# discussion_HW3

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We are to calculate the eigenvector and values for a 3x3 matrix in Upper Triangular form, which is helpful. I'll do most of this work by-hand but with some machine computation. First set up the $det(A - \lambda I)$ equation.

$$det(\begin{bmatrix}1 & 2 & 3\\0 & 4 & 5\\0 & 0 & 6\end{bmatrix} - \begin{bmatrix}\lambda & 0 & 0\\0 & \lambda & 0\\0 & 0 & \lambda\end{bmatrix}) = 0$$

Which expands to the following:

$$(1-\lambda)det(\begin{bmatrix}4-\lambda & 5\\0 & 6-\lambda\end{bmatrix}) - (2)det(\begin{bmatrix}0 & 5\\0 & 6-\lambda\end{bmatrix}) + (3)det(\begin{bmatrix}0 & 4-\lambda\\0 & 0\end{bmatrix}) = 0$$

Thankfully, the last two terms of this expansion reduce to zero. Thus we are left with the following:

$$(1-\lambda)(24 - 10\lambda + \lambda^2) = 0$$

This expands to the following,

$$24 - 34\lambda + 11\lambda^2 - \lambda^3 = 0$$

Which can be factored via the fundamental theorem of algebra (luckily $\lambda = 1$ is a root!). We see that this reduces to,

$-\lambda^2 + 10\lambda - 24 = 0$. Which factors our last two remaining eignevalues and our set of eigenvalues are as follows:

$\lambda_1 = 1, \lambda_2 = 4, and \lambda_3 = 6.$

If we return the first eigenvalue in the original matrix $A$ we apply $\lambda * I - A$ as the following,

$$\begin{bmatrix}0 & 2 & 3\\0 & 3 & 5\\0 & 0 & 5\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} = 0$$

This is as far as I got but I am struggling to calculate the eigenvectors by hand. Has anyone computed these?