Two-sample test

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1 Two-sample t test

The two-sample t test is used to test the hypothesis that two samples may be assumed to come from distributions with the same mean. Data are now from two groups, $x_{11}, ..., x_{1n1}$ and $x_{21}, ..., x_{2n2}$, which we assume are sampled from the normal distributions $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$, and it is desired to test the null hypothesis $\mu_1 = \mu_2$.
EXAMPLE: consider the problem of comparing energy expenditures between lean and obese women:

```
> attach(energy)
> t.test(expend~stature)  ############t.test for two samples

Welch Two Sample t-test

data:  expend by stature
t = -3.8555, df = 15.919, p-value = 0.001411
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
-3.459167 -1.004081
sample estimates:
mean in group lean mean in group obese
8.066154           10.297778
```

Notice the use of the tilde ($\sim$) operator to specify that expend is described by stature.
To get the usual (textbook) t test, you must specify that you are willing to assume that the variances are the same. This is done via the optional argument $var.equal = T$; that is:

```
############################ Assumption: equal variances
> t.test(expend~stature, var.equal=T)
Two Sample t-test

data:  expend by stature
t = -3.9456, df = 20, p-value = 0.000799
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
-3.411451 -1.051796
sample estimates:
mean in group lean mean in group obese
8.066154           10.297778
```

1.1 Comparison of variances

R provides the var.test function for that purpose, implementing an F test on the ratio of the group variances.

```
> var.test(expend~stature)            ###################### Check if variances are equal

F test to compare two variances

data:  expend by stature
F = 0.78445, num df = 12, denom df = 8, p-value = 0.6797
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
0.1867876 2.7547991
sample estimates:
ratio of variances
0.784446
```

1.2 Two-sample Wilcoxon test

You might prefer a nonparametric test if you doubt the normal distribution assumptions of the t test. The two-sample Wilcoxon test is based on replacing the data by their rank (without regard to grouping) and calculating the sum of the ranks in one group.

```
> wilcox.test(expend~stature)

Wilcoxon rank sum test with continuity correction

data:  expend by stature
W = 12, p-value = 0.002122
alternative hypothesis: true location shift is not equal to 0

Warning message:
In wilcox.test.default(x = c(7.53, 7.48, 8.08, 8.09, 10.15, 8.4,  :
cannot compute exact p-value with ties
```

The test statisticW is the sum of ranks in the first group minus its theoretical minimum (i.e., it is zero if all the smallest values fall in the first group).

1.3 The paired t test

Paired tests are used when there are two measurements on the same experimental unit. The theory is essentially based on taking differences and thus reducing the problem to that of a one-sample test. The data on pre- and postmenstrual energy intake in a group of women are considered several times which is available as a data set in the ISwR package.

```
> attach(intake)
> intake
    pre post
1  5260 3910
2  5470 4220
3  5640 3885
4  6180 5160
5  6390 5645
6  6515 4680
7  6805 5265
8  7515 5975
9  7515 6790
10 8230 6900
11 8770 7335
The point is that the same 11 women are measured twice.

> t.test(pre, post, paired=T)

Paired t-test

data:  pre and post
t = 11.941, df = 10, p-value = 3.059e-07
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
1074.072 1566.838
sample estimates:
mean of the differences
1320.455

> t.test(pre, post) ################ WRONG!

Welch Two Sample t-test

data:  pre and post
t = 2.6242, df = 19.92, p-value = 0.01629
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
270.5633 2370.3458
sample estimates:
mean of x mean of y
6753.636  5433.182
```

1.4 The matched-pairs Wilcoxon test

```
> wilcox.test(pre, post, paired=T) ############################ Wilcox.test

Wilcoxon signed rank test with continuity correction
```

```
data:  pre and post
V = 66, p-value = 0.00384
alternative hypothesis: true location shift is not equal to 0

Warning message:
In wilcox.test.default(pre, post, paired = T) :
cannot compute exact p-value with ties
```

R-Code

```
attach(energy)
t.test(expend~stature)  ###########t.test for two samples
t.test(expend~stature, var.equal=T) ###################### Assumption on equal variances
var.test(expend~stature)            ###################### Check if variances are equal
?wilcox.test                        ################### Droping the Normality assumption
wilcox.test(expend~stature)
############################################## Paired t-test
attach(intake)
t.test(pre, post, paired=T)
t.test(pre, post) #WRONG!
wilcox.test(pre, post, paired=T) ########################### Wilcox.test
```