EM 215 - Lab Assignment 1

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E/18/397

a) Instead of the velocity at time t = 0, if it is assumed that the velocity at some time t = tx was known as v = vx, derive the analytical solution for the velocity, v.

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

The terminal velocity is $\frac{mg}{c}$

$$\frac{dv}{\left(g - \frac{cv}{m}\right)} = dt$$

$$\int \frac{dv}{g - \frac{c}{m}v} = \int dt$$

$$-\frac{m}{c}\ln\left(g - \frac{c}{m}v\right) = t + K \qquad K \text{ is arbitrary constant}$$

We know that $t = t_x$ and $v = v_x$

$$K = -\frac{m}{c}\ln\left(g - \frac{c}{m}v_x\right) - t_x$$

$$-\frac{m}{c}\ln\left(g - \frac{c}{m}v\right) + \frac{m}{c}\ln\left(g - \frac{c}{m}v_x\right) + t_x = t$$

$$\left(\frac{g - \frac{c}{m}v_x}{g - \frac{c}{m}v}\right)^{\frac{m}{c}} = e^{t - t_x}$$

$$\left(g - \frac{c}{m}v\right) = \left(g - \frac{c}{m}v_x\right)e^{-\frac{c}{m}(t - t_x)}$$

$$\underline{\underline{v = \frac{mg}{c} - \frac{m}{c}\left(g - \frac{c}{m}v_x\right)e^{-\frac{c}{m}(t - t_x)}}}$$

b) If tx = 10 s and vx = 44.87 m/s, derive a numerical scheme to calculate velocities of the body from time t = 0 – 10 s

$$\frac{dv}{dt} = g - \frac{c}{m}v \longrightarrow (1)$$

The terminal velocity is $\frac{mg}{c}$

For two points in time. $t_2 > t_1$, where $t_2 - t_1 = \Delta t$ small, the derivative $\frac{dv}{dt}$ can be approximated as

$$\frac{dv}{dt} \simeq \frac{v_2 - v_1}{t_2 - t_1} = \frac{v_2 - v_1}{\Delta t} \longrightarrow (2)$$

by (1) and (2)

$$\frac{v_2 - v_1}{\Delta t} = g - \frac{c}{m}v$$

$$\therefore v_2 = \left(g - \frac{c}{m}v_1\right)\Delta t + v_1$$

c) Using the analytical solution, you derived in part (a) above, show graphically, the variation of velocity of the body for t = 0 – 10 s. Use black colour for the graph.

analytical solution code.m

```
clear
clc

% variables
m = 68.1;
c = 12.5;
g = 9.8;
vx = 44.87;
tx = 10;

% analytical solution
t=[10:-1:0];
v = m*g/c-(m/c)*(g-c*vx/m)*exp(-c*(t-tx)/m);
plot(t,v,'-k');
grid on;
legend('Analytical Solution');
xlabel('Time(s)');
ylabel('Velocity(m/s)');
```

