

# Binary Search Fix

## Understanding the "Kth Missing Positive Number" Problem

### Problem Statement Recap

You are given a strictly increasing array `vec[]` and a positive integer `k`. Your task is to find the `k`-th missing positive integer from the sequence.

### How to Identify Missing Numbers?

A strictly increasing array means every number is greater than the previous one. However, some numbers are missing in between the sequence, and we need to find the `k`-th missing one.

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## Examples to Understand the Problem

### Example 1

**Input:**

```
plaintext
```

```
vec = [4, 7, 9, 10], k = 1
```

**Missing numbers:**

```
plaintext
```

```
1, 2, 3, 5, 6, 8, 11, 12, ...
```

- The 1st missing number is `1`.

**Output:**

```
plaintext
```

## Example 2

### Input:

```
plaintext
```

```
vec = [4, 7, 9, 10], k = 4
```

### Missing numbers:

```
plaintext
```

```
1, 2, 3, 5, 6, 8, 11, 12, ...
```

- The 4th missing number is 5.

### Output:

```
plaintext
```

```
5
```

## Example 3

### Input:

```
plaintext
```

```
vec = [2, 3, 5, 8], k = 2
```

### Missing numbers:

```
plaintext
```

1, 4, 6, 7, 9, 10, 11, ...

- The 2nd missing number is 4.

#### Output:

plaintext

4

## Understanding the Two Key Cases

Now, let's analyze what happens when we go through the array while searching for the k-th missing number.

### Case 1: When $\text{vec}[i] < k$

⚠️ The current element in the array is smaller than k.

#### Effect:

- Since this number is present, it means one of the earlier missing numbers is accounted for.
- The missing number shifts forward by 1 because the current number reduces the missing count.
- So, increase k by 1.

### Example for Case 1

plaintext

```
vec = [2, 3, 5, 8], k = 2
```

- Initially, k = 2 (we are looking for the 2nd missing number).
- First element  $\text{vec}[0] = 2$ , which is less than  $k = 2$ , so we shift k to 3.
- Second element  $\text{vec}[1] = 3$ , which is less than  $k = 3$ , so we shift k to 4.

- Next element `vec[2] = 5`, which is greater than `k = 4`, so 4 is our answer.

### Final Output:

```
plaintext
```

```
4
```

### Case 2: When `vec[i] > k`

⚠️ The current element in the array is greater than `k`.

#### Effect:

- Since `k` is missing before this number appears, it must be one of the missing numbers.
- No need to shift `k`, as `k` is still missing.
- So, return `k` as the answer.

### Example for Case 2

```
plaintext
```

```
vec = [7, 10, 12], k = 4
```

- The first number is `7`, which is greater than `k = 4`.
- This means `4` is still missing and should be returned.

### Final Output:

```
plaintext
```

```
4
```

## Conclusion

- When a number smaller than `k` is present in `vec[]`, we shift `k` forward.

- When a number greater than  $k$  appears,  $k$  is the missing number and should be returned.
- This approach helps us efficiently find the  $k$ -th missing number without iterating through every number one by one.

Let me know if you need further clarification! 