Decentralized Control of Power System Zones based on Probabilistic Constrained Load Flow

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Abstract— **Modern power systems are characterized by high uncertainty. The paper presents a method for a network constrained setting of control variables in independent or weakly coupled power zones based on probabilistic load flow (plf) analysis. Plf is applied in order to provide the complete spectrum of all probable values of the state and output variables, such as bus voltages. The network is divided into clusters in order to create zones with the minimum power exchange and interaction with their neighboring zones. The zone controllers are adjusted in order to act independently from other zone controllers in order to eliminate the limit violations of the constrained variables. The method is applied to the IEEE 118-bus system. The results show the performance of graph partitioning and the ability of the method to provide the appropriate control actions in order to satisfy all physical and operating constraints of the power system.**

Keywords — Contrsained load flow, graph theory, partitioning algorithm, particle swarm optimization, power network

I. INTRODUCTION

Modern power systems are characterized by high integration of stochastic resources, such as wind turbines and photovoltaic systems. This adds significant uncertainty in the load demand predictions. Therefore, the deterministic approach is not sufficient for the analysis of modern power systems, since their operation is characterized by many uncertainties. Probabilistic load flow (plf) provides the complete spectrum of all probable values of the state and output variables, such as bus voltages and power flows, with their respective probabilities. Plf has been used to examine power system integrating renewable energy sources [2].

The constrained load flow (clf) problem deals with the adjustment of the power system control variables, in order to satisfy all physical and operating constraints. Appropriate corrective actions take place, with respect to the control variables, that eliminate the limit violations of the variables that have to be constrained. In [1] an algorithm is employed providing adjustments of the control variables based on sensitivity analysis of the constrained variables with respect to the control variables.

This paper uses the probabilistic constrained load flow technique to provide control settings of controllers operating in each zone without affecting the controllers of the neighboring zones. The power system is divided into clusters, in order to create zones with the minimum power exchange and interaction amongst them. The scope of power network partitioning is to improve the performance of control actions. Each zone has its own set of controllers, which act independently from the controllers of the others zones. Control actions within a zone should not affect the other zones. Controllers can operate in parallel, reducing the duration of control actions for the whole power system.

In [3] the power system is divided into regions and then decentralized controllers for each zone are designed in order to provide voltage regulation. A multi-attribute method for dividing a power system into electrically coherent zones, using electrical distances is presented in [7]. Partition algorithms of the system have also have been used for controlled islanding [4], [5].

Particle Swarm Optimization Algorithm (PSO) is used to find the appropriate corrective actions, with respect to the control variables and their upper and lower bounds that eliminate the limits violation of the variables that has to be constrained. PSO has been used to provide secondary voltage control using regional information [6].

This paper is organized as follows: Section II introduces the constrained probabilistic flow problem. Section III describes the partitioning algorithm. Section IV presents the design of zone controller using the particle swarm optimization algorithm. In Section V the method is applied to the IEEE-118 bus system to demonstrate its performance. Section VI concludes the paper.

II. CONSTRAINED LOAD FLOW PROBLEM

A. *Probabilisitc Load Flow*

The load flow model can be expressed mathematically by two sets of nonlinear equations.

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$$Y = g(X, U) \tag{1}$$

$$Z = h(X, U) \tag{2}$$

where Y is the input vector (comprising nodal power injections), Z is the output vector (comprising power flows, PV bus reactive powers, etc.), X is the state vector (comprising voltage magnitudes and angles) and U is the control vector (comprising the control variables).

Loads are modeled with their pdfs reflecting their variation in a specific time period. The normal distribution is typically assumed to describe the variation of the loads. Consider that Y_0 is vector of the loads expected values and X_0 *is* the state vector that satisfies the equation $Y_0 = g(X_0)$. The vector X_0 results from the power flow solution, where the input is the expected values of input vector Y. The linearization of the power flow equations at the points (X_0, Y_0) is defined as:

$$\begin{gathered} Y \approx g(X_0) + J(X - X_0) \Rightarrow \\ Y \approx Y_0 + J(X - X_0) \Rightarrow \\ X \approx X_0 + J^{-1}(Y - Y_0) \Rightarrow \\ X \approx X_0' + AY \\ A = J^{-1} \end{gathered} \tag{3}$$

where J is the Jacobian matrix of g at the given point Y_0. The linearization of h at the given point Y_0 can be calculated respectively.

