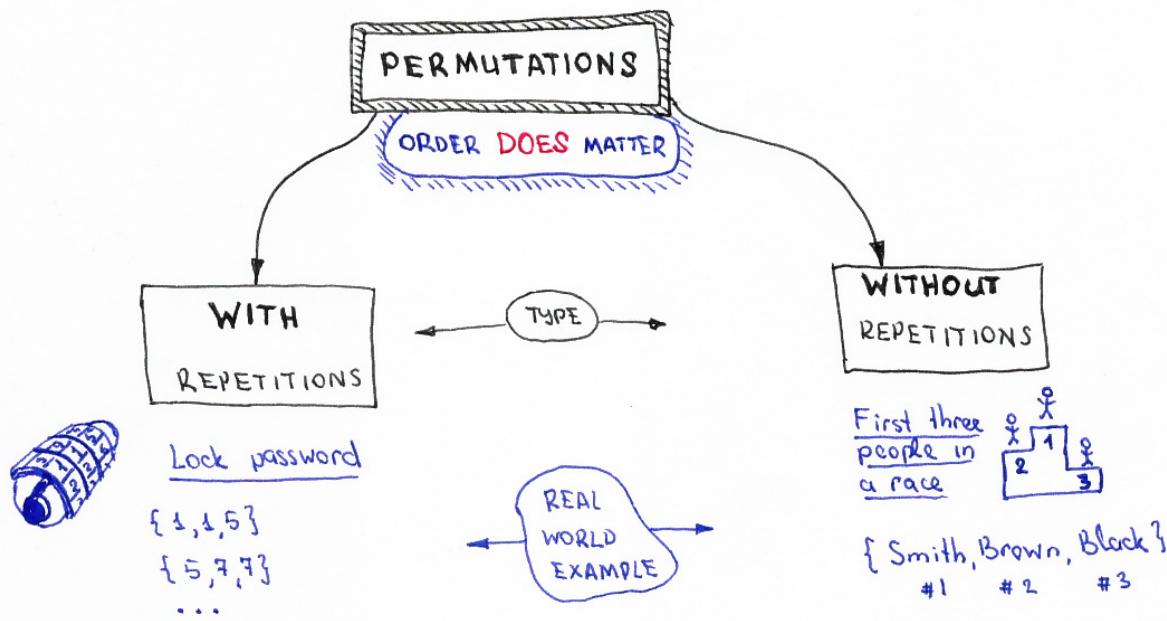
- Combination is the selection of items in which **order does not matter**.
- Number of ways of **selection of items** in which Order does not Matter

$${}^n C_r = \frac{n!}{r! (n-r)!}$$

Combinations with Repetition

TABLE 1 Combinations and Permutations with and without Repetition.

Type	Repetition Allowed?	Formula
r -permutations	No	$\frac{n!}{(n-r)!}$
r -combinations	No	$\frac{n!}{r! (n-r)!}$
r -permutations	Yes	n^r
r -combinations	Yes	$\frac{(n+r-1)!}{r! (n-1)!}$



