Assignment - 1

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1. (i) Proposition, The truth value is False

(ii) Not a proposition, Because 'n' value is unknown.

2. A truth table contains 2^n rows where "n" is number of variables.

(i) $(q \to \sim p) \vee (\sim p \to \sim q)$

(p, q) two variables

$\therefore 2^{(2)} = 4$ Rows

(ii) $(p \vee \sim t) \wedge (p \vee \sim s)$

Here '3' variables (p, t, s)

$\Rightarrow 2^3 = 8 \Rightarrow \therefore 8$ Rows.

3. (i) $(p \vee q) \to (p \oplus q)$

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \to (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

(ii) $(p \oplus q) \to (p \wedge q)$

p	q	$p \oplus q$	$p \wedge q$	$(p \oplus q) \to (p \wedge q)$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

4. $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r) \to r]$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

As observed above, All the truth values are true, hence it is a tautology.

5. $[p \vee q] \wedge (\sim p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$(p \vee q)$	$(\sim p \wedge \sim q)$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

All the truth values are false, hence it is a Contradiction

6. $\sim(p \oplus q)$ And $p \leftrightarrow q$ are logically Equivalent:

p	q	$p \oplus q$	$\sim(p \oplus q)$	$p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

From the above observation, $\sim(p \oplus q)$ and $p \leftrightarrow q$ have same truth values for the same values of p and q.

$\therefore$ They are logically Equivalent.

7. p: it is below freezing. (Given)

q : it is snowing (Given)

(i) $p \wedge q$: it is below freezing and snowing.

(ii) $p \wedge \sim q$: it is below freezing And not snowing.

(iii) $\sim p \wedge \sim q$: It is not below freezing And not snowing.

(iv) $p \vee q$: it is either below freezing or snowing.

(v) $p \rightarrow q$: If it is below freezing then it is snowing.

(vi) $(p \vee q) \wedge (p \rightarrow \sim q)$: it is either below freezing or snowing and if it is below freezing then it is not snowing.

(vii) $p \leftrightarrow q$: it is below freezing if and only if it is snowing.

8. Given statement;

if it snows today, I will ski tomorrow

Converse : If I ski tomorrow, it will snow today.

Contrapositive : If I don't ski tomorrow, it won't snow today.

Inverse : If it doesn't snow today, I won't ski tomorrow.

9. (i) Given,

$\forall x\,(P(x) \wedge Q(x)) \equiv \forall x\,P(x) \wedge \forall x\,Q(x)$

Let $\forall x(P(x) \wedge Q(x))$ be true,

$\Rightarrow$ Now for all 'a' in the domain, $P(a) \wedge Q(a)$ is true

$\Rightarrow$ so both $P(a)$ and $Q(a)$ are true.

$\therefore$ For all 'a' in the domain,

$P(a)$ & $Q(a)$ are true.

$\therefore$ $\forall x(P(x) \wedge Q(x))$ is true

$\therefore$ $\forall x\,P(x) \wedge \forall x\,Q(x)$ is true.

Let $\forall x\,P(x) \wedge \forall x\,Q(x)$ be true,

Now, $\forall x\,P(x)$ and $\forall x\,P(x)$ are true.

Hence for each element a in the domain, both $P(a)$ & $Q(a)$ are true

$\Rightarrow$ $P(a) \wedge Q(a)$ is true for each element a in the domain.

$\therefore$ $\forall x(P(x) \wedge Q(x)$ is true.

Hence, $\forall x(P(x) \wedge Q(x))$ and $\forall x\,P(x) \wedge \forall x\,Q(x)$ are logically Equivalent.

(ii) Find, if $\forall x\,(P(x) \vee Q(x))$ and $\forall x\,P(x) \vee \forall x\,Q(x)$ are logically Equivalent.

Now, Let x represent all humans

Let $P(x)$: x is a male

$Q(x)$: x is a female

$P(x) \vee Q(x) \Rightarrow$ x is either a male or a female.

Hence, $\forall x\,(P(x) \vee Q(x))$ is true {for all humans, they can either be a male or female]

Now, $\forall x\,P(x)$ represents Every human is a male

$\forall x\,Q(x)$ represents Every human is a female

$\therefore$ Both $\forall x\,P(x)$ and ~~Qut~~ $\forall x\,Q(x)$ are false.

