Project - Math 310 - Probability & Statistics

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1 Approaches

1.1 Task 1 - Discrete Random Variables

To compute expected position, we simulate a large number (roughly 1000) of random walking bots. A user-defined probability for moving left, moving right and not moving at all is assigned. Initially, all bots stand at $x = 0$ and a computer generated random value determines its next position. This computer-generated random value is generated via a function that respects a user-defined distribution. The average of the final positions of bots give us good approximation for expected position, provided that number of bots in the simulation is sufficiently large.

1.2 Task 2 - Discrete Random Variables

To compute expected time of meeting of two random-walking bots separated with a given distance, we perform the same simulation on each of two initial points (x_i). To differentiate between the two groups of bots, let us call them red and blue for convenience. Whenever any of the 1000 red bots meets with any blue bot ($1000 + 1000 = 2000$ total bots), we record the time of meeting. A red-blue bot pair, that has been met, will not be recorded in a meeting again. Thus, a unique pair of red-blue bot can meet only once, which is their first meeting, or they don't meet at all. The simulation is required to run for a sufficiently long time to record a large number of meetings. Then the average of the meeting times give us a good approximation of expected meeting time.

1.3 Task 3 - Discrete Random Variables

In this task, our bot performs a 2D random walk. The bot can take a step-size within discrete set $\{0, 0.5, 1\}$ and can choose to take one direction of North, South, East or West. The walk is to be confined inside a circular region. The goal is to define a good boundary condition that enables random walk's tendency to make "ordered pictures". Since the domain of walk is circular discrete lattice, we will use a scheme proposed by H. Ciftci and M. Cakmak called restriction by completely probabilistic rule. This rule says that whenever our bot should step outside boundary, the rule will return the bot to some neighbor of origin as suggested in the letter of Ciftci(a) and M. Cakmak.

1.4 Task 5 - Continuous Random Variables

In this task, we use the same technique used in task 3 with difference of using continuous variables instead of discrete. We use continuous uniform values in the range $[0, 1]$ for the step size and $[0, 2\pi]$ in case of orientation. This continuous random walk is to be confined inside a circular region and such that this confinement does not effect the tendency of RW to make a regular pattern. Ciftci and Cakmak Random Walk (CCRW) is a suitable scheme that can be used here. In this heuristic, the bot after crossing the boundary will acquire new coordinates $(f(\theta), g(\theta))$ near origin and then continues. Here $\theta = \tan^{-1}\dfrac{y}{x}$ and the curve $x = f(\theta)$, $y = g(\theta)$ is called the Base Curve (BC). In this case, we take the simplest case of:

$$f(\theta) = \cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

$$g(\theta) = \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

1.5 Task 8 - Continuous Random Variable

1.5.1 Picking Uniform Random Point from a Circular Region

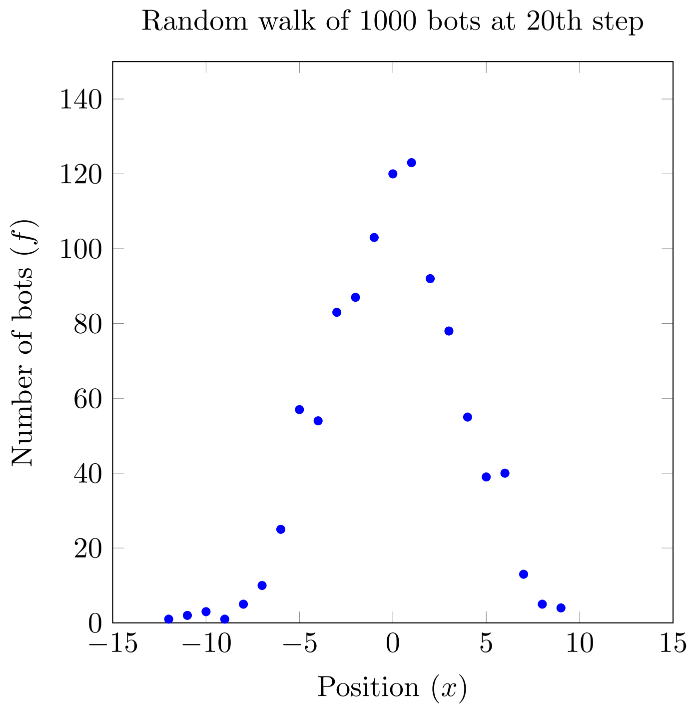
We assume our machine is only capable of generating continuous RV in range [0, 1]. We will generate this function twice to get two such RVs namely u_1 and u_2. Then our point in polar coordinates (r, θ) is given by $r = R\sqrt{u_1}$ and $\theta = 2\pi u_2$. Here R is the radius of our circular boundary of which this point is intended to be confined in.