Figure 01: matlab code for analytical solution

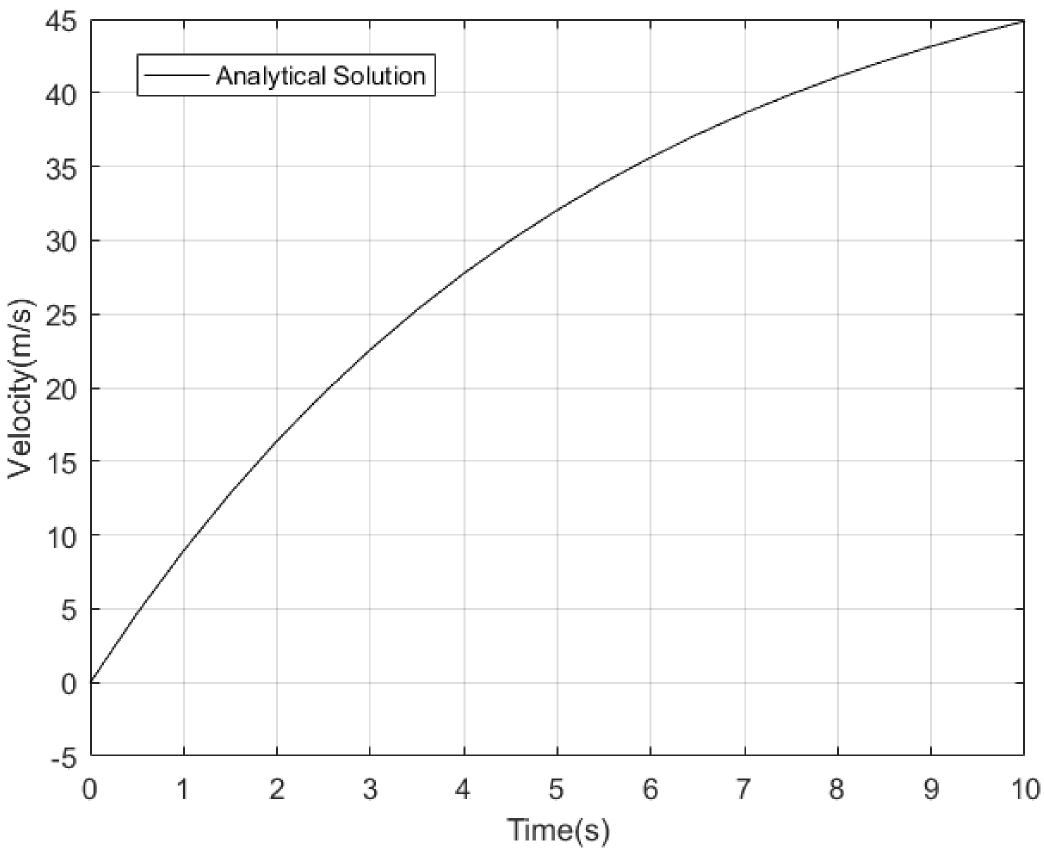

Figure 02: matlab code output

d) Use computer to solve the numerical scheme in part (b) above to give the velocity of the body for t = 0 – 10 s. Use red colour for the graph, and plot it on the same axes as those of (c).

full code.m

```
clear
clc

% variables
m = 68.1;
c = 12.5;
g = 9.8;
vx = 44.87;
tx = 10;

% analytical solutio0n
t=[10:-0.1:0];
v = m*g/c-(m/c)*(g-c*vx/m)*exp(-c*(t-tx)/m);


% Numerical solution
v1 = vx;
t1 = tx;
delt =0.01;
TV = [t1,v1];

while 1
    t2 = t1 - delt;
    v2 = (g-c/m*v1)*(t2-t1)+v1;
    if t2<0
        break
    end
    TV = [TV;[t2,v2]];
    v1 = v2;
    t1 = t2;
end

% ploting
plot(t,v,'-k'); hold on;
plot(TV(:,1),TV(:,2),'.-r');
grid on;
legend('Analytical Solution','Numerical Solution')
xlabel('Time(s)');
ylabel('Velocity(m/s)');
```

Figure 03: Full code for analytical and numerical solutions.