In case the pdfs of the input vector Y elements follow the normal distribution, the elements of the state vector X will also follow the normal distribution. The equivalent normal distribution of $x_i \in X$ can be calculated by the convolution of vector Y elements, defined as:

$$f_{x_i} = a_{1i} f_{y_1} * a_{2i} f_{y_2} * \dots * a_{ni} f_{y_n} \tag{4}$$

$$f_{x_i} = N(\mu_{x_i}, \sigma_{x_i}) \tag{5}$$

$$\mu_{x_i} = a_{1i}\mu_{y_1} + a_{2i}\mu_{y_2} + \dots + a_{ni}\mu_{y_n} \tag{6}$$

$$\sigma_{x_i} = \sqrt{(a_{1i}\sigma_{y_1})^2 + (a_{2i}\sigma_{y_2})^2 + \dots + (a_{ni}\sigma_{y_n})^2} \tag{7}$$

where $y_1, y_2, \dots, y_n \in Y$, a_{ij} is the sensitivity of j to i. μ and σ are the mean value and standard deviation of normal distribution respectively.

B. Probabilistic Assessment of Constraint Violations

Suppose that $f(x_i, U)$ is the probability density function of the random variable x_i, obtained from plf, and has to be constrained. E_{min} and E_{max} are the extreme values of the probability density function. The limits of the interval where this variable is allowed to vary are L_{min} and L_{max}. In case $E_{min} < L_{min}$ and $E_{max} > L_{max}$ then the lower and upper limits are not satisfied. $prob\_v_{x_i}$ is the respective probability of limit violations, obtained from the cumulative pdf.

In case of a normal distribution the values $\mu + 3\sigma, \mu - 3\sigma$ are considered as the extreme values E_{max} and E_{min} respectively.

III. Network Partitioning

A. Graph Notation

A power system can be described by an undirected graph $G(V, E, W)$ with vertex set V and edge set E. The matrix W is the weighted adjacency matrix and must satisfy the following properties: $w_{ij} = w_{ji}$, $w_{ii} = 0$ and $w_{ij} = 0$ if i is not adjacent to j in G.

The Laplacian matrix of G is defined as $\mathrm{L} = D - \mathrm{W}$. The degree matrix D is a diagonal matrix that contains elements D_{ii} that are equal to the total weight of the edges connected to vertex i [8], [9], [10].

B. Graph Cut Problem

The graph cut problem is defined as finding a partition that separates vertices in different groups according to their similarities. The partition must ensure that the edges between different groups have very low weights and the edges within a group have high weights. For a set of groups $(1,2,\dots,n)$ the following properties must be satisfied: $V_i \cap V_j = \emptyset$ $(i \neq j, 1 \leq i,j \leq n)$ and $V_1 \cup V_2 \cup \dots \cup V_n = V$.

A way of measuring the "size" of subset $V_i \subset V$ is $vol(V_i) = \sum_{j=1}^{m} d_{jj}$, where $j \in V_i$. $vol(V_i)$ measures the size of V_i by summing over the weights of all edges attached to vertices in V_i. Normalized spectral clustering uses the measure $vol(V_i)$ as a way to prevent the partition of a single vertex. Using normalized spectral clustering the creation of "equal" partitions is achieved. The normalized graph partition is defined as [8]:

$$Ncut(V_1, V_2, \dots, V_n) = \frac{1}{2}\sum_{i=1}^{n} \frac{W(V_i, \overline{V_i})}{vol(V_i)} \tag{8}$$

where W is the weighted adjacency matrix and $\overline{V_i}$ contains the vertices not contained in V_i.

Unfortunately, introducing balancing conditions, the graph cut problem becomes NP hard. Spectral clustering is a way to solve relaxed versions of those problems. The spectral clustering algorithm follows these steps:

- Compute Laplacian Matrix L
- Compute the first generalized eigenvectors $u_1, u_2, \dots, u_k$ of the generalized eigenproblem $Lu = \lambda Du$
- Let $U \in R^{n \times k}$ be the matrix containing the vectors $u_1, u_2, \dots, u_k$ as columns
- For $i = 1,2,\dots,n$ let $y_i \in R^k$ be the vector corresponding to the i-th row of U

- Cluster the points y_i, $i = 1,2,\dots,n$ in R^k with the k-medoids algorithm into clusters $C_1, C_2, \dots, C_k$ in order to create "equal" clusters
- Output: $V_1, V_2, \dots,, V_k$ with $V_i = \{j | y_j \epsilon V_i\}$

IV. Control Design

A. Zone Controllers

Controllers in each zone are used to eliminate the limits violation of the variables that have to be constrained. Particle Swarm Optimization Algorithm is used to find the appropriate corrective actions, with respect to the control variables and their upper and lower bounds that eliminate the limit violations of the variables that have to be constrained. Each zone has its own set of controllers, which act independently from the controls of the others zones. Also, control actions within a zone should not affect other zones.

