HW 8 Data 605

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Problem 11 Page 303

In general the expeccted value for exponential distributions are as follows:

$E[X_i] = \frac{1}{\lambda_i}$.

For X_i independent random variables we know that the expected value is simply:

$E[X_i] = \frac{1}{1000}$

If we are looking for the *first* lightbuld to burn out then we are interested in the minimum of the X_i from n=1 to n=100. For each lightbulb we have an expected value of $\frac{1}{1000}$ and using the ideas of CDF we can sum those values across all members of X_i to find the minimum expected value of X_i Resulting in:

$\sum_1^{100} \lambda_i = \frac{100}{1000} = \frac{1}{10}$

Thus, the minimum expected value is;

$min(E[X_i]) = \frac{1}{\frac{1}{10}} = 10$. We can expect the first lightbulb to burn out in 10 hours.

Problem 14 page 303

This is a convolution problem for a difference of two random variable with the same exponential density, $f_z(z) = \lambda e^{(-\lambda z)}$

Using the PDF ideas from the video we see that this be treated similarly,

$\int_{-\infty}^{\infty} f_{x1}(z-x) f_{x2}(x_1) dx_2$

Given that we already know f_{x1} and f_{x2} are the exponential density functions we can rewritte the above as the following.

$\int_{-\infty}^{\infty} \lambda^2 e^{-\lambda(z+2x_{x2})} e^{\lambda z} dx_1$

As much as I love to do this integral by hand, the internet has told me that this nicely reduces to the following states;

$f_z(z) = \begin{cases} \frac{\lambda}{2} e^{\lambda z}, z < 0 \\ \frac{\lambda}{2} e^{-\lambda z}, z > 0 \end{cases}$

This defines the desired outcome $\frac{\lambda}{2} e^{-\lambda |z|}$ *Note that λ is negative for the positive z.

Problem 1 page 320

Let X be a continuous random variable with mean µ = 10 and variance $\sigma^2 = 100/3$. Using Chebyshev's Inequality, find an upper bound for the following probabilities.

    a. $P(|X - 10| \geq 2)$

```
var <- 100/3
k_square<-4
upper<- var/k_square
upper
```

```
## [1] 8.333333
```

Since this is a probability, we take it to be 1.

    b. $P(|X - 10|) \geq 5)$

```
k_square<-25
upper<- var/k_square
upper
```

```
## [1] 1.333333
```

Again, this is a probability so we take it to be 1.

    c. $P(|X - 10| \geq 9)$

```
k_square<-81
upper<- var/k_square
upper
```

```
## [1] 0.4115226
```

Upper bound is around 40%.

    d. $P(|X - 10| \geq 20)$

```
k_square<-400
upper<- var/k_square
upper
```

```
## [1] 0.08333333
```

Upper bound is about 8%