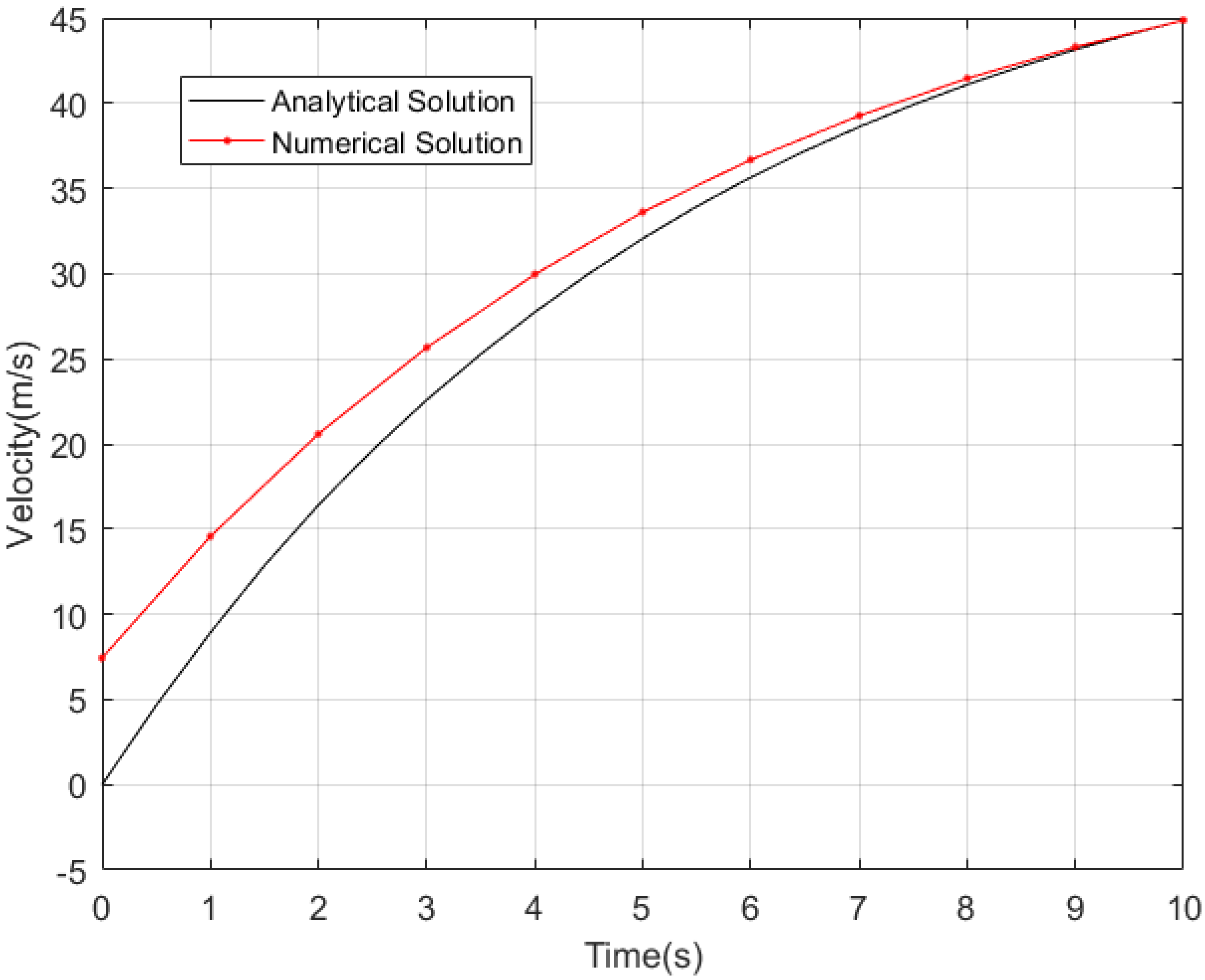

Figure 04: matlab code output for above code (when Δt = 1s)

e) Discuss the possible reasons for any discrepancies of the two solutions.

When both graphs are plotted together there were some small discrepancies can be seen. This could be happened due to some reasons.

In numerical method resultant graph was varied with Δt value. Therefore, some discrepancies may be due to Δt not being small enough. But if we decrease Δt value it may also lead to increase the time taken to complete the solution. Therefore, we should choose proper Δt value as our need. (see Figures 05,06 and 07)