$\therefore$ $\forall x\,P(x) \vee \forall x\,P(x)$ is false

Hence, $\forall x\,(P(x) \vee Q(x))$ and $\forall x\,P(x) \vee \forall x\,Q(x)$ are not logically equivalent.

10. Given,

Let $C(x)$ represent "x is in this class"

$J(x)$ represent "x knows Java Programming"

$H(x)$ represent "x can get a high Paying Job"

The Premises are $\exists x(C(x) \wedge J(x))$ & $\forall x(J(x) \rightarrow H(x))$

The conclusion is $\exists x(C(x) \wedge H(x))$

Step No	Statement	Reason
1.	$\exists x(C(x) \wedge J(x))$	P
2.	$C(a) \wedge J(a)$	Existential Specification
3.	$C(a)$	T (Simplification)
4.	$J(a)$	T (Simplification)
5.	$\forall x(J(x) \rightarrow H(x))$	P
6.	$\cancel{\forall x}\ J(a) \rightarrow H(a)$	Universal Specification
7.	$H(a)$	T Modus ponens
8.	$C(a) \wedge H(a)$	T Conjunction
9.	$\exists x(C(x) \wedge H(x))$	Existential Generalisation

11. (i) Equivalent form of $\sim(\forall x \in N\,(x \text{ is prime} \rightarrow x^2+1 \text{ is even}))$

$x \in N$

$P(x) \Rightarrow x$ is prime

$Q(x) \Rightarrow x^2+1$ is even

1) $\sim(\forall x\,(P(x) \rightarrow Q(x))$

2) $\sim\forall x\,(P(x) \rightarrow Q(x))$

3) $\exists x \sim(P(x) \rightarrow Q(x))$ $[\sim\forall x\,(R(x)) \equiv \exists x \sim(R(x))]$

4) $\exists x \sim(\sim P(x) \vee Q(x))$ $[p \rightarrow q \Rightarrow \sim p \vee q]$

5) $\exists x \in N\,(P(x) \wedge \sim Q(x)))$ [Morgan's Law]

Hence, The Equivalent form is

$(\exists x \in N\,(x \text{ is prime and } x^2+1 \text{ is odd}))$

(ii) Equivalent form of $\sim(\exists x \in Q\,(x>0 \wedge x^3=2))$

$x \in Q$

$P(x) \Rightarrow x>0$

$Q(x) \Rightarrow x^3=2$

① $\sim(\exists x\,(P(x) \wedge Q(x))$

② $\sim\exists x\,(P(x) \wedge Q(x))$

③ $\forall x\,(\sim((P(x) \wedge Q(x)))$ $[\sim\exists x\,(P(x) \equiv \exists x \sim(P(x))]$

④ $\forall x \in Q\,(\sim P(x) \vee \sim Q(x))$ Morgan's Law

⑤ $\forall x \in Q\,(x \leq 0 \vee x^3 \neq 2)$

$\therefore$ The Equivalent form is $\forall x \in Q\,(x \leq 0 \vee x^3 \neq 2)$

12. Given,

Premises $\Rightarrow$ $\forall x(Q(x) \to P(x))$ & $\exists x\, Q(x)$

① $\forall x(Q(x) \to P(x))$ Premise

② $Q(a) \to P(a)$ for some values of a, $Q(a)$ is True

③ $Q(a) \to P(a)$ is True

$\therefore$ if $Q(a)$ is True then $P(a)$ is true by Modus Ponens.

Hence, $\exists x\, P(x)$ is True.

13. Given,

$r^3 + r + 1 = 0$

To Prove that r is not a rational number.

$\Rightarrow$ By Proof of Contradiction.

Let r be a rational number, $r = \frac{a}{b}$

Now, substitute.

$$\left(\frac{a}{b}\right)^3 + \left(\frac{a}{b}\right) + 1 = 0$$

$$\Rightarrow \frac{a^3 + ab^2 + b^3}{b^3} = 0$$

$$\Rightarrow a^3 + ab^2 + b^3 = 0$$

Case ① a is even & b is odd then

a^3 will be even, ab^2 is even, b^3 is odd.

$\therefore a^3 + ab^2 + b^3$ is odd.

But $a^3 + ab^2 + b^3 = 0$, & 0 is Even.

