Chapter 18. Power and Sample Size Calculation

18.1. Statistical Power

- Power of a test
  - Probability of rejecting the null hypothesis (H_0)
  - Power = 1 – P(Type II error): Correctly reject H_0 and identify a truly significant result.
  - Determine the *usefulness* of a test.
  - Usually, 80% is considered a 'decent' power.
- Various factors affect the power of the test:
  - The larger the significance level (α), the higher power of the test.
  - The larger the sample size, the higher the power of the test.
  - The larger the size of the discrepancy between hypothesized and true values, the higher the power.

18.2. Power and Sample Size Analysis

- Optimize the design of a study. (Save money and time.)
- Improve chances of conclusive results with maximum efficiency.
- Achieve a desired balance between Type I and Type II errors.
- Minimize risks for subjects.
- Primary objectives
  - Determine the sample size to achieve a certain power.
  - Determine the power of a test for a given sample size.
  - Characterize the power of a study to detect a minimum meaningful effect.
- Planning for a future (prospective) study

Sample size	Description
Too small	Insufficient power Difference between groups is clinically important, but it may not reject H_0.
Too large	Excess power Difference between groups is not clinically important, but it may reject H_0.

- Parameters needed for power and sample size calculation[15]

Parameter	Description
Type I error (α)	Usually set at 5%. Power increases as α increases.
Standard deviation (σ)	Variance of the data If small, the power will be greater.
Effect size (Δ)	Minimum (clinically) significant difference (e.g. means, proportions) Big effect sizes are easier to detect and thus have greater power.
Sample size (*n*)	Usually driven by cost, time, etc.

Example: If we are interested in conducting a level α test and wish to have 100(1-β) % power,

Test	One-sample t-test (One-sided)	One-sample t-test (Two-sided)	Two-sample t-test[16] (Two-sided)
Sample size	$n \geq \left(\frac{(z_\alpha + z_\beta)\sigma}{\Delta}\right)^2$	$n \geq \left(\frac{(z_{\alpha/2} + z_\beta)\sigma}{\Delta}\right)^2$	$n \geq 2 \times \left(\frac{(z_{\alpha/2} + z_\beta)\sigma}{\Delta}\right)^2$

delta = distance from H0 and H1

[15] Effect size and standard deviation are usually obtained through pilot studies or previously published data.
[16] Sample size in each group (Assume equal-sized groups)

18.3. PROC POWER

General Syntax

```
proc power <options>;
     <statements>  <options>;
run;
```

Statement	Description
ONESAMPLEFREQ	Tests, confidence interval precision, and equivalence tests of a single binomial proportion TEST = Z / EXACT
ONESAMPLEMEANS	One-sample t-test, confidence interval precision, or equivalence test TEST = T
PAIREDFREQ	McNemar's test for paired proportions DIST = NORMAL
PAIREDMEANS	Paired t-test, confidence interval precision, or equivalence test TEST = DIFF / EQUIV_DIFF
TWOSAMPLEFREQ	Chi-square, likelihood ratio, and Fisher's exact tests for two independent proportions TEST = PCHI / LRCHI / FISHER
TWOSAMPLEMEANS	Two-sample t-test, confidence interval precision, or equivalence test TEST = DIFF / EQUIV / DIFF_SATT

TWOSAMPLEWILCOXON	Wilcoxon-Mann-Whitney (rank-sum) test for 2 independent groups
TWOSAMPLESURVIVAL	Log-rank, Gehan, and Tarone-Ware tests for comparing two survival curves TEST = LOGRANK / GEHAN / TARONEWARE
ONEWAYANOVA	One-way ANOVA including single-degree-of-freedom contrasts TEST = OVERALL / CONTRAST
MULTREG	Tests of one or more coefficients in multiple linear regression
ONECORR	Fisher's z-test and t-test of (partial) correlation DIST = FISHERZ / T
PLOT	Display plots for previous sample size analysis

- PROC GLMPOWER: Prospective power and sample size analysis for linear models.
- For analyses not supported directly in SAS, write your own program.
- PASS: Specialized software for power and sample size analysis

can never split one individual, so that's why the powe

Example

SAS Code

```
* One-sample t-test;
* Sample size
calculation with power
= 80%;
proc power;
onesamplemeans
test = t
mean = 5
stddev = 20
ntotal = .
power = 0.8
;
run;
```

```
* Two-sample t-test;
* Power calculation
with sample size = 200;
proc power;
twosamplemeans
test = diff
meandiff = 5
stddev = 12
ntotal = 200
power = .
;
run;
```

```
* One-way ANOVA:
Balanced groups;
* Power (overall test);
proc power;
onewayanova test =
overall
groupmeans = 59|66|42
std = 12
nperg = 4
power = .
;
run;
```

Output

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Mean	5
Standard Deviation	20
Nominal Power	0.8
Number of Sides	2
Null Mean	0
Alpha	0.05

Computed N Total	
Actual Power	N Total
0.802	128

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Mean Difference	5
Standard Deviation	12
Total Sample Size	200
Number of Sides	2
Null Difference	0
Alpha	0.05
Group 1 Weight	1
Group 2 Weight	1

Computed Power
Power
0.834

Fixed Scenario Elements	
Method	Exact
Group Means	59 66 42
Standard Deviation	12
Sample Size Per Group	4
Alpha	0.05

Computed Power
Power
0.585

SAS Code

```
* Chi-squared test;
* Power calculation
with a series of
different npergroup;
proc power;
twosamplefreq test=pchi
groupproportions =
(0.6 0.8)
nullproportiondiff = 0
npergroup = 25 50 75
100 200
power = .;
run;
```

```
* Multiple linear
regression;
proc power;
multreg
model = fixed
nfullpredictors = 7
ntestpredictors = 3
rsquarefull = 0.9
rsquarediff = 0.1
ntotal = .
power = 0.9;
run;
```

```
* Survival analysis;
* Compare two groups -
based on median
survivals;
proc power;
twosamplesurvival
accrualtime = 12
followuptime = 24
groupmedsurvtimes = 15
| 20 22 24
npergroup = .
power = 0.8
;
run;
```

Output

Fixed Scenario Elements

Distribution	Asymptotic normal
Method	Normal approximation
Null Proportion Difference	0
Group 1 Proportion	0.6
Group 2 Proportion	0.8
Number of Sides	2
Alpha	0.05

Computed Power

Index	N per Group	Power
1	25	0.335
2	50	0.590
3	75	0.767
4	100	0.876
5	200	0.993

Fixed Scenario Elements

Method	Exact
Model	Fixed X
Number of Predictors in Full Model	7
Number of Test Predictors	3
R-square of Full Model	0.9
Difference in R-square	0.1
Nominal Power	0.9
Alpha	0.05

Computed N Total

Actual Power	N Total
0.903	20

Fixed Scenario Elements

Method	Lakatos normal approximation
Form of Survival Curve 1	Exponential
Form of Survival Curve 2	Exponential
Accrual Time	12
Follow-up Time	24
Group 1 Median Survival Time	15
Nominal Power	0.8
Number of Sides	2
Number of Time Sub-Intervals	12
Group 1 Loss Exponential Hazard	0
Group 2 Loss Exponential Hazard	0
Alpha	0.05

Computed N Per Group

Index	Med Surv Time 2	Actual Power	N Per Group
1	20	0.801	273
2	22	0.800	158
3	24	0.802	108