

Simplified IBM 1 computation by hand.

* Initialisation step:

$$t(\text{la}|\text{the}) = \frac{1}{2}$$

$$t(\text{maison}|\text{the}) = \frac{1}{2}$$

$$t(\text{la}|\text{house}) = \frac{1}{2}$$

$$t(\text{maison}|\text{house}) = \frac{1}{2}$$

derived from a uniform distribution
based on the source vocabulary

ITER 1

* counts

$$c(\text{la}|\text{the}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$c(\text{maison}|\text{the}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

$$c(\text{la}|\text{house}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

$$c(\text{maison}|\text{house}) = \frac{\frac{1}{2}}{\frac{1}{2}} + \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{3}{2}$$

↑
from sent. 1

↑
from sent. 2

* translation probabilities:

$$t(\text{la}|\text{the}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

$$t(\text{maison}|\text{the}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}$$

$$t(\text{la}|\text{house}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{2}} = \frac{1}{2} \times \frac{2}{4} = \frac{1}{4}$$

$$t(\text{maison}|\text{house}) = \frac{\frac{3}{2}}{\frac{1}{2} + \frac{3}{2}} = \frac{3}{2} \times \frac{2}{4} = \frac{3}{4}$$

ITER 2:

* counts

$$c(\text{la}|\text{the}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{4}} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$

$$c(\text{maison}|\text{the}) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{4}} = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$$

$$c(\text{la}|\text{house}) = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$$

$$c(\text{maison}|\text{house}) = \frac{\frac{3}{4}}{\frac{1}{2} + \frac{3}{4}} + \frac{\frac{3}{4}}{\frac{3}{4}} = \frac{3}{5} + 1 = \frac{8}{5}$$

↑
from sent. 1
↑
from sent. 2

This step combines all counts as is in the cond formula $\sum_{e,f} c(e|f; e, f)$
 e, f - words,
 e, f - sentences.

* translation probabilities

$$t(\text{la}|\text{the}) = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{2}{5}} = \frac{\frac{2}{3}}{\frac{16}{15}} = \frac{2}{3} \times \frac{15}{16} = \frac{5}{8} = 0,625$$

$$t(\text{maison}|\text{the}) = \frac{\frac{2}{5}}{\frac{2}{3} + \frac{2}{5}} = \frac{2}{5} \times \frac{15}{16} = \frac{3}{8} = 0,375$$

$$t(\text{la}|\text{house}) = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{8}{5}} = \frac{\frac{1}{3}}{\frac{5}{15} + \frac{24}{15}} = \frac{1}{3} \times \frac{15}{29} = \frac{5}{29} \approx 0,172$$

$$t(\text{maison}|\text{house}) = \frac{\frac{8}{5}}{\frac{1}{3} + \frac{8}{5}} = \frac{8}{5} \times \frac{15}{29} = \frac{24}{29} \approx 0,828$$