Case ② a is odd and b is even then:

a^3 is odd, ab^2 is even, b^3 is even

$\therefore$ $a^3+ab^2+b^3$ is odd

but $a^3+ab^2+b^3=0$ and 0 is even

Case ③ a and b both are odd and have no common factor other than 1

a^3 is odd, b^3 is odd, ab^2 is odd

but, $a^3+ab^2+b^3=0$ & 0 is even

$\therefore$ All the cases Assumptions leads to Contradictions.

Hence, There is no rational number for r in $r^3+r+1=0$

(ii) Given,

$p \Rightarrow n^3+5$ is odd

$q \Rightarrow n$ is even

To Prove that $p \rightarrow q$

we have to prove this by proof of Contrapositive.

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

$\sim q$: n is odd

$\sim p$: n^3+5 is even

$\Rightarrow$ n is odd

$n = 2k+1$ (For odd numbers)

Now in n^3+5

$\Rightarrow (2k+1)^3+5$

$\Rightarrow 8k^3+4k^2+2k+1+5$

$\Rightarrow 8k^3+4k^2+2k+1+5$

16 $\Rightarrow 8k^3+4k^2+2k+6$

$\Rightarrow 2(4k^3+2k^2+k+3)$

let $4k^3+2k^2+k+3$ be 'm'.

$\Rightarrow n^3+5 = 2m$

Hence, we proved that if n^3+5 is odd, then n is even by using Proof of contrapositive.

14.

(i) To Find,

if x is a real number, $[2x] = [x] + [x + 1/2]$

Case(i)

Let $x = n+d$ and $0 \le d < 0.5$ and $n \in I$

$[2x] = [2(n+d)] = [2n+2d] = 2n \quad (2d < 1)$

$[x] = [n+d] = n$

$[x+\frac{1}{2}] = [n+d+1/2] = n \left((d+\frac{1}{2}) < 1 \right)$

$\Rightarrow 2n = n+n$

L.H.S = R.H.S

Hence, we Proved for this case.

Case(ii)

Let $x = n+d$ and $0.5 \le d < 1$ and $n \in I$

$[2x] = [2(n+d)] = [2n+2d] = 2n+1 \quad (2d > 1)$

$\Rightarrow [x] = [n+d] = n$

$\Rightarrow [x+\frac{1}{2}] = [n+d+\frac{1}{2}] = n+1 \left((d+1/2) > 1 \right)$

$\therefore 2n+1 = n+n+1$

L.H.S = R.H.S

Hence, we Proved for this case.

$\therefore$ we proved that $\lceil 2x \rceil - \lceil x \rceil + \lceil x+\frac{1}{2} \rceil$

(ii) To find,

for real numbers x and y, if $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$

case(i) : x and y are integers

if x, y are integers, then xy also an integer.

$\lceil xy \rceil = xy$ [Floor of an integer is the integer itself]

$\lceil x \rceil = x$

$\lceil y \rceil = y$

$\therefore xy = xy$ [L.H.S = R.H.S]

Hence, we proved for this case.

case(ii) x & y are not integers

Now, Let $x = 3.1$ $y = 2.5$

$\lceil xy \rceil = \lceil 3.1 \times 2.5 \rceil = \lceil 7.75 \rceil = 7$

$\lceil x \rceil = \lceil 3.1 \rceil = 3$

$\lceil y \rceil = \lceil 2.5 \rceil = 2$

$\lceil xy \rceil = 7$

$\lceil x \rceil \lceil y \rceil = 3 \times 2 = 6$

$7 \neq 6$

L.H.S $\neq$ R.H.S

Hence, when either x, y are integers or both the statement is false.

$\therefore$ we proved that $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$ is not true for all cases.

(iii) To find,

for all real numbers x & y, if $[x+y] = [x] + [y]$

Case (i) x and y are integers

if x, y are integers, then $x+y$ is also an integer.

$[x+y] = x+y$

$[x] = x$ [floor of an integer is the integer itself]

$[y] = y$

$x+y = x+y$

L.H.S = R.H.S

Hence, we proposed it is true for this case

Case (ii) x and y are not integers.

Let $x = 2.5$, $y = 3.75$

$[x+y] = [2.5+3.75] - [6.25] = 6$

$[x] = [2.5] = 2$

$[y] = [3.75] = 3$

$6 \neq 5$

L.H.S $\neq$ R.H.S

Hence, we proved that the statement is false

if both x and y are not integers.