1.5.2 Expected Time to Meet of two Points AB

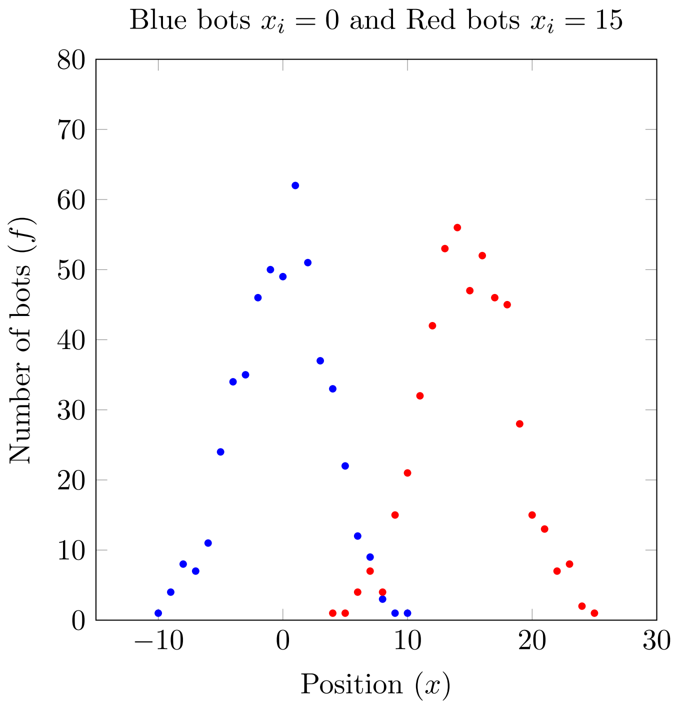
To compute expected-meeting time of two points, we place a large number (say 1000) of red-bots on one point and another 1000 blue-bots on the other point. That makes 2000 total bots. We then simulate random walk of all these 2000 bots simultaneously. Whenever a blue-bot meets with a red-bot, we record their time of meeting. Only first meeting of a pair will be recorded, so that a red-blue pair can have a unique meeting time. We simulate the process for sufficiently long time until almost all of pair-meetings have occurred. Then the expected meeting-time is roughly given by averaging all meeting times.

