Problem 1

Suppose we have the following statistical model: $Y_{ij} = \mu + b_i + e_{ij}$, where $i = 1, ..., m$ and $j = 1, ..., n$, $b_i \sim N(0, \sigma_b^2)$, $e_{ij} \sim N(0, \sigma_e^2)$, b_i and e_{ij} are statistically independent for each i and j, and e_{ij} and e_{ik} are statistically independent for any two values $j, k = 1, ..., n$. Find the variance of Y_{ij} and the covariance and correlation between any two values Y_{ij} and Y_{ik}, $j \neq k$. Which one of the popular covariance patterns does this correspond to?

Problem 2

A study was conducted involving 27 children, 16 boys and 11 girls. On each child, the distance (mm) from the center of the pituitary to the pterygomaxillary fissure was made at ages 8, 10, 12, and 14. The goal was to study how distance is affected by age and gender. The data file has 5 columns: (1) observation number, (2) child number (1-27), (3) age, (4) distance measurement, and (5) indicator of gender (0=girl, 1=boy).

1. Make a spaghetti plot to infer how distance is affected by age and gender.

2. Assume the model is

$$Y_{ij} = \beta_0 + a_i + b_0 * I_{(sex_i=0)} + b_1 * I_{(sex_i=1)} + \beta_1 * age_{ij} + e_{ij}$$

   where $i = 1, \cdots, 27$ and $j = 1, \cdots, 4$; β_0, β_1 are fixed coefficients; $a_i \sim N(0, \sigma_a^2)$ and $b_k \sim N(0, \sigma_b^2)$ $(k = 0, 1)$ are random coefficients, and $e_{ij} \sim N(0, \sigma_e^2)$ is the random error; a_i $(i = 1, \cdots, n)$, b_k $(k = 0, 1)$ and e_{ij} $(i = 1, \cdots, n; j = 1, \cdots, p)$ are mutually independent. Write the model in the marginal form.

3. Consider the following marginal model:

$$E(Y_{ij}) = \beta_0 + \beta_1 * sex_i + \beta_2 * age_{ij}$$

$$Var(Y_i) = \Sigma$$

   Fit a model with (a) compound symmetry covariance, (b) exponential covariance, (c) autoregressive covariance. Compare the coefficient parameter estimates and the covariance estimates.