Data 605 HW 7

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Problem Set #1

Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

This is kind of an amazing result. The distribution of the minimum number of Xi's can expressed in a simple quotient.

The sample space here is the k^n since there are Xn variables uniformly distributed from 1 to k. We can then use, effectively, the *compliment principal* to determine an expression for each Y value. If Y=1 we get $k^n - (k-1)^n$. This the sample space minus all the Xi's that are NOT equal to Y=1. We can pull a similar trick for Y=2, that is find all the Xi's that are not Y=1 or Y=2 and subtract that form the sample space. Thus, $k^n - (k-2)^n - (k^n - (k-1)^n)$. If you distribute the negative sign through the last term (for Y=1), we see that the k^n terms cancel out (which makes sense, it's the entire space) and we are left with $(k-1)^n - (k-2)^n$.

We can do this again for Y=3 and get $k^n - (k-3)^n - ((k-1)^n - (k-2)^n) - (k^n - (k-1)^n)$. Again, distribute the negative and combine like terms and this reduces to $(k-2)^n - (k-3)^n$. We can safely generalize this result to $(k-j+1)^n - (k-j)^n$ for all j between 1 and k integers.

To find the distribution, we simply divide by the space of possiblities. $m(j) = \frac{(k-j+1)^n - (k-j)^n}{k^n}$ for all j between 1 and k integers. So much of math is pattern matching…

Problem Set #2

Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part).

What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

$E(X) = \frac{1}{p}$ and $\sigma = \sqrt{\frac{1-p}{p^2}}$

```
p<-1/10
E_x<-1/p
sd<-sqrt((1-p)/p^2)
paste("Expected value:",E_x, "sd: ",sd)
```

```
## [1] "Expected value: 10 sd:  9.48683298050514"
```

R has a nice built in function to evaluate probabilities.

```r
paste("probability of failure in 8 years: ",pgeom(8, p, lower.tail = F))
```

```
## [1] "probability of failure in 8 years:  0.387420489"
```

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

$\lambda e^{-(\lambda x)}$

```r
lambda<-1/10
x<-8
paste("probability of failure in 8 years: ",exp(-lambda*x))
```

```
## [1] "probability of failure in 8 years:  0.449328964117222"
```

```r
E_x<-1/lambda
sd<-sqrt(1/lambda^2)
paste("Expected value:",E_x, "sd: ",sd)
```

```
## [1] "Expected value: 10 sd:  10"
```

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

```r
n <- 8
p <- 1/10
comp <- 1-p
k <- 0
E_x<-n*p
sd<-sqrt(n*p*comp)
paste("probability of failure in 8 years: ",dbinom(k, n, p))
```

```
## [1] "probability of failure in 8 years:  0.43046721"
```

```r
paste("Expected value:",E_x, "sd: ",sd)
```

```
## [1] "Expected value: 0.8 sd:  0.848528137423857"
```

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

```r
lambda <- 0.8
paste("probability of failure in 8 years: ",ppois(0, lambda = lambda))
```

```
## [1] "probability of failure in 8 years:  0.449328964117222"
```

For Poisson lambda is the expected value and variance. Thus,

```r
paste("Expected value:",lambda, "sd: ",sqrt(lambda))
```

```
## [1] "Expected value: 0.8 sd:  0.894427190999916"
```