Assume that U is the vector of the zone control variables that have upper and lower limits. Control variables can be generator buses voltages, tap position of transformer, reactive power by capacitor banks and load curtailment. In this paper generator buses voltages and capacitor banks are chosen as control variables. The cost function that has to be minimized is defined as

$$\begin{aligned} &minCF(V_1, \dots, V_n, C_1, \dots, C_m) = \\ &A_1 \textstyle\sum_{i \in R} prob\_v_i + A_2 \sum_{i \epsilon B} (V_{i,bc} - V_{i,ac})^2 + \\ &A_3 \textstyle\sum_{i \epsilon G} (V_{i,bc} - V_{i,ac})^2 + A_4 \sum_{i \epsilon C} (C_{i,bc} - C_{i,ac})^2 \end{aligned} \tag{9}$$

where $V_1, \dots, V_n$ are the generator bus voltages, $C_1, \dots, C_m$ are the amount of reactive power injected by capacitor banks. $prob\_v_i$ is the probability of variable i to be violated. R, B, G, C represent zone buses, boundary buses, generator buses and capacitor banks buses respectively. A_{1-4} are weighting factors. The indices bc and ac denote the value before and after control actions respectively.

The first term of the cost function eliminates limit violations inside the zone. The second minimizes the deviation of boundary buses voltage expected values in order to avoid that the control actions affect other zones. The third and fourth term minimize the generator bus voltage deviation and the reactive power generated by capacitor banks respectively, in order to eliminate limit violations with the minimum control actions.

The following bounds are considered for the control variables

$$V_G^{MIN} \le V_G \le V_G^{MAX} \quad \text{for generator buses}$$
$$Q_C^{MIN} \le Q_C \le Q_C^{MAX} \quad \text{for capacitor banks}$$

B. Particle Swarm Optimization

Particle Swarm Optimization is a stochastic, population-based evolutionary computer algorithm for problem solving inspired by social behavior of bird flocking or fish schooling. The swarm is typically modeled by particles in multidimensional space that have a position and a velocity, where each particle represents a candidate solution to the optimization problem. During the optimization procedure, particles communicate good positions to each other and adjust position according to their history experience and the experience of neighboring particles [11].

The objective of the algorithm when applied to a minimization problem is to find a solution X^* out of a set $X \subseteq R^d$ such that: $X^* = argminf(x)$, where $x \in X$ and $f(x)$ is the objective function. The procedure is organized in the following sequence of steps:

1. A population of N particles is created uniformly distributed over X

$$x_i^{(0)} = x_{min} + rand(x_{max} - x_{min}) \tag{10}$$

$$u_i^{(0)} = x_{min} + \frac{rand(x_{max} - x_{min})}{\Delta t} \tag{11}$$

where $x_i^{(t)}$, $u_i^{(t)}$, are the position and velocity of particle i at iteration t respectively defined as $x_i^{(t)} = [x_{i,1}^{(t)} \dots x_{i,d}^{(t)}]$ and $u_i^{(t)} = [u_{i,1}^{(t)} \dots u_{i,d}^{(t)}]$, x_{min} and x_{max} are vectors of lower and upper limit values respectively.

2. The positions of each particle is evaluated according to the objective function $f(x)$ and each particle records its best previous position (local best, $pbest_i$) and the best previous position among all particles (global best, $gbest$)

$$pbset_i = (pbest_{i,1}, pbest_{i,2}, \dots, pbest_{i,d}) \tag{12}$$

$$gbset_i = (gbest_1, gbest_2, \dots, gbest_d) \tag{13}$$

The global best is known to all particles and immediately updated when a new best position is found.