$\therefore$ $[x+y] = [x] + [y]$ is not true for all cases.

15) To Prove that, $3^n + 7^n - 2$ is Divisible by 8 using principle of mathematicccal induction.

Let $P(n) = 3^n + 7^n - 2$

$P(1) = 3^1 + 7^1 - 2$

$= 8$

8 is Divisible by 8

Hence, the P(1) is true.

Induction step: Let P(k) be true

$P(k) = 3^k + 7^k - 2$ is Divisible by 8

T.P.T $P(k+1) = 3^k + 7^k - 2$ is divisible by 8.

Let $3^k + 7^k - 2 = 8k$

$\Rightarrow 3(3^k) + 7(7^k) - 2$

$\Rightarrow 3(3^k + 7^k - 2) + 4(7^k) + 4$

$\Rightarrow 3(P(k)) + 4(7^k + 1)$

W.K.T, $3^k + 7^k - 2$ is Divisible by 8.

W.K.T, 7^k is odd for all values of k

$\therefore 7^k + 1$ is even

Let $7^k + 1 = 2m$

$\therefore 8(3k) + 4(2m)$

$\Rightarrow 8(3k + m)$ is Divisible by 8.

Hence Proved.

16. Given,

N is set of positive integers between 100 and 999 inclusive

$n(N) = 900$

i) Numbers divisible by 7

$\Rightarrow \left[\frac{n(N)}{7}\right] = \left[\frac{900}{7}\right] = 128$

$\therefore$ 128 integers are divisible by 7.

ii) Numbers which are odd

$\Rightarrow \left[\frac{n(N)}{2}\right] = \left[\frac{900}{2}\right] = 450$

Total numbers − No of even numbers = odd numbers

$\Rightarrow$ 450 integers are odd.

iii) Numbers not Divisible by 4

$\Rightarrow n(N) - \left[\frac{n(N)}{4}\right] = 900 - \left[\frac{900}{4}\right] = 900 - 225 = 675$

$\therefore$ 675 integers are not Divisible by 4

(iv) Numbers Divisible by 3 (or) 4

$\Rightarrow$ Subration Rule

$\Rightarrow |A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

$\Rightarrow$ No of Divisible by 3 + No of Divisible by 4 − No of divisible by (3 & 4)

$\Rightarrow \left[\frac{n(N)}{3}\right] + \left[\frac{n(N)}{4}\right] - \left[\frac{n(N)}{12}\right] \Rightarrow 300 + 225 - 75 = 450$

$\Rightarrow \therefore$ 450 integers are Divisible by 3 (or) 4.

v) Numbers not Divisible by 3 or 4

$\Rightarrow \lceil n(N) \rceil - \left[\lceil \frac{n(N)}{3} \rceil + \lceil \frac{n(N)}{4} \rceil - \lceil \frac{n(N)}{12} \rceil \right]$

$\Rightarrow 900 - [300 + 225 - 75]$

$\Rightarrow 450$

$\therefore$ 450 integers are not Divisible by 3 (or) 4

vi) Numbers Divisible by 3 but not 4

$\Rightarrow \lceil \frac{n(N)}{3} \rceil - \lceil \frac{n(N)}{3\times 4} \rceil = 300 - 75 = 225$

$\therefore$ 225 integers are Divisible by 3 but not 4.

vii) Numbers Divisible by 3 & 4

$\lceil \frac{n(N)}{3\times 4} \rceil = \lceil \frac{900}{12} \rceil = 75$

$\therefore$ 75 integers are divisible by both 3 & 4.

17) There are 2^7 bit strings of length 7

① $\underset{1\,way}{0}\ \underset{1\,way}{0}\ \underbrace{-\ -\ -\ -\ -}_{2\,ways\,each}$ $\Rightarrow 2^5$ ways

② $\underbrace{-\ -\ -\ -}_{2\,ways\,each}\ \underbrace{1\ 1\ 1}_{1\,way\,each}$ $\Rightarrow 2^4$ ways

③ $\underset{1\,way}{0}\ \underset{1\,way}{0}\ \underbrace{-\ -}_{2\,ways\,each}\ \underbrace{1\ 1\ 1}_{1\,way\,each}$ $\Rightarrow 2^2$ ways

Bitstrings that start with 2 0's + Bitstrings that end with 3 1's − Bitstrings that start with 2 0's and end with 3 1's

$\Rightarrow 2^5 + 2^4 - 2^2 = 32 + 16 - 4$

$= 44$ ways

$\therefore$ Bit strings that either start with 2 0's or end with 3 1's are in 44 ways.