$\{A, B\} \rightarrow \{A, A\}$

$\{A, B\}$

$\{B, A\}$

$\{B, B\}$

EXAMPLE

$\{A, B, C\} \rightarrow \{A, B, C\}$

$\{A, C, B\}$

$\{C, A, B\}$

$\{B, A, C\}$

$\{B, C, A\}$

$\{C, B, A\}$

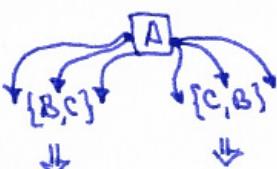
$$n^r$$

NUMBER OF PERMUTATIONS

r - number of slots
 n - number of options

$$n!$$

• PERMUTATIONS •

WITH REPETITIONS	WITHOUT REPETITIONS
<p>[Input] $\{A, B\} \times 2$ slots</p> <p>.....</p> <p>① \boxed{A} : Remember \boxed{A}</p> <p>② $\{A, B\} \times 1$: Find all smaller permutations ↓ $\{A\}, \{B\}$</p> <p>③ $\boxed{A} + \{A\} = \{A, A\}$ $\boxed{A} + \{B\} = \{A, B\}$: Concatenate \boxed{A} to all smaller permutations</p> <p>④ Repeat steps ①, ②, ③ for \boxed{B} and $\{A, B\} \times 1$</p> <p>.....</p> <p>OUTPUT</p> <p>$\{A, A\}$ $\{A, B\}$ $\{B, A\}$ $\{B, B\}$</p>	<p>[Input] $\{A, B, C\} \times 3$ slots</p> <p>.....</p> <p>① $\leftarrow \boxed{A}$: Take \boxed{A} out</p> <p>② $\{B, C\} \times 2$: Find all smaller permutations ↓ $\{B, C\}, \{C, B\}$</p> <p>③  \downarrow $\{A, B, C\}^{\vee}$ $\{A, C, B\}^{\vee}$ $\{B, A, C\}^{\vee}$ $\{C, A, B\}^{\vee}$ $\{B, C, A\}^{\vee}$ $\{C, B, A\}^{\vee}$</p> <p>④ Repeat steps ①, ②, ③ in case if input set is bigger. In current example the 3rd step already includes recursion.</p> <p>.....</p> <p>OUTPUT</p> <p>$\{A, B, C\}$ $\{A, C, B\}$ $\{B, A, C\}$ $\{C, A, B\}$ $\{B, C, A\}$ $\{C, B, A\}$</p>

COMBINATIONS

ORDER DOES NOT MATTER

WITH REPETITIONS



Coins in your pocket
 $\{1, 5, 5, 50, 50\}$
 $\{1, 1, 1\}$
 \dots

$$\{A, B, C\} \rightarrow \begin{aligned} &\{A, A\} \\ &\{A, B\} \\ &\{A, C\} \\ &\{B, B\} \\ &\{B, C\} \\ &\{C, C\} \end{aligned}$$

$$\frac{(r+n-s)!}{r!(n-s)!}$$

TYPE

REAL WORLD EXAMPLE

EXAMPLE
for 2 slots

WITHOUT REPETITIONS

Fruit Salad ingredients
 $\{ \text{banana, apple} \}$
 $\{ \text{apple, grape} \}$
 \dots

$$\{A, B, C\} \rightarrow \begin{aligned} &\{A, B\} \\ &\{A, C\} \\ &\{B, C\} \end{aligned}$$

$$\frac{n!}{r!(n-r)!}$$

NUMBER OF COMBINATIONS
 r - number of slots
 n - number of options

• COMBINATIONS •

WITH REPETITIONS	WITHOUT REPETITIONS
<p>Input $\{A, B, C\} \times 2$ slots</p> <hr/> <p>① A : Remember A</p> <p>② $\{A, B, C\} \times 1$: Find all smaller repetitions (combinations) \downarrow $\{A\}, \{B\}, \{C\}$</p> <p>③ $A + \{A\} = \{A, A\} \checkmark$ $A + \{B\} = \{A, B\} \checkmark$ $A + \{C\} = \{A, C\} \checkmark$: Concatenate A to all smaller combinations</p> <p>④ Repeat steps ①, ②, ③ for B and $\{B, C\} \times 1$</p> <hr/> <p>Output</p> <p>$\{A, A\}$ $\{A, B\}$ $\{A, C\}$ $\{B, B\}$ $\{B, C\}$ $\{C, C\}$</p>	<p>Input $\{A, B, C\} \times 2$ slots</p> <hr/> <p>① $\leftarrow A$: Take A out</p> <p>② $\{B, C\} \times 1$: Find all smaller combinations \downarrow $\{B\}, \{C\}$</p> <p>③ $A + \{B\} = \{A, B\} \checkmark$ $A + \{C\} = \{A, C\} \checkmark$: Concatenate A to all smaller combinations</p> <p>④ Repeat steps ①, ②, ③ for B and $\{C\} \times 1$</p> <hr/> <p>Output</p> <p>$\{A, B\}$ $\{A, C\}$ $\{B, C\}$</p>