$E(y) = b'(\theta) = n \cdot \frac{e^\theta}{1+e^\theta} = n \cdot \pi$ (Replace $\frac{e^\theta}{1+e^\theta}$ by π)

$$Var(y) = b''(\theta) \cdot \phi = n \cdot \frac{e^\theta(1+e^\theta) - e^\theta \cdot e^\theta}{[(1+e^\theta)]^2}$$

$$= n \cdot \frac{e^\theta}{(1+e^\theta)^2} = n \cdot \frac{\frac{\pi}{1-\pi}}{\frac{1}{(1-\pi)^2}} = n\pi(1-\pi)$$

For canonical link function:

we set $\theta = \eta$, which is eqivalent to

$\log(\frac{\pi}{1-\pi}) = \eta$ (link) $\pi = \frac{e^\theta}{1+e^\theta}$

As $b'(\theta) = n\pi = \mu = n \cdot \frac{e^\theta}{1+e^\theta} \Rightarrow [b'(\theta)]^{-1}$

$g(\mu) = [b'^{-1}(\mu)] = \log(\frac{\mu}{n-\mu})$ $e^\theta = (\frac{n}{\mu} - 1)^{-1} = \frac{\mu}{n-\mu}$

that is $g(\mu) = \log[\frac{\mu}{n-\mu}]$ $\Rightarrow \theta = -\log(\frac{n}{\mu} - 1)$

3 Poisson Distribution.

$$f(y;\lambda) = \frac{1}{y!} \lambda^y \cdot e^{-\lambda};$$

$$f(y;\lambda) = \exp\{-\log y! + y \cdot \log\lambda - \lambda\}$$

$$= \exp\{y \cdot \log\lambda - \lambda - \log(y!)\}$$

Natural parameter $\theta = \log\lambda$.

Scale parameter $\phi = 1$

Convex function: $b(\theta) = \lambda = e^\theta$

Expectation $E(y) = b'(\theta) = e^\theta = e^{\log\lambda} = \lambda$

Variance $Var(y) = b''(\theta) \cdot \phi = e^\theta \cdot 1 = e^\theta = e^{\log\lambda} = \lambda$

Canonical link function:

$b'(\theta) = e^\theta$

$[b'(\theta)]^{-1} = \log\mu$

$g(\mu) = [b'^{-1}(\mu)] = \log\mu$

4. Chi-squared Distribution. $\chi^2_{(k)}$

$$f(y;k) = \frac{1}{\Gamma(\frac{k}{2}) 2^{\frac{k}{2}}} y^{\frac{k}{2}-1} \cdot e^{-\frac{y}{2}}$$

$$= \frac{1}{\Gamma(\frac{k}{2}) \cdot 2^{\frac{k}{2}}} \exp\{(\frac{k}{2} - 1) \cdot \log y - \frac{y}{2}\}$$

$$= \exp\{(\frac{k}{2} - 1)\log y - \frac{y}{2} - \log(\Gamma(\frac{k}{2})) - \frac{k}{2}\log 2\}$$