2 Simulation Graphs

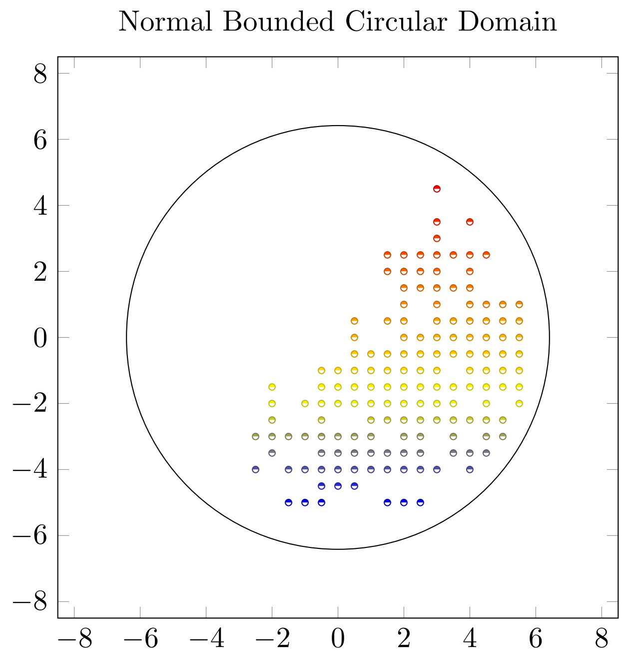
2.1 Task 1 Simulation



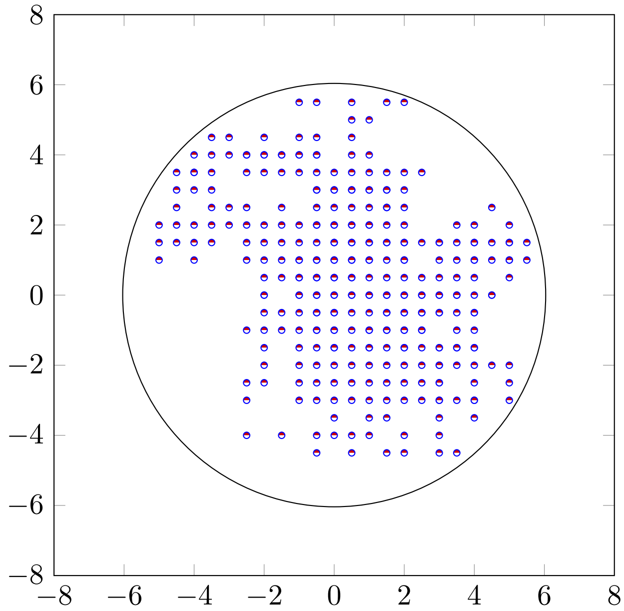
2.2 Task 2 Simulation



2.3 Task 3 Simulation

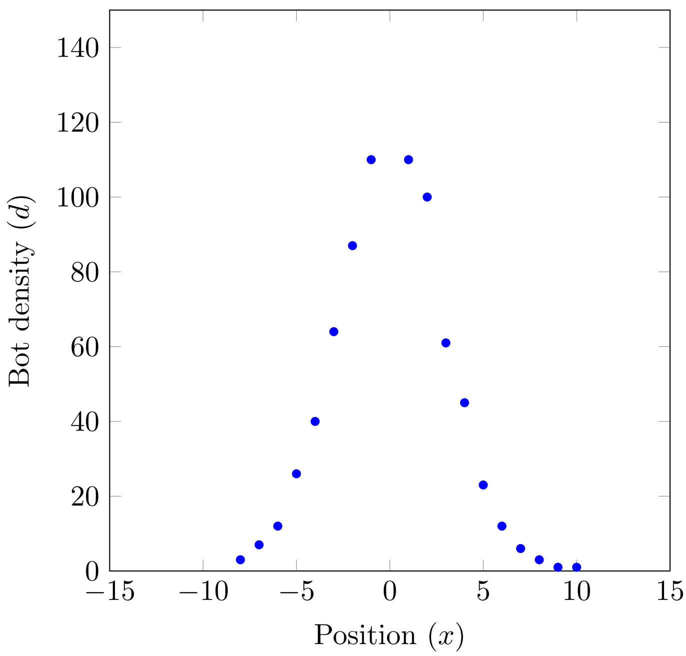


H. Ciftci(a) and M. Cakmak's Completely Probabilistic Restriction

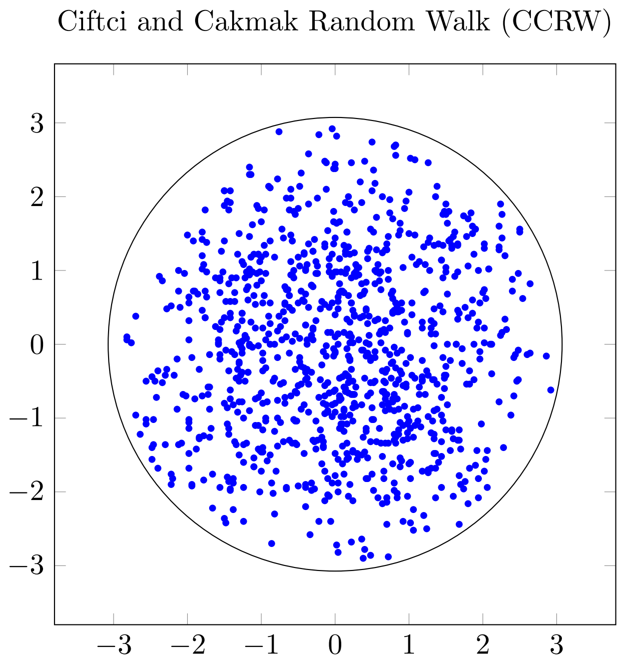


2.4 Task 4 Simulation

Continuous Random walk

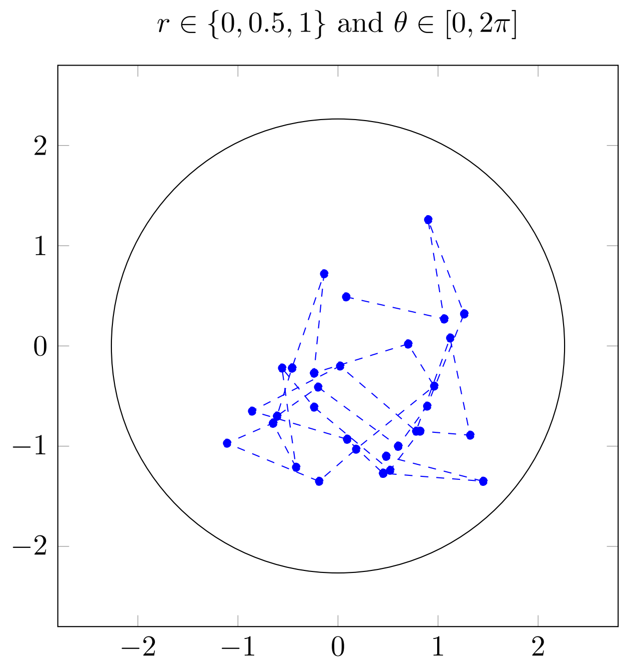


2.5 Task 5 Simulation



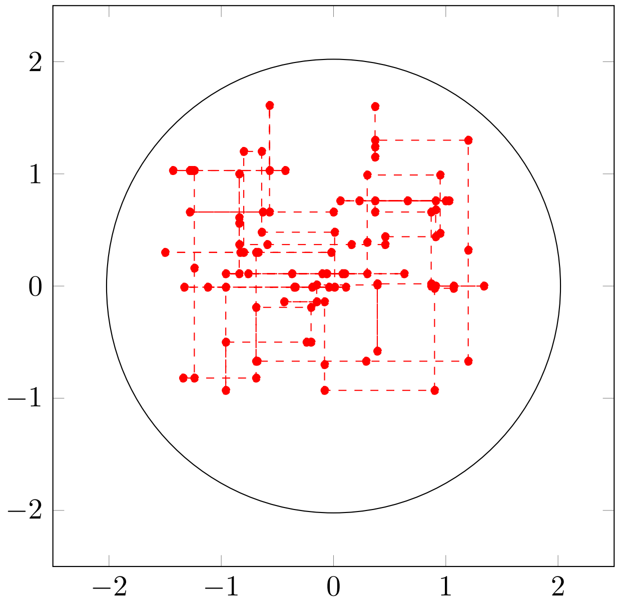
2.6 Task 7 Simulation

Following graph plots for discrete step-size (r) and continuous orientation (θ).



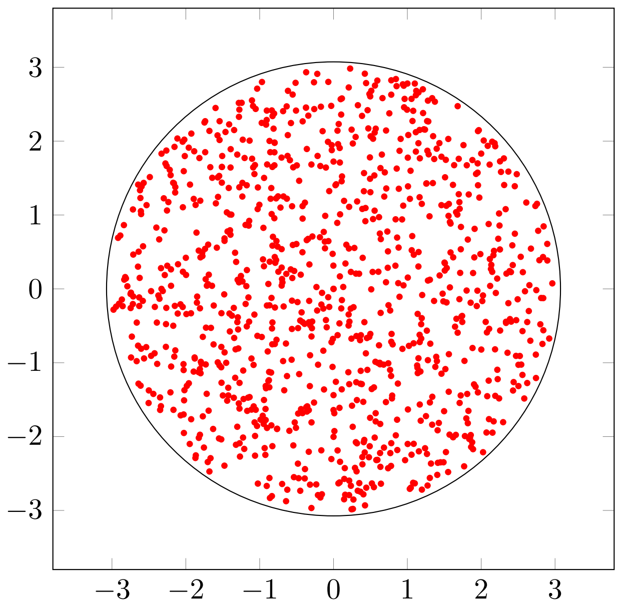
Following graph plots for continuous step-size (r) and discrete orientation (θ).

$r \in [0, 1]$ and $\theta \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$



2.7 Task 8 Simulation for Uniform Bounded Domain

Picking Random Points Uniformly



3 Github Repository

Please, refer to our Github repository for simulation codes.

Click here to browse the repository.

4 References

- Mahashweta Basu and P. K. Mohanty. (August 12, 2013). Two-dimensional random walk in a bounded domain.
- H. Ciftci(a), M. Cakmak. (October 6, 2009). The confined random walks in two-dimensional bounded domain.