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18. i) To Prove that, if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.

Let set $I = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Four pairs with sum equal to 9 : $(1,8), (2,7), (3,6), (4,5)$

we use pigeonhole principle here.

Pigeonhole Principle :- if N objects are placed into k boxes, then there is atleast one box containing at last $\left\lceil \frac{N}{K} \right\rceil$ objects.

$\therefore$ We are taking 5 integers from 4 pairs here,

$N = 5, \; k = 4$

$\Rightarrow \left\lceil \frac{5}{4} \right\rceil = 2$

Hence we can select a guaranteed pair of integers whose sum is 9.

ii) if we choose the integers as $\{1, 2, 3, 4\}$ as Example,

No two integers sum is equal to 9.

Hence, the condition is not true in all cases.

iii) Given,

$N = 677$

$k = 38$

$\Rightarrow \left\lceil \frac{N}{k} \right\rceil = \left\lceil \frac{677}{38} \right\rceil = 18$

Hence, 18 different rooms will be needed.

19. Given,

Number of men = 10

Number of women = 15

Maximum members in Committee = 6.

(i) No. of women and No. of men are equal = 3

$\Rightarrow$ There are ${}^{15}C_3 \times {}^{10}C_3$ ways to form a comittee

$\Rightarrow \frac{15!}{3!(15-3)!} \times \frac{10!}{(10-3)!\,3!} = \frac{15\times14\times13}{6} \times \frac{10\times9\times8}{6} = 54600$

ii) Number of women greater than no. of men

$\Rightarrow$ ① 6 W, 0 Men

② 5 women, 1 men

③ 4 women, 2 men

$\Rightarrow ({}^{15}C_6 \times {}^{10}C_0) + ({}^{15}C_5 \times {}^{10}C_1) + ({}^{15}C_4 \times {}^{10}C_2)$

$\Rightarrow \left(\frac{15!}{6!\times9!} \times \frac{10!}{0!\times10!}\right) + \left(\frac{15!}{5!\times10!} \times \frac{10!}{1!\times4!}\right) + \left(\frac{15!}{4!\times1!} \times \frac{10!}{2!\times8!}\right)$

$\Rightarrow \left(\frac{15\times14\times13\times\cancel{12}^{2}\times11\times\cancel{10}^{8}}{\cancel{1}\times\cancel{2}\times\cancel{3}\times\cancel{4}\times\cancel{5}\times\cancel{6}}\right) + \left(\frac{\cancel{15}^{3}\times\cancel{14}^{7}\times13\times\cancel{12}\times11\times\cancel{10}^{5}}{\cancel{5}\times\cancel{4}\times3\times2\times1}\right)$

$+ \left(\frac{15\times14\times13\times12\times45}{4\times3\times2\times1}\right)$

$\Rightarrow (5005) + (30030) + (61425) = 96460$

$\therefore$ There are 96460 ways to arrange a Commitee where women are greater than men.

20 (i) The Binomial theorem states

$$(a+b)^n = \sum_{k=0}^{n} a^k \binom{n}{k} b^{n-k}$$

Here, $a = 2x$

$b = -3y$

$n = 200$

$k = 101$

$n-k = 199$

$\therefore$ Coefficient of $x^{101} y^{99}$

$$\Rightarrow {}^{200}C_{99} \times (2x)^{101} \times (-3y)^{99}$$

$$\Rightarrow \left(\left(\frac{200!}{99! \times 101!} \right) \times (2)^{101} \times -(3)^{99} \right) \left(x^{101} y^{99} \right)$$

$\therefore$ The Coefficient is $-\left(\frac{200! \times (2)^{101} \times (3)^{99}}{99! \times 101!} \right)$

ii) We prove that

$3^n = \sum_{k=0}^{n} 2^k \binom{n}{k}$ using Binomial theorem

Binomial theorem states that:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

if $x = 2$ and $y = 1$,

$$(2+1)^n = \sum_{k=0}^{n} \binom{n}{k} 2^k (1)^{n-k}$$

$$3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k$$

Hence proved,,.

