

# Contingency Tables

Classification of subjects based on two (or more) factors.

A/B	1	...	J
1	$Y_{11}$	...	$Y_{1J}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
I	$Y_{I1}$	...	$Y_{IJ}$

Table: An example of two-way contingency table

- ▶ Data are organized in a cross-classified table
- ▶ Variables are categorical

**Primary goal:** Test whether the two variables are associated (for more factors, test different types of association)

# Sampling Distributions

The probability model for a contingency table depends on the study design.

- ▶ Poisson model
  - ▶ Number of new cases in gender-by-age groups ( $2 \times J$ ) during a year in an epidemiological study.
- ▶ Multinomial model
  - ▶ Cross-sectional study of patients with melanoma. Patients are classified based on tumor types and locations.
- ▶ Product multinomial model
  - ▶ Case control study of lung cancer and smoking. Subjects in each group are further divided into the smoking group and non-smoking group.

- ▶ The three probability models are highly related.
- ▶ In fact, they are equivalent in modeling cell means.
- ▶ For two-way contingency table, they all lead to the same  $\chi^2$  test.

# Poisson Model

$$(\alpha_1, \alpha_2, \beta_1, \beta_2)$$

$$(\alpha_1 + c, \alpha_2 + c, \beta_1 - c, \beta_2 - c)$$

$$\alpha_1 + \alpha_2 = 0 \quad \beta_1 + \beta_2 = 0$$

Ex

	$\beta_1$	$\beta_2$
$\alpha_1$		
$\alpha_2$		

$$\log \mu_{ij} = \alpha_i + \beta_j$$

~~$$\log \mu_{ij} = \alpha_i + \beta_j$$~~

- Observe a set of Poisson process, one for each cell of the contingency table. No prior knowledge regarding the total number of observations.

$$Y_{ij} \sim \text{Poisson}(\mu_{ij})$$

- If no constraint on  $\mu_{ij}$ , this is a saturated model with  $IJ$  observations and  $IJ$  parameters.

$$\hat{\mu}_{ij} = y_{ij}$$

- Under the independent assumption, we have  $\mu_{ij} = \mu_{i.} \cdot \mu_{.j}$ . Therefore, we have ( $i = 1, \dots, I; j = 1, \dots, J$ )

H/O: row & col

$$\log \mu_{ij} = \log \mu_{i.} + \log \mu_{.j}$$

$$\log \mu_{ij} = (\mu + \alpha_i) + (\beta_j)$$

$i = 1, \dots, I \quad j = 1, \dots, J$

where  $\sum \alpha_i = \sum \beta_j = 0$

$$\frac{I \cdot J}{n} - \frac{(I+J+1)}{n} = \frac{IJ - I - J + 1}{n} = \frac{(I-1)(J-1)}{n}$$

- ▶ This leads to a Poisson regression model with  $I + J - 1$  parameters.
- ▶ Testing the association is equivalent to checking the goodness-of-fit of the additive model.

$$D(\text{or } G) \sim \chi^2((I - 1)(J - 1)), \text{ under } H_0$$

- ▶ If  $D$  (or  $G$ ) is larger than some threshold, we reject the null, and claim there is significant association between the two factors.

# Multinomial Model

If the total counts  $Y_{++} = \sum_{i=1}^I \sum_{j=1}^J Y_{ij} = n$  is fixed, the joint distribution of  $(Y_{11}, \dots, Y_{IJ})$  is multinomial.

$$f(\mathbf{y} | y_{++} = n) = n! \prod_{i=1}^I \prod_{j=1}^J \frac{\pi_{ij}^{y_{ij}}}{y_{ij}!}.$$

For a saturated model, the number of parameters is  $IJ - 1$ .

$H_0: \text{indep}$

- Under the independent assumption, we have  $\pi_{ij} = \pi_{i.} \pi_{.j}$ . Therefore, we can obtain the MLE as

$$\hat{\pi}_{i.} = \frac{\sum_{j=1}^J Y_{ij}}{n}, \quad \hat{\pi}_{.j} = \frac{\sum_{i=1}^I Y_{ij}}{n}$$

$\pi_{i.} \quad i=1, \dots, I$   
 $\pi_{.j} \quad j=1, \dots, J$   
 $I-1+J-1$

- Use the likelihood ratio test, we have

$$\underline{LR} \sim \underline{\chi^2((I-1)(J-1))}, \text{ under } H_0$$

$(I-1)(J-1) = IJ - I - J + 1$

- If  $LR$  is larger than some threshold, we reject the null, and claim there is significant association between the two factors.

# Product Multinomial Model

If there are more fixed marginal totals than just the overall total  $n$ , then an appropriate model is a product multinomial model.

1 - - J

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## Example: Malignant Melanoma Study

The following data are from a cross-sectional study of patients with a form of skin cancer called malignant melanoma. For a sample of 400 patients, the site of the tumor and its histological type were recorded.

Tumor	Site		
	HeadNeck	Trunk	Extremities
Hutchinson	22	2	10
Superficial	16	54	115
Nodular	19	33	73
Indeterminate	11	17	28

The question of interest is whether there is any association between tumor type and site.