EM215: Numerical Methods

Lab Assignment 2

You may use either Python or Matlab to solve the followings.

(1) (a) Estimate the first derivative of the function,

$$f(x) = -0.1x^4 - 0.5x^2 - 0.5x + 1.2$$

at $x = 0.5$ with forward, backward and centered difference approximation at $x = 0.5$. In each case, start with a step size of 1 and halve the step size and repeat calculations until the step size reaches $\frac{1}{2^{15}}$. Given that $f'(0.5) = -1.05$, tabulate the values: step size, derivative approximations using the three methods and the absolute errors of the estimates of the three methods. Plot the errors versus step size, in log-log scale, for the three methods.

Explain how you identify the orders of errors in the methods using the gradients of the graphs drawn.

(b) Estimate the second derivative of the function given in part (a) above at $x = 0.5$. Use the centered difference approximation and repeat the calculations as before until the step size reaches $\frac{1}{2^{17}}$. Given that $f''(0.5) = -1.3$, tabulate the values: step size, the second derivative approximation and the absolute error. As earlier, plot the error versus step size in log-log scale. Explain the behavior of the graph.

(2) Consider the graphical method to approximate the roots of $f(x) = 4\log x - x = 0$ (page 27 of the lecture note). Considering the graphs of,

$$f_1(x) = x$$
$$f_2(x) = 4\log x$$
$$f_2'(x) = \frac{4}{x}$$

explain how/why the fixed point iteration converges/diverges for starting values,

$x_0 < x_{r1}$, $x_{r1} < x_0 \leq 4$, and $x_0 > 4$. $x_{r1} < x_{r2}$ are the two roots of $f(x) = 0$.

Deadline for submission: 4.00PM on 24th of December 2021