3. Particles position and velocity is updated according to equations (14) and (15) respectively

$$x_i^{(t+1)} = x_i^{(t)} + u_i^{(t+1)} \tag{14}$$

$$\begin{aligned} u_i^{(t+1)} = \omega u_i^{(t)} &+ c_1 rand_1 \left(pbest_i - x_i^{(t)}\right) \\ &+ c_1 rand_2 \left(gbest - x_i^{(t)}\right) \end{aligned} \tag{15}$$

where ω is the inertial constant, c_1, c_2 are constants, which indicate how much the particles are directed towards the good positions and $rand_1$, $rand_2$ are uniform random values in the range [0,1].

4. Go to step 2 until stopping criteria are satisfied. The stopping criterion set is usually the maximum change in best fitness to be smaller than a specified tolerance for a specified number of moves.

$$\left| f(gbest^{(t)} - gbest^{(t-q)}) \right| \le \varepsilon \tag{16}$$

The fitness function can be modified to include functional constraints, by imposing penalties when the constraints are not satisfied:

$$f(x) = f(x) + \sum_{i=1}^{Ncon} r_i \qquad (17)$$

As the swarm iterates, the fitness of the global best solution improves. However PSO iteration is a heuristic method and it is not able to guarantee the convergence to a global minimum, it is able to give a good solution.

V. Application Of Method

A. Test Case: IEEE 118-Bus System

The partitioning algorithm is applied to the IEEE 118-bus system. The system is consisted of 118 buses (vertices) and 186 branches (edges). The loads are independent and normal distribution is assumed to represent their pdfs. The system is divided with respect to the expected values of the loads. Matpower Software is used to solve power flow problem [12].

The weighted adjacent matrix W of the power system is defined as

$$w_{ij} = \frac{|S_{ij}| + |S_{ji}|}{2} \qquad (18)$$

where S_{ij} is the apparent power transferred by bus i to j.

Figure 1 shows the eigenvalue plot calculated by the Laplacian matrix. For systems with no clear separation, the largest gap between two successive eigenvalues λ_r and λ_{r+1} is a criterion for finding the optimal number of zones. This procedure ensures that the connections between the zones will be weak. For the 118-bus system, three to five zones are used. In this case, three areas seem to be appropriate since ε_3 is smaller than ε_4 or ε_5, where $\varepsilon = |\lambda_r|/|\lambda_{r+1}|$. Table 1 presents the first eigenvalues of Laplacian matrix and value ε of them [8],[9].

Figure 2 shows the partition of the power system. Figure 3 presents the result of partition algorithm when the expected values of loads are increased by 10%. Small changes are observed between the two partitions.

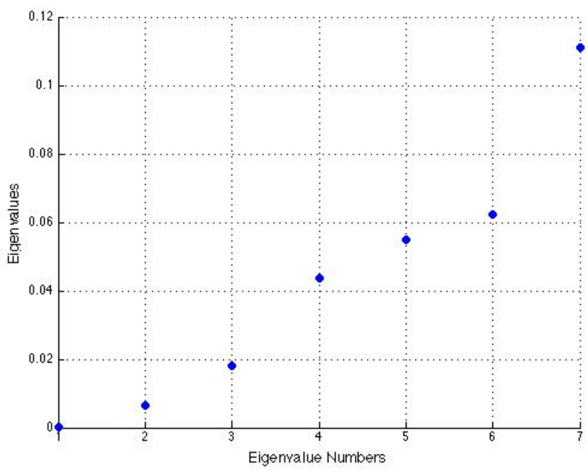

Fig. 1 First eigenvalues of Laplacian matrix

TABLE 1
FIRST EIGENVALUES OF LAPLACIAN MATRIX

	$\boldsymbol{\lambda}$ **(Eigenvalues)**	$\mathbf{e_r} = \|\lambda_r\|/\|\lambda_{r+1}\|$
1	0.0000	0.0000
2	0.0066	0.3626
3	0.0182	0.4174
4	0.0436	0.7913
5	0.0551	0.8859
6	0.0622	0.5609
7	0.1109	