(iii) Given,

$$\binom{n+1}{r} = {}^{n+1}C_r = \frac{(n+1)!}{r!(n+1-r)!}$$

$$\binom{n}{r-1} = {}^{n}C_{r-1} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\binom{n}{r} = {}^{n}C_r = \frac{n!}{r!(n-r)!}$$

To Prove that, ${}^{n+1}C_r = {}^{n}C_{r-1} + {}^{n}C_r$

R.H.S

$$\Rightarrow \frac{n!}{(r-1)!\,(n-r+1)!} + \frac{n!}{r!\,(n-r)!}$$

$$\Rightarrow \frac{n!}{(r-1)!\,(n-r+1)\,(n-r)!} + \frac{n!}{r(r-1)!\,(n-r)!} = \frac{n!}{(r-1)!\,(n-r)!}\left(\frac{1}{n-r+1} + \frac{1}{r}\right)$$

$$\Rightarrow \frac{n!}{(r-1)!\,(n-r)!}\left(\frac{r+n-r+1}{r(n-r+1)}\right) = \frac{(n+1)\,n!}{r(r-1)!\times(n-r+1)\cancel{\xi}(n-r)!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!} = L.H.S$$

L.H.S = R.H.S Hence Proved,,

21) Given, $(x,y) \in R$ if and only if $x = y+1$ or $x = y-1$

① **Reflexive**: R is reflexive if and only if $(x,x) \in R$

Now, $x \neq x+1$ & $x \neq x-1$

$\therefore$ R is not reflexive as $(x,x) \notin R$

② **Symmetric**: R is symmetric such that if $(x,y) \in R$ then $(y,x) \in R$

Now, $x = y+1 \Rightarrow y = x-1$ | $x = y-1 \Rightarrow y = x+1$

Hence, $(y,x) \in R$

$\therefore$ R is symmetric.

③ **AntiSymmetric**: R is antisymmetric such that if $(x,y) \in R$ and $(y,x) \in R$ then $x = y$

$(x,y) \in R$
$x = y+1$
$y = x-1$
$(y,x) \in R$ but $x \neq y$

$(x,y) \in R$
$x = y-1$
$y = x+1$
$(y,x) \in R$ but $x \neq y$

$\therefore$ R is not antisymmetric as $x \neq y$

④ **Transitive**: such that if $(x,y) \in R$ & $(y,z) \in R$ then $(x,z) \in R$

Now, $(x,y) \in R \Rightarrow x = y+1$
$(y,z) \in R \Rightarrow y = z+1$
$\therefore x = (z+1)+1$
$x = z+2$

$(x,y) \in R = x = y-1$
$(y,z) \in R \Rightarrow y = z-1$
$\therefore x = (z-1)-1$
$\boxed{x = z-2}$
but $x = z-1$ if $(x,z) \in R$
$\therefore (x,z) \notin R$

$\therefore$ R is not transitive.

Hence, R is not Reflective, Symmetric, not antisymmetric and not transitive.

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22) Given,

$R = \{(a,e), (b,a), (b,d), (c,d), (d,a), (d,c), (e,a), (e,b)$
$(e,c), (e,e)\}$

$A = \{a, b, c, d, e\}$

$$\therefore M_R = \begin{array}{c|c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Now, Matrix of $R^* = M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \cdots \vee M_R^{[n]}$

where n = no of elements in set A

$$\therefore M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee M_R^{[4]} \vee M_R^{[3]}$$

$$M_R^{[2]} = M_R \odot M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{[3]} = M_R^{[2]} \odot M_R = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$M_R^{[3]} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{[4]} = M_R^{[3]} \odot M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$M_R^{[4]} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{[5]} = M_R^{[4]} \odot M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$M_R^{[5]} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\therefore M_R^{*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee M_R^{[4]} \vee M_R^{[5]}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

∴ Transitive closure are:

$R^* = \{ (a,a), (a,b) (a,c), (a,d), (a,e), (b,a), (b,b)$
$(b,c), (b,d), (b,e), (c,a), (c,b), (c,c), (c,d),$
$(c,e), (d,a), (d,b), (d,b), (d,c), (d,d), (d,e)$
$(e,a), (e,b), (e,c), (e,d), (e,e). \}$

—— The End ——