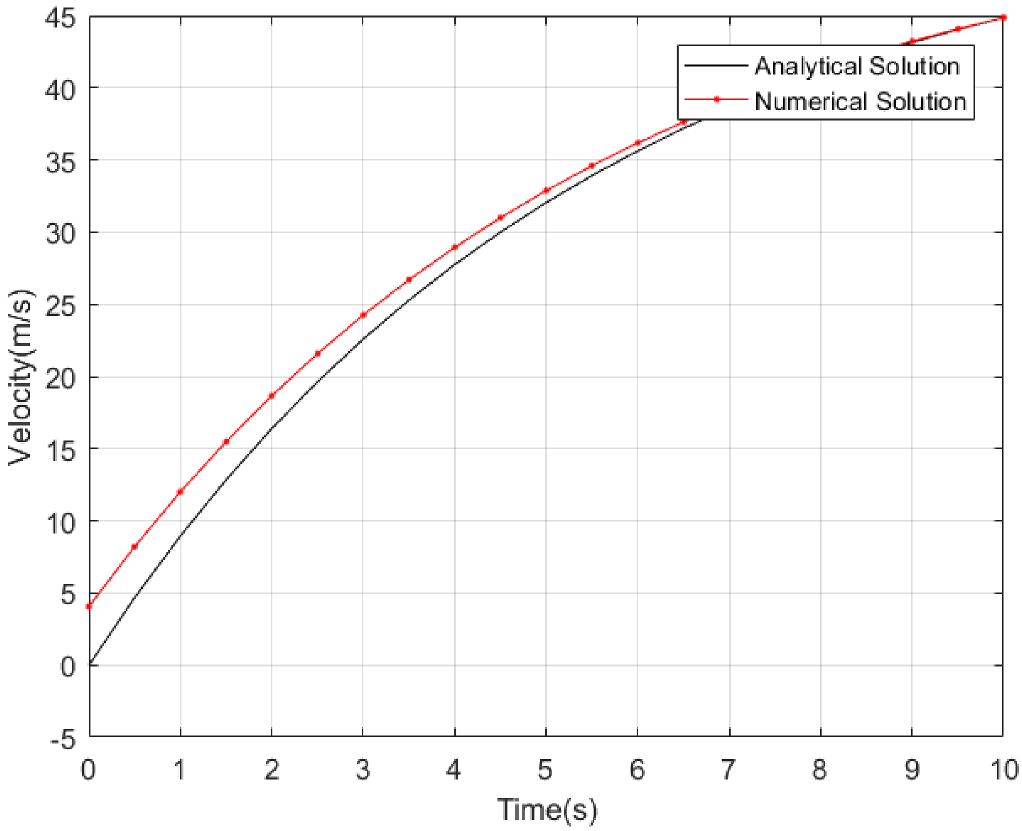

Figure 05: graph when Δt = 0.5s

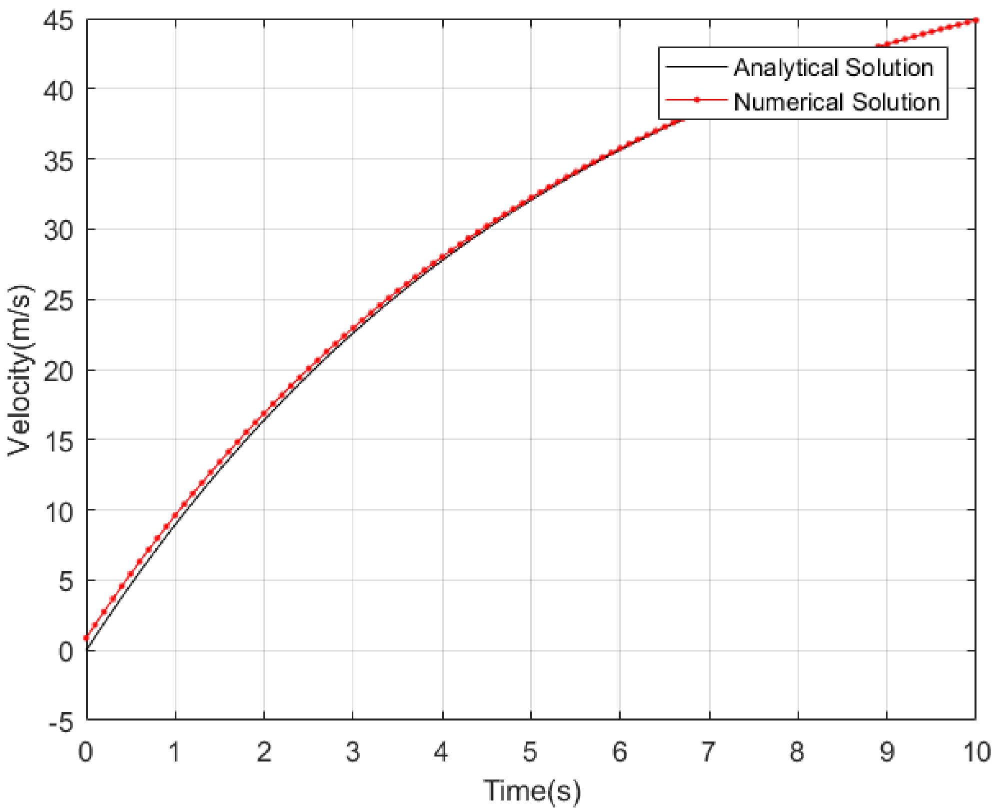

Figure 06: graph when Δt = 0.1s

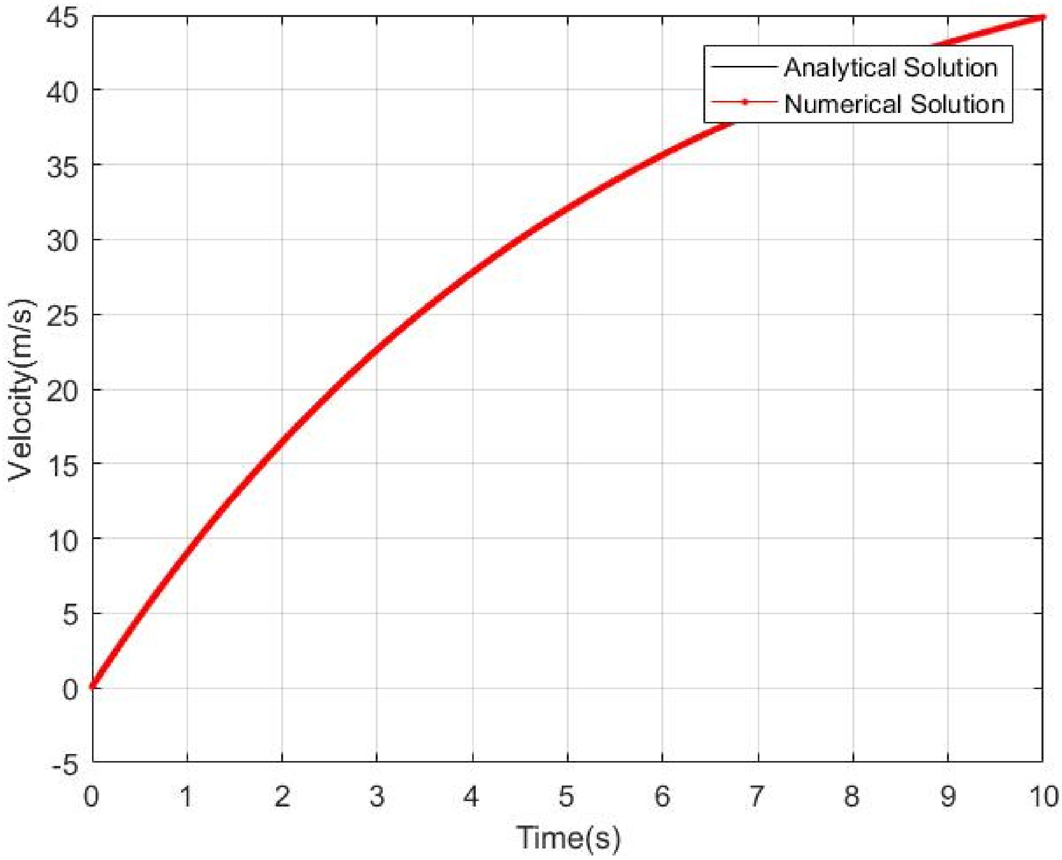

Figure 07: graph when Δt = 0.001s

And also, it may not possible to computers to represent the data accurately. At some cases computers make some approximations. This could be cause to small errors in calculated points. Therefore, point of the graph are slight offset from the analytical solution.