Week 8 discussion

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Chebyshev's Inequality

For the longest time I really never fully understood the value of Chebyshev's inequality. It just seemed like formalism without context to usefulness. For this week, I went in search of problem that could make this idea more meaningful.

I found this problem to be instructive for why Chebyshev's Inequality can be useful.

Suppose we extract an individual at random from a population whose members have an average income of \$40,000, with a standard deviation of \$20,000. What is the probability of extracting an individual whose income is either less than \$10,000 or greater than \$70,000? In the absence of more information about the distribution of income, we cannot compute this probability exactly. However, we can use Chebyshev's inequality to compute an upper bound to it. If X denotes income, then X is less than \$10,000 or greater than \$70,000 if and only if $|X - \mu| > k$.

This nice because it is a meaningful context to me. Here $\mu = \$40000$ and $k = \$30000$. Chebyshev's Inequality becomes the following;

$P(|X - \mu|) > k) \leq \frac{\sigma^2}{k^2} = \frac{4}{9}$

This is the upper bound on the probability that a randomly selected person will fall *outside* the income range of 10K to 70K. Thus, there is a 44% maximum chance a randomly selected person will be outside this range.

This is a really nice idea if you are interested in understand the maximum probability a given event can occur.