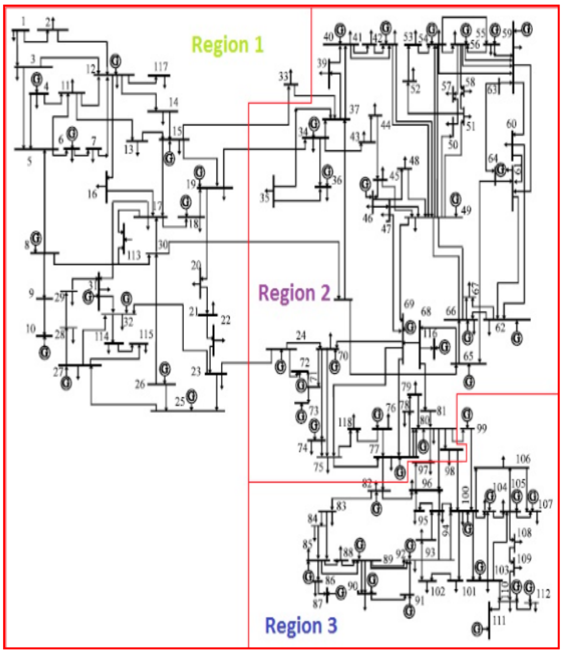

Fig. 2 Partition of IEEE 118-bus system

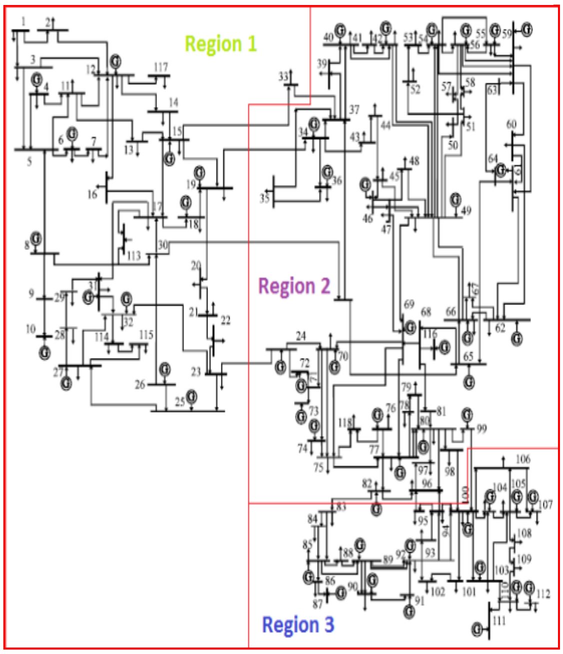

Fig. 3 Partition of IEEE 118-bus system (10% increase of load expected values)

Loads are independent and follow normal distribution. Their expected values are considered as the values of IEEE 118-bus system increased by 10%, in order to lead the system to operate nearest to its limits. Load standard deviations vary in [0, 14%].

In this work only bus voltage violations are examined. Bus voltage is constrained in the interval [0.95, 1.05]. Table 2 and Table 3 show which buses violate their voltage limits and the respective probability. In zone 3 there are no limit violations.

The nominal capacity of the capacitor banks is 12 MVar and they vary with step of 1 MVar. Table 4 shows the capacitor bank buses.

The output variables of PSO, corresponding to the amount of reactive power injected by capacitor banks, are relaxed to continuous values between 0 and the nominal capacity of capacitor banks. The solutions are rounded to the nearest intenger.

A probabilistic load flow is executed, for each output of PSO, in order to assess the probability of constraint violation after the corrective actions.

B. Simulation Results

Figures 4-8 show the probability density function of the bus voltages, which violate their limits, before and after control actions. $Prob\_min$ and $Prob\_max$ are the cumulative probabilities of lower and upper bounds violations.

TABLE 2 BUS VOLTAGE LIMIT VIOLATIONS - ZONE 1

Bus	$Prob\_min$ (%)	$Prob\_max$ (%)
20	0.4	0
21	4	0

TABLE 3 BUS VOLTAGE LIMIT VIOLATIONS - ZONE 2

Bus	$Prob\_min$ (%)	$Prob\_max$ (%)
52	19	0
53	100	0
118	93	0

TABLE 4 CAPACITOR BANKS BUSES

Zone	Capacitor Banks Buses
1	13,21,117
2	38,44,53,118
3	82,95

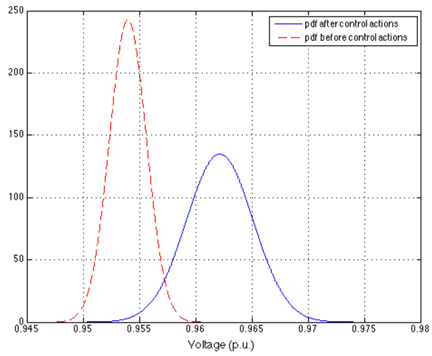

Fig. 4 Probability density of voltage magnitude at bus 20 (zone 1)

