Example: Beetle Mortality

- Study the relation between the mortality of beetles after 5 hours exposure to gaseous disulphide (CS_2) and the concentrations.

Log10(Dose), x_i	# of beetles, m_i	# killed, y_i
1.6907	59	6
1.7242	60	13
1.7552	62	18
1.7842	56	28
1.8113	63	52
1.8369	59	53
1.8610	62	61
1.8839	60	60

Table: Grouped data.

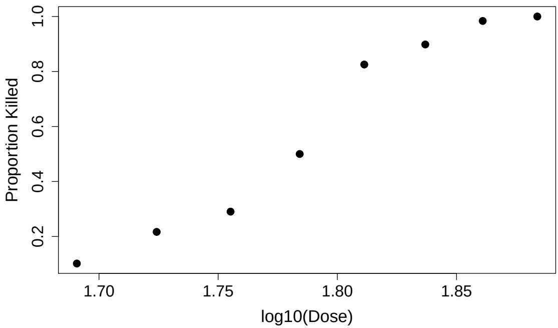

Figure: Beetle mortality data. Proportion killed: y_i/m_i against dose on the $\log_{10}$ scale: x_i.

Example: Median Lethal Dose

- Recall the beetle mortality example, where the primary goal is to study the relation between the mortality rate and the CS2 concentration.
- Suppose we are interested in finding the median lethal dose (LD50), i.e., the dose required to kill half the population.
- In the logistic regression framework, it is equivalent to predicting the dose x_0 that leads to a response rate $\pi_0 = 0.5$.

$$g(\pi) = \beta_0 + \beta_1 x$$

- This type of problem is common in toxicology.

We are interested in x_0 s.t. $\beta_0 + \beta_1 x_0 = g(0.5)$ in LD50 study

- Logit: $x_0 = -\frac{\beta_0}{\beta_1}$
- Point Estimate: $\hat{x}_0 = x_0(\hat{\beta}_0, \hat{\beta}_1) = -\hat{\beta}_0/\hat{\beta}_1$
- Asymptotic variance of $\hat{x}_0$:

$$\begin{aligned} \text{var}(\hat{x}_0) &= (\frac{\partial x_0}{\partial \beta_0})^2 \text{var}(\hat{\beta}_0) + (\frac{\partial x_0}{\partial \beta_1})^2 \text{var}(\hat{\beta}_1) \\ &\quad + 2(\frac{\partial x_0}{\partial \beta_0})(\frac{\partial x_0}{\partial \beta_1}) \text{cov}(\hat{\beta}_0, \hat{\beta}_1) \end{aligned}$$

- Then the asymptotic CI of x_0 is

$$\begin{aligned} x_0 &\in [x_L, x_R] \\ &= \left[\hat{x}_0 - z_{\alpha/2}\sqrt{\text{var}(\hat{x}_0)}, \hat{x}_0 + z_{\alpha/2}\sqrt{\text{var}(\hat{x}_0)}\right] \end{aligned}$$

- $(1-\alpha)100\%$ CI of LD50: $[10^{x_L}, 10^{x_R}]$