Discussion 15

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Taylor Series for sin(x)

It is possible to write a trigonometric function as a polynomial thanks to the Taylor series. As a reminder the Taylor series is,

$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$

and centered at zero we get,

$\sum_{n=0}^{\infty} \frac{f^n(0)}{n!}(x)^n = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots \frac{f^n(0)}{n!}x^n$

If we have a function, it's derviatives, and those derivatives evaluated at zero, we can piece together a Taylor expansion so as to develop a polynomial expression for f(x), our given function.

Let's look at $f(x) = sin(x)$

The first eight derivatives are (starting a zero) : $[sin(x), cos(x), -sin(x), -cos(x), sin(x), cos(x), -sin(x), -cos(x)]$

You might be thinking, it repeats, that is expected as sine is periodic. If we evaluate the above at x=0 get the following, $[0, 1, 0, -1, 0, 1, 0, -1]$

Now plug these values into our Taylor series we get

$sin(x) = 0 + x + 0 - \frac{x^3}{3!} - 0 + \frac{x^5}{5!} + 0 - \frac{x^7}{7!} + \ldots$

Which reduces to

$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

These first few terms probably do a good job approximating the sine function but we can re-write it in sumation form as well.

$sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} x^{2n+1}$