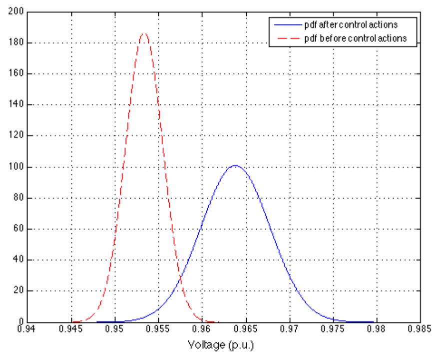

Fig. 5 Probability density function of voltage magnitude at bus 21 (zone 1)

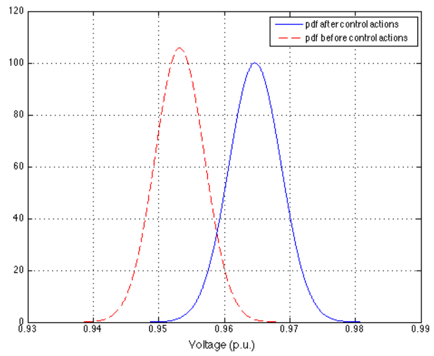

Fig. 6 Probability density function of voltage magnitude at bus 52 (zone 2)

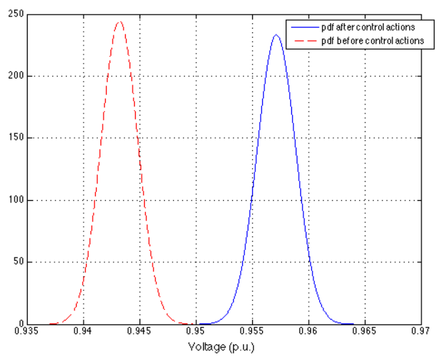

Fig. 7 Probability density function of voltage magnitude at bus 53 (zone 2)

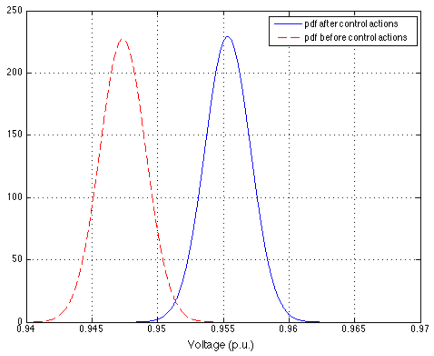

Fig. 8 Probability density function of voltage magnitude at bus 118 (zone 2)

TABLE 5 CAPACITOR BANKS POWER INJECTION - ZONE 1

Capacitor Bank Bus	Q(Mvar)
13	0
21	5
17	4

TABLE 6 CAPACITOR BANKS POWER INJECTION - ZONE 2

Capacitor Bank Bus	Q(Mvar)
38	1
44	11
53	4
118	3

Table 5 and Table 6 present the reactive power injection of capacitor banks according to zone controller's decisions. Table 7 shows the variation of expected value of bus, boundary bus and generator bus voltages in each zone, due to control actions. Percentage variation is calculated as

$$V_{var} = \frac{\left|V_{ac_{avg}} - V_{bc_{avg}}\right|}{V_{bc_{avg}}} \tag{19}$$

where $V_{ac_{avg}}$ and $V_{bc_{avg}}$ are the average voltage of buses after and before control actions respectively.

TABLE 7 VOLTAGE VARIATIONS DUE TO CONTROL ACTIONS

	Voltage Variation (%)		
Zone	Buses	Boundary Buses	Generator Buses
1	0.2	0.29	0.2
2	0.9	0.72	0.65
3	0.05	0.18	0

VI. Conclusion

Decentralized control for modern power systems operating with high uncertainty is proposed. Probabilistic load flow provides operating constraint violations. The power network is divided into zones and a decentralized control method is applied in order to eliminate limit violations by acting on controllers of each zone without affecting neighboring zones.

Particle swarm optimization is used to provide the adjustments of the control variables. The goal is achieved when all given constraints are satisfied (if possible). Alongside, corrective actions within a zone must affect as little as possible the other zones.

Further research is carried out in order to include all possible control actions and more accurate probabilistic representation of the system.

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